Title: Lattice Chirality and the Decoupling of Mirror Fermions

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Abstract: With LHC commissioned in just a few month ahead, all sorts of ideas about physics beyond the standard model are being explored intensively. A strong-coupling chiral theory appearing at TeV scale remains a possibility but also a very hard scenario to study. When it comes to strongly coupled theories, lattice regularization is by far the most reliable method. But defining exact chiral gauge theory on the lattice remains a difficult problem on its own. We show that the idea to use additional non-gauge, high-scale mirror-sector dynamics to decouple the mirror fermions without breaking the gauge symmetry might lead to a practically manageable solution. We demonstrate, using the exact lattice chirality, that partition functions of lattice gauge theories with vector like fermion representations can be split into \'light\' and \'mirror\' parts, and each contains a chiral representation. Such a splitting is only well defined when both sectors are separately anomaly free. We also prove that, the generating function and therefore the spectrum of an arbitrary chiral gauge theory is a smooth function of the background gauge field, if and only if the anomaly free condition is satisfied. We reached this conclusion by proving some very general properties of an arbitrary chiral gauge theory on lattice, and the results should be of importance for further studies in this field.

# Lattice Chirality and the Decoupling of Mirror Fermions

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Perimeter Institute, PASCOS '08

E. Poppitz and YS, arXiv:0801.0587.
E. Poppitz and YS, JHEP **0708**, 081 (2007) [arXiv:0706.1043].
J. Giedt and E. Poppitz, JHEP **0710**, 076 (2007) [arXiv:hep-lat/0701004].

# Outline

1. Motivation and idea
   - Why chiral, why lattice
   - Why need the idea of "decoupling of mirror fermions"
   - Does it work: some encouraging numerical results
2. Theoretical thoughts: more on why lattice chiral gauge theory is so hard
   - Naive problem: fermion doubling problem
   - Exact lattice chiral symmetry
   - Some general results and a powerful theorem

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# Why chiral and why lattice

- Currently most popular scenarios for LHC-scale physic involve weakly coupled models of electroweak symmetry breaking
- It, however, remains possible that strongly coupled dynamics is at work at the scale beyond SM.
- The kinds of strong-coupling gauge dynames we understand are only a few
  - 't Hooft anomaly matching
  - SUSY protected theories
  - Large-N
  - AdS/CFT type dualities, etc.
- Most don't work very well for chiral theories
- Lattice formulation remains the most reliable non-perturbative definition of strongly coupled QFT
- Non-perturbative definition of SM?

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- Defining chiral gauge theory on the lattice is really difficult, explained more later
- Defining vector-like gauge theories (e.g. QCD) is not as a problem
- Can we start with a vector like theory, for example:

| | | | |
|---|---|---|---|
| $SU(5)$ | $5^*$ | $5$ | all Weyl |
| | $10$ | $10^*$ | |
| | light | mirror | |

  and then deform the theory such that
  - mirror decouple from the low-energy spectrum
  - the gauge symmetry remains unbroken
- Normal continuum field theorists might say: no!
- Lattice may offer new possibilities

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# Strong-coupling symmetric phase

- Everybody knows that four-fermi interactions, if coupling taken strong enough, break chiral symmetries

$$\frac{g}{\Lambda^2}(\overline{\psi}\psi)(\overline{\psi}\psi), \quad gN > 8\pi^2$$

- However, if one takes coupling even stronger, the theory enters a "strong-coupling symmetric phase": with only massive excitations and unbroken chiral symmetry
- These phases are "lattice artifact" as the massive excitations are heavier than the UV cutoff
- Strong coupling expansion, finite range of convergence in $\frac{1}{g}$

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# Gauged XY model

- $$-S_\kappa = \sum_{\mathbf{x}} \left( \frac{\beta}{2} \prod_{\text{plaq}} U + \frac{\kappa}{2} \sum_{\hat{\mu}} \phi^*_{\mathbf{x}} U_{\mathbf{x},\mathbf{x}+\hat{\mu}} \phi_{\mathbf{x}+\hat{\mu}} \right) + \text{h.c.}$$

  where $\phi_{\mathbf{x}} = e^{i\eta_{\mathbf{x}}}$ is a unitary field.
- $\kappa < 1$, the theory is in a strong-coupling symmetric phase
- D. R. T. Jones, J. B. Kogut and D. K. Sinclair, Phys. Rev. D **19** (1979) 1882. …

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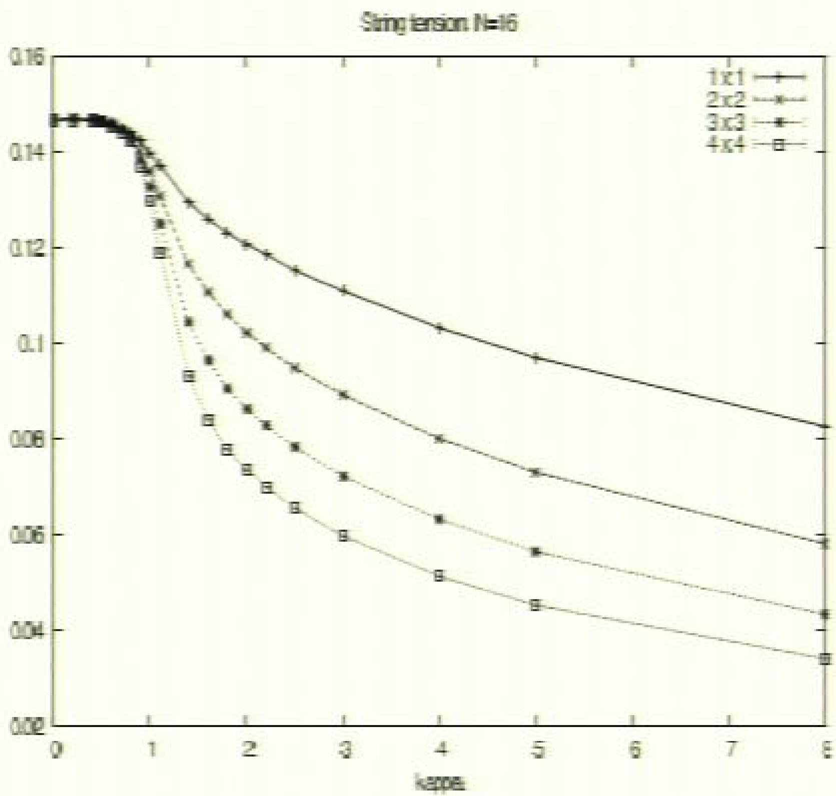


Figure: String tension vs $\kappa$ for $N = 16$.

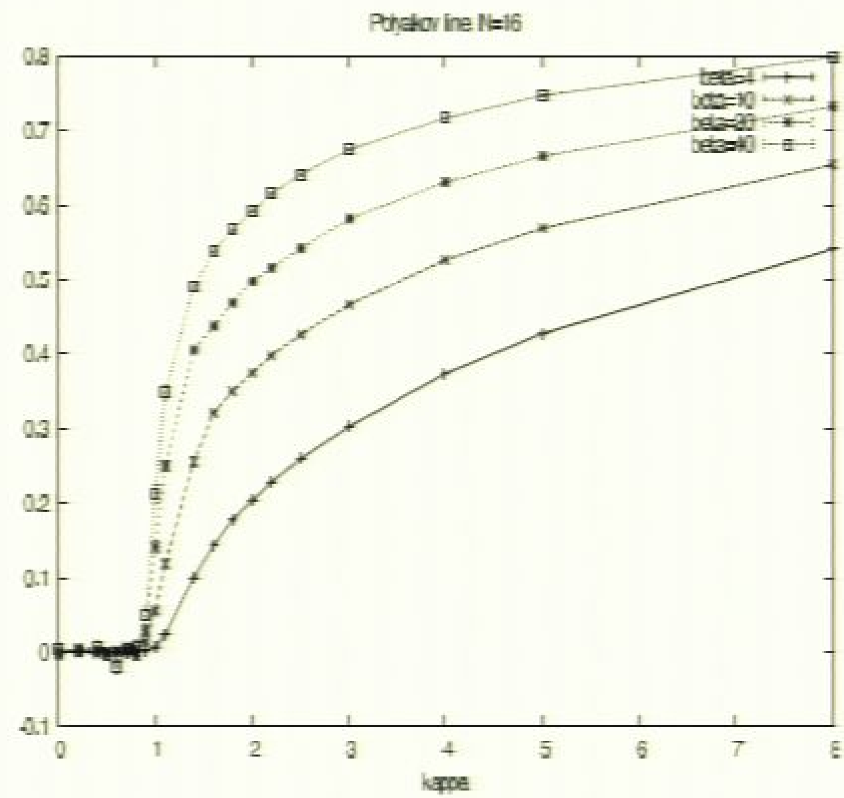


Figure: Polyakov line vs $\kappa$ for $N = 16$.

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# A toy model: 0–1 model

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\begin{aligned}
S &= S_{\rm light} + S_{\rm mirror} \\
S_{\rm light} &= (\overline{\psi}_-, D_1 \psi_-) + (\overline{\chi}_-, D_0 \chi_-) \\
S_{\rm mirror} &= (\overline{\psi}_+, D_1 \psi_+) + (\overline{\chi}_+, D_0 \chi_+) \\
&\quad + y\{(\overline{\psi}_+, \phi^* \chi_-) + (\overline{\chi}_-, \phi \psi_+) \\
&\quad + h[(\psi_+^T, \phi \gamma_2 \chi_-) - (\overline{\chi}_-, \gamma_2 \phi^* \overline{\psi}_+^T)]\} \\
S_\kappa &= \frac{\kappa}{2} \sum_{x,\mu} [2 - (\phi_x^* U_{x,x+\mu} \phi_{x+\mu} + \text{h.c.})]
\end{aligned}
$$

  Here $\phi_x = e^{i\eta_x}$ is a unitary higgs field and $(\psi, \chi) = \sum_x \psi_x^* \chi_x$
- Evidence for a symmetric phase while $y$ large and $h > 1$, mirror fermions $\psi_+$ are heavy.

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# Evidence: scalar is heavy

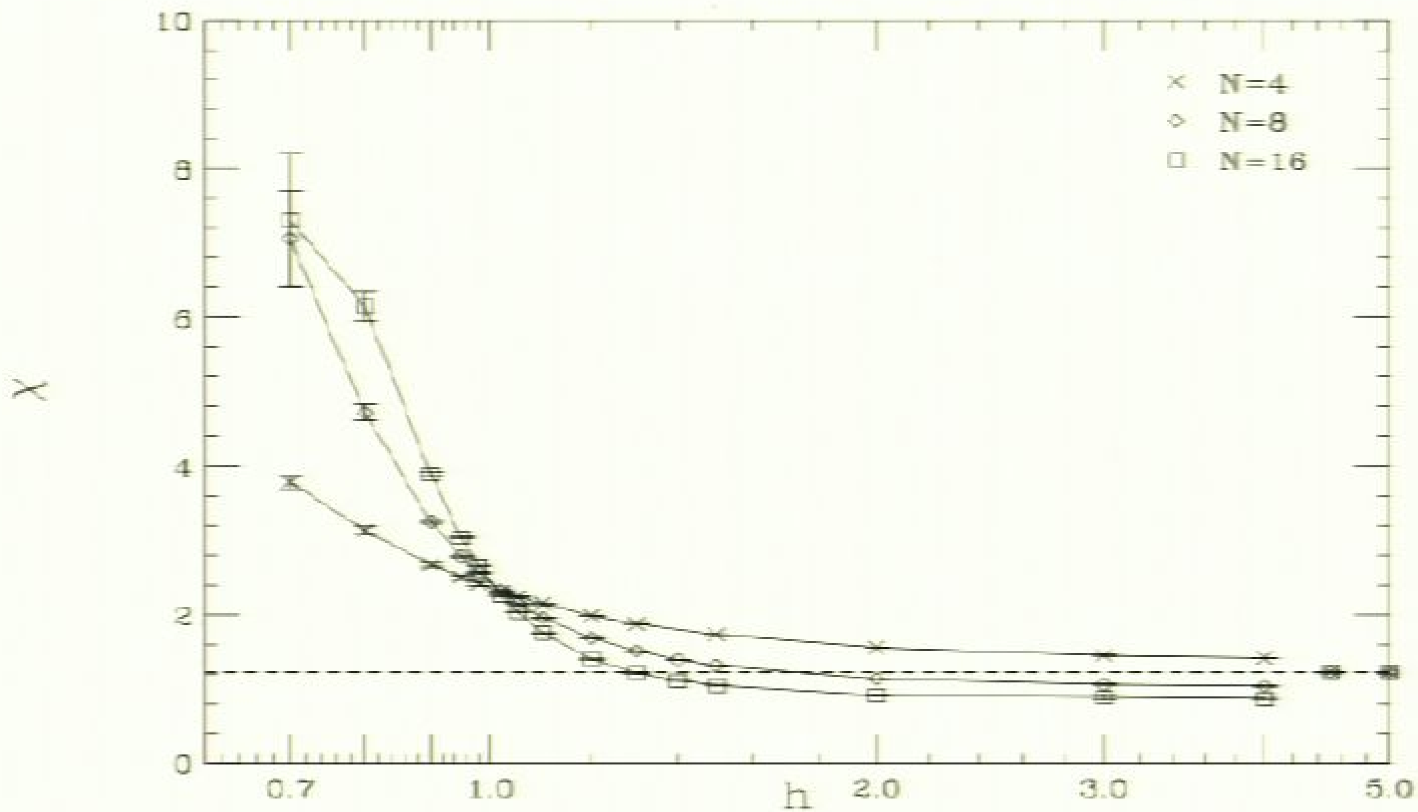


Figure: Susceptibilities of $\phi$ for $\kappa = 0.1$ and $N = 4, 8, 16$. Dash line indicates the susceptibility of $\phi$ in pure $XY$-model

# Evidence: fermions are heavy

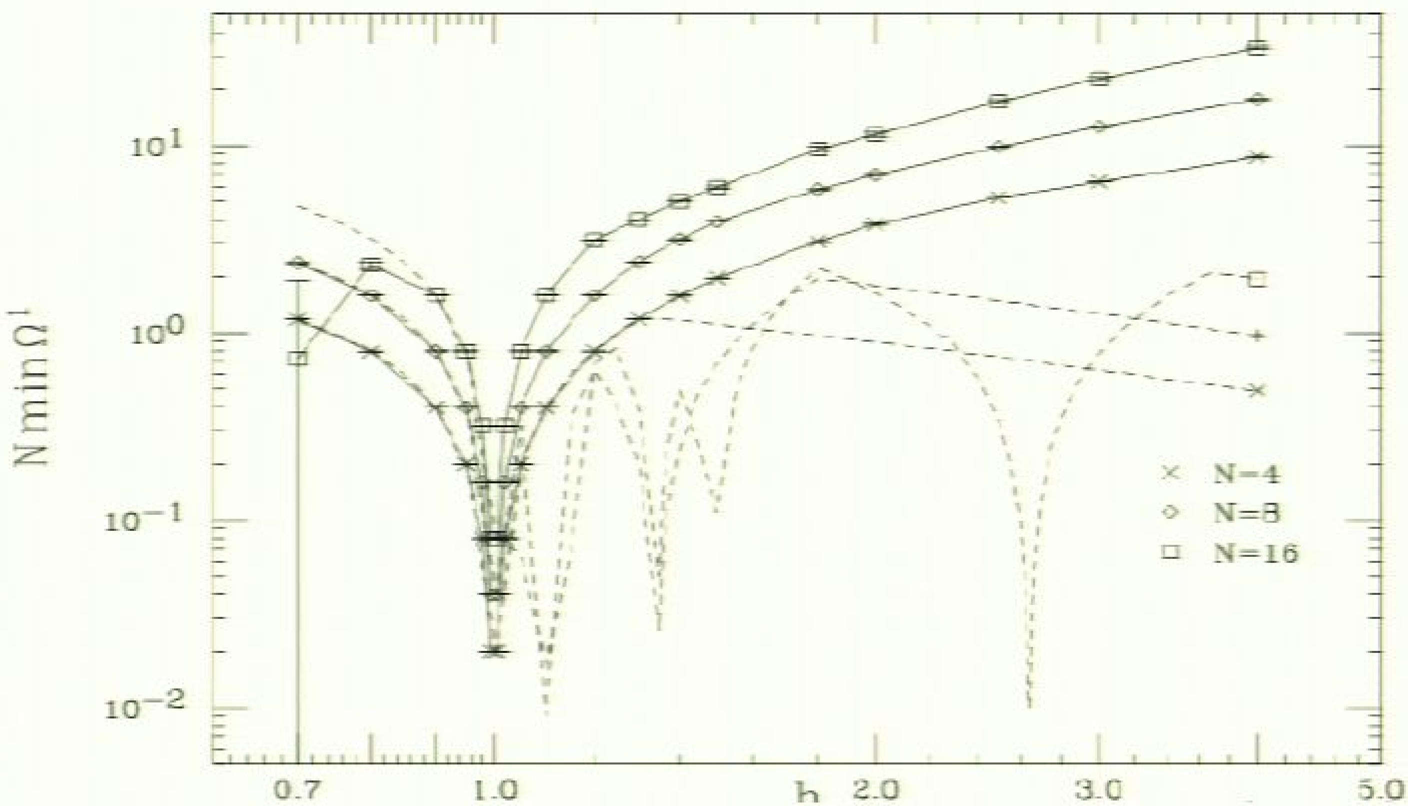


Figure: The lower bound on the charged mirror fermion mass for $\kappa = 0.1$

# So did the dream come true?

- If the mirror parts are all heavy, at the low energy we get a chiral gauge theory on the lattice automatically, circumventing the difficulty of defining it explicitly. Great!
- Are we sure?
  - That entire mirror sector is heavy?
  - Is the continuum limit unitary?
  - The light content is anomalous.

$$S_{\text{light}} = (\overline{\psi}_{-}, D_1 \psi_{-}) - (\overline{\chi}_{-}, D_0 \chi_{-})$$

    and same with $S_{\text{mirror}}$. Therefore, the splitting between light and mirror must NOT be consistent. Something has to go wrong, and what is it?
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# Naive problem:

## Fermion doubling problem

- Naive discretization of Dirac operator causes fermion species doubling in the continuum limit, each with opposite chirality
- No chiral theories, missing the essential features of the chiral theories, e.g. chiral anomaly never exists
- Solution: break chiral symmetry explicitly on lattice
- Difficulty: continuum limit hard to control when the symmetry is only approximate on lattice

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# Ginsparg-Wilson operator

- Ginsparg-Wilson, 1982: "A remnant of chiral symmetry on the lattice",

$$\{\, \mathrm{D}\, ,\ \gamma_5 \,\} = a\mathrm{D}\gamma_5\mathrm{D}$$

  Reminder: $a = 1$

- As

$$\mathrm{D} \sim \mathbf{k}$$

  In the continuum limit: $\mathbf{k} \to 0$, the usual anti-commutative relationship between Dirac operator and $\gamma_5$ recovered

- If we define: $\hat{\gamma}_5 = (1 - \mathrm{D})\gamma_5$, GW implies

$$\hat{\gamma}_5^2 = 1 \quad \text{and} \quad \hat{\gamma}_5\mathrm{D} = -\mathrm{D}\hat{\gamma}_5$$

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# A new kind exact "chiral symmetry" on the lattice

- Suppose the action:

$$S = \sum_{\mathbf{x}} \overline{\psi}_{\mathbf{x}} \mathrm{D}_{\mathbf{xy}} \psi_{\mathbf{y}}$$

invariant under the rotation:

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- Chiral fermions: define chiral projection operator on $\psi$ and $\overline{\psi}$ separately:

$$P_{\pm} = \frac{1 \pm \gamma_5}{2}, \quad \hat{P}_{\pm} = \frac{1 \mp \hat{\gamma}_5}{2}$$

and chiral spinors:

$$\psi_{\pm} = P_{\pm}\psi, \quad \overline{\psi}_{\pm} = \overline{\psi}\hat{P}_{\pm}$$

chiral theory:

$$S = \sum_{\mathbf{x}} \overline{\psi}_{+} \mathrm{D} \psi_{+}$$

# Fascinating theoretical achievement on lattice chiral gauge theory

Ginsparg, Wilson (1982); Callan, Harvey (1985); D.B. Kaplan (1992); Narayanan, Neuberger (1994); Neuberger (1997); P. Hasenfratz, Laliena, Niedermaier (1997); Luescher (1998); Neuberger (1998),

- No fermion doubling problem
- exact lattice chiral symmetry
- exact lattice gauge anomaly and lattice index theorem
- exact Ward identities, axial charge violation, ...

# Remain a hard problem

## Locality is not manifest

- Lüscher proved: $D_{\mathbf{x}\mathbf{x}'} \sim e^{-|\mathbf{x}'-\mathbf{x}|}$ while $|\mathbf{x}'-\mathbf{x}| >$ few, exponentially local.

## Something more serious

- Defining gauge chiral theory appears ambiguous
- Gauge anomaly and topological property of D

## We need study more general chiral theories on lattice

- Only theories well studied before is the free theory: $S = \bar{\psi}_- D \psi_-$
- Our model of "mirror fermions" is a lot more complicated!

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## A general definition of Chiral fermion theories on the lattice:

E. Poppitz and YS, "Lattice chirality and the decoupling of mirror fermions," JHEP **0708**, 081 (2007) [arXiv:0706.1043 [hep-th]]

- Chiral action $S$, a functional of the spinors that satisfies:

$$S[X, Y^\dagger, O] = S[PX, Y^\dagger, O] = S[X, Y^\dagger \hat{P}, O]$$

$X \sim \psi$, $Y^\dagger \sim \overline{\psi}$, and $O$ any other local operators. $P$ and $\hat{P}$ any two projection operators defined above

- Choose particular sets of orthonormal basis $\{u_i, v_j\}$:

$$P\, u_i = u_i\,, \quad v_j^\dagger \hat{P} = v_j^\dagger$$

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## A general definition of Chiral fermion theories on the lattice:

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- Chiral action $S$, a functional of the spiniors that satisfies:

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- Suppose we choose $v_i' = \mathcal{U}_{ij} v_j$, $\mathcal{U}$ unitary matrix, then $Z \to \det\mathcal{U} \cdot Z$
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- Usually not a problem because this phase is just an unphysical constant
- A serious problem in GW-formalism: "chiral projection" $P$ depends on the gauge backgroud U $\Rightarrow$ it seems that the effective action of the gauge field $U$ is completely arbitrary since $\mathcal{V}[U]$ is.
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More accurately: a unique gauge-invariant and smooth chiral partition function exists if and only if the fermion content is anomaly free, i.e.: 2-D: $\sum_i q_{i+}^2 = \sum_j q_{j-}^2$, 4-D: $\sum_i q_{i+}^3 = \sum_j q_{j-}^3$.

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we proved that

$$\delta \log Z = \mathcal{J}_\delta + \left\langle \frac{\delta S}{\delta O} \delta O \right\rangle$$

# Gauge invariance

- If under the gauge variation:

$$\delta_\omega X = i\omega X,\ \delta_\omega Y = i\omega Y,\ \delta_\omega O = i[\omega\,,\, O]$$

the chiral action $S[X, Y^\dagger, O]$ is invariant:

$$0 = \delta_\omega S = \frac{\delta S}{\delta X}\delta_\omega X + \delta_\omega Y^\dagger \frac{\delta S}{\delta Y^\dagger} + \frac{\delta S}{\delta O}\delta_\omega O$$

- then by the "splitting theorem", for any chiral partition function:

$$\delta_\omega \log Z = \mathcal{J}_\omega + \frac{i}{2}\mathrm{Tr}\,\omega\gamma_5$$

- Anomaly free: $\mathrm{Tr}\,\omega\gamma_5 = 0$

# Gauge invariance and smoothness

- $$\delta \log Z = \mathcal{J}_\delta + \left\langle \frac{\delta S}{\delta O} \delta O \right\rangle$$

  $\mathcal{J}_\delta$ captures all the ambiguity, and $\left\langle \frac{\delta S}{\delta O} \delta O \right\rangle$ is well defined. $\mathcal{J}$ has very curious topological properties, if one studies the "curvature" $\mathcal{F}_{\alpha\beta} \equiv \delta_\alpha \mathcal{J}_\beta - \delta_\beta \mathcal{J}_\alpha$
- Lüscher proved that the current $\mathcal{J}$ can be chosen uniquely as a smooth function of the gauge field $U(x)$, if and only if anomaly cancellation condition is satisfied (1999-2000)
- For general chiral action, by applying our "splitting theorem" recursively, we proved that $\log Z$ is gauge invariant and smooth as long as $\mathcal{J}$ is.
- Remarks:
  - although $\mathcal{J} = \sum_i \delta v_i^\dagger \cdot v_i$ is smooth, always some of the $v_i$ is singular
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# Several "big things" we know now

- Lüscher's proof of the smoothness of $\mathcal{J}$ is "constructive" but computationally unuseful, because The proof is inductive on the dimension of the gauge field configuration space ($N^2 + 2$ in 2-d).
- We have a manifest prescription of defining a smooth $\mathcal{J}$ while only homogeneous Wilson lines turned on, which becomes similarly complicated when general gauge field configurations are considered
- By the splitting theorem, splitting of any vector-like theory into chicral sectors: $\log Z = \log Z_{\text{light}} + \log Z_{\text{mirror}}$ is smooth iff each sector is anomaly free.
- In anolamous cases, the obstacle is topological, can always be circumvented locally (in gauge field configuraiton space) by tuning the boundary conditions. Therefore, it's reasonable to expect that local properties, such as the spectrum, will not be serously affected.

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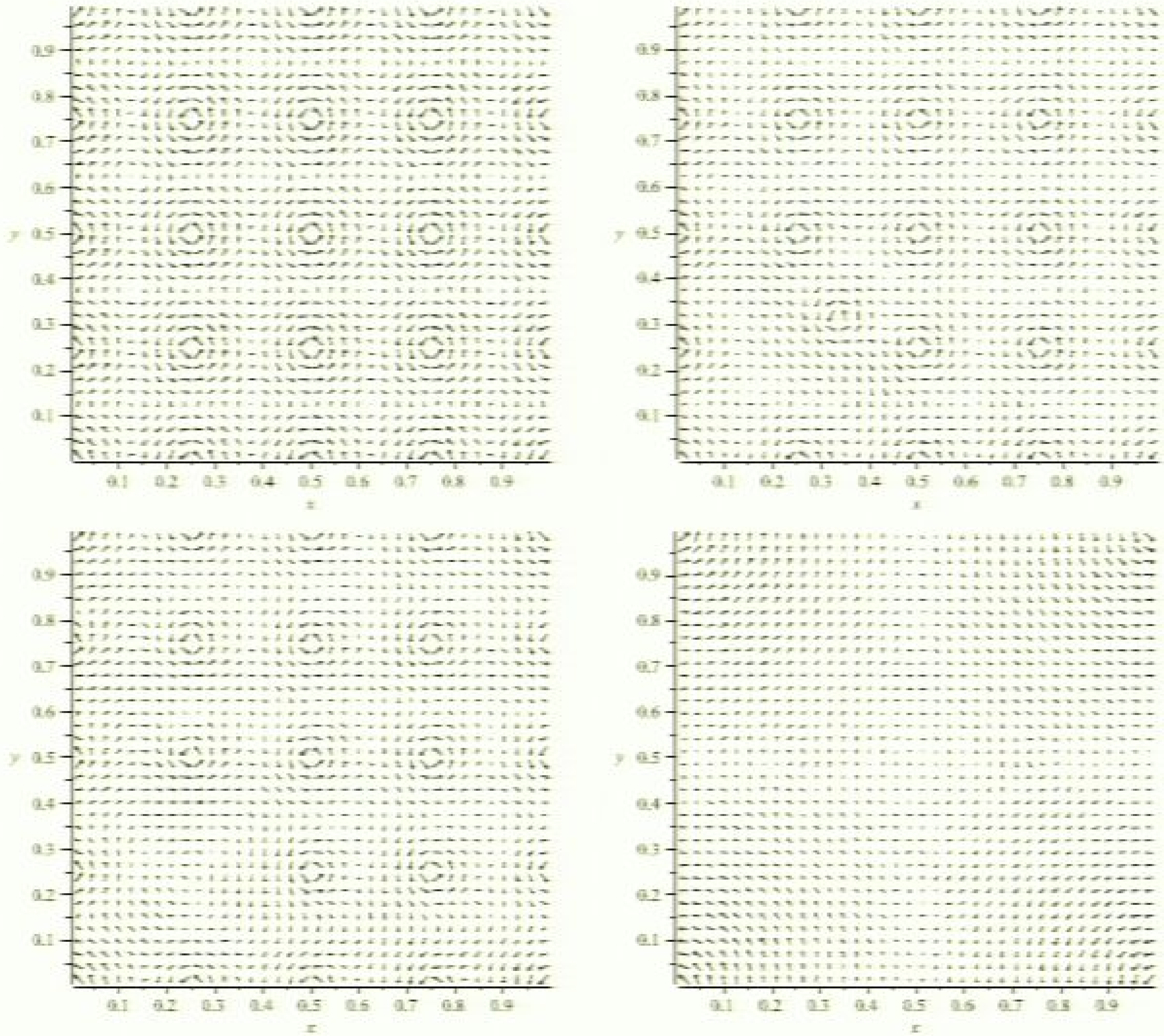

Figure: 1st panel: the 16 singularities of $\mathcal{J}_\mu^4$, 2nd: one singular vortex slightly shifted; 3rd: one vortex moved to $\mathbf{h} = (0, 0)$ so that two singularities coincide; 4th: all 16 vortices shifted to the corner.

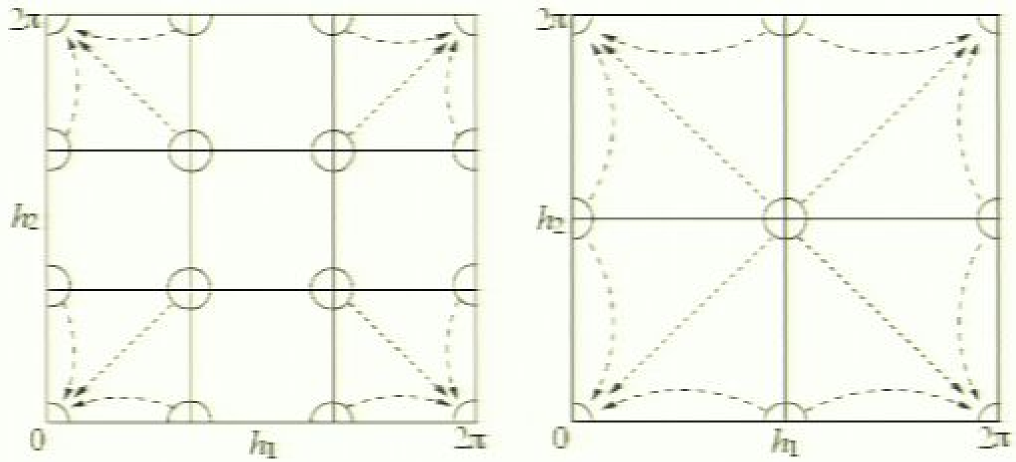


Figure: Moving the singularities of $\mathcal{J}_\mu^3$ and $\mathcal{J}_\mu^2$.

$$\begin{aligned}\sigma(h_1, h_2) =& \frac{1}{4}\Bigg[\tan^{-1}\frac{T(h_2)}{T(h_1-\pi)-T(h_1)} - \tan^{-1}\frac{T(2\pi-h_2)}{T(h_1-\pi)-T(h_1)} \\ & - \tan^{-1}\frac{T(h_2)}{T(\pi-h_1)-T(2\pi-h_1)} + \tan^{-1}\frac{T(2\pi-h_2)}{T(\pi-h_1)-T(2\pi-h_1)}\Bigg] \\ & + \frac{1}{4}\Bigg[-\tan^{-1}\frac{T(h_1)}{T(h_2-\pi)-T(h_2)} + \tan^{-1}\frac{T(h_1)}{T(\pi-h_2)-T(2\pi-h_2)} \\ & + \tan^{-1}\frac{T(2\pi-h_1)}{T(h_2-\pi)-T(h_2)} - \tan^{-1}\frac{T(2\pi-h_1)}{T(\pi-h_2)-T(2\pi-h_2)}\Bigg] \\ & - \frac{1}{2}\tan^{-1}\frac{T(h_2)}{T(h_1)} + \frac{1}{2}\tan^{-1}\frac{T(h_1)}{T(h_2)}, \quad T(x) \equiv \tan\left(\frac{x-\pi}{2}\right)\end{aligned}$$

# Summary

- GW formalism is theoretically elegant but practically difficult
- The idea of decoupling of the mirror fermions in GW formalism appears promising. Some preliminary numerical results are encouraging
- Our "splitting theorem" is a very general and powerful result for any lattice chiral gauge theory, which often leads to surprisingly strong conclusions. ("Smooth splitting" for example.)
- Reasonably hopeful that the spectra found in the toy model won't change qualitatively in anomaly free models
- Open questions
  - Are mirror fermions really all heavy and decoupled?
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