Title: Experimental limits on kappa-Minkowski field theories

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Abstract: The non-commutative kappa-Minkowski field theory, often tauted as a low-energy candidate for the quantum description of gravity, operates on a spacetime which possesses a modified Lorentz invariance, and as such can be probed in high-precision low-energy experiments. We study the first order corrections in kappa to the Standard Model, and show that they typically induce a coupling of nuclear spin to an external background at low energies. Strong limits on this type of interactions push the scale of non-commutativity to more than 10^7 Planck masses, thus exhibiting a severe naturalness problem for kappa-Minkowski Field Theory viewed as an effective description of quantum gravity.

EXPERIMENTAL LIMITS ON κ-MINKOWSKI FIELD THEORY

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Pavel A. Bolokhov

U. Pittsburgh & St. Petersburg State Univ.

We will be considering κ-Minkowski

{arising in Loop Quantum Gravity}

$$[x^0, x^k]_* = \frac{i}{\kappa} x^k$$

In more general form:

$$\Rightarrow \quad [x^\mu, x^\nu] = i C^{\mu\nu}_\rho x^\rho$$

where

$$C^{\mu\nu}_\rho = a^\mu \delta_\lambda{}^\nu - a^\nu \delta_\lambda{}^\mu$$

and

$$a^\mu = [1, 0, 0, 0]$$

in some reference frame

The most common tool for analysis of NC theory = Seiberg–Witten map

reflects a NC theory to a theory on normal space-time:

$$\phi(x)\cdot\psi(x) \to \phi(x) * \psi(x)$$

<u>$*$-product</u>

$*$-product is the (associative) operation which contains all information about NC

e.g.

$$\phi(x) * \psi(x) = \phi(x)\cdot\psi(x) + \frac{i}{2} C^{\mu\nu}_{\lambda} x^{\lambda} \partial_\mu \phi(x)\cdot\partial_\nu \psi(x) + \dots$$

The theory built on NC spacetime together with the relations

$$[x^\mu, x^\nu]_\star = i\Theta^{\mu\nu}$$

are invariant under quantum Poincaré symmetry group.

However Seiberg-Witten map produces an effective theory on an ordinary space-time

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + a^\mu \mathcal{O}_\mu + \dots$$

That is one can expect to see the effects of NC via the Lorentz-breaking operators $\mathcal{O}_\mu$

In different formulations of
κ-Minkowski Field Theory
one has the following effective
Lagrangian:

- Lukierski et al.

$$\frac{1}{2}\phi\left[\Box + m^2 - \frac{\partial_t^6}{\kappa^4}\right]\phi = \mathcal{L}$$

- Dimitrijević, Wess et al

$$\mathcal{L} = \frac{1}{2}\phi\left[\Box + m^2 - \frac{\partial_t^2}{\kappa^2}\Box\right]\phi$$

- Freidel et al

$$(\partial_\mu\phi)^*\sqrt{1 + \Box/\kappa^2}\,(\partial_\mu\phi) = \mathcal{L}$$

Cannot capture any effect from

It is a common feature of NC free Field Theories that the effect of NC does not show up at 2-point function level.

However in an interacting theory, 2-points can be modified at loop level

Or even at "tree" level for nucleons:

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We consider a $U(1)$ gauge theory on $\varkappa$-Minkowski, where Wess extended the notion of gauge invariance on NC space-time; at $1/\varkappa$ order the Lagrangian is

$$\mathcal{L}^{U(1)} = \bar{\psi}\left[i\not{D}-m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}C_{\lambda}^{\rho\sigma}\left[\bar{\psi}\gamma_{\rho}D_{\sigma}D^{\lambda}\psi + \overline{D_{\sigma}D^{\lambda}\psi}\gamma_{\rho}\psi\right]$$

There are x^{μ}-dependent terms in the action. However, by a choice of ambiguous params one can put them to zero.

$$\Rightarrow -\frac{1}{2} C_\lambda^{\rho\delta} \bar{\psi} \gamma_\rho D_\delta D_\lambda \psi$$

In a free theory this operator vanishes on the E.O.M.: no effect.

For an interacting theory, it reduces to

$$e\, a^\mu \bar{\psi} \tilde{F}_{\mu\nu} \gamma^\nu \gamma^5 \psi$$

In Phys. Rev. D77 (2008) we classified all dimension 5 operators CPT-odd.

This operator: T-even, C-odd

$\Rightarrow$ It can mix to the current $\bar{\psi}\gamma^\mu\psi$

Axial current $\bar{\psi}\gamma_\mu\gamma_5\psi$ protected by C, P.

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⇒ In the S.M. mixing is possible

Generalize to the Standard Model:

$\bar{Q} \gamma_\rho D_\delta D_\lambda Q$, $\bar{U} \gamma_\rho D_\delta D_\lambda U$,

$\bar{D} \gamma_\rho D_\delta D_\lambda D$, ...

$$\Rightarrow c\, a^\mu_e \tilde{F}_{\mu\nu} \bar{q} \gamma^\nu q + \hat{c}\, a^\mu_e \tilde{F}_{\mu\nu} \bar{q} \gamma^\nu \gamma^5 q$$

the same CPT quantum numbers as

$\bar{\psi} \gamma^\mu \gamma^5 \psi$

coupling of axial current to external direction $\Rightarrow$ yes effect

$\bar{\psi} \gamma^\mu \psi$

just a const addition to U(1) gauge field $\Rightarrow$ no effect

$\bar{Q}\gamma_\ell D_\delta \partial_\lambda Q$, $\bar{U}\gamma_\ell \partial_\delta \partial_\lambda U$,

$\bar{D}\gamma_\ell \partial_\delta \partial_\lambda D$, ...

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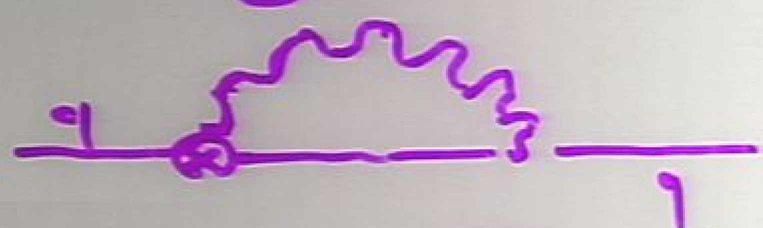
$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$
coupling of axial current to external direction $\Rightarrow$ yes effect	just a const addition to U(1) gauge field $\Rightarrow$ no effect

At loop level the diagrams involving $F_{\mu\nu}\bar{q}\gamma^{\nu}q$ are <u>divergent</u>



$\Rightarrow$ sensitive to UV scale via

$$\Lambda^2_{UV}\,\bar{\psi}\gamma^{\nu}\gamma^{5}\psi$$

We leave the problems connect- to the loop level aside and consider a "tree" level effect for <u>nucleons</u>.

For nucleon,

$$\langle N | a^\mu e \tilde{F}_{\mu\nu} \bar{q} \gamma^\nu q | N \rangle \simeq$$

$$\simeq \lambda a^\mu \cdot \bar{n} \gamma^\mu \gamma^5 n.$$

with

$$\lambda \sim \alpha_{QED} m_N^2 .$$

We use Vector Meson Dominance Model to estimate the order of λ.

$$\Rightarrow \lambda \simeq 10^{-4} \times GeV^{-2}$$

$$\lambda \sim 10^{-4}\ GeV^{-2}$$

for neutron.

Clock comparison experiments bound

$$\lambda a^{\kappa} < 10^{-31}\ GeV^{-1}$$

$\vec{a}$ is induced by $a^0 = 1/\varkappa$ via motion of Earth.

Then

$$\varkappa > 10^{23-24}\ GeV^{-1}$$

Conclusions

- low-energy exp sensitive to κ-noncommutativity
- Exp. bounds create serious problems for κ-Minkowski theory
- We take $U(1)$ {not St. Model} Gauge Theory to find effective interactions.

Effective theory depends on definition of $*$ product

- L. Freidel:

$$\phi(x) * \phi(x) =$$
$$= [1 + i\, C_\lambda^{\mu\nu} x_\mu \partial_\nu \partial^\lambda]\, \phi\, \phi$$

however cpx. conj. field $\phi^\dagger$ is defined such that this contribution is canceled for $\phi^\dagger \phi$.

- J. Wess

$$\phi * \phi =$$
$$= \phi \cdot \phi + \frac{i}{2} C_\lambda^{\mu\nu} x^\lambda \partial_\mu \phi \cdot \partial_\lambda \phi$$

At 2-point function level, effects are elusive

• in interacting theory, one cannot get rid of Lorentz-breaking.

⇨ • Dependence on reference frame $a^{\mu} = (1/\varkappa, 0, 0, 0)$

⇨ • Different values of $\varkappa$ correspond to physically different theories

$$\frac{e}{\varkappa}\,\bar{\psi}\,\tilde{F}_{0\nu}\gamma^{\nu}\gamma^{5}\psi$$