

Title: A covariant entropy bound conjecture on dynamical horizon

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Abstract: After introduction and motivation, I will present a covariant entropy bound conjecture on dynamical horizon in a general sense. Then I will show its validity in black hole case and cosmological context. Especially its power in constraint on the cosmological constant is addressed. I will end up with the outlook of this conjecture.

From mechanics to thermodynamics for black holes

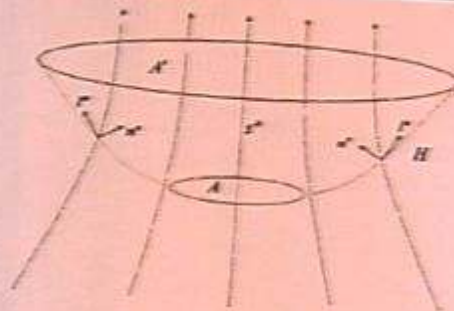
- *The four laws of black hole mechanics*
J. M. Bardeen et al.(1973)
- *Black holes and entropy*
J. D. Bekenstein(1973)
- *Generalized second law of thermodynamics in black-hole physics*
J. D. Bekenstein(1974)
- *Particle creation by black holes*
S. W. Hawking(1975)

From event horizon to dynamical horizon for black holes

- *Event horizon is a global concept, which means we can locate it only if we know the whole spacetime, especially the null infinity*
- *Dynamical horizon is essentially a foliation of apparent horizon, which agrees with the boundary of black holes discussed in astrophysics*
- *The first and second law has been developed for dynamical horizon*

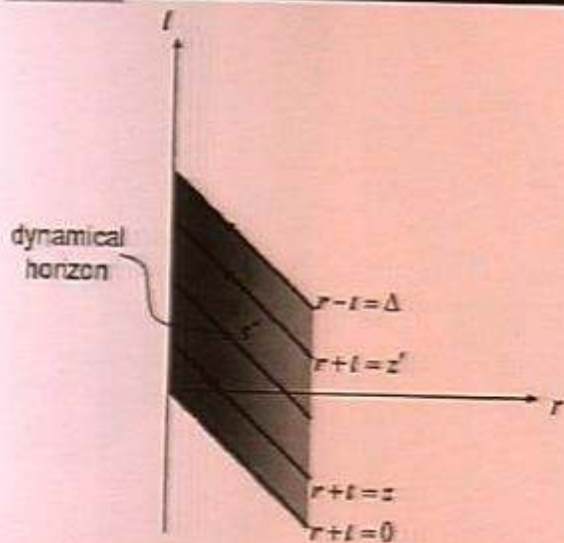
A. Ashtekar and B. Krishnan(2002,2003,2004)

CEB formulation of GSL for black hole dynamical horizon



$S \leq (A' - A)/4$, where
Planck units have been
used and dominant
energy condition has
been assumed.

CEBC tested in growing Vaidya black holes



$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 + u(dt + dr)^2$$

$$u = \begin{cases} 0, & \text{Minkowski region,} \\ \frac{2m}{r} F\left(\frac{r+t}{\Delta}\right), & \text{within light shells.} \\ \frac{2m}{r}, & \text{Schwarzschild region.} \end{cases}$$

$$T_{ab} = \frac{mF'}{4\pi\Delta r^2} k_a k_b.$$

$$\delta A = 16\pi m^2 \left[F^2\left(\frac{z'}{\Delta}\right) - F^2\left(\frac{z}{\Delta}\right) \right]$$

$$M_{eff} = m \left[F\left(\frac{z'}{\Delta}\right) - F\left(\frac{z}{\Delta}\right) \right]$$

$$S \leq 4\pi m^2 \left[F\left(\frac{z'}{\Delta}\right) - F\left(\frac{z}{\Delta}\right) \right]^2$$

CEB conjecture for general dynamical horizon

$\theta_l = 0$	$\theta_n > 0$	$\theta_n < 0$
$\mathcal{L}_n \theta_l > 0, \mathcal{L}_l \theta_l < 0$ (timelike)	expanding FRW universes with $-1 < w < \frac{1}{3}$	time reversal
	time reversal	growing Vaidya-De sitter black holes
$\mathcal{L}_n \theta_l > 0, \mathcal{L}_l \theta_l = 0$ (null generated by l^a)	expanding De sitter spacetime	time reversal
$\mathcal{L}_n \theta_l < 0, \mathcal{L}_l \theta_l < 0$ (spacelike)	time reversal	growing Vaidya black holes
	expanding FRW universes with $\frac{1}{3} < w \leq 1$	time reversal
$\mathcal{L}_n \theta_l < 0, \mathcal{L}_l \theta_l = 0$ (null generated by l^a)	time reversal	Schwarzschild black holes
$\mathcal{L}_n \theta_l = 0, \mathcal{L}_l \theta_l < 0$ (null generated by n^a)	expanding FRW universe with $w = \frac{1}{3}$	time reversal

In comparison with Bousso's covariant entropy conjecture

- Dominant energy condition
- Dynamical horizon versus light sheet

CEBC tested in expanding FRW universes

$$ds^2 = a^2(\eta)[-d\eta^2 + d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$\theta_{\pm} = \frac{\dot{a}}{a} \pm \frac{f'}{f}$$

$$T_{ab} = a^2(\eta) \{ \rho(\eta)(d\eta)_a(d\eta)_b + p(\eta)[(d\chi)_a(d\chi)_b + f^2(\chi)((d\theta)_a(d\theta)_b + \sin^2\theta(d\phi)_a(d\phi)_b)] \}$$

$$p = wp.$$

$$a = f^q \left(\frac{\eta}{q} \right),$$

$$\chi = \frac{\eta}{q}$$

$$s^a = \frac{s}{a^3} \left(\frac{\partial}{\partial t} \right)^a$$

$$q = \frac{2}{1+3w}$$

CEB conjecture for general dynamical horizon

$\theta_l = 0$	$\theta_n > 0$	$\theta_n < 0$
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CEBC tested in expanding FRW universes

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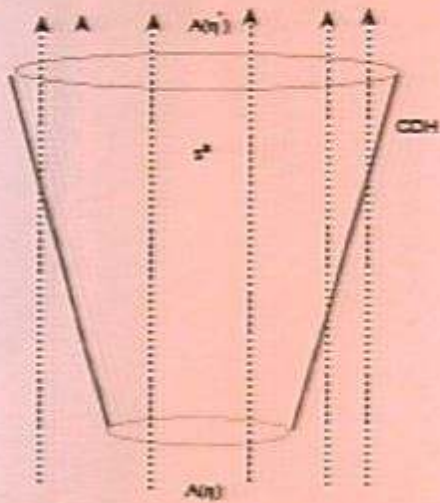
$$a = f^q\left(\frac{\eta}{q}\right),$$

$$\chi = \frac{\eta}{q}$$

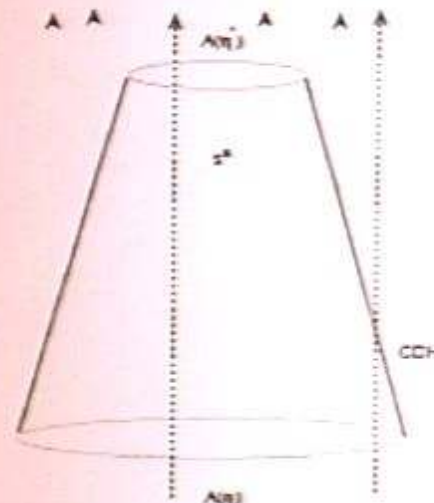
$$s^a = \frac{s}{a^3} \left(\frac{\partial}{\partial t}\right)^a$$

$$q = \frac{2}{1+3w}$$

CEBC tested in expanding FRW universes



$\Omega < 1/2$



$\Omega < 1$

CEBC tested in expanding FRW universes

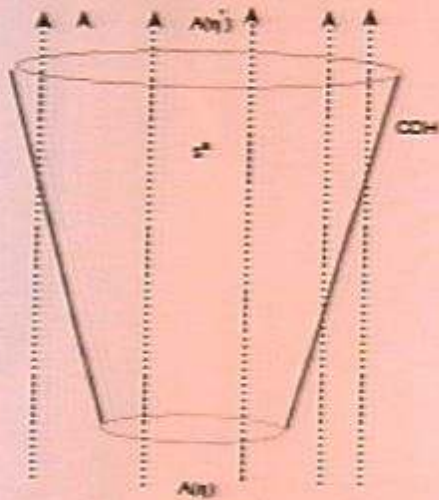
$$\frac{A(\eta')}{4} - S(\eta') \geq \frac{A(\eta)}{4} - S(\eta) \quad (q+1)f^{2q-1}\left(\frac{\eta}{q}\right) - 2s \geq 0$$

$$\frac{A(\eta')}{4} + S(\eta') \geq \frac{A(\eta)}{4} + S(\eta) \quad (q+1)f^{2q-1}\left(\frac{\eta}{q}\right) + 2s \leq 0$$

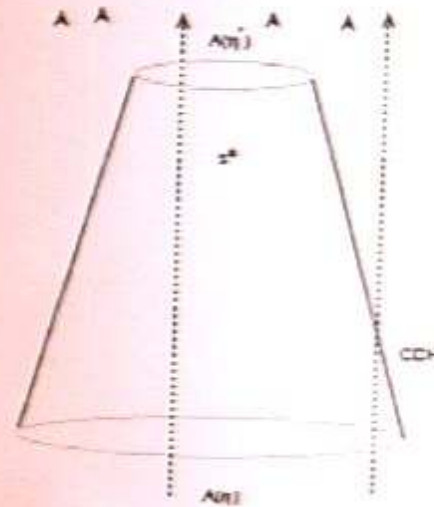
$$A(\eta) = 4\pi f^{2(q+1)}\left(\frac{\eta}{q}\right)$$

$$S(\eta) = 4\pi s \int_0^{\frac{\eta}{q}} d\chi f^2(\chi).$$

CEBC tested in expanding FRW universes



CS-12



CS-1

CEBC tested in expanding FRW universes

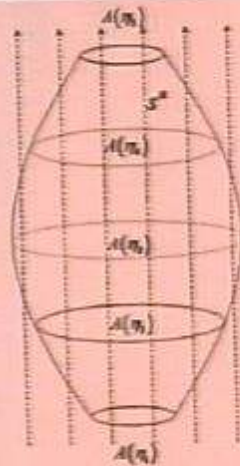
$$\frac{A(\eta')}{4} - S(\eta') \geq \frac{A(\eta)}{4} - S(\eta) \quad (q+1)f^{2q-1}\left(\frac{\eta}{q}\right) - 2s \geq 0$$

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$$A(\eta) = 4\pi f^{2(q+1)}\left(\frac{\eta}{q}\right)$$

$$S(\eta) = 4\pi s \int_0^{\frac{\eta}{q}} dx f^2(x)$$

As a Theoretical Constraint to Cosmological Physics



$$s \leq \frac{\sqrt{3(8\pi\rho a^2 + \Lambda a^2)}(1+w)^{1/2}}{2[\lambda - (1+3w)\rho]}$$

$$\rho = \frac{\rho_r^0 a_0^4}{a^4} + \frac{\rho_m^0 a_0^3}{a^3}$$

$$s\sqrt{\Lambda} \leq 2\sqrt{3\pi}\rho_m^0$$

Conclusions

- Our CEBC has been formulated to general cases and confirmed by various examples
- Our CEBC implies a novel but profound relation between cosmological constant and dust matter, which provides an alternative macroscopic approach to shed light on the longstanding cosmological constant problem

Outlook

- To investigate our CEBC in more realistic gravitational collapse
- To investigate our CEBC in universes with negative cosmological constant
- To investigate our CEBC in the inhomogeneous universes such as LTB universe
- To investigate our CEBC in the regime where quantum gravity effect becomes dominant such as loop quantum cosmology
- More importantly, the microscopic origin of our CEBC