Title: Bubbles of Nothing and Violations of the Energy Conditions in AdS-CFT

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Abstract: I describe a variety of bubbles of nothing which do not require a Kaluza-Klein circle but instead may be found in asymptotically flat or AdS spaces without any identifications. Â There are many such bubbles which expand outwards and threaten to destabilize spacetimes with more than four dimensions. Â In the AdS case, one can show there are both bubbles and topologically trivial smooth states which violate all of the energy bounds, both classical and quantum, in the corresponding gauge theory.

Bubbles of Nothing and Violations of the Energy Conditions in AdS-CFT

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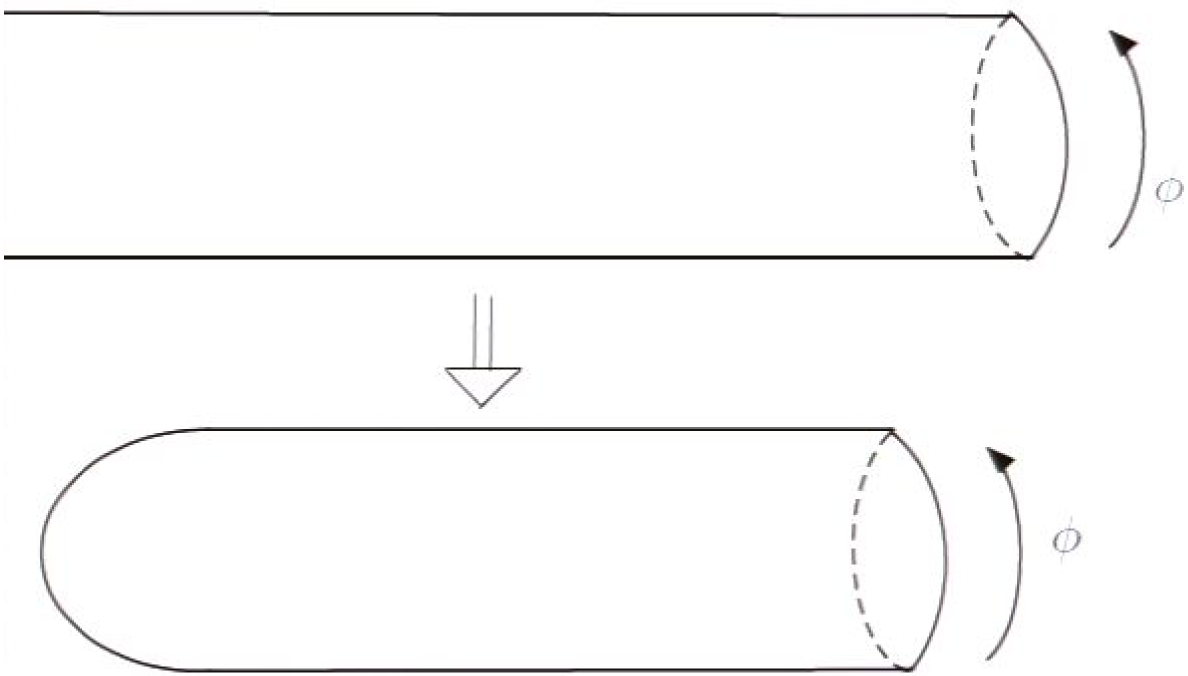
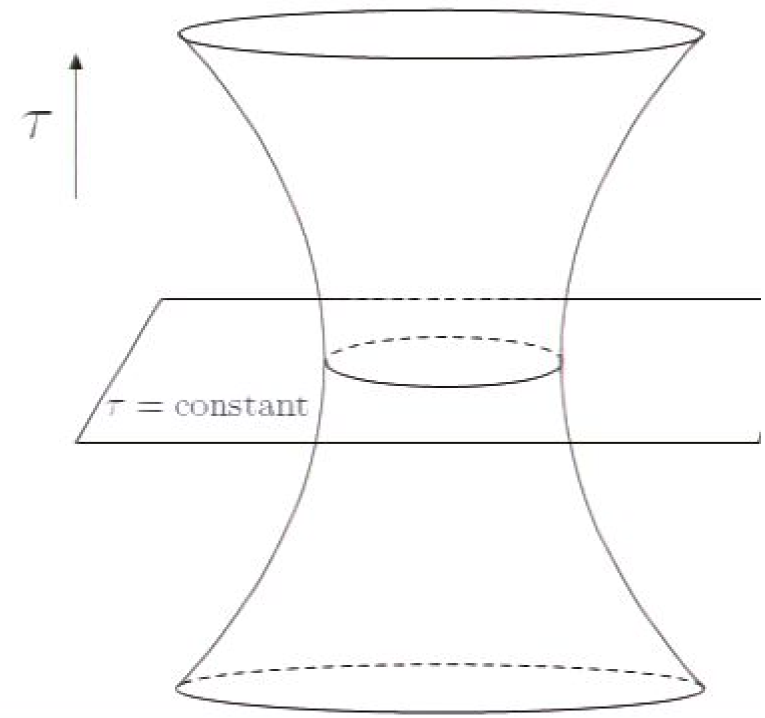
Of Bubbles past

- Witten's bubble: by analytically continuing 5d Schwarz. BH

$$ds^2 = -r^2 d\tau^2 + (1 - \frac{{r_s}^2}{r^2}) d\phi^2 + \frac{dr^2}{(1 - \frac{{r_s}^2}{r^2})} + r^2 \cosh^2(\tau) d\Omega_2$$





Are there bubbles in asymptotically flat and/or AdS space?

- Analytic continuation doesn't produce desired solutions;
  consider instead momentarily static states could produce quantum mechanicall

- Want time symmetric initial data satisfying constraints

- Can find asym. flat solutions inspired by BR consist of double bubbles which touch at a point
  - If one bubble much larger $(\gtrsim 2\times)$ then larger expands while smaller collapse
  - Can keep larger bubble size fixed and make arbitrarily light; as do so initial acceleration $\to \infty$

Single Bubbles

Consider metrics containing squashed sphere

$$ds^2 = \frac{dr^2}{W(r)} + \alpha(r)\left(\sigma_1{}^2 + \sigma_2{}^2\right) + \beta(r)\sigma_3{}^2$$

In terms of the Euler angles $(\bar{\psi}, \bar{\phi}, \bar{\theta})$:

$$\sigma_1 = \cos(\bar{\psi})d\bar{\theta} + \sin(\bar{\psi})\sin(\bar{\theta})d\bar{\phi}$$
$$\sigma_2 = -\sin(\bar{\psi})d\bar{\theta} + \cos(\bar{\psi})\sin(\bar{\theta})d\bar{\phi}$$
$$\sigma_3 = d\bar{\psi} + \cos(\bar{\theta})d\bar{\phi}$$

- Note $\sigma_1{}^2 + \sigma_2{}^2 = d\bar{\theta}^2 + \sin^2(\bar{\theta})d\bar{\phi}^2$

and $\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 = 4\left(d\theta^2 + \sin^2(\theta)d\psi^2 + \cos^2(\theta)d\phi^2\right)$

$\longrightarrow$ If $\alpha = \beta = \frac{r^2}{4}$ have undistorted S^3

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$\longrightarrow$ If $\alpha = \beta = \frac{r^2}{4}$ have undistorted S^3

Constraint gives a 1st order linear ODE for W(r) for any $\alpha(r)$ and $\beta(r)$

$$(4)\, R = \frac{2}{\alpha} - \frac{\beta}{2\alpha^2} - \left[\frac{\alpha'}{\alpha} + \frac{\beta'}{2\beta}\right]W' - 2\left[\frac{\alpha''}{\alpha} + \frac{\beta''}{2\beta} + \frac{\alpha'\beta'}{2\alpha\beta} - \frac{\alpha'^2}{4\alpha^2} - \frac{\beta'^2}{4\beta^2}\right]W = -\frac{12}{l^2}$$

- May find bubbles of any size or topologically trivial metrics with squashed S^3
- Can find asymptotically flat or global AdS sols
- Free of apparent horizons
- Gauge fixing:

$$ds^2 = \frac{dr^2}{W(r)} + \alpha(r)\left(\sigma_1{}^2 + \sigma_2{}^2\right) + \beta(r)\sigma_3{}^2$$

May take

$$\alpha = \frac{r^2}{4} \qquad \beta = \frac{r^2}{4}\gamma$$

$\longrightarrow$ Can view each bubble as different way γ go from 0 to 1

Dynamics

- Can use Hamiltonian evol. eqns $(N^i = 0)$
  to study initial dynamics near moment of time symmetry $(t = 0)$
- $\ddot{A}(0) = 8\pi N^2 \left(\frac{1}{r_0 \gamma'(r_0)} - 1 - \frac{{r_0}^2}{l^2} \right)$
- Bubbles which collapse (semiclassically) should produce black holes
- If bubble expand for any sig. period of time spacetime far away from initial disturbance is radically altered $\longrightarrow$ Instability!
- Single bubbles threaten to eat up whole space. Even if stop expanding, collision of multiple bubbles may produce singularities.
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- For asym. flat or small AdS bubbles ($r_0 \ll l$) $E \sim \frac{{r_0}^2}{G}$

  while for large AdS bubbles $E \sim \frac{{r_0}^4}{l^2 G}$

- Stress Tensor: (Balasubramanian & Kraus hep-th/9902121; de Haro, Skenderis & Solodukhin 0002230 ;.....)

$$\mathbf{T} = \rho\, e_t e_t + p_\alpha \left(\sigma_1\sigma_1 + \sigma_2\sigma_2\right) + p_\beta\, \sigma_3\sigma_3$$

$$\alpha(r,t) = \frac{r^2}{4}\left(1 + \delta\alpha\, \frac{l^4}{r^4} + \mathcal{O}\left(\frac{1}{r^{4+\epsilon}}\right)\right)$$

$$\beta(r,t) = \frac{r^2}{4}\left(1 + \delta\beta\, \frac{l^4}{r^4} + \mathcal{O}\left(\frac{1}{r^{4+\epsilon}}\right)\right)$$

$$p_\alpha = \frac{1}{16\pi G}\left(\frac{1}{16} + \frac{2GM}{3\pi l^2} + \frac{1}{3}(\delta\alpha - \delta\beta)\right)$$

$$p_\beta = \frac{1}{16\pi G}\left(\frac{1}{16} + \frac{2GM}{3\pi l^2} + \frac{2}{3}(\delta\beta - \delta\alpha)\right)$$

- In fact can fairly easily find examples where p_α or p_β is negative and has magnitude parametrically larger than M

e.g.

$$\gamma = 1 + \frac{1}{a + \epsilon \frac{r^4}{l^4}} \qquad \text{for} \quad r > r_1$$

where $|a| \gg 1, \ |\epsilon| \ll 1$ and $a\epsilon > 0$

Then

$$\delta\beta - \delta\alpha = \frac{1}{\epsilon}$$

while

$$M \sim \frac{1}{a\epsilon}\frac{l^2}{G} + \mathcal{O}\left(\frac{r_1^2}{G}\right)$$

Can find many γ with same leading order mass and pressures, i.e.

$$\gamma = 1 + \frac{1}{a e^{-\frac{\epsilon}{a}\frac{r^4}{l^4}} + \epsilon \frac{r^4}{l^4}} \qquad \text{for} \quad r > r_1$$

- Can violate all standard classical energy conditions (strong, weak, dominant) parametrically

  $\longrightarrow$ Boosted observers see regions of negative energy

- Note there are bubbles and topologically trivial metrics which violate energy conditions. There are also many bubbles which don't.

- Worry? Well known that can quantum mechanically can violate classical energy conditions for short time periods

$$\bar{\rho}\,(\Delta t)^4 \geq C_0$$

AdS Evolution

$$ds^2 = g_{tt}(r,t)dt^2 + \frac{dr^2}{W(r,t)} + \alpha(r,t)\left(d\bar{\theta}^2 + \sin^2(\bar{\theta})d\bar{\phi}^2\right) + \beta(r,t)\left(d\bar{\psi} + \cos(\bar{\theta})d\bar{\phi}\right)$$

$$\alpha(r,t) = \frac{r^2}{4}\left(1 + \delta\alpha\,\frac{l^4}{r^4} + \mathcal{O}\left(\frac{1}{r^{4+\epsilon}}\right)\right)$$

$$\beta(r,t) = \frac{r^2}{4}\left(1 + \delta\beta\,\frac{l^4}{r^4} + \mathcal{O}\left(\frac{1}{r^{4+\epsilon}}\right)\right)$$

Note if take a gauge ("static gauge") where g_{tt} indep. of t through order $\frac{1}{r^2}$

EOM at leading order and energy cons. implies to same order so is g_{rr} and

$$\delta\beta + 2\delta\alpha = \text{constant}$$

$\longrightarrow$ May view asym. dynamics entirely in terms of squashing

and energy conservation restricts form but not magnitude of squashing

Linearized evolution:

- First consider time dependence of asym. metric

- If consider $\gamma = 1 + \dfrac{1}{ae^{-\frac{\epsilon}{a}\frac{r^4}{l^4}} + \epsilon\frac{r^4}{l^4}}$ for $r > r_1$

at least until times of order $\frac{\pi l}{2}$, $\delta\alpha = 0$ and $\delta\beta = \frac{1}{\epsilon}$ (stat. gauge)

$\longrightarrow$ Recalling $M \sim \frac{1}{a\epsilon}\frac{1}{G}$, if $a \gg 1$,

$p_\beta \approx \frac{1}{16\pi G}\frac{2}{3\epsilon}$ and $p_\alpha \approx -\frac{1}{16\pi G}\frac{1}{3\epsilon}$

Generically pressures dynamic; e.g. take $\gamma = 1 + \frac{1}{ae^{-\frac{\epsilon}{a}\frac{r^6}{l^6}} + \epsilon\frac{r^6}{l^6}}$ for $r > r$

- $\delta\alpha = -\frac{1}{2}\delta\beta$ and

$$\delta\beta = \frac{2}{3\epsilon}\left[6\frac{t^2}{l^2} + 14\frac{t^4}{l^4} + \frac{108}{5}\frac{t^6}{l^6} + \frac{2734}{105}\frac{t^6}{l^6} + \frac{125444}{4725}\frac{t^8}{l^8} + \mathcal{O}\left(\frac{t^{10}}{l^{10}}\right)\right]$$

where can show all terms in series have same sign

- Coefficients in series determined iteratively but no simple pattern and even convergence unclear; before loose control at $t \approx \frac{\pi l}{2}$

$$|\delta\beta| \gtrsim \frac{10^7}{|\epsilon|}$$

- $M \sim \frac{1}{a^{2-\frac{2}{3}}\epsilon^{\frac{2}{3}}}\frac{l^2}{G}$

Mode Analysis:

- Mode frequencies are real and discrete and modes can be given in terms of the complete orthogonal Jacobi polynomials

- Resulting mode sum does not converge pointwise but only in a L^2 sense

- Generic L^2 normalizable function does not meet usual AdS bdy conditions;

  in particular allows perturbations which grow asymptotically as long as not do so as fast as r

- May impose stronger falloff conditions on initial data but doesn't seem to be any reason to believe such restrictions will be preserved under evolution.

  While resulting sum is oscillatory is strict mathematical sense,

  $\delta\alpha$ and $\delta\beta$ may diverge in finite time

Summary & Open Questions

- New solutions (single and double bubbles of nothing) which describe a new threat to stability of higher dimensions

- In regards to AdS/CFT would like to understand dual gauge theory states;

  Can make sense of QFT with matter which violates energy conditions ?

- Is rapid growth of gravity modes instability or just evolution into more generic state ?

  If diverge in finite time should be possible to show explicitly (work in progress)

- Appears both ansatz can be extended to more than five dimensions