Title: Bubbling AdS and Matrix Models

Date: Jun 05, 2008 03:30 PM

URL: http://pirsa.org/08060161

Abstract: In this talk we will focus on the supergravity duals of BPS states in N=4 Super Yang-Mills. In particular, we will describe how one can obtain a universal AdS bubbling picture for 1/4 and 1/8 BPS geometries, in analogy with the well-established 1/2 BPS droplet picture of LLM. In addition, we will show how interactions of two-matrix (1/4 BPS) states can be understood in terms of those of the much simpler single matrix (1/2 BPS) states.

10^{th} Anniversary of AdS/CFT Conjecture

passed many checks

Holographic Duality:

Gravity

CFT on boundary

In its original incarnation:

IIB string theory on $AdS_5 \times S^5 \longleftrightarrow$ $\mathcal{N}=4$ U(N) SYM in 4D

(fields) (operators)

Hints of Relation Between Theories:

Write $AdS_5 \times S^5$ as an embedding:

$$\boxed{-X_{-1}^2 - X_0^2 + X_1^2 + \ldots + X_4^2 = R^2_{AdS_5}}$$

$$\boxed{X_1^2 + X_2^2 + \ldots + X_6^2 = R^2_{S^5}}$$

$SO(2,4) \times SO(6)$

CONFORMAL GROUP IN 4D

R-SYMMETRY GROUP

6 adjoint scalars ϕ_i of $\mathcal{N}=4$ SYM

WILL PLAY CRUCIAL ROLE

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Early Stages of AdS/CFT:

Perturbations on $AdS_5 \times S^5$ $\longleftrightarrow$ Certain CFT states

Natural to ask:

- Can one go beyond perturbative description?
- CFT dual of more complicated geometries?

Larger questions:

- What geometries can we reconstruct from gauge theory data?
- More realistic strongly coupled field theories?

Outline

- Basics of AdS/CFT dictionary in 1/2 BPS sector
  - most developed case, nice features arise

- Reduced SUSY (1/4 and 1/8 BPS) ← hep-th/0704.2233
  - How crucial is SUSY?

    (push AdS/CFT towards more realistic scenarios)

- Matrix Models with reduced SUSY challenging
  - Interactions for two-matrix states ← hep-th/0712.4366

IF TIME

- Future Directions / Open Questions

On the Gravity Side:

- 10D geometries that are asymptotically $AdS_5 \times S^5$ (good candidates for dual states)
- Turning on R-charge (J_1, J_2, J_3) breaks isometries of S^5

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no R-charge	SO(6)	S^5	
J_1	SO(4)	S^3	1/2 BPS
J_1 J_2	SO(2)	S^1	1/4 BPS
J_1 J_2 J_3	no isometry left		1/8 BPS

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Keep S^3 inside AdS_5

½ BPS Geometries in Type IIB SUGRA

Lin, Lunin, Maldacena
hep-th/0409174

LLM:

- constructed type IIB geometries dual to 1/2 BPS states in CFT

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$S^3 \times S^3$

$$\boxed{ds_{10}^2 = \underbrace{g_{\mu\nu}dx_\mu dx_\nu}_{4D} + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2}$$

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- Large amount of symmetry
- Time-independent geometries

$\Longrightarrow$ Only 3D really matter !

$\Downarrow$

Solution depends on a single function
$z(x_1, x_2, y)$

$$\boxed{ds_{10}^2 = g_{\mu\nu}dx_\mu dx_\nu + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2}$$

$\Rightarrow$

On y=0 plane

each $S^3 \longrightarrow 0$

$$ds^2_{10} = g_{\mu\nu} dx_\mu dx_\nu + y e^G d\Omega_3^2 + y e^{-G} d\tilde{\Omega}_3^2$$

$\Longrightarrow$

On y=0 plane
each $S^3 \longrightarrow 0$

Regularity imposes boundary conditions on y=0 plane:

$$\boxed{z(\underbrace{x_1, x_2}_{\text{2D plane}}, y = 0) = \pm\frac{1}{2}}$$

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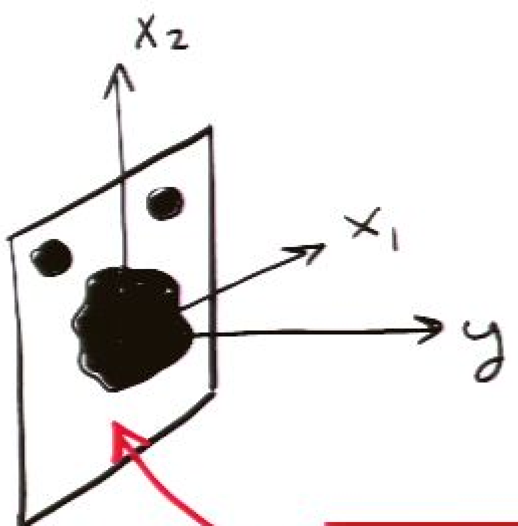


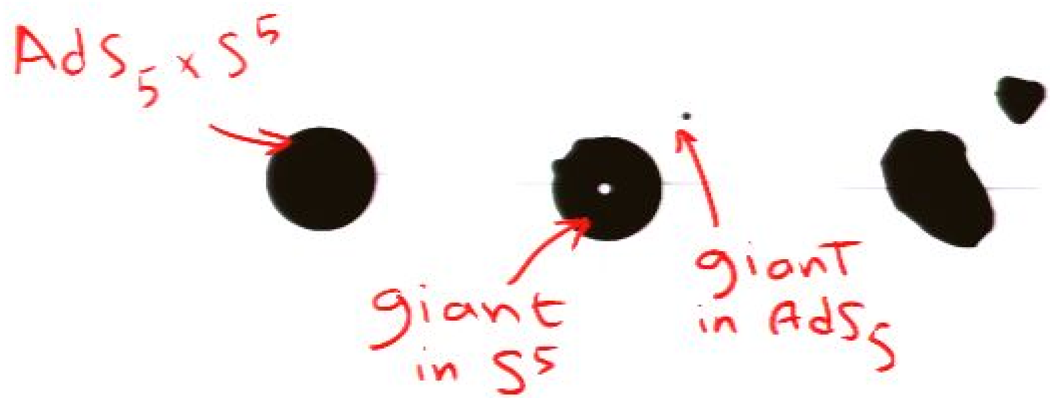
$$\boxed{z(\underbrace{x_1, x_2}_{\text{2D plane}}, y = 0) = \pm\frac{1}{2}}$$

phase space of FREE FERMIONS
(eigenvalues of $Z = \phi_5 + i\phi_6$)
in harmonic oscillator potential

"Simplicity" of ½ BPS sector arises from <u>linearity</u>

$$\left(\partial_1^2+\partial_2^2+y\partial_y\frac{1}{y}\partial_y\right)z(x_1,x_2,y)=0$$

2D droplet distribution $\longleftrightarrow$ Unique 10D geometry



Bubbling for More General States?

B.Chen, S.C., A.Donos, F.Lin, H.Lin, J.Liu, D.Vaman, W.Wen, hep-th/0704.223

Natural questions:

- What changes with less SUSY?
- Retain any nice structure?

Focus on 1/4 and 1/8 BPS sectors

General SUGRA analysis already done:

A. Donos
hep-th/0606199, hep-th/0610259

1/4 BPS

N. Kim,
hep-th/0511029

1/8 BPS

We gave evidence for a generic droplet picture:

- Analog of LLM's function z (crucial for BCs)
- Hyperplanes where BCs are imposed

  $\hookrightarrow$ give droplets

Equation for z is highly non-linear:

- General SUGRA solutions are not known
- A number of subtleties arise

multi-matrix models!

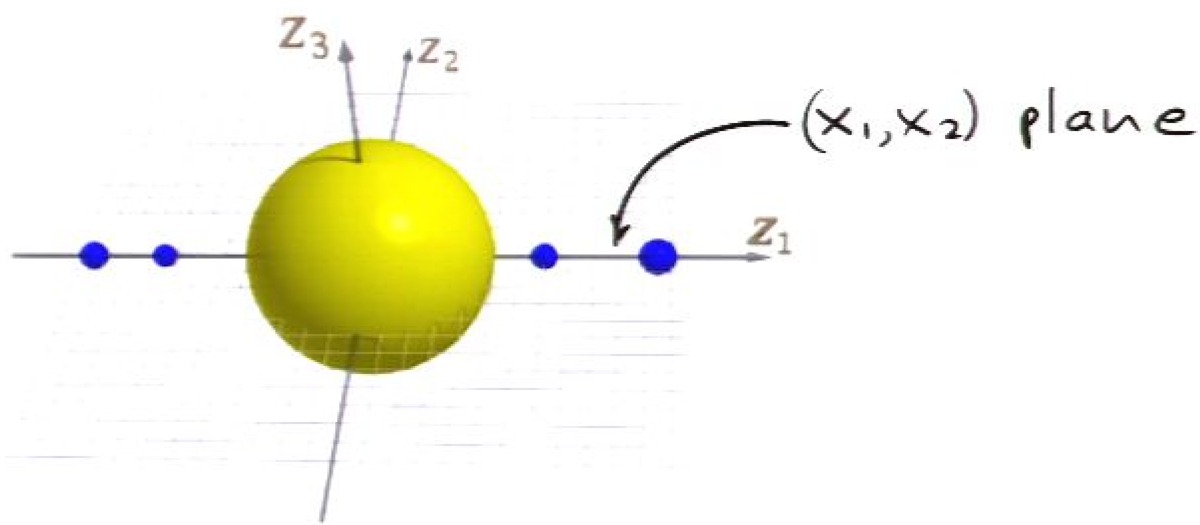
Gravity picture that has emerged :

1/2 BPS

$$ds^2_{1/2} = -h^{-2}(dt+V)^2 + h^2(dy^2 \boxed{+dx_1^2+dx_2^2)} + ye^{G} \underbrace{d\Omega_3^2}_{S^3} + ye^{-G} \underbrace{d\tilde{\Omega}_3^2}_{S^3}$$

2D plane
(FLAT)

$$\boxed{z(x_1, x_2, 0) = \pm\frac{1}{2}}$$



1/4 BPS states preserve S^3 x S^1 :

$$ds^2_{1/4} = -h^{-2}(dt+W)^2 + h^2 dy^2 + \frac{1}{ye^G}\boxed{h_{ij}dx^i dx^j} + ye^G \overbrace{d\Omega_3^2}^{S^3} + ye^{-G}(\overbrace{d\psi+}^{S^1}\mathcal{A})^2$$

4D Plane (Kahler)

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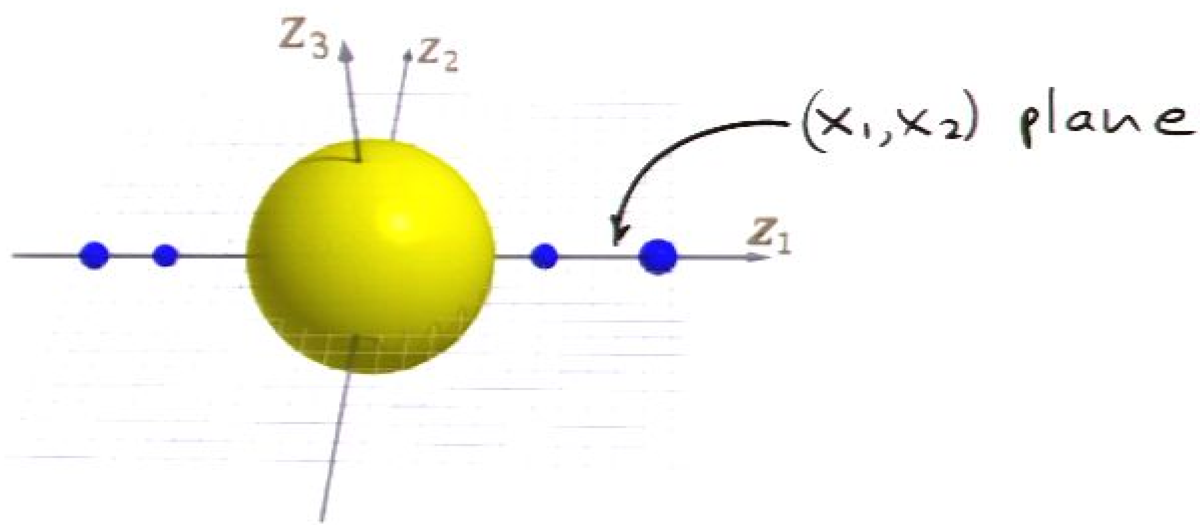

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S^3 S^1

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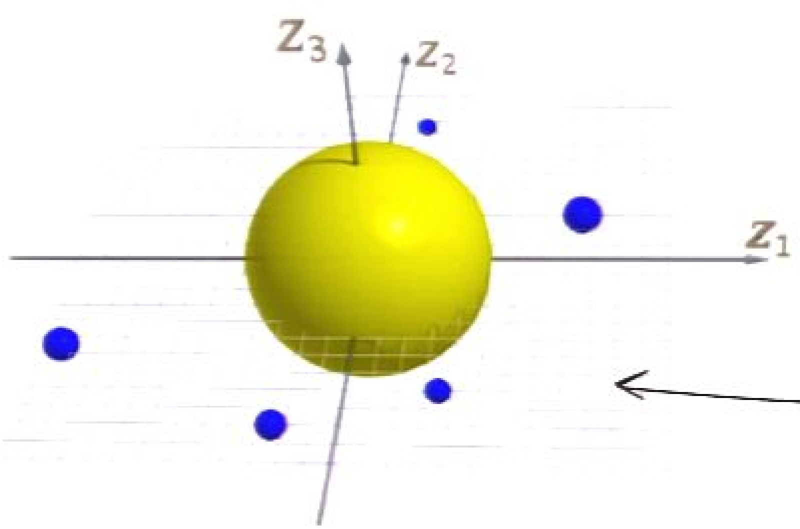
4D Plane (Kahler)

Potential singularity when either $S^3 \to 0$ or $S^1 \to 0$

$z(x_i, y = 0) = \pm\frac{1}{2} \quad (i = 1, \ldots, 4)$ **ensure regularity**



Droplets in 4D (y=0) hyperplane

Even less SUSY...

1/8 BPS states preserve S^3 :

$$ds^2_{1/8} = -e^{2\alpha}(dt + w)^2 + e^{-2\alpha} \boxed{ds^2_6} + e^{2\alpha} \underbrace{d\Omega^2_3}_{}$$

S^3

6D plane
(Kahler)

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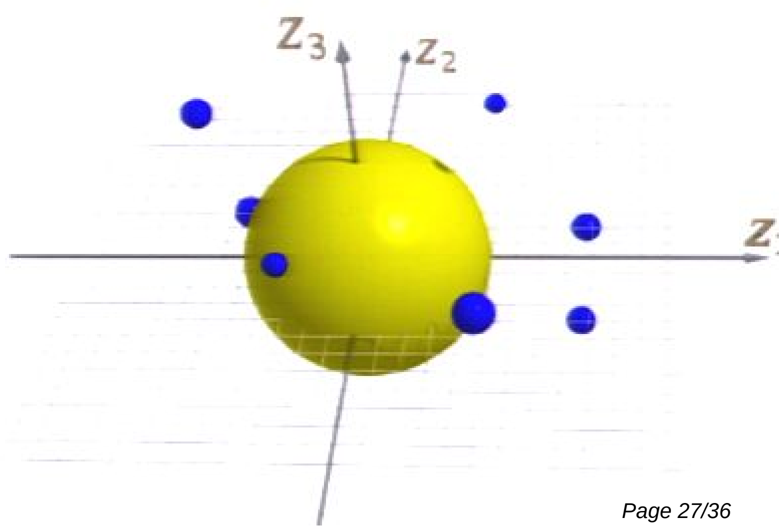
S^3

$$ds^2_{1/8} = -e^{2\alpha}(dt+w)^2 + e^{-2\alpha}\boxed{ds^2_6} + e^{2\alpha} d\Omega^2_3$$

6D plane
(Kahler)

Potential singularity when $S^3 \to 0$

Droplets in 6D
(interiors are unphysical)



Still possible to develop robust bubbling picture
(even w/out complete knowledge of SUGRA solutions)

Non-linearity leads to some open issues:

- Given a droplet, is the 10D geometry unique? (yes for LLM)
- Require asymptotically $AdS_5 \times S^5$, and regularity near droplets, but is that enough?
- Is 1/4 BPS sector integrable?

$\longrightarrow$ Need better understanding of regularity conditions

Multi-Matrix Models

$$\phi_1, \phi_2, \ldots, \phi_6 \rightarrow \boxed{X = \phi_1 + i\phi_2\,,\ Y = \phi_3 + i\phi_4\,,\ Z = \phi_5 + i\phi_6}$$

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1/2 BPS

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1/2 BPS

1/4 BPS, 1/8 BPS → **multi-matrix models**

challenging program

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1/2 BPS

1/4 BPS, 1/8 BPS $\longrightarrow$ multi-matrix models

challenging program

Cases w/ less SUSY admit matrix model description
BUT <u>no longer free fermions</u>

It turns out that symmetries can help

Interactions for Two-Matrix States

SC, A. Jevicki, R. de Mello Koch
hep-th/0712.4366

Goal:

Reconstruct cubic interaction vertex for general (multi-matrix) states

Simple strategy:

Take vertex for highest-weight states (1/2 BPS states)

$$V_{j_1 j_2 j_3} \sim \int \psi_{j_1} \psi_{j_2} \psi_{j_3}$$

one-matrix wavefunction

Use RAISING AND LOWERING OPERATORS (SL(2,R) generators of AdS) to "generate" the two-matrix states vertex

End result:

Identity that reconstructs the full two-matrix vertex from the simpler, one-matrix vertex

$$V_{j_1 j_2 j_3} \sim \int \psi_{j_1} \psi_{j_2} \psi_{j_3}$$

$$\Big\updownarrow$$

$$V_{j_1 n_1 j_2 n_2 j_3 n_3} \sim \int \underbrace{\psi_{j_1 n_1}} \psi_{j_2 n_2} \psi_{j_3 n_3}$$

Donos, Jevicki, Rodrigues
hep-th/0507124

Conclusions

- Lots of nice structure in 1/2 BPS sector
- Robust bubbling picture even with less SUSY
- Exploit symmetries to reconstruct matrix models interactions

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- Future Directions / Open Questions
  - Integrability with lower SUSY? Uniqueness?
  - Push AdS/CFT to better understand time-dependent geometries and singularities?
  - Non-BPS sector: any nice features? ← Work in progress

    e.g. isometries of LLM but no SUSY (non-trivial 2D plane? Deformed Hamiltonian?)