Title: String Theory Effects on Five-Dimensional Black Hole Physics

Date: Jun 10, 2008 11:00 AM

URL: http://pirsa.org/08060189

Abstract: We discuss recent developments in the study of black holes and similar compact objects in string theory. The focus is on how these solutions are effected by higher-derivative terms in an effective action. The setting of this investigation is an off-shell formulation of five-dimensional supergravity, including terms of order four-derivatives whose precise form are determined by embedding this theory in M-theory. We find that certain singular solutions are fully regularized by the higher-derivative terms and that generic solutions receive calculable corrections to the entropy, or other relevant quantities such as the dual central charge. A particular solution studied corresponds to the geometry sourced by a fundamental string and may set the stage for a new and exciting example of holography.

Outline

Introduction

String theory has generated much interest for a number of reasons

- My interest today is quantum gravity

Wherefore quantum gravity?

- Information paradox
- Singularities
  - Cosmological
  - Black hole interior
  - Naked
- Micro black holes

Would expect that string theory should contribute at least some partial answers to the above

We focus on topics related to black hole physics

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Black Holes and String Theory

Microscopic description of black holes a great success of string theory

- But story is incomplete!

The classic work:

- Note that SUGRAs are low-energy limits of string theories
- Known SUSY black hole solutions in SUGRA
  - Bekenstein-Hawking entropy is $S_{BH} = \frac{A}{4G_N}$
- Compute # of configs of D-branes, etc. with same asymp. charges
  - Done at weak coupling but SUSY allows continuation to finite coupling
- To compare with SUGRA result, take limit of large charges
  - Thermo limit of a stat mech result
- They match!
- Get tenure at Harvard (Strominger & Vafa)

This was hailed as a great success (and it is) but some of us still need tenure so let us see if we can do better.

Corrections to SUGRA

Stat mech does more than reproduce thermo – provides corrections due to finite system size, etc.

- Quantum gravity should do more than *reproduce* S_{BH}

In quantum gravity, SUGRA Lagrangian is only leading order terms in derivative expansion

- Expect ∞ sequence of higher-∂ terms like R^2, $R_{\mu\nu}R^{\mu\nu}$, etc.
- Cannot be computed from SUGRA since non-renormalizable
- String theory provides, in principle, definite set of corrections

Goal : Investigate SUSY solutions to effective SUGRA action with leading-order string theory corrections

- Corrections to entropy?
- Effects on singularities?
- Consistency checks on string theory
- This program has been pursued in 4D SUGRA

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1. Introduction

2. Physical Setting – 5D SUGRA

3. Off-shell Formalism and Incorporating Stringy Corrections

4. Corrected SUGRA Solutions

5. Singularities and the Rather Small

6. Closing Comments

Physics in five dimensions

Setting will be string-corrected SUGRA in 5D. But why 5D?

- Richer set of solutions than 4D
  - Rotating single-center SUSY black holes
    - ★ Single-center SUSY black holes in 4D have $J = 0$
  - Magnetically charged strings
    - ★ Magnetic charges in 4D are pointlike
  - More exotic solutions– black rings, saturns . . .
    - ★ No black rings in 4D – SUSY or otherwise
  - Two distinct classes of attractor geometries
- Chern-Simons terms
- 4D-5D connection
  - Randall-Sundrum
  - Kaluza-Klein
- Powerful tools from AdS_3/CFT_2

$\mathcal{N}=2$ 5D Supergravity Spectrum

Minimal 5D SUGRA includes

- Graviton $g_{\mu\nu}$ and gravitino ψ_μ
- Graviphoton A_μ^{grav}

Can couple to vector supermultiplets which include

- Gauge field A_μ^I
- Moduli M^I
- Gaugino Ω^I

Convenient formulation – extra vector multiplet contains graviphoton

- Too many moduli – need a constraint
  - $\mathcal{N} \equiv \frac{1}{6} c_{IJK} M^I M^J M^K = 1$
  - c_{IJK} symmetric and $c_{IIJ} = 0$
  - Known as very special geometry
- Also a gaugino constraint
- Yields $A_\mu^{grav} = \partial_I \mathcal{N} A_\mu^I \equiv \mathcal{N}_I A_\mu^I$

M-theory on CY_3

The Kähler moduli, M^I, are volumes of 2-cycles, ω^I

$$M^I = \int_{\omega^I} J$$

Corresponding vector comes from wrapping A_3 on ω^I

CY_3 volume is $\mathcal{N}$. The c_{IJK} are intersection numbers of the ω^I.

M-theory contains non-perturbative states, the $M2$ and $M5$-branes

- 11D SUGRA solns promoted to dynamical objects

On CY_3 can wrap these branes, fully or partially, while preserving SUSY

- $M2$-brane on 2-cycle $\longrightarrow$ 5D electric point charge
- $M5$-brane on 4-cycle $\longrightarrow$ 5D magnetic string charge

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M-theory on CY_3

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Off-shell supersymmetry

Previous discussion on-shell SUSY

- SUSY x-forms close up to EOMs
  - Corrections to action correct BPS conditions

An off-shell SUSY would be useful

- SUSY transformations purely representation theory
  - Unaffected by form of action
    - Conditions on BPS solutions easy to obtain
- SUSY actions easy to construct once irreps known
- Fixes field-redefinition ambiguity ($g_{\mu\nu} \to g_{\mu\nu} + aR_{\mu\nu} + \ldots$)

To obtain such a formulation:

- Begin with 5D conformal SUGRA
  - Off-shell but too much symmetry
- Couple to "compensator" fields
- Gauge fix extraneous fields and symmetries
- End result is Poincarè SUGRA with physical and auxiliary fields

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Off-shell supersymmetry

Multiplets:

Weyl: $e_\mu{}^a$, ψ_μ, v^{ab}, χ, and D

Vector: A^I_μ, M^I and Ω^I

SUSY transformations(around bosonic bkgd):

$$\delta\psi_\mu = \left(\mathcal{D}_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}\gamma_\mu\gamma\cdot v\right)\varepsilon\ ,$$

$$\delta\chi = \left(D - 2\gamma^c\gamma^{ab}\mathcal{D}_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(\gamma\cdot v)^2\right)\varepsilon\ ,$$

$$\delta\Omega^I = \left(-\frac{1}{4}\gamma\cdot F^I - \frac{1}{2}\gamma^a\partial_a M^I - \frac{1}{3}M^I\gamma\cdot v\right)\varepsilon$$

These variations are valid at every order. BPS conditions simply

$$\boxed{\delta\psi_\mu = \delta\chi = \delta\Omega^I = 0}$$

Classical action

Can construct an invariant two-derivative Lagrangian by combining multiplets. The bosonic part is

$$\frac{1}{2}\mathcal{L}_0 = -2\left(\frac{1}{8}D + \frac{3}{16}R - \frac{1}{4}v^2\right) + \mathcal{N}_I v^{ab} F^I_{ab} + \frac{1}{48} c_{IJK} A^I_a F^J_{bc} F^K_{de} \epsilon^{abcde}$$
$$+\mathcal{N}\left(\frac{1}{4}D - \frac{1}{8}R + \frac{3}{2}v^2\right) + \mathcal{N}_{IJ}\left(\frac{1}{8}F^I_{ab}F^{Jab} + \frac{1}{4}\partial_a M^I \partial^a M^J\right)$$

To obtain conventional on-shell description, integrate out non-dynamical fields

- D Lagrange multiplier enforcing volume constraint $\mathcal{N} = 1$
- v_{ab} is roughly graviphoton field strength

$$v_{ab} = -\tfrac{1}{4}\mathcal{N}_I F^I_{ab}$$

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Higher derivative corrections

Can construct action with higher derivatives by considering other off-shell SUSY invariants (Hanaki, Ohashi, Tachikawa)

SUSY completion of mixed gauge-gravitational Chern-Simons term,

$$\alpha_I \epsilon_{abcde} A^{Ia} R^{bcfg} R^{de}{}_{fg}$$

This term is special in that its coefficient in M-theory is determined by $M5$-brane anomaly cancellation

We use the HOT Lagrangian, instead of computing every term in M-theory and then dim reducing

- Can't do that anyway in practice
- Thought to be unique $N = 2$ Lagrangian at this order

Four-derivative Lagrangian

$$\mathcal{L}_1 = \frac{c_I}{24}\left\{\frac{1}{16}\epsilon_{abcde}A^{Ia}R^{bcfg}R^{de}{}_{fg} + \frac{1}{8}M^I C^{abcd}C_{abcd} + \frac{1}{12}M^I D^2 \right.$$
$$+ \frac{1}{6}F^{Iab}v_{ab}D + \frac{1}{3}M^I C_{abcd}v^{ab}v^{cd} + \frac{1}{2}F^{Iab}C_{abcd}v^{cd}$$
$$+ \frac{8}{3}M^I\left(v_{ab}\mathcal{D}^b\mathcal{D}_c v^{ac} - \frac{2}{3}v^{ac}v_{cb}R_a{}^b - \frac{1}{12}v_{ab}v^{ab}R\right)$$
$$+ \frac{4}{3}M^I\left(\mathcal{D}^a v^{bc}\mathcal{D}_a v_{bc} + \mathcal{D}^a v^{bc}\mathcal{D}_b v_{ca} - \frac{1}{2}\epsilon_{abcde}v^{ab}v^{cd}\mathcal{D}_f v^{ef}\right)$$
$$+ \frac{2}{3}F^{Iab}\epsilon_{abcde}v^{cd}\mathcal{D}_f v^{ef} + F^{Iab}\epsilon_{abcde}v^c{}_f\mathcal{D}^d v^{ef}$$
$$\left. -\frac{4}{3}F^{Iab}v_{ac}v^{cd}v_{db} - \frac{1}{3}F^{Iab}v_{ab}v^2 + 4M^I v_{ab}v^{bc}v_{cd}v^{da} - M^I(v^2)^2\right\}$$

where the $c_I \in \mathbb{Z}$ are expansion coeffs of the second Chern class of CY_3

The equations of motion rather formidable.

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Obtaining solutions

Would like to avoid solving the equations of motion

Strategy:

- Make an *ansatz* with the desired symmetry properties
  - SUSY implies $\exists$ causal Killing vector $\propto \bar{\varepsilon}\gamma_\mu\varepsilon$
    - ★ Analysis splits into timelike and null cases
    - ★ Killing spinor obeys a projection – $1/2$ BPS
- Solve SUSY conditions – indep of $\mathcal{L}$
  - Gravitino variation eliminates v_{ab}
  - Gaugino variation determines gauge fields
  - Auxino variation eliminates D
- Solve some equations of motion as necessary
  - D eqn gives constraint on scalar moduli space
  - Maxwell eqn relate moduli and metric functions to electric charges
    - ★ For magnetic solutions, use Bianchi identity instead
  - Avoid Einstein eqns and v_{ab} EOM if possible

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Example solution - Electric black hole

Start with spherically symmetric ansatz

$$ds^2 = e^{4U_1(r)}dt^2 - e^{-2U_2(r)}dx^i dx^i$$
$$v_{ij} = F^I_{ij} = 0$$

Killing spinor satisfies

$$\gamma_{\hat{t}}\varepsilon = -\varepsilon$$

The supersymmetry conditions then eliminate auxiliary fields and imply

$$U_1(r) = U_2(r) \equiv U(r)$$

Also determines gauge fields in terms of scalars

$$A^I_t = e^{2U}M^I$$

This is all valid regardless of higher-derivative terms in action.

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Example solution - Electric black hole

Press further and investigate Maxwell equation which is affected by corrections. Nevertheless yields compact condition

$$\nabla^2\left[e^{-2U}M_I - \frac{c_I}{8}(\nabla U)^2\right] = 0$$

where $M_I = \frac{1}{2}c_{IJK}M^J M^K$. Solve to obtain

$$e^{-2U}M_I - \frac{c_I}{8}(\nabla U)^2 = M_I^\infty + \frac{q_I}{r^2}$$

Supplement this with modified special geometry constraint(from D EOM)

$$\mathcal{N} - 1 + \frac{c_I}{24}\left[\nabla^2 U - 4(\nabla U)^2)M^I + \nabla U \nabla M^I\right] = 0$$

Given a choice of charges, q_I, and boundary conditions, M_I^∞, the system of highlighted equations can now be addressed numerically.

Magnetic solutions – Strings

Obtained similarly to black hole but important differences

- Killing spinor obeys light-like projection
- Magnetic charges – Use Bianchi not Maxwell
  - Much simpler since Bianchi topological

Solution determined by harmonic functions

$$H^I = M^I_\infty + \frac{p^I}{2r}$$

to be

$$ds^2 = e^{2U}(dt^2 - dy^2) - e^{-4U} dx^i dx^i$$

$$F^I = -\frac{p^I}{2}\epsilon_{S^2}$$

$$M^I = e^{2U} H^I$$

$$e^{-6U} = \frac{1}{6} c_{IJK} H^I H^J H^K + \frac{c_I}{24}\left(\nabla U \cdot \nabla H^I + 2H^I \nabla^2 U\right)$$

Given M^I_∞ and p^I, determine U numerically

Magnetic attractor geometry

Standard aspect of SUSY black holes is attractor mechanism

- Near-horizon regime has enhanced SUSY, $AdS_p \times S^q$ geometry
- M^I_∞ arbitrary but near-horizon moduli determined by charges
- Scale of geometry also determined by charges

Assume geometry near string is $AdS_3 \times S^2$

$$e^{-2U} = \frac{\ell_S}{r}\,, \quad M^I_\infty = 0$$

This is an exact solution if

$$\ell_A^3 = 8\ell_S^3 = \frac{1}{6}c_{IJK}p^I p^J p^K + \frac{1}{12}c_I p^I$$

So we have an exact attractor solution to our corrected eqns

- Must investigate numerically to match onto asymptotically flat space

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Small strings

Have found scale of geometry for string solutions

$$\ell_A^3 = \frac{1}{6} c_{IJK} p^I p^J p^K + \frac{1}{12} c_I p^I$$

Consider $c_{IJK} p^I p^J p^K = 0$ solutions, or small strings

- Occurs if there are < 3 non-zero charges ($c_{IIJ} = 0$)
- Nakedly singular at leading order – size of AdS vanishes
- Correction term smooths out singularity
  - 1 and 2 charge solutions now generically regular

Caveat is in order

- The derivative expansion breaks down at singularities
- We appeal to AdS/CFT

Non-renormalization

Symmetries powerful in AdS_3/CFT_2

- Isometries are $SL(2) \times SL(2)$, but get promoted to infinite dimensional Virasoro algebra at boundary
- Bulk $N = 2$ SUSY $\rightarrow (4,0)$ SCFT on bndy
- Central charges related to AdS scale at leading order
  - $c_L = c_R = \frac{3\ell_A}{2G_3}$
  - Use c as proxy for ℓ_A

Careful treatment of anomalies (Kraus & Larsen)

$$c = \frac{c_L + c_R}{2} = -6 \left(\ell_A^3 \ell_S^2 \mathcal{L} \right)_{ext}$$

This is exact, *i.e.* valid for all Lagrangians. Applied to magnetic attactor obtain

$$c = c_{IJK} p^I p^J p^K + \frac{3}{4} c_I p^I$$

Matches the known central charge of CFT for $M5$'s on CY_3. Higher order terms must not contribute

The Fundamental Heterotic String

Let $CY_3 = T^2 \times K3$

$$c_0 = 24, c_i = 0, c_{0ij} = c_{ij}, c_{ijk} = 0$$

Consider N $M5$-branes wrapping $K3$. Previous formula yields

$$c = 18N$$

But config is dual to N heterotic strings in T^5 compactification

- Heterotic-IIA duality in 6D
- Central charges – $c_L = 24N$ & $c_R = 12N$
- Agreement as expected: $c = \frac{c_L + c_R}{2}$

Fundamental String Holography

Since F-string geometry has an AdS factor we should think about holography. This example of holography would have some interesting features. On gravity side:

- Geometry is string scale, so cannot neglect α' corrections
- Can be weakly coupled since $g_s \sim N^{-1/2}$
- Gravity side at large N is *classical string theory*

Have a CFT/CFT duality:

- Worldsheet CFT with critical central charge
  - Compute perturbative string "scattering" amplitudes in AdS geometry
- Dual CFT at boundary of AdS_3
  - Dual to complete quantum gravity theory
- Should exist regime in which both are manageable
  - Qualitatively new example of AdS/CFT

Unfortunately, this is not that easy

- Non-linear algebras – unitarity issues

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Other Solutions

Have developed general black hole solutions

$$ds^2 = e^{4U}\left(dt + \omega_i dx^i\right)^2 - e^{-2U} h_{ij} dx^i dx^j$$

- Rotating black hole solutions
- Base space metric h_{ij} HyperKähler
  - Interesting example is Taub-NUT
    - Interpolates between 4D and 5D
- Can compute entropy for these solutions
  - Currently no microscopic model for comparison
- Small black holes problematic
  - No protected quantity since no AdS_3

Future directions and work in progress

- Asymptotically AdS_5 solutions (WIP)
  - Requires gauged SUGRA – but formalism is ready
  - Make statements about dual gauge theory
  - Superstar – naked singularity
- NSP system – String with momentum (WIP)
  - Need to use Einstein's eqn
  - Small black hole in 4D
- Black Ring (WIP)
  - Need v_{ab} EOM
- Cosmological singularities
  - Non-SUSY very difficult
  - SUSY toy cosmology, but with "null time"
- F-string holography (WIP by several groups)
  - Various proposals but status unclear

Conclusions

- Off-shell formalism very useful
  - Construction of 5D theory
  - Finding SUSY solutions
  - Can we examine non-SUSY solutions?
- Have seen an example of singularity resolution
  - Cannot keep controlled expansion
    - Have to rely on some external facts – AdS/CFT, etc.
- 5D gravity interesting laboratory
  - Complements understanding of 4D
  - May be of pheno relevance