Title: Remarks on the Currie-Jordan-Sudarshan no interaction theorem and the status of position operators in Lorentz covariant quantum theory

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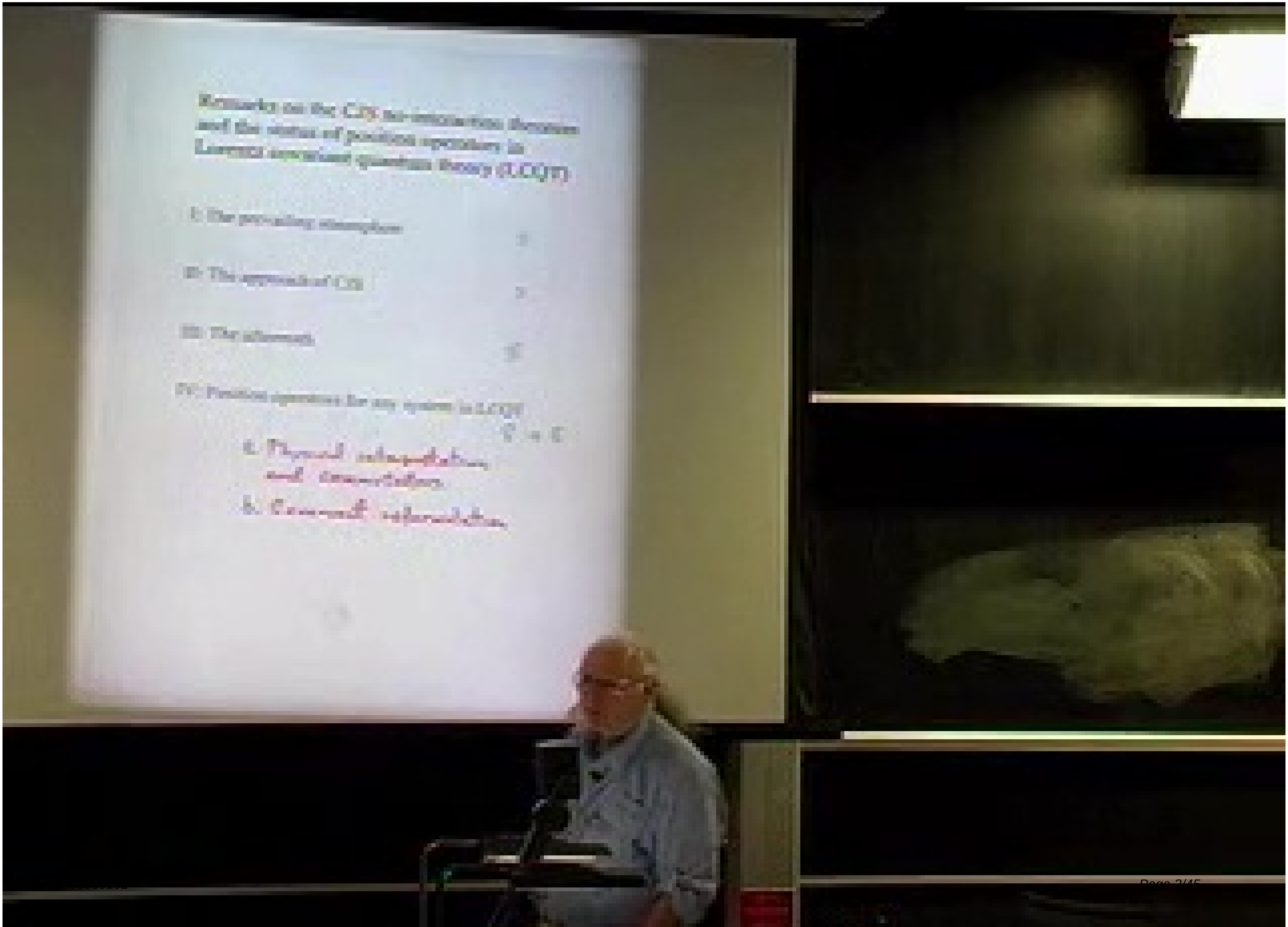
Abstract: I will comment on the prevailing atmosphere and attitudes that provoked the CJS theorem, aspects of the theorem itself, some features of the aftermath following the theorem and, finally, a critique of the relevance of the theorem based on my own research on position operators in Lorentz covariant quantum theory.


Remarks on the CSS no-interaction theorem
and the status of position operators in
Lorentz covariant quantum theory (LCQT)
I: The prevailing atmosphere
II: The appearance of CSS
III: The aftermath
IV: Position operators for any system in LCQT
a. Physical interpretation and connotation
b. Covariant reformulation


I. The prevailing atmosphere (prior to 1965)
D. Alternative models of interactions
Proc. Roy. Soc.
Rev. Mod. Phys. 21
Rev. Mod. Phys. 21
Rev. Mod. Phys. 21
Phys. Rev. 92
Phys. Rev. 122

D. Alternative models of interactions

1. Pryce's study of position in LCQT
   Proc. Roy. Soc. A195 621 (1948)
2. Dirac's forms of dynamics
   Rev. Mod. Phys. 21 392 (1949)
3. The Newton-Wigner position
   Rev. Mod. Phys. 21 400 (1949)
4. Wheeler-Feynman electrodynamics
   Rev. Mod. Phys. 21 425 (1949)
5. The Bakamjian-Thomas model
   Phys. Rev. 92 1300 (1953)
6. Foldy's model
   Phys. Rev. 122 275 [illegible]61)

1.′ C. Møller
Comm. Dublin Inst
A no. 5 (194

The Newton-Wigner position RMP 21 400 (1949)

"It may be well to remember at this point that the position operators to which our postulates lead necessarily commute with each other [i.e., have commuting components] - - - [and] that a state which is localized at the origin in one coordinate system [at t = 0], is not localized in a moving coordinate system, even if the origins coincide at t = 0. Hence our operators q^k have no simple covariant meaning under relativistic transformations."

The Bakamjian-Thomas model PR 92 1300 (1953)

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"It is - - - not implied, nor is it true, that, even in non-quantum mechanics, there exist world lines $r_i = q_i(t)$, for the particles, that are the same loci in space-time when transformed by the corresponding Lorentz transformations, unless there is no interaction and the particles (are without spin)."

"We have carried through a - - - transformation [from particle to (CM) and relative motion variables] in relativity mechanics, at the cost of giving up the assumption of invariant world lines."

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Foldy's model PR 122 275 (1961)

"We feel that the direct application of the Lorentz transformation equations for space and time coordinates to the coordinates of a *particle* at a particular time moment, while relevant in classical mechanics, cannot be naively carried over into quantum mechanics where - - - "

What the [illegible] and [illegible] generators look like

$$\mathbf{P} = \mathbf{p}^{(1)} + \mathbf{p}^{(2)}, \quad \mathbf{J} = \mathbf{q}^{(1)} \times \mathbf{p}^{(1)} + \mathbf{q}^{(2)} \times \mathbf{p}^{(2)}$$

$$\equiv \mathbf{Q} \times \mathbf{P} + \mathbf{S}$$

$$\mathbf{H} = \mathbf{h}^{(1)} + \mathbf{h}^{(2)} + V = [\mathbf{P}^2 + M^2 c^2]^{1/2}$$

What the B-T and F Poincare generators look lik

$$P = p^{(1)} + p^{(2)}, \quad J = q^{(1)} \times p^{(1)} + q^{(2)} \times p^{(2)}$$

$$= Q \times P + S$$

$$H = h^{(1)} + h^{(2)} + V = [\ \ldots + M^2c^2]^{1/2}$$

$$M = M_0 + \Delta M$$

$$K = h^{(1)}{:}q^{(1)} + h^{(2)}{:}q^{(2)} - Pct + U$$

$$= H{:}Q + [P \times S/(H + Mc)] - Pct$$

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Q. $q^{(1)}$ and $q^{(2)}$ are each the NW position for their system.

F improves on B-T by imposing separability.

II: The approach of CJS RMP 35 350 (1963)

"We distinguish two different kind of assumptions that occur under the heading of 'relativistic invariance' - - - . The first of these reflects the principle - - that the laws of physics should be invariant under changes of reference frame;- - - . The second is an assumption - - that certain quantities transform under changes of reference frame in a particular manner ('manifest invariance')- - -."

"The requirement of symmetry under the relativistic transformation group can be satisfied in a - - simple manner - -. A requirement of 'manifest invariance' - - also may be satisfied quite - - simply. But the transform under changes of reference frame in a particular manner ['manifest invariance']- - -."

"The requirement of symmetry under the relativistic transformation group can be satisfied in a - - simple manner - -. A requirement of 'manifest invariance' - - also may be satisfied quite - - simply. But the combined requirements - - - may restrict the theory so - - that it is capable only of describing non-interacting particles. We - - show - - this is - - the case in a Lorentz symmetric - - - theory - - - [for] a pair of particles."

A. The universal, canonical formalism

B. Relativistic symmetry

A. The universal, canonical formalism

1. Galilean, Lorentzian, classical and quantum all treated together

2. Real, linear space, R, of quantities, including subspace, S, of states.

classical: real functions over phase space

quantum: s. a. operators on a Hilbert space

3. Introduce Lie bracket on R x R

classical: Poisson brackets

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classical: Poisson brackets

quantum: commutators / $i\hbar$

4. One parameter automorphism groups

$$e^{[H]t}(A) := A + [A, H]t + \tfrac{1}{2}[[A, H], H]t^2 + \text{—}$$

$$(\partial/\partial t)e^{[H]t}(A) = [e^{[H]t}(A), H]$$

$$e^{[H]t}(A)|_{t=0} = A$$

B. Relativistic symmetry

1. The Lie algebra

$$[P_a, P_b] = 0\,, \quad [P_a, H] = 0\,, \quad [J_a, H] = 0$$

$$[J_a, J_b] = \varepsilon_{abc} J_c\,, \quad [J_a, P_b] = \varepsilon_{abc} P_c$$

$$[J_a, K_b] = \varepsilon_{abc} K_c\,, \quad [K_a, H] = P_a$$

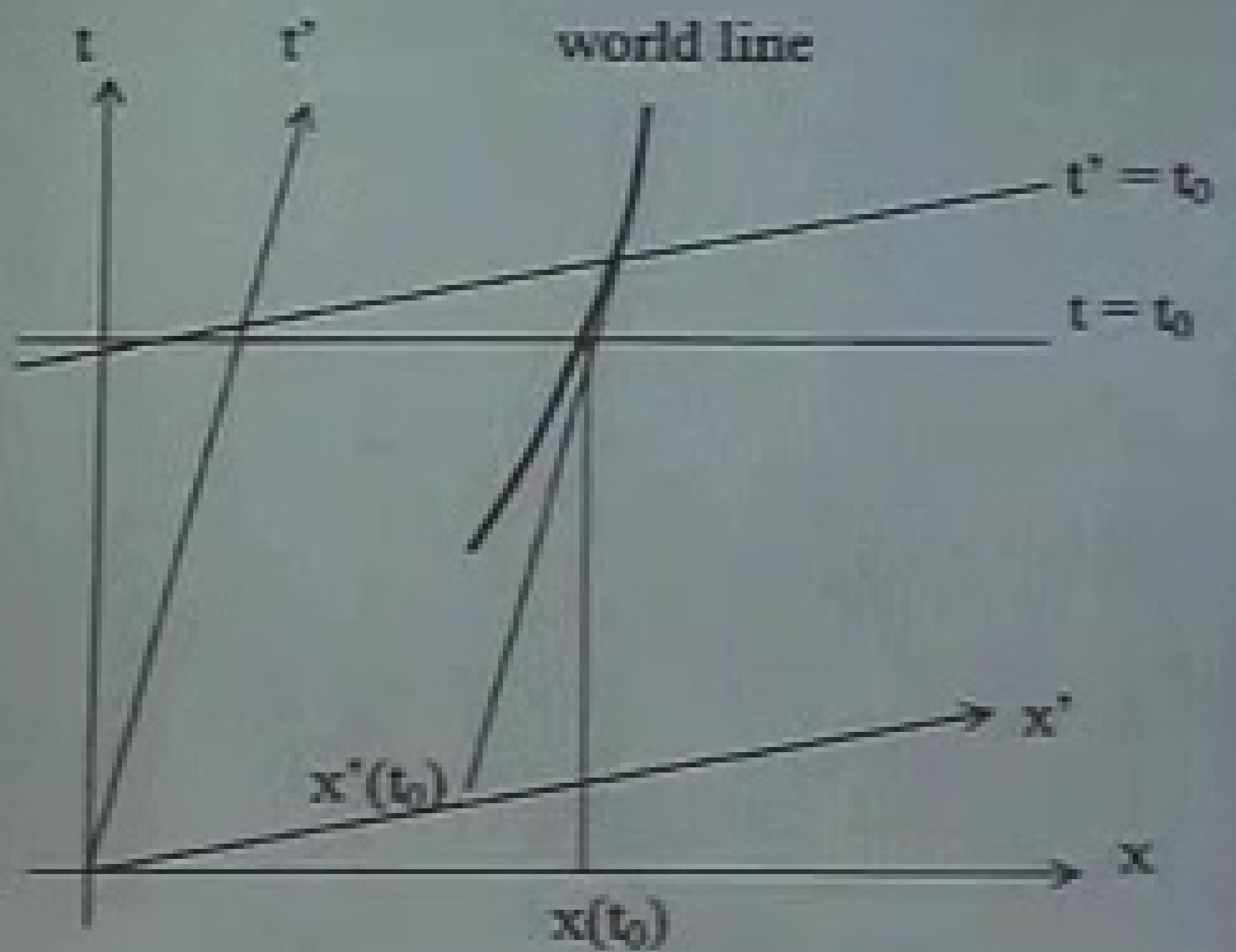
$$[K_a, K_b] = -\varepsilon_{abc} (J_c \text{ or } 0)$$

$$[K_a, P_b] = \delta_{ab} (H \text{ or } M)$$

2. Composite system

$$G_a = G_a^{(1)} + G_a^{(2)}$$

C. The transformation of particle position



$$x'(t') = x(t) - \delta\beta ct\,,\quad ct' = ct - \delta\beta x(t)\,.$$

$$x'(t') = x(t) - \delta\beta ct\,, \quad ct' = ct - \delta\beta x(t)\,,$$

$$\delta t := t' - t = -\,\delta\beta x(t)/c$$

$$x'(t') = x'(t + \delta t) = x'(t) + v(t)\,\delta t$$

$$\delta x(t) := x'(t) - x(t) = \delta\beta[x(t)(v(t)/c) - ct]$$

$$[x_a^{(n)}(t)\,, K_b] = x_b^{(n)}(t)[x_a^{(n)}(t)\,, H] - \delta_{ab}ct$$

establishing the equations for the Lie brackets of the particle positions - - with the total momentum, total angular momentum, and generators of Galilean transformations, but that this method breaks down for generators of Lorentz transformations. This failure - - - could be viewed as a reason to proceed with the construction of quantum mechanical theories - - - not including [the boost Lie bracket equations] in the case of Lorentz symmetry.[24] [24: It seems to us that this attitude is evident in the work of L. L. Foldy, - -] However, we would tend to view such an attitude as having only the temporary validity of expediency."

"Our proof makes use of the requirement that all of the generators be sufficiently regular functions of the canonical variables. This rules out any generators that this attitude is evident in the work of L. L. Foldy. - -] However, we would tend to view such an attitude as having only the temporary validity of expediency."

"Our proof makes use of the requirement that all of the generators be sufficiently regular functions of the canonical variables. This rules out any generators that may contain singular terms, such as delta functions, that could represent contact interactions."

A. Coester's model
Helv. Phys. Acta 38 7 (1965)

B. Ekstein's theorem
Comm. Math. Phys. 1 6 (1965)

C. The Wigner-van Dam model
Phys. Rev. 138 B1576 (1965)

D. The Kalnay industry
Many papers, mid 60's to mid 70's

E. The Fleming industry
Many papers, mid 60's to - - -

F. Peres' criticism
Phys. Rev. Lett. 27 1666 (1971)

F. Rohrlich's generalization
Ann. Phys. 117 292 (1979)

"The assumption that the individual particle coordinates are observable at all times leads to the requirement that they form invariant world lines."

"In a quantum theory of strongly interacting particles, it is reasonable to follow Heisenberg (8) and assert that the only observables are S-matrix elements and the masses and dynamical variables of free particles."

C imposes separability (cluster decomposition) but succeeds only up to three particles.

B. Ekstein's theorem

Comm. Math. Phys. 1 6 (1965)

"Relativistic quantum mechanical particle theories [1 – 4] – as contrasted to field theories – have recently attracted great attention, - - - . - - - Closer inspection shows that the success of quantum mechanical particle theories is bought for a price: the particle variables q_i are the Newton-Wigner position operators which do not 'transform covariantly' - - - ."

"Instead of investigating specific alternatives for position operators, we examine - - - particle theories in which the single particle observables are not necessarily given a physical interpretation. - - - We

... that the success of quantum mechanical particle theories is bought for a price: the particle variables q_i are the Newton-Wigner position operators which do not 'transform covariantly'- - -."

"Instead of investigating specific alternatives for position operators, we examine - - - particle theories in which the single particle observables are not necessarily given a physical interpretation. - - - We give a definition of the particle concept which - - - is more general. - - - our definition of particle requires only that it preserves its individuality under Lorentz - - transformations."

E demands ... under Lorentz boosts, just like Euclidean transformations, single particle give a definition of the particle concept which - - - is more general. - - - our definition of particle requires only that it preserves its individuality under Lorentz - - transformations."

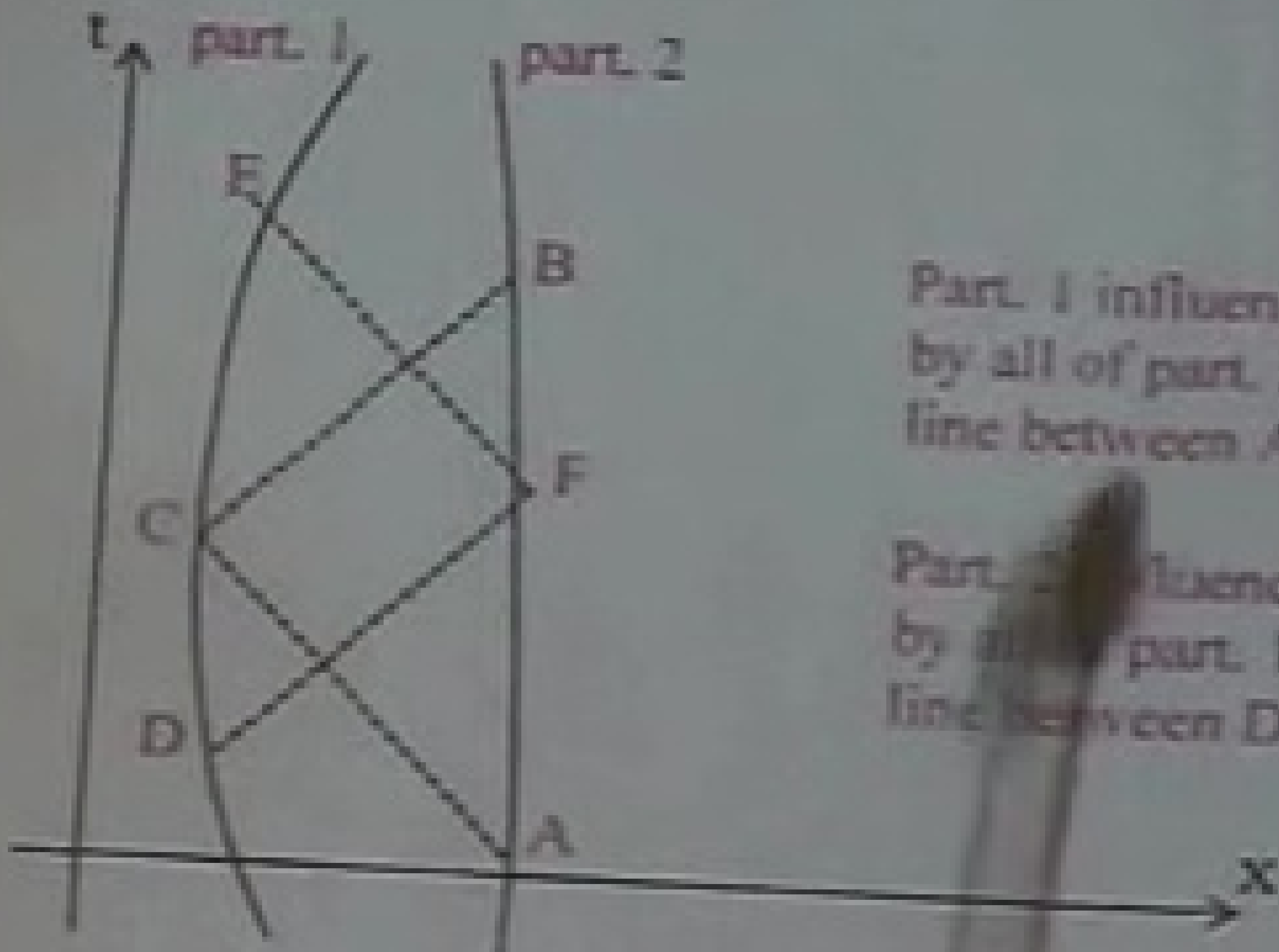
E demands that under Lorentz boosts, just like Euclidean transformations, single particle observables do not mix with other particle observables, i.e., boosts are purely kinematical. Again, this leads to no interaction.

C. The Wigner-van Dam model

Phys. Rev. 138 B1576 (1965)

"There appears to be a body of opinion according to which an interaction between point particles must - - - be via a field if the theory is to be [Lorentz invariant]. - - - The present considerations show that these opinions must be revised. The equations which we shall consider postulate that a particle 'interacts' at a definite space-time point with those points of the other orbits which are spacelike with respect to the point under considertion (see Fig. 1)."

(also Fig. 1).



Part. 1 influenced at C by all of part. 2 world-line between A and B.

Part. [illegible] [illegible]fluenced at F by a[illegible] [illegible] part. 1 world-line [illegible]ween D and E.

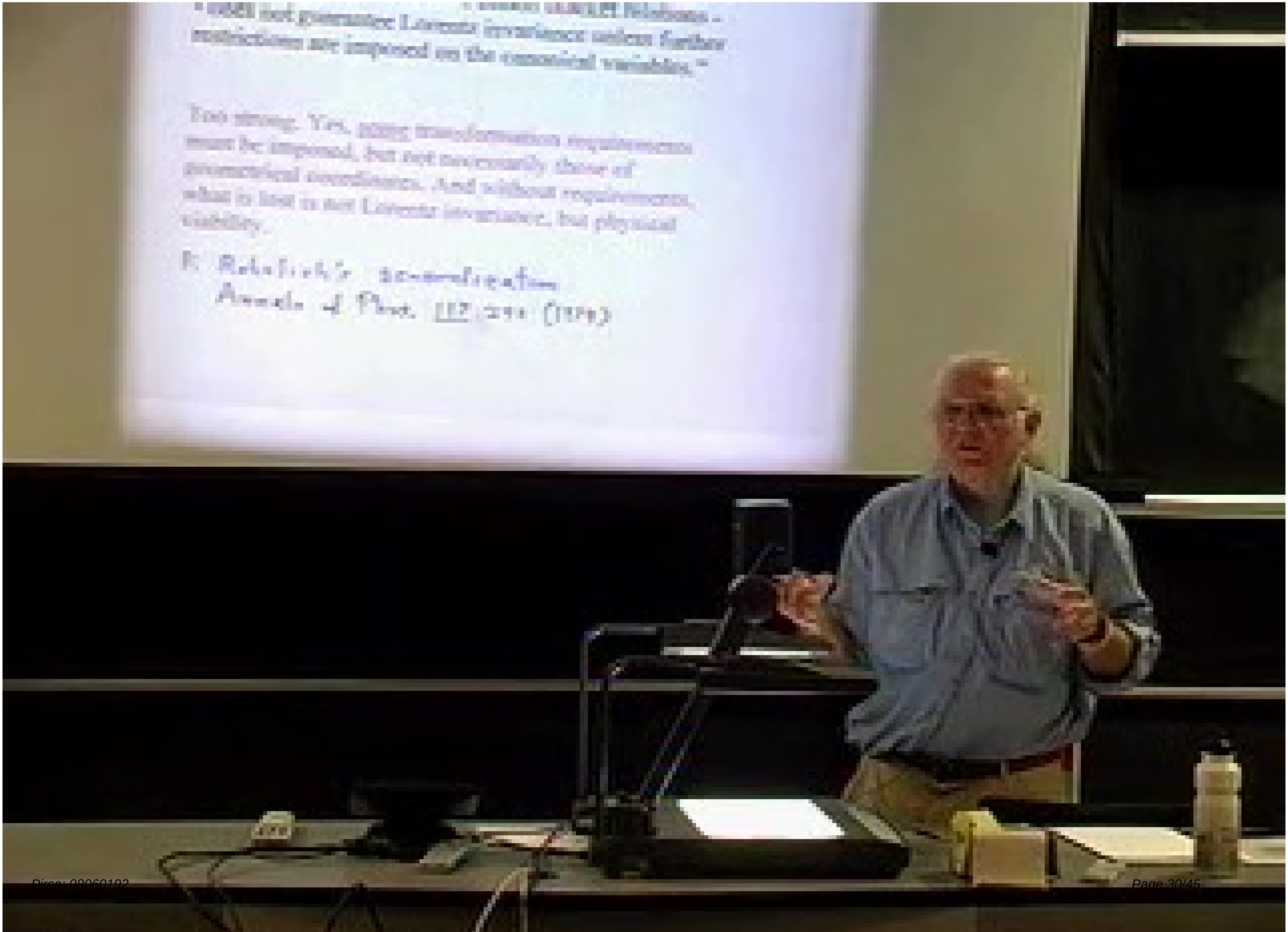
Classical and non-canonical.

Phys. Rev. Lett. 27 1666 (1971)

"- - - some authors[10 - 12] proposed to relax the traditional identification of the canonical coordinates q_i with the physical positions x_i of the particles. - - - It thus seems that one can easily forego the requirement that the canonical q_i transform like geometrical coordinates.

However, it is not so. The purpose of this note is to show that if no restriction is put on the transformation law of the canonical coordinates q_i, then the principle of relativity becomes vacuous."

"- - - the fulfillment of the Poisson bracket relations - - does not guarantee Lorentz invariance unless further restrictions are imposed on the canonical variables"


not guarantee Lorentz invariance unless further
restrictions are imposed on the canonical variables."
Too strong. Yes, some transformation requirements
must be imposed, but not necessarily those of
geometrical coordinates. And without requirements,
what is lost is not Lorentz invariance, but physical
viability.

A. Global position operators for any system Constructible out of the system's Poincare generators.

B. The center of energy (CE)

C. The Newton-Wigner center of spin (CS)

D. The geometrical centroid (CG) (?)

E. Hyperplane dependence and covariance

B. The center of energy (CE)

$[X_a , P_b] = i\hbar\, \delta_{ab} = [X_a , P_b^{(tot)}]$

$[X_a , J_b] = i\hbar\, \varepsilon_{abc} X_c = [X_a , J_b^{(tot)}]$

$[X_a , H] = i\hbar\, P_a / H$, $\quad [X_a , H^{(tot)}] = i\hbar\, dX_a/dt$

These hold for <u>all</u> positions considered here

Definition of CE:

$[X_s, H] = i\hbar P_s / H$, $\quad [X_s, H^{(tot)}] = i\hbar\, dX_s/dt$

These hold for <u>all</u> positions considered here

Definition of CE:

$$\mathbf{X}(t) := \mathbf{H}^{-1} : (\mathbf{K} + ct\, P) \qquad \text{closed system}$$

$$\mathbf{X}(t) := \mathbf{H}(t)^{-1} : (\mathbf{K}(t) + ct\, \mathbf{P}(t)) \qquad \text{open system}$$

Reason for the name:

$$(\mathbf{H}^{(1)} + \mathbf{H}^{(2)} + \mathbf{V}) : \mathbf{X}^{(12)} = \mathbf{H}^{(1)} : \mathbf{X}^{(1)} + \mathbf{H}^{(2)} : \mathbf{X}^{(2)} + \mathbf{U}$$

For quantum field theoretic systems:

$$\mathbf{X}(t) := \mathrm{H}(t)^{-1}:(\mathbf{K}(t) + ct\,\mathbf{P}(t))$$ open system

Reason for the name:

$$(\mathrm{H}^{(1)} + \mathrm{H}^{(2)} + \mathrm{V}):\mathbf{X}^{(12)} = \mathrm{H}^{(1)}:\mathbf{X}^{(1)} + \mathrm{H}^{(2)}:\mathbf{X}^{(2)} + \mathbf{U}$$

For quantum field theoretic systems:

$$\mathrm{V} = \int d^3x\, \mathcal{V}(\mathbf{x}), \qquad \mathbf{U} = \int d^3x\, \mathbf{x}\, \mathcal{V}(\mathbf{x})$$

$$\mathrm{H} = \int d^3x\, \theta_{00}(\mathbf{x}), \qquad \mathrm{H}:\mathbf{X} = \int d^3x\, \mathbf{x}\, \theta_{00}(\mathbf{x}).$$

Pitt PhilSci Archive: Quant. Mech. (2002) 663

Angular momentum decomposition

$$\mathbf{J} := \mathbf{X} \times \mathbf{P} + \boldsymbol{\Sigma} = \mathbf{L}_{CE} + \boldsymbol{\Sigma} \quad \text{where} \quad [\Sigma_a , L_{CEb}] \neq 0$$

$$[X_a , \Sigma_b] \neq 0 \quad \text{and} \quad [X_a , X_b] = -i\hbar \, \varepsilon_{abc} \Sigma_c / H^2 .$$

Ordinary "spin":

$$\mathbf{S} := \boldsymbol{\Sigma}_{\parallel} + (H/Mc)\, \boldsymbol{\Sigma}_{\perp} \quad \text{where} \quad \boldsymbol{\Sigma}_{\parallel} := \mathbf{P}(\mathbf{P} \cdot \boldsymbol{\Sigma})/P^2 .$$

Pauli-Lubanski 4-vector:

$$-W_0 = \mathbf{P} \cdot \boldsymbol{\Sigma} = \mathbf{P} \cdot \mathbf{S} , \quad \mathbf{W} = \mathbf{K} \times \mathbf{P} - \mathbf{J} H = -H \boldsymbol{\Sigma}$$

$$W_0^2 - \mathbf{W}^2 = -M^2 c^2 \mathbf{S}^2$$

decomposition

$$J := X \times P + \Sigma = L_{CE} + \Sigma \quad \text{where} \quad [\Sigma_a , L_{CE\,b}] \neq 0$$

$$[X_a , \Sigma_b] \neq 0 \quad \text{and} \quad [X_a , X_b] = -i\hbar\, \epsilon_{abc}\, \Sigma_c / H^2 .$$

Ordinary "spin":

$$S := \Sigma_{\|} + (H/Mc)\, \Sigma_{\perp} \quad \text{where} \quad \Sigma_{\|} := P(P \cdot \Sigma)/P^2 .$$

Pauli-Lubanski 4-vector:

$$-W_0 = P \cdot \Sigma = P \cdot S , \quad W = K \times P - J\, H = -H\, \Sigma$$

$$W_0^2 - W^2 = -M^2 c^2 S^2 , \quad \text{P-L Casimir invariant.}$$

Ordinary "spin":

$$\mathbf{S} := \Sigma_{\parallel} + (H/Mc)\,\Sigma_{\perp} \quad \text{where} \quad \Sigma_{\parallel} := \mathbf{P}(\mathbf{P}\cdot\Sigma)/\mathbf{P}^2 .$$

Pauli-Lubanski 4-vector:

$$-W_0 = \mathbf{P}\cdot\Sigma = \mathbf{P}\cdot\mathbf{S}, \quad \mathbf{W} = \mathbf{K}\times\mathbf{P} - \mathbf{J}\,H = -H\,\Sigma$$

$$W_0^2 - \mathbf{W}^2 = -M^2c^2\,\mathbf{S}^2, \text{ P-L Casimir invariant.}$$

Commutator with Lorentz boost generator:

$$[X_a, K_b] = i\hbar\,\{X_b(P_a/H) - \delta_{ab}\,ct + \epsilon_{abc}\Sigma_c/H\}$$

invariant world line terms

Notice: In the presence of spin, the invariant world line condition fails even without interactions!

C. The Newton-Wigner center of spin (CS)

Definition:

$$Q := X - P \times S / [H(H + Mc)]$$

Reason for name:

$$[Q_a, Q_b] = 0, \quad J = Q \times P + S = L_{CS} + S,$$

$$[Q_a, S_b] = 0 = [L_{CS\,a}, S_b], \; [S_a, S_b] = i\hbar\, \varepsilon_{abc} S_c$$

Commutator with Lorentz boost generator:

$$[Q_a, K_b] = i\hbar\, \{Q_b, (P_a/H)\} - \delta_{ab}\, ct$$

Commutator with Lorentz boost generator:

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invariant world line terms

$$+ (H + Mc)^{-1}[\varepsilon_{abc}S_c - P_a\, \varepsilon_{bcd}P_cS_d\, H^{-1}(H + Mc)^{-1}]\}$$

Distance between CE and CS:

$$(X - Q)^2 = (P \times S)^2[H(H + Mc)]^{-2}$$

$$= [P^2S^2 - (P\cdot S)^2][H(H + Mc)]^{-2} \;<\; S^2/H^2$$

D. The geometrical centroid (CG) (?)

Definition:

$$Y := Q - P \times S / [Mc(H + Mc)]$$

$$= X - P \times S / McH$$

Reason for "name":

$$[Y_a , K_b] = i\hbar \{Y_b:(P_a/H) - \delta_{ab}\, ct\} \;!!!$$

Operator valued invariant world line in the absence of interactions

Decomposition of angular momentum

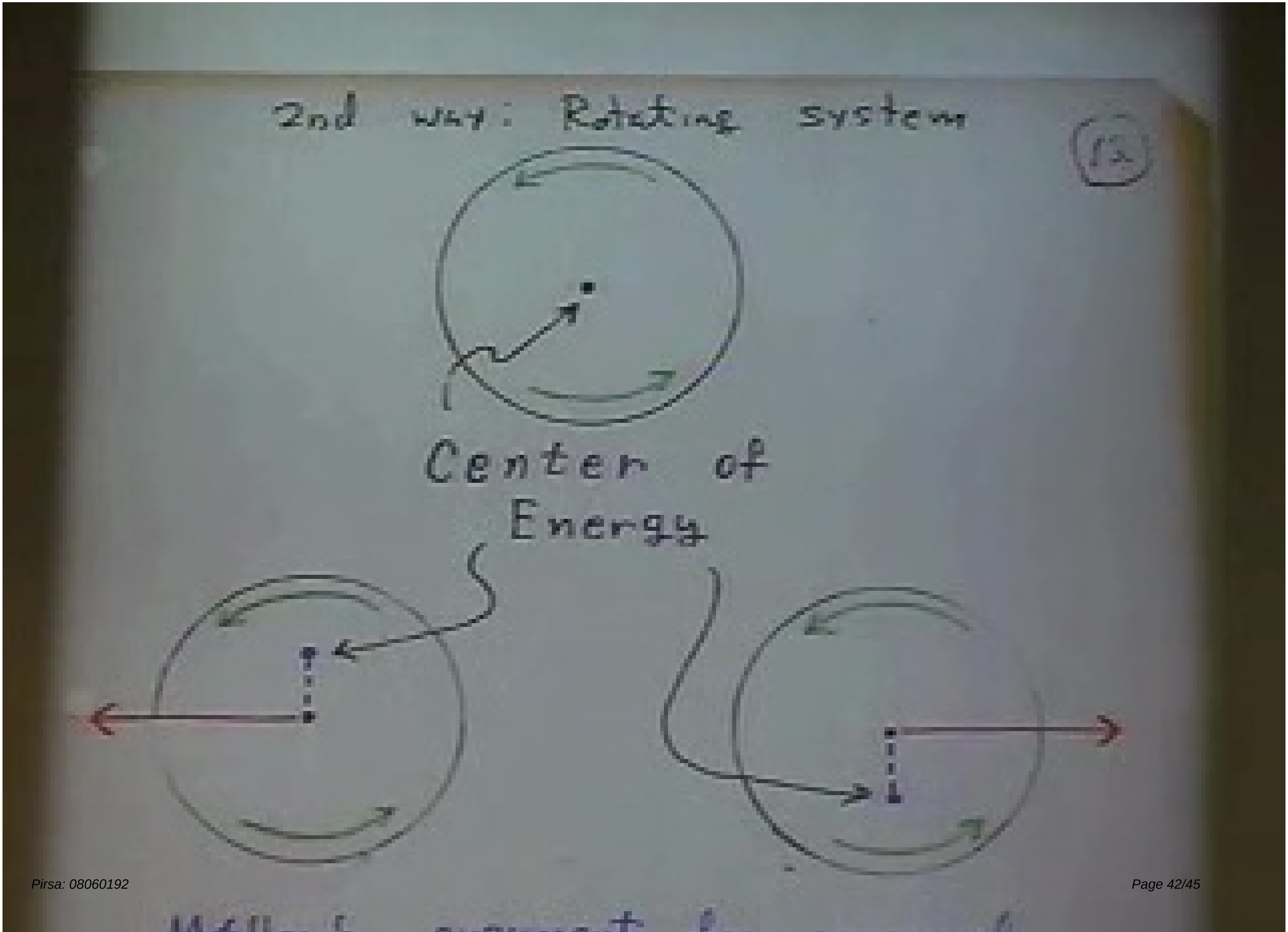
$$\mathbf{J} := \mathbf{Y} \times \mathbf{P} + \mathbf{Z} := \mathbf{L}_{CG} + \mathbf{Z}\,, \quad [Y_a\,, Z_b] \neq 0\,,$$

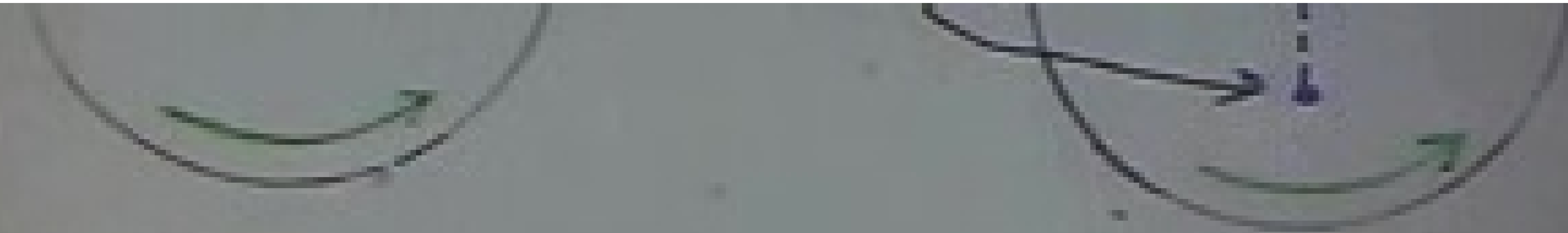
$$[L_{CG\,a}\,, Z_b] \neq 0\,, \qquad [Y_a\,, Y_b] = i\hbar\, \varepsilon_{abc} \Sigma_c \,/(Mc)^2$$

$$\mathbf{Z} = \mathbf{S}_{\parallel} + (H/Mc)\, \mathbf{S}_{\perp} = \mathbf{\Sigma}_{\parallel} + (H/Mc)^2\, \mathbf{\Sigma}_{\perp}$$

Distances between positions

$$(\mathbf{X} - \mathbf{Q})^2 \leq (\mathbf{Q} - \mathbf{Y})^2 \leq (\mathbf{X} - \mathbf{Y})^2 \leq S^2/(Mc)^2$$


2nd way: Rotating system
12
Center of
Energy

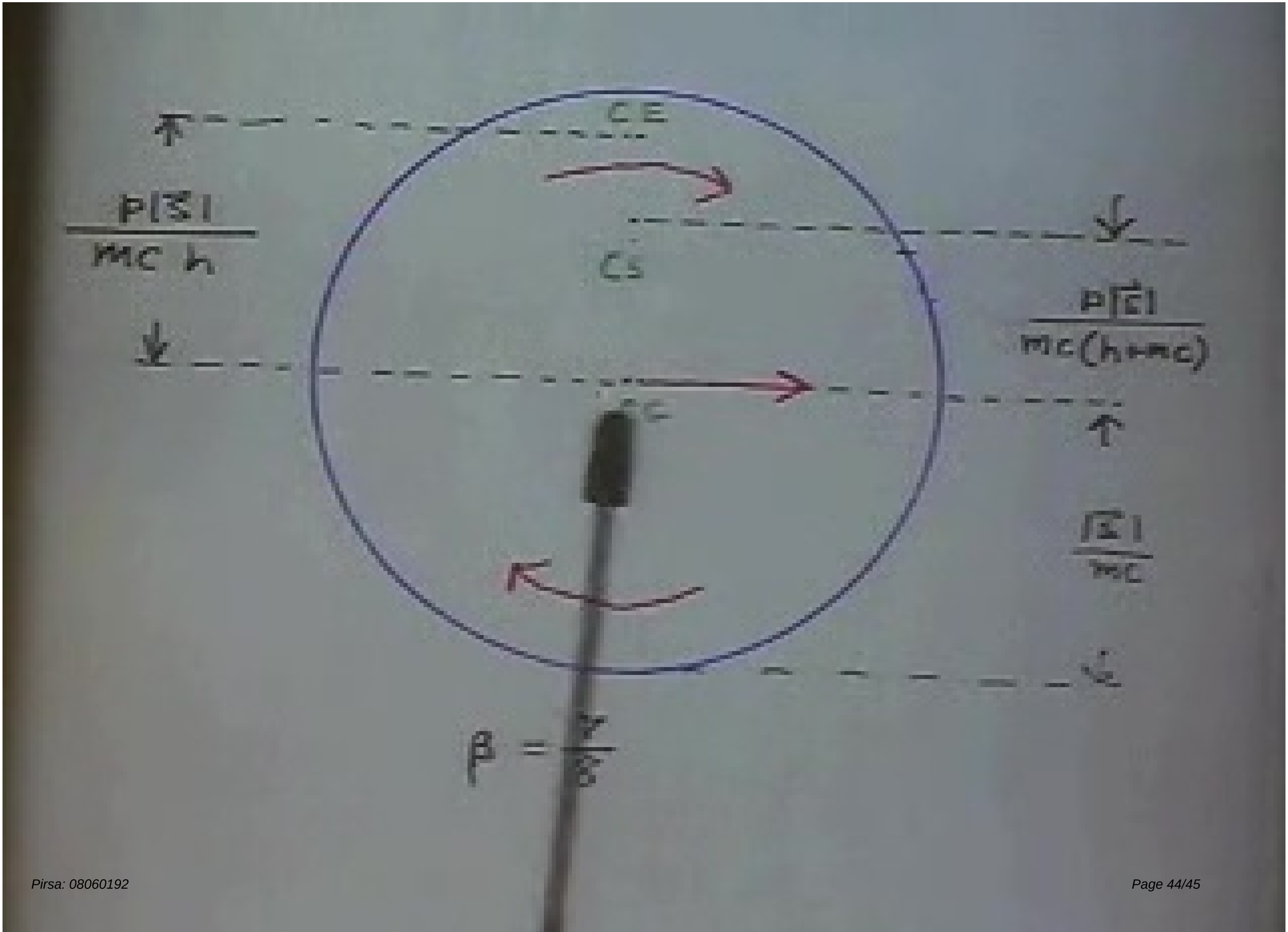


Møller's argument for minimal extension of spinning systems

$$\vec{S} = \int \vec{r} \times \vec{v} \, d\rho \; ; \; |\vec{S}| \le RcM$$

$$\therefore \; R \ge |\vec{S}|/Mc$$

$$\left(\frac{|\vec{S}|}{Mc} \right)_{Earth} \sim 10 \, m.$$


CE
CS

GF, Pitt PhilSci Archive, QM 2004 ID 2085

$$\hat{\theta}^{00}(x^0, \vec{x}) = \frac{hc}{2} :\left\{ \frac{\partial}{\partial x^0} \hat{\phi}^\dagger(x) \frac{\partial}{\partial x^0} \hat{\phi}(x) + \frac{\partial}{\partial \vec{x}} \hat{\phi}^\dagger(x) \cdot \frac{\partial}{\partial \vec{x}} \hat{\phi}(x) + \kappa^2 \hat{\phi}^\dagger(x) \hat{\phi}(x) \right\}:$$

$$[\hat{\theta}^{00}(x)]_\eta = \frac{hc}{2} :\left\{ \eta\partial \hat{\phi}^\dagger(x)\, \eta\partial \hat{\phi}(x) - D_{\eta,\mu} \hat{\phi}^\dagger(x) D_\eta^\mu \hat{\phi}(x) + \kappa^2 \hat{\phi}^\dagger(x) \hat{\phi}(x) \right\}$$

$D_{\eta,\mu} = \partial_\mu - \eta_\mu(\eta\partial)$