Title: Dark Energy, Induced Gravity and Broken Scale Invariance.

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Abstract: We study the cosmological evolution of an induced gravity model with a self-interacting scalar field $sigma$ and in the presence of matter and radiation. Such model leads to Einstein Gravity plus a cosmological constant as a stable attractor among homogeneous cosmologies and is therefore a viable dark-energy (DE) model for a wide range of scalar field initial conditions and values for its positive $gamma$ coupling to the Ricci curvature $gamma sigma^{2}R$.

Dark Energy, Induced Gravity and Broken Scale Invariance

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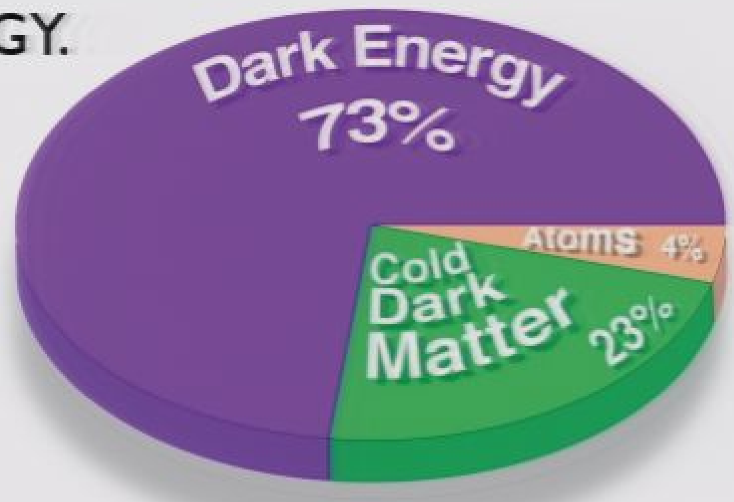
Introduction

- Cosmological observations indicate that the Universe has entered a phase of accelerated expansion at a redshift $z \approx 0.5$ suggesting that it is currently dominated by some form of DARK ENERGY.



- Currently ΛCDM is in agreement with data and has the minimum number of parameters. It is therefore preferred to many other "dynamical" alternatives:
  - Modified gravity models
  - Quintessence models
  - Stringy inspired models
  -

Scalar Tensor Quintessence

- In general one has (in the Jordan Frame)

$$\mathcal{L}_{ST} = \sqrt{-g}\left[\frac{1}{2}\left(F(\phi)R - Z(\phi)\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) - U(\phi)\right]$$

- Scalar-Tensor Quintessence has non minimal coupling and the parameters can be constrained partially by Solar System observations.
- These models have a richer phenomenology and some of them violate the NEC $\rho + p \geq 0$.
- NO DIRECT INTERACTION with ordinary matter in the Jordan Frame, which is the physical one.
- In ST theories the meaning of the Newton Constant is not unique

$$G_N = \frac{1}{8\pi F}$$

Effective cosmological Newton constant

$$G_{eff} = G_N \frac{F + 2\left(\frac{dF}{d\phi}\right)^2}{F + \frac{3}{2}\left(\frac{dF}{d\phi}\right)^2}$$

Effective Newton constant measured by Cavendish experiments

$U(\phi)$ negligible

The equation of state

- A natural definition of the equation of state in ST quintessence model does not exist. A consistent definition can be given in Einstein Gravity framework:

$$\begin{cases} \rho_{\rm DE} \equiv 3\gamma\sigma_0^2 H^2 - \sum_{i=R,M} \rho_i \\ p_{\rm DE} \equiv -2\gamma\sigma_0^2 \left(\dot{H} + \frac{3}{2}H^2\right) - \frac{\rho_R}{3} \end{cases}$$

- In EG framework one can also define relative energy densities

$$\Omega_R = \frac{\rho_R}{3\gamma\sigma_0^2 H^2}, \ \Omega_M = \frac{\rho_M}{3\gamma\sigma_0^2 H^2} \quad \Omega_{\rm DE} = \frac{\rho_{\rm DE}}{3\gamma\sigma_0^2 H^2}$$

- On using above definition one recovers standard dynamics:

$$H^2 = \frac{1}{3\gamma\sigma_0^2} \sum_i \rho_i\,, \ \dot{\rho}_i = -3H(1+w_i)\rho_i\,, \ (i = \mathrm{R, M, DE})$$

De Sitter Fixed Point

- We consider $Z = 1 \quad F(\sigma) = \gamma\,\sigma^2$
- Check stability of the fixed points of the dynamics (σ_F, H_F^2)

$$4U(\sigma_F) - \sigma_F \frac{dU(\sigma_F)}{d\sigma_F} = 0 \qquad H_F^2 = \frac{U(\sigma_F)}{3\gamma\sigma_F^2}$$ E.o.m.

$$12\gamma H_F^2 - \frac{d^2U(\sigma_F)}{d\sigma_F^2} \leq 0$$ Stability condition

- Different choices for the potential (n even)

$$U(\sigma) = \frac{\lambda}{|n|}\mu^{4-n}\sigma^n + V_0 \qquad \lambda > 0 \qquad [\mu] = \left[V_0^{1/4}\right]$$

- Coleman-Weinberg quantum correction effective potential

$$U(\sigma) = \frac{\lambda}{4}\sigma^4(1 + \alpha\,\ln\frac{\sigma^2}{\sigma_0^2}) + V_0 \qquad \lambda > 0 \qquad n = 4$$

Comparison of Potentials

	$\lvert\sigma_F\rvert$	H_F^2	V_0	Asy. Geo.	Stab ility
$n > 4$	$\mu\left(\frac{n}{n-4}\frac{V_0}{\lambda\mu^4}\right)^{1/n}$	$\frac{\lambda}{12\gamma}\mu^{4-n}\sigma_F^{n-2}$	≥ 0	MK/ DS	Y
$n = 4$	unconstrained	$\frac{\lambda}{12\gamma}\sigma_F^2$	0	DS	Y
$n = 2$	$\frac{2}{\mu}\sqrt{-\frac{V_0}{\lambda}}$	$\frac{\lambda\mu^2}{12\gamma}$	< 0	DS	N
$n < 0$	∞	0	0	MK	Y
CW	$\left(\frac{8V_0}{\lambda\alpha}\right)^{1/4}$	$\frac{\lambda}{12\gamma}\sigma_F^2\left(1+\frac{\alpha}{2}+\alpha\ln\frac{\sigma_F^2}{\sigma_0^2}\right)$	$\geq \frac{\alpha\lambda\sigma_0^4}{8\exp\left(1+\frac{2}{\alpha}\right)}$	MK/ DS	Y

The model I

- The model we consider was introduced in the 1981 by G.Venturi et al.
- In the Jordan Frame:

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(-g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \gamma\sigma^2 R - \frac{\lambda}{2}\sigma^4\right) + \sum_{j=R,M}\mathcal{L}_j$$

Motivations

- The model contains a stable, renormalizable potential
- It contains only 2 dimensionless parameters $\gamma,\ \lambda$
- It has a de Sitter attractor in the future
- Stable against inhomogeneous perturbation near the fixed point

The model II

- It corresponds to an interacting Brans-Dicke model

$$\mathcal{L} = -\frac{\omega}{2\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4\gamma^2}\phi^2 + \frac{\phi}{2}R + \sum_{j=R,M}\mathcal{L}_j$$

with $\omega = \frac{1}{4\gamma} \quad \phi = \sigma^2$

Super-acceleration

- It admits a super-accelerated regime $\dot{H} > 0$

$$\dot{H} = -\frac{1}{\gamma\sigma^2}\left[\sum_{j=R,M}\frac{(1+8\gamma+w_j)}{2(1+6\gamma)}\rho_j + \frac{\dot{\sigma}^2}{2} - 4\gamma H\sigma\dot{\sigma}\right]$$

Dynamics of quintessence I

- Consider the homogeneous mode $\sigma(t)$ on the spatially flat RW background $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2$
- The dynamics of quintessence is that of a scalar field

$$\mathcal{L} = \sqrt{-g}\,(T - V)\,, \quad V = -\frac{\gamma}{2}R\sigma^2 + \frac{\lambda}{4}\sigma^4$$

- The potential has one or two minima depending on

$$R = \frac{1}{\gamma\sigma^2}\left(\lambda\sigma^4 - \dot{\sigma}^2 + \sum_{j=R,M}\frac{\rho_j - 3P_j}{1+6\gamma}\right)$$

- The independent equations can be chosen as

$$H^2 = \sum_{j=R,M}\frac{\rho_j}{3\gamma\sigma^2} + \frac{1}{6\gamma}\frac{\dot{\sigma}^2}{\sigma^2} - 2H\frac{\dot{\sigma}}{\sigma} + \frac{\lambda}{12\gamma}\sigma^2$$ Friedmann Like eq.

$$\frac{d}{dt}\left(a^3\sigma\dot{\sigma}\right) = a^3\sum_{j=R,M}\frac{(\rho_j - 3P_j)}{(1+6\gamma)}$$ Klein-Gordon Like eq.

$$\dot{\rho}_j = -3H\,(\rho_j + P_j)$$ Continuity eq.

Dynamics of quintessence II

de Sitter solutions $\Rightarrow$

$$\begin{cases} \rho_i = 0 \\ a = a_0 \exp(H_0 t) \\ \sigma = \pm\sqrt{12\frac{\gamma}{\lambda}} H_0 \end{cases}$$

- If $\dot{\sigma} \sim 0$ nowadays

$$H^2 = \sum_{j=R,M} \frac{\rho_j}{3\gamma\sigma^2} + \frac{1}{6\gamma}\frac{\dot{\sigma}^2}{\sigma^2} - 2H\frac{\dot{\sigma}}{\sigma} + \frac{\lambda}{12\gamma}\sigma^2$$

$$\frac{\rho_j}{3\gamma\sigma^2} \to \frac{8\pi G_N}{3}\rho_j \qquad \frac{\lambda}{12\gamma}\sigma^2 \to \frac{8\pi G_N}{3}\rho_\Lambda$$

$\Rightarrow$

$$\sigma \sim M_{Pl}$$

$$\frac{\lambda}{\gamma} \ll 1$$

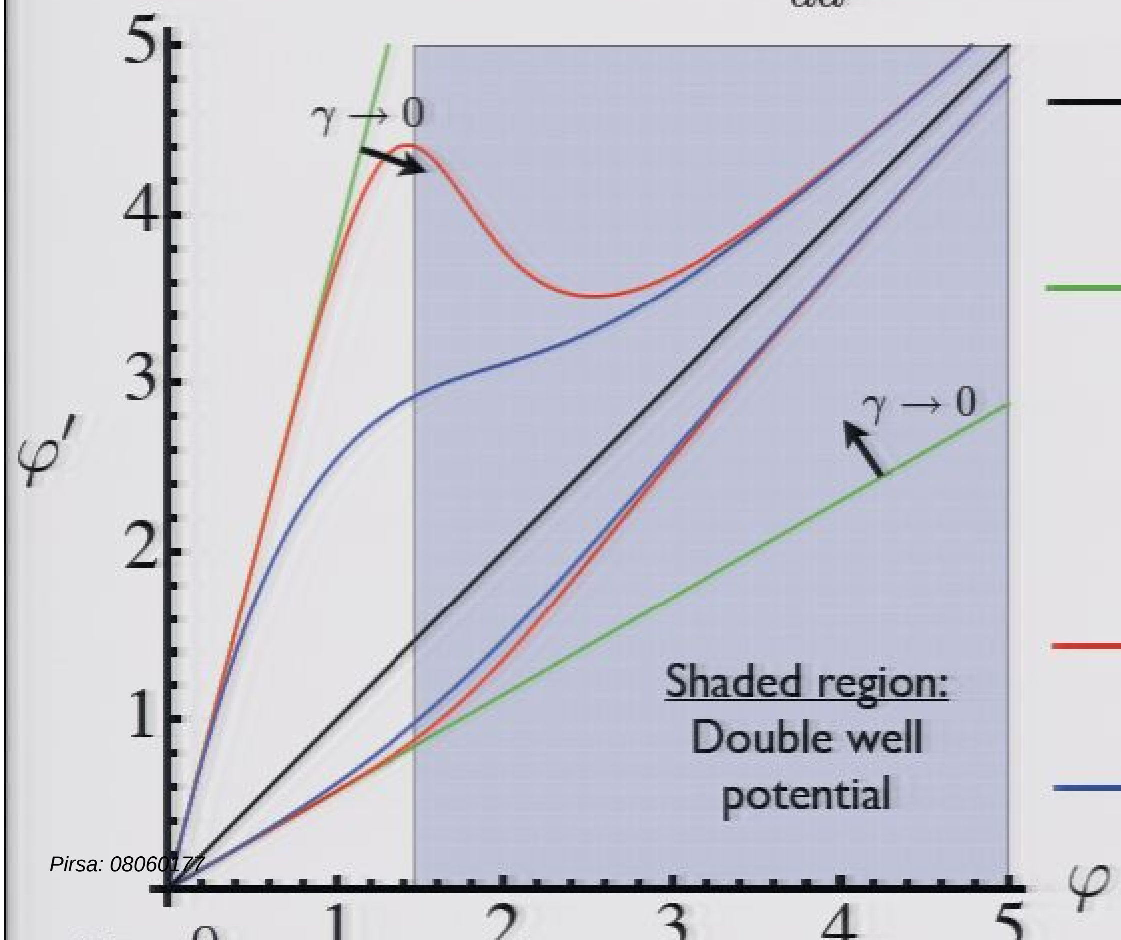
- λ is extremely small (in our calculations $\lambda \leq 10^{-128}$)

Dynamics: $\rho_{\mathrm{M}} = 0$ case

- One can examine phase-space trajectories

$$\gamma\, c_0^2 \left[\varphi^2 - \frac{1}{6\gamma}\left(\varphi' - \varphi\right)^2 + 2\varphi\left(\varphi' - \varphi\right)\right] = \frac{\varphi^4}{3}\left(\frac{\rho_{R,0}}{\varphi^2} + \frac{\lambda}{4}\varphi^2\right)\left(\varphi' - \varphi\right)^2$$

$$\varphi \equiv a\sigma, \quad ' \equiv a\frac{d}{da} \quad c_0 = a^3\sigma\dot{\sigma}, \quad \rho_R \xrightarrow{a\to 1} \rho_{R,0}$$



- (black line) $\sigma = \mathrm{cost}$,constant solution and de Sitter attractor;
- (green line) $\sigma(a) \propto a^{6\gamma\left(1\pm\sqrt{1+\frac{1}{6\gamma}}\right)}$, NOT solutions, phase space boundaries; correspond to $H^2(a) \sim a^{-\left[6+24\gamma\left(1\pm\sqrt{1+\frac{1}{6\gamma}}\right)\right]}$
- (red line) $\rho_{R,0} = 0$ Solutions
- (blue line) $\rho_{R,0} = 5\cdot 10^{-1}$ Solutions

$\gamma = 2\cdot 10^{-1},\ \lambda = 10^{-1},\ c_0 = 1$

Dynamics: matter domination

- No exact analytical predictions can be done...
- Approximating the dynamics: $H^2 \simeq \frac{\rho_M}{3\gamma\sigma^2}\,,\ \frac{d}{dt}\left(a^3\sigma\dot{\sigma}\right) \simeq a^3\frac{\rho_M}{1+6\gamma}$

$$\frac{x''}{x} + 3\frac{x'}{x} + \frac{1}{2}\frac{x'}{x}\left(\frac{\beta'}{\beta} + \frac{x'}{x} - 3\right) = \frac{6\gamma}{6\gamma+1}\beta$$

where: $x = \sqrt{\lambda}\,\sigma^2\,,\ \beta = 1 - \frac{1}{24\gamma}\frac{x'^2}{x^2} + \frac{x'}{x}\,,\ {}' \equiv a\frac{d}{da}$

- The above equation can be solved with the ansatz: $\frac{x'}{x} = \alpha$
- It is possible to verify numerically that x rapidly evolves toward the above solution with:

$$\alpha = \frac{-3-6\gamma+\sqrt{3}\sqrt{3+68\gamma+300\gamma^2}}{7+36\gamma} \xrightarrow{\gamma<<1} 4\gamma + \mathcal{O}\left(\gamma^2\right)$$

Cosmological Evolution

- Consider matter+radiation ⇨ numerical solutions are needed
- More convenient to solve for $x = \lambda^{1/2}\sigma^2, \quad \mathcal{H} = \lambda^{-1/4} H$

β

Evolution FORWARD in time!

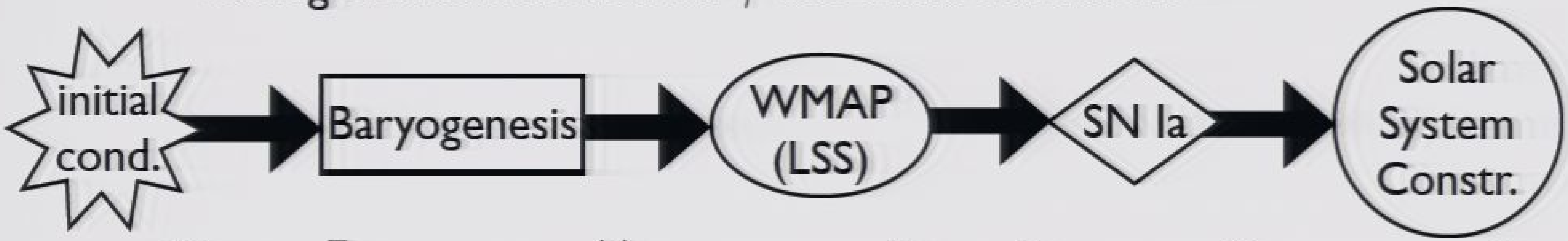
$$\begin{cases} \mathcal{H}^2 \left[1 - \frac{1}{24\gamma}\left(\frac{x'}{x}\right)^2 + \frac{x'}{x} \right] = \frac{\rho_M + \rho_R}{3\gamma x} + \frac{x}{12\gamma} \\ \mathcal{H}^2 x'' + \left[3\mathcal{H}^2 + \frac{1}{2}\left(\mathcal{H}^2\right)' \right] x' = \frac{2\rho_M}{6\gamma + 1} \end{cases}$$

where $' = a\frac{d}{da} \equiv \frac{d}{dN} \rightarrow N = \ln\frac{a}{a_0}, \; a_0 = 1$

- matter and radiation: $\rho_M = \rho_{M,0} e^{-3N}; \quad \rho_R = \rho_{R,0} e^{-4N}$
- Back in time evolution: $N \rightarrow -N = \ln(1+z) \equiv l_z$

Analysis Overview

- COMPARISON OF DYNAMICS WITH OBSERVATIONS
  testing different choices of γ and initial conditions



$z \sim 10^{15}$ $\quad 10^{7} < z < 10^{11}$ $\quad z \simeq 10^{3}$ $\quad 0 < z < 2$ $\quad z = 0$

- INITIAL CONDITION chosen to approximate $\Lambda\mathrm{CDM}$ today;

$$x_i, x_i', \gamma \longrightarrow \boxed{\Delta z \sim 10^{15}} \longrightarrow \begin{array}{c} x_0, x_0' \ll 1 \\ \rho_{DE,0} \simeq x_0^2/4 \end{array}$$

- PARAMETERS are taken from WMAP3 fit with $\Lambda\mathrm{CDM}$ model
  identifying $\rho_{DE,0} \leftrightarrow \rho_{\Lambda,0}$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G_N} = 8.0992\, h^2\, 10^{-47}\mathrm{GeV}^4 \qquad \rho_{c,0} \simeq \rho_{M,0} + \rho_{\Lambda,0}$$

$$h^2 \stackrel{\star}{=} 0.5329 \qquad \rho_{M,0} \stackrel{\star}{=} 0.27\,\rho_{c,0} \qquad \rho_{\Lambda,0} \stackrel{\star}{=} 0.73\,\rho_{c,0} \qquad \rho_{R,0} \stackrel{\star}{=} \frac{\rho_{M,0}}{[illegible]}$$

Constraints Summary

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Constraints			
	Solar system observations	Post Newtonian parameters	$\beta_{PN} - 1 = (0 \pm 1)10^{-4} \stackrel{\star}{=} 0$ $\gamma_{PN} - 1 = (2.1 \pm 2.3)10^{-5} \stackrel{\star}{=} -\frac{4\gamma}{1+8\gamma}$
		Time dependence of Newton constant	$\dot{G}_{eff,0}/G_{eff,0} = (-0.2 \pm 0.5)10^{-13} y^{-1}$
	Supernovae data	Luminosity distance	$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}$
	Early cosmology: WMAP and Nucleosynthesis		

- Post Newtonian parameter bounds $\gamma \leq \gamma_M = 5 \cdot 10^{-7}$
- The value of the effective Newton constant today fixes λ:

$$G_{eff,0} = \frac{\sqrt{\lambda}}{8\pi\gamma x_0} \cdot \frac{8\gamma+1}{6\gamma+1} \to x_0 = x_0(\gamma, \lambda)$$

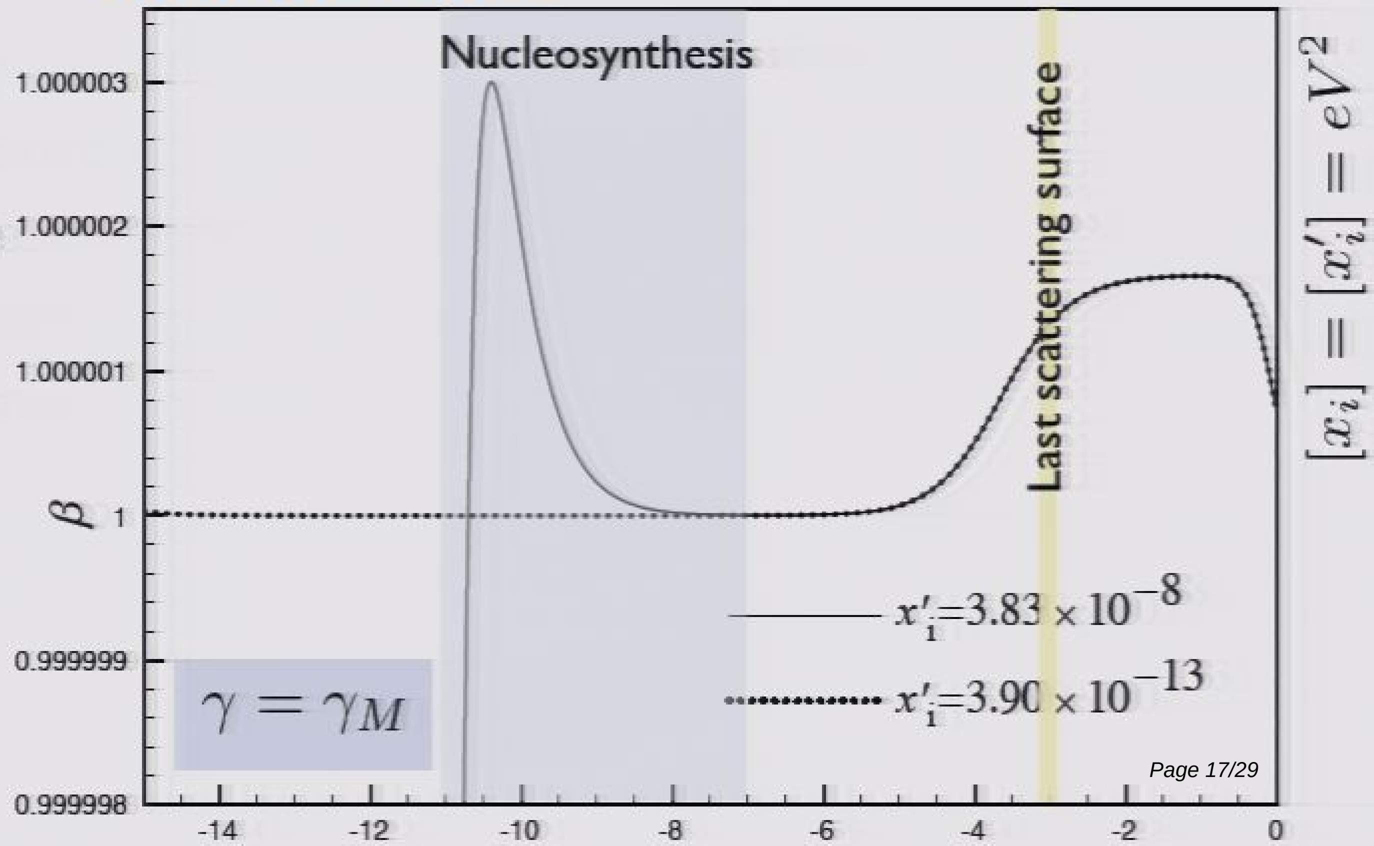
Numerical Analysis I

- The dynamics shows to be insensitive to initial conditions after an initial stage independently of γ;
- Kinetic energy is dissipated as foreseen in the Phase Space analysis;

$$\beta = 1 - \frac{1}{24\gamma}\left(\frac{x'}{x}\right)^2 + \frac{x'}{x} \longrightarrow 0$$

- Kinetic energy of quintessence must be dissipated before Nucleosynthesis; otherwise one has:

$$H^2 \propto a^{-6}$$



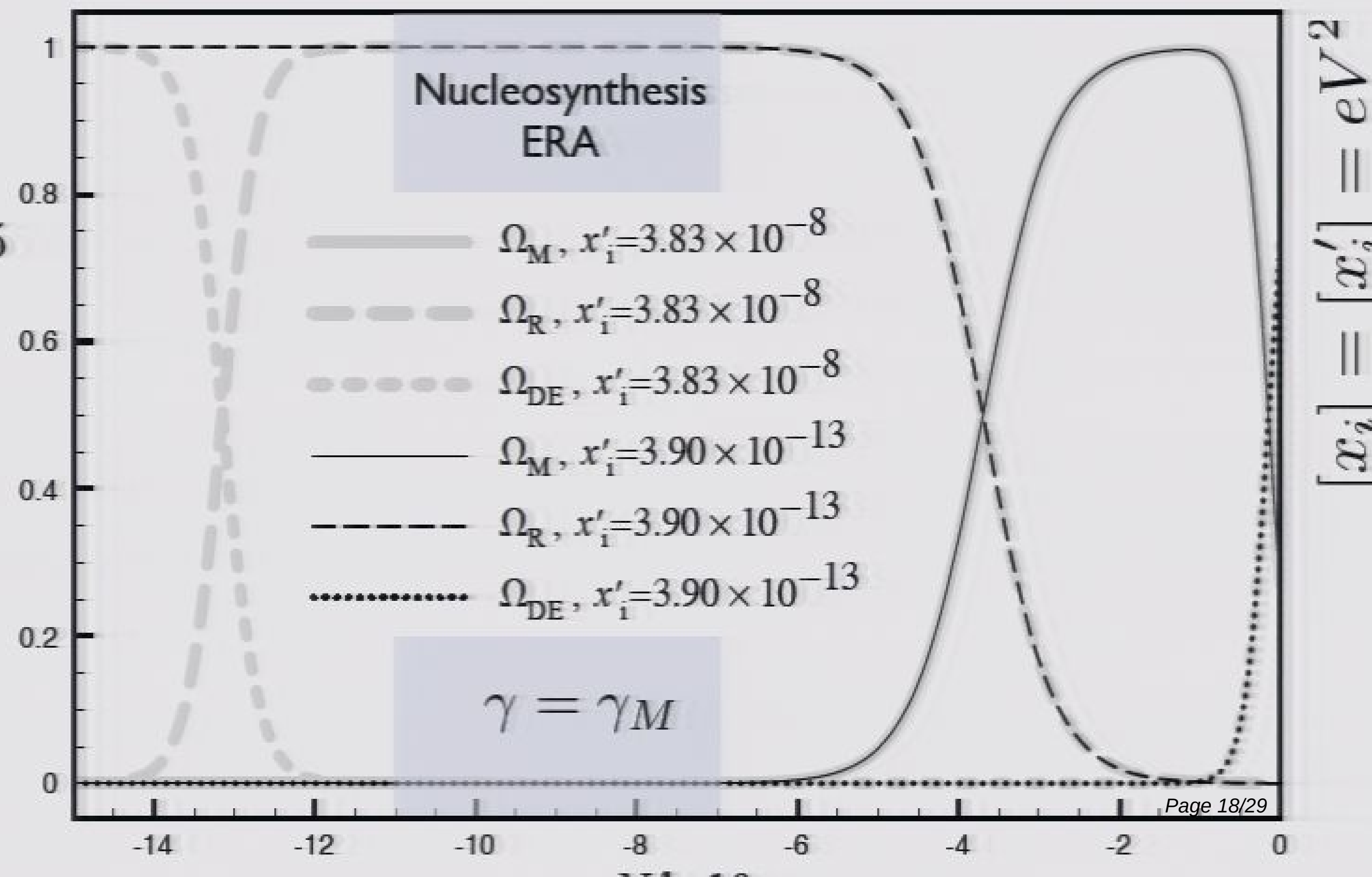
Numerical Analysis II

- The domination of radiation during Nucleosynthesis is necessary to make standard predictions for the light elements abundance;
- values of gamma compatible with Solar System Observations gives a RD ERA indistinguishable to that of the $\Lambda\mathrm{CDM}$ model.

a ratio

$$\left|\frac{H}{H_{\Lambda\mathrm{CDM}}} - 1\right| \leq 0.05$$

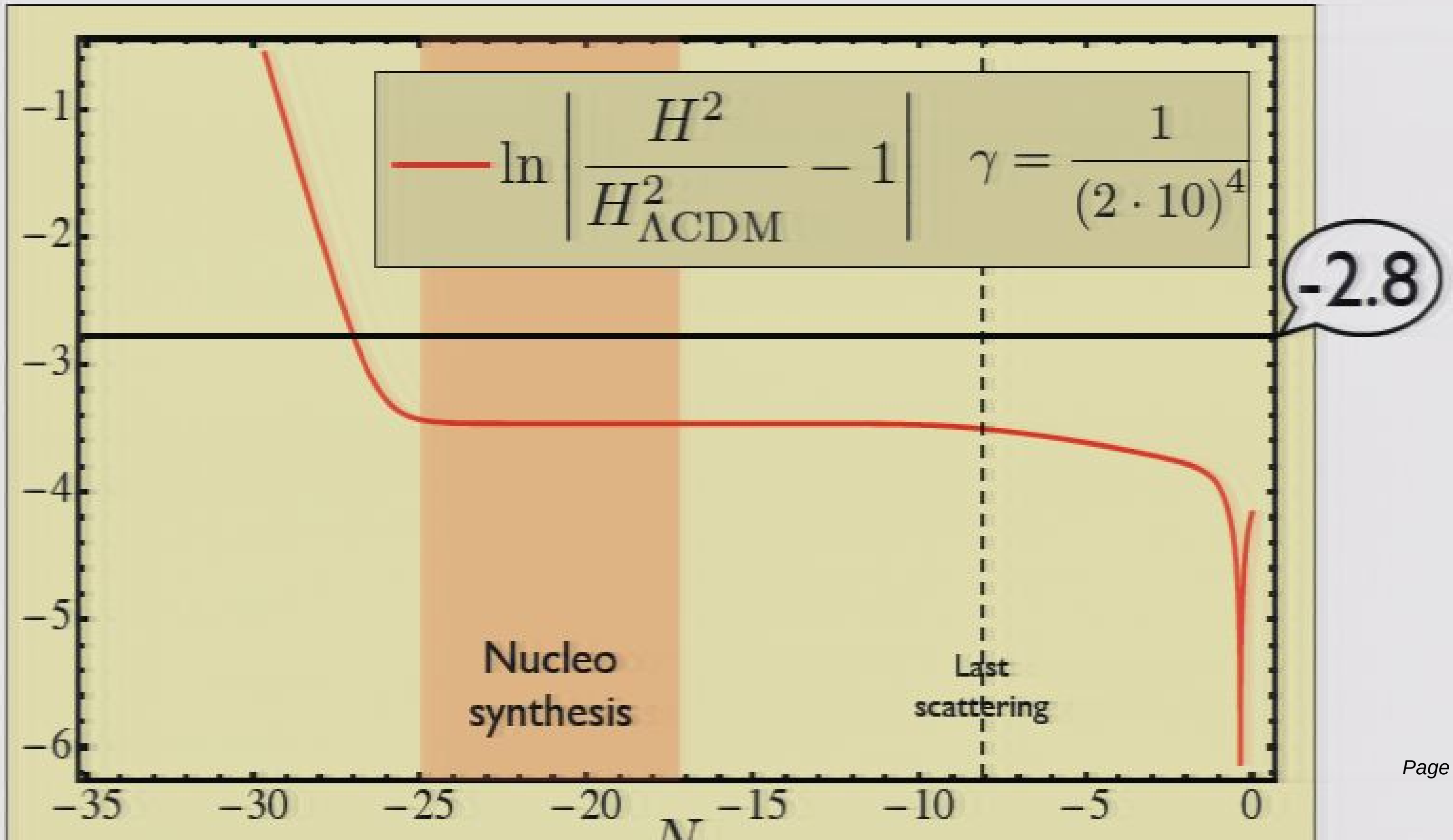
is required at the beginning of nucleosynthesis



Nucleosynthesis

- Even releasing the Solar system constraints on gamma one obtains evolution compatible with more severe theoretical predictions: $\ln\left|\frac{H^2}{H^2_{\Lambda CDM}}-1\right| \leq -2.8$
- Note that even milder constraint are usually imposed on the nucleosynthesis:

$$\ln\left|\frac{H^2}{H^2_{\Lambda CDM}}-1\right| \leq -1$$



SN Ia χ^2 TEST

- We compare the evolution with the "gold" set of 182 SNe Ia by Riess et al. (*Astrophys. J.* **659**, 98 (2006))
- In ST quintessence model the Chandrasekhar mass varies with redshift: $\mathrm{M}_{Ch}(z) \propto (G_{eff,z})^{-3/2}\ m_{prot}^{-2}$
- The distance modulus-redshift is modified since SN Ia luminosity varies with Chandrasekhar mass

$$\mu(z) = 5\log_{10}\frac{d_L(z)}{10\,\mathrm{pc}} + \frac{15}{4}\log_{10}\frac{x_0}{x(z)}$$

- We observe that SN data can not resolve values of $\gamma \leq \mathcal{O}\left(10^{-3}\right)$

#	γ/γ_M	x_i	x'_i	$w_{\mathrm{DE}\,0}+1$	χ^2_{snIa}	$(d_{\mathrm{L}}^{\mathrm{LSS}} - d_{\mathrm{L}\Lambda}^{\mathrm{LSS}})/d_{\mathrm{L}\Lambda}^{\mathrm{LSS}}$
1	1	$1.10 \cdot 10^{-5}$	$3.83 \cdot 10^{-8}$	-10^{-6}	186	$+8.2 \cdot 10^{-6}$
2	1	$1.12 \cdot 10^{-5}$	$3.90 \cdot 10^{-13}$	-10^{-6}	186	$-6.6 \cdot 10^{-7}$
3	$2^{-3} \cdot 10^2$	$1.056 \cdot 10^{-5}$	$1.30 \cdot 10^{-7}$	-10^{-5}	186	$-1.1 \cdot 10^{-5}$
4	$2^{-3} \cdot 10^2$	$1.112 \cdot 10^{-5}$	$1.38 \cdot 10^{-11}$	-10^{-5}	186	$-1.4 \cdot 10^{-5}$
5	$2 \cdot 10^2$	$1.12 \cdot 10^{-5}$	0	$-2 \cdot 10^{-4}$	186	$-1.8 \cdot 10^{-4}$
6	$2^{-3} \cdot 10^4$	$2.01 \cdot 10^{-5}$	$-2.32 \cdot 10^{-6}$	-10^{-3}	188	$-1.0 \cdot 10^{-3}$
7	$2 \cdot 10^3$	$1.09 \cdot 10^{-5}$	0	$-2 \cdot 10^{-3}$	189	$-2.0 \cdot 10^{-3}$
8	$2 \cdot 10^4$	$8.22 \cdot 10^{-6}$	0	0.02	224	$1.7 \cdot 10^{-2}$

Rows 1–5: 99% confidence level

Rows 6–7: 95% confidence level

Last Scattering Surface

- CMB anisotropies angular power spectrum is sensitive to LSS:

$$z_{LSS} = 1089 \pm 1$$

- LSS distance enters in the definition on the SHIFT PARAMETER and is related to the position of the FIRST ACOUSTIC PEAK in CMB spectrum.

$$R \simeq \sqrt{\Omega_{M,0}} \frac{H_0 d_L^{LSS}}{1+z_{LSS}}$$

Nearly model independent

- the SHIFT PARAMETER is measured with a precision of $\simeq 1\%$
- values of $\gamma < 10^{-2}$ give deviations of less than 1% and are thus compatible with observations.

#	γ/γ_M	x_i	x_i'	$w_{DE0}+1$	χ^2_{snIa}	$(d_L^{LSS} - d_{L\Lambda}^{LSS})/d_{L\Lambda}^{LSS}$
1	1	$1.10 \cdot 10^{-5}$	$3.83 \cdot 10^{-8}$	-10^{-6}	186	$+8.2 \cdot 10^{-6}$
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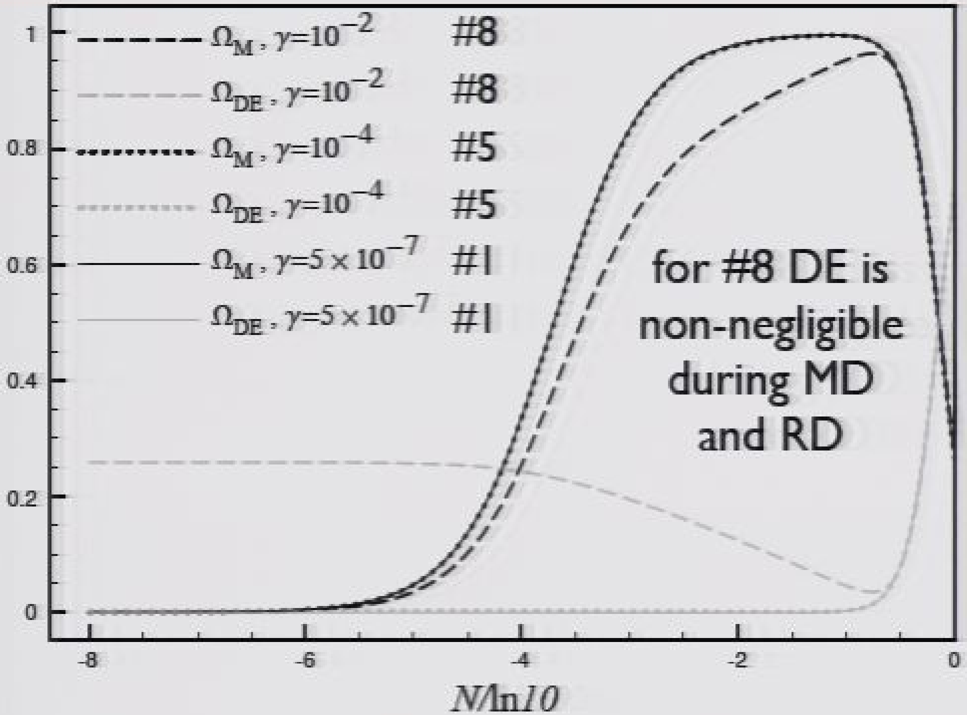
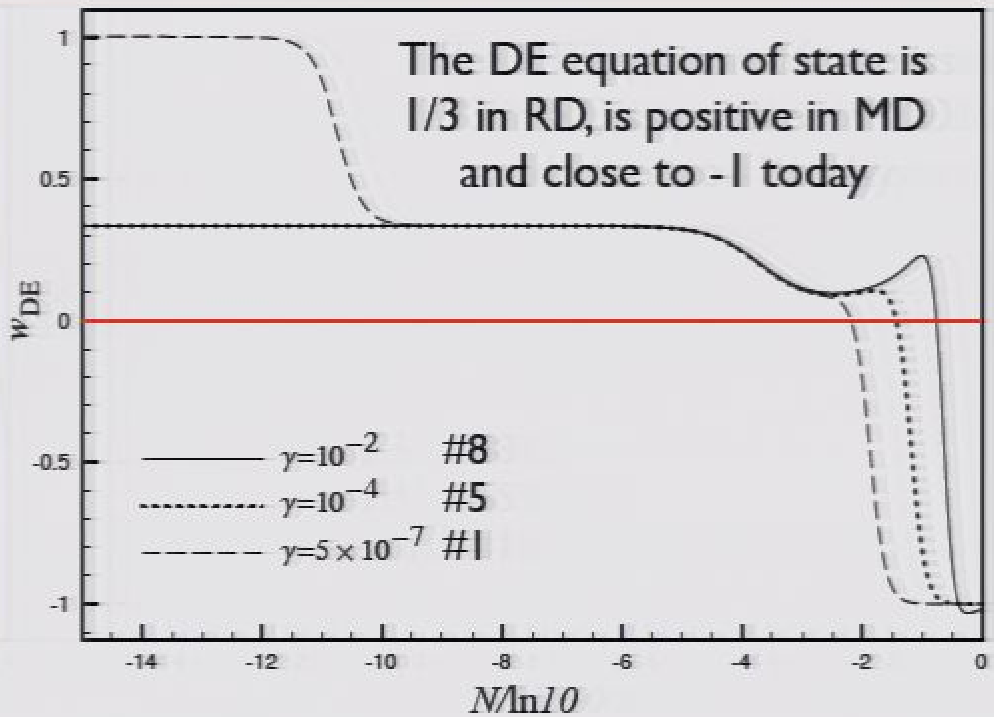
8	$2 \cdot 10^4$	[illegible]	0	[illegible]	[illegible]	[illegible]

NEC violation





- We find a **violation of NEC** of order $\mathcal{O}(\gamma)$

#	γ/γ_M	x_i	x_i'	$w_{DE0}+1$	χ^2_{snIa}	$(d_L^{LSS}-d_{L\Lambda}^{LSS})/d_{L\Lambda}^{LSS}$
1	1	$1.10\cdot10^{-5}$	$3.83\cdot10^{-8}$	-10^{-6}	186	$+8.2\cdot10^{-6}$
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8	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]

Compatible with observations

Conclusions

Pascos '08

- First model of Dark Energy proposed (1981) indeed works! Checked by accurate numerical methods.
- The scalar-tensor model does not need a runaway potential but just a quartic (renormalizable) self-interaction.
- The dynamics tends to Einstein Gravity plus a cosmological constant term in the future. It has a global attractor toward an accelerated Universe independently from the initial conditions for the field and its time derivative.
- Richer phenomenology with respect to DE in Einstein gravity since it can be constrained also on Solar System scales.
- For the parameters allowed by Solar System observations the our analysis shows that the cosmological backgroud expansion is in good agreement with current observations. Cosmological data are indeed compatible with bigger values of gamma.
- Our simulations showed evidence of possible NEC violation.

Work in progress...

Coleman-Weinberg potential

- One can consider modifications of the dynamics of CW type

$$\frac{\lambda}{4}\sigma^4 \rightarrow V_{CW} \equiv V_0 + \frac{\lambda}{4}\sigma^4\left(1 + \alpha \ln\frac{\sigma^2}{\sigma_0^2}\right)$$

- These modifications were invoked in the original article by *G. Venturi et al.* (1981) to dynamically generate a unique fixed point σ_F

- The potential can be cast in the form:

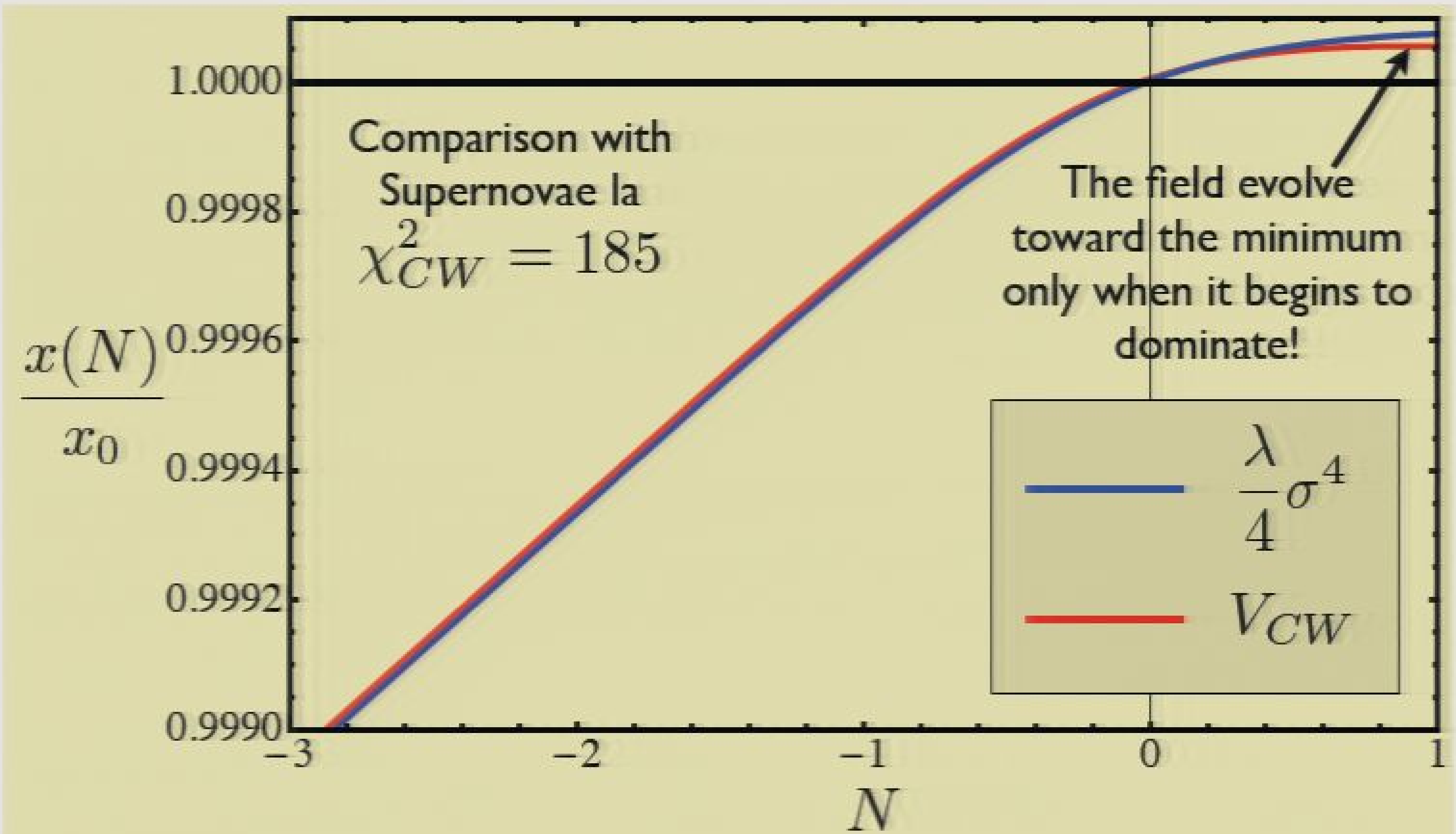
needed to have a regular evolution near the fixed point

$$V_{CW} = \frac{\alpha\lambda}{8}\sigma_F^4\left(1 - \frac{\sigma^4}{\sigma_F^4} + \frac{\sigma^4}{\sigma_F^4}\ln\frac{\sigma^4}{\sigma_m^4}\right), \quad \sigma_F^2 \geq \sigma_m^2$$

- <u>Differences in the dynamics when quintessence energy</u> dominates the Universe! RD and MD ERAS are unaffected!

Coleman-Weinberg example I

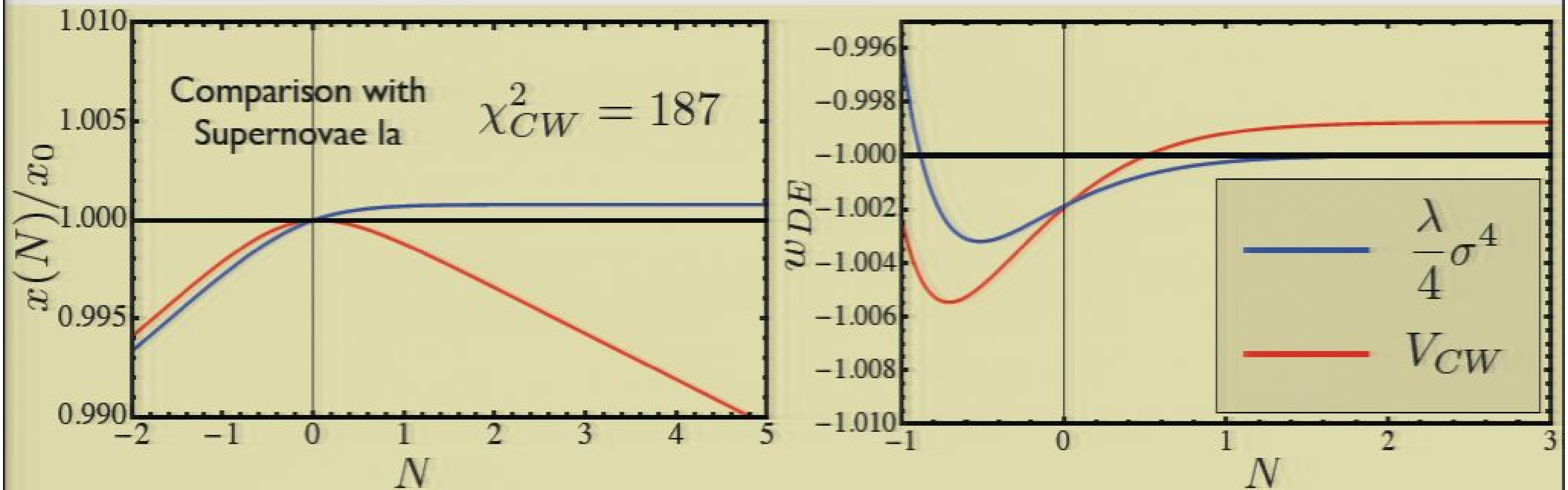
$$\frac{\sigma_m^2}{\sigma_F^2} = e^{-\frac{1}{2}-\frac{1}{\alpha}},\ \gamma = 10^{-4},\ \alpha = 10^{-1},\ x_F = \frac{1}{2}x_0$$



- The evolution begins to deviate today from the original potential pointing toward the fixed point!

Coleman-Weinberg example II

Extreme choice! $\frac{\sigma_m^2}{\sigma_F^2} = e^{-\frac{1}{2}}, \ \gamma = 10^{-3}, \ \alpha = 10^{-1}, \ x_F = \frac{1}{5} x_0$



- Equation of state stabilizes around a nearly constant value different from -1 in the FUTURE;
- Time variation of Newton constant constraints are satisfied in spite of the large value of gamma.

Work in progress...

Coleman-Weinberg potential

- One can consider modifications of the dynamics of CW type

$$\frac{\lambda}{4}\sigma^4 \to V_{CW} \equiv V_0 + \frac{\lambda}{4}\sigma^4 \left(1 + \alpha \ln \frac{\sigma^2}{\sigma_0^2}\right)$$

- These modifications were invoked in the original article by *G. Venturi et al.* (1981) to dynamically generate a unique fixed point σ_F

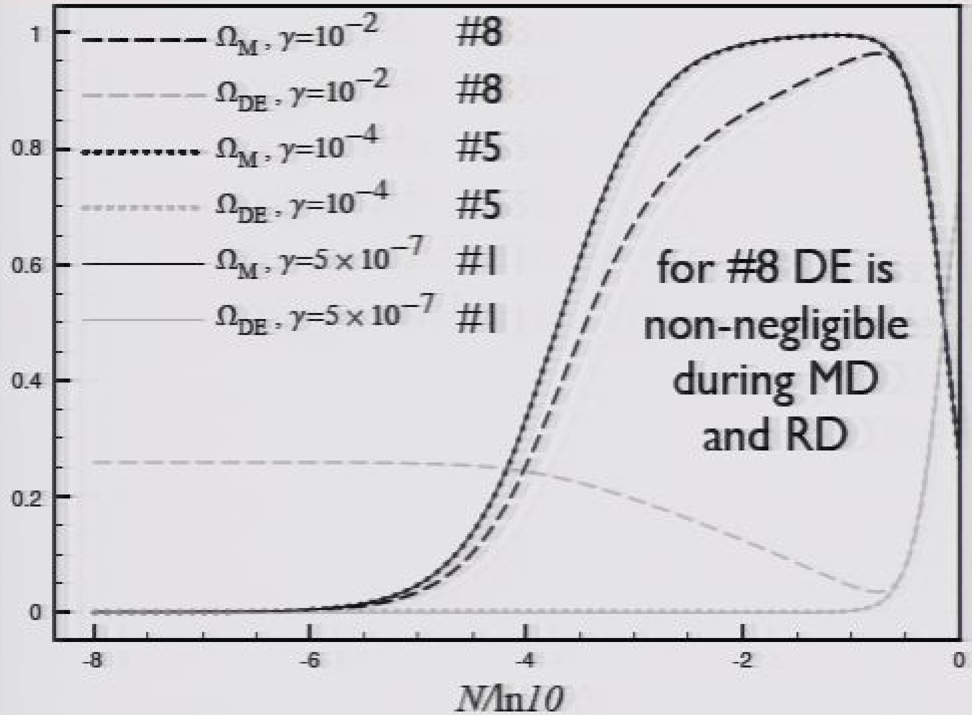
- The potential can be cast in the form:

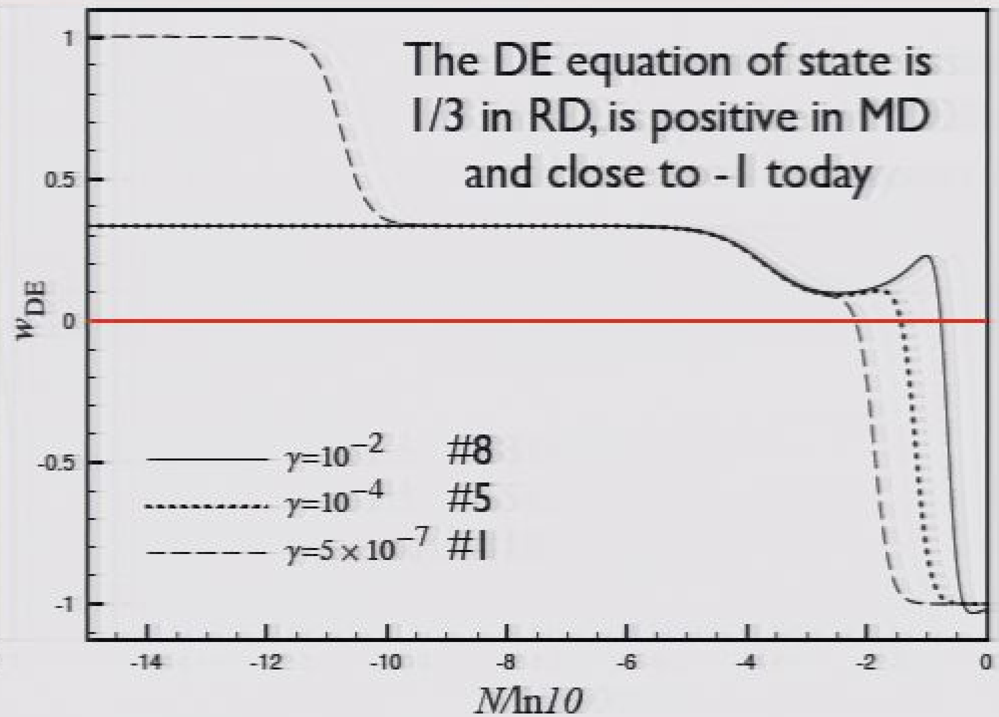
needed to have a regular evolution near the fixed point

$$V_{CW} = \frac{\alpha\lambda}{8}\sigma_F^4 \left(1 - \frac{\sigma^4}{\sigma_F^4} + \frac{\sigma^4}{\sigma_F^4} \ln \frac{\sigma^4}{\sigma_m^4}\right) , \sigma_F^2 \geq \sigma_m^2$$

- <u>Differences in the dynamics when quintessence energy</u> dominates the Universe! RD and MD ERAS are unaffected!

NEC violation





- We find a **violation of NEC** of order $\mathcal{O}(\gamma)$

#	γ/γ_M	x_i	x'_i	$w_{DE\,0}+1$	χ^2_{snIa}	$(d_L^{LSS}-d_{L\Lambda}^{LSS})/d_{L\Lambda}^{LSS}$
1	1	$1.10\cdot10^{-5}$	$3.83\cdot10^{-8}$	-10^{-6}	186	$+8.2\cdot10^{-6}$
2	1	$1.12\cdot10^{-5}$	$3.90\cdot10^{-13}$	-10^{-6}	186	$-6.6\cdot10^{-7}$
3	$2^{-3}\cdot10^2$	$1.056\cdot10^{-5}$	$1.30\cdot10^{-7}$	-10^{-5}	186	$-1.1\cdot10^{-5}$
4	$2^{-3}\cdot10^2$	$1.112\cdot10^{-5}$	$1.38\cdot10^{-11}$	-10^{-5}	186	[illegible]
5	$2\cdot10^2$	$1.12\cdot10^{-5}$	0	$-2\cdot10^{-4}$	[illegible]	[illegible]
6	$2^{-3}\cdot10^4$	$2.01\cdot10^{-5}$	$-2.32\cdot10^{-6}$	-10^{-3}	188	$-1.0\cdot10^{-3}$
7	$2\cdot10^3$	$1.09\cdot10^{-5}$	0	$-2\cdot10^{-3}$	189	$-2.0\cdot10^{-3}$
8	$2\cdot10^4$	[illegible]	0	[illegible]	[illegible]	[illegible]

Compatible with observations

SN Ia χ^2 TEST

- We compare the evolution with the "gold" set of 182 SNe Ia by Riess et al. (*Astrophys. J.* **659**, 98 (2006))
- In ST quintessence model the Chandrasekhar mass varies with redshift: $\mathrm{M}_{Ch}(z) \propto (G_{eff,z})^{-3/2}\ m_{prot}^{-2}$
- The distance modulus-redshift is modified since SN Ia luminosity varies with Chandrasekhar mass

$$\mu(z) = 5\log_{10}\frac{d_L(z)}{10\,\mathrm{pc}} + \frac{15}{4}\log_{10}\frac{x_0}{x(z)}$$

- We observe that SN data can not resolve values of $\gamma \leq \mathcal{O}\left(10^{-3}\right)$

#	γ/γ_M	x_i	x_i'	$w_{\mathrm{DE}\,0}+1$	χ^2_{snIa}	$(d_{\mathrm{L}}^{\mathrm{LSS}} - d_{\mathrm{L}\Lambda}^{\mathrm{LSS}})/d_{\mathrm{L}\Lambda}^{\mathrm{LSS}}$
1	1	$1.10\cdot10^{-5}$	$3.83\cdot10^{-8}$	-10^{-6}	186	$+8.2\cdot10^{-6}$
2	1	$1.12\cdot10^{-5}$	$3.90\cdot10^{-13}$	-10^{-6}	186	$-6.6\cdot10^{-7}$
3	$2^{-3}\cdot10^2$	$1.056\cdot10^{-5}$	$1.30\cdot10^{-7}$	-10^{-5}	186	[illegible] (99% confidence level, rows 1–5)
4	$2^{-3}\cdot10^2$	$1.112\cdot10^{-5}$	$1.38\cdot10^{-11}$	-10^{-5}	186	$-1.4\cdot10^{-5}$
5	$2\cdot10^2$	$1.12\cdot10^{-5}$	0	$-2\cdot10^{-4}$	186	$-1.8\cdot10^{-4}$
6	$2^{-3}\cdot10^4$	$2.01\cdot10^{-5}$	$-2.32\cdot10^{-6}$	-10^{-3}	188	$-1.0\cdot10^{-3}$
7	$2\cdot10^3$	$1.09\cdot10^{-5}$	0	$-2\cdot10^{-3}$	189	[illegible] (95% confidence level, rows 6–7)
8	$2\cdot10^4$	[illegible]	0	[illegible]	224	[illegible]