

Title: Beyond the big bang in Loop Quantum Cosmology

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Abstract: Loop quantum cosmology is a non-perturbative canonical quantization of simple cosmological models based on loop quantum gravity. In recent years, a greater control on the underlying quantum theory has revealed a picture where the big bang is replaced by a quantum bounce at Planck scale. The evolution across the bounce is unitary and non-singular without a need of choice of exotic potential or matter. By analysis of an exactly solvable model of a homogeneous and isotropic spacetime we will describe how the backward evolution of our universe with the quantum constraint leads to a pre big bang branch. We will also discuss the way it predicts modifications to Friedman dynamics at high curvatures, contrasts with the Wheeler-DeWitt scenario and semi-classicality across the bounce.

Beyond the Big Bang in Loop Quantum Cosmology

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PASCOS 08

Based on works in collaboration with Abhay Ashtekar (PSU), Alejandro Corichi (UNAM),
Tomasz Pawłowski (CSIC)

- Universe at low curvature is extremely well described by the Friedman dynamics. However, GR is inadequate at high curvatures.

Evolve the Universe backwards: For $a \rightarrow 0$, energy density and curvature $\propto a^n$ ($n < 0$) $\rightarrow \infty$.

\Rightarrow **Big Bang singularity. Evolution Stops.**
Result of powerful singularity theorems.

Classical GR fails to describe the birth of our Universe. **Need of new physics.**

- **Example from Quantum Theory:**

- Rutherford's model of Atom is unstable.

- **Bohr's model:** Energy levels discrete. Finite minimum energy
 $E_{min} = -(me^4/2\hbar^2)$. **As $\hbar \rightarrow 0$, $E_{min} \rightarrow -\infty$.**

**Can a quantum theory of gravity/geometry resolve the Big Bang singularity?
If yes, What is on the other side?**

Penrose, Hawking (1960's)

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Wheeler-DeWitt Quantum Cosmology

Based on Metric based canonical (Hamiltonian) quantization.^a

- Basic variables: Metric g_{ab} , Momentum p^{ab}
- To extract physics – isolate a ‘time’ variable, find an inner product, physical Hilbert space and Dirac observables, study their evolution.
- Hamiltonian constraint non-polynomial, difficult to quantize.

Simplifications for cosmological models (only finitely many degrees of freedom).

→ Standard quantum mechanical quantization possible.

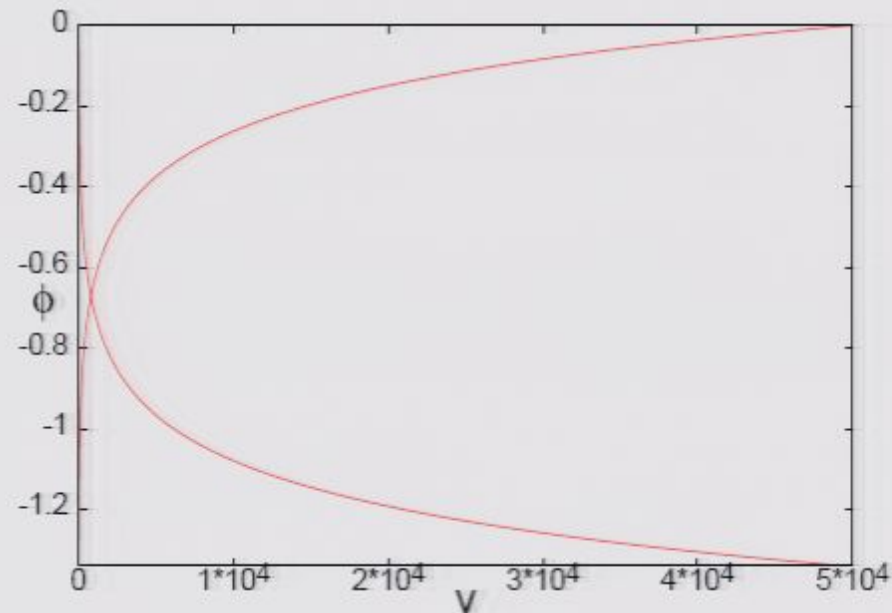
Geometry $\rightarrow a, p_a (\propto \dot{a}(t))$, Matter $\rightarrow \phi, p_\phi$.

Quantum States: $\Psi(a, \phi)$, $\hat{a} \Psi(a, \phi) = a \Psi(a, \phi)$, ...

Quantum Hamiltonian:

$$\hat{p}_a^2 \hat{a}^2 \Psi(a, \phi) = \text{const. } \hat{\mathcal{H}}_\phi \Psi(a, \phi)$$

Example: Massless Scalar Field $\mathcal{H}_\phi = p_\phi^2/2a^3$. ϕ is monotonic – a good clock.
Relational dynamics.



All classical solutions are singular.

Wheeler-DeWitt Equation:

$$\frac{\partial^2}{\partial \alpha^2} \Psi(\alpha, \phi) = \frac{\partial^2}{\partial \phi^2} \Psi(\alpha, \phi), \quad \alpha = \log a$$

All states follow the classical trajectories into the big bang.

Loop Quantum Cosmology

Canonical quantization of cosmological spacetimes based on loop quantum gravity.

New phase space variables:

- **Connection (SU(2))** A_a^i : Matrix valued vector potential (encodes time derivative of spatial metric)
- **Triad** E_i^a : Three orthonormal vectors (encode metric). Analogous to Electric field.

Hamiltonian constraint:

$$C_{\text{grav}} = - \int_V d^3x N \varepsilon_{ijk} F_{ab}^i (E^{aj} E^{bk} / \sqrt{|\det E|})$$

Elementary variables:

- **Holonomies** of connection along a curve: $h(A)$ (Fundamental excitations of quantum geometry)
- **Flux** across surface: $F(E)$

Express the constraint in terms of holonomies and fluxes and quantize.

An important distinction from WDW: Geometrical operators have discrete spectra

Massless scalar Model: Use ϕ as a clock. Observables $- p_\phi, V|_\phi, \rho|_\phi$

Quantum constraint:^a $\partial_\phi^2 \Psi(v, \phi) = -\Theta \Psi(v, \phi)$

$$\Theta \Psi(v, \phi) := \left[C^+(v) \Psi(v+4, \phi) + C^o(v) \Psi(v, \phi) + C^-(v) \Psi(v-4, \phi) \right]$$

Constraint similar to the massless Klein-Gordon equation in static spacetime.
 $\Theta \rightarrow$ Laplacian-type operator (Is self-adjoint and positive definite).

Hilbert space can be constructed as in Klein-Gordon theory (Positive frequency solutions). Features:

- Difference equation in constant steps of eigenvalues of the volume operator.
- Non-singular for all states.
- $\hat{C}_{\text{grav}} \longrightarrow \hat{C}_{\text{grav}}^{\text{WDW}}$ with natural factor ordering for $|v| \gg 1$.
- We obtain GR at low curvatures, departures from GR at high curvatures.

^aAshtekar, Pawłowski, PS (06)

Exactly Solvable Model

In the connection representation:^a

$$\Theta(b)\chi(b, \phi) = -12\pi G \frac{\sin(\lambda b)}{\lambda} \frac{\partial}{\partial b} \frac{\sin(\lambda b)}{\lambda} \frac{\partial}{\partial b} \chi(b, \phi) = -\partial_\phi^2 \chi(b, \phi)$$

λ is the discreteness parameter from LQG, $\lambda = \alpha \ell_P$, $\alpha \sim O(1)$.

– Introduce $x := (12\pi G)^{-1/2} \ln(\tan(\lambda b/2))$

\Rightarrow

$$\partial_\phi^2 \chi(x, \phi) = \partial_x^2 \chi(x, \phi)$$

Features:

Volume observable:

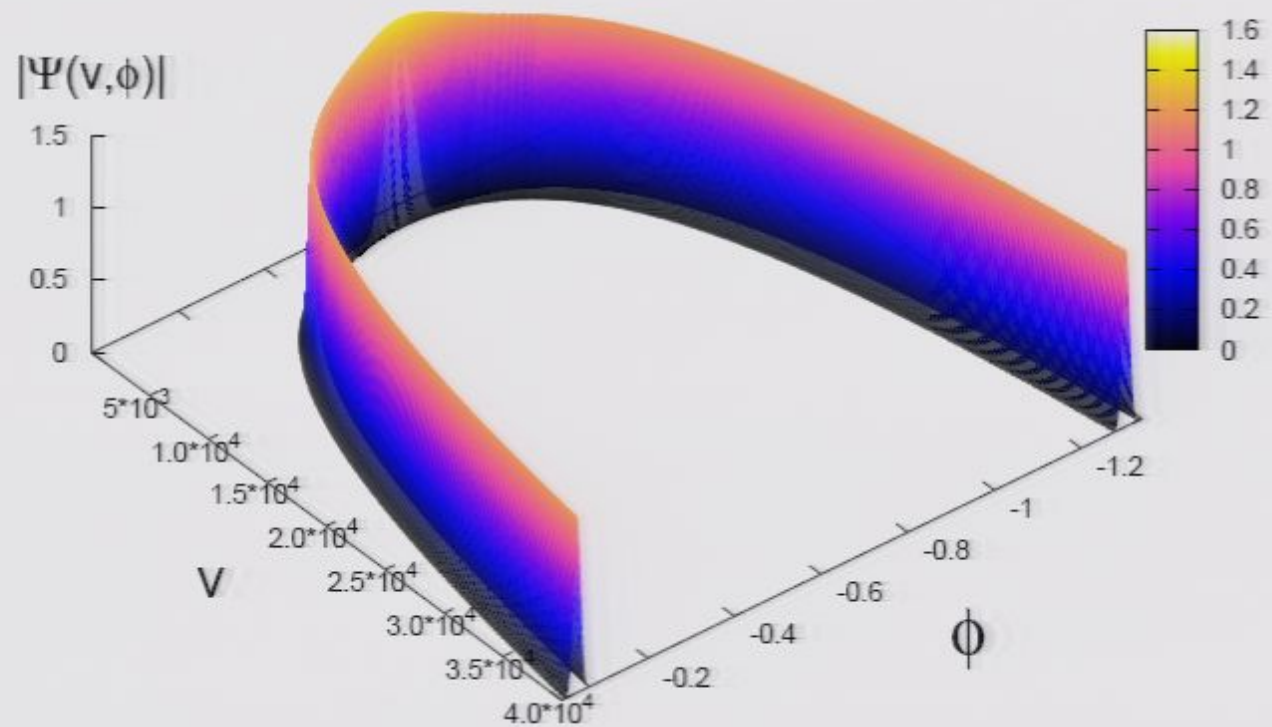
$$(\chi, \hat{V}|_\phi \chi)_{\text{phy}} = V_+ e^{\sqrt{12\pi G}\phi} + V_- e^{-\sqrt{12\pi G}\phi}, \quad (V_\pm \text{ are constants } > 0)$$

– As $\phi \rightarrow \pm\infty$, $\langle \hat{V}|_\phi \rangle \rightarrow \infty$. The Universe is infinitely large in asymptotic past and future. **There exists a non-zero minimum volume.** The universe bounces!

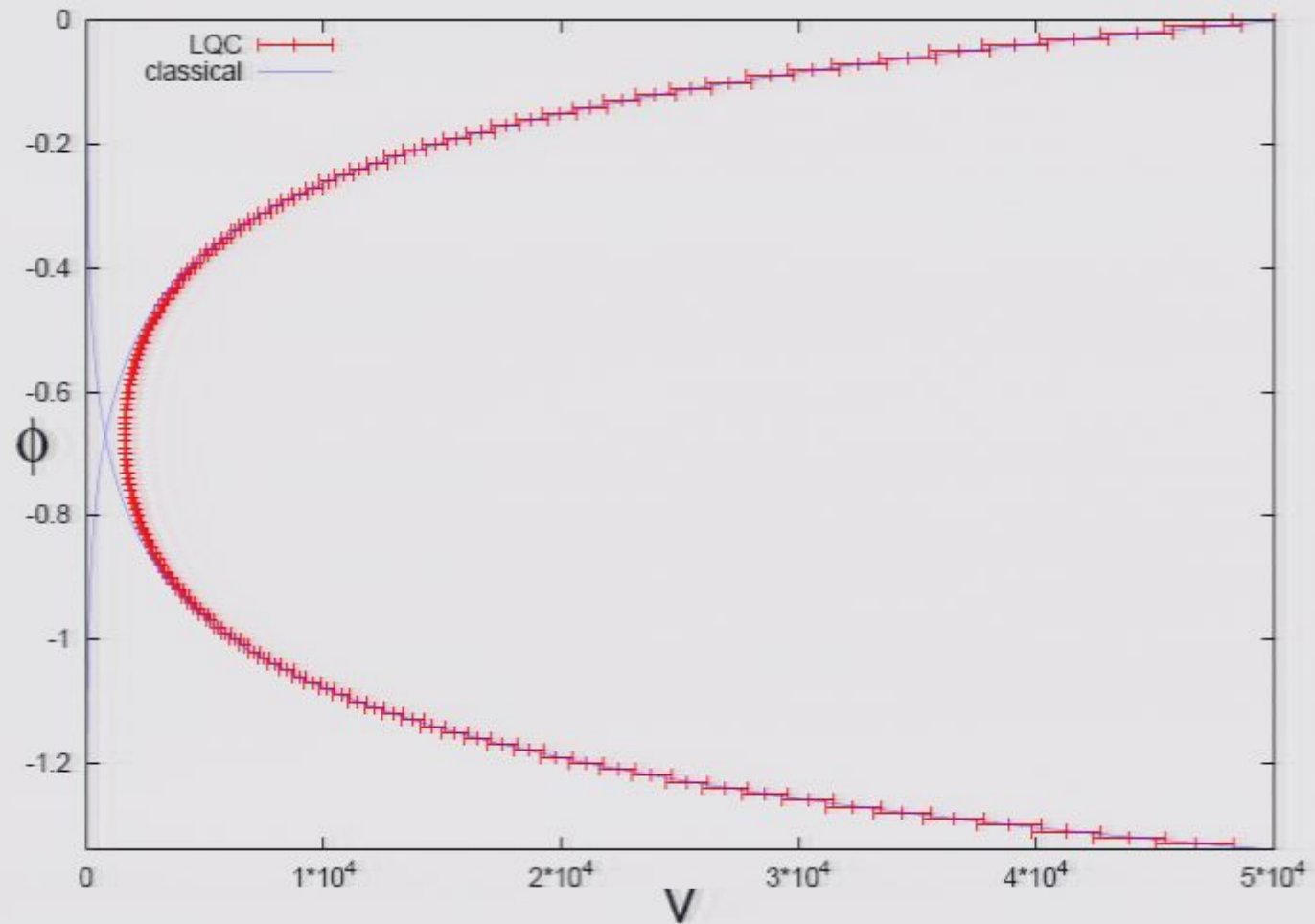
Energy density: There exists a supremum $\rho_{\text{crit}} = 0.82\rho_{\text{Pl}}$.

Fluctuations: Remain small across the bounce, semiclassicality is preserved.^b

Numerical Simulations: Quantum Bounce



Comparison of Evolution



Some Features of New Physics:

- Quantum dynamics described by an effective Hamiltonian. Leads to a modified Friedman^a and Raichaudhuri equation:

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}} \right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \left(1 - 4\frac{\rho}{\rho_{\text{crit}}} \right) - 4\pi G P \left(1 - 2\frac{\rho}{\rho_{\text{crit}}} \right)$$

- Rich phenomenology.^{b c d e f g h i}

^aCoincidentally also in some braneworld models: Sahni, Shtanov (02)

^bCyclic & Pre-Big Bang models: PS, Vandersloot, Vereshchagin (06); De Risi, Maartens, PS (07)

^cBig Rip avoidance: Sami, PS, Tsujikawa (06)

^dScaling solutions: PS (06)

^eInflationary models: Zhang, Ling (07); Copeland, Mulryne, Nunes, Shaeri (07)

^fTachyon & Quintom Models: Sen (06); Wei, Zhang (07); Xiong, Qiu, Cai, Zhang (07)

^gPhantom Models: Samart, Gumjudpai (07); Naskar, Ward (07)

^hScale invariant thermal fluctuations: Magueijo, PS (07)

ⁱEinstein Static Universes: Parisi, Bruni, Maartens, Vandersloot (07)

Summary

- Unlike singular Wheeler-DeWitt quantization, non-perturbative loop quantization of homogeneous models reveals quantum bounce at the Planck scale. Emerging picture from simple models: **Big bang not the beginning, big crunch not the end.** Two classical regions of spacetime joined by a quantum geometric bridge.
- Quantum gravity makes curvature non-local at the Planck scale. This plays an important role to yield a non-singular evolution across the classical singularity. No need to introduce any exotic matter/ad-hoc assumptions/fine tuning.
- Bounce occurs for states in a dense subspace of the physical Hilbert space when $\rho = \rho_{\text{crit}} = 0.82\rho_{\text{Pl}}$. When quantum discreteness i.e. $\lambda \sim G\hbar \rightarrow 0$, $\rho_{\text{crit}} \rightarrow \infty$. Bounce disappears if no quantum geometry!
- A very similar picture in presence of massive scalars (inflaton) and cosmological constant.
- Work in progress: Include anisotropies and inhomogenities. (Bianchi models and Gowdy spacetimes). Quantum geometric effects on cosmological perturbations.