Title: LARGE Volume Scenarios and String Loops

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Abstract: We study the necessary and sufficient topological conditions for general Calabi-Yaus to get a non-supersymmetric AdS exponentially large volume minimum of the scalar potential in flux compactifications of IIB string theory. It turns out that string loop corrections play a crucial role to realise exponentially large volume minima for fibration Calabi-Yaus and to stabilise 4-cycles which support chiral matter. The robustness of these results is due to the \'extended no-scale structure\': for arbitrary Calabi-Yaus, the leading contribution of these corrections to the scalar potential is always vanishing. We use the Coleman-Weinberg potential to motivate this cancellation from the viewpoint of low-energy field theory.

LARGE Volume Scenarios and String Loops

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Based on:
1) M. Cicoli, J. Conlon, F. Quevedo arXiv:0708.1873 [hep-th]
2) M. Cicoli, J. Conlon, F. Quevedo arXiv:0805.1029 [hep-th]
3) M. Cicoli, F. Quevedo (in preparation)

Type IIB CY Flux Compactifications

- Low energy limit to N=1 4D Type IIB SUGRA via dimensional reduction

→ Need to know f, W, K!

- Ubiquitous presence of moduli: massless uncharged scalar particles with effective gravitational coupling that would give rise to long range unobserved fifth forces

→ Need to stabilise them!

- Closed string moduli: U (complex structure), S (axio-dilaton), T (Kähler)
- Open string moduli: Wilson lines, D3 and D7 moduli
- Turn on background 3-form fluxes (GKP)

→ $D_U W = D_s W = 0$ fixes U and S moduli supersymmetrically

- No-scale structure → flat potential for T moduli at tree level

→ T moduli still unfixed!

- Need to study perturbative versus non-perturbative corrections!

Perturbative vs Non perturbative

- In general: $\begin{cases} K = K_{tree} + K_p + K_{np}, \\ W = W_{tree} + W_{np}. \end{cases}$ ← negligible

where $K_p = \delta K_{(\alpha')} + \delta K_{(g_s)}$. with $K_0 + \delta K_{(\alpha')} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\,\mathrm{Re}\,(S)^{3/2}\right) =$

$$= -2\ln(\mathcal{V}) - \frac{\xi \mathrm{Re}\,(S)^{3/2}}{\mathcal{V}} + \mathcal{O}\left(1/\mathcal{V}^2\right)$$

and $W_{np} = \sum_i A_i(S,U)e^{-a_i T_i}$.

$\xi \sim -\chi = -2\,(h_{11} - h_{12})$

Neglect loop corrections and get

$$\begin{aligned} V &= V_{np} + V_{(\alpha')} = \\ &= e^K \Big[K^{jk} \left(a_j A_j a_k \bar{A}_k e^{-(a_j T_j + a_k \bar{T}_k)} - \left(a_j A_j e^{-a_j T_j} \bar{W} K_k + a_k \bar{A}_k e^{-a_k \bar{T}_k} W K_j \right) \right) \\ &\quad + 3\xi \frac{(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} |W|^2 \Big] \end{aligned}$$

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LARGE Volume

- Fix T moduli in a natural way
- Set $W_0 \sim$ 1-10 and $\mathcal{V} \sim \exp(a_i \tau_i)$

➡ α' and non-perturbative corrections compete naturally to give an exponentially large volume AdS minimum that breaks SUSY

- Need to up-lift to dS (anti-D3, D-terms, F-terms, non-perturbative α' corr.)
- Simplest realisation of LVS: $CP^4_{[1,1,1,6,9]}$ with $h_{11}=2$

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}}\right) \qquad \tau_4 \propto \xi^{\frac{2}{3}} \quad \text{and} \quad \langle \mathcal{V} \rangle \propto W_0 e^{\frac{a_4 \tau_4}{gs}}.$$

- Generalisation: Swiss-cheese CY (F_{11}, $CP^4_{[1,3,3,3,5]}$, $CP^4_{[1,1,3,10,15]}$ for $h_{11}=3$)

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left[\alpha\left(\tau_1^{3/2} - \sum_{i=2}^{n}\lambda_i \tau_i^{3/2}\right) + \frac{\xi}{2}\right],$$

$$W = W_0 + \sum_{i=2}^{n} A_i e^{-a_i T_i},$$

➡

$$V = \sum_i \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i \alpha} e^{-2a_i\tau_i} - \sum_i 4\frac{a_i A_i}{\mathcal{V}^2} W_0 \tau_i e^{-a_i\tau_i} + \frac{3\xi W_0^2}{4\mathcal{V}^3}.$$

Mass scales

The mass scales present are: (for $\mathcal{V} \sim 10^{15} \ell_s^6$)

- Planck scale: $M_P = 2.4 \times 10^{18}$GeV.
- String scale: $M_S = \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV.
- KK scale $M_{KK} = \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^9$GeV.
- Gravitino mass $m_{3/2} = \frac{M_P}{\mathcal{V}} \sim 30$TeV.
- Small Kähler moduli $m_{\tau_s} \sim m_{3/2} \ln(M_P/m_{3/2}) \sim 1000$TeV.
- Complex structure moduli $m_U \sim m_{3/2} \sim 30$TeV.
- Soft terms $m_{susy} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})} \sim 1$TeV.
- Volume modulus $m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.

NB No FCNC!

NB Get a robust effective field theory and generate hierarchies (axionic, weak and neutrino scale)!!!

$f_a \sim M_s \sim 10^{11}$ GeV $\quad M_{\nu R} \sim \mathcal{V}^{1/3} M_P^{-1} \sim 10^{14}$ GeV

Some open questions in LVS

- No general analysis: what happens for an arbitrary CY?
- No inclusion of loop corrections for an arbitrary CY (except Swiss-cheese CY by BHP)
- Tension between moduli stabilisation and chirality
- Inflation flatness spoiled by loop corrections
- No detectable tensor modes

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  $0.009<r<0.01 \quad 0.966<n<0.971 \quad \mathcal{V} \sim 10^2$

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  $0.009<r<0.01$ $0.966<n<0.971$ $\mathcal{V} \sim 10^2$ See Quevedo's talk

LARGE Volume Claim

Let X be an arbitrary CY and take the following large volume limit

$$\begin{cases} \tau_j \ remains \ small, \ \forall j = 1, ..., N_{small}, \\ \mathcal{V} \to \infty \quad for \ \ \tau_j \to \infty, \ \forall j = N_{small}+1, .., h_{1,1}(X). \end{cases}$$

Form of K and W - neglect string loop corrections at this point!

$$\begin{cases} K = K_{cs} - 2\ln\left(\mathcal{V} + \hat{\xi}\right), \\ W = W_0 + \sum_{j=1}^{N_{small}} A_j e^{-a_j T_j}. \end{cases}$$

→ there is an AdS non-supersymmetric minimum at $\mathcal{V} \sim e^{a_j \tau_j} \ \forall j = 1, ..., N_{small}$ IFF

i) $h_{12} > h_{11} > 1$ ⟷ $\xi > 0$

ii) τ_j is a blow-up mode (point-like singularity) $\forall j = 1, ..., N_{small}$
non-perturbative superpotential guaranteed since the cycle is rigid!

- N_{small} blow-up modes fixed by non-perturbative effects, $\mathcal{V}$ by α' corrections + W_{np}
- There are still $L=(h_{11}-N_{small}-1)$ moduli which are sent large (e.g. fibration moduli)
  → their non-perturbative corrections are switched off
- Get L flat directions!
- These directions are usually lifted by string loop corrections
  → L moduli lighter than the volume!

String Loop Corrections to K

- Explicit calculation known only for unfluxed toroidal orientifolds as $N{=}1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ (BHK)

$$\delta K_{(g_s)} = \delta K_{(g_s)}^{KK} + \delta K_{(g_s)}^{W},$$

where

$$\delta K_{(g_s)}^{KK} = -\frac{1}{128\pi^4} \sum_{i=1}^{3} \frac{\mathcal{E}_i^{KK}(U,\bar{U})}{\mathrm{Re}\,(S)\,\tau_i}$$

is due to the exchange of KK strings between D7s and D3s and

$$\delta K_{(g_s)}^{W} = -\frac{1}{128\pi^4} \sum_{i\neq j\neq k=1}^{3} \frac{\mathcal{E}_i^{W}(U,\bar{U})}{\tau_j \tau_k}$$

is due to the exchange of Winding strings between intersecting D7s

<u>NB</u> Complicated dependence on the U moduli BUT simple dependence on the T moduli!

Generalisation to CY

- Generalisation to Calabi-Yau three-folds (BHP)

$$\langle V_U V_{\bar{U}} \rangle_s \sim g(U,T,S) \Longleftrightarrow \langle V_U V_{\bar{U}} \rangle_E \sim g(U,T,S) \frac{e^{\varphi/2}}{\mathcal{V}_E}$$

where either $g(U,T,S)$ originates from KK modes as m_{KK}^{-2}

or $g(U,T,S)$ goes as $m_W^{-2} \sim t^{-1}$

In fact for $T^6/(Z_2 \times Z_2)$:

$$\mathcal{V} = t_1 t_2 t_3 = \sqrt{\tau_1 \tau_2 \tau_3} \quad \Longrightarrow \quad \tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = t_2 t_3 = \frac{\mathcal{V}}{t_1}$$

$$\delta K^{KK}_{(g_s),\tau_1} \sim \frac{1}{\tau_1} = \frac{t_1}{\mathcal{V}} = \frac{m_{KK}^{-2}}{\mathcal{V}}, \qquad \delta K^{W}_{(g_s),\tau_2\tau_3} \sim \frac{1}{\tau_2\tau_3} = \frac{1}{\mathcal{V} t_1} = \frac{m_W^{-2}}{\mathcal{V}}$$

Conjecture for an arbitrary CY! $\Longrightarrow$

$$\delta K^{KK}_{(g_s)} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, m_{KK,i}^{-2}}{\mathrm{Re}(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, t_i}{\mathrm{Re}(S)\,\mathcal{V}}$$

$$\delta K^{W}_{(g_s)} \sim \sum_i \frac{C_i^{W}(U)\, m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i}$$

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Conjecture for an arbitrary CY! $\Longrightarrow$

$$\begin{cases} \delta K_{(g_s)}^{KK} \sim \displaystyle\sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) m_{KK,i}^{-2}}{\mathrm{Re}(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) t_i}{\mathrm{Re}(S)\,\mathcal{V}} \\ \delta K_{(g_s)}^{W} \sim \displaystyle\sum_{i} \frac{C_i^{W}(U) m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_{i} \frac{C_i^{W}(U)}{\mathcal{V} t_i} \end{cases}$$

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Conjecture for an arbitrary CY! ⟹

$$\delta K^{KK}_{(g_s)} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, m_{KK,i}^{-2}}{\mathrm{Re}(S)\, \mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, t_i}{\mathrm{Re}(S)\, \mathcal{V}}$$

← transverse to D7

$$\delta K^{W}_{(g_s)} \sim \sum_i \frac{C_i^{W}(U)\, m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i}$$

← intersection of D7s

Low Energy Approach

- The reduced DBI action contains a term like

$$\delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}}\, \tau F^{\mu\nu} F_{\mu\nu}$$

- Expand τ around its VEV

$$\tau = \langle \tau \rangle + \tau'. \quad \Longrightarrow \quad \delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}} \left(\langle \tau \rangle F^{\mu\nu} F_{\mu\nu} + \tau' F^{\mu\nu} F_{\mu\nu} \right)$$

- Read off the gauge coupling $g^2 = \dfrac{1}{M_s^4 \tau}$

- Loop corrected kinetic terms

$$S_{Einstein} \supset \int d^4x \sqrt{-g^{(4)}} \left(\frac{\partial^2 (K_{tree})}{\partial \tau^2} + \frac{\partial^2 \left(\delta K^{KK}_{(g_s)} \right)}{\partial \tau^2} \right) (\partial \tau)^2$$

- Analogy with charged scalar fields

$$\int d^4x \sqrt{-g^{(4)}} \frac{1}{2} (1 + A)\, \partial_\mu \varphi \partial^\mu \varphi. \qquad A \simeq \frac{g^2}{16\pi^2}$$

$$\Longrightarrow \quad \frac{\partial^2 \left(\delta K^{KK}_{(g_s)} \right)}{\partial \tau^2} \sim \frac{f(\mathrm{Re}(S))}{16\pi^2} \frac{1}{\tau} \frac{\partial^2 (K_{tree})}{\partial \tau^2}$$

Example: $CP^4_{[1,1,1,6,9]}$

- Match the conjecture for *N=1* $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$
- Loop corrections from the conjecture for $CP^4_{[1,1,1,6,9]}$

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_5^{3/2} - \tau_4^{3/2}\right) \simeq \tau_5^{3/2}$$

$$\delta K^{KK}_{(g_s)} \sim \frac{\mathcal{C}_4^{KK}\sqrt{\tau_4}}{\mathrm{Re}(S)\,\mathcal{V}} + \frac{\mathcal{C}_5^{KK}\sqrt{\tau_5}}{\mathrm{Re}(S)\,\mathcal{V}} \simeq \frac{\mathcal{C}_4^{KK}\sqrt{\tau_4}}{\mathrm{Re}(S)\,\mathcal{V}} + \frac{\mathcal{C}_5^{KK}}{\mathrm{Re}(S)\,\tau_5}$$ (BHP)

NB Loops are leading with respect to the α' corrections!

Low Energy Interpretation: $\frac{\partial^2(K_{tree})}{\partial\tau_4^2} \simeq \frac{1}{\sqrt{\tau_4}\mathcal{V}}$. $\frac{\partial^2(K_{tree})}{\partial\tau_5^2} \simeq \frac{1}{\tau_5^2}$.

$$\Longrightarrow \begin{cases} \dfrac{\partial^2\left(\delta K^{KK}_{(g_s)}\right)}{\partial\tau_4^2} \sim \dfrac{1}{16\pi^2}\dfrac{1}{\mathrm{Re}(S)}\dfrac{1}{\tau_4^{3/2}\mathcal{V}} \\ \dfrac{\partial^2\left(\delta K^{KK}_{(g_s)}\right)}{\partial\tau_5^2} \sim \dfrac{1}{16\pi^2}\dfrac{1}{\mathrm{Re}(S)}\dfrac{1}{\tau_5^3} \end{cases}$$

OK!

- Match the conjecture also for $CP^4_{[1,1,2,2,6]}$

Extended No-scale Structure

Let X be a Calabi-Yau three-fold and consider type IIB $N = 1$ 4D SUGRA where the Kähler potential and the superpotential in the Einstein frame take the form:

$$\begin{cases} K = K_{tree} + \delta K, \\ W = W_0. \end{cases} \tag{4.1}$$

If and only if the loop correction δK to K is a homogeneous function in the 2-cycles volumes of degree $n = -2$, then at leading order

$$\delta V_{(g_s)} = 0. \tag{4.2}$$

Proof: Expand K^{-1} and use homogeneity! $\delta V_{(g_s)} = V_0 + \varepsilon \delta V_1 + \varepsilon^2 \delta V_2 + \mathcal{O}(\varepsilon^3)$

$$\delta V_1 = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}\left(3n + n(n-1)\right)\delta K = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}n(n+2)\delta K.$$

$$\begin{cases} n = -2 \quad \text{for} \quad \delta K^{KK}_{(g_s)}, \\ n = -4 \quad \text{for} \quad \delta K^{W}_{(g_s)}, \end{cases} \Longrightarrow \begin{cases} \delta V^{KK}_{(g_s),1} = 0, \\ \delta V^{W}_{(g_s),1} = -2\delta K^{W}_{(g_s)}\frac{|W|^2}{\mathcal{V}^2}. \end{cases}$$

The loop corrections to V are subleading with respect to the α' ones BUT are crucial to stabilise the SM cycle or to lift the L flat directions!!!

General formula for the 1 loop corrections to V

$$\delta V_2 = \left(K_0^{ij} \delta K_i \delta K_j - 2 K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2}$$

Homogeneity allows us to simplify it to:

$$\boxed{\delta V_{(g_s)}^{1loop} = \sum_i \left(\frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 - 2 \delta K_{(g_s),\tau_i}^W \right) \frac{W_0^2}{\mathcal{V}^2}}$$

NB Everything in terms of K_{ii} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}} \left(0 \cdot \frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)} K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3} \right) \right) \frac{W_0^2}{\mathcal{V}^2}$$

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$$\begin{cases} n=-2 \quad \text{for} \quad \delta K_{(g_s)}^{KK}, \\ n=-4 \quad \text{for} \quad \delta K_{(g_s)}^{W}, \end{cases} \Longrightarrow \begin{cases} \delta V_{(g_s),1}^{KK} = 0, \\ \delta V_{(g_s),1}^{W} = -2\delta K_{(g_s)}^{W}\frac{|W|^2}{\mathcal{V}^2}. \end{cases}$$

The loop corrections to V are subleading with respect to the α' ones BUT are crucial to stabilise the SM cycle or to lift the L flat directions!!!

General formula for the 1 loop corrections to V

$$\delta V_2 = \Big(K_0^{ij} \delta K_i \delta K_j - 2 K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \Big) \frac{|W|^2}{\mathcal{V}^2}$$

Homogeneity allows us to simplify it to:

$$\boxed{\delta V_{(g_s)}^{1loop} = \sum_i \left(\frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 - 2\delta K_{(g_s),\tau_i}^W \right) \frac{W_0^2}{\mathcal{V}^2}}$$

NB Everything in terms of K_{ii} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}} \left(0 \cdot \frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)} K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3} \right) \right) \frac{W_0^2}{\mathcal{V}^2}$$

Extended No-scale Structure

Let X be a Calabi-Yau three-fold and consider type IIB $N = 1$ 4D SUGRA where the Kähler potential and the superpotential in the Einstein frame take the form:

$$\begin{cases} K = K_{tree} + \delta K, \\ W = W_0. \end{cases} \tag{4.1}$$

If and only if the loop correction δK to K is a homogeneous function in the 2-cycles volumes of degree $n = -2$, then at leading order

$$\delta V_{(g_s)} = 0. \tag{4.2}$$

Proof: Expand K^{-1} and use homogeneity! $\delta V_{(g_s)} = V_0 + \varepsilon \delta V_1 + \varepsilon^2 \delta V_2 + \mathcal{O}(\varepsilon^3)$

$$\delta V_1 = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}\left(3n + n(n-1)\right)\delta K = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}n(n+2)\delta K.$$

$$\begin{cases} n = -2 \text{ for } \delta K^{KK}_{(g_s)}, \\ n = -4 \text{ for } \delta K^{W}_{(g_s)}, \end{cases} \Longrightarrow \begin{cases} \delta V^{KK}_{(g_s),1} = 0, \\ \delta V^{W}_{(g_s),1} = -2\delta K^{W}_{(g_s)}\frac{|W|^2}{\mathcal{V}^2}. \end{cases}$$

The loop corrections to V are subleading with respect to the α' ones BUT are crucial to stabilise the SM cycle or to lift the L flat directions!!!

General formula for the 1 loop corrections to V

$$\delta V_2 = \left(K_0^{ij}\delta K_i \delta K_j - 2K_0^{im}\delta K_{ml}K_0^{lj}K_i^0\delta K_j + K_0^{im}\delta K_{mp}K_0^{pq}\delta K_{ql}K_0^{lj}K_i^0K_j^0\right)\frac{|W|^2}{\mathcal{V}^2}$$

Homogeneity allows us to simplify it to:

$$\boxed{\delta V_{(g_s)}^{1loop} = \sum_i \left(\frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2}K_{ii}^0 - 2\delta K_{(g_s),\tau_i}^W\right)\frac{W_0^2}{\mathcal{V}^2}}$$

NB Everything in terms of K_{ii} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}}\left(0\cdot\frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)}K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2}K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3}\right)\right)\frac{W_0^2}{\mathcal{V}^2}$$

General formula for the 1 loop corrections to V

$$\delta V_2 = \Big(K_0^{ij}\delta K_i \delta K_j - 2K_0^{im}\delta K_{ml} K_0^{lj} K_i^0 \delta K_j + K_0^{im}\delta K_{mp} K_0^{pq}\delta K_{ql} K_0^{lj} K_i^0 K_j^0 \Big) \frac{|W|^2}{\mathcal{V}^2}$$

Homogeneity allows us to simplify it to:

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NB We know the sign of these corrections!!!

NB Everything in terms of K_{ij} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}} \left(0 \cdot \frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)} K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3} \right) \right) \frac{W_0^2}{\mathcal{V}^2}$$

Field Theory Interpretation

- Use the Coleman-Weinberg Potential

$$\delta V_{1loop} = \frac{1}{64\pi^2}\left[\Lambda^4 STr\left(M^0\right)\ln\left(\frac{\Lambda^2}{\mu^2}\right) + 2\Lambda^2 STr\left(M^2\right) + STr\left(M^4\ln\left(\frac{M^2}{\Lambda^2}\right)\right)\right]$$

where $STr\left(M^0\right) = 0$ due to SUSY

→ SUSY is the physical explanation for the extended no-scale structure!

- Single modulus example

$$\mathcal{V} = \tau^{3/2}.$$

$$\delta V^{KK}_{(g_s),1loop} = \left[0\cdot\mathcal{C}^{KK}\frac{\partial K_0}{\partial\tau} + \alpha_2\left(\mathcal{C}^{KK}\right)^2\frac{\partial^2 K_0}{\partial\tau^2}\right.$$
$$\left. + \alpha_3\left(\mathcal{C}^{KK}\right)^3\frac{\partial^3 K_0}{\partial\tau^3} + \mathcal{O}\left(\frac{\partial^4 K_0}{\partial\tau^4}\right)\right]\frac{W_0^2}{\mathcal{V}^2}$$
$$= \left(0\cdot\frac{-3\mathcal{C}^{KK}}{\mathcal{V}^{8/3}} + \frac{3\alpha_2\left(\mathcal{C}^{KK}\right)^2}{\mathcal{V}^{10/3}} - \frac{6\alpha_3\left(\mathcal{C}^{KK}\right)^3}{\mathcal{V}^4} + \mathcal{O}\left(\frac{1}{\mathcal{V}^{14/3}}\right)\right)W_0^2$$

Evaluation of the Coleman-Weinberg

$STr\,(M^2) \simeq m_{3/2}^2.$ where $m_{3/2}^2 = e^K W_0^2 \simeq \frac{1}{\mathcal{V}^2} \implies STr\,(M^2) \simeq \frac{1}{\mathcal{V}^2}.$

$$\Lambda = m_{KK} \simeq \frac{M_s}{R} = \frac{M_s}{\tau^{1/4}} = \frac{1}{\tau^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}$$

$$\delta V_{1loop} \simeq 0 \cdot \Lambda^4 + \Lambda^2 STr\,(M^2) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right) \simeq$$
$$\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}$$

Perfect matching!!!

NB It is possible to get a matching with the Coleman-Weinberg also for the $CP^4_{[1,1,1,6,9]}$ and the $CP^4_{[1,1,2,2,6]}$ case

Inclusion of loop corrections

Loop corrections are subleading w.r.to V_{np} and $V_{\alpha'}$

→ do not destroy the exponentially large volume minimum $\mathcal{V} \sim \exp(a_1 \tau_1)$

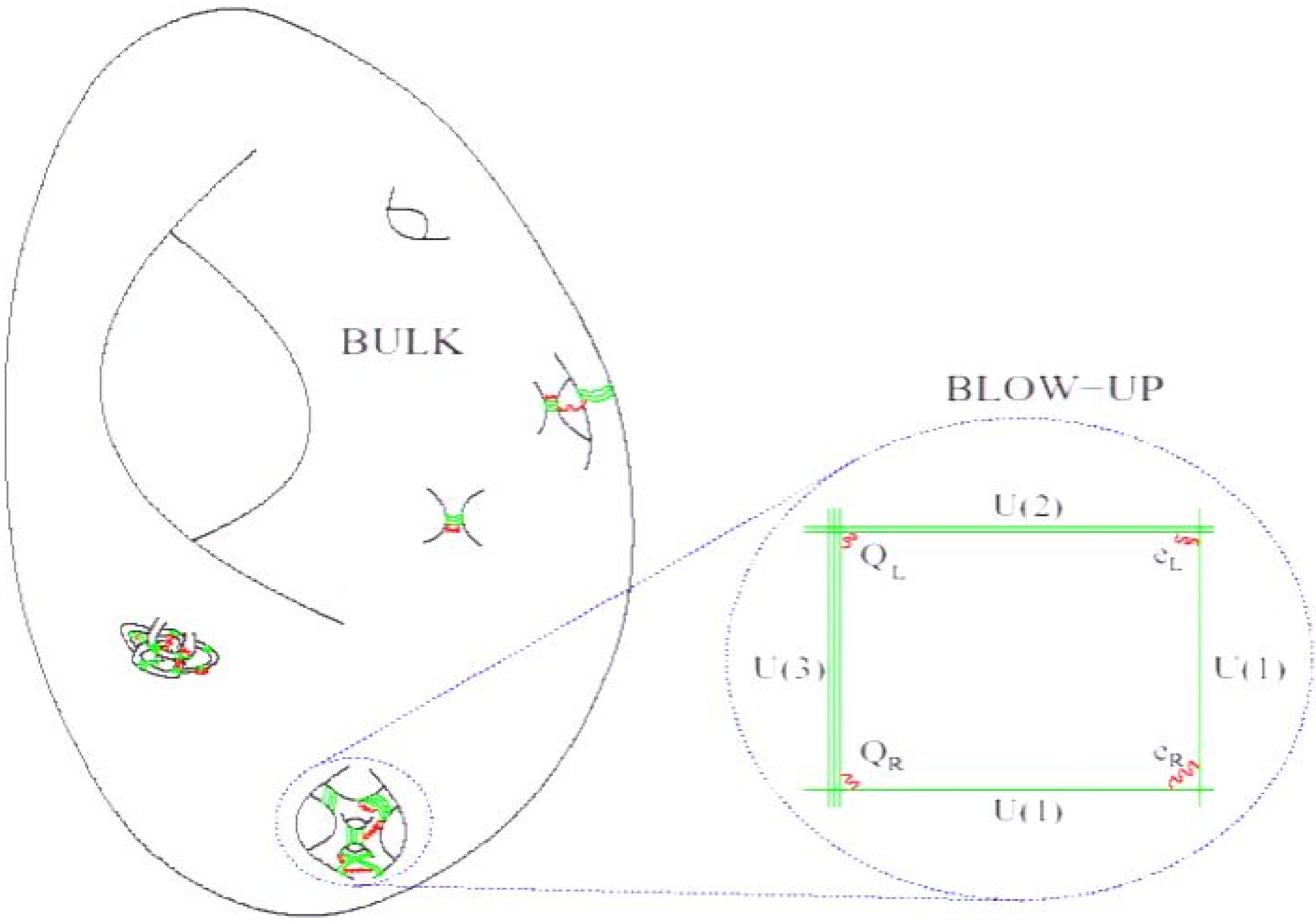
BUT they can:

- Stabilise the L non-blow moduli LARGE
- Fix the SM cycle which does not get any non-perturbative W

$$W_{\text{string}} \sim \left[\prod_i \Phi_i\right] e^{-S_{\text{inst}}} = 0 \qquad \text{for } \langle\Phi_{SM}\rangle = 0 \qquad \text{(BMP)}$$

The Standard Model in the CY

- SM on a small cycle as $g^2=1/\tau_{SM}$



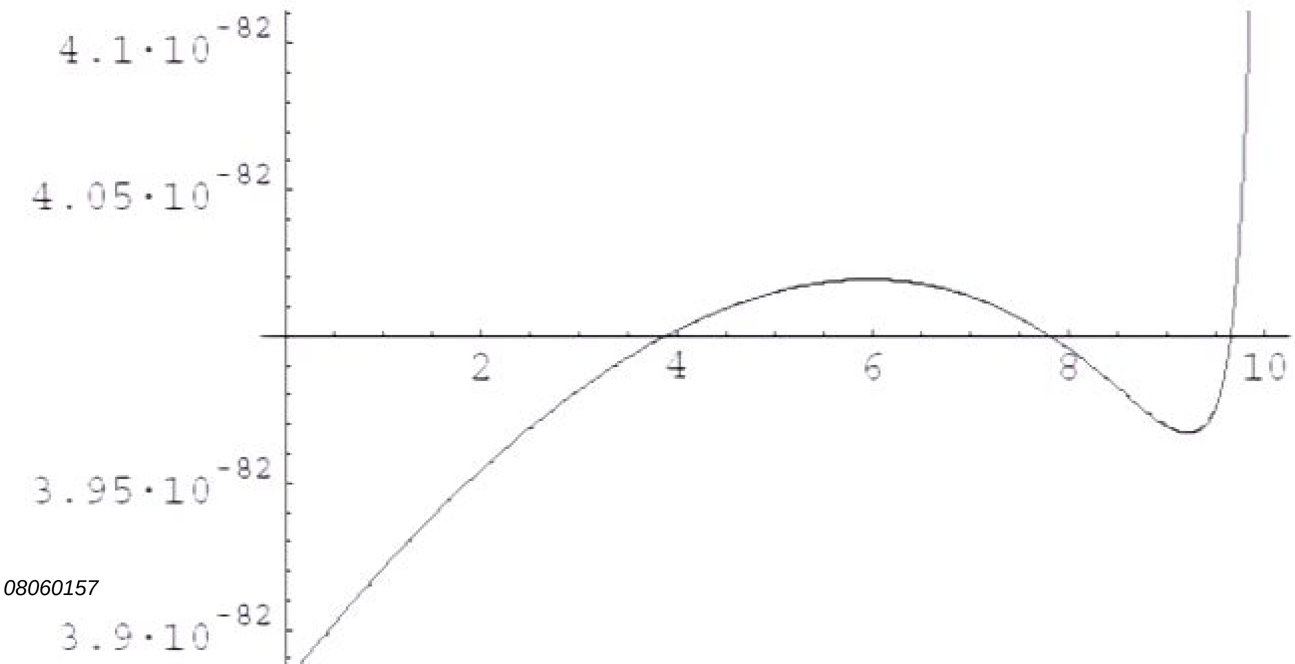
- BUT τ_{SM} cannot have any non-perturbative superpotential!

Example 1: SM cycle fixed by loops

- Solution: fix τ_{SM} via string loops
- $CP^4_{[1,3,3,3,5]}$ for $h_{11}=3$

$$V+\delta V_{(g_s)} = \frac{\lambda_1\left(\sqrt{5\left(2\tau_{E3}+\tau_{SM}\right)}+\sqrt{\tau_{E3}-\tau_{SM}}\right)e^{-4\pi\tau_{E3}}}{\mathcal{V}} - \frac{3\lambda_2\tau_{E3}e^{-2\pi\tau_{E3}}}{\mathcal{V}^2} + \frac{\lambda_3}{\mathcal{V}^3} + \left(\frac{\lambda_4}{\sqrt{\tau_{E3}-\tau_{SM}}}+\frac{\lambda_5}{\sqrt{2\tau_{E3}+\tau_{SM}}}\right)\frac{1}{\mathcal{V}^3}.$$

$$\mathcal{V} \sim \sqrt{\tau_{E3}}e^{2\pi\tau_{E3}}$$



Example 2:Flat directions lifted by loops

- K3 Fibration with h_{11}=2: $CP^4_{[1,1,2,2,6]}$

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1}\left(\tau_2 - \frac{2}{3}\tau_1\right)$$

- No blow-up mode ➡ No LARGE Volume

$$V = -\frac{4}{\mathcal{V}^2}W_0 a_1 \tau_1 e^{-a_1\tau_1} + \frac{3}{4}\frac{\xi}{\mathcal{V}^3}W_0^2$$

- K3 Fibration with h_{11}=3

$$\mathcal{V} = \alpha\left[\sqrt{\tau_1}(\tau_2 - \beta\tau_1) - \gamma\tau_3^{3/2}\right]$$

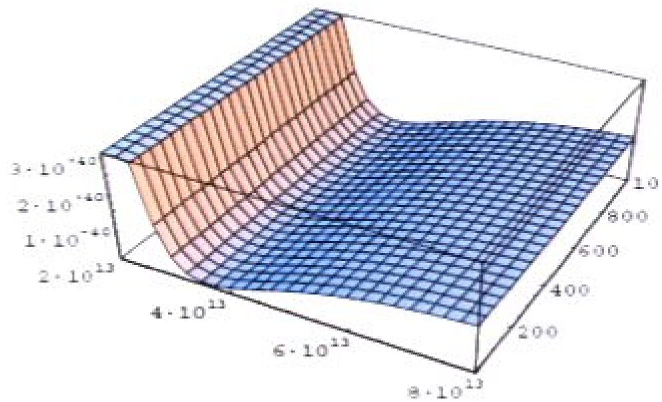
- Now τ_3 is a blow-up mode ➡ LARGE Volume

- Field redefinition

$$(\tau_1, \tau_2) \longrightarrow (\mathcal{V}, \Omega): \qquad \begin{cases} \mathcal{V} \simeq \alpha\left[\sqrt{\tau_1}\,(\tau_2 - \beta\tau_1)\right] \\ \Omega = \alpha\left[\sqrt{\tau_1}\,(\tau_2 + \beta\tau_1)\right] \end{cases}$$

- Scalar potential without loop corrections

$$V = \frac{16a_3^2}{3\mathcal{V}}\sqrt{\tau_3}e^{-2a_3\tau_3} - \frac{4}{\mathcal{V}^2}a_3\tau_3 e^{-a_3\tau_3} + \frac{3}{2g_s^{3/2}\mathcal{V}^3}$$



➡ Ω is a flat direction, $\mathcal{V} \sim \exp(a_3\tau_3)$!

- Consider loop corrections
- Wrap D7s around τ_1, τ_2 and τ_3
- KK corrections in τ_3 do not depend on Ω and are subleading w.r.to the α' ones

$$\delta V^{KK}_{(g_s),\tau_3} \sim \frac{1}{\mathcal{V}^3\sqrt{\tau_3}}$$ ➡ negligible!

- The 4-cycle τ_3 is a blow up mode ➡ it does not intersect with the other 4-cycles ➡ No winding corrections!

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- No blow-up mode ➡ No LARGE Volume

$$V = -\frac{4}{\mathcal{V}^2}W_0 a_1 \tau_1 e^{-a_1\tau_1} + \frac{3}{4}\frac{\xi}{\mathcal{V}^3}W_0^2$$

- K3 Fibration with h_{11}=3

$$\mathcal{V} = \alpha\left[\sqrt{\tau_1}(\tau_2 - \beta\tau_1) - \gamma\tau_3^{3/2}\right]$$

- Now τ_3 is a blow-up mode ➡ LARGE Volume

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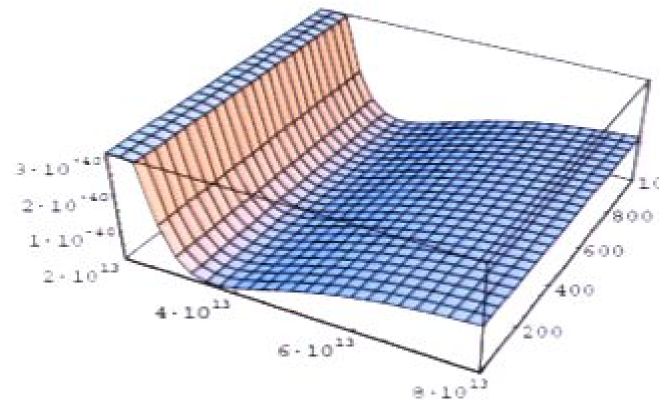
$$(\tau_1, \tau_2) \longrightarrow (\mathcal{V}, \Omega): \qquad \begin{cases} \mathcal{V} \simeq \alpha \left[\sqrt{\tau_1} (\tau_2 - \beta\tau_1) \right] \\ \Omega = \alpha \left[\sqrt{\tau_1} (\tau_2 + \beta\tau_1) \right] \end{cases}$$

- Scalar potential without loop corrections

$$V = \frac{16 a_3^2}{3\mathcal{V}} \sqrt{\tau_3} e^{-2a_3\tau_3} - \frac{4}{\mathcal{V}^2} a_3 \tau_3 e^{-a_3\tau_3} + \frac{3}{2 g_s^{3/2} \mathcal{V}^3}$$



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- Consider loop corrections
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- KK corrections in τ_3 do not depend on Ω and are subleading w.r.to the α' ones

$$\delta V^{KK}_{(g_s),\tau_3} \sim \frac{1}{\mathcal{V}^3 \sqrt{\tau_3}}$$ ➡ negligible!

- The 4-cycle τ_3 is a blow up mode ➡ it does not intersect with the other 4-cycles ➡ No winding corrections!

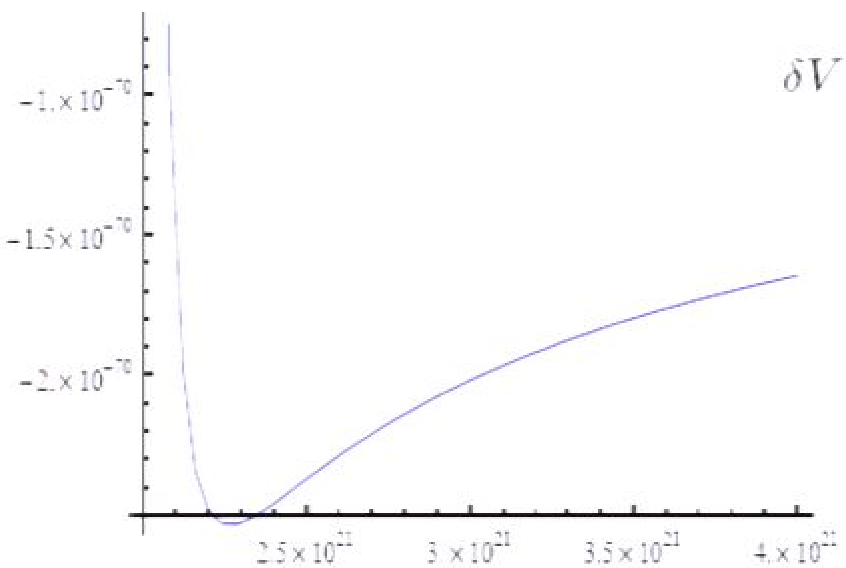
- Relevant loop corrections

$$\delta V_{(g_s)} = \delta V^{KK}_{(g_s),\tau_1} + \delta V^{KK}_{(g_s),\tau_2} + \delta V^{W}_{(g_s),\tau_1\tau_2}$$

$$\Rightarrow \quad \delta V_{(g_s)} = \frac{A}{\tau_1^2 \mathcal{V}^2} + \frac{C}{\sqrt{\tau_1}\mathcal{V}^3} + \frac{D\tau_1}{\mathcal{V}^4}$$

NB A>0, D>0 !!!

- Loop corrections for A=1, C=-20, D=2/9



$$\delta V = \left(\frac{2}{3}\right)^{1/3} \frac{\Omega^2 - 62\Omega\mathcal{V} + 63\mathcal{V}^2}{3\left(\Omega - \mathcal{V}\right)^{4/3}\mathcal{V}^4}$$

NB The minimum lies within the Kähler cone

$$\mathcal{V} < \Omega < 2\,\mathcal{V}$$

NO D7 wrapping τ_1 $\Rightarrow$ NO minimum

NO D7 wrapping τ_2 $\Rightarrow$ Minimum at $\Omega = 3\,\mathcal{V}$ $\Rightarrow$ out of the Kähler cone

Conclusions

- Non-perturbative effects fix only blow-up Kähler moduli
- Then α' effects + W_{np} fix the Volume exponentially large
- All the other Kähler moduli are flat directions
- Loop corrections to K are leading w.r. to the α' ones
- Loop corrections to V are SUB-leading w.r. to the α' ones due to the "extended no-scale structure"
- Coleman-Weinberg potential gives a nice physical understanding of this cancellation: SUSY!
- Loop corrections needed to fix the rest of Kähler moduli!
- Loop corrections crucial to:

1) Extend LVS to fibration CYs
2) Fix SM cycle
3) Get natural inflation with r ~ 0.01?
4) Give a forest of monochromatic lines?

Need to compute explicit g_s corrections for arbitrary CY!!

Evaluation of the Coleman-Weinberg

$STr\left(M^2\right) \simeq m_{3/2}^2.$ where $m_{3/2}^2 = e^K W_0^2 \simeq \frac{1}{\mathcal{V}^2} \implies STr\left(M^2\right) \simeq \frac{1}{\mathcal{V}^2}.$

$$\Lambda = m_{KK} \simeq \frac{M_s}{R} = \frac{M_s}{\tau^{1/4}} = \frac{1}{\tau^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}$$

$$\delta V_{1loop} \simeq 0 \cdot \Lambda^4 + \Lambda^2 STr\left(M^2\right) + STr\left(M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right) \simeq$$
$$\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}$$

Perfect matching!!!

NB It is possible to get a matching with the Coleman-Weinberg also for the $CP^4_{[1,1,1,6,9]}$ and the $CP^4_{[1,1,2,2,6]}$ case

General formula for the 1 loop corrections to V

$$\delta V_2 = \Big(K_0^{ij}\delta K_i \delta K_j - 2K_0^{im}\delta K_{ml} K_0^{lj} K_i^0 \delta K_j + K_0^{im}\delta K_{mp} K_0^{pq}\delta K_{ql} K_0^{lj} K_i^0 K_j^0 \Big) \frac{|W|^2}{\mathcal{V}^2}$$

Homogeneity allows us to simplify it to:

$$\boxed{\delta V_{(g_s)}^{1loop} = \sum_i \left(\frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 - 2\delta K_{(g_s),\tau_i}^W \right) \frac{W_0^2}{\mathcal{V}^2}}$$

NB We know the sign of these corrections!!!

NB Everything in terms of K_{ij} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}} \left(0 \cdot \frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)} K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3} \right) \right) \frac{W_0^2}{\mathcal{V}^2}$$

General formula for the 1 loop corrections to V

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NB Everything in terms of K_{ii} and δK^W!!!

$$\delta V_{(g_s),1loop}^{KK} = \sum_{i=1}^{h_{1,1}} \left(0 \cdot \frac{\left(\mathcal{C}_i^{KK}\right)}{\mathrm{Re}(S)} K_i^0 + \frac{\left(\mathcal{C}_i^{KK}\right)^2}{\mathrm{Re}(S)^2} K_{ii}^0 + \mathcal{O}\left(\frac{\left(\mathcal{C}_i^{KK}\right)^3}{\mathrm{Re}(S)^3} \right) \right) \frac{W_0^2}{\mathcal{V}^2}$$

Example: $CP^4_{[1,1,1,6,9]}$

- Match the conjecture for $N=1\ T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$
- Loop corrections from the conjecture for $CP^4_{[1,1,1,6,9]}$

$$\mathcal{V}=\frac{1}{9\sqrt{2}}\left(\tau_5^{3/2}-\tau_4^{3/2}\right)\simeq\tau_5^{3/2}$$

$$\delta K^{KK}_{(g_s)}\sim\frac{\mathcal{C}_4^{KK}\sqrt{\tau_4}}{\mathrm{Re}\left(S\right)\mathcal{V}}+\frac{\mathcal{C}_5^{KK}\sqrt{\tau_5}}{\mathrm{Re}\left(S\right)\mathcal{V}}\simeq\frac{\mathcal{C}_4^{KK}\sqrt{\tau_4}}{\mathrm{Re}\left(S\right)\mathcal{V}}+\frac{\mathcal{C}_5^{KK}}{\mathrm{Re}\left(S\right)\tau_5}$$ (BHP)

NB Loops are leading with respect to the α' corrections!

Low Energy Interpretation: $\frac{\partial^2\left(K_{tree}\right)}{\partial\tau_4^2}\simeq\frac{1}{\sqrt{\tau_4}\mathcal{V}}$, $\frac{\partial^2\left(K_{tree}\right)}{\partial\tau_5^2}\simeq\frac{1}{\tau_5^2}$.

$$\Longrightarrow\left\{\begin{array}{l}\frac{\partial^2\left(\delta K^{KK}_{(g_s)}\right)}{\partial\tau_4^2}\sim\frac{1}{16\pi^2}\frac{1}{\mathrm{Re}(S)}\frac{1}{\tau_4^{3/2}\mathcal{V}}\\ \frac{\partial^2\left(\delta K^{KK}_{(g_s)}\right)}{\partial\tau_5^2}\sim\frac{1}{16\pi^2}\frac{1}{\mathrm{Re}(S)}\frac{1}{\tau_5^3}\end{array}\right.$$ OK!

- Match the conjecture also for $CP^4_{[1,1,2,2,6]}$

Generalisation to CY

- Generalisation to Calabi-Yau three-folds (BHP)

$$\langle V_U V_{\bar{U}} \rangle_s \sim g(U,T,S) \Longleftrightarrow \langle V_U V_{\bar{U}} \rangle_E \sim g(U,T,S)\frac{e^{\varphi/2}}{\mathcal{V}_E}$$

where either $g(U,T,S)$ originates from KK modes as m_{KK}^{-2}

or $g(U,T,S)$ goes as $m_W^{-2} \sim t^{-1}$

In fact for $T^6/(Z_2 \times Z_2)$:

$$\mathcal{V} = t_1 t_2 t_3 = \sqrt{\tau_1 \tau_2 \tau_3} \quad \Longrightarrow \quad \tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = t_2 t_3 = \frac{\mathcal{V}}{t_1}$$

$$\delta K^{KK}_{(g_s),\tau_1} \sim \frac{1}{\tau_1} = \frac{t_1}{\mathcal{V}} = \frac{m_{KK}^{-2}}{\mathcal{V}}, \qquad \delta K^{W}_{(g_s),\tau_2\tau_3} \sim \frac{1}{\tau_2\tau_3} = \frac{1}{\mathcal{V}t_1} = \frac{m_W^{-2}}{\mathcal{V}}$$

Conjecture for an arbitrary CY! ⟹

$$\delta K^{KK}_{(g_s)} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, m_{KK,i}^{-2}}{\mathrm{Re}\,(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U)\, t_i}{\mathrm{Re}\,(S)\,\mathcal{V}}$$

$$\delta K^{W}_{(g_s)} \sim \sum_i \frac{C_i^{W}(U)\, m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i}$$

← transverse to D7

Generalisation to CY

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where either $g(U,T,S)$ originates from KK modes as m_{KK}^{-2}

or $g(U,T,S)$ goes as $m_W^{-2} \sim t^{-1}$

In fact for $T^6/(Z_2 \times Z_2)$:

$$\mathcal{V} = t_1 t_2 t_3 = \sqrt{\tau_1 \tau_2 \tau_3} \quad \Longrightarrow \quad \tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = t_2 t_3 = \frac{\mathcal{V}}{t_1}$$

$$\delta K_{(g_s),\tau_1}^{KK} \sim \frac{1}{\tau_1} = \frac{t_1}{\mathcal{V}} = \frac{m_{KK}^{-2}}{\mathcal{V}}, \qquad \delta K_{(g_s),\tau_2\tau_3}^{W} \sim \frac{1}{\tau_2 \tau_3} = \frac{1}{\mathcal{V} t_1} = \frac{m_W^{-2}}{\mathcal{V}}$$

Conjecture for an arbitrary CY! $\Longrightarrow$

$$\delta K_{(g_s)}^{KK} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) m_{KK,i}^{-2}}{\mathrm{Re}(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) t_i}{\mathrm{Re}(S)\,\mathcal{V}}$$

$$\delta K_{(g_s)}^{W} \sim \sum_i \frac{C_i^{W}(U) m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i}$$

String Loop Corrections to K

- Explicit calculation known only for unfluxed toroidal orientifolds as $N=1\ T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ (BHK)

$$\delta K_{(g_s)} = \delta K_{(g_s)}^{KK} + \delta K_{(g_s)}^{W},$$

where

$$\delta K_{(g_s)}^{KK} = -\frac{1}{128\pi^4} \sum_{i=1}^{3} \frac{\mathcal{E}_i^{KK}(U,\bar{U})}{\mathrm{Re}(S)\,\tau_i}$$

is due to the exchange of KK strings between D7s and D3s and

$$\delta K_{(g_s)}^{W} = -\frac{1}{128\pi^4} \sum_{i\neq j\neq k=1}^{3} \frac{\mathcal{E}_i^{W}(U,\bar{U})}{\tau_j \tau_k}$$

is due to the exchange of Winding strings between intersecting D7s

NB Complicated dependence on the U moduli BUT simple dependence on the T moduli!

LARGE Volume Claim

Let X be an arbitrary CY and take the following large volume limit

$$\begin{cases} \tau_j \;\; remains\; small,\; \forall j = 1, ..., N_{small}, \\ \mathcal{V} \to \infty \quad for \;\; \tau_j \to \infty,\; \forall j = N_{small}+1, .., h_{1,1}(X). \end{cases}$$

Form of K and W - neglect string loop corrections at this point!

$$\begin{cases} K = K_{cs} - 2\ln\left(\mathcal{V} + \hat{\xi}\right), \\ W = W_0 + \sum_{j=1}^{N_{small}} A_j e^{-a_j T_j}. \end{cases}$$

⟹ there is an AdS non-supersymmetric minimum at $\mathcal{V} \sim e^{a_j \tau_j} \;\; \forall j = 1, ..., N_{small}$ IFF

i) $h_{12} > h_{11} > 1$ ⟺ $\xi > 0$

ii) τ_j is a blow-up mode (point-like singularity) $\forall j = 1, ..., N_{small}$

non-perturbative superpotential guaranteed since the cycle is rigid!

- N_{small} blow-up modes fixed by non-perturbative effects, $\mathcal{V}$ by α' corrections + W_{np}
- There are still $L=(h_{11}-N_{small}-1)$ moduli which are sent large (e.g. fibration moduli)

  ⟹ their non-perturbative corrections are switched off
- Get L flat directions!
- These directions are usually lifted by string loop corrections

  ⟹ L moduli lighter than the volume!

String Loop Corrections to K

- Explicit calculation known only for unfluxed toroidal orientifolds as $N=1\ T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ (BHK)

$$\delta K_{(g_s)} = \delta K_{(g_s)}^{KK} + \delta K_{(g_s)}^{W},$$

where

$$\delta K_{(g_s)}^{KK} = -\frac{1}{128\pi^4} \sum_{i=1}^{3} \frac{\mathcal{E}_i^{KK}(U,\bar{U})}{\mathrm{Re}\,(S)\,\tau_i}$$

is due to the exchange of KK strings between D7s and D3s and

$$\delta K_{(g_s)}^{W} = -\frac{1}{128\pi^4} \sum_{i\neq j\neq k=1}^{3} \frac{\mathcal{E}_i^{W}(U,\bar{U})}{\tau_j \tau_k}$$

is due to the exchange of Winding strings between intersecting D7s

NB Complicated dependence on the U moduli BUT simple dependence on the T moduli!

Generalisation to CY

- Generalisation to Calabi-Yau three-folds (BHP)

$$\langle V_U V_{\bar{U}} \rangle_s \sim g(U,T,S) \Longleftrightarrow \langle V_U V_{\bar{U}} \rangle_E \sim g(U,T,S) \frac{e^{\varphi/2}}{\mathcal{V}_E}$$

where either $g(U,T,S)$ originates from KK modes as m_{KK}^{-2}

or $g(U,T,S)$ goes as $m_W^{-2} \sim t^{-1}$

In fact for $T^6/(Z_2 \times Z_2)$:

$$\mathcal{V} = t_1 t_2 t_3 = \sqrt{\tau_1 \tau_2 \tau_3} \quad \Longrightarrow \quad \tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = t_2 t_3 = \frac{\mathcal{V}}{t_1}$$

$$\delta K_{(g_s),\tau_1}^{KK} \sim \frac{1}{\tau_1} = \frac{t_1}{\mathcal{V}} = \frac{m_{KK}^{-2}}{\mathcal{V}}, \qquad \delta K_{(g_s),\tau_2\tau_3}^{W} \sim \frac{1}{\tau_2 \tau_3} = \frac{1}{\mathcal{V} t_1} = \frac{m_W^{-2}}{\mathcal{V}}$$

Conjecture for an arbitrary CY! $\Longrightarrow$

$$\begin{cases} \delta K_{(g_s)}^{KK} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) m_{KK,i}^{-2}}{\mathrm{Re}(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) t_i}{\mathrm{Re}(S)\,\mathcal{V}} \\ \delta K_{(g_s)}^{W} \sim \sum_i \frac{C_i^{W}(U) m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i} \end{cases}$$

Generalisation to CY

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$$\delta K^{KK}_{(g_s),\tau_1} \sim \frac{1}{\tau_1} = \frac{t_1}{\mathcal{V}} = \frac{m_{KK}^{-2}}{\mathcal{V}}, \qquad \delta K^{W}_{(g_s),\tau_2\tau_3} \sim \frac{1}{\tau_2\tau_3} = \frac{1}{\mathcal{V} t_1} = \frac{m_W^{-2}}{\mathcal{V}}$$

Conjecture for an arbitrary CY! $\Rightarrow$

$$\delta K^{KK}_{(g_s)} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) m_{KK,i}^{-2}}{\mathrm{Re}\,(S)\,\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C_i^{KK}(U) t_i}{\mathrm{Re}\,(S)\,\mathcal{V}}$$

$$\delta K^{W}_{(g_s)} \sim \sum_i \frac{C_i^{W}(U) m_{W,i}^{-2}}{\mathcal{V}} \sim \sum_i \frac{C_i^{W}(U)}{\mathcal{V} t_i}$$

t_i: transverse to D7

Low Energy Approach

- The reduced DBI action contains a term like

$$\delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}} \tau F^{\mu\nu} F_{\mu\nu}$$

- Expand τ around its VEV

$$\tau = \langle \tau \rangle + \tau'. \quad \Longrightarrow \quad \delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}} \left(\langle \tau \rangle F^{\mu\nu} F_{\mu\nu} + \tau' F^{\mu\nu} F_{\mu\nu} \right)$$

- Read off the gauge coupling $g^2 = \frac{1}{M_s^4 \tau}$
- Loop corrected kinetic terms

$$S_{Einstein} \supset \int d^4x \sqrt{-g^{(4)}} \left(\frac{\partial^2 (K_{tree})}{\partial \tau^2} + \frac{\partial^2 \left(\delta K^{KK}_{(g_s)} \right)}{\partial \tau^2} \right) (\partial \tau)^2$$

- Analogy with charged scalar fields

$$\int d^4x \sqrt{-g^{(4)}} \frac{1}{2} (1 + A) \partial_\mu \varphi \partial^\mu \varphi. \qquad A \simeq \frac{g^2}{16\pi^2}$$

$$\Longrightarrow \quad \frac{\partial^2 \left(\delta K^{KK}_{(g_s)} \right)}{\partial \tau^2} \sim \frac{f(\mathrm{Re}(S))}{16\pi^2} \frac{1}{\tau} \frac{\partial^2 (K_{tree})}{\partial \tau^2}.$$