Title: Beyond the big bang in Loop Quantum Cosmology

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Abstract: Loop quantum cosmology is a non-perturbative canonical quantization of simple cosmological models based on loop quantum gravity. In recent years, a greater control on the underlying quantum theory has revealed a picture where the big bang is replaced by a quantum bounce at Planck scale. The evolution across the bounce is unitary and non-singular without a need of choice of exotic potential or matter. By analysis of an exactly solvable model of a homogeneous and isotropic spacetime we will describe how the backward evolution of our universe with the quantum constraint leads to a pre big bang branch. We will also discuss the way it predicts modifications to Friedman dynamics at high curvatures, contrasts with the Wheeler-DeWitt scenario and semi-classicality across the bounce.

Beyond the Big Bang in Loop Quantum Cosmology

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PASCOS 08

Based on works in collaboration with Abhay Ashtekar (PSU), Alejandro Corichi (UNAM), Tomasz Pawlowski (CSIC)

- Universe at low curvature is extremely well described by the Friedman dynamics. However, GR is inadequate at high curvatures.

  Evolve the Universe backwards: For $a \to 0$, energy density and curvature $\propto a^n$ (n < 0) $\to \infty$.

  $\Rightarrow$ Big Bang singularity. Evolution Stops.
  Result of powerful singularity theorems.

  Classical GR fails to describe the birth of our Universe. Need of new physics.

- Example from Quantum Theory:
  – Rutherford's model of Atom is unstable.

  – Bohr's model: Energy levels discrete. Finite minimum energy $E_{min} = -(me^4/2\hbar^2)$. As $\hbar \to 0$, $E_{min} \to -\infty$.

  Can a quantum theory of gravity/geometry resolve the Big Bang singularity? If yes, What is on the other side?

Penrose, Hawking (1960's)

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Wheeler-DeWitt Quantum Cosmology

Based on Metric based canonical (Hamiltonian) quantization.[a]

– Basic variables: Metric g_{ab}, Momentum p^{ab}

– To extract physics – isolate a 'time' variable, find an inner product, physical Hilbert space and Dirac observables, study their evolution.

– Hamiltonian constraint non-polynomial, difficult to quantize.

Simplifications for cosmological models (only finitely many degrees of freedom).
$\rightarrow$ Standard quantum mechanical quantization possible.

Geometry $\rightarrow a, p_a(\propto \dot{a}(t))$, Matter $\rightarrow \phi, p_\phi$.

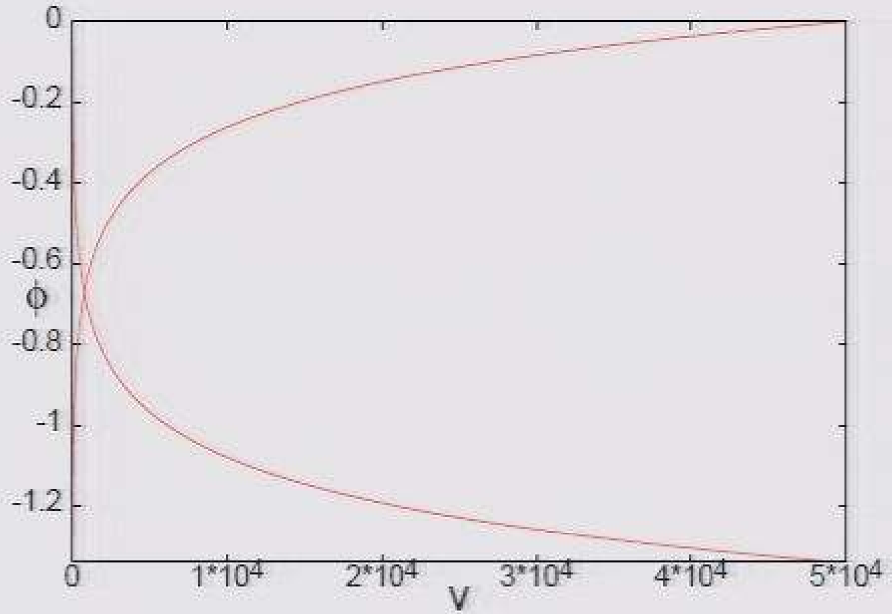
Quantum States: $\Psi(a,\phi)$, $\hat{a}\,\Psi(a,\phi) = a\Psi(a,\phi)$, ...

Quantum Hamiltonian:

$$\hat{p}_a^2 \hat{a}^2\, \Psi(a,\phi) = \text{const.}\, \hat{\mathcal{H}}_\phi\, \Psi(a,\phi)$$

[a] Misner, Wheeler, DeWitt (70's)

Example: Massless Scalar Field $\mathcal{H}_\phi = p_\phi^2/2a^3$. ϕ is monotonic – a good clock. Relational dynamics.



All classical solutions are singular.

Wheeler-DeWitt Equation:

$$\frac{\partial^2}{\partial \alpha^2}\Psi(\alpha,\phi) = \frac{\partial^2}{\partial \phi^2}\Psi(\alpha,\phi), \quad \alpha = \log a$$

All states follow the classical trajectories into the big bang.

Loop Quantum Cosmology

Canonical quantization of cosmological spacetimes based on loop quantum gravity.

New phase space variables:

– Connection (SU(2)) A_a^i: Matrix valued vector potential (encodes time derivative of spatial metric)

– Triad E_i^a: Three orthonormal vectors (encode metric). Analogous to Electric field.

Hamiltonian constraint:

$$C_{\rm grav} = -\int_{\mathcal{V}} d^3x\, N\, \varepsilon_{ijk}\, F^i_{ab}\, (E^{aj} E^{bk} / \sqrt{|\det E|})$$

Elementary variables:

- Holonomies of connection along a curve: $h(A)$ (Fundamental excitations of quantum geometry)
- Flux across surface: $F(E)$

Express the constraint in terms of holonomies and fluxes and quantize.

An important distinction from WDW: Geometrical operators have discrete spectra

Massless scalar Model: Use ϕ as a clock. Observables – $p_\phi, V|_\phi, \rho|_\phi$

Quantum constraint: [a] $\partial_\phi^2 \Psi(v,\phi) = -\Theta\Psi(v,\phi)$

$$\Theta\Psi(v,\phi) := \left[C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi)\right]$$

Constraint similar to the massless Klein-Gordon equation in static spacetime.
$\Theta \rightarrow$ Laplacian-type operator (Is self-adjoint and positive definite).

Hilbert space can be constructed as in Klein-Gordon theory (Positive frequency solutions). Features:

– Difference equation in constant steps of eigenvalues of the volume operator.

– Non-singular for all states.

– $\hat{C}_{\rm grav} \longrightarrow \hat{C}^{\rm WDW}_{\rm grav}$ with natural factor ordering for $|v| \gg 1$.

– We obtain GR at low curvatures, departures from GR at high curvatures.

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[a] Ashtekar, Pawlowski, PS (06)

Exactly Solvable Model

In the connection representation: [a]

$$\Theta(b)\chi(b,\phi) = -12\pi G \frac{\sin(\lambda b)}{\lambda}\frac{\partial}{\partial b}\frac{\sin(\lambda b)}{\lambda}\frac{\partial}{\partial b}\chi(b,\phi) = -\partial_\phi^2\chi(b,\phi)$$

λ is the discreteness parameter from LQG, $\lambda = \alpha\ell_{\rm P}$, $\alpha \sim O(1)$.

– Introduce $x := (12\pi G)^{-1/2}\,\ln(\tan(\lambda b/2))$

$\Rightarrow$

$$\partial_\phi^2\chi(x,\phi) = \partial_x^2\chi(x,\phi)$$

Features:

Volume observable:

$(\chi, \hat{V}|_\phi\,\chi)_{\rm phy} = V_+\,e^{\sqrt{12\pi G}\phi} + V_-\,e^{-\sqrt{12\pi G}\phi}, \qquad (V_\pm \text{ are constants} > 0)$

– As $\phi \to \pm\infty$, $\langle\hat{V}|_\phi\rangle \to \infty$. The Universe is infinitely large in asymptotic past and future. There exists a non-zero minimum volume. The universe bounces!

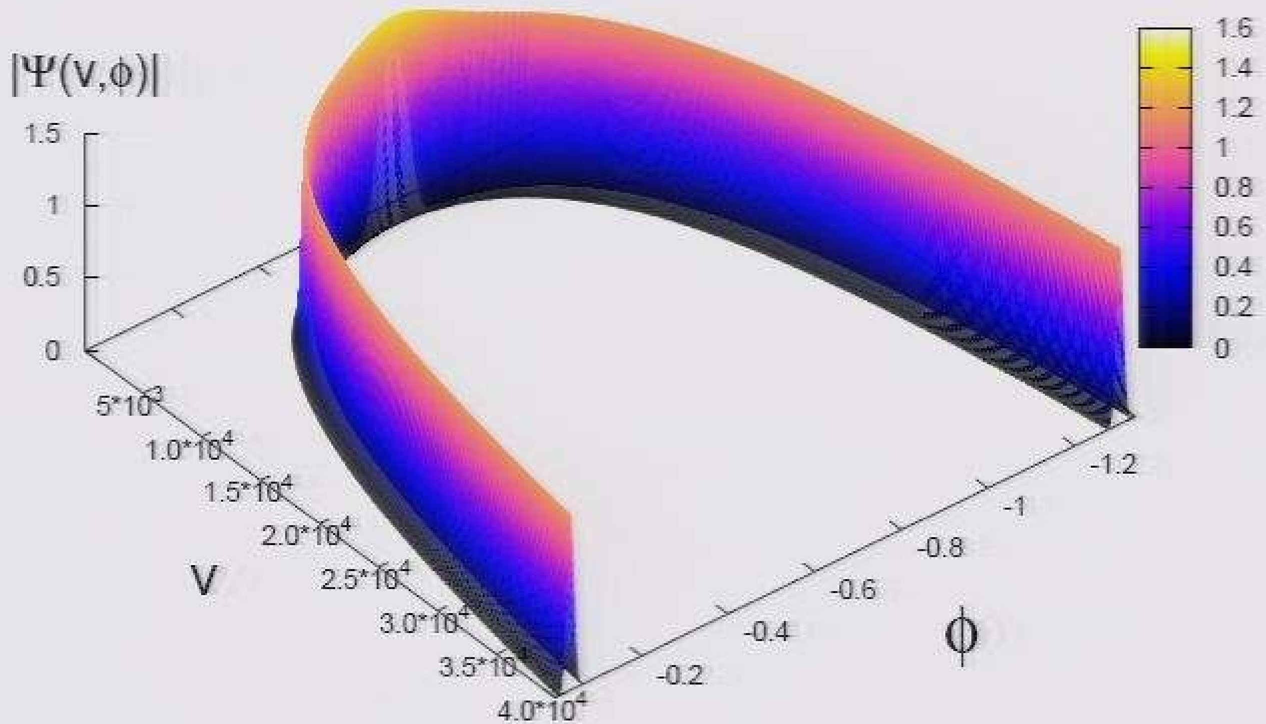
Energy density: There exists a supremum $\rho_{\rm crit} = 0.82\rho_{\rm Pl}$.

Fluctuations: Remain small across the bounce, semiclassicality is preserved. [b]

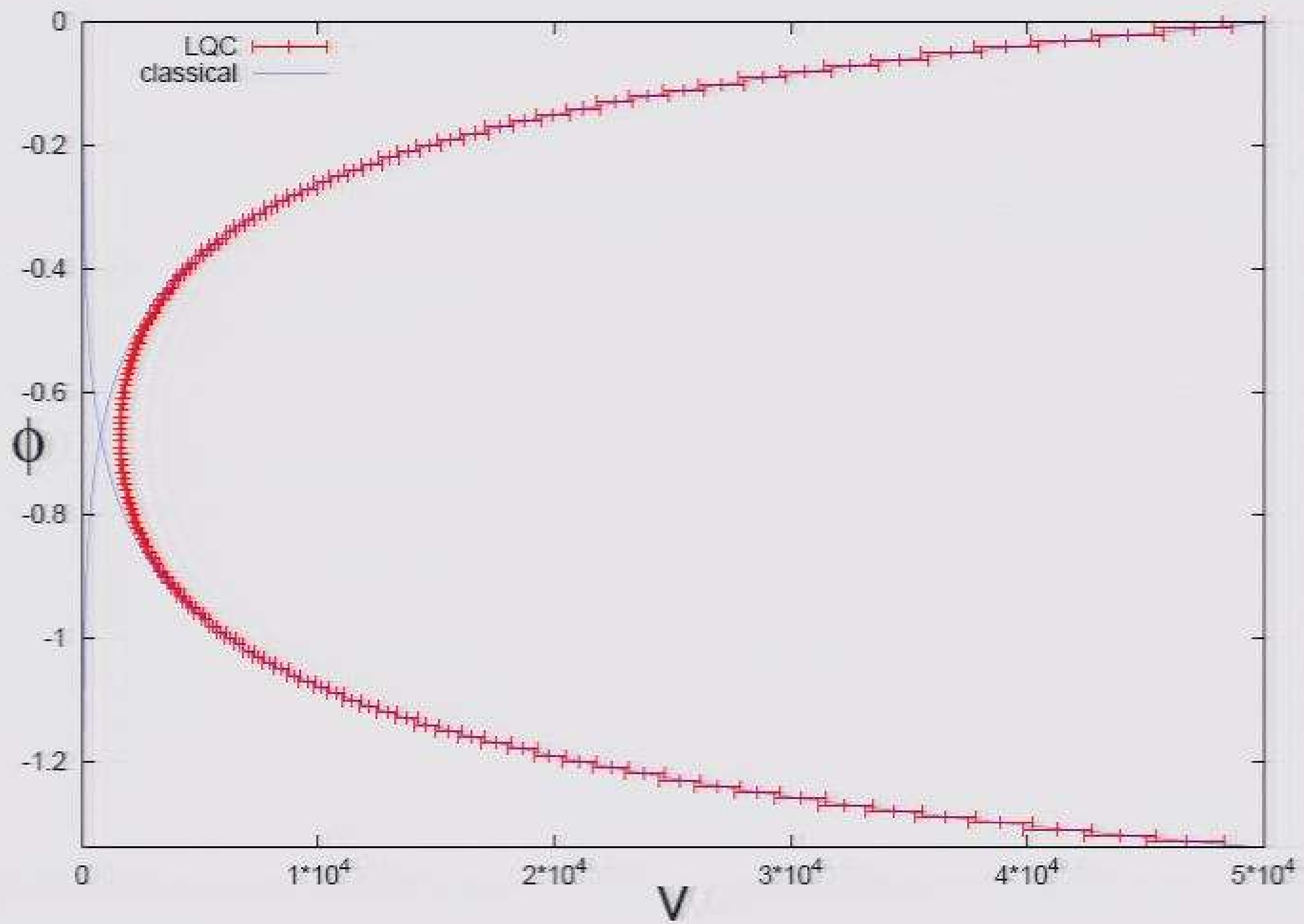
[a] Ashtekar, Corichi, PS (07)

[b] Corichi, PS (07)

Numerical Simulations: Quantum Bounce



Comparison of Evolution



Some Features of New Physics:

– Quantum dynamics described by an effective Hamiltonian. Leads to a modified Friedman[a] and Raichaudhuri equation:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1 - 4\frac{\rho}{\rho_{\rm crit}}\right) - 4\pi G\, P\left(1 - 2\frac{\rho}{\rho_{\rm crit}}\right)$$

– Rich phenomenology.[b c d e f g h i]

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[a] Coincidentally also in some braneworld models: Sahni, Shtanov (02)

[b] Cyclic & Pre-Big Bang models: PS, Vandersloot, Vereshchagin (06); De Risi, Maartens, PS (07)

[c] Big Rip avoidance: Sami, PS, Tsujikawa (06)

[d] Scaling solutions: PS (06)

[e] Inflationary models: Zhang, Ling (07); Copeland, Mulryne, Nunes, Shaeri (07)

[f] Tachyon & Quintom Models: Sen (06); Wei, Zhang (07); Xiong, Qiu, Cai, Zhang (07)

[g] Phantom Models: Samart, Gumjudpai (07); Naskar, Ward (07)

[h] Scale invariant thermal fluctuations: Magueijo, PS (07)

[i] Einstein Static Universes: Parisi, Bruni, Maartens, Vandersloot (07)

Summary

- Unlike singular Wheeler-DeWitt quantization, non-perturbative loop quantization of homogeneous models reveals quantum bounce at the Planck scale. Emerging picture from simple models: Big bang not the beginning, big crunch not the end. Two classical regions of spacetime joined by a quantum geometric bridge.
- Quantum gravity makes curvature non-local at the Planck scale. This plays an important role to yield a non-singular evolution across the classical singularity. No need to introduce any exotic matter/ad-hoc assumptions/fine tuning.
- Bounce occurs for states in a dense subspace of the physical Hilbert space when $\rho = \rho_{\rm crit} = 0.82 \rho_{\rm Pl}$. When quantum discreteness i.e. $\lambda \sim G\hbar \to 0$, $\rho_{\rm crit} \to \infty$. Bounce disappears if no quantum geometry!
- A very similar picture in presence of massive scalars (inflaton) and cosmological constant.
- Work in progress: Include anisotropies and inhomogenities. (Bianchi models and Gowdy spacetimes). Quantum geomteric effects on cosmological perturbations.