Title: A Singularity Problem with f(R) Dark Energy

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Abstract: I point out that there is a curvature singularity problem appearing on non-linear level that generally plagues f(R) models that modify Einstein gravity in the infrared. It is caused by the fact that for the effective scalar degree of freedom, the curvatur

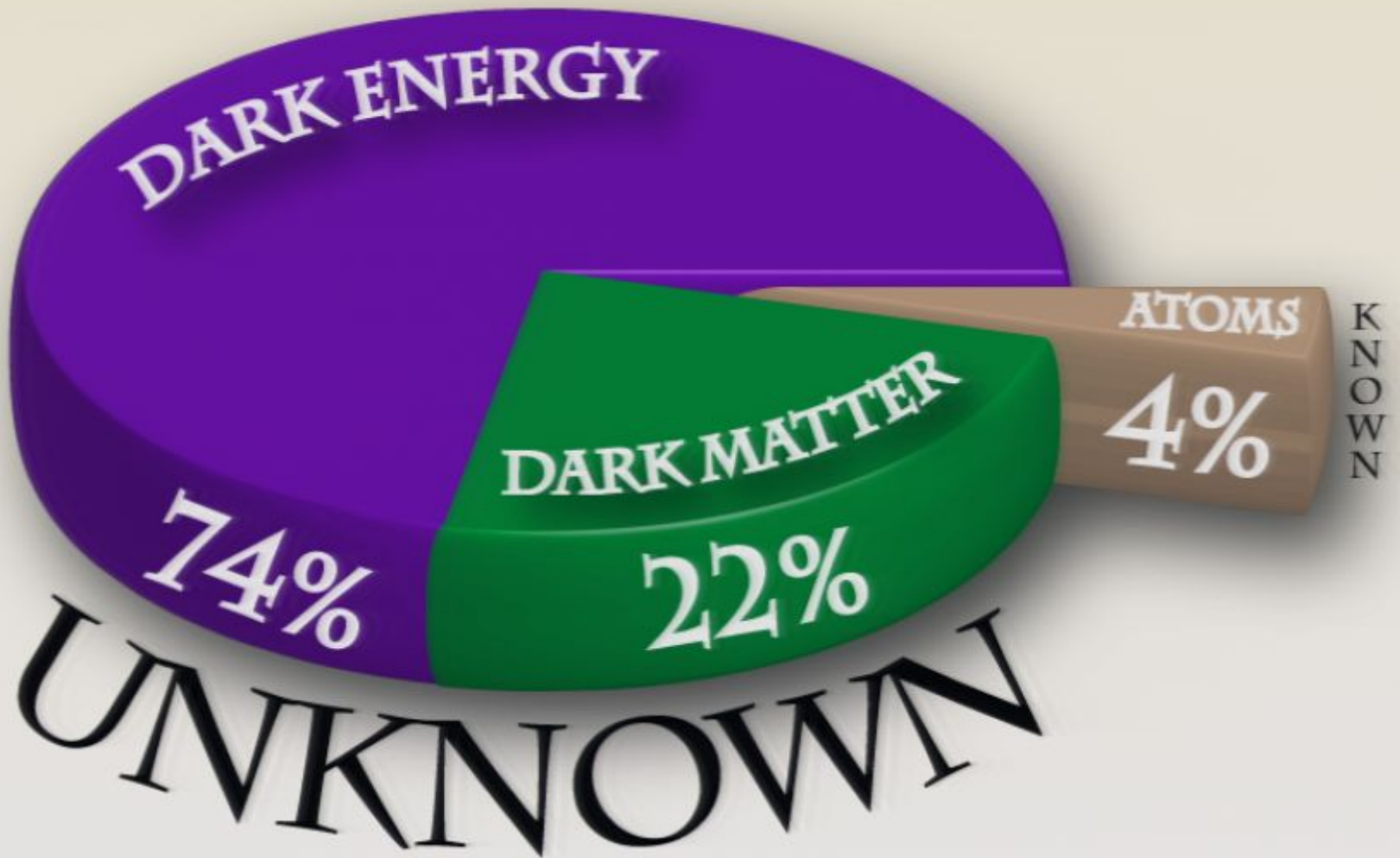# A Singularity Problem with F(R) Dark Energy

arXiv:0803.2500

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Particles, Strings, and Cosmology
Perimeter Institute
6 June 2008

Maybe it's Gravity we Don't Understand...

What if instead of curvature in Einstein-Hilbert action we had

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_m \right\} \sqrt{-g}\, d^4 x$$

UV Modification:

$$f(R) = R + \frac{R^2}{M^2}$$

Starobinsky (1980)

IR Modification:

$$f(R) = R - \frac{\mu^4}{R}$$

Capozziello et. al. [astro-ph/0303041]
Carroll et. al. [astro-ph/0306438]

For F(R) theory to make sense we need:

- $f' > 0$ – otherwise gravity is a ghost
- $f'' > 0$ – otherwise gravity is a tachyon

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AFTER A ROCKY START...

Linearly stable models
   have been found:

     Hu and Sawicki
     [0705.1158]

   Starobinsky [0706.2041]

   various "designer" models

LSS growth studied:

    Pogosian & Silvestri
    [0709.0296]

But something is
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AFTER A ROCKY START... A NEW TROUBLE?

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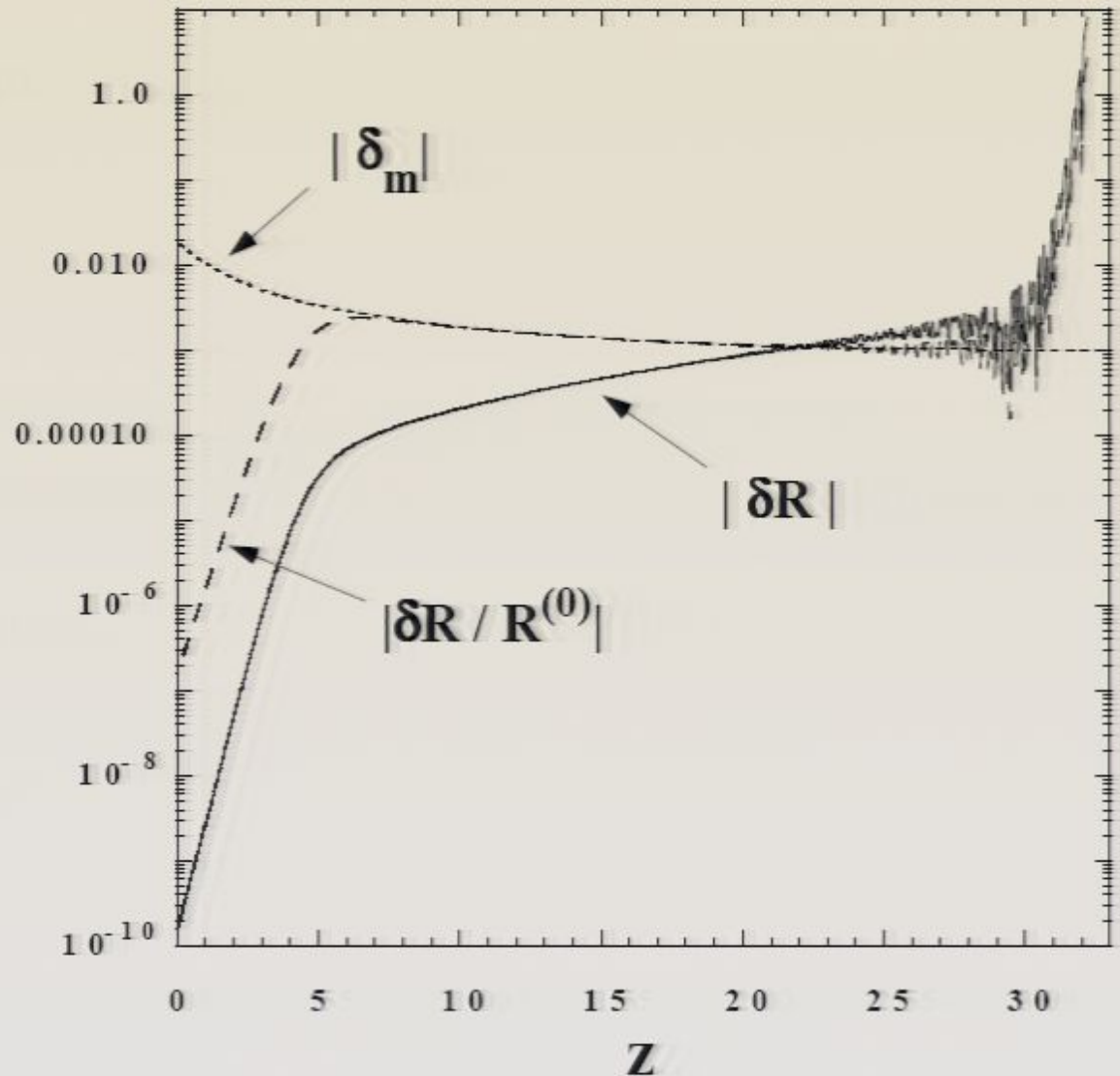
Tsujikawa [0709.1391]

WHY DOESN'T F(R) DARK ENERGY WORK?

FIELD EQUATIONS IN F(R) GRAVITY

- Vary the action with respect to the metric:

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathcal{L}_m \right\} \sqrt{-g} \, d^4x$$

- Einstein equations turn into a fourth-order equation:

$$f' R_{\mu\nu} - f'_{;\mu\nu} + \left(\Box f' - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

- A new scalar degree of freedom $\phi \equiv f' - 1$ appears:

$$\Box f' = \frac{1}{3}(2f - f'R) + \frac{8\pi G}{3} T$$

- Can rewrite fourth-order field equation as two second order ones!

A New Scalar Degree of Freedom

- Equation for $\phi \equiv f' - 1$ is just a scalar wave equation:

$$\Box\phi = V'(\phi) - \mathscr{F}$$

- Matter directly drives the field ϕ by a force term:

$$\mathscr{F} = \frac{8\pi G}{3}(\rho - 3p)$$

- Effective potential can be found by integrating

$$V'(\phi) \equiv \frac{dV}{d\phi} = \frac{1}{3}(2f - f'R)$$

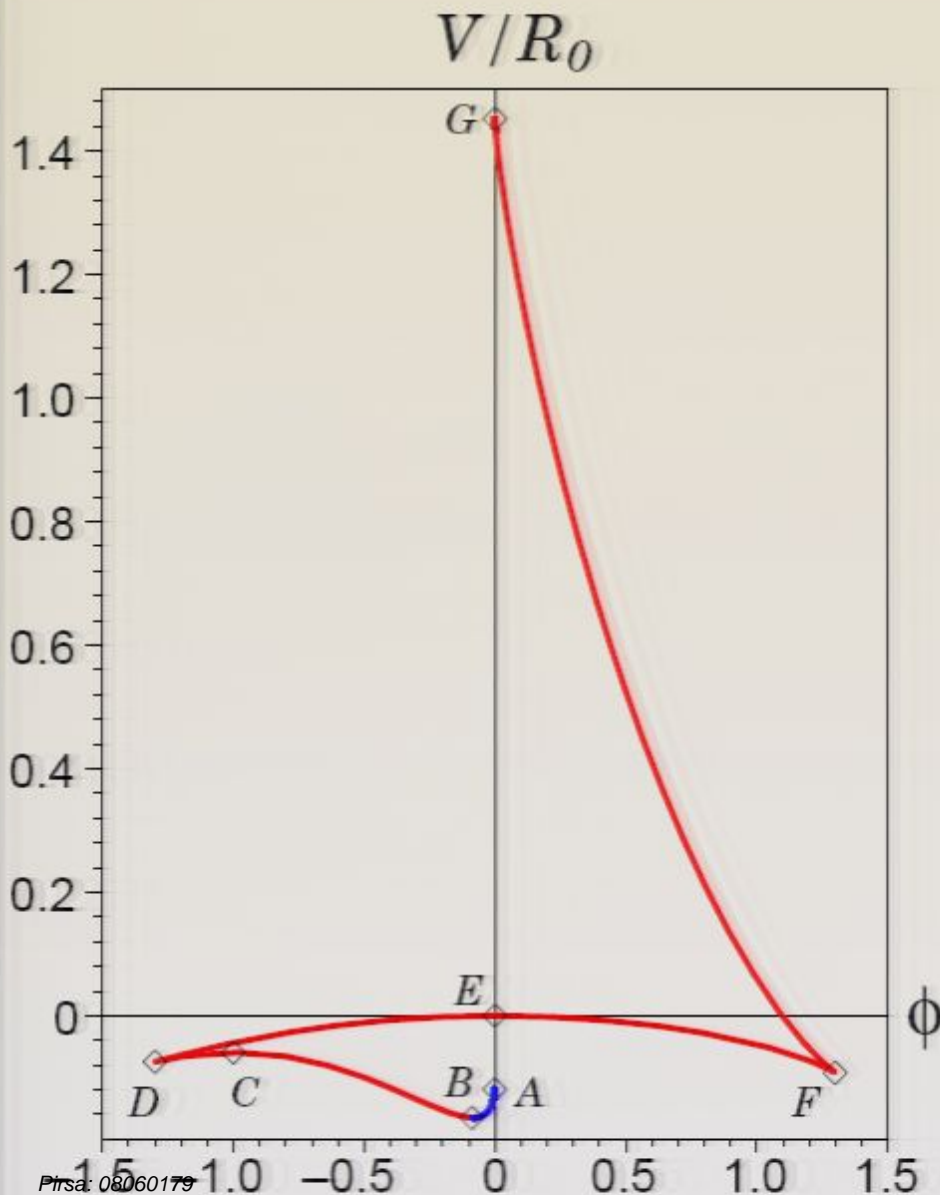- In practice, easier to obtain in parametric form:

$$\frac{dV}{dR} \equiv \frac{dV}{d\phi}\frac{d\phi}{dR} = \frac{1}{3}(2f - f'R)f''$$

Disappearing Cosmological Constant Model

Starobinsky [0706.2041]

$$f(R) = R + \lambda \left[\left(1 + R^2\right)^{-1} - 1 \right]$$
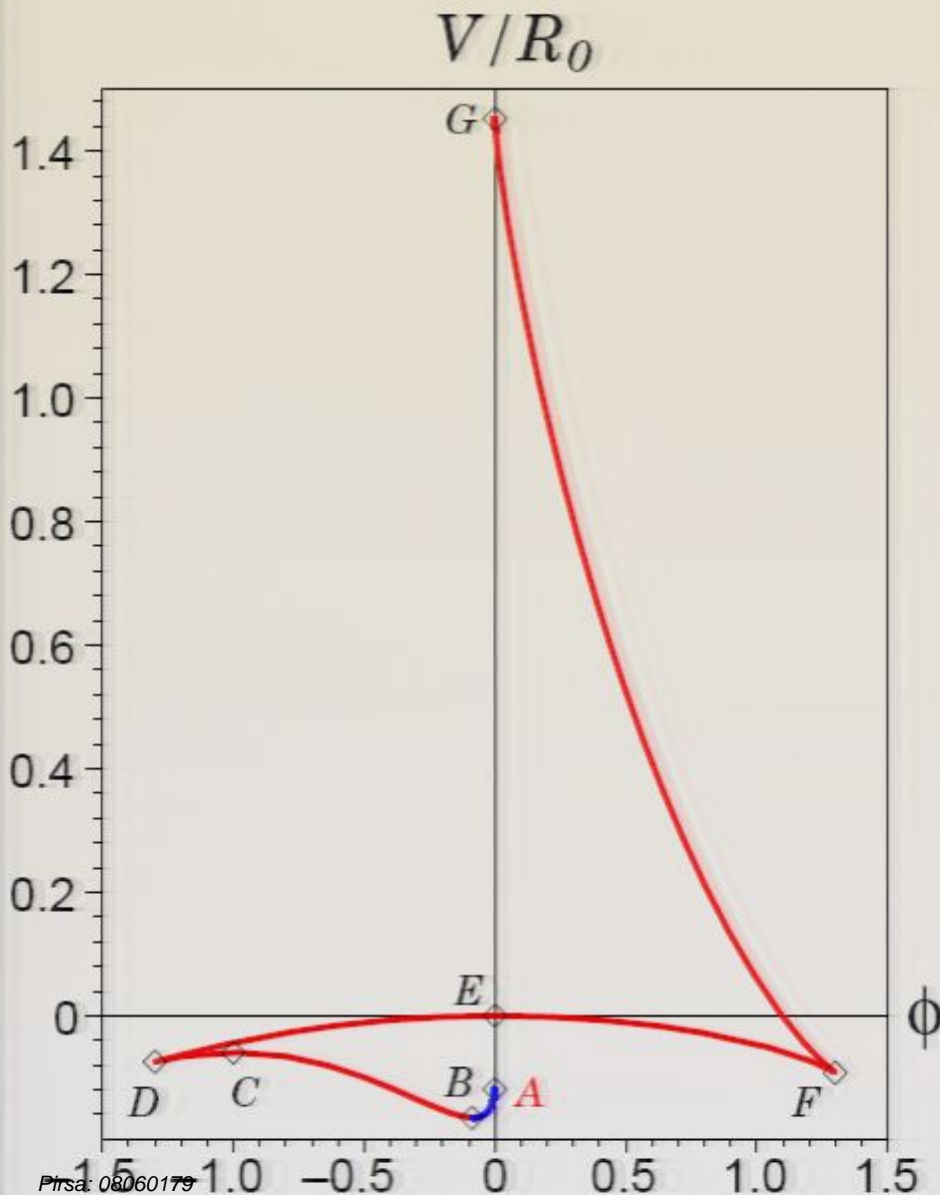
$$\phi = -\frac{2\lambda R}{(1 + R^2)^2}$$

A singularity $(R = +\infty)$

B stable dS min $(\; f' = 0 \;)$

C unstable dS max $(\; f' = 0 \;)$

D critical point $(\; f'' = 0 \;)$

E flat spacetime $(\; f' = 0 \;)$

F critical point $(\; f'' = 0 \;)$

G singularity $(R = -\infty)$

DISAPPEARING COSMOLOGICAL CONSTANT MODEL

V/R_0
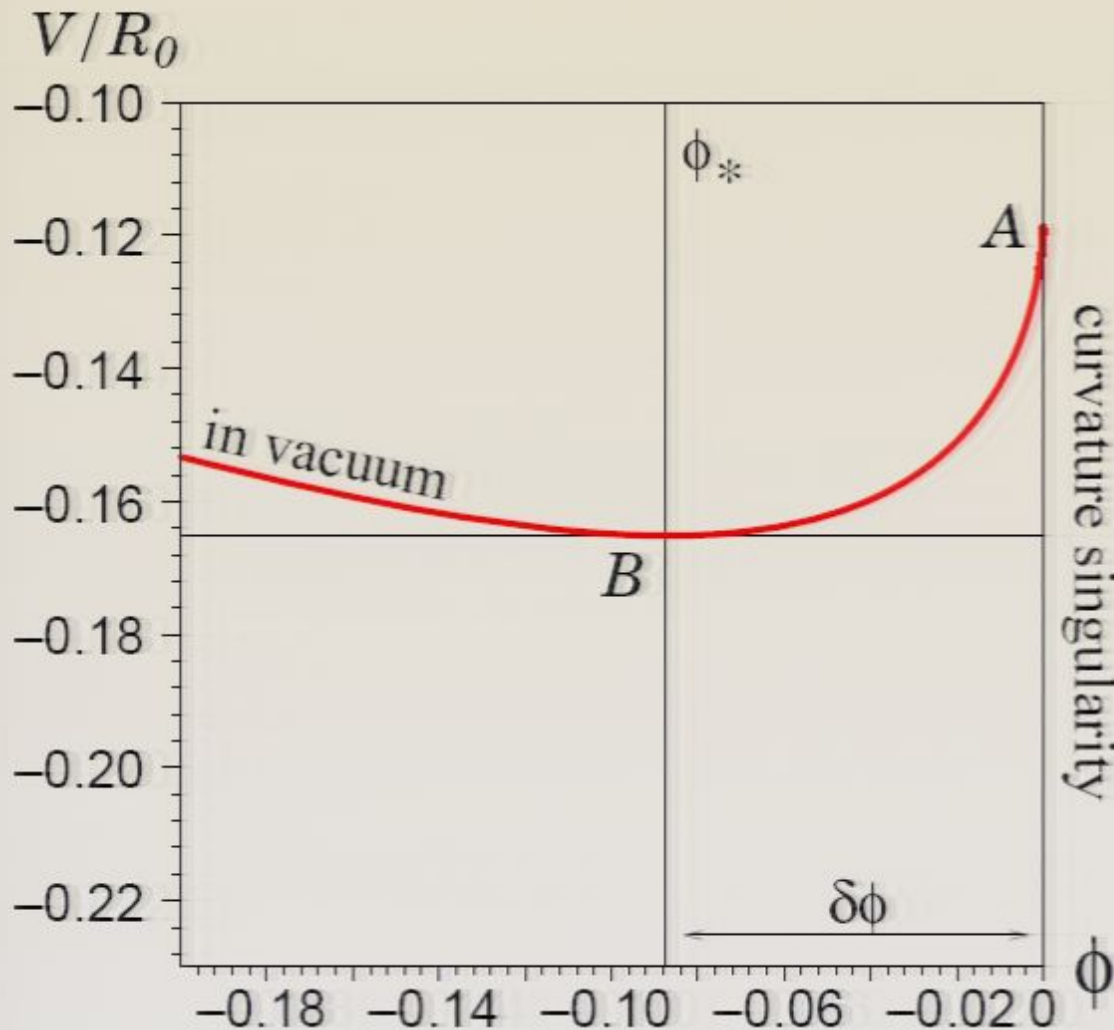
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SINGULARITY IS FINITE DISTANCE AWAY!

V/R_0

ϕ_*

A

in vacuum

B

curvature singularity

$\delta\phi$

ϕ

in large R limit:

$$f(R) = R + \Lambda + \frac{1}{R^\alpha} \sum_{n=0}^{\infty} \frac{\mu_n}{R^n}$$

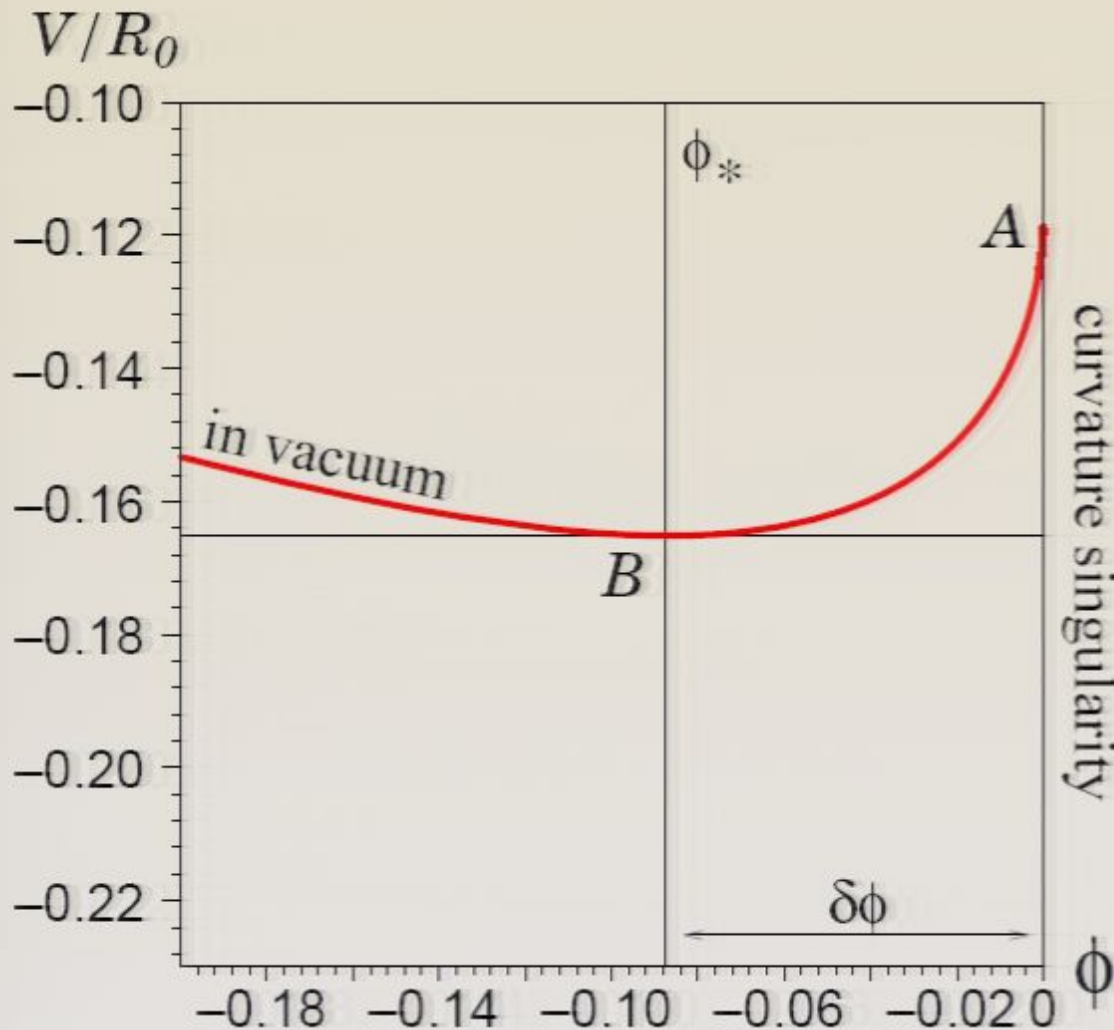$$\phi \equiv f' - 1 \simeq -\frac{\alpha\mu_0}{R^{\alpha+1}}$$

$$\frac{dV}{dR} \simeq \frac{Rf''}{3} = \frac{\alpha(\alpha+1)\mu_0}{3\,R^{\alpha+1}}$$

weak power-law singularity:

$$V(\phi) \simeq \text{const} - \frac{(\alpha+1)\mu_0}{3|\alpha\mu_0|^\gamma}|\phi|^\gamma$$

$$\gamma = \frac{\alpha}{\alpha+1}$$

$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

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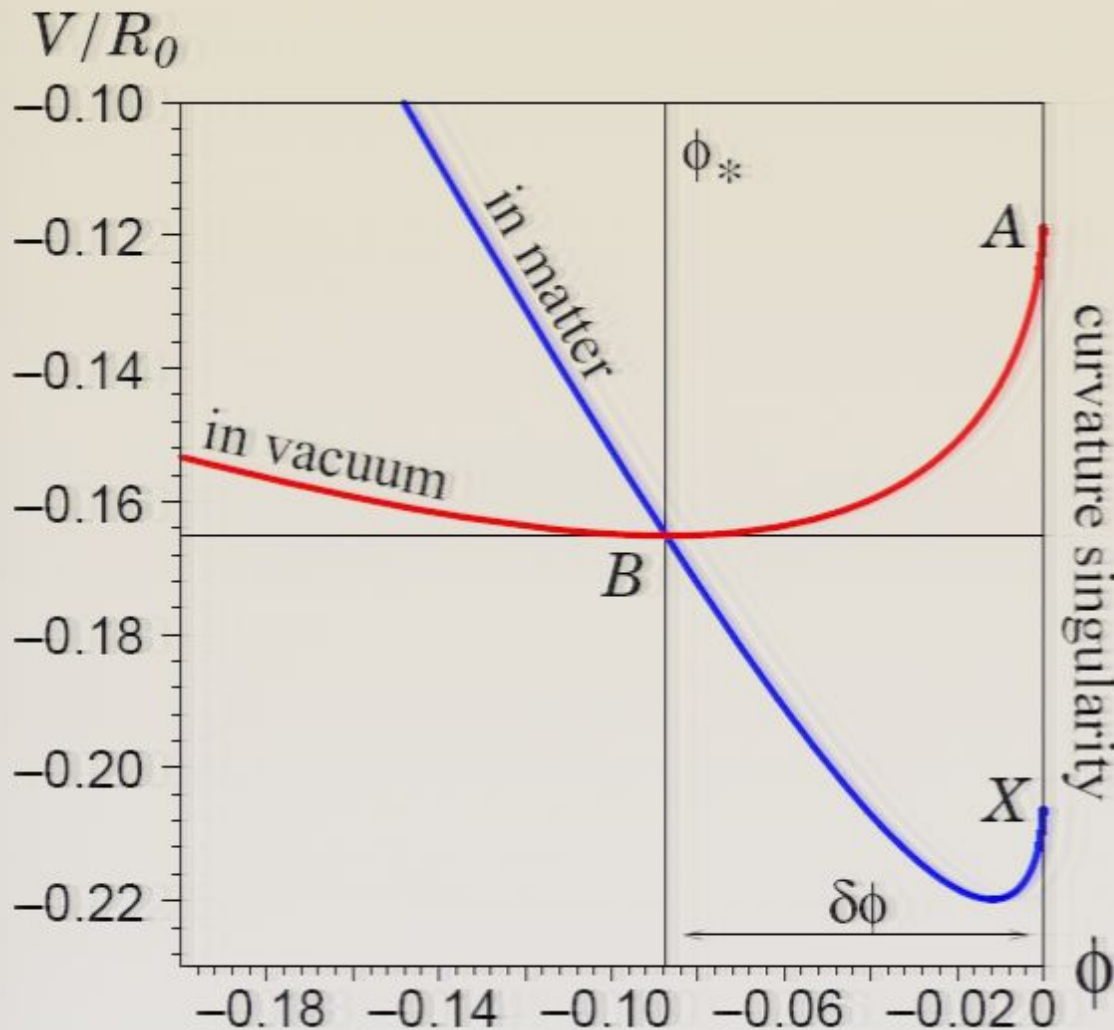$$\phi \equiv f' - 1 \simeq -\frac{\alpha \mu_0}{R^{\alpha+1}}$$

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COSMOLOGY IN F(R) GRAVITY

- Homogeneous flat cosmology is described by FRW metric:

$$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$$

- Scalar degree of freedom looks as usual (albeit with a force term):

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \mathscr{F}$$

- What about Friedman equation? It looks strange...

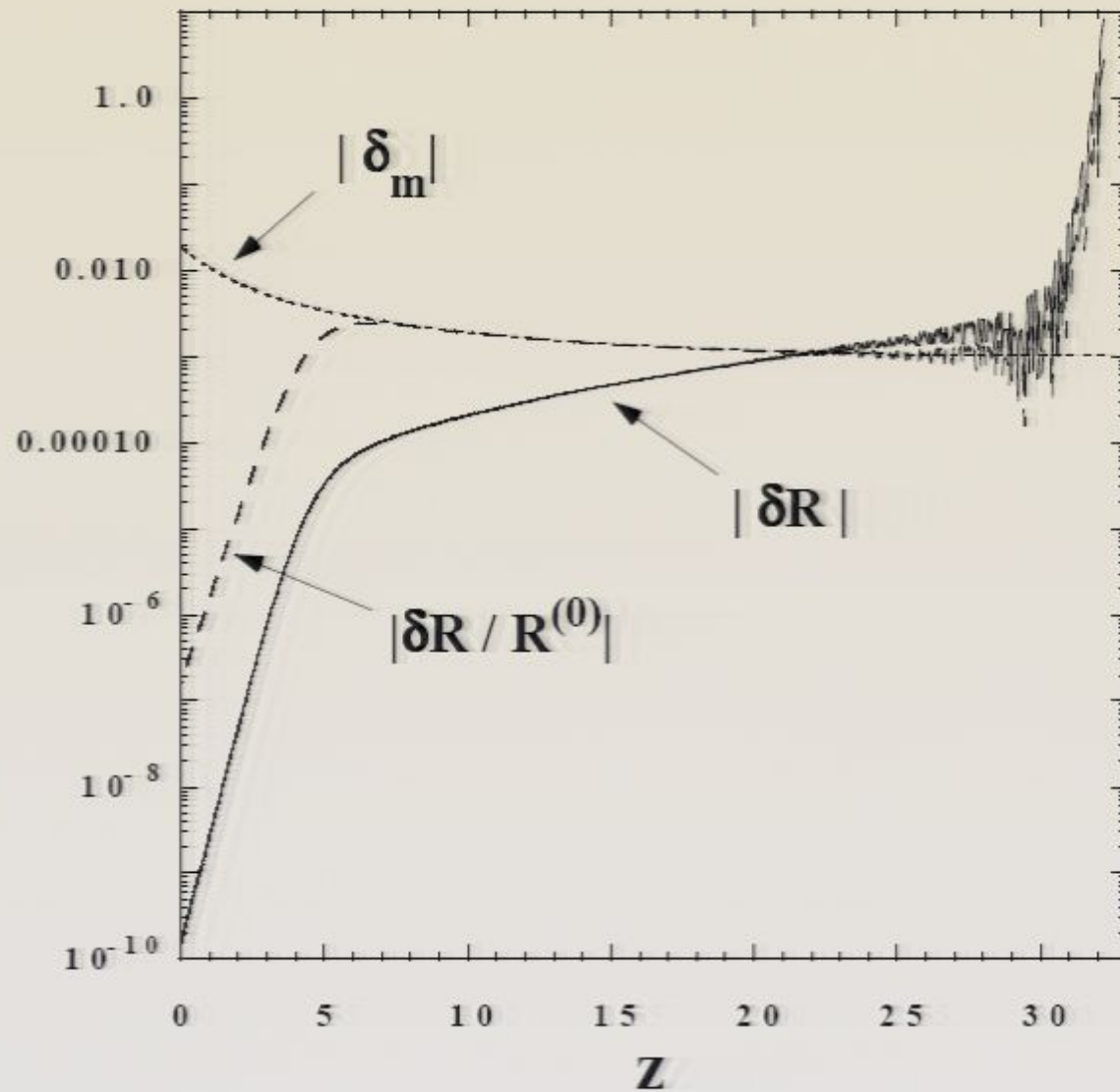$$3H(f')\dot{} - 3\frac{\ddot{a}}{a}f' + \frac{1}{2}f = 8\pi G\rho$$

- ... but it isn't! Eliminating $\ddot{a}$ in favor of $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ does the trick:

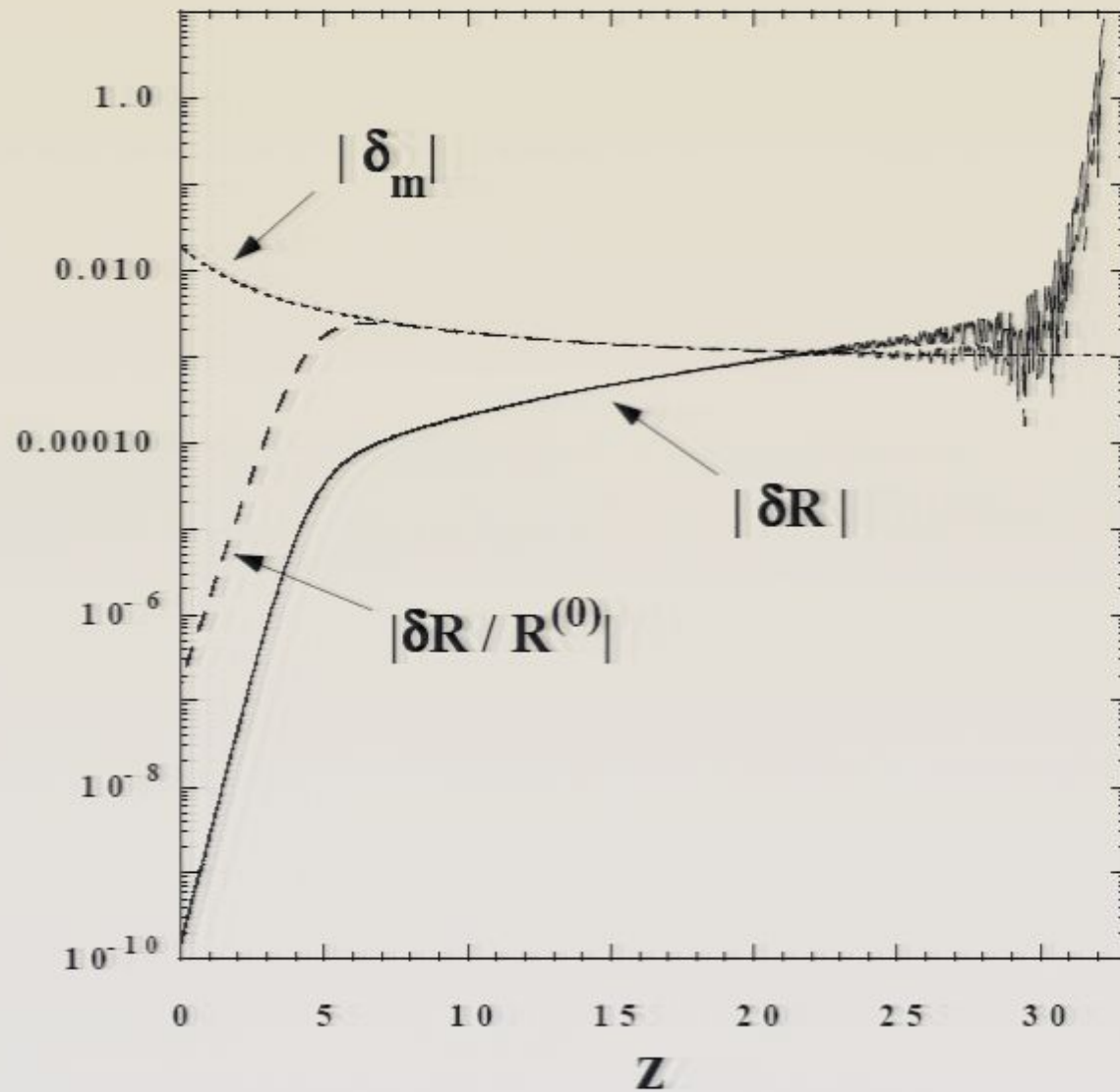$$H^2 + (\ln f')\dot{}\,H + \frac{1}{6}\frac{f - f'R}{f'} = \frac{8\pi G}{3f'}\rho$$

NOW WE UNDERSTAND WHAT'S GOING ON HERE!

Tsujikawa [0709.1391]

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BIGGER PROBLEM: SINGULAR COMPACT OBJECTS!

ϕ

curvature singularity

$$\Delta\phi = -\frac{8\pi}{3}G\rho \quad \text{[f(R) degree of freedom]}$$

$$\Delta\Phi = 4\pi G\rho \quad \text{[Newtonian potential]}$$

$$\text{need } \delta\phi \gtrsim \frac{1}{3} \text{ separating UV from IR}$$

asymptotic curvature

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