Title: The Wheeler-De Witt equation forÂ brane gravity

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Abstract: We consider the gravity in the system consisting of the BPS D3-brane embedded in the flat background geometry, produced by the solutions of the supergravity. The effective action for this system is represented by the sum of the Hilbert-Einstein and DBI actions. We derive the Wheeler-De Witt equation for this system and obtain analytical solutions in some special cases. We also calculate tunneling probability from Planckian size of D3-brane to the classical regime. This paper appeared in Phys. Rev. D 77, 066017 (2008)

The Wheeler-De Witt equation for brane gravity

Based on Phys. Rev. D **77**, 066017 (2008)

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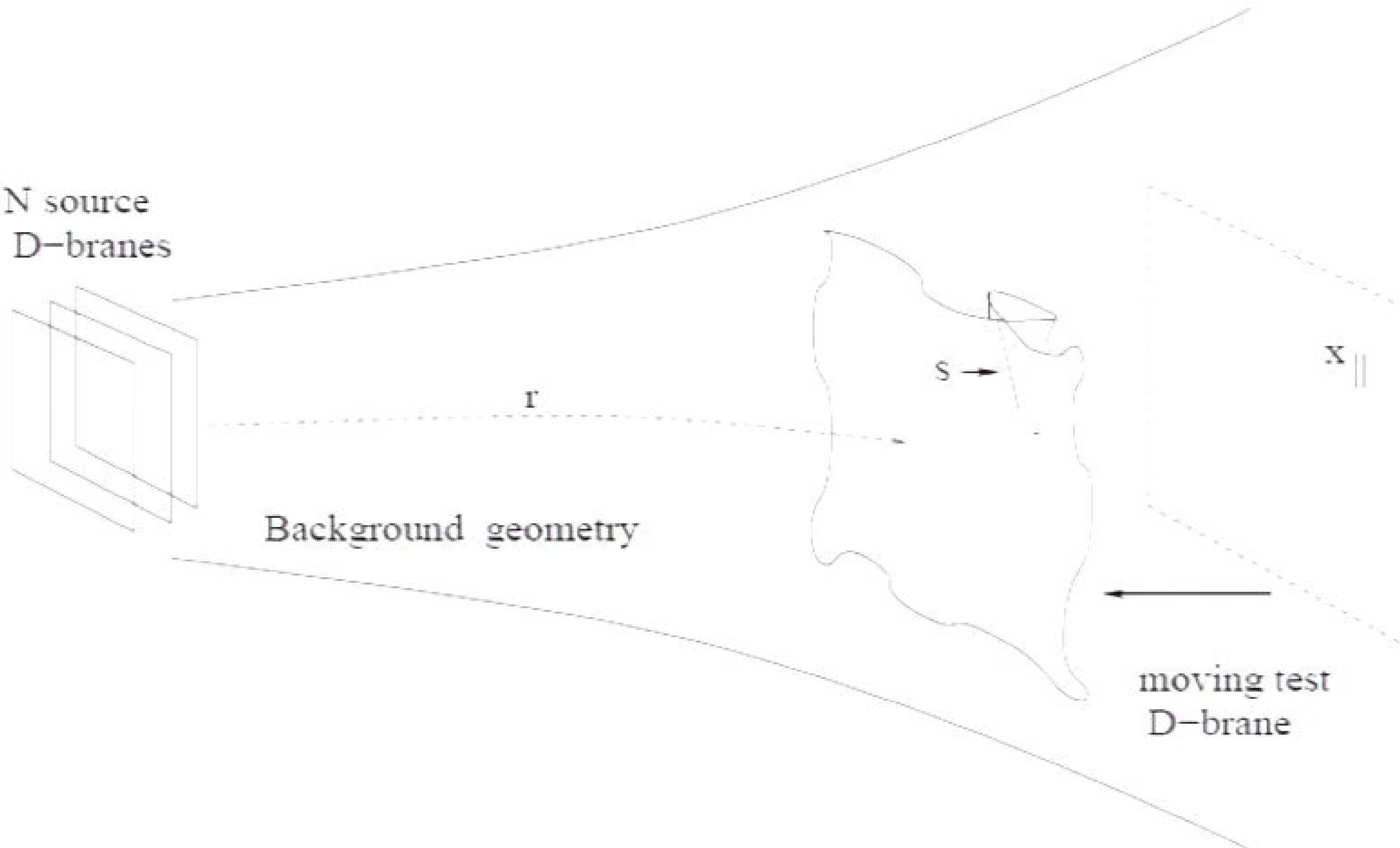
Outlook

- Classical action for D3-brane with gravity
- The Wheeler-de Witt equation for D3-brane
- The wave function
- The probability of tunneling
- Conclusions



Motion of a probe D-brane in a background produced by N D-branes

DBI action for a D3-brane

$$S_3 = -T_3 \int d^4x e^{-\phi} \sqrt{-\det(\gamma_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} + B_{\alpha\beta})}$$

Equivalent form:

$$S_3' = -\frac{\Lambda T_3}{2} \int d^4x e^{-\phi} \sqrt{-\det\left(h_{\alpha\beta}\right)} \left[h^{\alpha\beta}\gamma_{\alpha\beta} - 2\Lambda\right]$$

Gravity on the D3-brane

$$S = \frac{m_P^2}{2}\int d^4x\sqrt{-\det\left(h_{\alpha\beta}\right)}R\left(h\right)-\frac{\Lambda T_3}{2}\int d^4x e^{-\phi}\sqrt{-\det\left(h_{\alpha\beta}\right)}\left[h^{\alpha\beta}\gamma_{\alpha\beta}-2\right.$$

ADM construction: $\mathbf{R}^1 \times \Sigma_3$

FRW metric: $$ds^2 \;=\; -dt^2 \;+\; a^2(t)\;\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\right]\;.$$

This leads to (k=1):

$$S = 6\pi^2 m_P^2\int_{\mathbf{R}^1} dta^3N\left[\frac{1}{a^2}-\left(\frac{\dot{a}}{Na}\right)^2\right]+6\pi^2\Lambda T_3\int_{\mathbf{R}^1} dta^3e^{-\phi}N\left[\frac{\gamma_{00}}{N^2}+2\Lambda\right]$$
$$-\frac{\Lambda T_3}{2}\int_{\mathbf{R}^1} dta\int_{S^3} d\mu e^{-\phi}NTr(\gamma). \qquad (2.6)$$

The low-energy solution in type II B with the Poincare symmetry group:

$$ds_{10}^2 = g_{MN} dX^M dX^N = H_p^{-1/2} \eta_{\mu\nu} dX^\mu dX^\nu + H_p^{1/2} dX_I dX^I.$$

$$e^{2\phi} = H_p^{(3-p)/2}.$$

$$C = \left(H_p^{-1} - 1\right) dX^0 \wedge ... \wedge dX^p.$$

$$H_p = 1 + \frac{N g_s}{r^{7-p}}. \quad \left(r = \left(X_I X^I\right)^{1/2}\right)$$

The embedding field:

$$X(x) = \left(t, x^1, ..., x^3, X^4(t), ..., X^9(t)\right)$$

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$$S = 6\pi^2 \int_{\mathbf{R}^1} dt \left[-m_P^2 a \dot{a}^2 + \Lambda T_3 a^3 H_p^{(p-1)/4} \dot{r}^2 - U(a, r) \right]$$

$$U(a, r) = a^3 \Lambda T_3 \left(H_p^{(p-5)/4} - 2\Lambda \right) - a \left(m_P^2 + \Lambda T_3 \gamma H_p^{(p-5)/4} \right) + J^2 a^{-3} H_p^{(5-p)/4}.$$

Equations of motion:

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\Lambda T_3}{m_P^2} \left(H_p^{(p-5)/4} - 2\Lambda \right) - \frac{1}{a^2} \left(1 + \frac{\Lambda T_3 \gamma}{m_P^2} H_p^{(p-5)/4} \right) + \frac{1}{a^6} \frac{J^2}{m_P^2} H_p^{(5-p)/4}.$$

$$\frac{\ddot{a}}{a} = -\frac{1}{m_P^2} \left[\Lambda T_3 \left(2\Lambda - H_p^{(p-5)/4} \right) + \frac{2}{a^6} J^2 H_p^{(5-p)/4} \right].$$

The energy density: $$\rho = 3\Lambda T_3 \left(H_p^{(p-5)/4} - 2\Lambda \right) + \frac{3}{a^6} J^2 H_p^{(5-p)/4}.$$

The pressure $$p = -3\Lambda T_3 \left(H_p^{(p-5)/4} - 2\Lambda \right) + \frac{3}{a^6} J^2 H_p^{(5-p)/4}$$

The curvature parameter: $$k = 1 + \frac{\Lambda T_3}{m_P^2} H_p^{(p-5)/4}.$$

The state equation:

$$w = \frac{p}{\rho} = -\left(1 - \frac{2J^2 H_p^{(5-p)/4}}{\Lambda T_3 \left(H_p^{(p-5)/4} - 2\Lambda \right) a^6 + J^2 H_p^{(5-p)/4}} \right)$$

If $a \to \infty$, then:

$$w = \frac{p}{\rho} = -1.$$

The Wheeler-De Witt equation:

$$\left[-\frac{\hbar^2}{2}\frac{1}{\sqrt{-G}}\partial_\Theta\left(\sqrt{-G}G^{\Theta\Pi}\partial_\Pi\right)+U\left(Q\right)\right]\Psi\left(Q\right)=0.$$

where: $Q=\left(Q_1,\ldots,Q_N\right)$ and $G_{\Theta\Pi}$ is a metric on the superspace.

In the considered system Q=*(a,r)*, so we get:

$$\left[\frac{\partial^2}{\partial a^2}-\frac{\gamma}{a}\frac{\partial}{\partial a}+\frac{\delta}{a^2}\right]\Psi+$$

$$-\frac{m_P^2}{\Lambda T_3 a^2 H_p^{(p-1)/4}}\left[\frac{\partial^2}{\partial r^2}+\frac{\mu\left(1-p\right)}{4}\frac{H_p'}{H_p}\frac{\partial}{\partial r}+\frac{\nu\left(p-1\right)}{4H_p}\left(\frac{p+3}{4}\frac{H_p'}{H_p}-H_p''\right)\right]\Psi+\frac{2m_P^2}{\hbar^2}U_{eff}\left(a,r\right)\Psi=0.$$

The effective potential:

$$U_{eff}\left(a,r\right)=a^4\Lambda T_3\left(H_p^{(p-5)/4}-2\Lambda\right)-a^2\left(m_P^2+\Lambda T_3\gamma H_p^{(p-5)/4}\right)+J^2a^{-2}H_p^{(5-p)/4}.$$

In the fixed possition *r* of the D3-brane we get:

$$\left[\frac{d^2}{da^2} - \frac{\gamma}{a}\frac{d}{da} - \frac{\eta}{a^2} - \frac{2m_P^4 k}{\hbar^2}a^2 + \frac{2m_P^2\lambda}{3\hbar^2}a^4\right]\Psi = 0.$$

where: $$\lambda = 3\Lambda T_3\left(H_p^{(p-5)/4} - 2\Lambda\right).$$

By the change of variables $\Psi(z) = z^s F(z)$ we have:

$$s = (1+\gamma)/4$$

$$\frac{d^2F}{dz^2} + \frac{1}{z}\frac{dF}{dz} + \left(1 - \frac{n^2}{z^2} + mz\right)F = 0 \qquad \text{(S.E.)}$$

$$z = -ia^2\frac{m_P^2}{\hbar}\sqrt{\frac{k}{2}}, \qquad m = -i\cdot\frac{\hbar\lambda}{3m_P^4 k}\sqrt{\frac{2}{k}}.$$

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Solutions

1. Comological constant λ=0 (*m=0*)

$$\Psi\left(a;r_0\right)=a^{(1+\gamma)/2}\left[EI_n\left(\frac{a^2}{l_{Pl}^2}\sqrt{\hbar^2k/2}\right)+FK_n\left(\frac{a^2}{l_{Pl}^2}\sqrt{\hbar^2k/2}\right)\right]$$

Physical interpretation: instantons and the decay of a fallse vacuum. In the world-volum of D3 –brane there exists a matter with the state equation: *w=+1* .

2. When the parameter *n=0* the function

$$\Psi\left(a;r_0\right)=\frac{e^{i\pi s}}{3}\left(\frac{1}{l_{Pl}^2}\sqrt{\frac{\hbar^2k}{2}}\right)^s a^{2s}\sqrt{1-\frac{\lambda l_{Pl}^4}{3\hbar^2k}\frac{a^2}{l_{Pl}^2}}\times$$

$$\times\left[AI_{-1/3}\left(\frac{4}{l_{Pl}^4\lambda}\left(\frac{\hbar^2k}{2}\right)^{3/2}\left(1-\frac{\lambda l_{Pl}^4}{3\hbar^2k}\frac{a^2}{l_{Pl}^2}\right)^{3/2}\right)+BI_{1/3}\left(\frac{4}{l_{Pl}^4\lambda}\left(\frac{\hbar^2k}{2}\right)^{3/2}\left(1-\frac{\lambda l_{Pl}^4}{3\hbar^2k}\frac{a^2}{l_{Pl}^2}\right)^{3/2}\right)\right] \quad (3.19)$$

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The probability of tunneling

The Schrödinger equation (S.E.) takes the form:

$$\frac{d^2F}{dz^2} - V(z)\,F = 0.$$

where the potential V for real variable y reads:

$$V(y) = -1 - \frac{n^2}{y^2} + xn^2\left(\frac{2}{\hbar^2 k}\right)^{3/2} y$$

and $y = -\frac{a^2}{l_{Pl}^2}\sqrt{\hbar^2 k/2} < 0$, $x = l_{Pl}^4\lambda/\left(6n^2\right)$

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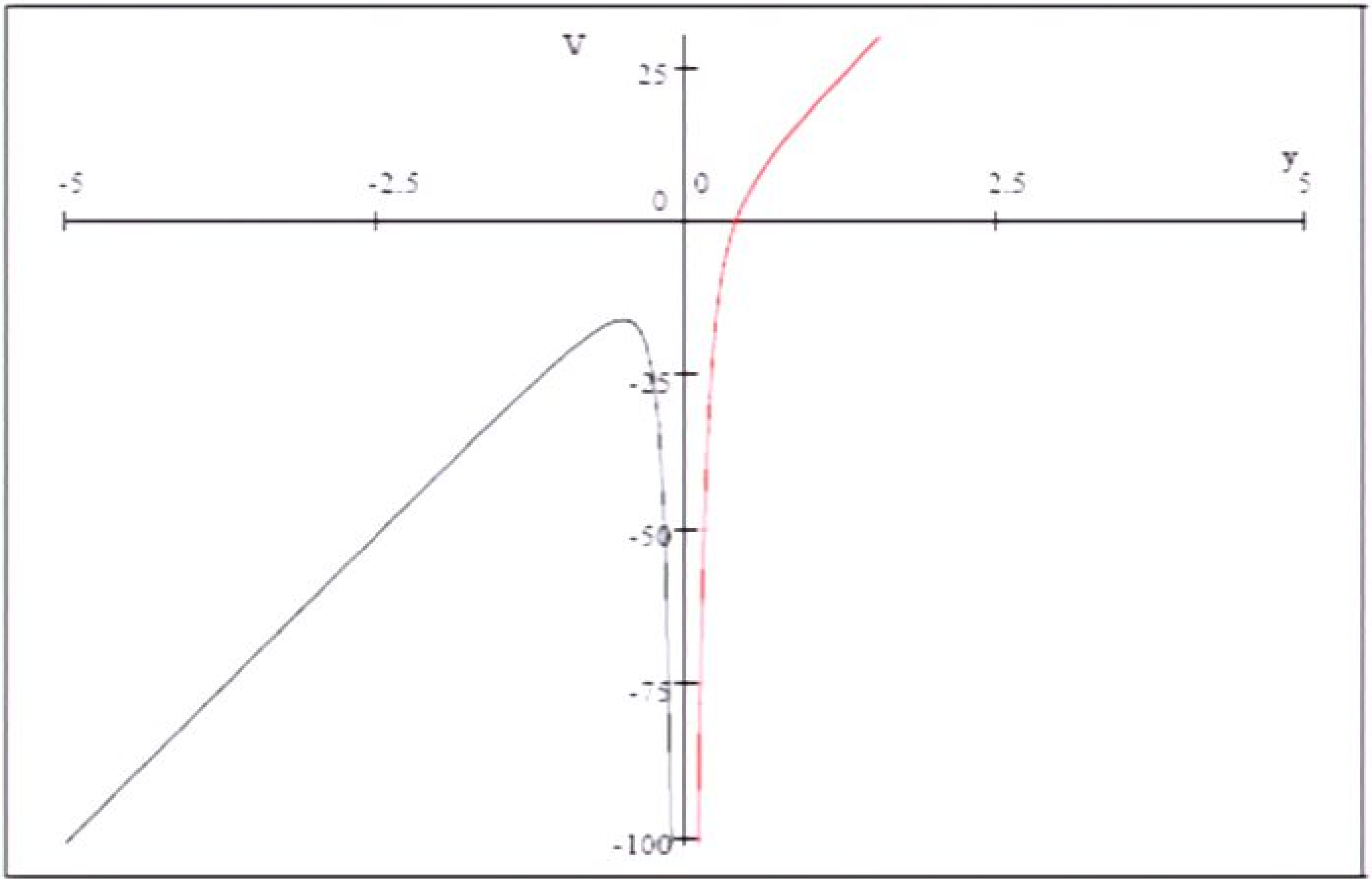

where

for :

$$V_{\max} = -1 - 3n^2 \left(\frac{x}{2}\right)^{2/3} \frac{2}{\hbar^2 k}.$$

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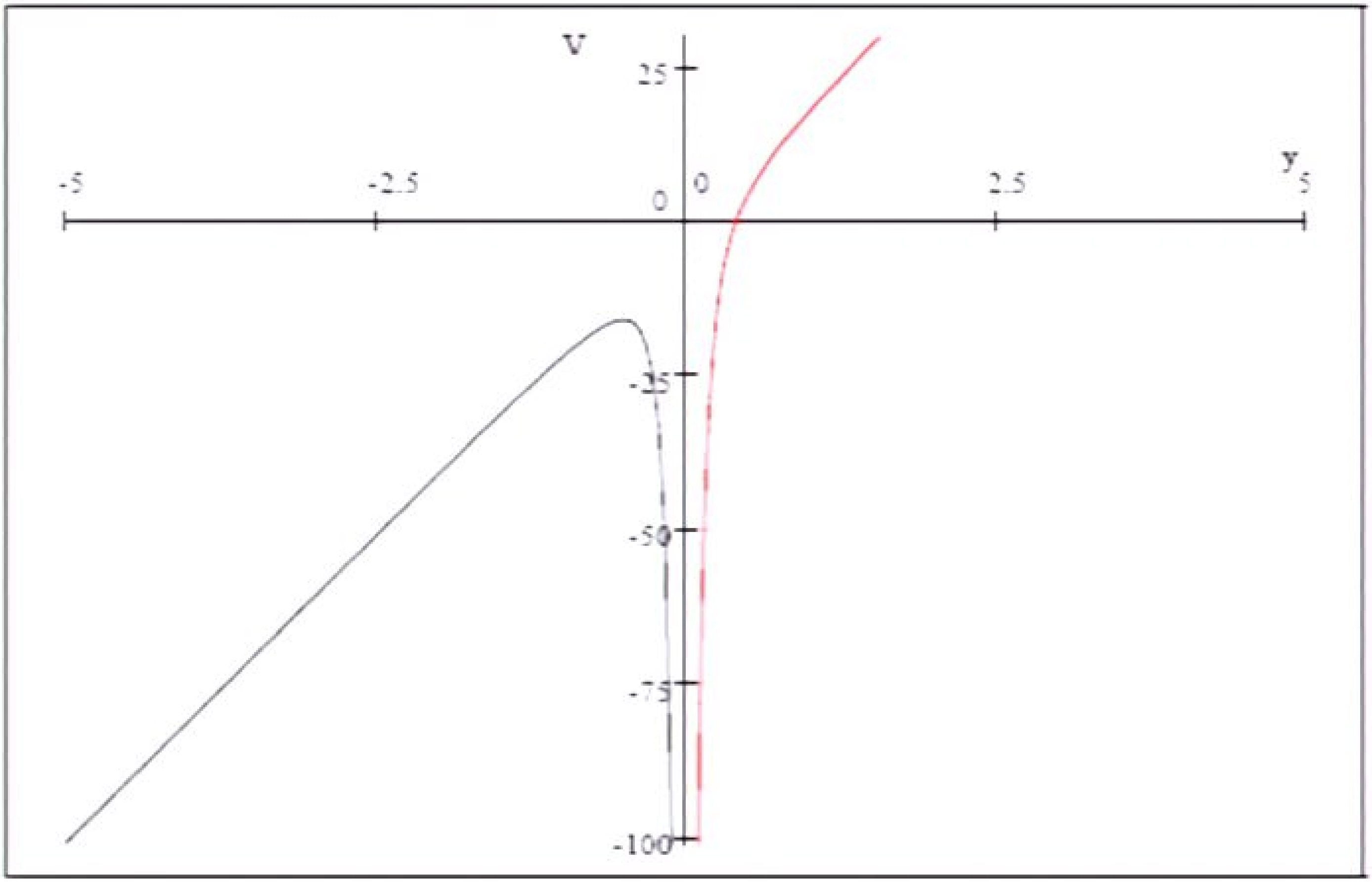

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The tunneling probability in the quasiclassical approximation is given by :

$$\Gamma \simeq \exp\left[-\frac{2}{\hbar}\int_{y_2}^{y_3}\sqrt{-E+V(y)}dy\right]$$

The limits of the integration are solutions of: $E - V(y) = 0.$

$y_3 < y_2 < 0.$

The bottom limit corresponding to the Planck length, is:

$$y_{Pl} = -\hbar\sqrt{k/2}\ .$$

So: $V(y_{Pl}) = -1 - 2n^2\frac{1+x}{\hbar^2 k}.$ and: $V_{\max} > V(y_{Pl}).$

The upper limit is: $$y_3 = -\frac{1}{2x}\sqrt{\frac{\hbar^2 k}{2}}\left(1+\sqrt{1+4x}\right)$$

The barrier width is: $$b = \sqrt{\frac{\hbar^2 k}{2}}\left(-1+\frac{1+\sqrt{1+4x}}{2x}\right)$$

Let I denotes the exponent in Γ :

$$I = \int_{y_2}^{y_3} \sqrt{-E + V(y)}dy$$

thus we get by the integration:

$$I(x,n) = n\frac{1+x}{6x(1+4x)^{1/4}}\left[4\sqrt{1+4x}E(\kappa) + \left(1-\sqrt{1+4x}\right)K(\kappa)\right] +$$

$$+n\frac{1-\sqrt{1+4x}}{(1+4x)^{1/4}}\Pi(c,\kappa).$$

where:

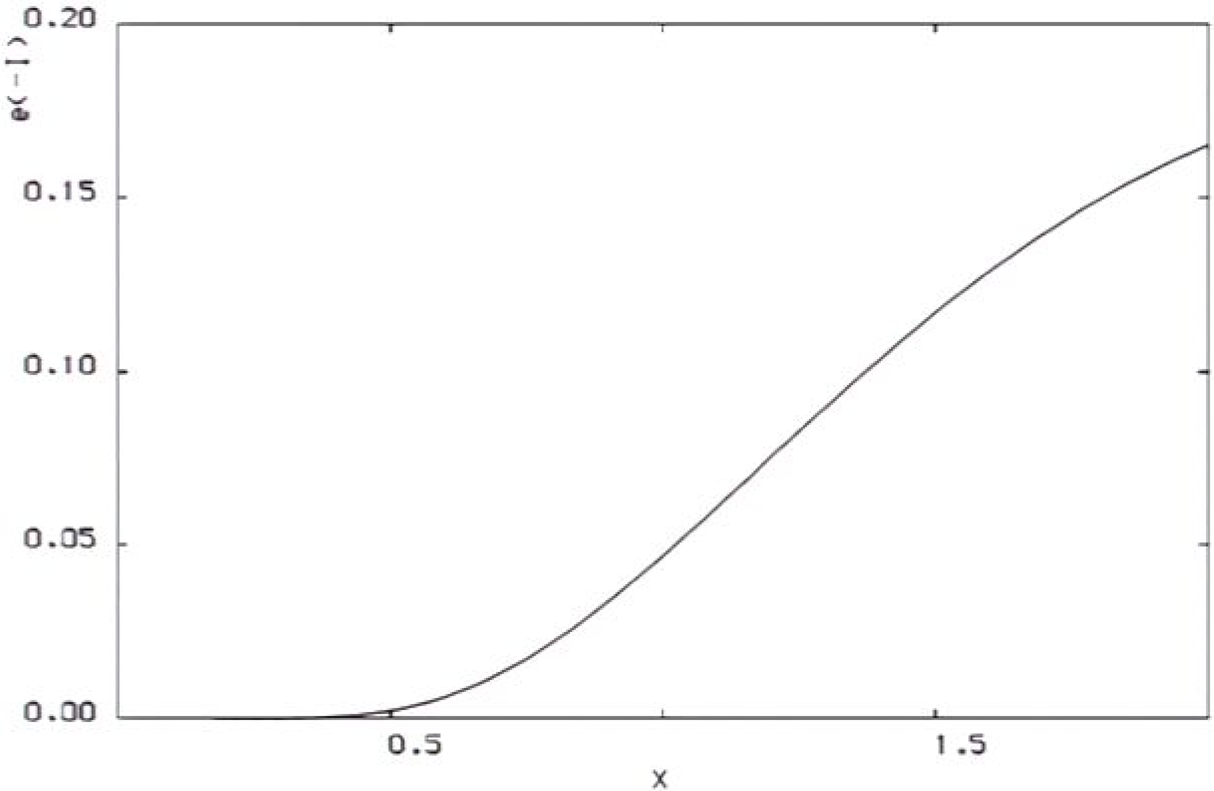
$$\kappa = \frac{y_{Pl} - y_3}{y_1 - y_3} = \frac{1}{2}\left(1 + \frac{1-2x}{\sqrt{1+4x}}\right).$$

$$c = -\frac{y_{Pl} - y_3}{y_3} = \frac{1}{2}\left(3 - \sqrt{1+4x}\right) > 0.$$

$$0 < \kappa^2 < 1 \text{ and } 0 < c < 1 \text{ for } x \in (0,2).$$

Thus we obtain the probability: $\Gamma(x, n) = \exp\left[-\frac{2}{\hbar} I(x, n)\right]. \quad (n = n(r))$

The plot of the probability for fixed n is given :



For x=2 $\Gamma(2, n) = \exp\left(-\frac{\pi l_{PL}^2}{4\hbar}\sqrt{\frac{\lambda}{6}}\right)$

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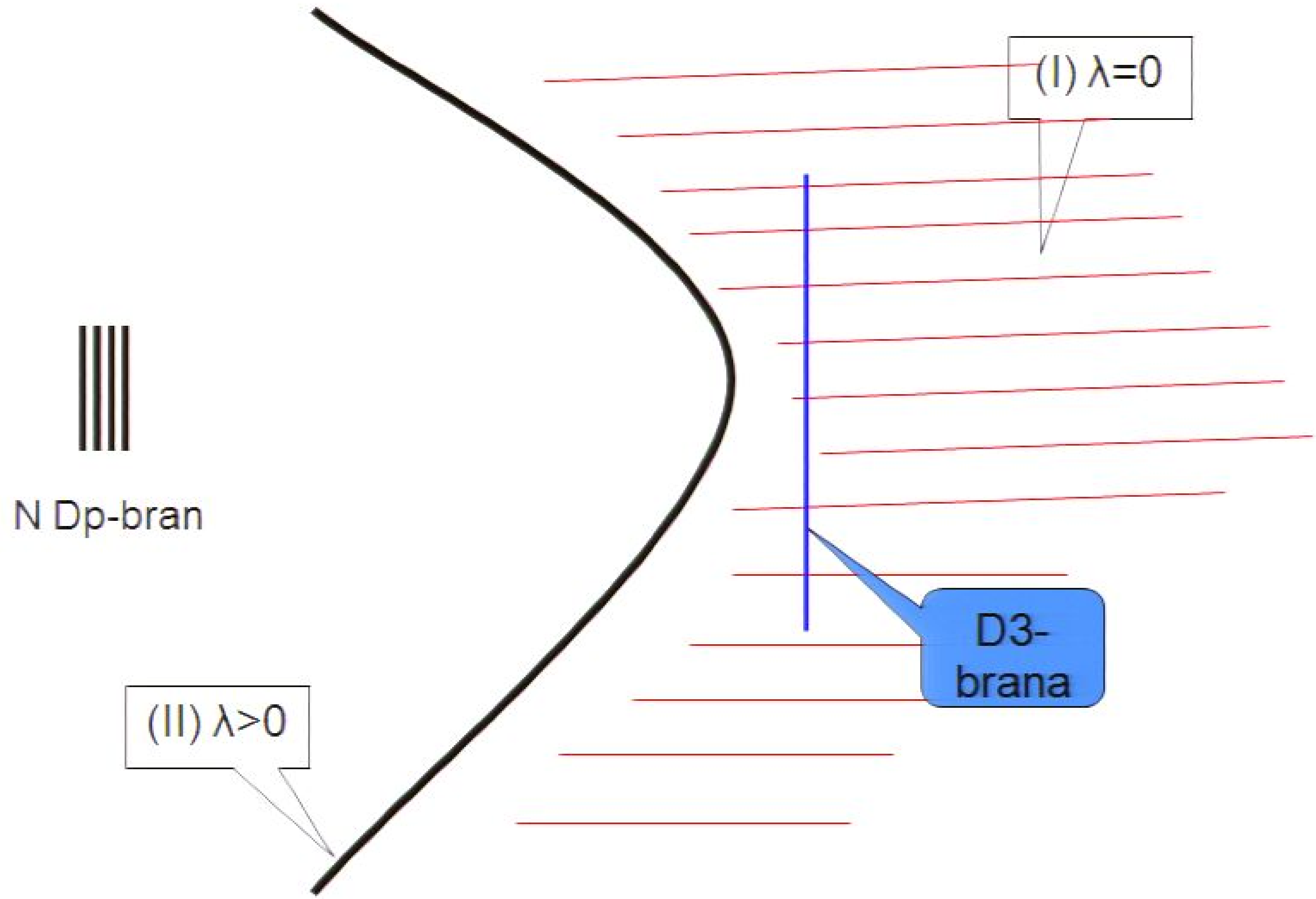
- In the calssical regime we get accelerated expansions of the world-volume for the big scale factor. The state equation corresponds to the dark energy.
- Wave function of the gravity on the world-volume depends on the position of the D3-brane in the 10-dimensional background.
- The explicit forms of the wave function in the two region (I) and (II) in the10-dimensional spacetime are given. In the region (I) the wave function corresponds to the decay of the false vacuum with the vanishing cosmological constant. In the region (II) the wave function corresponds to the Hartle-Hawking function with $\lambda>0$. Moreover the region (II) is the boundary of (I).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N Dp-bran
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(II) λ>0

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$$w = \frac{p}{\rho} = -1.$$

The low-energy solution in type II B with the Poincare symmetry group:

$$ds_{10}^2 = g_{MN}dX^M dX^N = H_p^{-1/2}\eta_{\mu\nu}dX^\mu dX^\nu + H_p^{1/2}dX_I dX^I.$$

$$e^{2\phi} = H_p^{(3-p)/2}.$$

$$C = \left(H_p^{-1} - 1\right) dX^0 \wedge ... \wedge dX^p.$$

$$H_p = 1 + \frac{Ng_s}{r^{7-p}}. \quad \left(r = \left(X_I X^I\right)^{1/2}\right)$$

The embedding field:

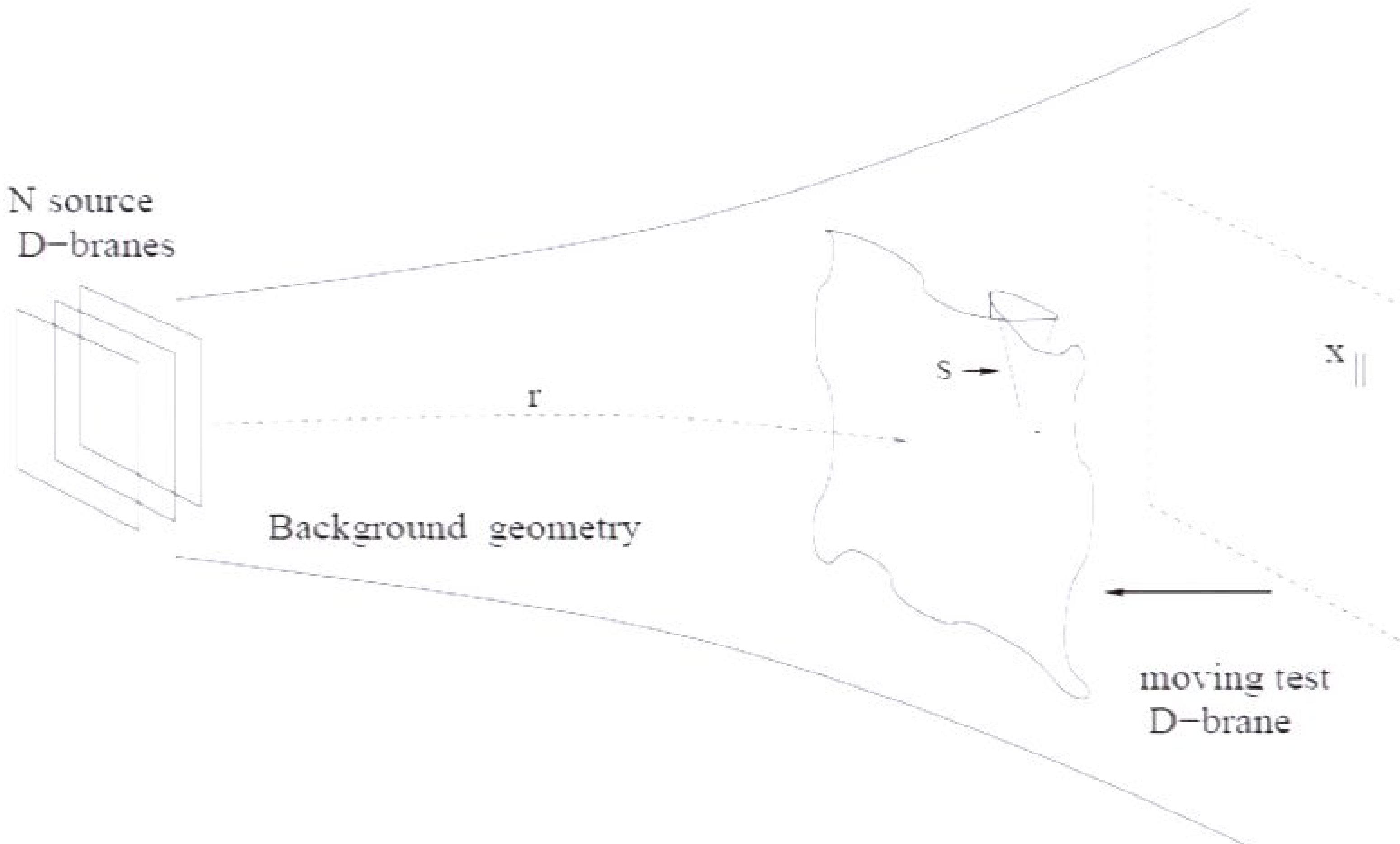
$$X(x) = \left(t, x^1, ..., x^3, X^4(t), ..., X^9(t)\right)$$

DBI action for a D3-brane

$$S_3 = -T_3 \int d^4x e^{-\phi} \sqrt{-\det(\gamma_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} + B_{\alpha\beta})}$$

Equivalent form:

$$S_3' = -\frac{\Lambda T_3}{2} \int d^4x e^{-\phi} \sqrt{-\det\left(h_{\alpha\beta}\right)} \left[h^{\alpha\beta}\gamma_{\alpha\beta} - 2\Lambda\right]$$



Motion of a probe D-brane in a background produced by N D-branes