Title: Inflationary Constraints on String Theory

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Abstract: It is an important task to embed inflation in a fundamental microphysical theory such as string theory. Since string theory possesses a vast landscape of 4-dimensional theories, we would like to know which portions contain inflation and which do not. I prove a no-go theorem that inflation and de Sitter vacua are forbidden in an exponentially large number of infinite families of simple and well understood compactifications of type IIA string theory. I also mention more complicated and less well understood compactifications, which may have the ingredients for our cosmology.

Inflationary Constraints on String Theory

PASCOS 08

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- hep-th/0711.2512: MH, Kachru, Taylor, Tegmark

Cosmology & String Theory

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  What is the inflaton potential?

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Cosmologists and String theorists can learn from eachother!

Stringlish to English

Symbol	Name	Approximate meaning
α'	Regge parameter	Inverse string tension
l_s	String length	$= 2\pi\sqrt{\alpha'}$ (in our convention)
κ_{10}	10-d gravitational strength	$= \sqrt{8\pi G_{10}} = l_s^4/\sqrt{4\pi}$, gravitational strength in 10 dims
$\bar{m}_{\rm P}$	(Reduced) Planck mass	$= 1/\sqrt{8\pi G}$, mass scale of quantum gravity in 4 dims
ϕ	Dilaton	Scalar field that rescales the strength of gravity
a_i	Axions	Pseudo-scalars that appear in the 4-d theory
b_i	Geometric moduli	Scalar fields describing ϕ and the size & shape of the compact space
	– Dilaton modulus[a]	$\sim e^{-\phi}$ (explicit form is model dependent)
	– Kähler moduli	Scalar fields that specify the *size* of the compact space
	– Complex structure moduli	Scalar fields that specify the *shape* of the compact space
ψ_i	Complex moduli	$= a_i + i\, b_i$
ψ	Complex inflaton vector	$= (\psi_1, ..., \psi_n)$, the complex moduli-vector that can evolve during inflation
ϕ	Real inflaton vector	$= (a_1, b_1, ..., a_n, b_n)$, the real moduli-vector that can evolve during inflation
g_s	String coupling	$= e^{\phi}$, the string loop expansion parameter
F_p	p-form field strength	Generalized electromagnetic field strength carrying p-indices
f_p	Flux	$\propto \int F_p$, (normally integer valued) equivalent to a generalized electric or magnetic charge, but can arise purely due to non-trivial topology
g_{10}/R_{10}	10-d string metric/Ricci scalar	Metric/Ricci scalar in the fundamental 10-d action in string frame
g_4/R_4	4-d string metric/Ricci scalar	Metric/Ricci scalar in the effective 4-d action in string frame
g_E/R_E	4-d Einstein metric/Ricci scalar	Metric/Ricci scalar in the effective 4-d action after a conformal transformation to Einstein frame
g_6	Metric on compact space	2nd block of $g_{10} = \mathrm{diag}(g_4, g_6)$, describing the geometry of compact space
Vol	6-d volume of compact space	$= \int_{cs} d^6x\sqrt{g_6}$ (cs $\equiv$ compact space)

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- Compactification

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- Outlook: Evading the No-Go Theorem

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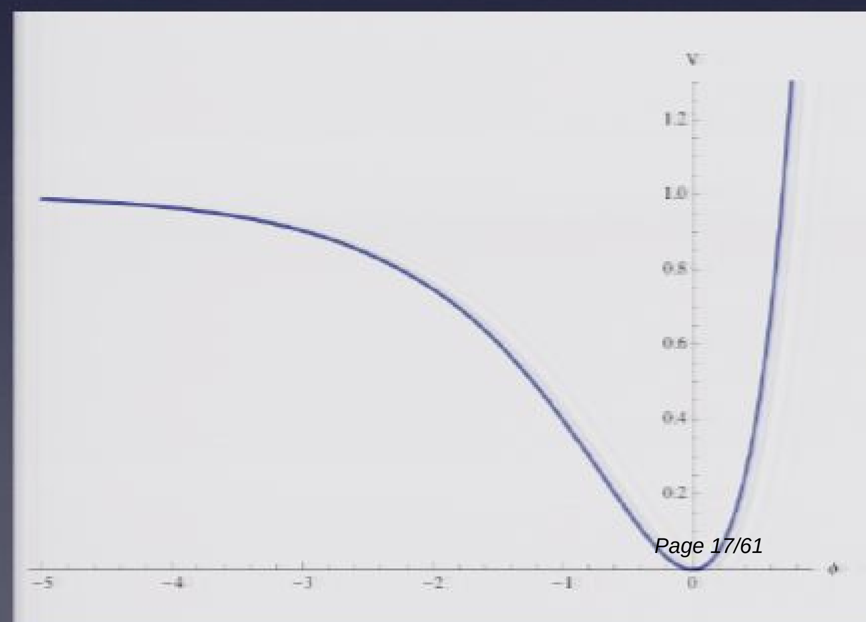

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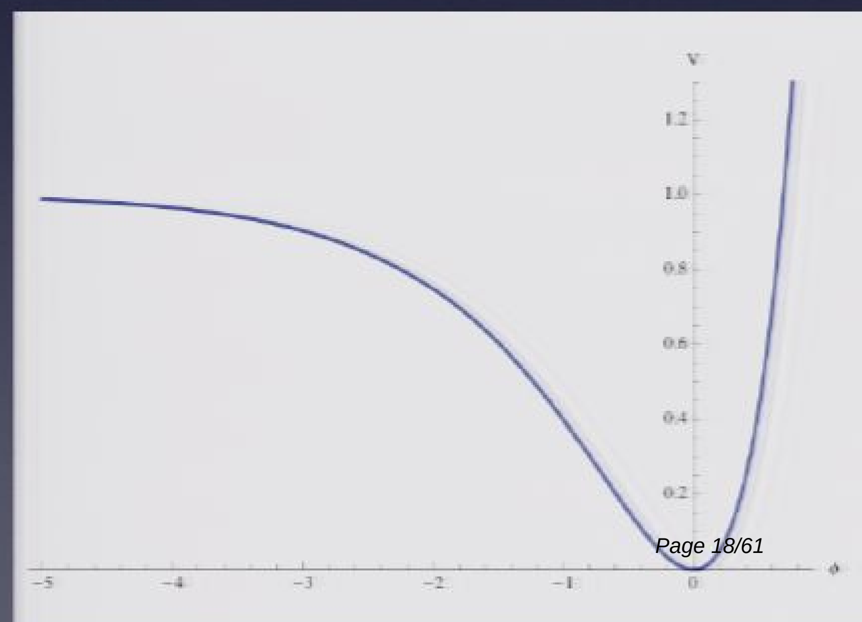

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- IIA with D6/O6 branes on CYs are "semi-realistic"

e.g., can build MSSM

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- We proved 4D theory has the Lagrangian:

$$\mathcal{L} = \frac{1}{16\pi G} R_E - \left(\frac{1}{2}(\partial_\mu \phi_d)^2 + \frac{1}{2}(\partial_\mu \phi_v)^2 + \ldots \right) - V(\phi_i)$$

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Einstein gravity

Dilaton

6D Volume

All other “moduli”
size & shape

Fluxes, Branes & Planes

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Or perhaps just get lucky...

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In Summary; we explore an exponentially large number of infinite (10^500) families of 4D models

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- We proved rigorously (using K and W, or direct dimensional reduction) that in the Einstein frame: $(R \Leftrightarrow \psi_v)$

$$V = V_3 + \sum_p V_p + V_{D6} + V_{O6}$$

$$= \frac{A_3(\phi_j)}{\psi_d^2 \psi_v^3} + \sum_p \frac{A_p(\phi_j)}{\psi_d^4 \psi_v^{p-3}} + \frac{A_{D6}(\phi_j)}{\psi_d^3} - \frac{A_{O6}(\phi_j)}{\psi_d^3}$$

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You are not here

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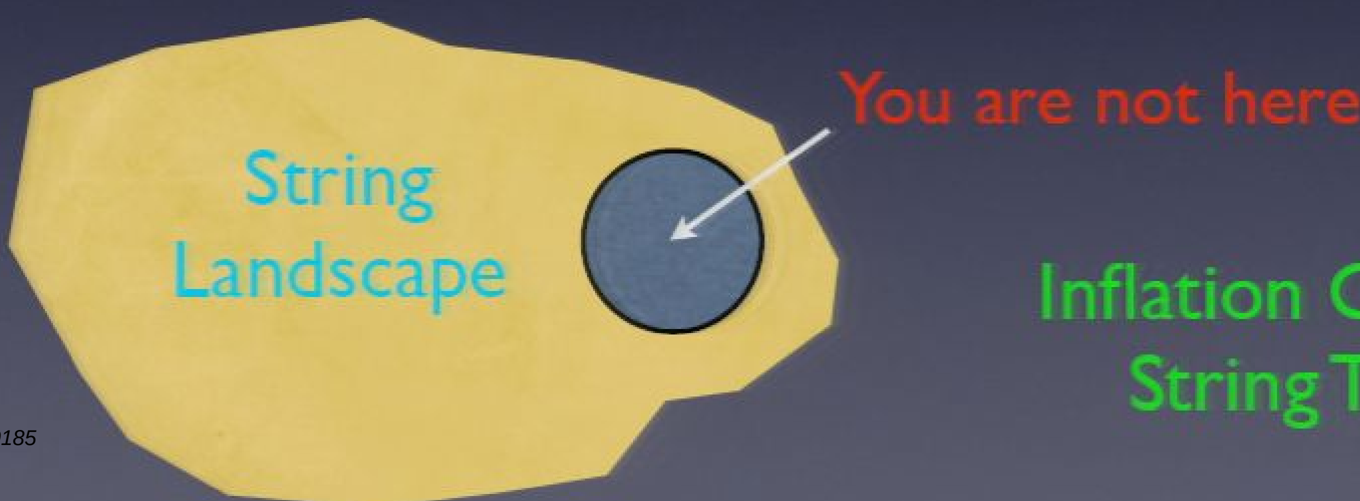

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