Title: Limits from the Perturbative Regime of Inflation and Non-Gaussianity

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Abstract: The validity of the perturbative analysis during inflation imposes bounds on the inflationary parameters. For single field inflation, the current experimental bounds on non-Gaussianity necessarily imply that the physics is weakly coupled at CMB scales. In this talk, I will show that for models with a scale dependent sound speed, the system can become strongly coupled at lower scale. I will also discuss multiple field models which can produce non-Gaussianity at CMB scales. In these scenarios, the extra scalar fields are strongly coupled in a large part of the parameter space.

Limits from the Perturbative Regime of Inflation and Non-Gaussianity.

Louis Leblond
Texas A&M

Pascos 08
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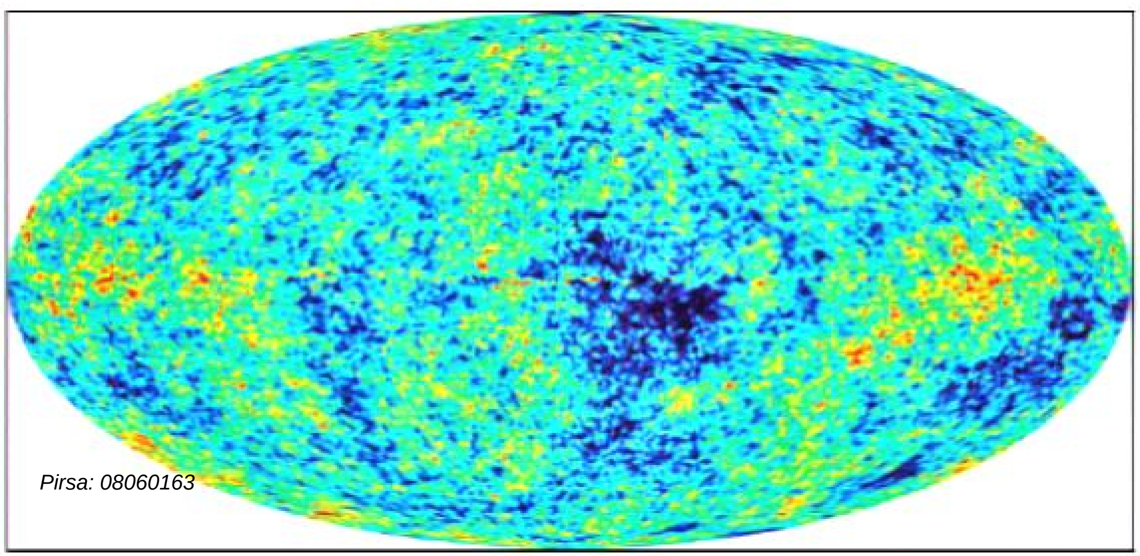
arXiv:0802.2290
arXiv:0805.1229

L.L. and Sarah Shandera
Bhaskar Dutta, Jason Kumar, L.L.

Non-Perturbative Physics in the CMB?

- CMB experiment tells us that anisotropies are small, Gaussian and nearly scale invariant.
- Pointing towards a weakly coupled system.
- Can we have non-Perturbative physics in Inflation?



Basic Idea and Procedure

- We can think of inflation as an effective field theory for the fluctuations around a nearly de-Sitter background. For a given action, we can expand in power of fluctuations

$$S = S_0 + S_2 + S_3 + \dots$$

- where S_0 is the background action, S_1 vanishes when you satisfy equation of motion, S_2 is the free field part and S_3 represents the interactions terms.

$$S_0 \quad > \quad S_2$$ Gradient Energy Conditions

$$S_2 \quad > \quad S_n,\ n \geq 3$$ Perturbative Regime

Single Field, General Sound Speed - 1

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2 R + 2P(X,\phi)\right)$$

e.g. K-inflation,
DBI-inflation
Ghost inflation

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad c_s^2 = \frac{dP}{d\rho} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

- This action is highly non-linear, there should be extra terms in addition. EFT in inflation

  Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore
  Weinberg
  Armendariz-Picon, Fontanini, Penco, Trodden

- Use comoving gauge where we set the perturbations of the inflaton to zero and only remains the curvature perturbations.

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

- Using the ADM formalism, we can write the action purely in terms of a single field.

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Single Field, General Sound Speed - 2

$$S_0 \sim V(\phi) \sim H^2 M_p^2$$

$$S_2 = M_p^2 \int dt d^3x \left(a^3 \frac{\epsilon}{c_s^2} \dot{\zeta}^2 - a\epsilon(\partial\zeta)^2 \right)$$

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Maldacena,
Seery Lidsey,
Chen, Huang, Kachru, Sh

- The usual procedure is to calculate the variance of ζ assuming
  - the perturbations are small compared to the background $S_0 > S_2$
  - the interaction terms are small $S_2 > S_3$

Single Field, General Sound Speed - 3

- Under these assumptions,

$$\mathrm{P}^{\zeta} \sim < \zeta^2 >$$

$$< \zeta^2 >^{1/2} \sim \frac{H}{2\pi M_p \sqrt{2\epsilon c_s}}$$

- Plug back the answer and check the assumptions again

	Slow-roll	Small Sound Speed
Gradient Energy Bound	$\frac{H^2}{M_p^2} < 1$	$\frac{H^2}{M_p^2} < c_s^3$
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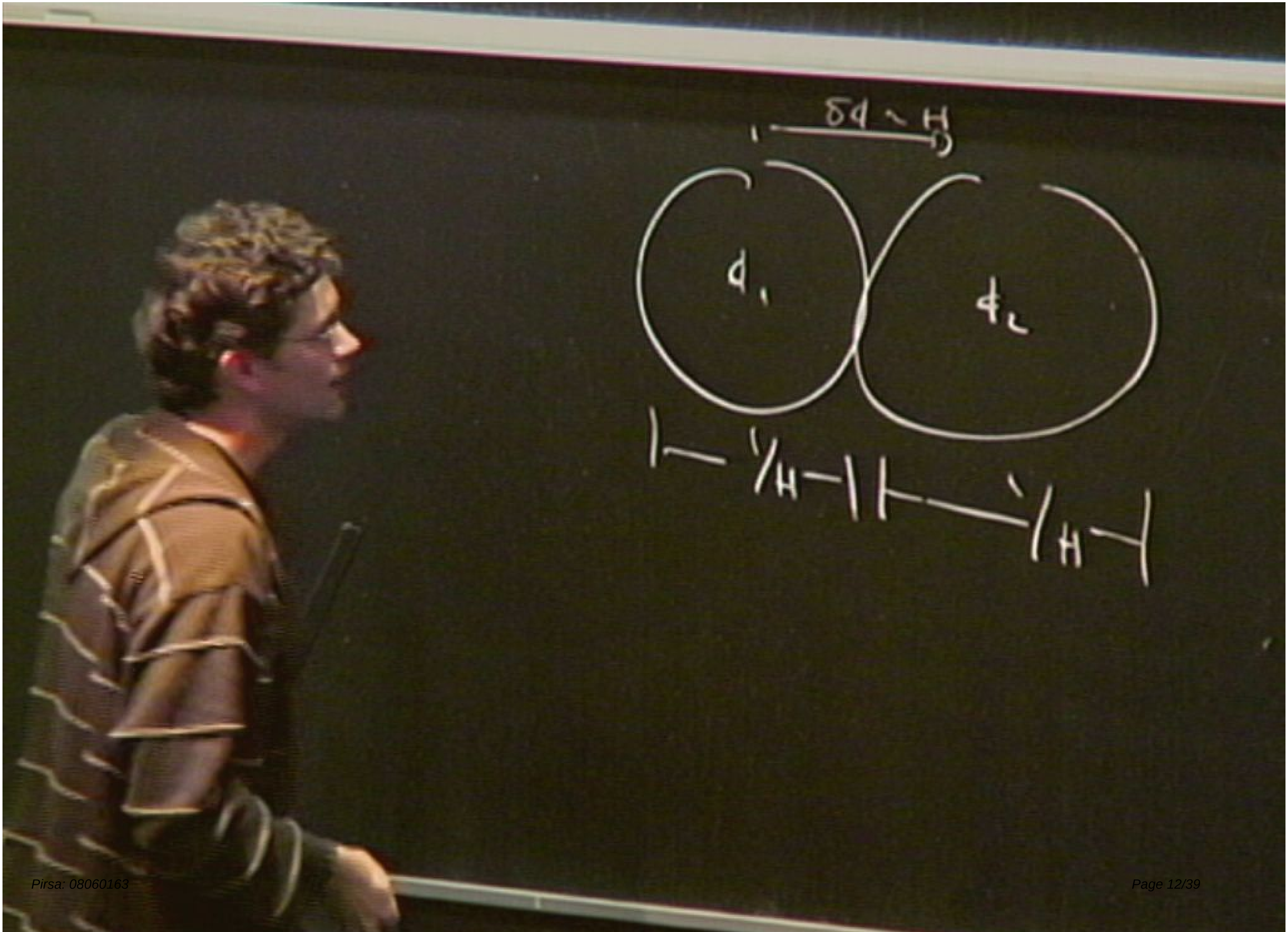
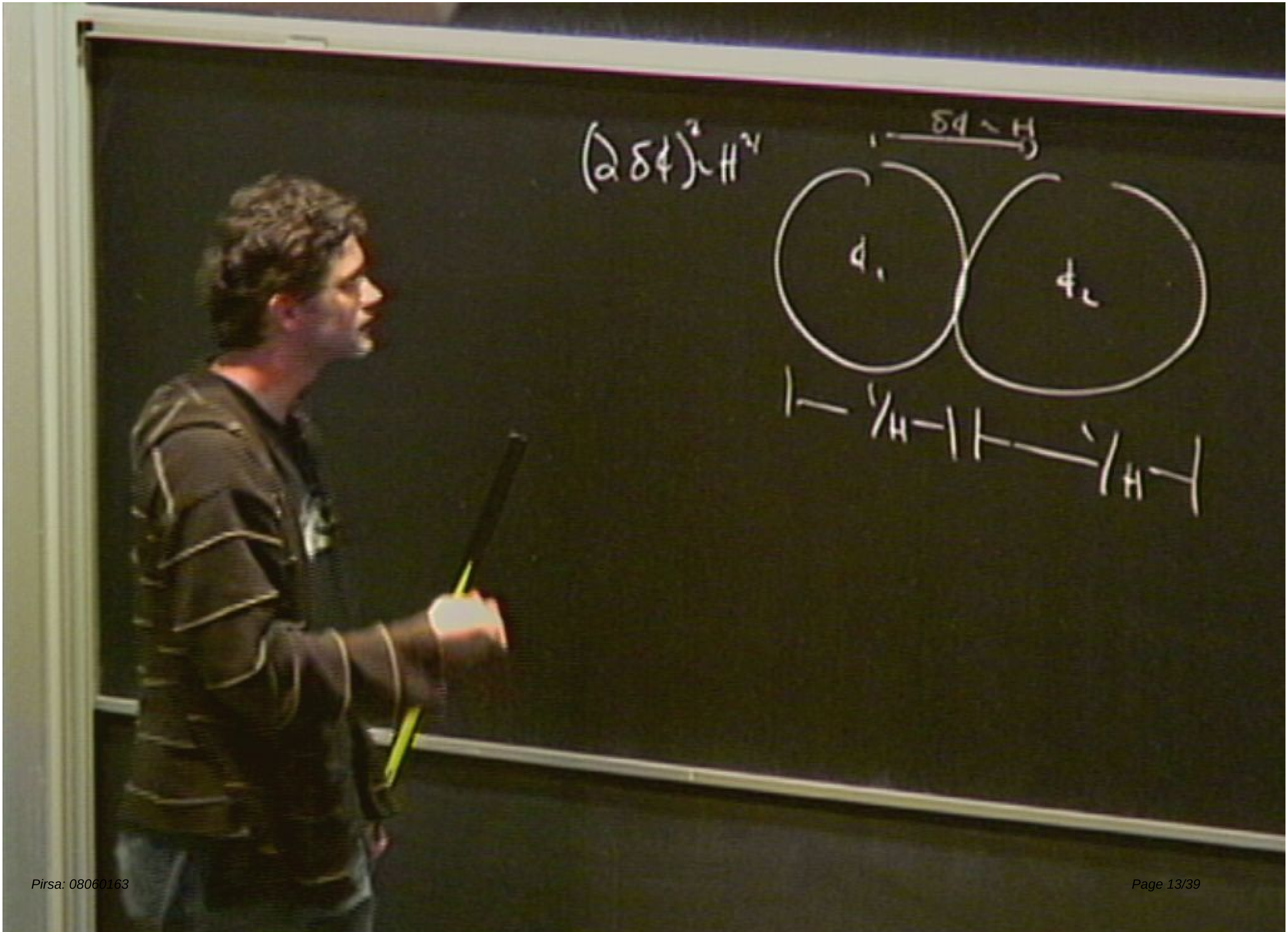

δφ ~ H
φ₁
φ₂
1/H
1/H



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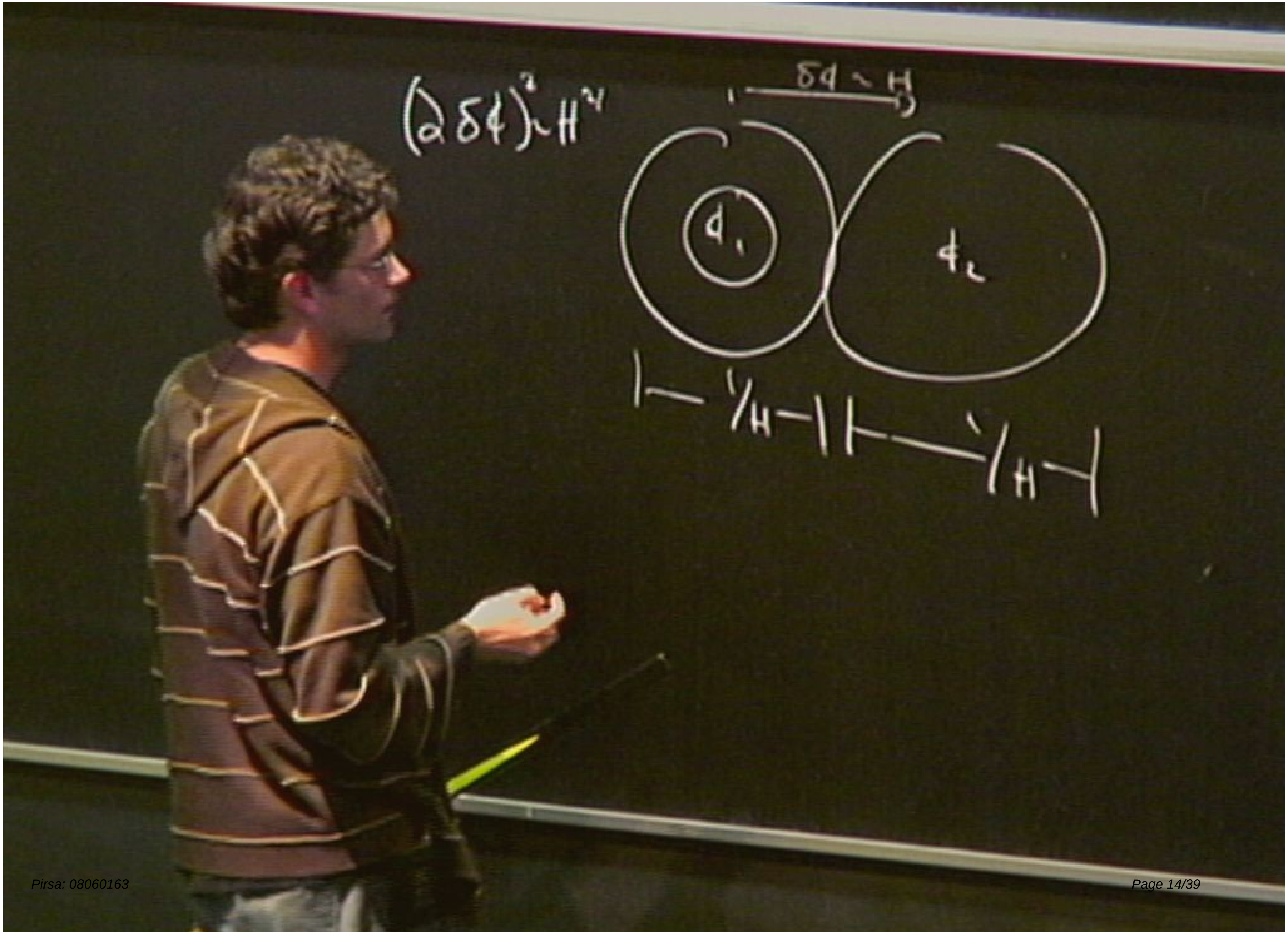

$(2\delta\phi)^2 \sim H^4$
$\delta\phi \sim H$
ϕ_1
ϕ_2
$1/H$
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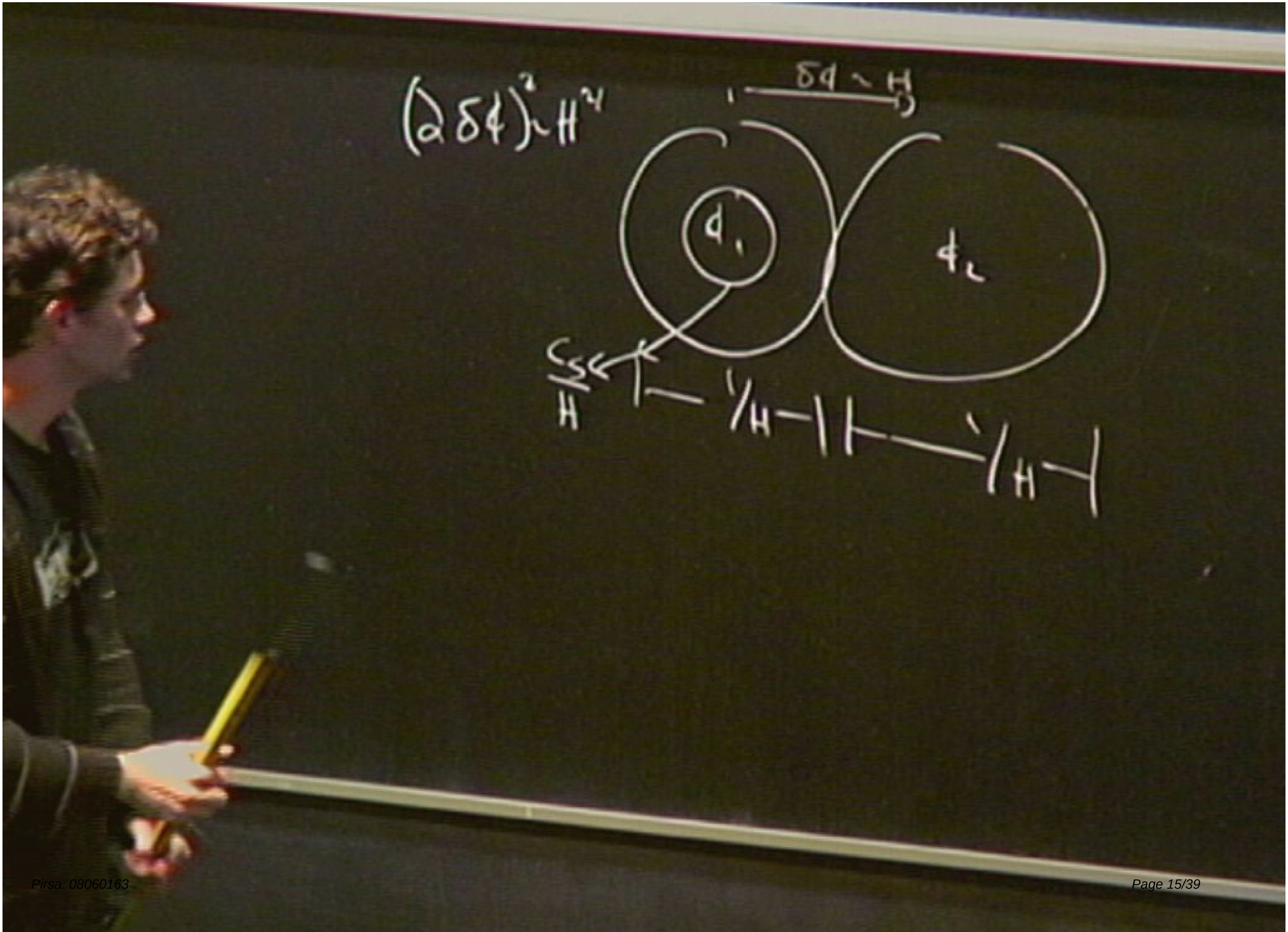
$(\partial\delta\phi)^2 \sim H^4$
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Comments on the method

- This bounds are the minimum required, they become even more restrictive when one include the fact that they should be valid for all time.

Mukhanov, Abramo, Brandenberger

- For the backreaction, include the stochastic behavior. Loops calculations (get in addition log or power law factors) ...

Weinberg, Seery, Sloth, Lyth,...

- Main interesting bound, there exists a minimum sound speed before losing perturbative control.

$$\zeta(\vec{x},t) = \zeta_{Gauss} + \frac{3}{5} f_{NL}(\zeta^2_{Gauss} - <\zeta^2_{Gauss}>)$$

$$c_s^4 \quad > \frac{H^2}{M_p^2 \epsilon c_s} \sim P_\zeta$$

$$f_{NL} \sim \frac{1}{c_s^2}$$

$$\mathrm{P}^\zeta \sim 10^{-9} \qquad |f_{NL}| < 10^{9/2}$$

$$|f_{NL}| < 300$$

Creminelli et a

But experimental bound is weaker

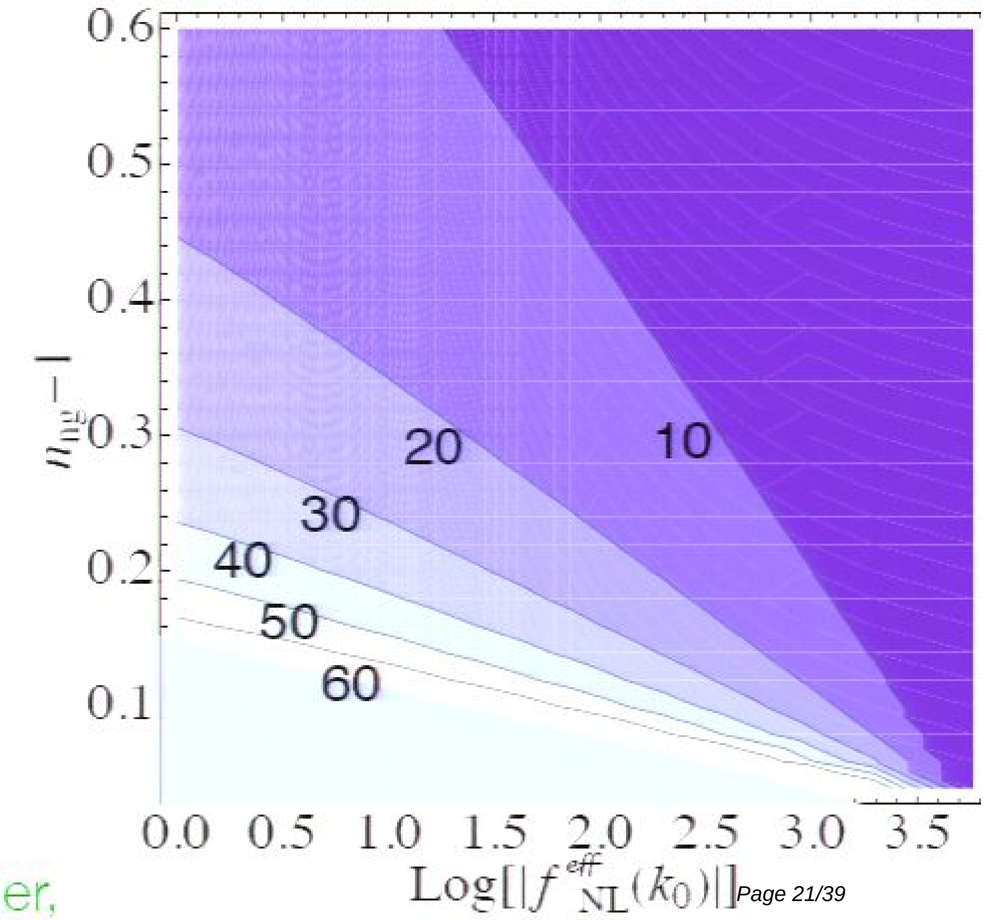
Scale Dependence and DBI

- If the sound speed decreases with scales, the system can become non-perturbative on larger scales

$$c_s(k) = c_s(k_0)\left(\frac{k}{k_0}\right)^s$$

$$f_{NL} = f_{NL}(k_0)\left(\frac{k}{k_0}\right)^{n_{NG}-1}$$

In model of DBI inflation, the sound speed is usually very strongly scale dependent



Could be important for large scale structure

LoVerde, Miller, Shandera, Verde

Louis Leblond, Pascos 08, Perimeter

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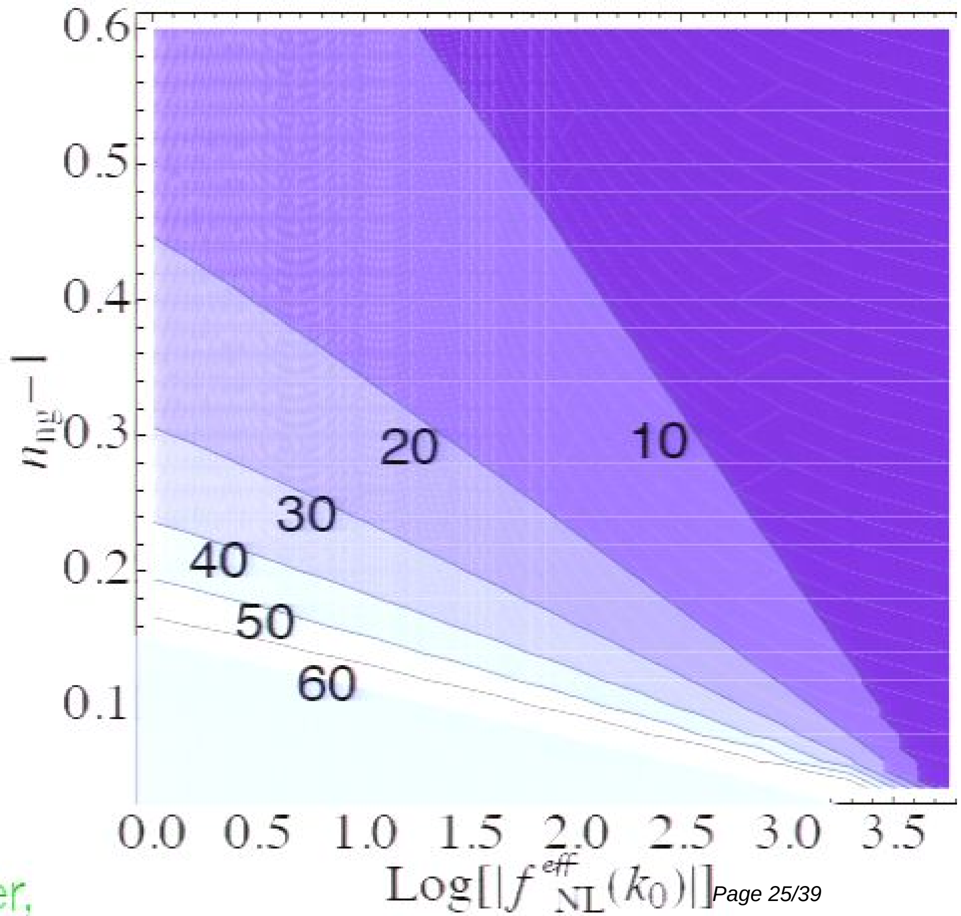

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Comments on Eternal Inflation

- Eternal inflation occur when the quantum fluctuations of the inflaton are of the same order as the classical motion. This translate to order 1 curvature perturbations $\zeta \sim 1$ $P^{\zeta} \sim 1$ Creminelli, Dubovsky, Nicoli Senatore, Zaldarriaga
- For slow-roll, eternal inflation is possible $$P_{\zeta} < \frac{1}{\epsilon^2}$$
- But for small sound speed

$$P^{\zeta} < c_s^4 < 1$$

- There is no point locally, where one can eternal inflate in the perturbative regime if the sound speed is small. Tolley, Wymar Helmer Winitzl
- eternal inflation is naively easier (sound speed enhancement)

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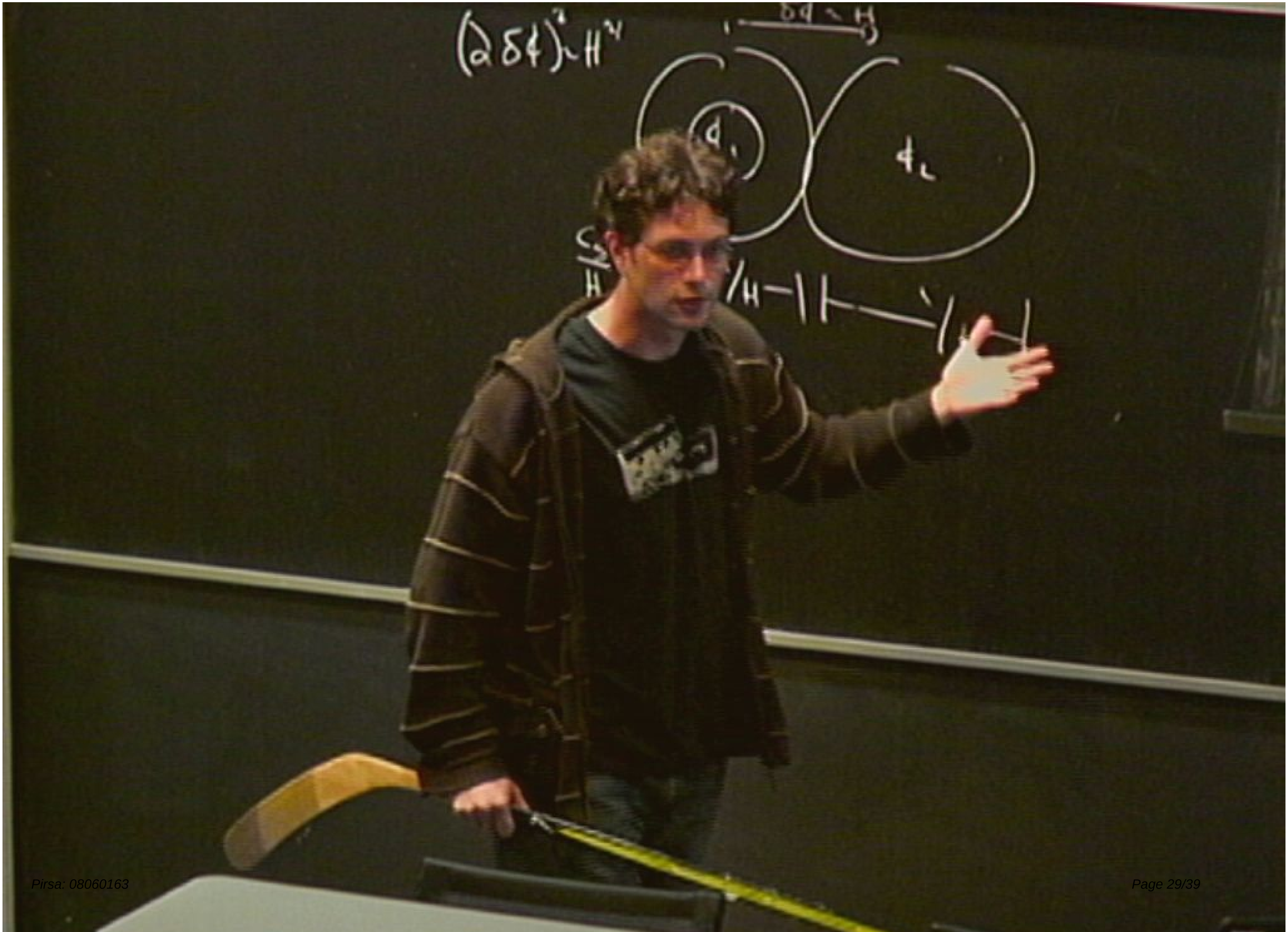

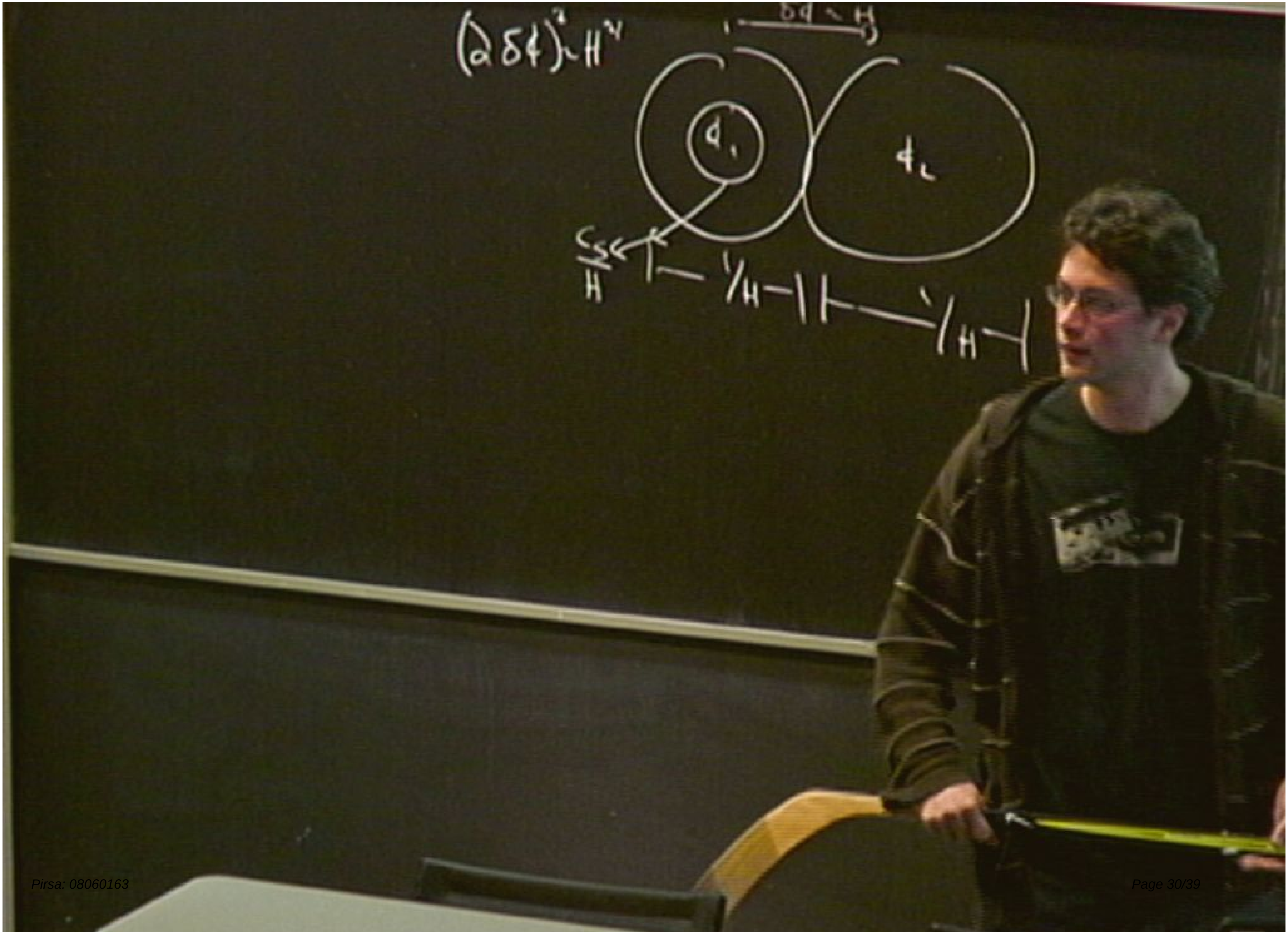

Multi-fields Large N bound

- Can also look a multiple field inflation and put bounds on the number of fields.

$$S = \int d^4x \sqrt{g} \sum_i \left(\frac{1}{2} \dot{\phi}_i^2 \right) - W(\phi_i)$$

$$\frac{H\sqrt{N}}{M_p} < 1$$

$$\frac{H\sqrt{N\epsilon}}{M_p} < 1$$

Agree with entropy bound and black hole bounds

Fischler, Susskind
Easther, Lowe

Dvali
Dvali,Lust

Limits loop contribution

Weinberg

Louis Leblond, Pascos 08, Perimeter

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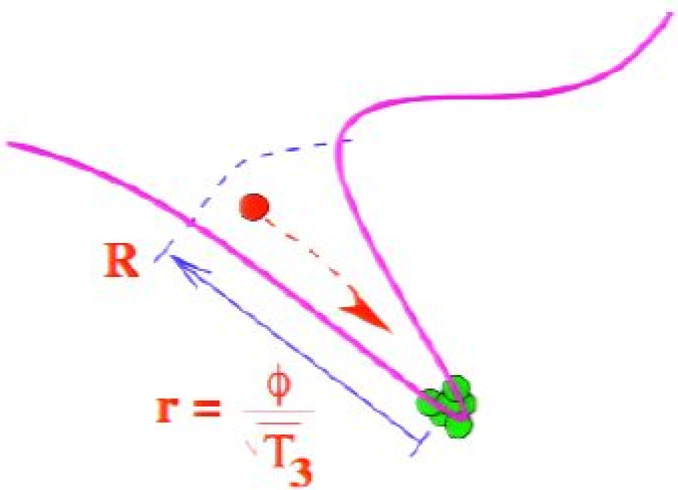
A duality between small sound speed and large N?

- Small sound speed system is dual to a large field (DBI-inflation)

Alishahiha, Silverstein, Tong



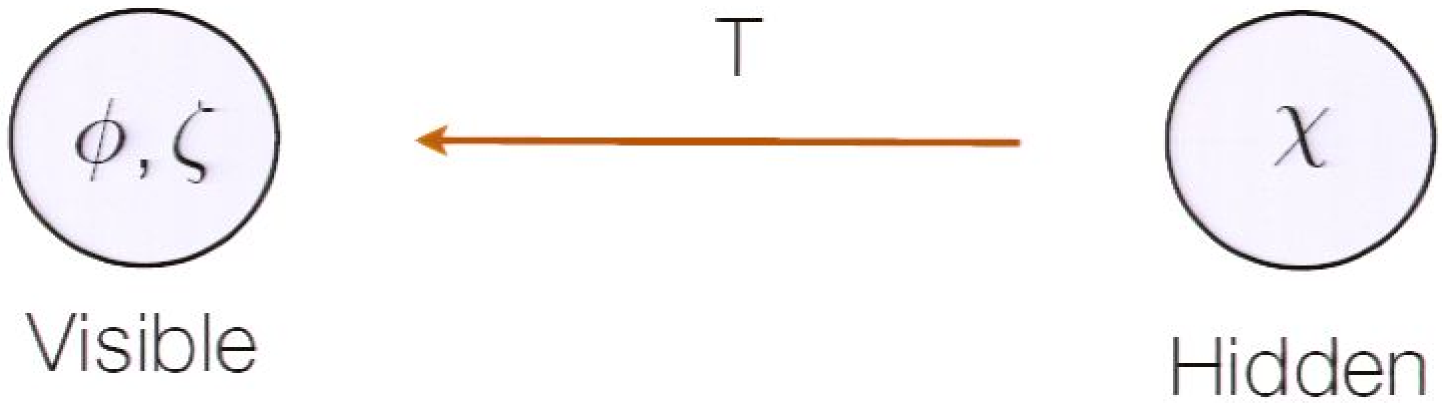
⟷ Large N (color) SYM theory

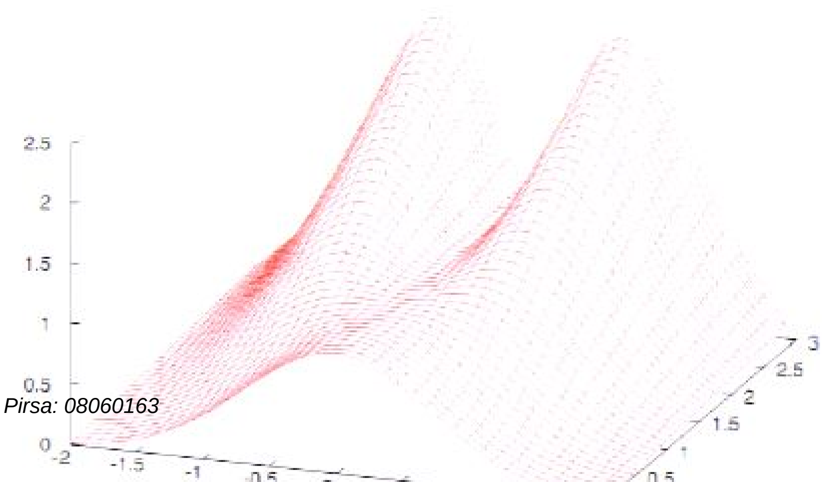
Minimum sound speed ⟷? Maximum number of fields

$$\frac{H^2}{M_p^2 \epsilon} < c_s^5 \qquad \frac{H\sqrt{N\epsilon}}{M_p} < 1$$

Tachyon Mediated Non-Gaussianity

- The inflaton sector needs to be perturbative on CMB scale





Natural in hybrid inflation

Bernardeau, Brunie
Alabidi, Lyth,
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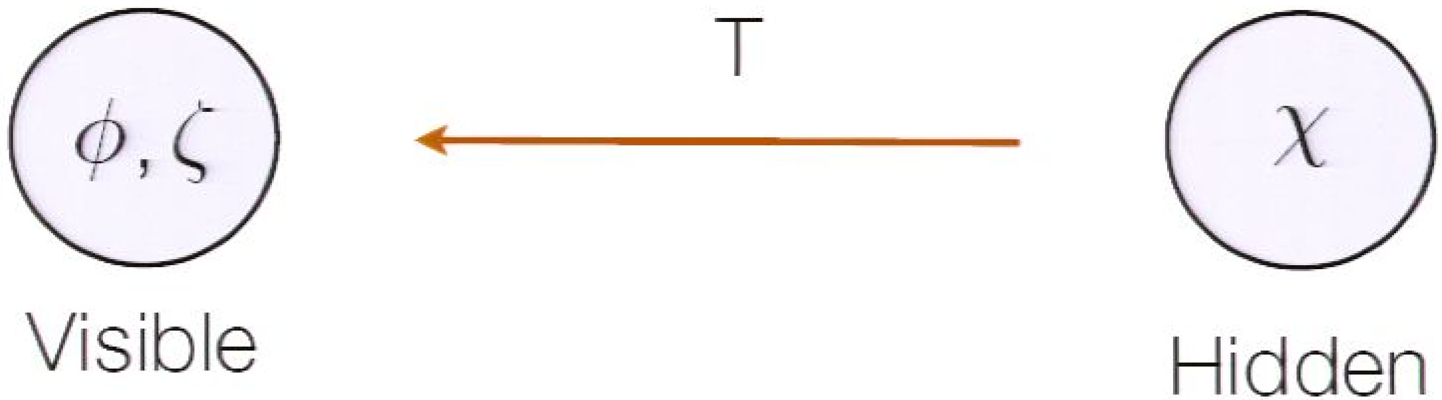

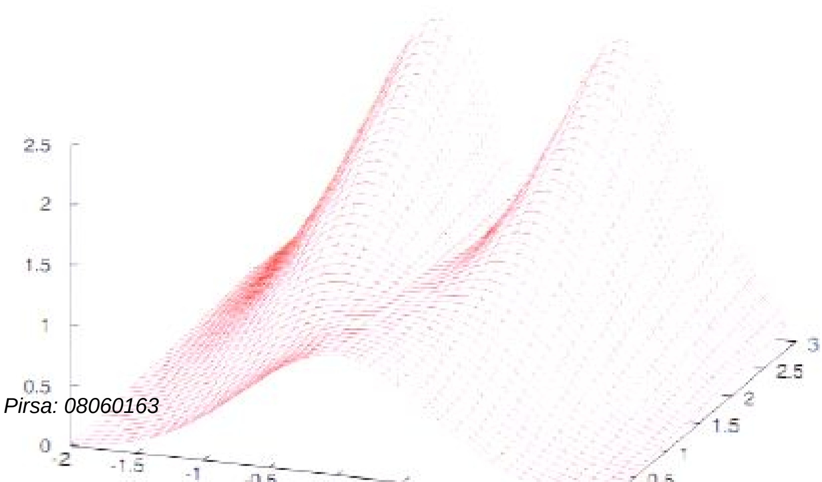

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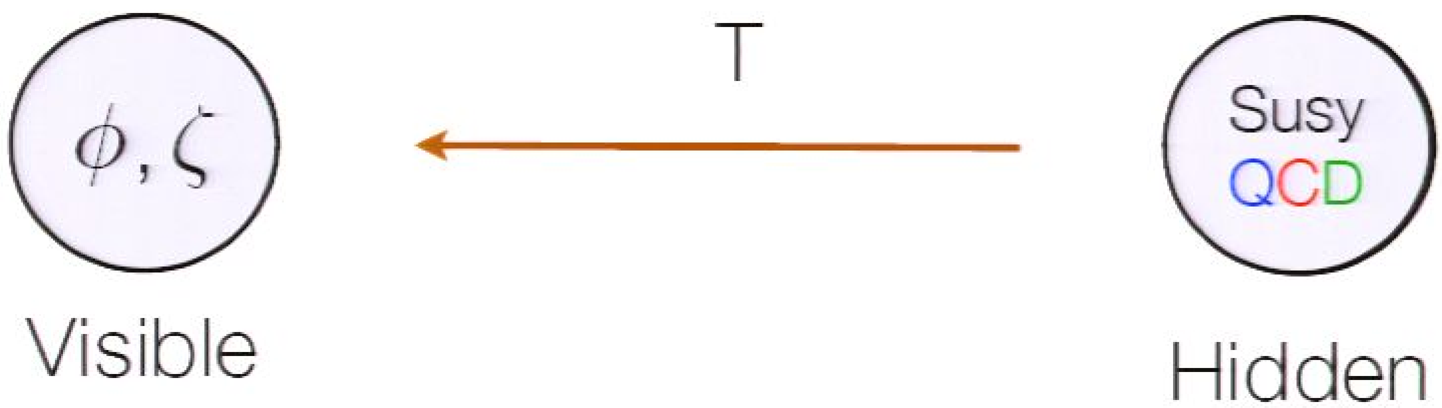

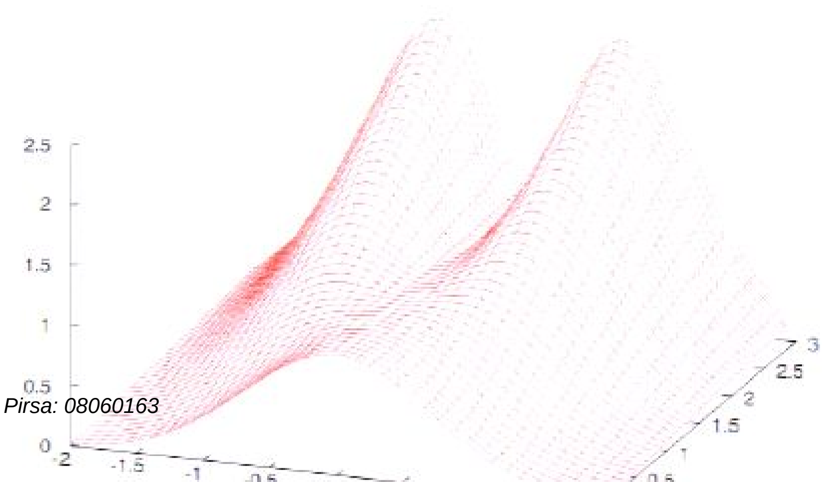

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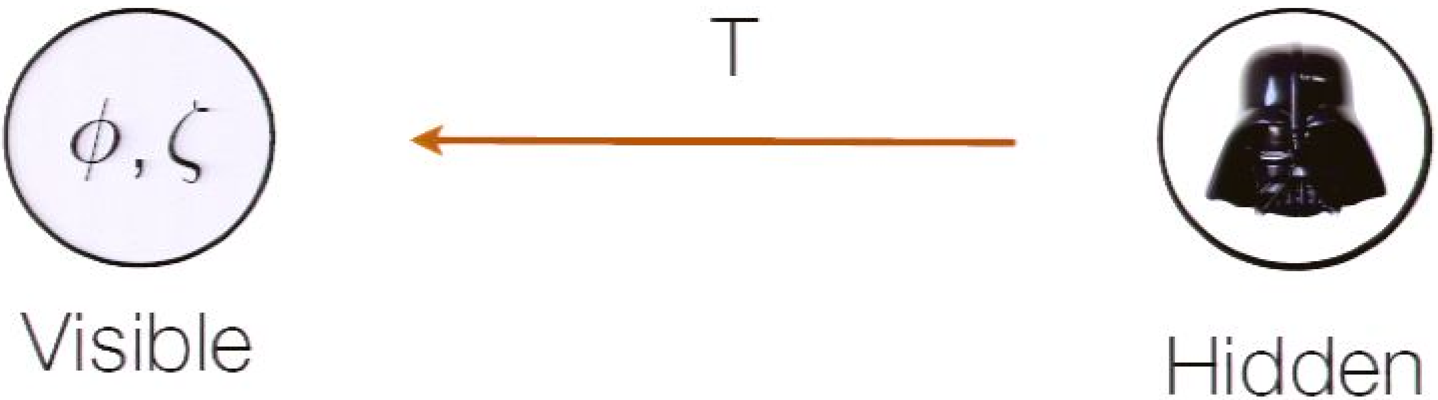

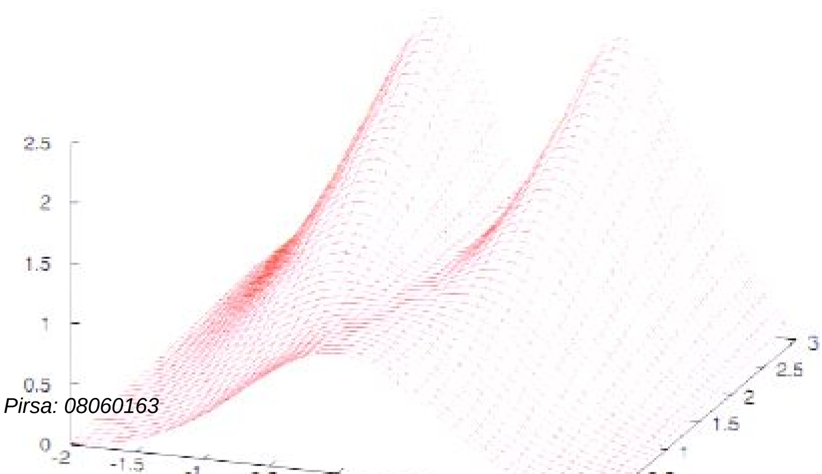

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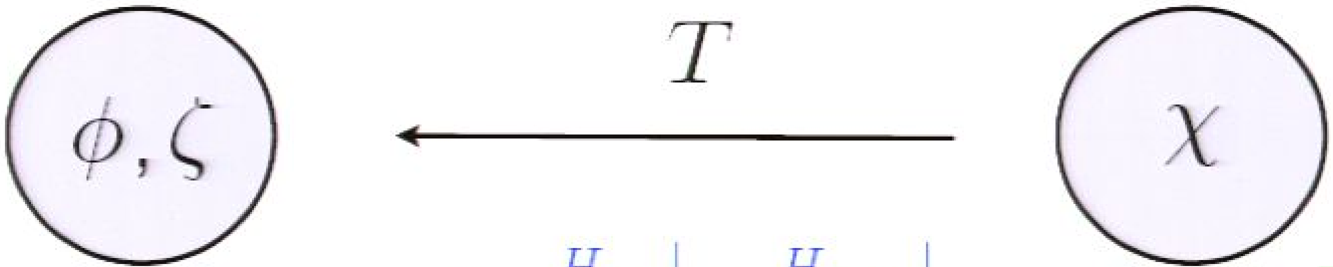
A contribution to f_NL



$$\zeta = -\frac{H}{\dot\phi}\delta\phi\Big|_* + \frac{H}{\dot\phi}\gamma\,\delta\chi\Big|_{\phi_c}$$

Bernardeau, Bruni

$$f_{NL} \sim \frac{\epsilon_f^{1/2} N_e M_p \gamma^3}{H^2} V_{,\chi\chi\chi} \longleftarrow F(\vec{k}_1, \vec{k}_2, \vec{k}_3) \sim \frac{H^2 N_e}{(k_1 k_2 k_3)^2} V_{,\chi\chi\chi}$$

“transfer function” $\gamma \equiv \frac{\partial \phi_c}{\partial \chi}\Big|_{\phi_c}$

Falk, Rangarajan, Srednicki, '93
Zaldarriaga
Lyth, Malik Seery

Barnaby, Cline

- In order to get NG of order 100 for the curvature spectrum, we need bigger NG for χ (for $\gamma < 1$)

Bounds $\gamma > 2(P_2^{\zeta})^{1/6}\left(\frac{f_{NL}}{N_e}\right)^{1/3} \sim 0.07$

Louis Leblond, Pascos 08, Perimeter

Conclusion

- It could be possible to see non-perturbative physics in inflation.
- CMB bound on Non-Gaussianity is telling us that for single field we need to be in a perturbative regime at CMB scales
- but it could become strongly interacting on larger scales (important fact for DBI!)
- or we could have strongly interacting physics in a Hidden sector
- Validity of the perturbative analysis imposes bounds.