Title: Improved ring potential of QED at finite temperature in the limit of weak and strong magnetic fields

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Abstract: Using the general structure of the vacuum polarization tensor at non-zero temperature T and finite magnetic field B, the ring contribution to QED effective potential is determined beyond the static (zero momentum) limit. In the limit of weak magnetic field and at high temperature, the improved ring potential consists of a term proportional to T4Â®5=2, in ad-dition to the well-known T4Â®3=2 term. In the limit of strong magnetic field, where QED dynamics is dominated by the lowest Landau level (LLL), the ring potential consists of a novel term proportional to 2Â¼eB m2 ln Â¡2Â®Â¼ eB m2 Â¢. Using the full effective potential including both the one-loop effective and the improved ring potentials, QED gap equation is determined and the dynamical fermion mass generation is studied in the regime of LLL dominance at non-zero temperature. It is shown that at high temperature limit, where the thermal fluctuations dominate the magnetic catalysis of dynamical chiral symmetry breaking in LLL, a chiral symmetry restoration occurs at certain critical temperature Tc. But, comparing to Tc in the static limit, the critical temperature arising from the improved ring potential is lower.The improved ring contribution is also relevant in studying the electroweak phase transition in the presence of external (strong) magnetic fields [1]. PACS numbers: 11.10.Wx, 11.15.Ex, 12.38.Gc

Improved ring potential of QED at finite temperature and in the presence of weak an strong magnetic field

Talk prepared for PASCOS-08, Waterloo, ON, Canada
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Tehran-Iran

Talk based on:

arXiv: 0805.0078 [hep-ph]. N.S. and K. Sohrabi

The General Framework:

- ***The Problem of Baryogenesis***

The Problem of Baryogenesis:

- **Questions:**
  - Why is the density of baryons much less than the density of photons?
    - 9 order of magnitude difference between theory and observation
  - Why is in the observable part of the universe, the density of baryons many orders of magnitude greater than the density of antibaryons?
    - In cosmic rays, protons outnumber antiproton 10000 to 1

3 Sakharov conditions for baryogenesis (1967):

- Violation of C and CP symmetries
- Non-conservation of baryonic charge
- Deviation from thermal equilibrium, 1st order phase transition

- *In the framework of mininal Standard Model (SM)*

  - Electroweak baryogenesis
  - Electroweak baryogenesis in a constant magnetic field
  - …
  - …

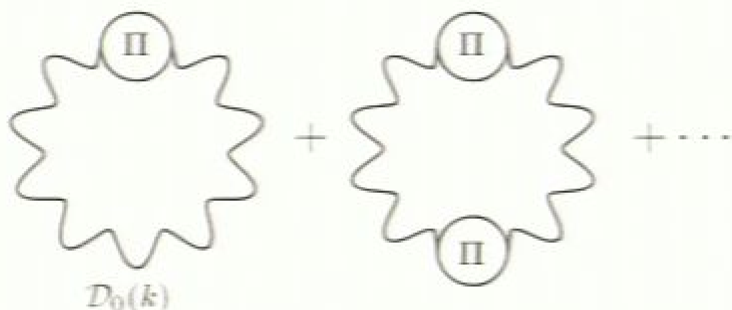
Electroweak (EW) Phase transition at Finite T; History

- *Kirzhnits ('72), Dolan and Jackiw ('74), Weinberg ('74) etc.*
- Look at the EW phase transition
  - Effective potential including only one-loop corrections
  - Result: 2nd order phase transition at Tc > 200 GeV

- *Carrington ('92)*
- Look at the EW phase transition
  - Effective potential includes one-loop *and* ring corrections
  - Result: 1st order phase transition at Tc ~ 140 GeV

Ring (Plasmon) Diagrams in Finite T Field Theory

- Nonperturbative contribution to the effective potential
- They capture the IR behavior of the theory



$$V_{ring} = -\frac{T}{2}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\sum_{N=2}^{\infty}\frac{(-1)^{N-1}}{N}\left[\mathcal{D}_0^{\mu\rho}(n,\mathbf{k})\,\Pi_{\rho\mu}(n,\mathbf{k})\right]^N$$

$$= -\frac{T}{2}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\left\{\ln[1+\mathcal{D}_0^{\mu\rho}(n,\mathbf{k})\Pi_{\rho\mu}(n,\mathbf{k})] - \mathcal{D}_0^{\mu\rho}(n,\mathbf{k})\Pi_{\rho\mu}(n,\mathbf{k})\right\}$$

Ring Diagrams

- Ring potential → In the static limit

$$V_{ring} = -\frac{T}{2}\int \frac{d^3k}{(2\pi)^3} \ln[1 + \mathcal{D}_0^{\mu\rho}(n=0,\mathbf{k})\Pi_{\rho\mu}(n=0,\mathbf{k}=\mathbf{0})]$$

- Effective potential of EW Standard Model

$$V_{ring}(v) = \frac{T}{12\pi}\mathrm{Tr}\left(\left[m^2(v) + \Pi_{00}(0)\right]^{3/2} - m^3(v)\right)$$

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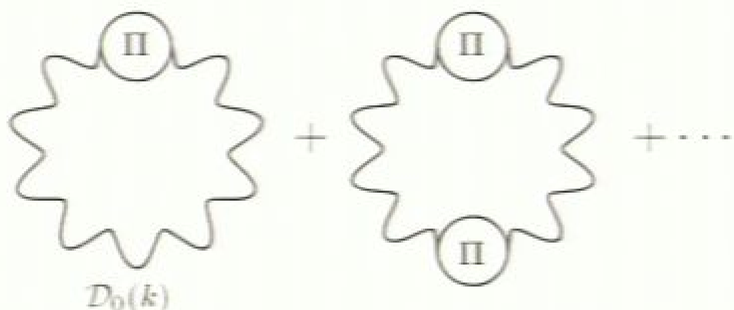

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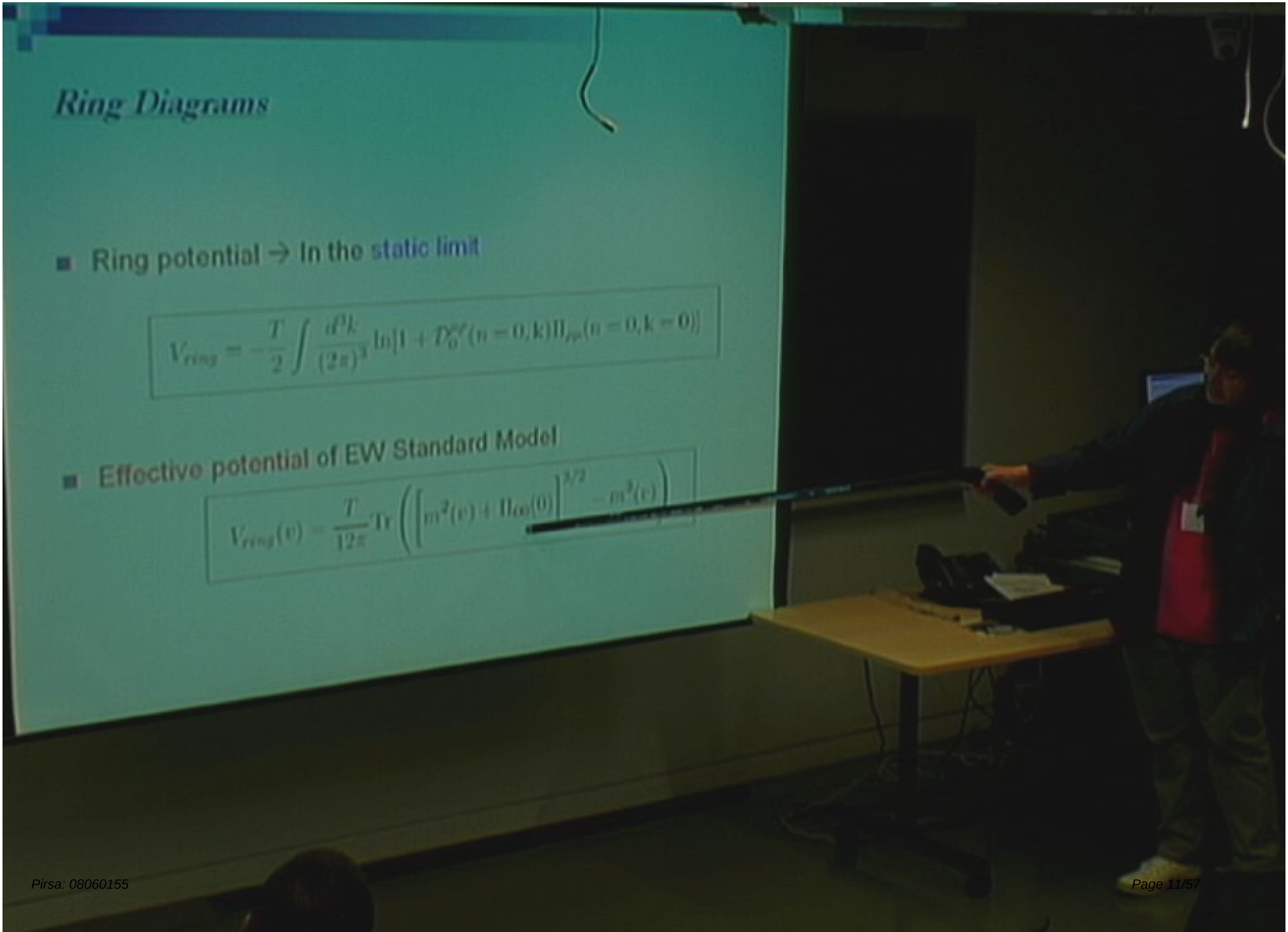
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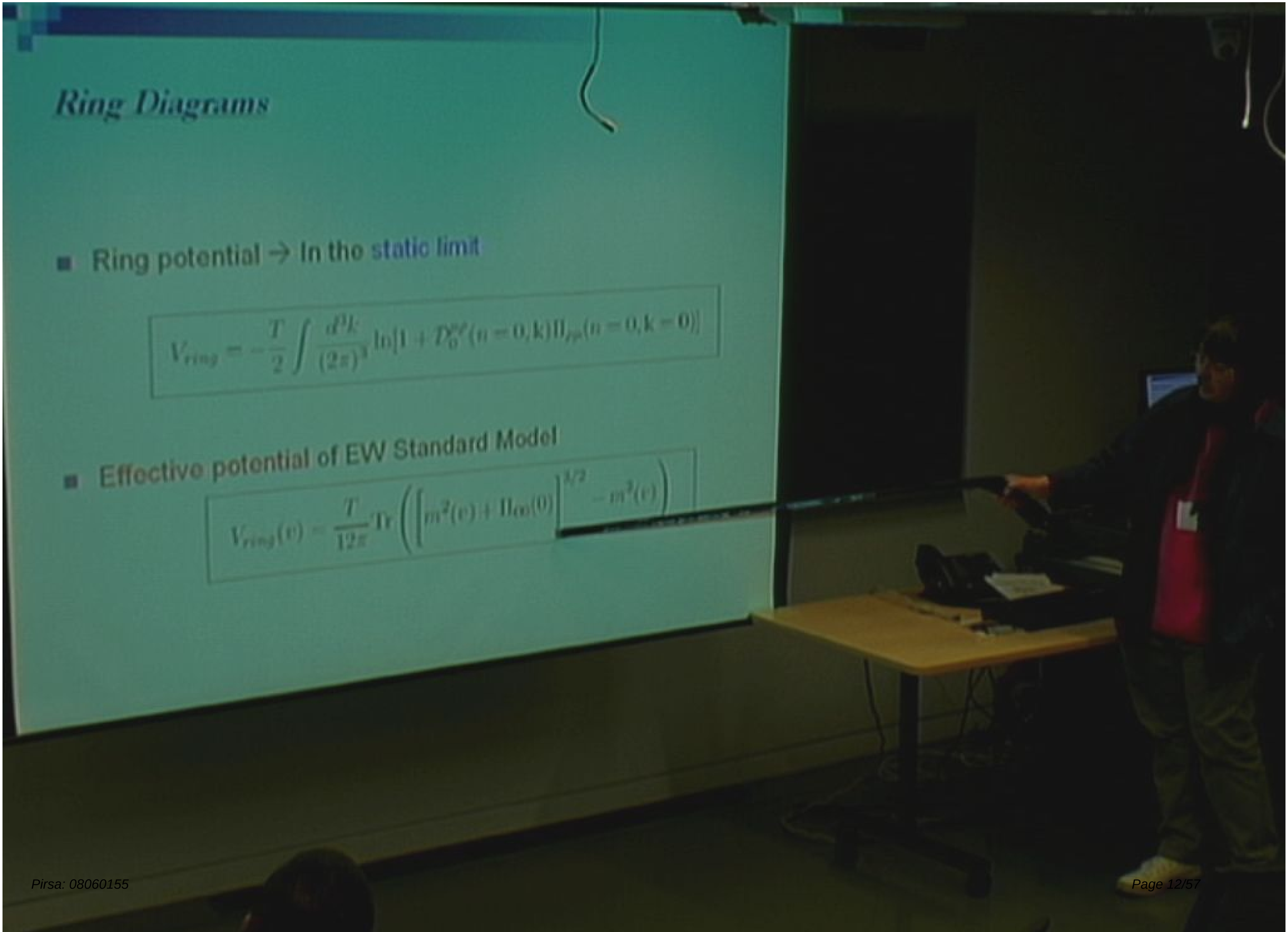
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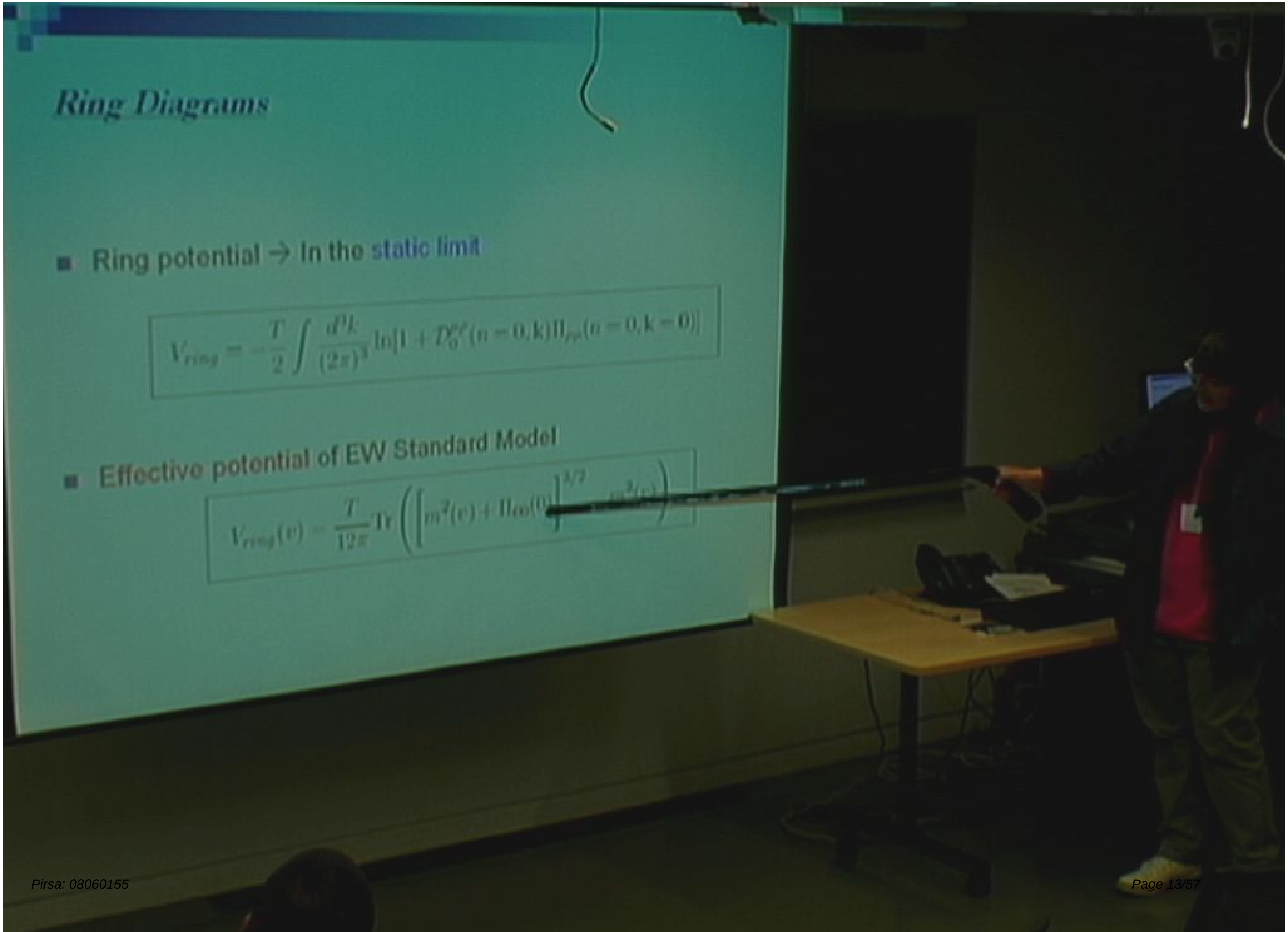
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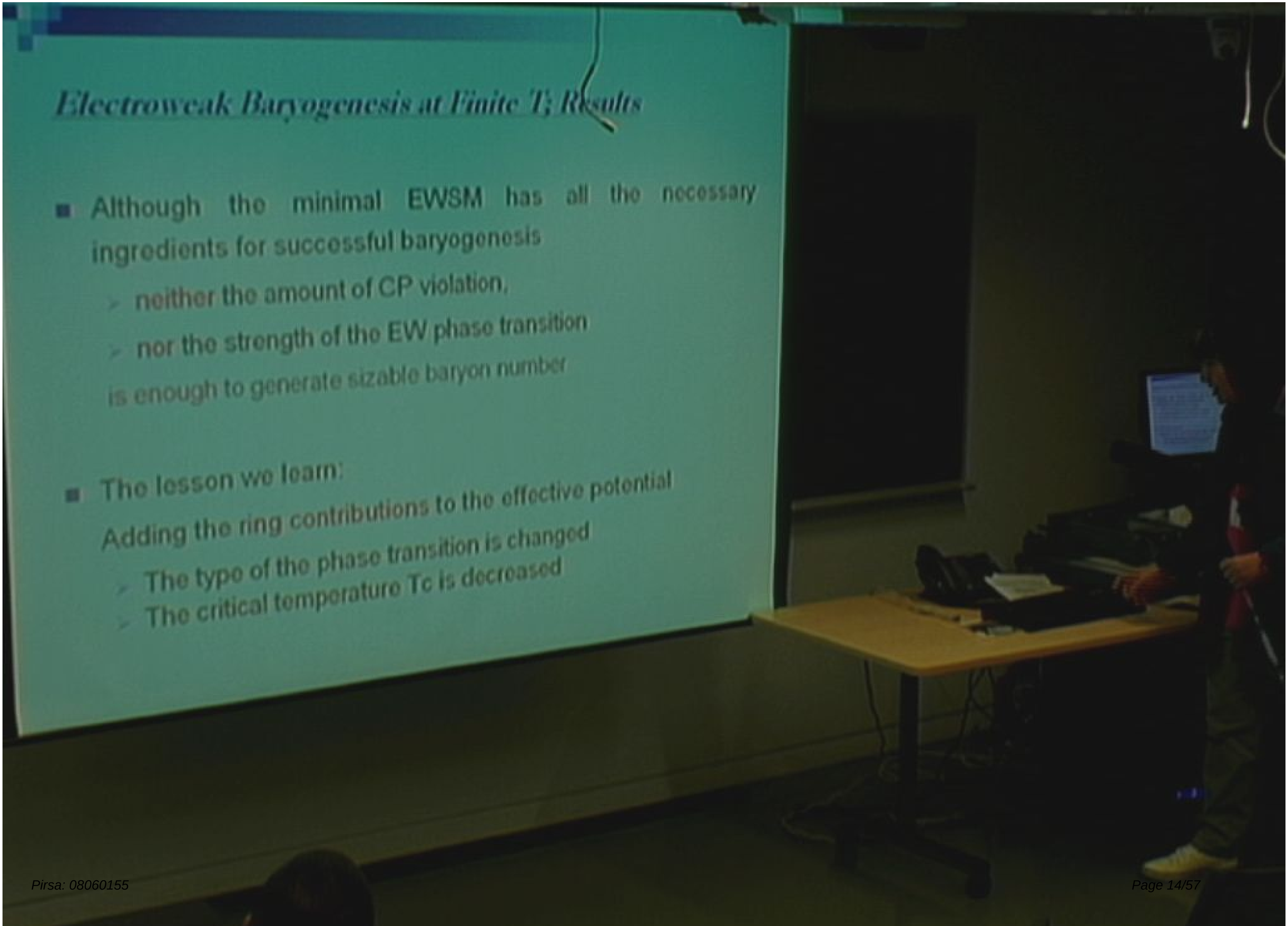
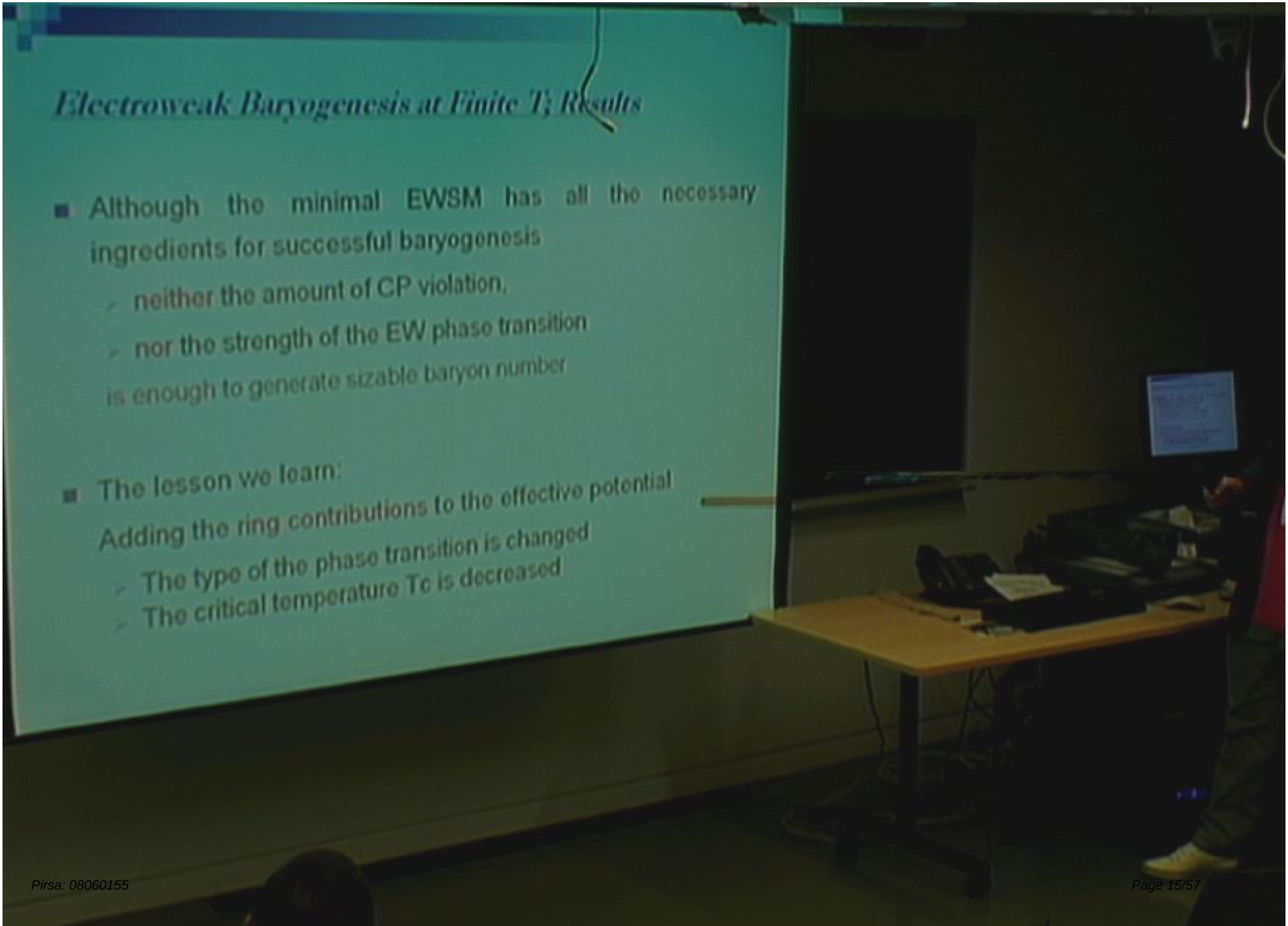

Electroweak Baryogenesis at Finite T; Results
Although the minimal EWSM has all the necessary ingredients for successful baryogenesis
neither the amount of CP violation,
nor the strength of the EW phase transition
is enough to generate sizable baryon number
The lesson we learn:
Adding the ring contributions to the effective potential
The type of the phase transition is changed
The critical temperature Tc is decreased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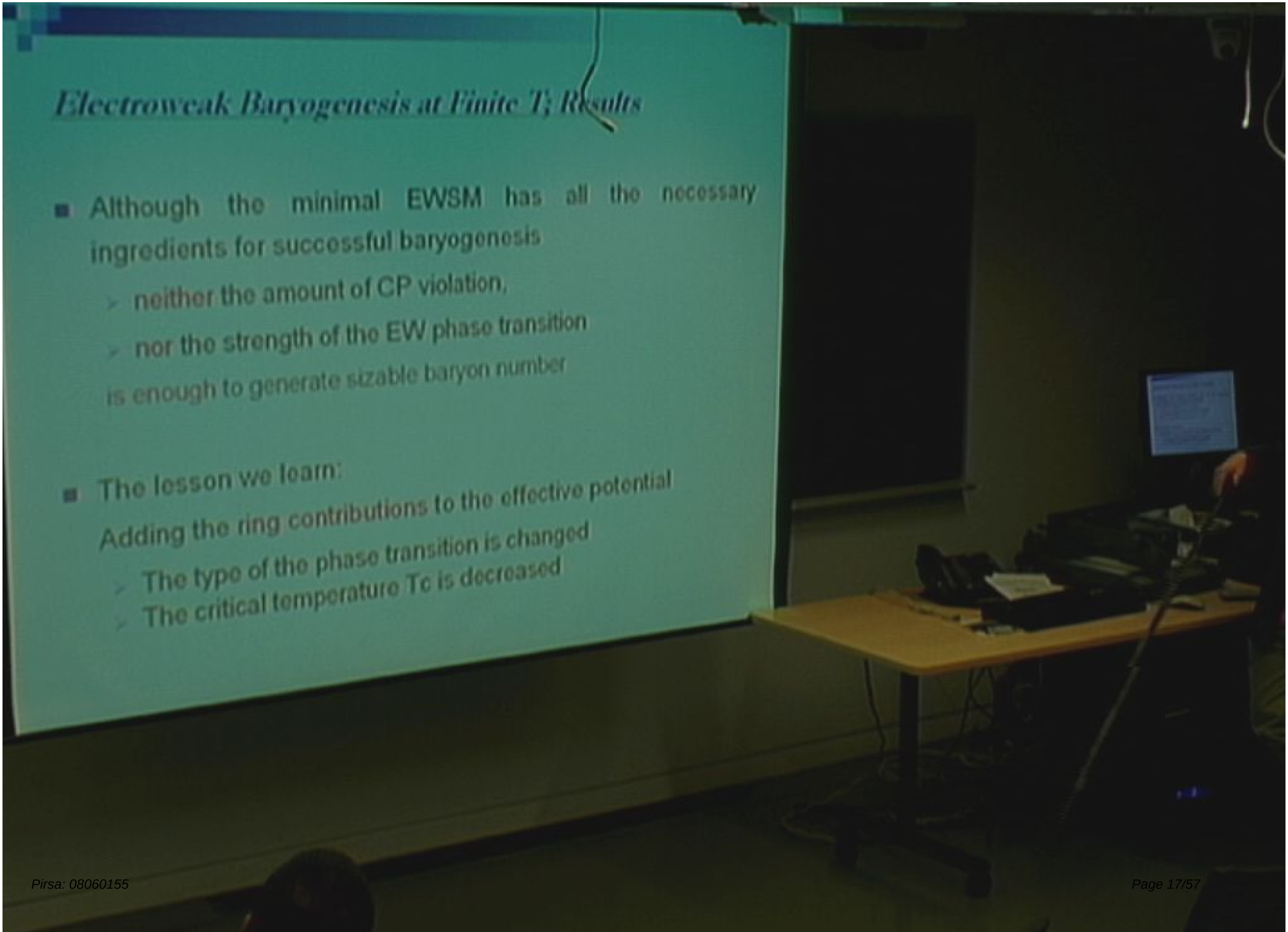
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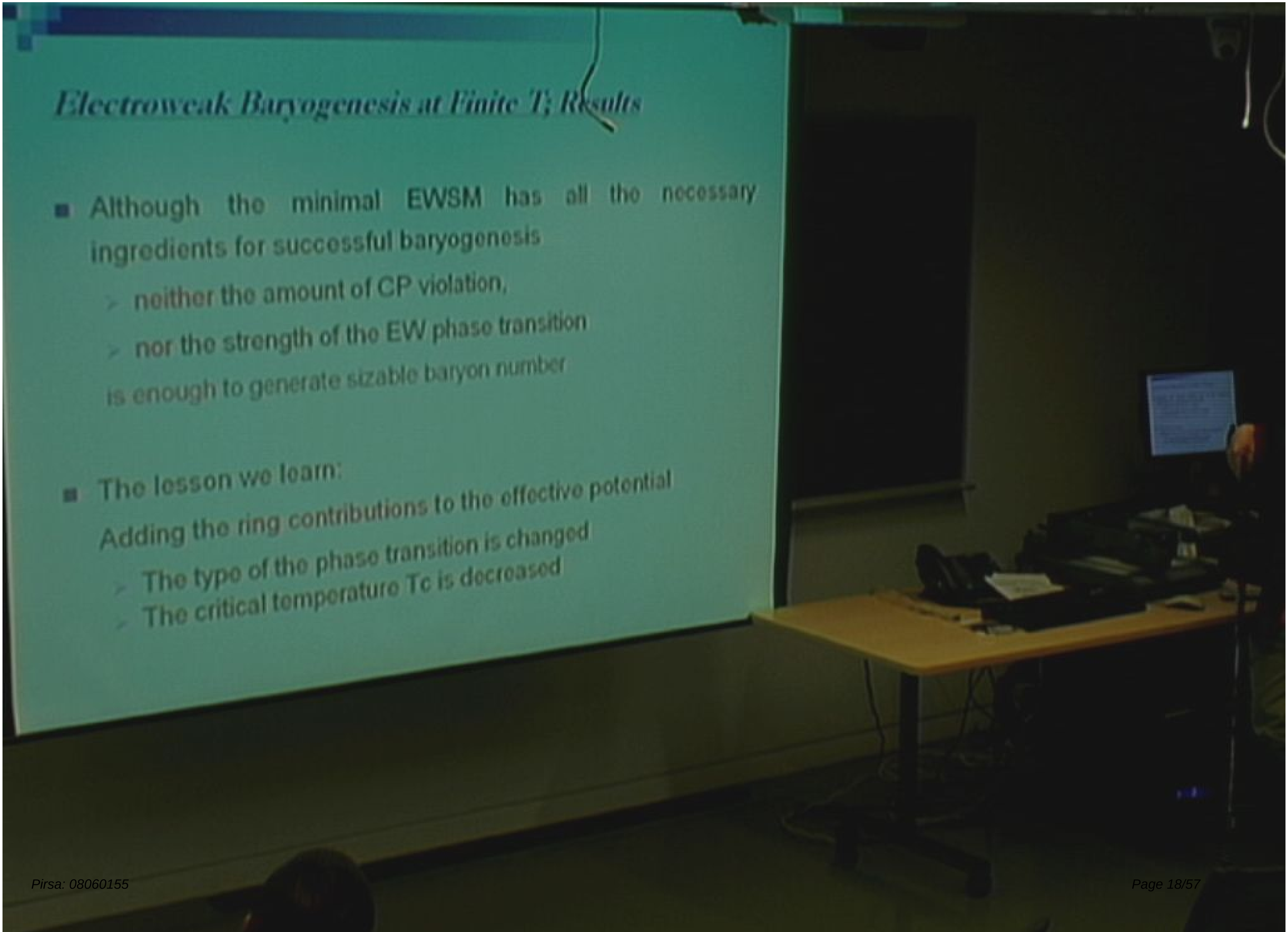
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EW Baryogenesis in Strong Hypermagnetic Field

Series of papers by:

- *Skalozub & Bordag (1998-2006), Ayala et al. (2004-2008)*
  - Electroweak phase transition in a strong magnetic field
  - Effective potential in one-loop + ring contributions in the static limit

Result:

- The phase transition is of 1st order for strong magnetic field

$$10^{23} - 10^{24}\text{G}$$

- The baryogenesis conditions are still not satisfied !!!

Suggestion:

- To improve these results, we suggest to use a certain IR limit to calculate the ring potential

$$V_{ring} = -\frac{T}{2}\int \frac{d^3k}{(2\pi)^3} \ln[1 + \mathcal{D}_0^{\mu\rho}(n=0,\mathbf{k})\Pi_{\rho\mu}(n=0,\mathbf{k}\neq\mathbf{0}; eB)]$$

- To check the efficiency of this novel approximation

→ Test: Dynamical chiral symmetry breaking of QED in LLLA

N.S. and K Sohrabi, 0805.0078 [hep-ph]

Outline of the rest of the talk

- QED effective (thermodynamic) potential in the improved IR limit
  - One loop contribution
  - Ring contribution
- QED ring potential for weak and strong magnetic field
- Dynamical chiral symmetry breaking of QED in the LLL
  - Dynamically generated fermion mass
  - Critical temperature in the static as well as in the IR limit
- Numerical calculation to check the efficiency of the IR limit

- *QED Effective Potential at Nonzero T and B*
    - *One loop*
    - *Ring contribution in the IR limit*

QED One-Loop Effective Potential at Finite T and B

$$V^{(1)}(m, eB; T) = -\frac{2eB}{\beta}\int_0^\infty ds\ \frac{\Theta_2(0|is\frac{4\pi}{\beta^2})}{(4\pi s)^{\frac{3}{2}}}\coth(seB)e^{-sm^2}.$$

Sato ('98)

- T independent part

$$V_0^{(1)}(m, eB; \Lambda) = -\frac{eB}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^\infty \frac{ds}{s^2}\left[e^{-sm^2} + 2\sum_{\ell=1}^\infty e^{-s(m^2+2eB\ell)}\right],$$

$$= -\frac{eB}{8\pi^2}\left[m^2\Gamma\left(-1, \frac{m^2}{\Lambda^2}\right) + 2\sum_{\ell=1}^\infty (m^2+2eB\ell)\ \Gamma\left(-1, \frac{(m^2+2eB\ell)}{\Lambda^2}\right)\right]$$

- T dependent part

$$V_T^{(1)}(m, eB) = -\frac{eB}{4\pi^2}\sum_{n=1}^\infty(-1)^n\left[\int_0^\infty \frac{ds}{s^2}e^{-\left(sm^2+\frac{n^2\beta^2}{4s}\right)} + 2\sum_{\ell=1}^\infty\int_0^\infty \frac{ds}{s^2}e^{-\left(s(m^2+2eB\ell)+\frac{n^2\beta^2}{4s}\right)}\right]$$

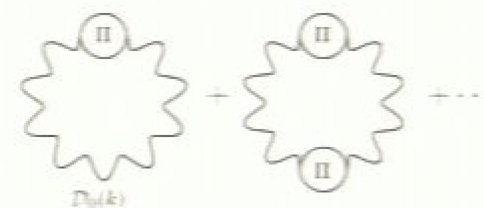
$$= -\frac{eB}{\pi^2}\sum^\infty(-1)^n\left[\frac{m}{n\beta}K_1(n\beta m) + 2\sum^\infty \frac{\sqrt{(m^2+2eB\ell)}}{n\beta}K_1\left(n\beta\sqrt{(m^2+2eB\ell)}\right)\right]$$

QED Ring Potential at Finite T and B

- QED ring potential

$$V_{ring} = \frac{T}{2}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\sum_{N=1}^{\infty}\frac{(-1)^N}{N}\left[\mathcal{D}_0^{\mu\rho}(n,\mathbf{k})\,\Pi_{\rho\mu}(n,\mathbf{k})\right]^N$$

$$= -\frac{T}{2}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\ln[1+\mathcal{D}_0^{\mu\rho}(n,\mathbf{k})\Pi_{\rho\mu}(n,\mathbf{k})].$$



- Using a certain basis vectors $b_\mu^{(i)}$ defined by the eigenvalue equation of the VPT (→Perez Rojas & Shabad '79)

$$\Pi_{\mu\nu}(k)b_\nu^{(i)} = \kappa_i(k)b_\mu^{(i)}.$$

- The free photon propagator in the Euclidean space

$$\mathcal{D}^{(0)}_{\mu\nu}(k_0, \mathbf{k}) = -\sum_{i=1}^{4} \frac{1}{k_E^2} \frac{b^{(i)}_{\mu} b^{\star(i)}_{\nu}}{\left(b^{(i)}_{\rho} b^{\star\rho(i)}\right)}$$

- VPT at finite T and in a constant B field (→ Perez Rojas et al. '79)

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- Orthonormality properties of eigenvectors $b^{(i)}_{\mu}$ → Ring potential

$$V_{ring}(m, eB; T) = -\frac{T}{2} \sum_{n=-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3} \sum_{i=1}^{4} \ln\left(1 - \frac{\kappa_i(k_0, \mathbf{k})}{k_E^2}\right).$$

→ Ring potential in the IR limit (*n=0*)

QED Ring Potential at Finite T and B

- QED ring potential

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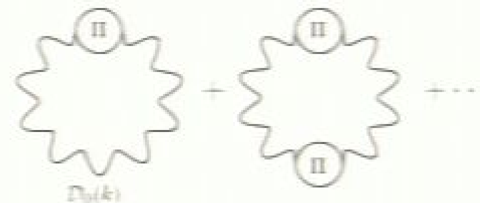

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→ Ring potential in the IR limit (*n=0*)

Ring Potential of QED for Finite B and T

- IR limit (*n=0*)

$$V_{ring}^{\text{IR limit}}(m,eB;T) = -\frac{T}{2}\int\frac{d^3k}{(2\pi)^3}\sum_{i=1}^{4}\ln\left(1-\frac{\kappa_i(k_0\to 0,\mathbf{k})}{\mathbf{k}^2}\right)$$

$$\begin{aligned}
\kappa_1(k_0\to 0,\mathbf{k}) &= -\mathbf{k}^2 I_1\\
\kappa_2(k_0\to 0,\mathbf{k}) &= -\mathbf{k}_\perp^2 I_1 + I_2 - k_3^2 I_3\\
\kappa_3(k_0\to 0,\mathbf{k}) &= -k_3^2 I_1 - 2\mathbf{k}_\perp^2 I_4.\\
\kappa_4(k_0\to 0,\mathbf{k}) &= 0.
\end{aligned}$$

$$I_i = I_i^0 + I_i^T,\ i=1,\cdots,4$$

The integrals $I_i = I_i^0 + I_i^T, i = 1, \cdots, 4$ (→ Alexandre 2001)

$$I_1^0 = -\frac{\alpha e B}{4\pi} \int_{\frac{1}{\Lambda^2}}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} \frac{(\cosh \bar{u}v - v \coth \bar{u} \sinh \bar{u}v)}{\sinh \bar{u}}.$$

$$I_1^T = -\frac{\alpha e B}{2\pi} \int_{0}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} \sum_{\ell=1}^{\infty} (-1)^{\ell} e^{-\frac{\ell^2}{4uT^2}} \frac{(\cosh \bar{u}v - v \coth \bar{u} \sinh \bar{u}v)}{\sinh \bar{u}}.$$

$$I_2^0 = 0.$$

$$I_2^T = -\frac{\alpha e B}{2\pi} \int_{0}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} \sum_{\ell=1}^{\infty} (-1)^{\ell} \frac{\ell^2}{T^2} \; e^{-\frac{\ell^2}{4uT^2}} \; \frac{\coth \bar{u}}{u^2}.$$

$$I_3^0 = -\frac{\alpha e B}{4\pi} \int_{\frac{1}{\Lambda^2}}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} (1 - v^2) \coth \bar{u}.$$

$$I_3^T = -\frac{\alpha e B}{2\pi} \int_{0}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} \sum_{\ell=1}^{\infty} (-1)^{\ell} e^{-\frac{\ell^2}{4uT^2}} (1 - v^2) \coth \bar{u}.$$

$$I_4^0 = -\frac{\alpha e B}{4\pi} \int_{\frac{1}{\Lambda^2}}^{\infty} du \int_{-1}^{+1} dv \; e^{\phi(u,v)} \; \frac{(\cosh \bar{u} - \cosh \bar{u}v)}{\sinh^3 \bar{u}}.$$

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IR vs. Static Limit

- Ring potential in the IR limit

$$V_{ring}^{\text{IR limit}}(m, eB; T) = -\frac{T}{2}\int \frac{d^3k}{(2\pi)^3}\sum_{i=1}^{4}\ln\left(1 - \frac{\kappa_i(k_0 \to 0, \mathbf{k})}{\mathbf{k}^2}\right).$$

- In the static limit $k \to 0$

→ $\kappa_i(0, \mathbf{0}) = 0, \quad \text{for} \quad i = 1, 3, 4 \quad \text{and} \quad \kappa_2(0, \mathbf{0}) = -\Pi_{00}$

$$V_{ring}^{\text{static limit}}(m, eB; T) = -\frac{T}{4\pi^2}\int_0^\Lambda \mathbf{k}^2 d\mathbf{k}\ln\left(1 + \frac{\Pi_{00}(0, \mathbf{0})}{\mathbf{k}^2}\right) = \frac{T}{12\pi}\left[\Pi_{00}\,(0, \mathbf{0})\right]^{3/2}$$

- *QED Ring Potential in Weak B Field Limit*

Weak B Field Limit

- Characterized by: $eB \ll m^2 \ll T^2$ and $\mathbf{k}_\perp^2 \ll eB \ll k_3^2$
- Evaluating $I_i = I_i^0 + I_i^T, i = 1, \cdots, 4$ in $eB \to 0$ limit

$$\tilde{I}_i^0 = a_i^0 + \frac{\mathbf{k}^2}{m^2} b_i^0 \quad \text{for} \quad i = 1, 3, \quad \text{as well as} \quad \tilde{I}_i^T = a_i^T + \frac{\mathbf{k}^2}{m^2} b_i^T, \quad \text{for} \quad i = 1, 2.$$

- In the IR limit

$$V_{ring}^{\text{IR limit/weak}} \approx T \frac{(a_2^T)^{3/2}}{\left(1 - \frac{b_2^T}{m^2}\right)^{3/2}} + \text{Cutoff dependent terms}$$

- In the static limit

$$V_{ring}^{\text{static limit}} \approx T \left(a_2^T\right)^{3/2}$$

QED ring potential in the IR limit and weak magnetic field

$$V_{ring}^{\text{IR limit/weak}} \approx T \frac{(a_2^T)^{3/2}}{\left(1 - \frac{b_2^T}{m^2}\right)^{3/2}} + \text{Cutoff dependent terms}$$

→ In the high temperature expansion $m\beta \to 0$

$$V_{ring}^{\text{IR limit/weak}} \approx T^4 \left(\frac{\alpha}{1+\frac{\alpha}{2\pi}}\right)^{3/2} \left[1 - \frac{7\zeta(3)}{8\pi^3} \frac{\alpha}{(1+\frac{\alpha}{2\pi})}(m\beta)^2\right] + \mathcal{O}\left((m\beta)^3\right)$$

→ In the limit $\alpha \to 0$

$$V_{ring}^{\text{IR limit/weak}} \approx T^4 \alpha^{3/2} \left[1 - \frac{\alpha}{2\pi}\left(1 - \frac{7\zeta(3)}{4\pi^2}(m\beta)^2\right)\right] + \mathcal{O}\left(\alpha^{7/2}, (m\beta)^3\right)$$

- Comparing to the static limit, an additional term $\alpha^{5/2}$ appears
- Well-known terms in QCD at finite T ← HTL expansion

Braaten+Pisarski ('90)

- *QED Ring Potential in Strong B Field Limit*

QED in a Strong Magnetic Field at zero T

- Characterized by Landau levels as in non-relativistic QM
- For strong enough magnetic fields the levels are well separated and Lowest Landau Level (LLL) approximation is justified

→ In the LLLA, an effective QFT replaces the full QFT

Properties at zero T; Magnetic Catalysis (Miransky et al. '95)

- Dynamical chiral symmetry breaking
  - Dynamical mass generation

$$m_{dyn.} = C\sqrt{eB}\exp\left(-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right)$$

- Dimensional reduction from $D \to D\text{-}2$
  - Two regimes of dynamical mass
    - $|\mathbf{q}_{\parallel}^2| \ll m_{dyn.}^2 \ll |eB|$
    - $m_{dyn.}^2 \ll |\mathbf{q}_{\parallel}^2| \ll |eB|$
  - Photon is massive in the 2nd regime:

$$M_\gamma^2 = \frac{2\alpha|eB|N_f}{\pi}$$

QED Ring Potential in Strong B Field Limit at nonzero T

- Characterized by:

$$m^2 \ll T^2 \ll |eB| \quad \text{and} \quad \mathbf{k}_3^2 \ll m^2 \ll |eB| \quad \text{or} \quad m^2 \ll \mathbf{k}_3^2 \ll |eB|$$

- Evaluating $I_i = I_i^0 + I_i^T, i = 1, \cdots, 4$ in $\frac{eB}{m^2} \to \infty$ limit
- QED ring potential in the IR limit

$$V_{ring}^{\text{IR limit/strong}} = -\frac{mTeB}{8\pi^2}\left(\text{Li}_2\left(-\frac{M_\gamma^2}{m^2}\left(1-\frac{5}{3}C_1(m\beta,\pi)\right)\right) - \text{Li}_2\left(-\frac{M_\gamma^2}{m^2}\right)\right)$$

with
$$C_1(z,\phi) \equiv \frac{\pi}{2}\sum_{\ell=-\infty}^{\infty}\frac{z^2}{\left(z^2+(\phi-2\pi\ell)^2\right)^{3/2}}$$

$$M_\gamma^2 = \frac{2\alpha|eB|N_f}{\pi}$$

$z \equiv m\beta$ as well as $\phi \equiv \pi$

→ In the high temperature limit $m\beta \to 0$

$$V_{ring}^{\text{IR limit/strong}}\Big|_{\frac{eB}{m^2}\to\infty,\, m\beta\to 0} \approx -\frac{35 m^4 \zeta(3)}{384\pi^3 \alpha}\left[1 + \frac{2\alpha}{\pi}\frac{eB}{m^2}\ln\left(\frac{2\alpha}{\pi}\frac{eB}{m^2}\right)\right](m\beta)$$

→ From QCD at finite T → $g_s^4 \ln g_s$ **Toimela ('83)**

→ Comparing to the static limit

$$V_{ring}^{\text{static limit/strong}} = \frac{T}{12\pi}\left(\Pi_{00}(0,\mathbf{0})\right)^{3/2} = \frac{T}{12\pi}\left(I_2^T(0,\mathbf{0})\right)^{3/2}$$
$$\xrightarrow{eB\to\infty} \frac{T}{12\pi}\left[M_\gamma^2\left(1 - 2C_1(m\beta,\pi)\right)\right]^{3/2}.$$

- *Dynamical Chiral Symmetry Breaking in the LLL*

QED Gap Equation in the LLL

- QED in the LLL → Dynamically generated fermion mass
  - The corresponding (mass) gap equation $$\frac{\partial V}{\partial \langle \bar{\psi}\psi \rangle} = Gm$$
  - Using $\langle \bar{\psi}\psi \rangle \equiv \frac{\partial V}{\partial m}$
  - Gap equation →

$$\frac{\partial \tilde{V}}{\partial m} = Gm \frac{\partial^2 \tilde{V}}{\partial m^2} \qquad \text{where} \qquad \tilde{V} = V_T^{(1)} + V_{ring}$$

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One-loop Contribution:

- Dynamical mass

$$m^{(1)}(T) = \Lambda_B \exp\left(\ln\frac{4\pi T}{\Lambda_B} - \gamma - 2C_0 + \frac{G}{1-G}(1-2C_1)\right), \qquad \Lambda_B \equiv \sqrt{eB}$$

$$C_0(z,\phi) \equiv \frac{\pi}{2}\sum_{\ell}{}' \left(\frac{1}{\sqrt{z^2+(\phi-2\pi\ell)^2}} - \frac{1}{2\pi|\ell|}\right), \qquad z \equiv m\beta \text{ as well as } \phi \equiv \pi$$

- Critical temperature Tc of DSB is determined by $m(T_c) = 0$

$$T_c^{(1)} = \frac{\Lambda_B}{4\pi}\exp\left(\gamma + 2\ln 2 - \frac{G}{1-G}\right)$$

Ring Contribution

- Dynamical mass

$$m(T) = m^{(1)}(T)\exp\left(-\frac{2\pi^2}{eBm(1-G)}\left[\frac{\partial V_{ring}}{\partial m} - Gm\frac{\partial^2 V_{ring}}{\partial m^2}\right]\right).$$

- Critical temperature of dynamical chiral symmetry restoration

$$T_c \sim T_c^{(1)}$$

- Tc in the:
  - IR Limit
  - Static Limit → →

Ring Contribution

- Dynamical mass

$$m(T) = m^{(1)}(T)\exp\left(-\frac{2\pi^2}{\epsilon B m(1-G)}\left[\frac{\partial V_{ring}}{\partial m} - Gm\frac{\partial^2 V_{ring}}{\partial m^2}\right]\right).$$

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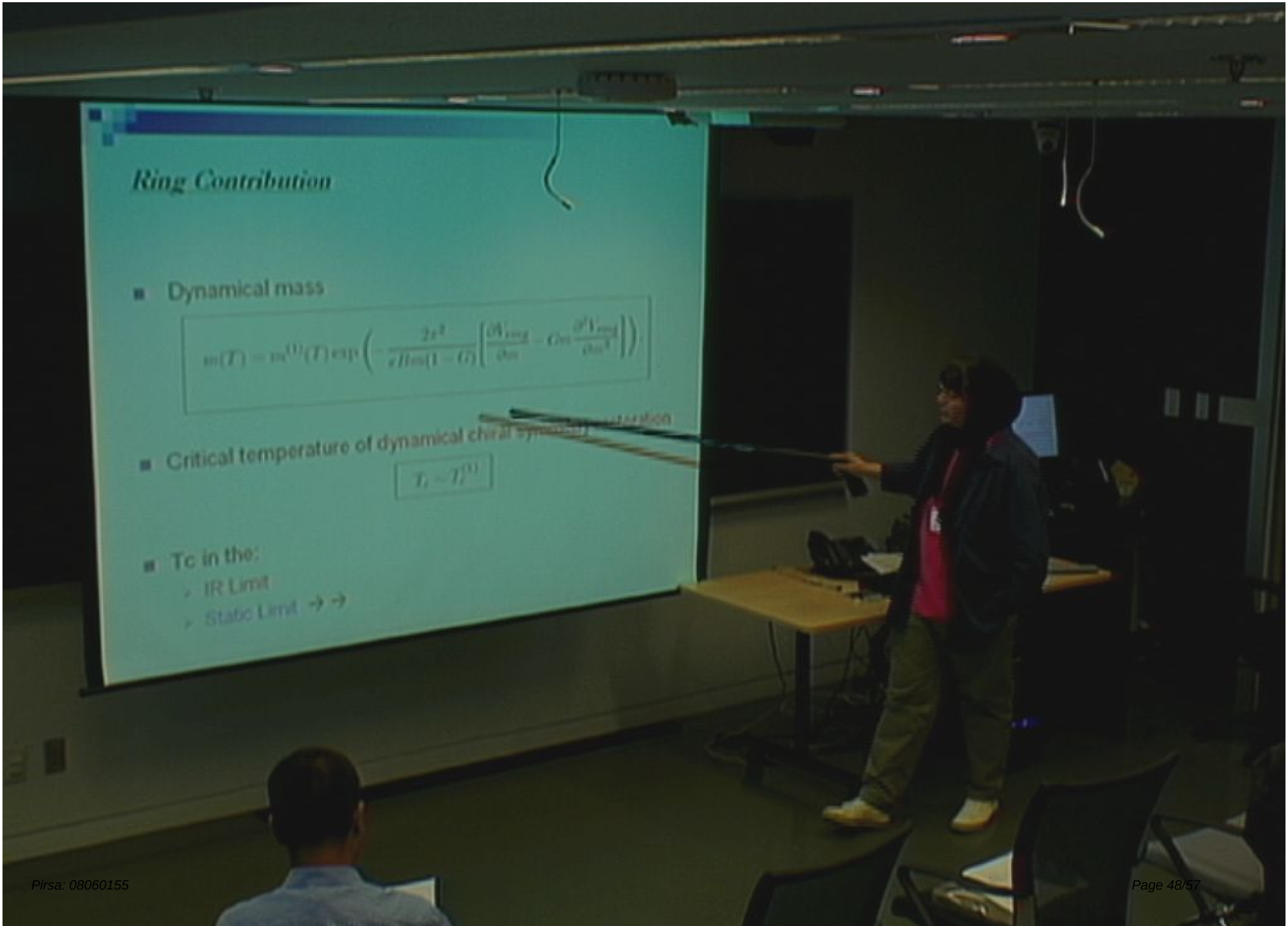
Ring Contribution
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Tc in the:
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Ring Contribution

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- Critical temperature of dynamical chiral symmetry restoration

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- Tc in the:
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Critical Temperature of DSB in the IR Limit

- Using

$$V_{ring}^{\text{IR limit/strong}} = -\frac{mTeB}{8\pi^2}\left(\mathrm{Li}_2\left(-\frac{M_\gamma^2}{m^2}\left(1-\frac{5}{3}C_1(m\beta,\pi)\right)\right)-\mathrm{Li}_2\left(-\frac{M_\gamma^2}{m^2}\right)\right)$$

- The critical temperature Tc in the IR limit

$$T_c^{\text{IR}} = T_c^{(1)}\exp\left(-\frac{35\zeta(3)}{48\pi^2}(m_0\beta_c)\ln\left(1+z_0^2\right)\right)$$

Critical Temperature of DSB in the Static Limit

- Using
$$V_{ring}^{\text{strong limit}} = \frac{T}{12\pi}\left[M_\gamma^2\left(1-2C_1\left(m\beta,\pi\right)\right)\right]^{3/2}.$$

- The critical temperature Tc in the static limit

$$\boxed{T_c^{\text{strong}} = T_c^{(1)}\exp\left(-\frac{7\zeta(3)\alpha}{4\pi^2}(m_0\beta_c)z_0\right)}$$

m_0 is a fixed, T independent mass (a necessary IR cutoff) and

$$z_0^2 \equiv \frac{M_\gamma^2}{m_0^2} = \frac{2\alpha}{\pi}\frac{eB}{m_0^2}$$

IR vs. Static Limit

- The general structure of Tc →

$$T_c = T_c^{(1)} \exp\left(\frac{m_0}{T_c}\kappa\left(z_0\right)\right)$$

- IR limit

$$\kappa^{\mathrm{IR}} = \frac{35\zeta(3)}{48\pi^2}\ln(1+z_0^2)$$

- Static limit

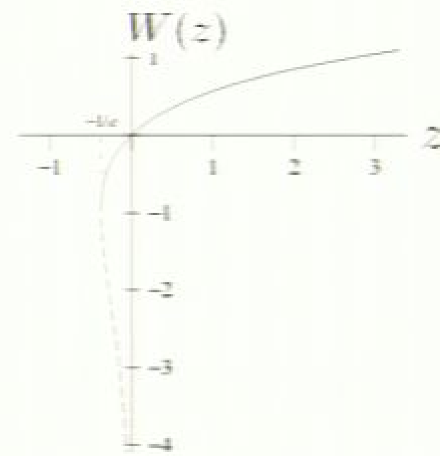
$$\kappa^{\mathrm{strong}} = \frac{7\zeta(3)\alpha}{4\pi^2}z_0.$$

- Defining

$$u \equiv \frac{T_c^{(1)}}{T_c}, \qquad a \equiv \frac{m_0}{T_c^{(1)}} \Longrightarrow \kappa = \frac{1}{au}\ln u$$

- This equation can now be solved for *u*

$$u \equiv \frac{T_c^{(1)}}{T_c} = \frac{W(-a\kappa)}{-a\kappa} \qquad \text{with} \qquad a \equiv \frac{m_0}{T_c^{(1)}},$$



with the Lambert *W(z)* function, that satisfies

$$W(z)e^{W(z)} = z.$$

- Question: How efficient is the ring contribution in the IR or static limits in decreasing the Tc of DSB arising from one-loop effective potential?

- Define the efficiency factor $\eta \equiv 1 - \frac{1}{u} = \frac{T_c^{(1)} - T_c}{T_c^{(1)}}$

Numerical Results

$$\eta \equiv 1 - \frac{1}{u} \quad \text{with} \quad u = \frac{W(-a\kappa(z_0))}{-a\kappa(z_0)}$$

Choosing $m_0 = 0.5\ \text{MeV}$ and $\alpha = \frac{1}{137}$ $\quad a = 1$

- Astrophysics of neutron stars $\quad B \sim 10^{13} - 10^{15}$ Gauß
- RHIC experiment (heavy ion collisions) $\quad B \sim 10^{16} - 10^{17}$ Gauß

eB in GeV^2	B in Gauß	$z_0^2 \equiv \frac{2\alpha}{\pi}\frac{eB}{m_0^2}$	u^{IR}	η^{IR} in %	u^{strong}	η^{strong} in %
10^{-6}	1.691×10^{14}	1.86×10^{-2}	1.002	0.16%	1.0002	0.02%
10^{-5}	1.691×10^{15}	1.86×10^{-1}	1.02	1.53%	1.0006	0.07%
10^{-4}	1.691×10^{16}	1.86	1.11	9.83%	1.0021	0.21%
10^{-3}	1.691×10^{17}	1.86×10^{1}	1.48	32.32%	1.0068	0.68%
10^{-2}	1.691×10^{18}	1.86×10^{2}	$1.82 - 1.45i$	–	1.0219	2.14%
10^{-1}	1.691×10^{19}	1.86×10^{3}	$0.89 - 1.58i$	–	1.0747	6.95%
1	1.691×10^{20}	1.86×10^{4}	$0.47 + 1.43i$	–	1.3243	24.49%
10^{1}	1.691×10^{21}	1.86×10^{5}	$0.25 - 1.28i$	–	$0.89 - 1.58i$	
10^{2}	1.691×10^{22}	1.86×10^{6}	$0.11 - 1.15i$		$0.19 - 0.89i$	

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To summarize:

- QED effective potential is calculated at finite T and in the presence of weak/strong magnetic field

- In particular, the ring contribution to the EP is calculated in the improved IR limit

- The results are compared with the static limit

- Dynamical chiral symmetry breaking of QED in the LLL at finite T is studied

→ Comparing to the static limit, the IR limit is more efficient in decreasing the Tc arising from one-loop effective potential