Title: Non-Gaussianity from preheating

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Abstract: If light scalar fields are present at the end of inflation, their nonequilibrium dynamics can produce non-Gaussian density perturbations. Lattice field theory simulations show that this effect can be very strong in the massless preheating model. It is therefore an important factor in assessing the viability of inflationary models. I present a phenomenological model that can be used to calculate the perturbations analytically.

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Non-Gaussianity from Massless Preheating

Arttu Rajantie

A. Chambers and AR, PRL 100(2008)041302; arXiv:0805.4795

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Non-Gaussianity

- Gaussian random field: Everything determined by the two-point function

$$\langle\zeta(k_1)\zeta(k_2)\rangle = P(k_1)(2\pi)^3\delta(k_1+k_2)$$

- Define $f_{\rm NL}$ using the three-point function

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_2)\rangle = -\frac{5}{6}f_{\rm NL}\left[P(k_1)P(k_2)+\text{cyclic}\right] \times(2\pi)^3\delta(k_1+k_2+k_3)$$

  - Generally $f_{\rm NL} = f_{\rm NL}(k_1,k_2,k_3)$
- Single field: $f_{\rm NL}\sim\epsilon\ll 1$ (Maldacena 2003)
- Observations:
  - WMAP $-9 < f_{\rm NL} < 111$ (Dunkley's talk)
  - LSS $-29 < f_{\rm NL} < 69$ (Slosar et al 2008)

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Massless Preheating

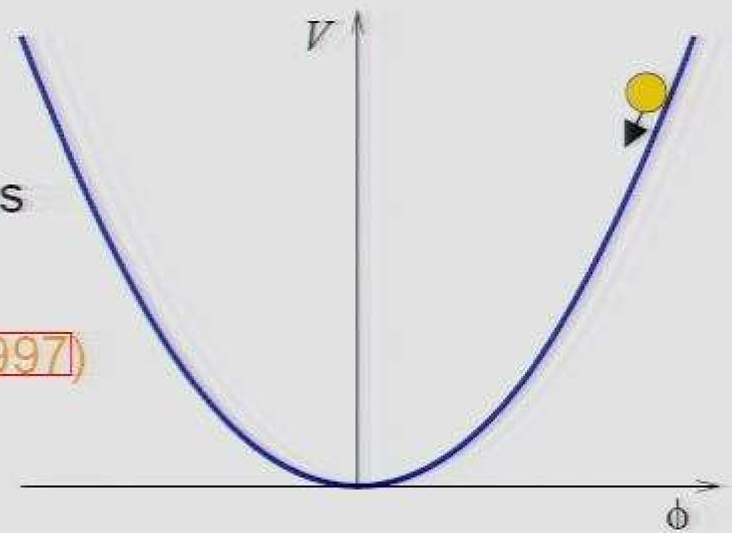
- Chaotic inflation + massless scalar χ (Prokopec&Roos 1997)

$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2, \quad \lambda \sim 7\times 10^{-14}$$

- Inflation: $\phi > 2\sqrt{3}M_{\rm Pl}$
- Mass of χ: $m_\chi = g\phi \ll H$ if $g^2 \lesssim \lambda$

  $\Rightarrow$ Nearly scale-invariant Gaussian perturbations
- Convert $\delta\chi \to \zeta$

  (as in the curvaton model) (Linde&Mukhanov 1997)
- Jokinen&Mazumdar 2006: $f_{\rm NL} \gtrsim O(1000)$



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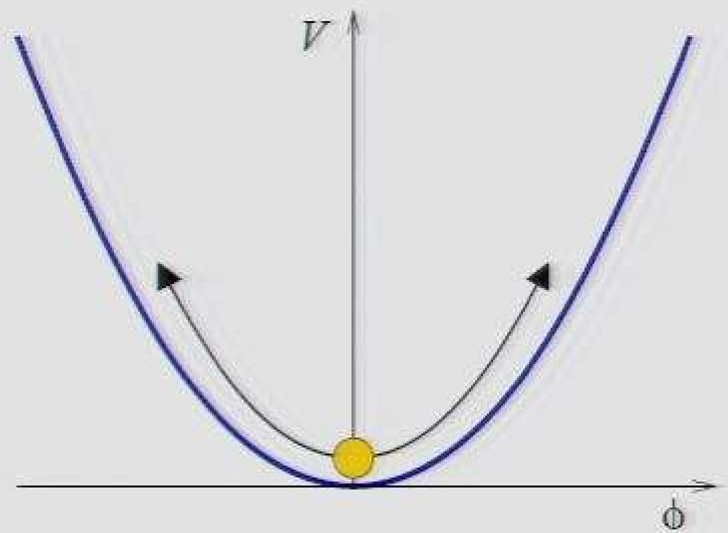
- After inflation: Radiation domination $a \propto t^{1/2}$



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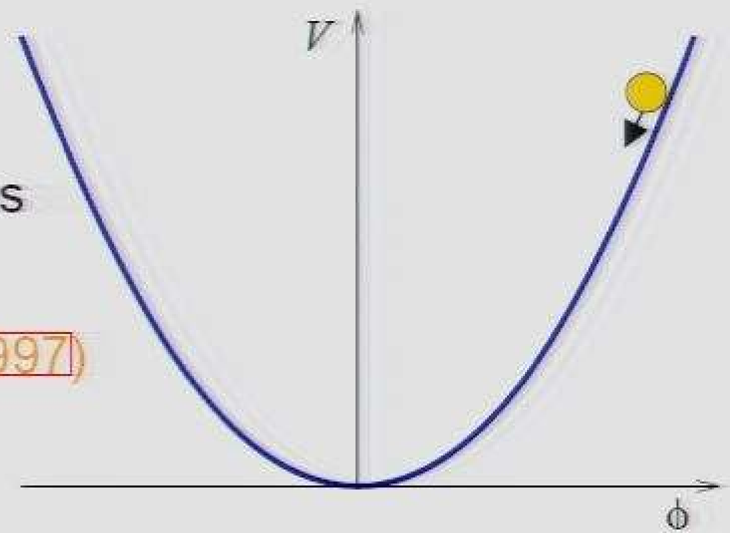

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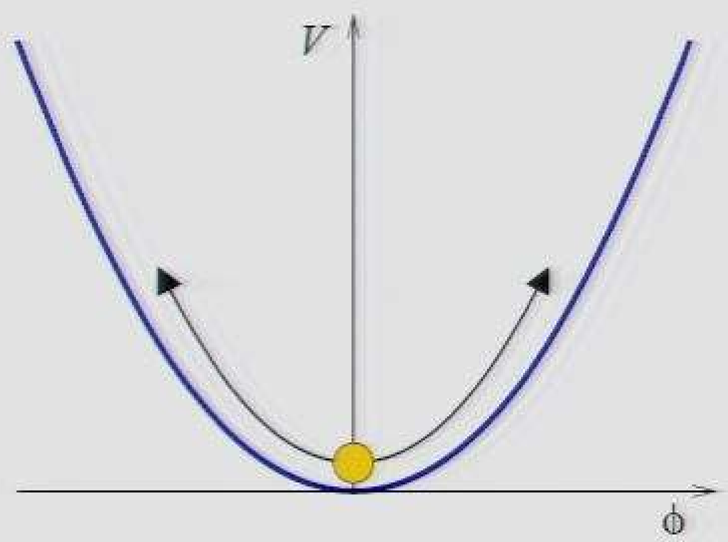

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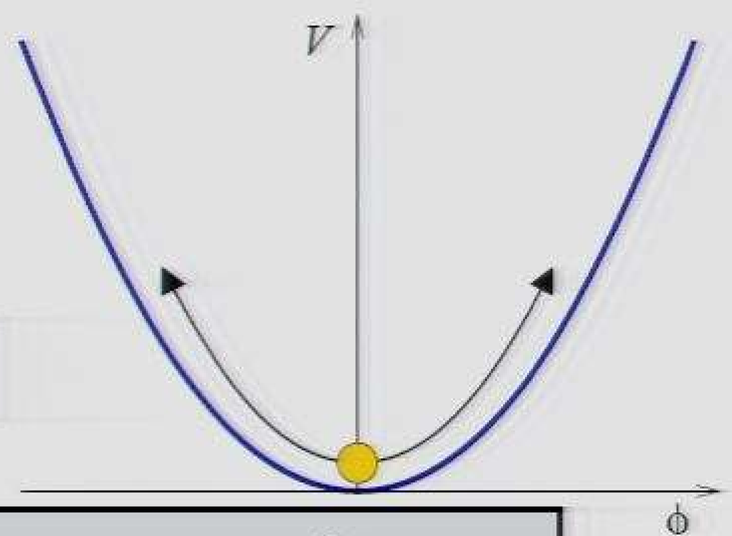
- After inflation: Radiation domination $a \propto t^{1/2}$
- Inflaton zero mode $\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 = 0$
  - Rescale the field $\phi = a^{-1}\tilde{\phi}$
  - Rescale time $d\tau = a^{-1}\lambda^{1/2}\tilde{\phi}_{\rm ini}dt$

$\Rightarrow$ $\tilde{\phi}'' + \lambda\tilde{\phi}^3 = 0$ $\Rightarrow$ $\tilde{\phi}(\tau) = \tilde{\phi}_{\rm ini}\,{\rm cn}(\tau; 1/\sqrt{2})$



- Inhomogeneous χ modes $\chi_k = a^{-1}\tilde{\chi}_k$

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}{\rm cn}^2(\tau;1/\sqrt{2})\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda\tilde{\phi}^2_{\rm ini}}$$

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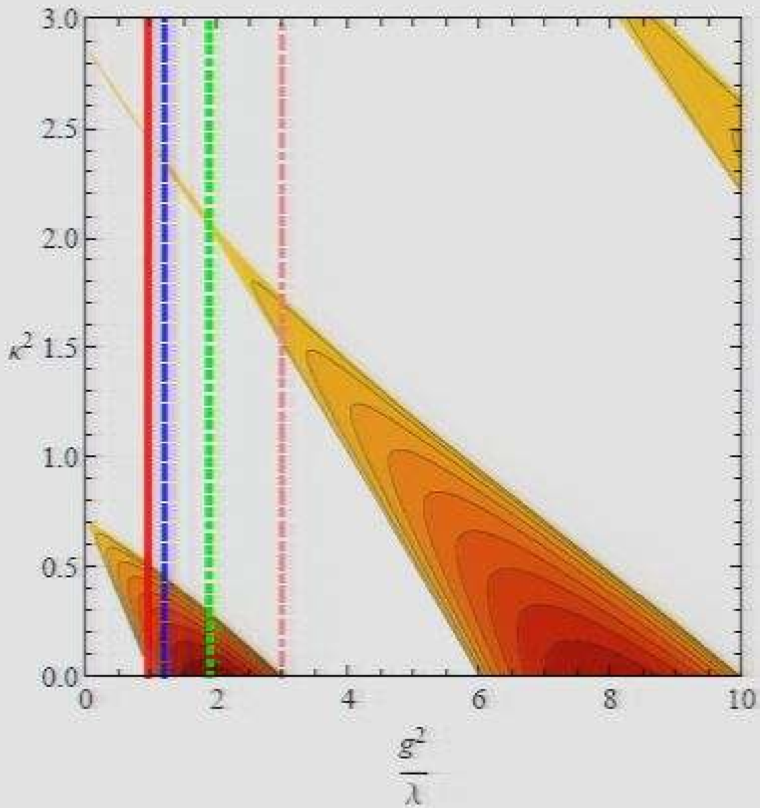
Resonance Bands

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(\tau; 1/\sqrt{2})\right]\tilde{\chi}_k = 0$$



- $\tilde{\chi}_k(\tau) = e^{\mu\tau} f(\tau)$ with periodic $f(\tau)$

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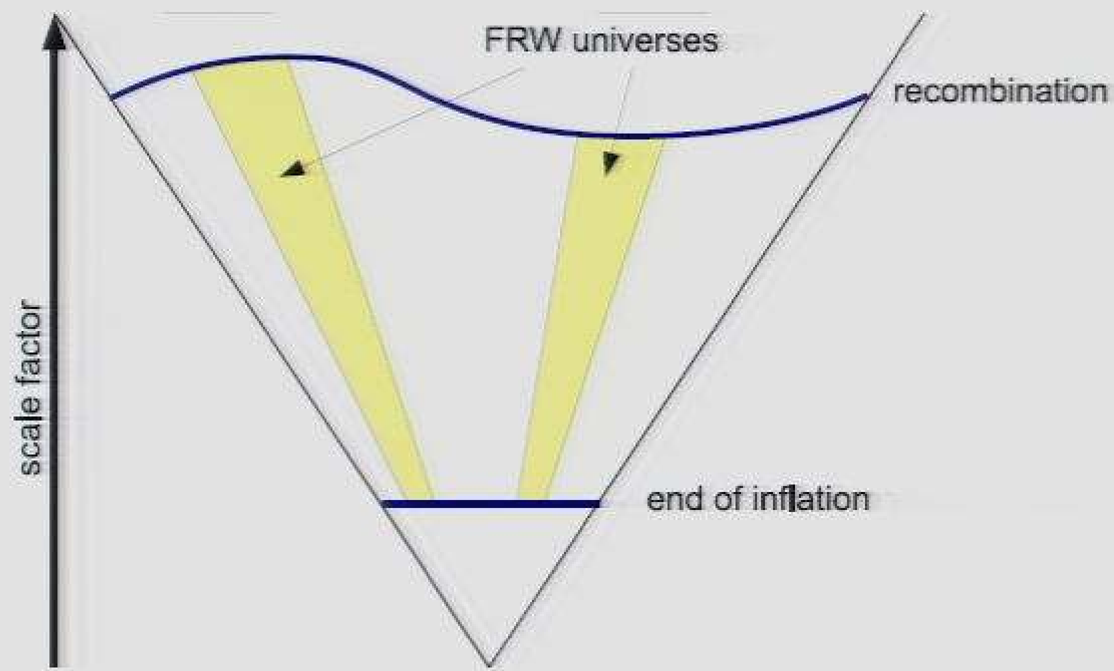
Separate Universes



- Each Hubble patch described by a separate FRW universe (Salopek&Bond 1990)
- Curvature perturbation $\zeta = \delta \ln a|_{H=H_{\rm rec}}$

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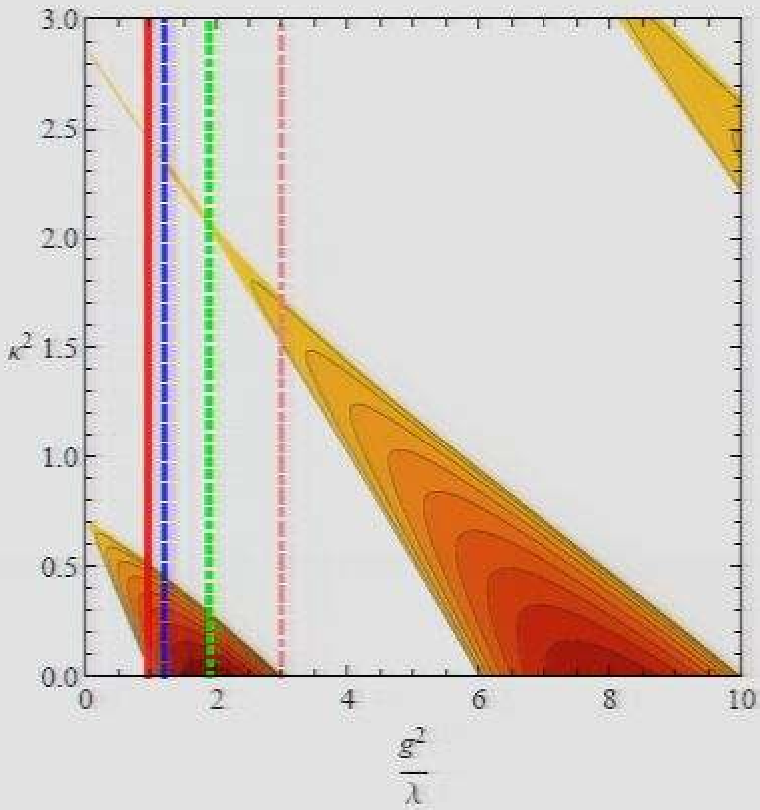

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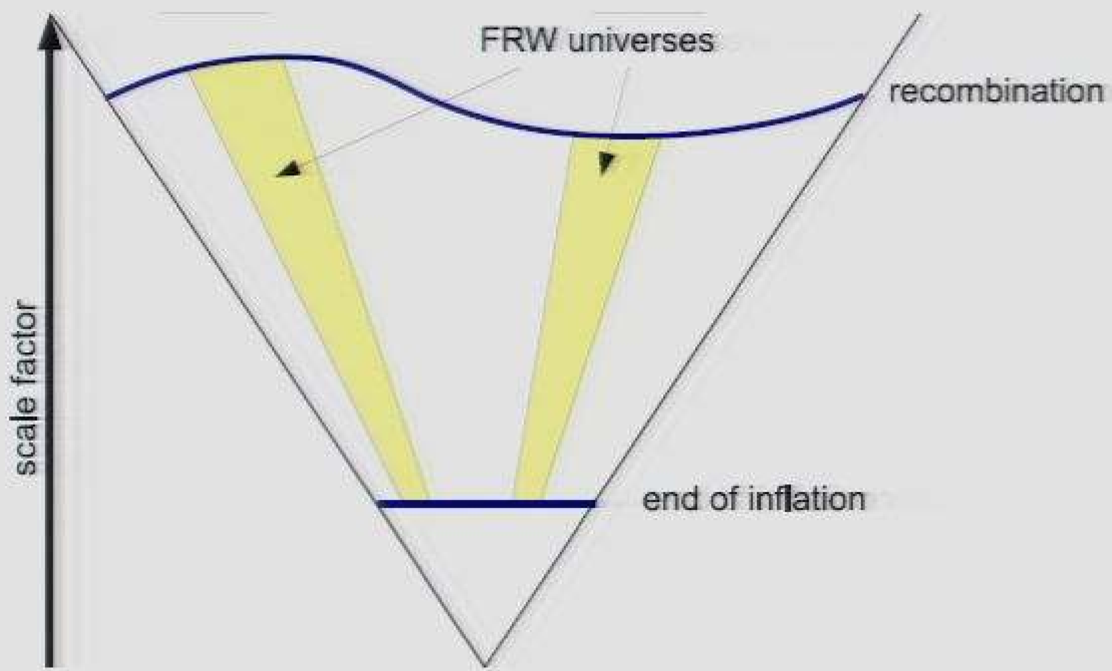

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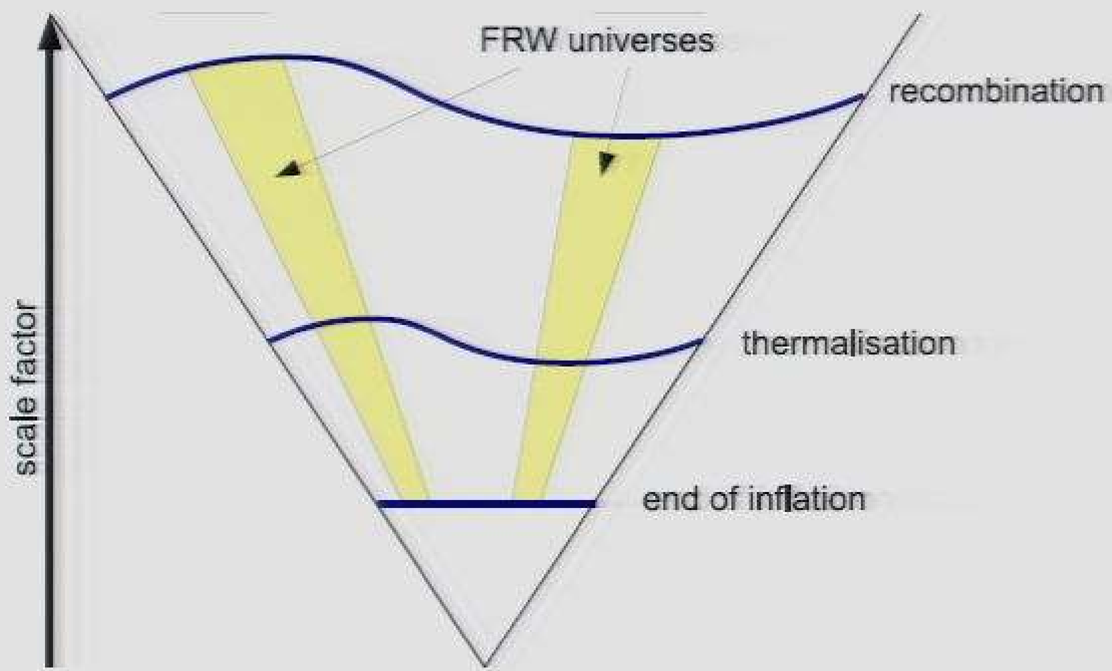


- After thermalisation: Unique equation of state $p = p(\rho)$

$\Rightarrow$ Curvature perturbation conserved $\zeta_{\rm rec} = \delta \ln a|_{H=H_{\rm rec}} = \zeta_{\rm th} = \delta \ln a|_{H=H_{\rm th}}$

Separate Universes

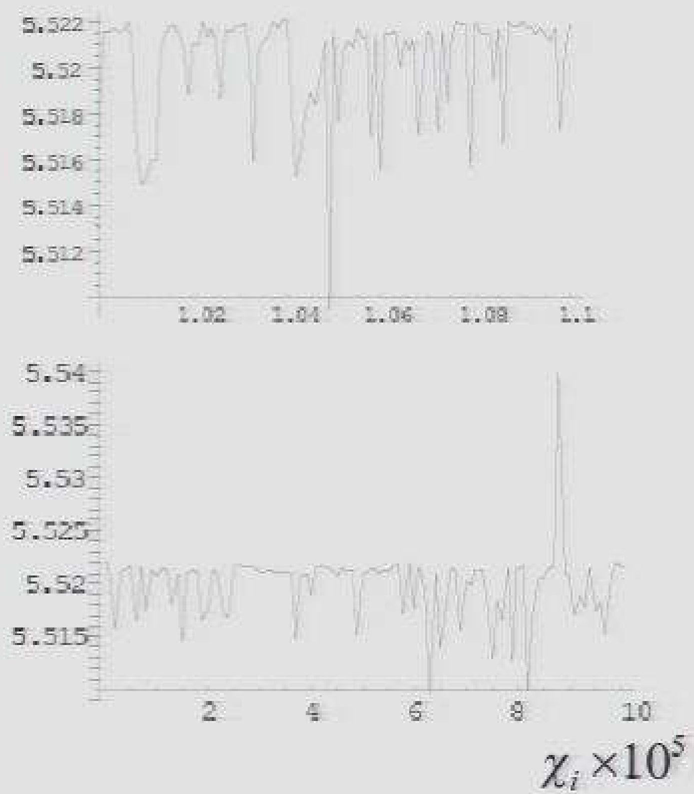
- Regions separated by $d \gg 1/H$ treated as independent FRW universes, but with different initial data (Salopek&Bond 1990)
- Friedmann eq. $\Rightarrow a(t), H(t)$
  - Calculate curvature perturbation $\zeta = \delta N = \delta \ln a|_{H=H_*}$
  - Non-linear, includes gravity
- Dependence on initial conditions $(\phi_{\rm ini}, \chi_{\rm ini})$:

$$\boxed{\zeta = \zeta_\phi(\phi_{\rm ini}) + \frac{1}{2}\frac{\partial^2 N}{\partial \chi_{\rm ini}^2}\chi_{\rm ini}^2 + \ldots}$$

  - Vary $\phi_{\rm ini} \Rightarrow$ Usual inflationary perturbations
  - Vary $\chi_{\rm ini} \Rightarrow$ New contribution

$\Rightarrow$ Need $\partial^2 N/\partial \chi_{\rm ini}^2$

Earlier Results



(Bassett and Tanaka 2003)

- Homogeneous fields $\Rightarrow$ No sub-horizon dynamics

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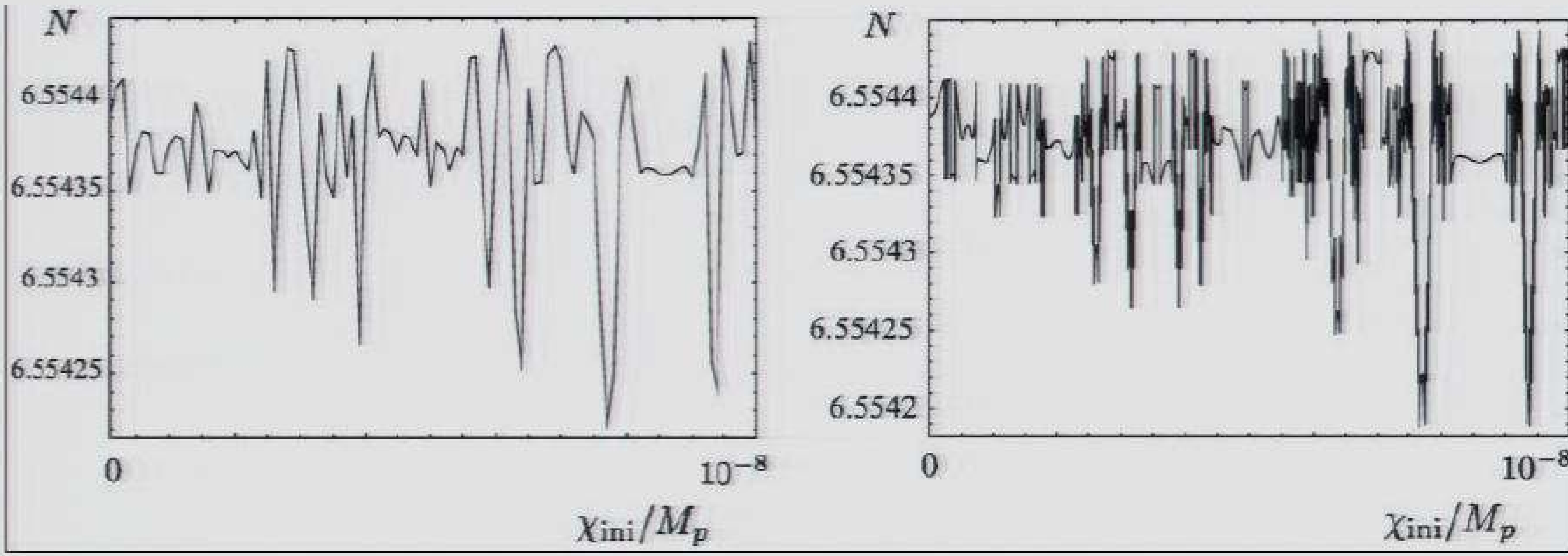
Earlier Results



- Homogeneous fields (Suyama and Yokoyama 2006)
- Could not measure $\partial^2 N/\partial\chi_{\rm ini}^2$
- Chaotic (Podolsky and Starobinsky 2002)

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Inhomogeneous Fields

- Make ϕ and χ position-dependent (Khlebnikov&Tkachev 1996)

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0\end{aligned}$$

- Couple to Friedmann eq with avg energy density (Suyama et al 2005)

$$\begin{aligned}H^2 &= \frac{1}{3M_{\rm Pl}^2 V}\int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2 \right. \\ &\left. + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right)\end{aligned}$$

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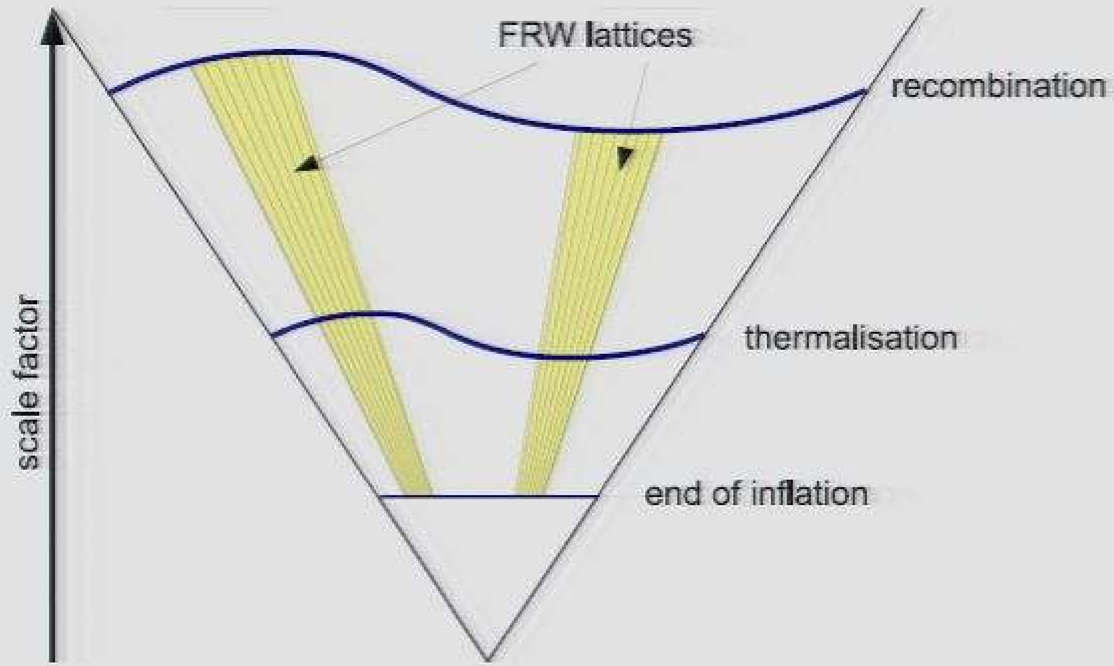
Inhomogeneous Fields



- Calculate dependence on the initial super-horizon modes

  $\approx$ Averages over the lattice $\chi_{\text{ini}} \equiv \overline{\chi}(t=0)$

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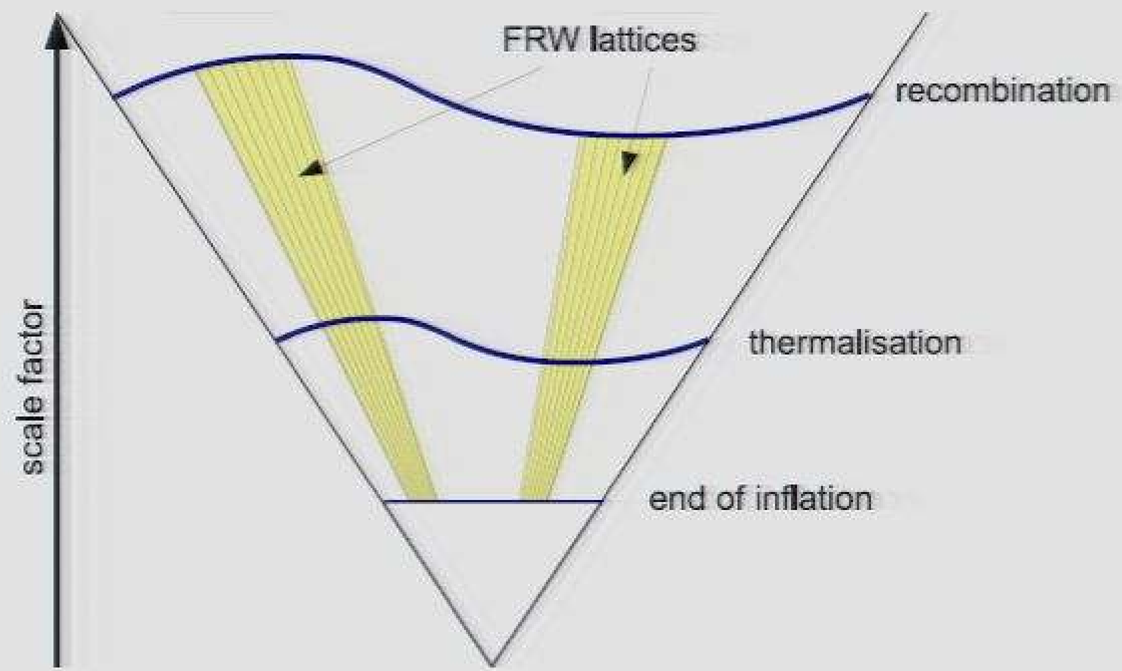
Inhomogeneous Fields



- Dynamical scales must be smaller than the lattice $m^{-1} \ll L$
- Lattice must be smaller than the horizon $L \ll H^{-1}$
- Wavelength of perturbations must be longer than the horizon $k \ll H$

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Parameters

- $\lambda = 7 \times 10^{-14}$, vary $0 < g^2/\lambda < 10$
- $\phi_{\rm ini} = 5M_{\rm Pl}$, vary $\chi_{\rm ini}$
- Lattice size 32^3, $\delta x = 1.25 \times 10^5 M_{\rm Pl}^{-1}$
- Statistics: up to 250 runs for each $\chi_{\rm ini}$

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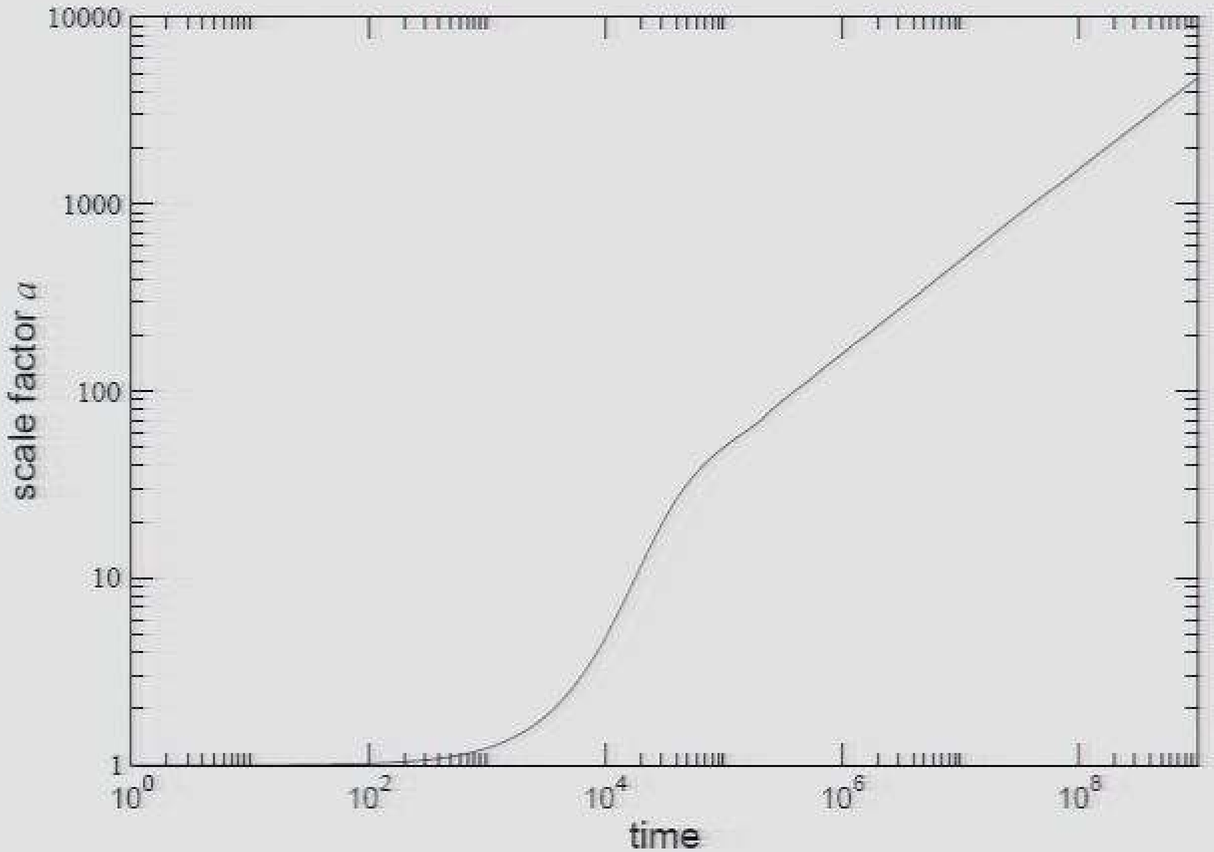
Inflation



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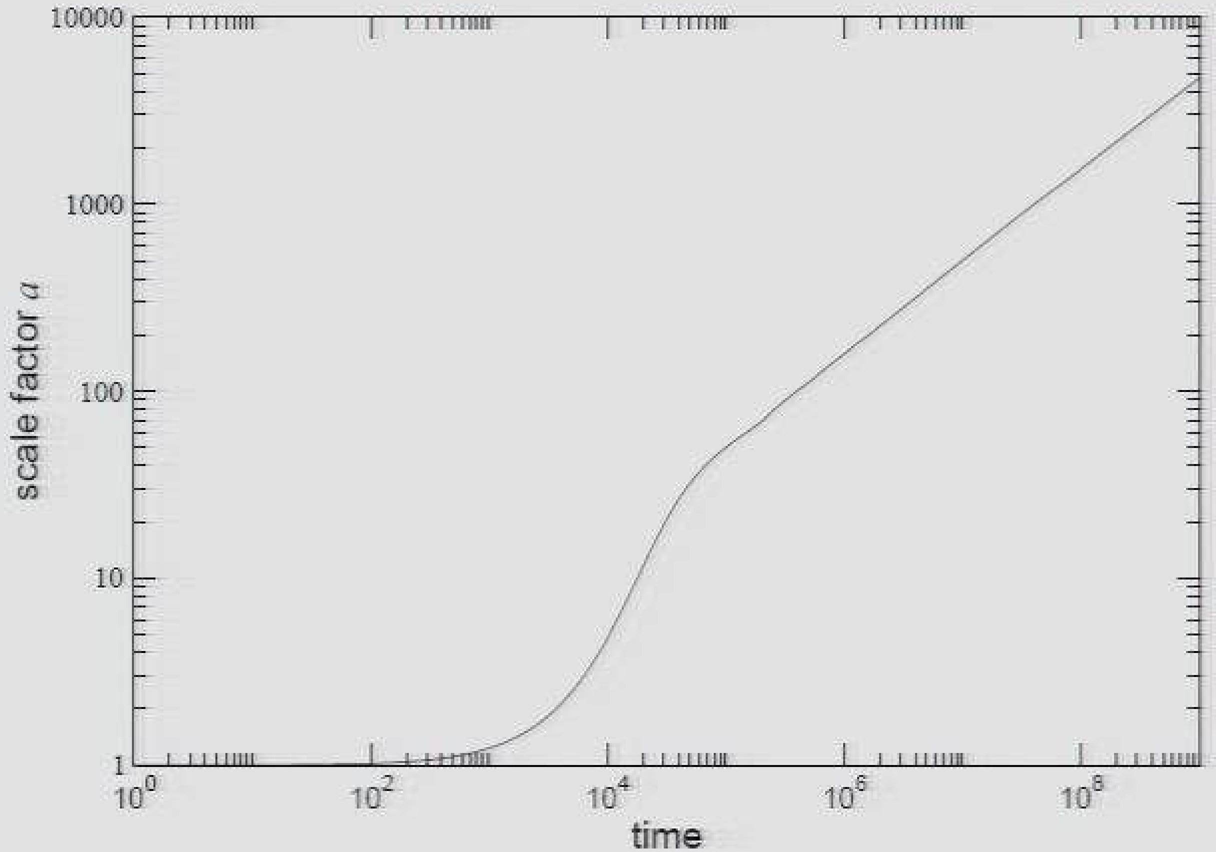

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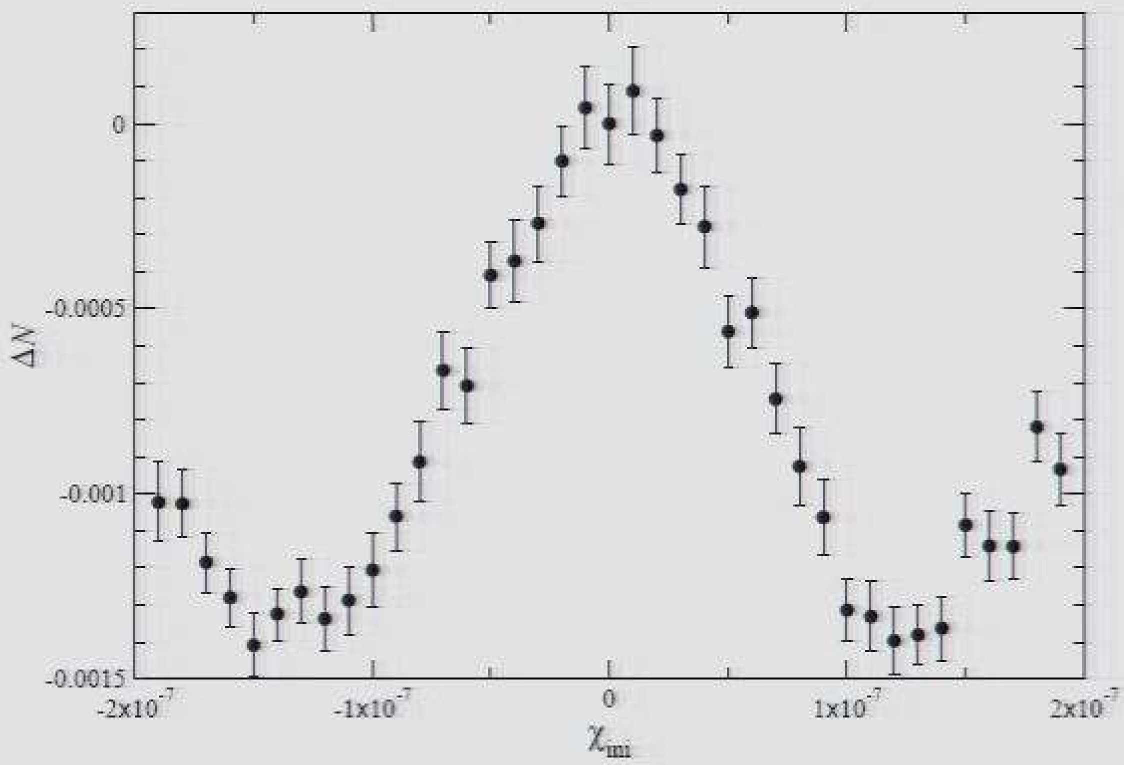
Dependence on $\chi_{\rm ini}$



- $g^2/\lambda = 1.875$, measured at $H = 5.53 \times 10^{-12} M_{\rm Pl}$

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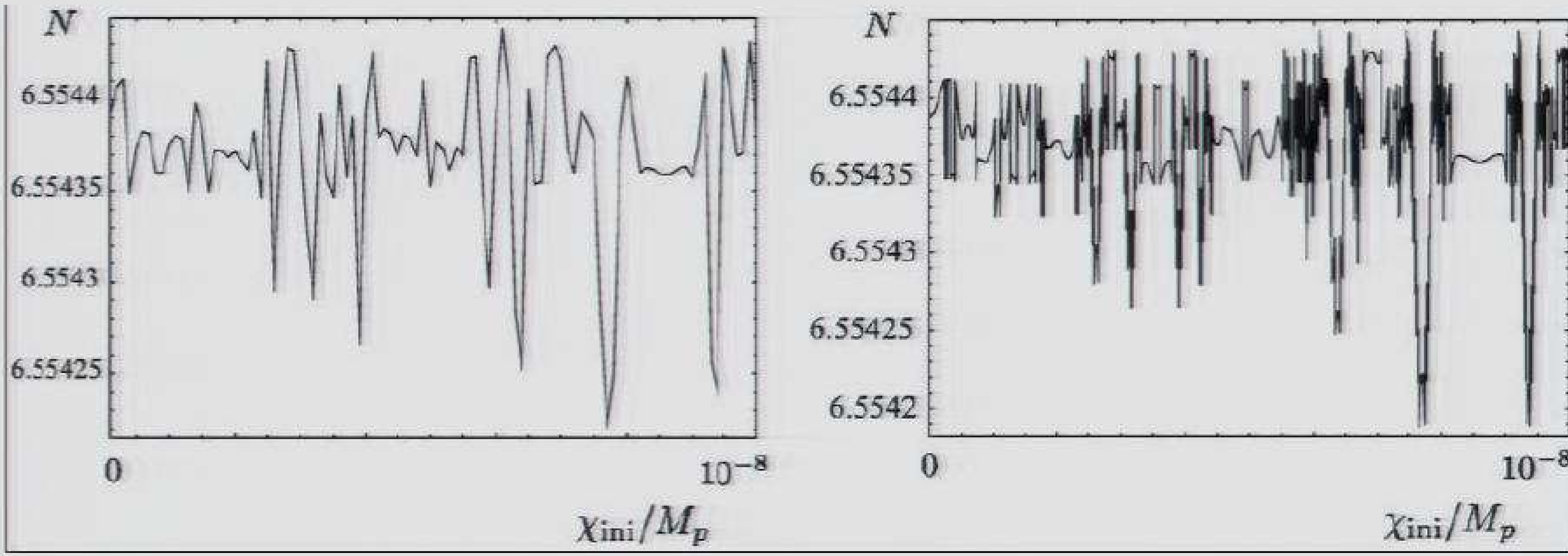
Earlier Results



(Suyama and Yokoyama 2006)

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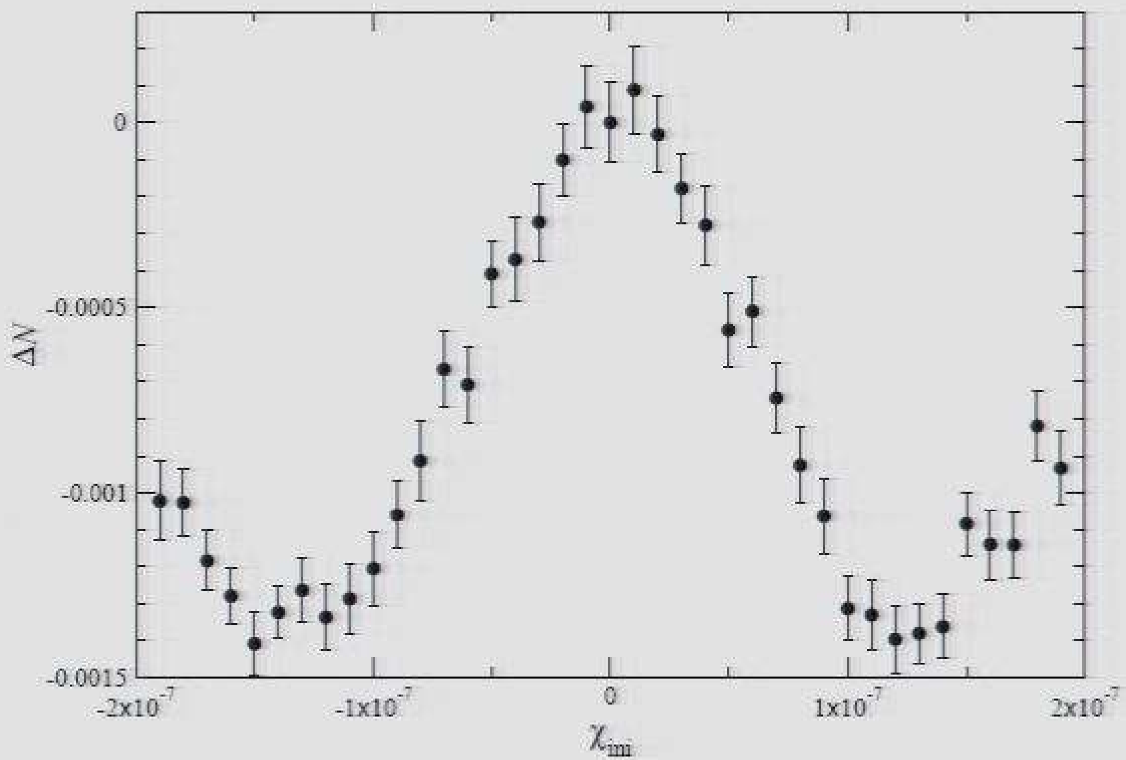
Fitting $N(\chi_{\rm ini})$



- Regular at small $\chi_{\rm ini}$ – Chaotic at larger values
- Analyticity $\Rightarrow N(\chi_{\rm ini}) = N(0) + c\chi_{\rm ini}^2 + O(\chi_{\rm ini}^4)$

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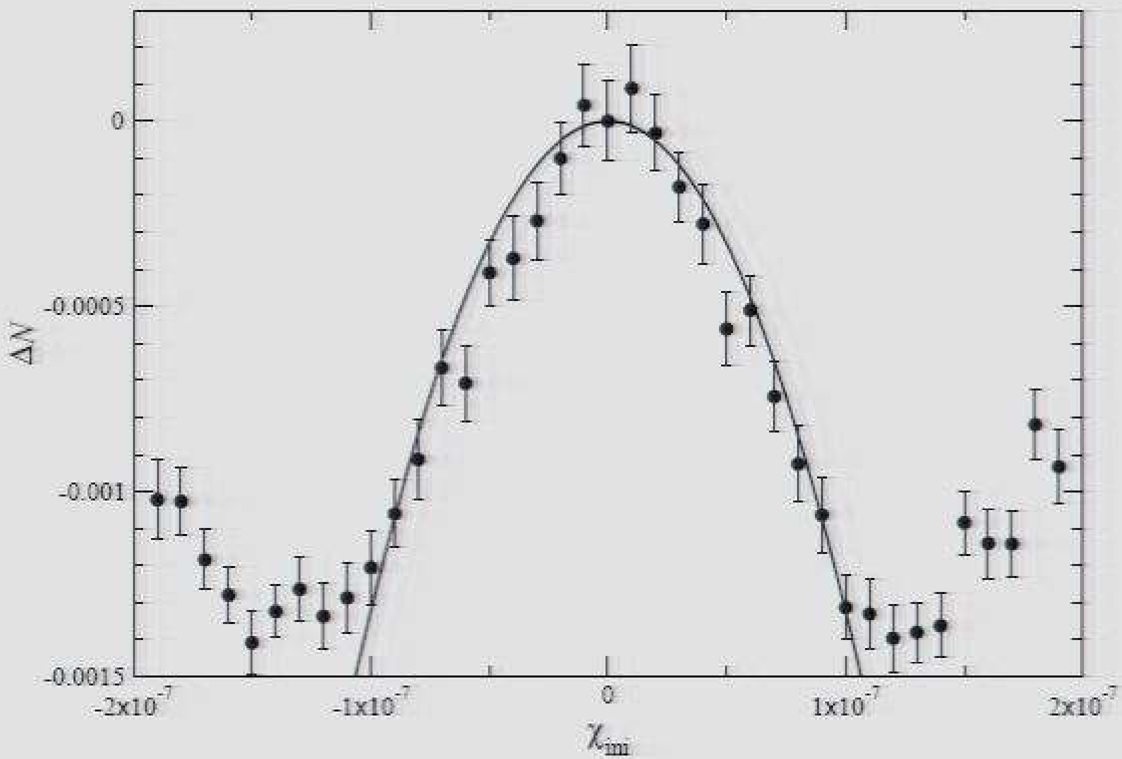

- Fit $N(\chi_{\rm ini}) = N(0) + c\chi_{\rm ini}^2 + O(\chi_{\rm ini}^4)$

  $\Rightarrow$ $\boxed{c = -10^{10.96\pm0.05} M_{\rm Pl}^{-2}}$ for $g^2/\lambda = 1.875$

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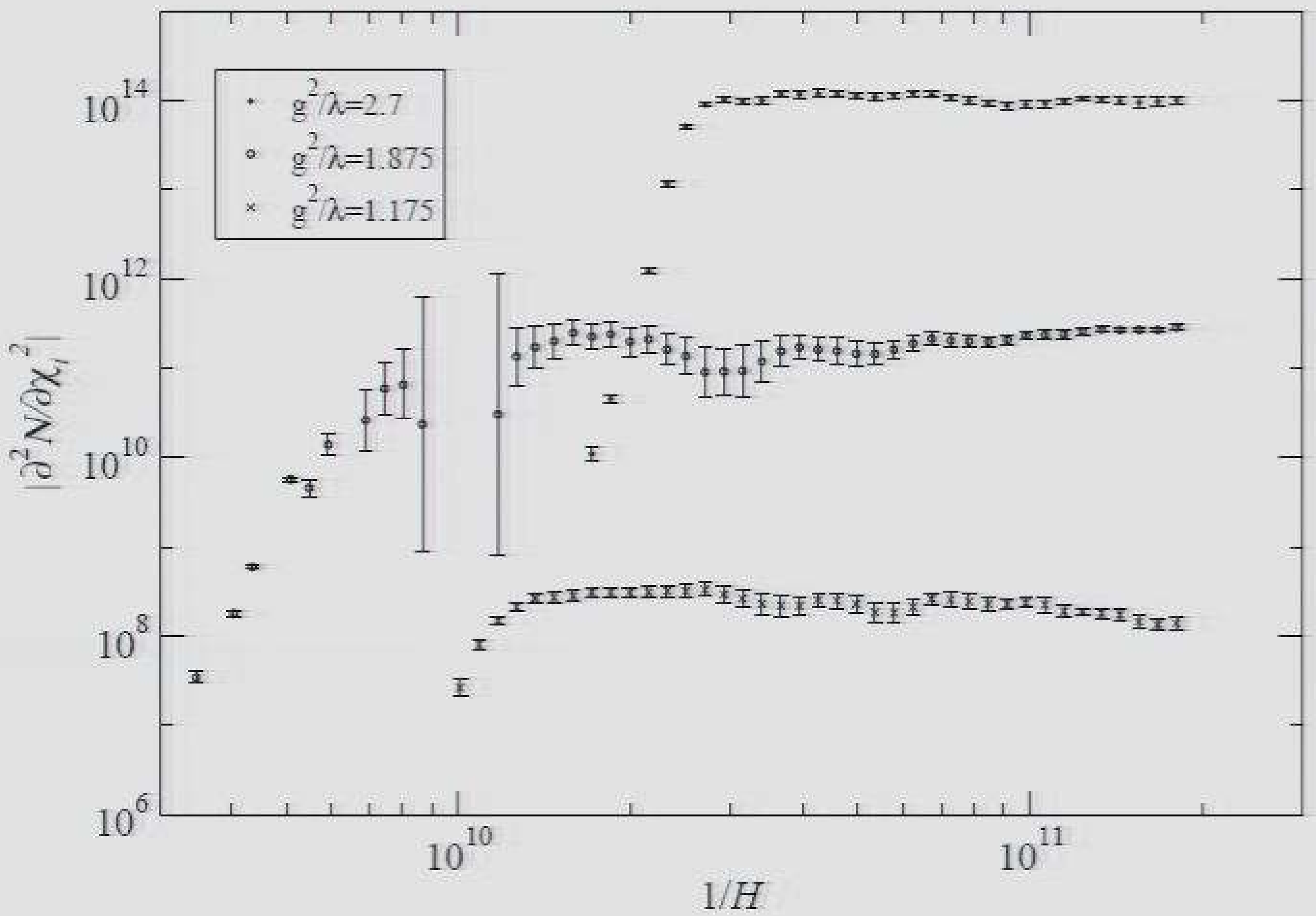
Dependence on H



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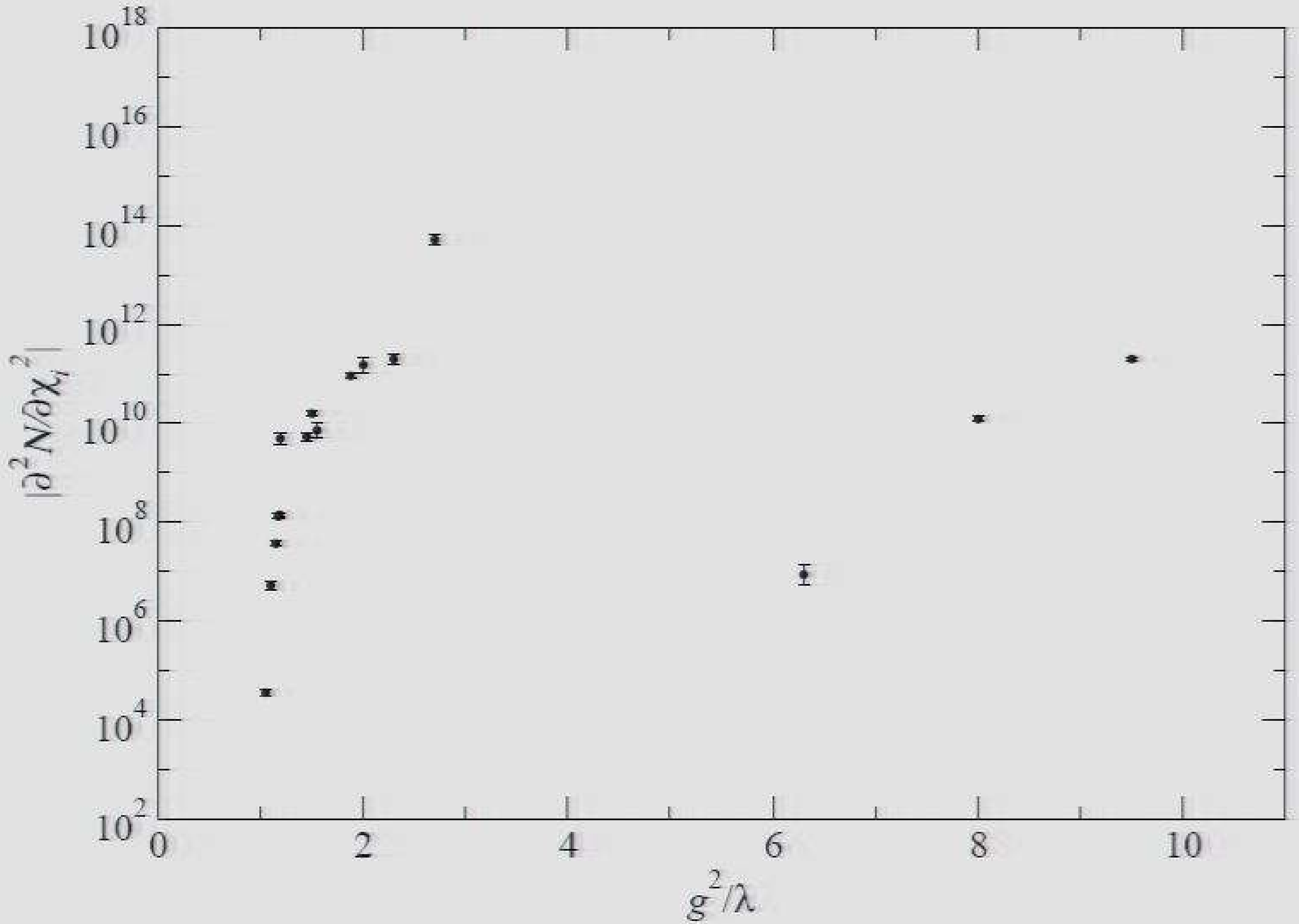
Dependence on g^2/λ



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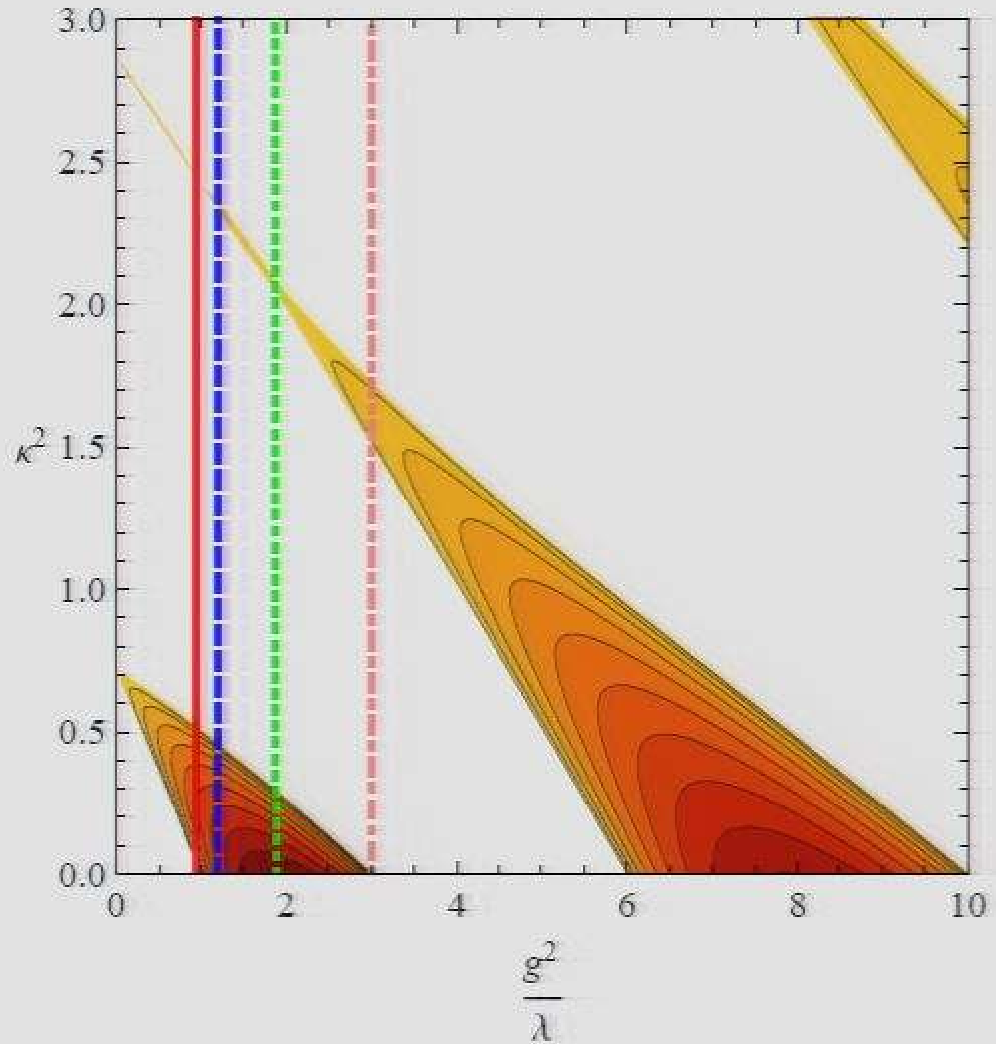

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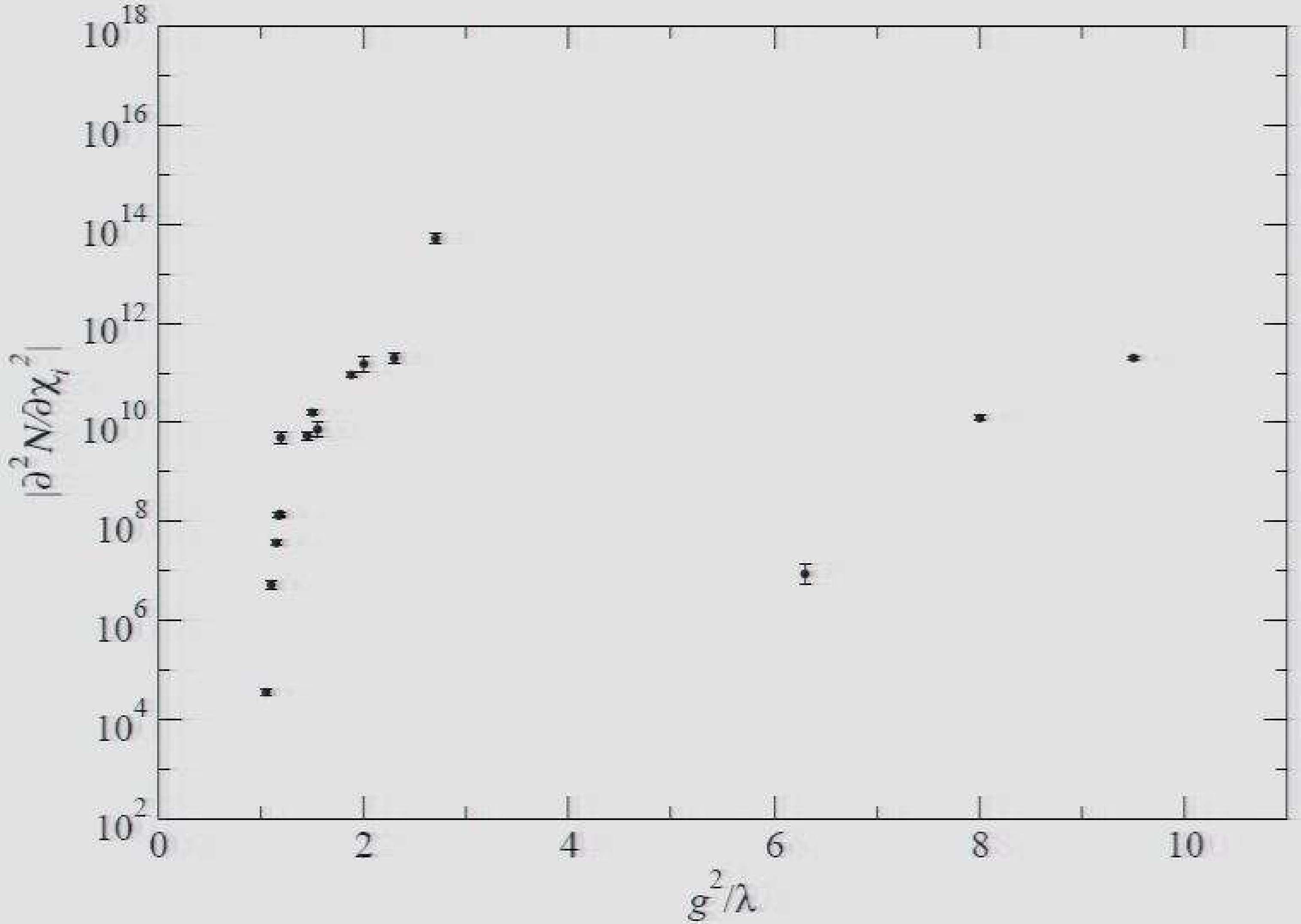

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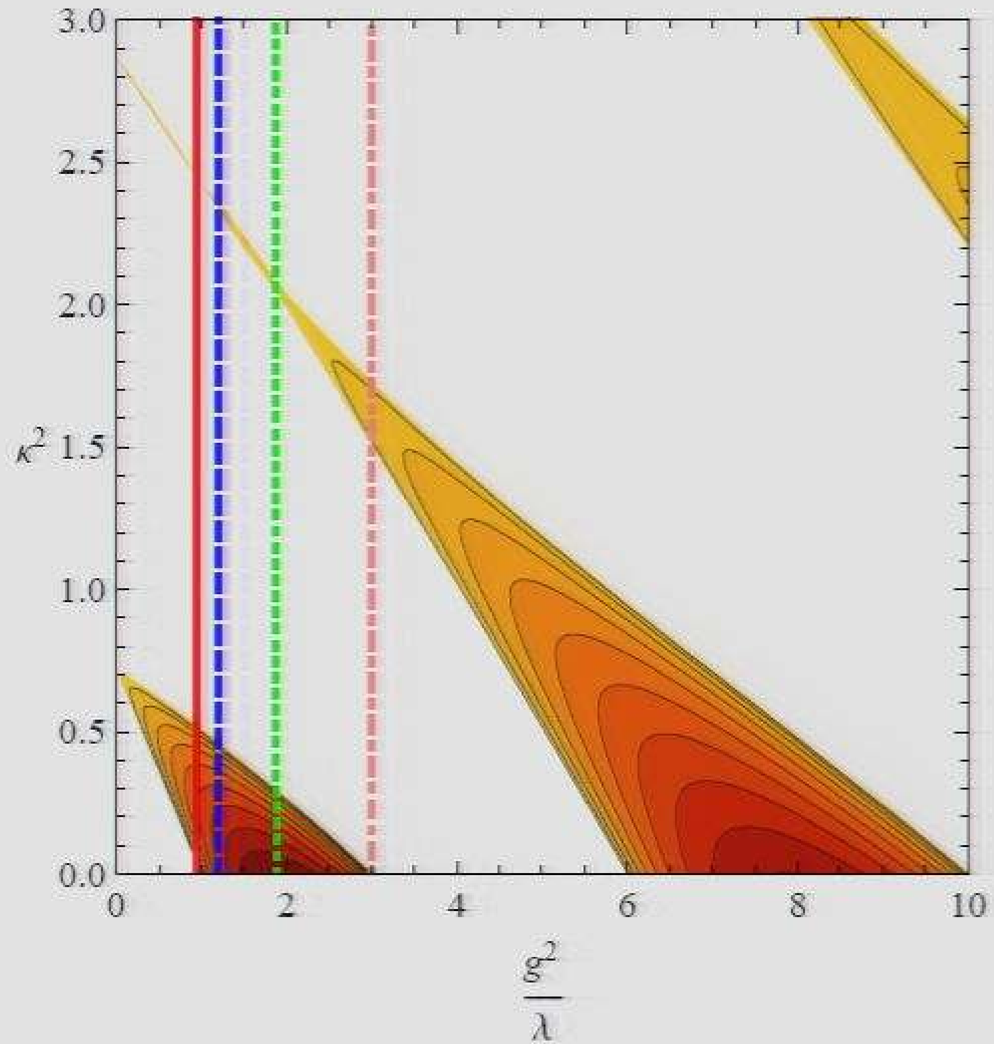

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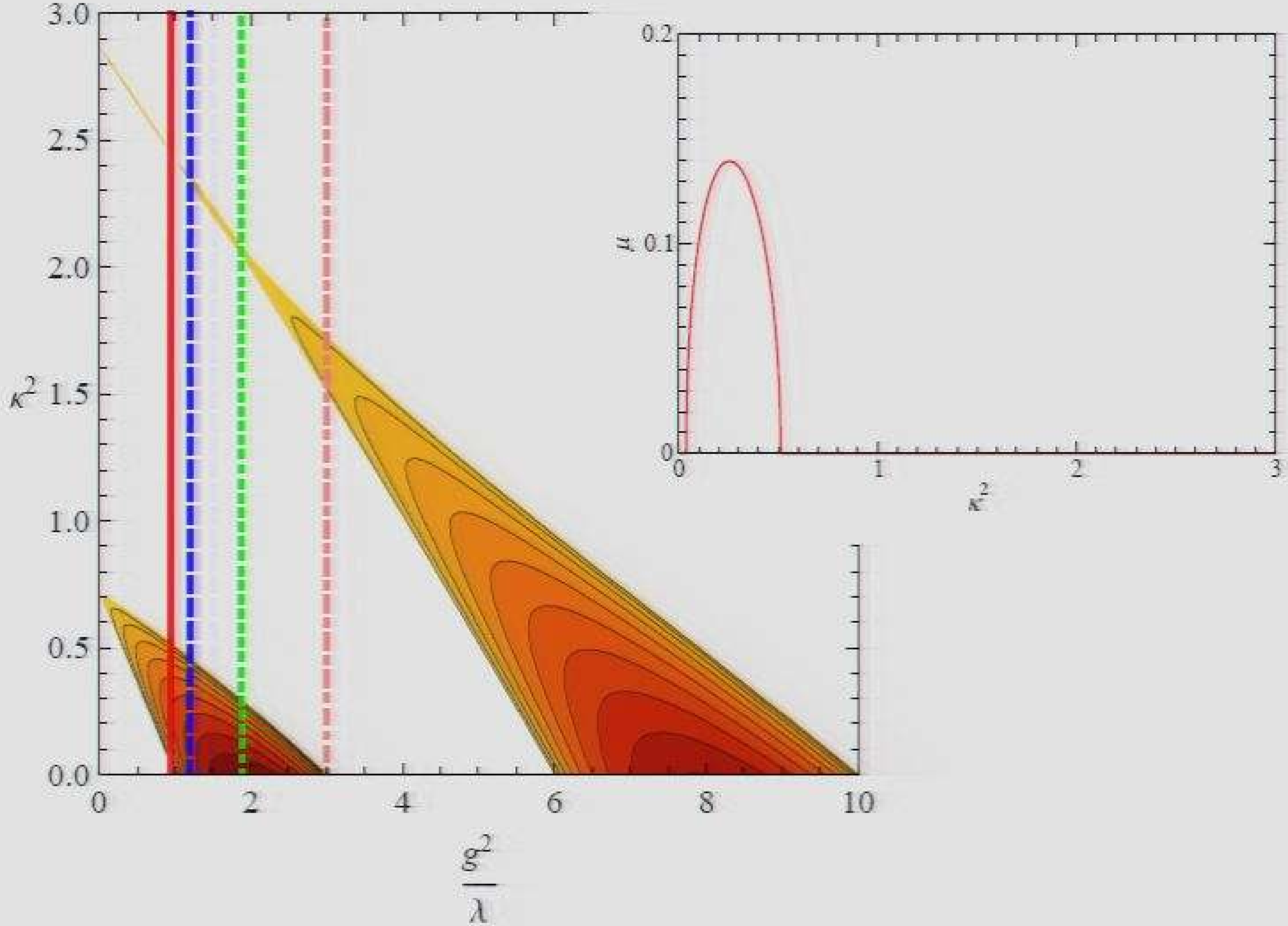

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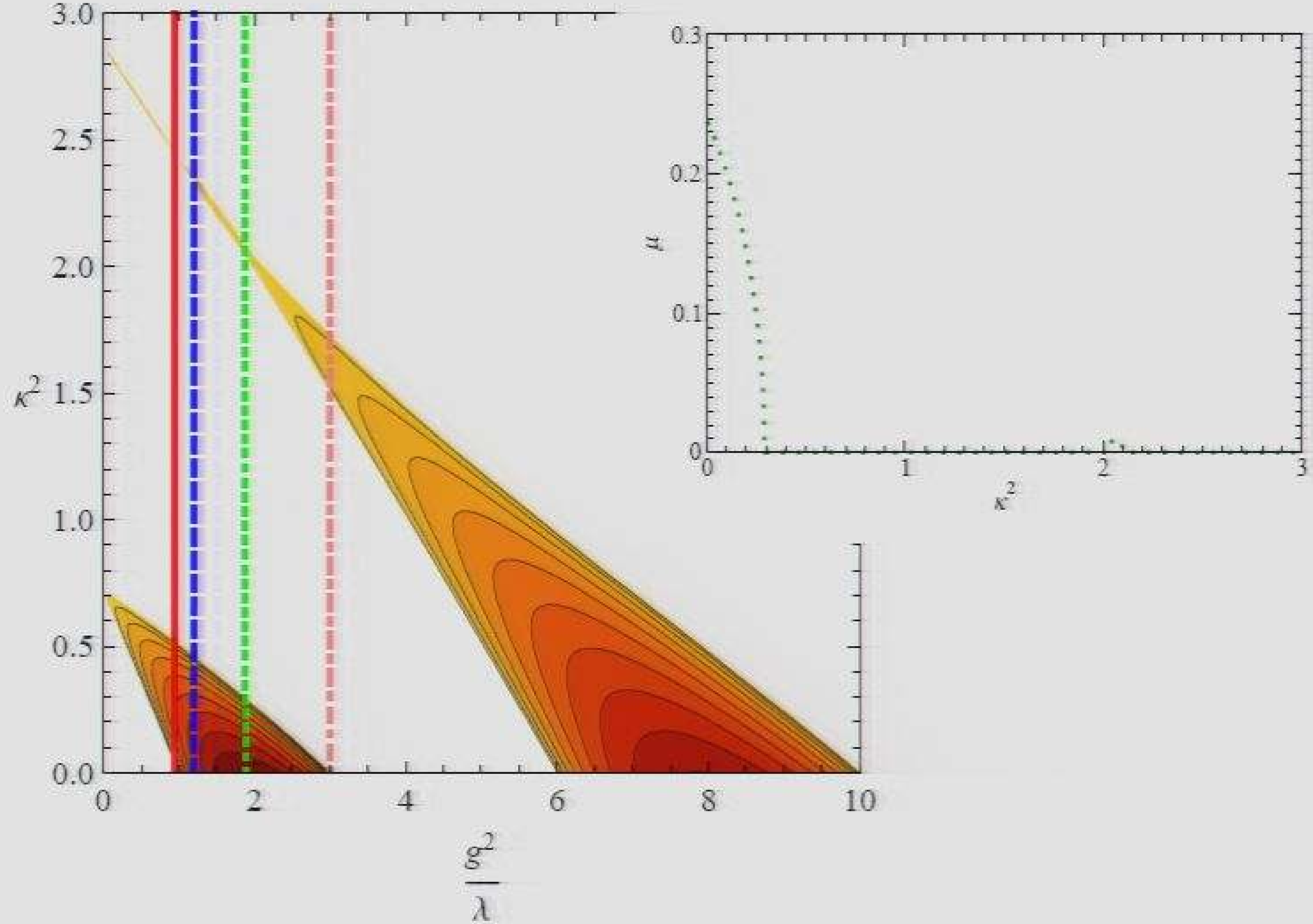

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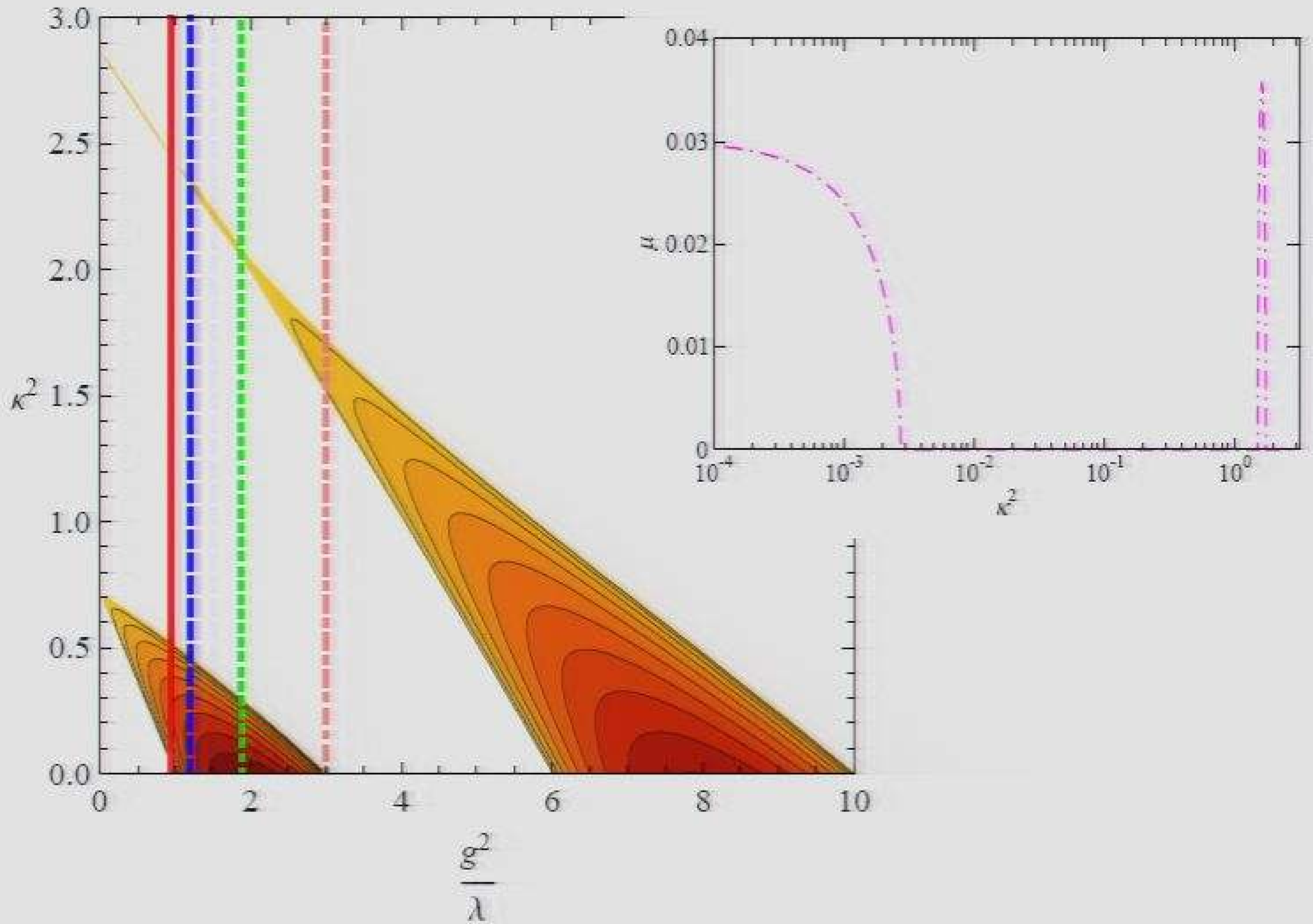

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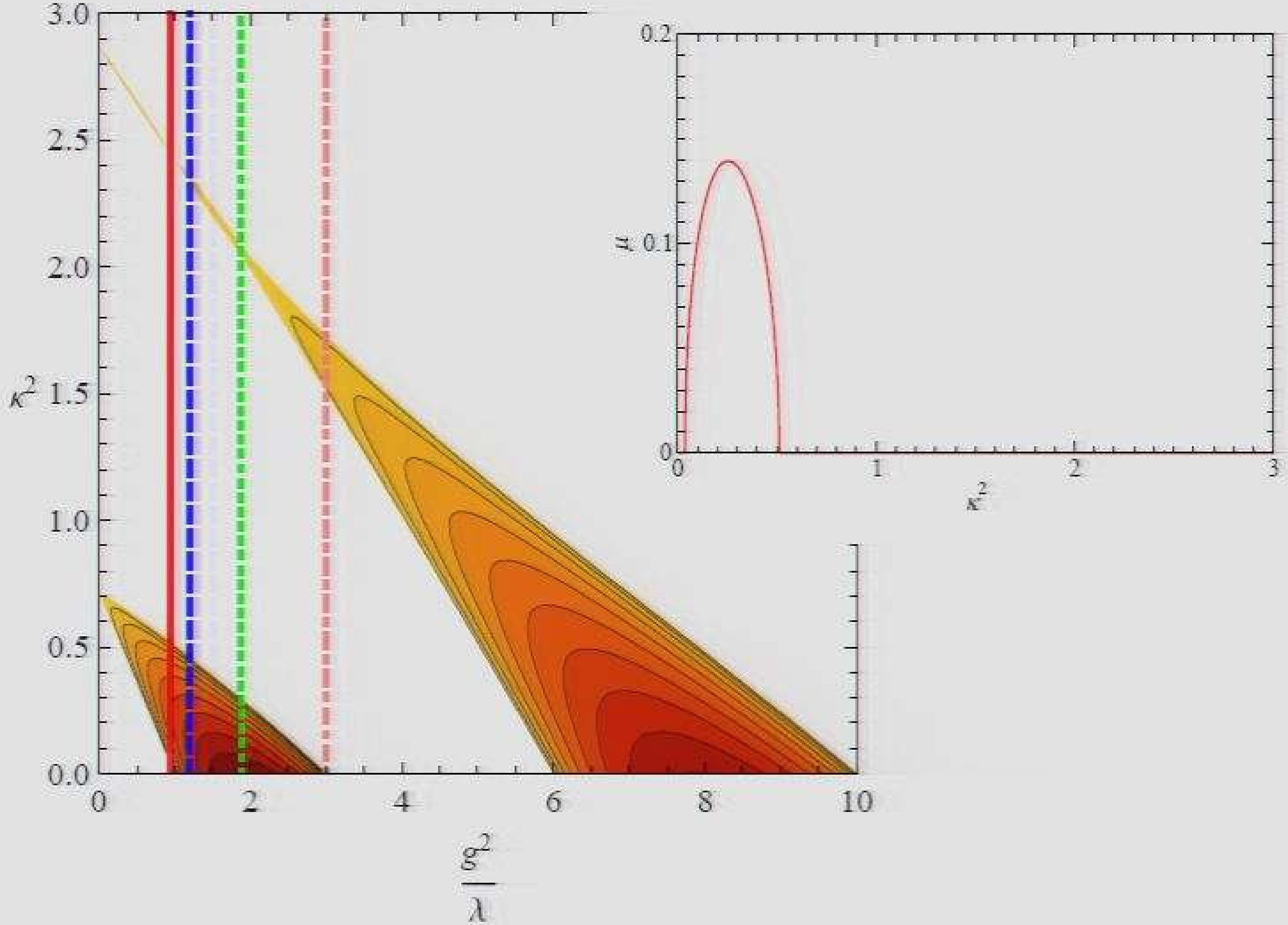

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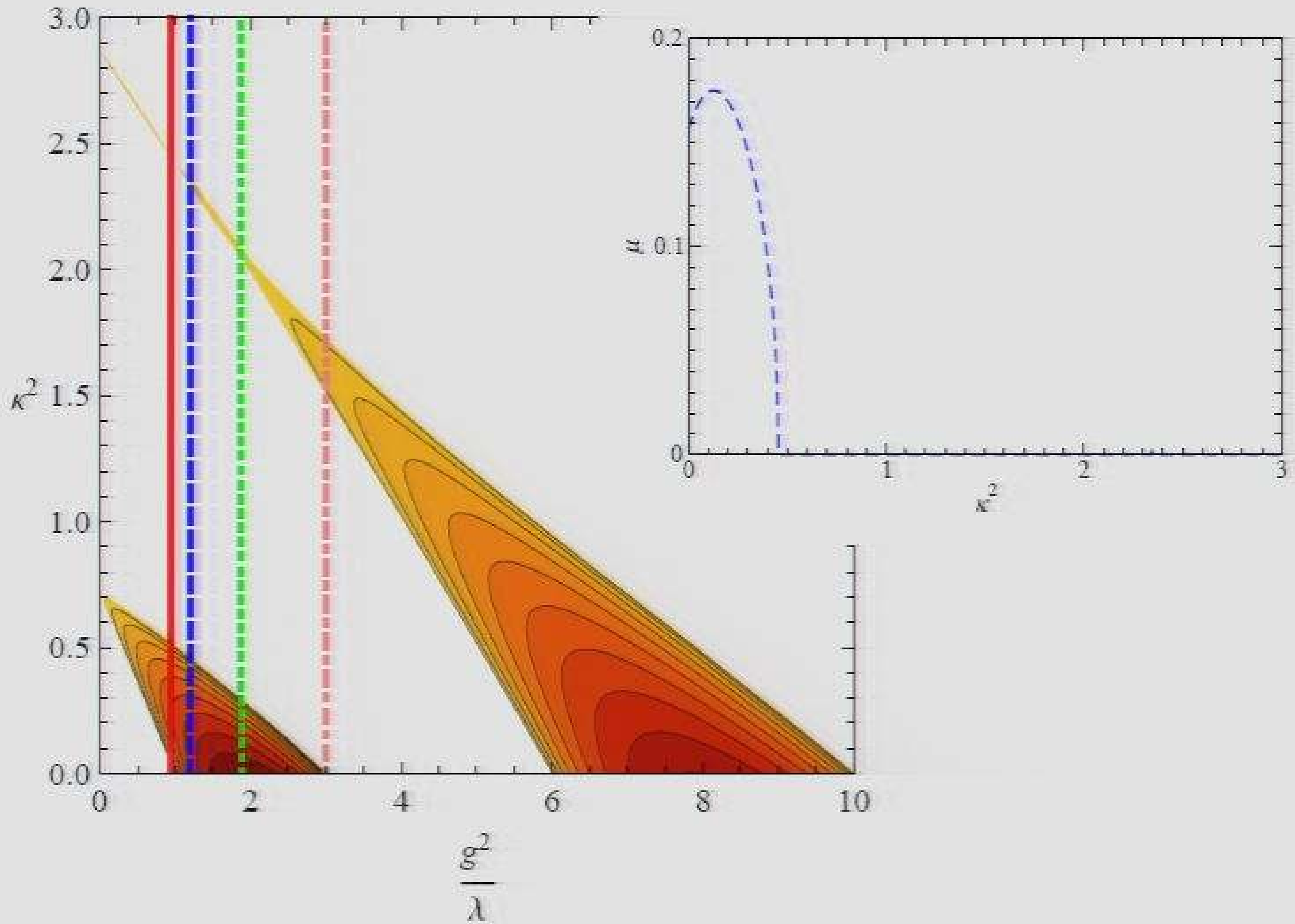

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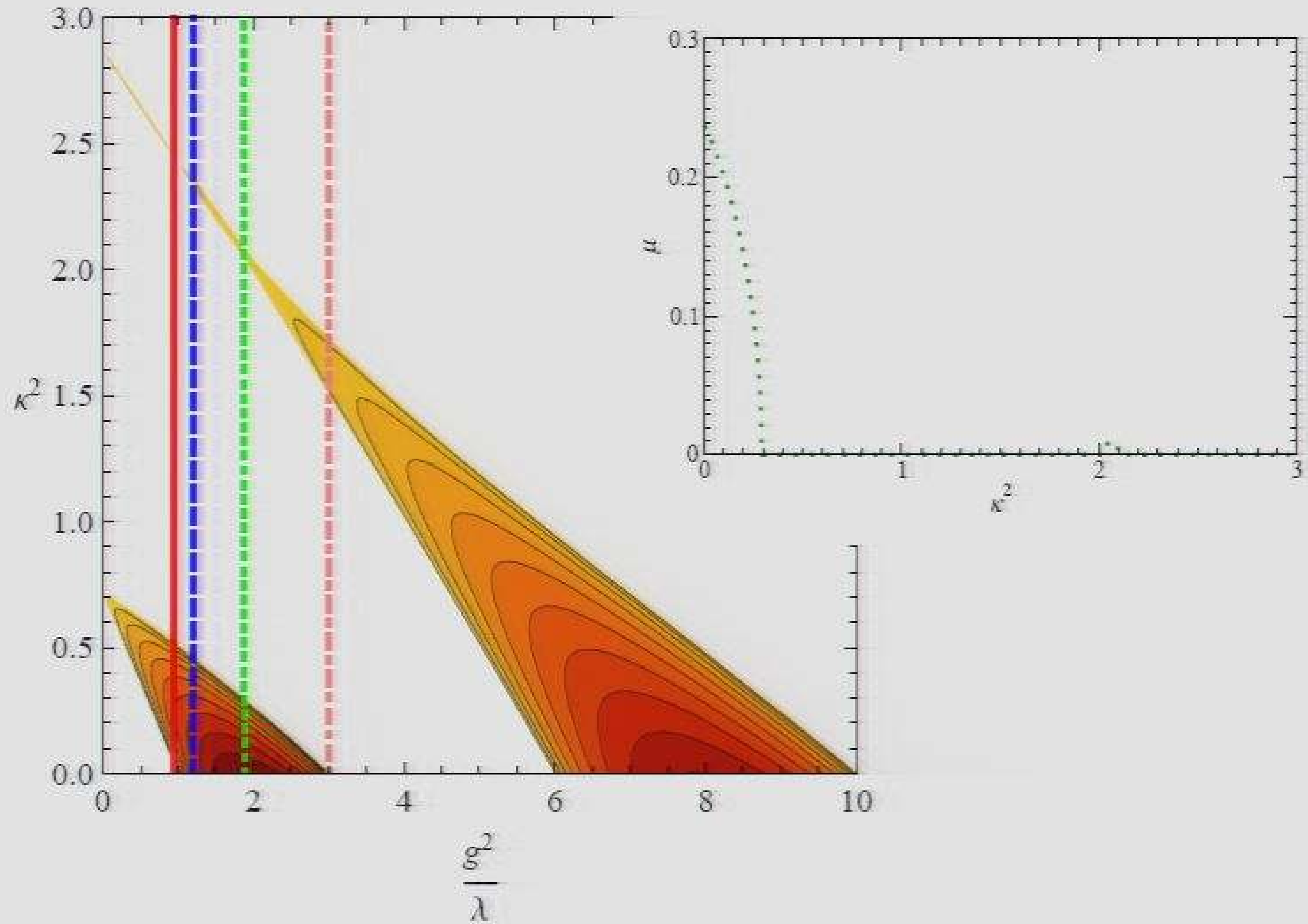

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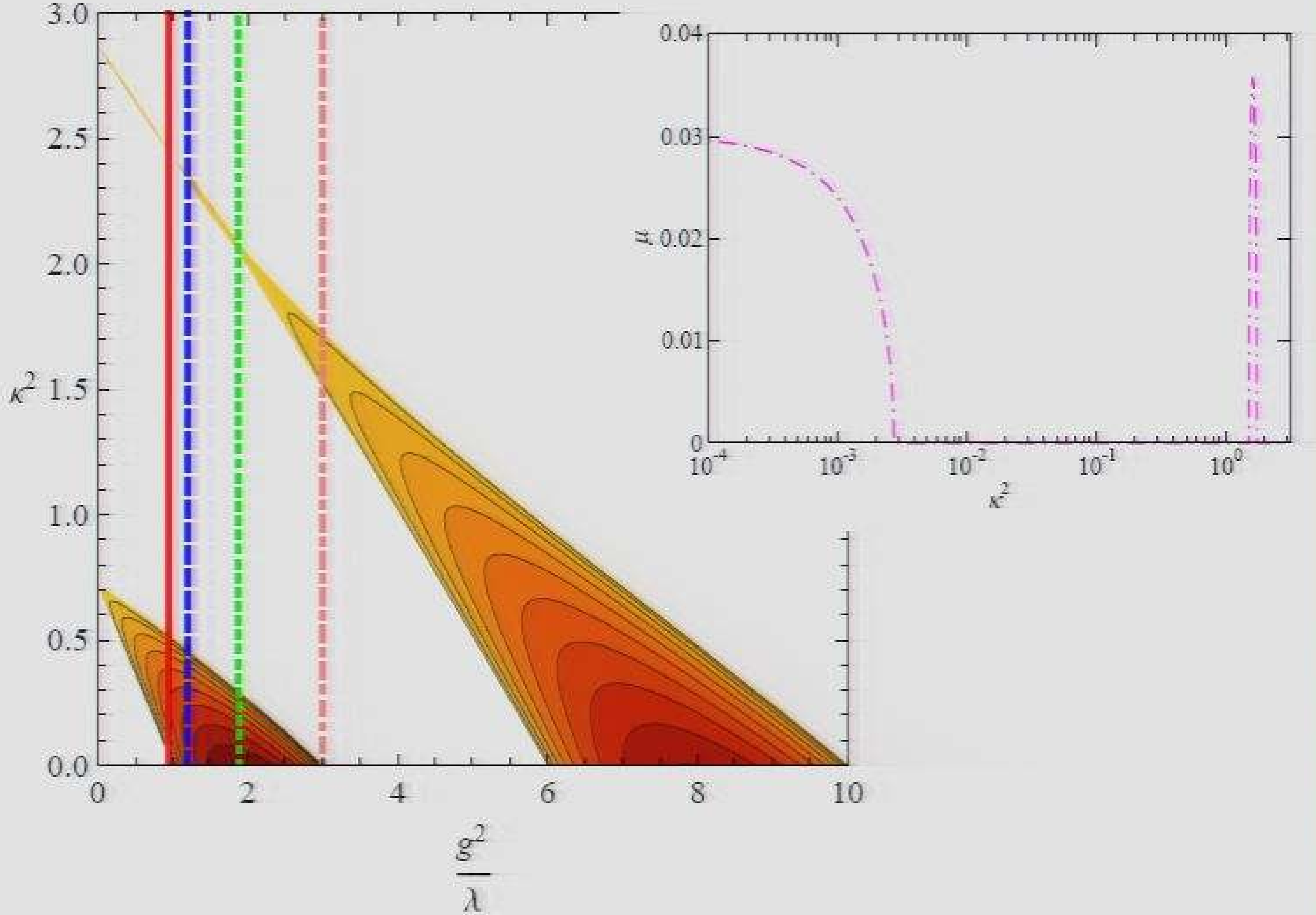

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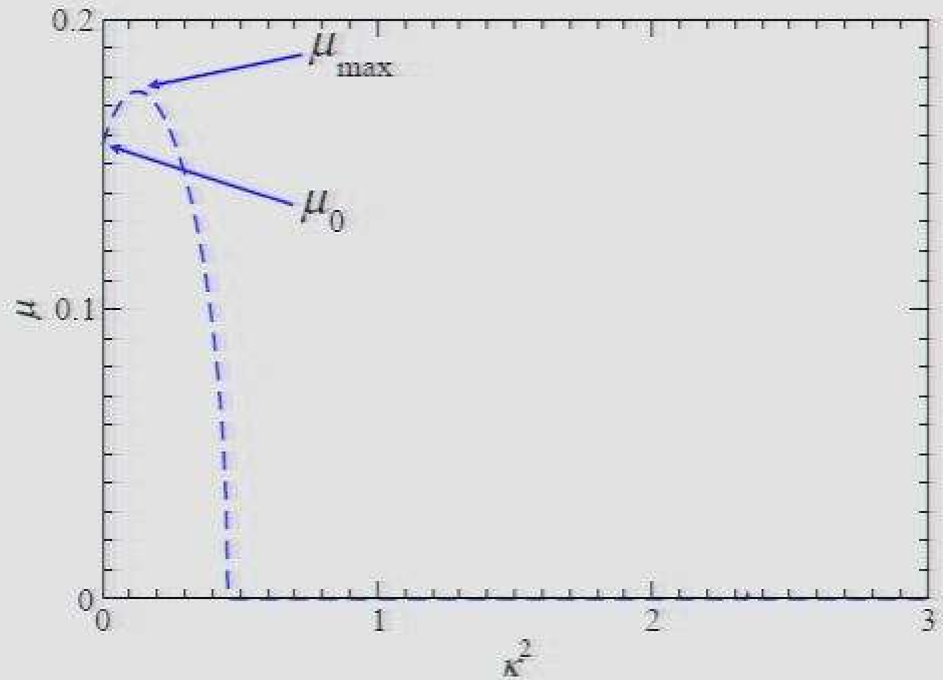
Analytic Calculation

- Zero mode grows as $\propto \exp(\mu_0 \tau)$
- Fluctuations grow as $\propto \exp(\mu_{\max} \tau)$



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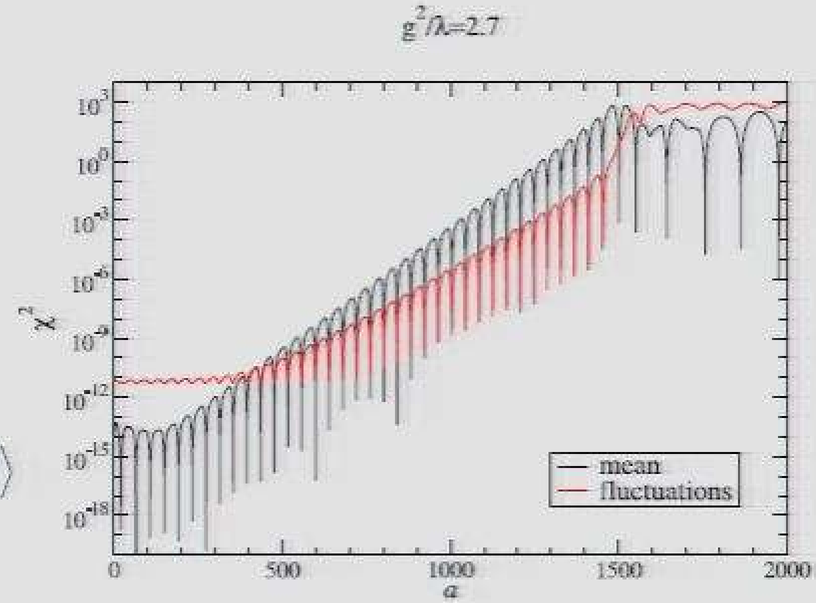

- Zero mode grows as $\propto \exp(\mu_0 \tau)$
- Fluctuations grow as $\propto \exp(\mu_{\rm max} \tau)$
- Becomes non-linear when $g^2\langle\chi^2\rangle \approx \lambda\langle\phi^2\rangle$

$$\ln a_{\rm nl}(\chi_{\rm ini}) = \ln a_{\rm nl}(0) - \frac{g^2}{24\lambda M_{\rm Pl}^2 \ln(1/g)} g^{-2\mu_0/\mu_k} \chi_{\rm ini}^2 + O(\chi_{\rm ini}^4)$$

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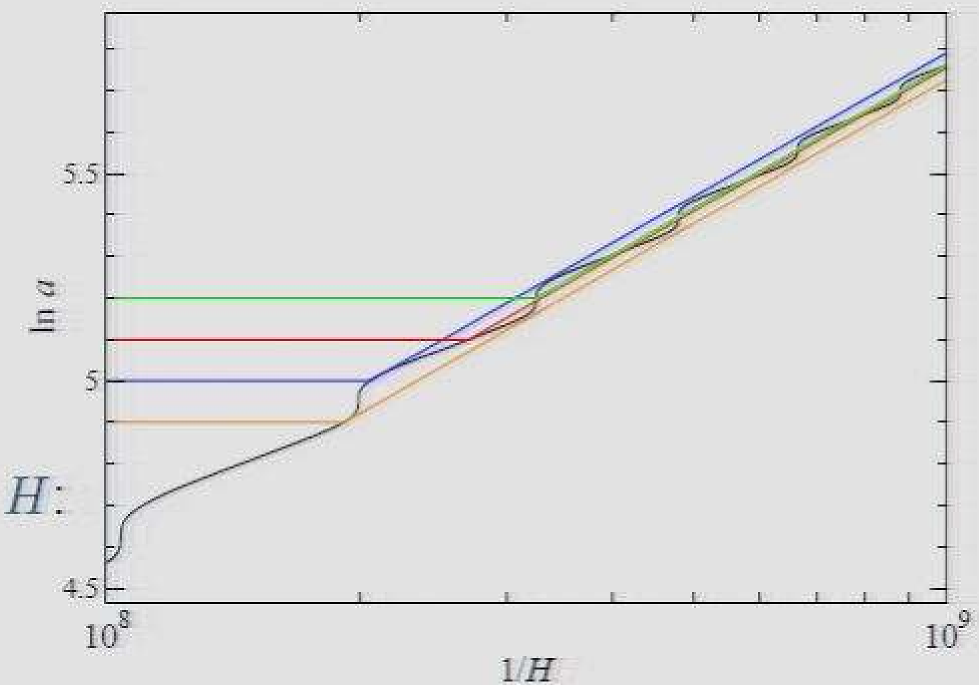
Analytic Calculation



- Zero mode grows as $\propto \exp(\mu_0 \tau)$
- Fluctuations grow as $\propto \exp(\mu_{\max} \tau)$
- We need the scale factor at constant H:

$$\Delta \ln a|_{H=H_*} = \left(1 + \frac{1}{2}\frac{d\ln H}{d\ln a}\right)\Delta \ln a_{\rm nl}$$

$$\Rightarrow \quad \left|\frac{\partial^2 N}{\partial \chi_{\rm ini}^2}\right| \approx \frac{1}{12 M_{\rm Pl}^2 \ln(1/g)} \frac{g^2}{\lambda} g^{-2\mu_0/\mu_{\max}}$$

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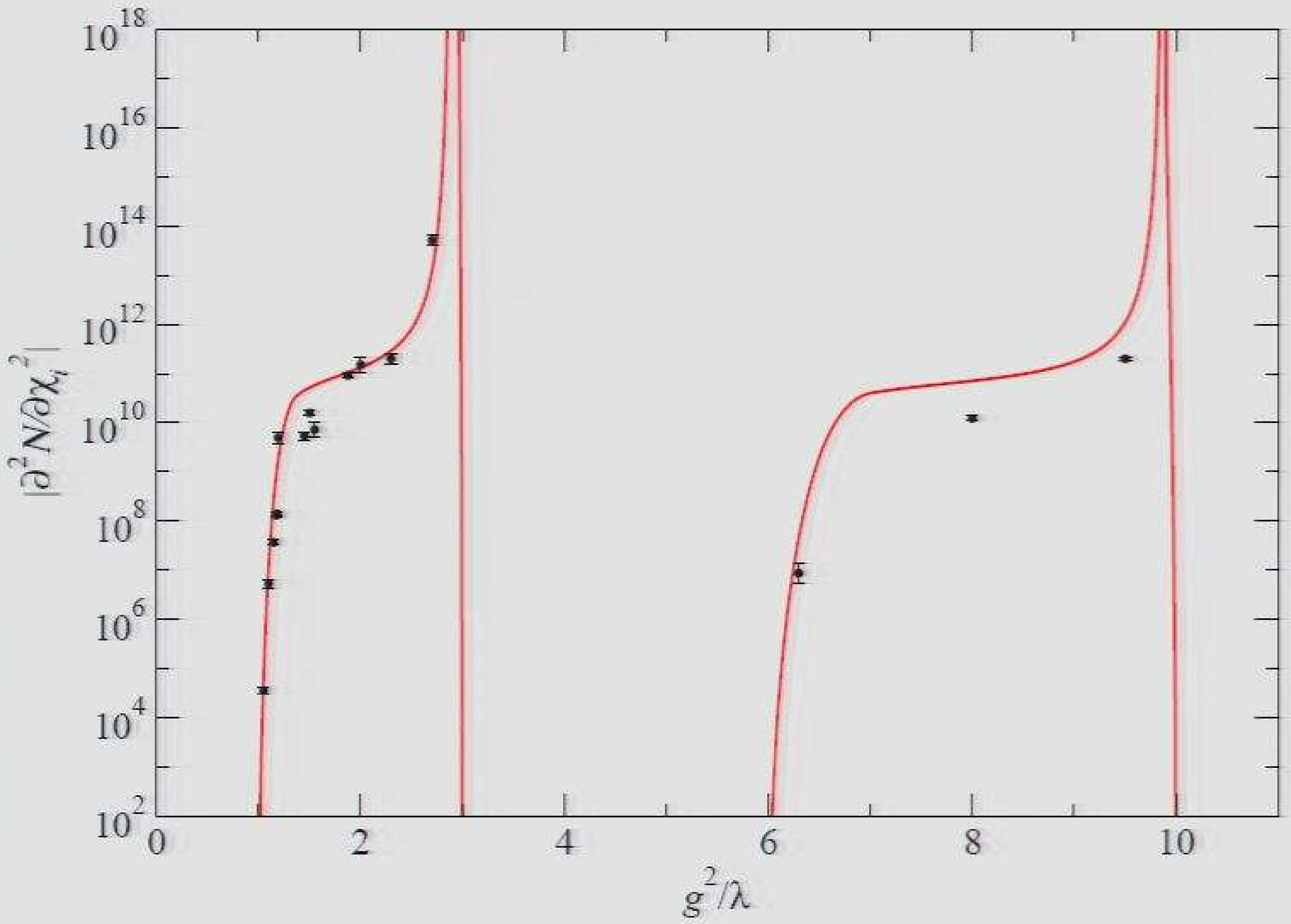
Dependence on g^2/λ



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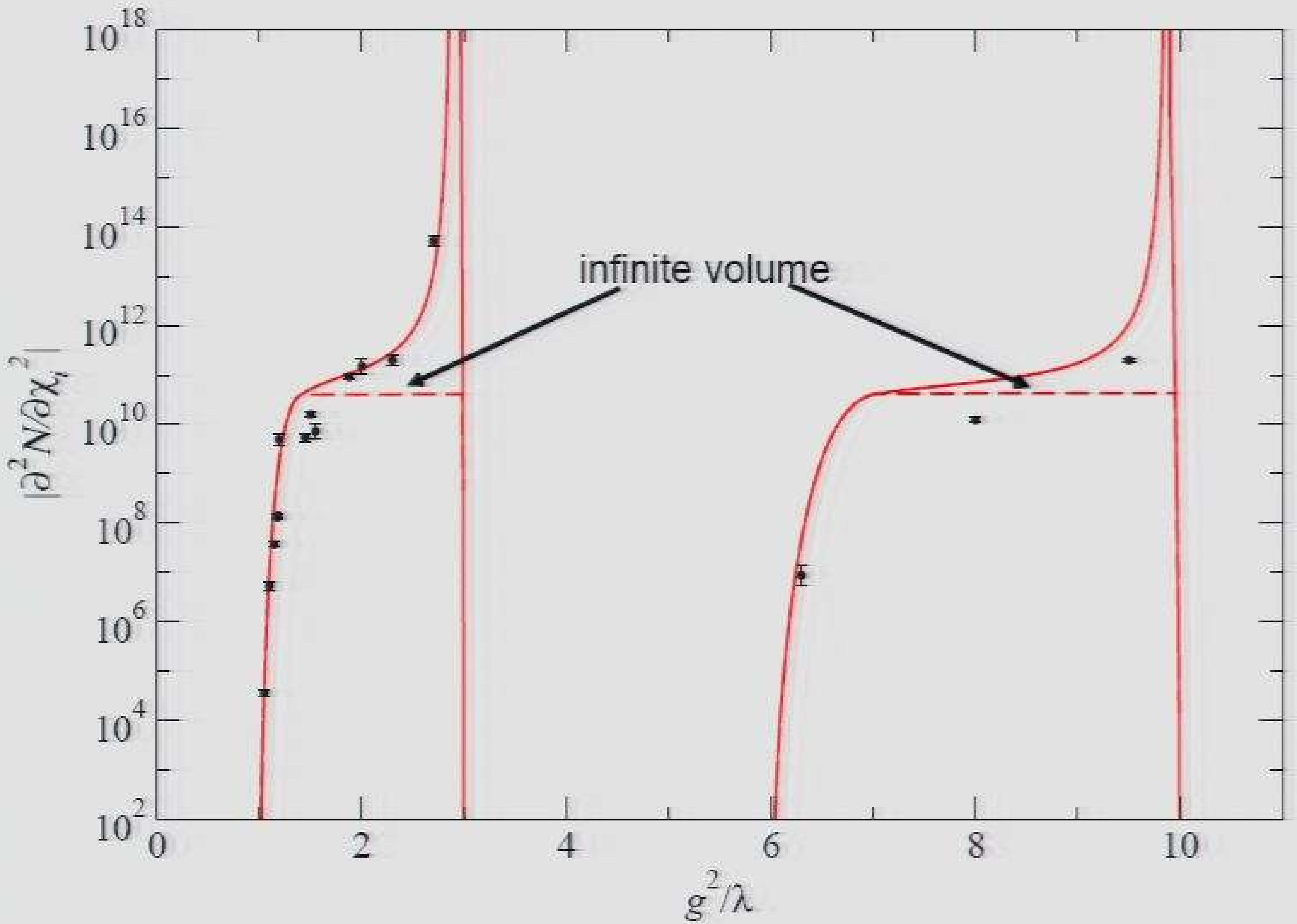

Calculating $f_{\rm NL}$

- Boubekeur&Lyth: For flat spectrum, $\zeta = \zeta_0 + (\partial^2 N/\partial\chi_{\rm ini}^2)\chi_{\rm ini}^2$ gives

$$f_{\rm NL} = -\frac{5}{9\pi^2}\left(\frac{\partial^2 N}{\partial\chi_{\rm ini}^2}\right)^3 \ln\frac{k}{a_0 H_0}$$

- Tilted spectrum

$$f_{\rm NL} \approx -\left(\frac{\partial^2 N}{\partial\chi_{\rm ini}^2}\right)^3 \lambda M_{\rm Pl}^6 \left(\frac{N_k}{N_{\rm sim}}\right)^{3(2-g^2/\lambda)}$$

with $N_k \sim 60$ and $N_{\rm sim} = 25/8$

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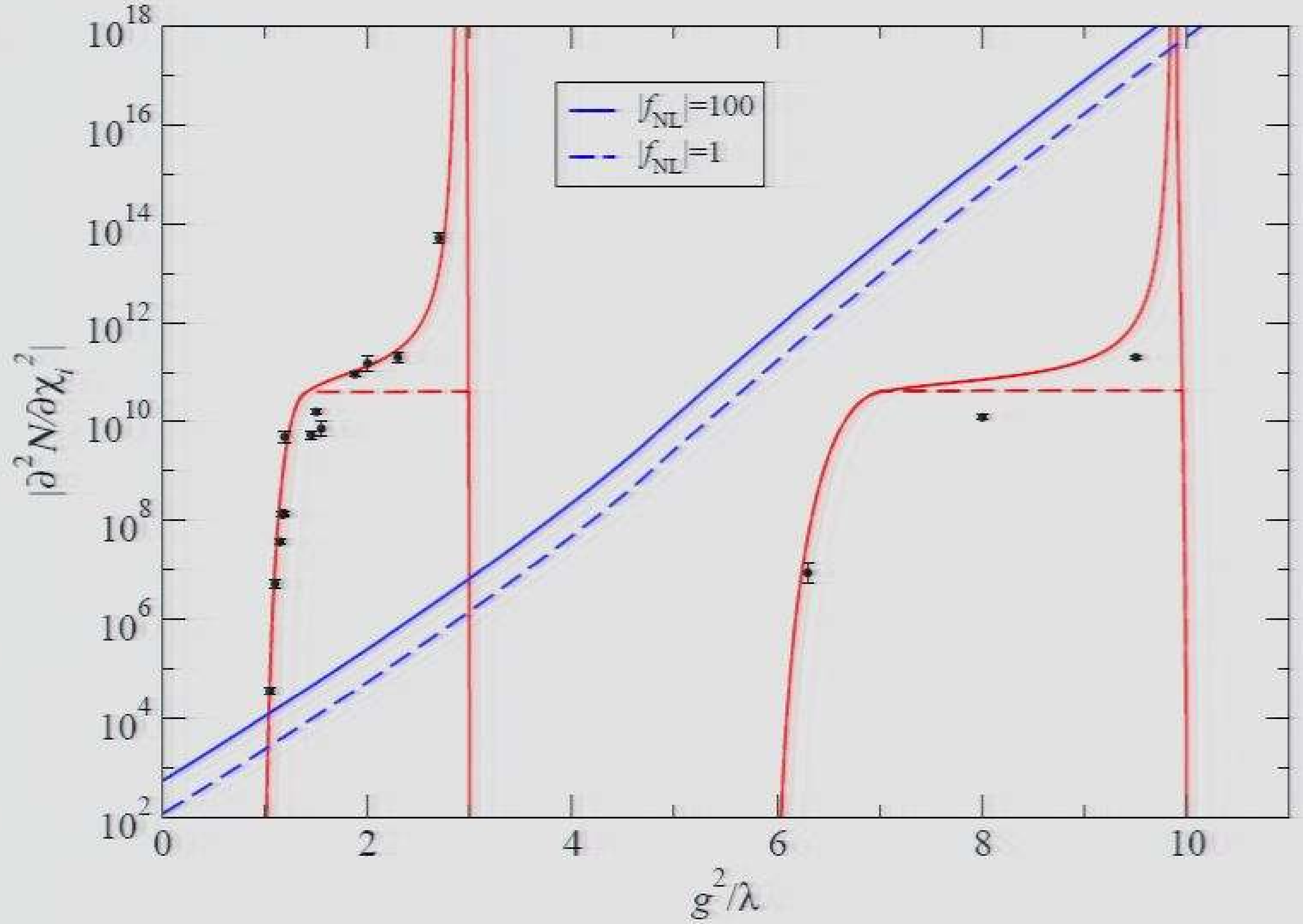
Dependence on g^2/λ



Amplitude of Perturbations

- Gaussian contribution from χ

$$\Delta\zeta = \frac{\partial^2 N}{\partial \chi_{\rm ini}^2} \overline{\chi_{\rm ini}} \Delta\chi_{\rm ini}$$

where $\overline{\chi_{\rm ini}}$ is the average of our currently observable universe

- If this is more than 10^{-5}, it dominates over the inflaton

$\Rightarrow$ Incompatible with observations, but can be cured by tuning λ

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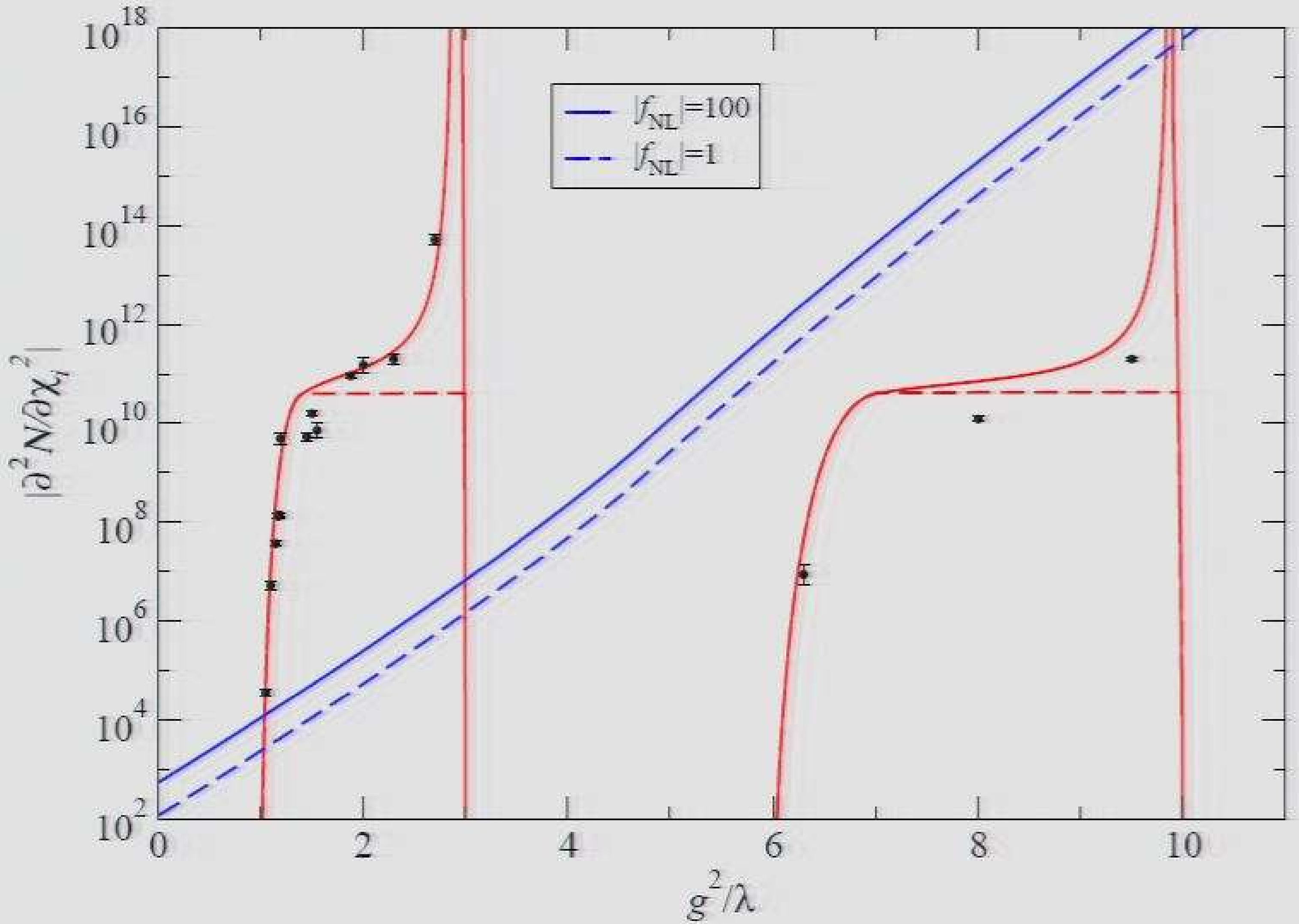
Dependence on g^2/λ



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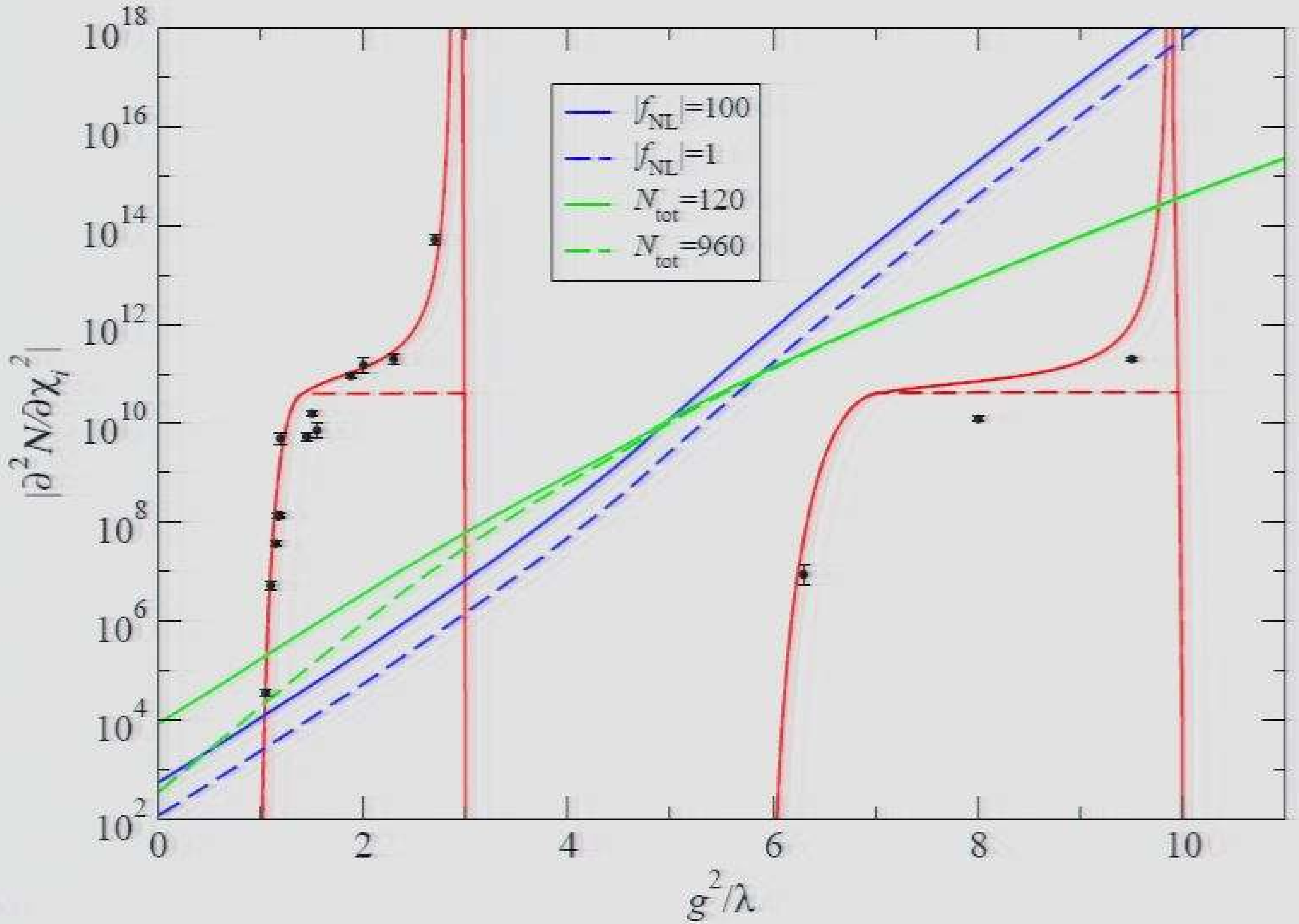

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Conclusions

- Massless preheating:
  - Strong effect if homogeneous mode dominates the resonance
  - Parts of parameter space ruled out
  - Other parameters fine, even if preheating takes place
- Simulations:
  - Non-linear calculation of curvature perturbation due to non-equilibrium physics
  - Works with any (bosonic) field dynamics
  - Box size dependence - Combine with metric perturbations?
- Analytic approximation:
  - Reproduces numerical results
  - Generalise to other models?

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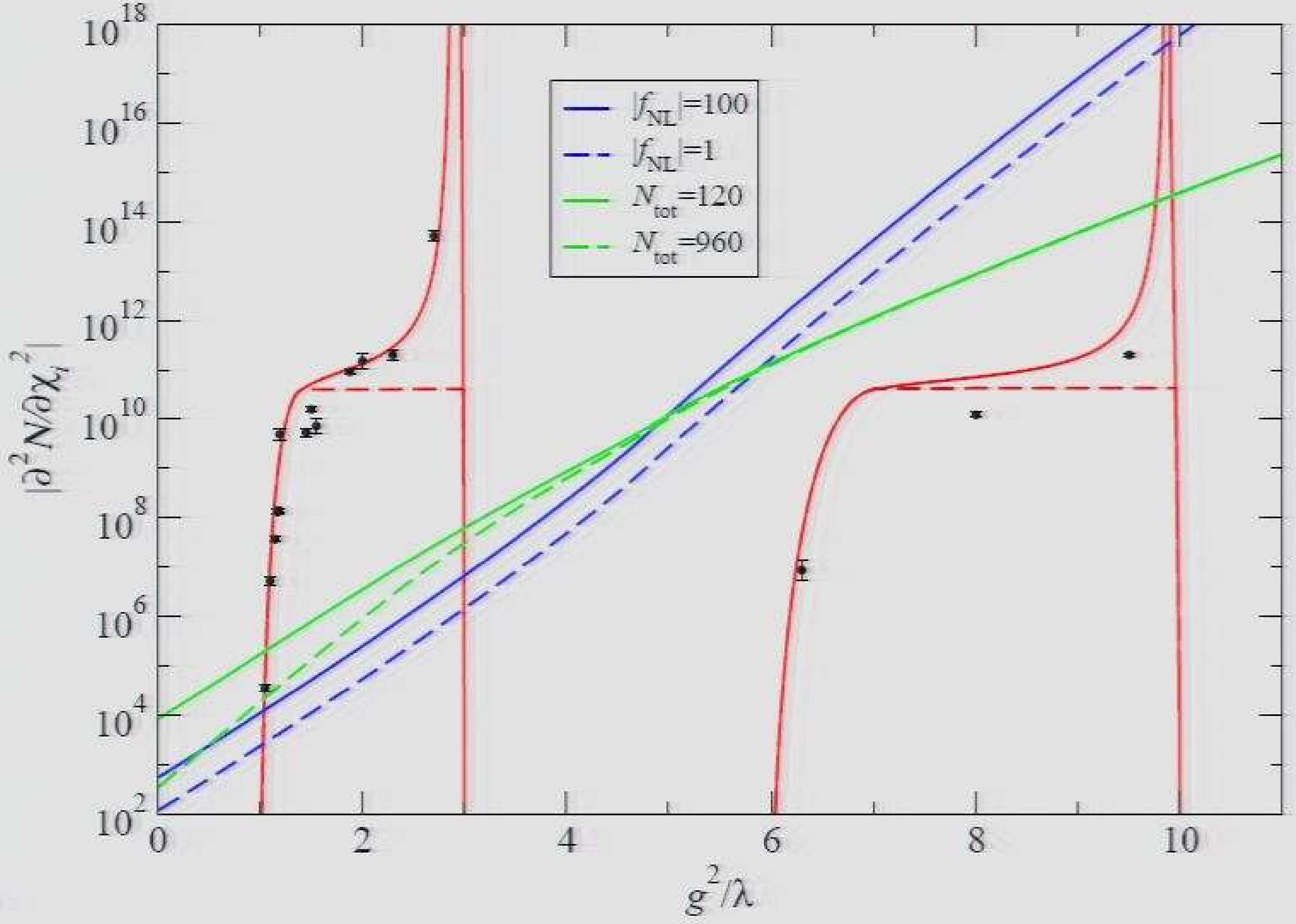

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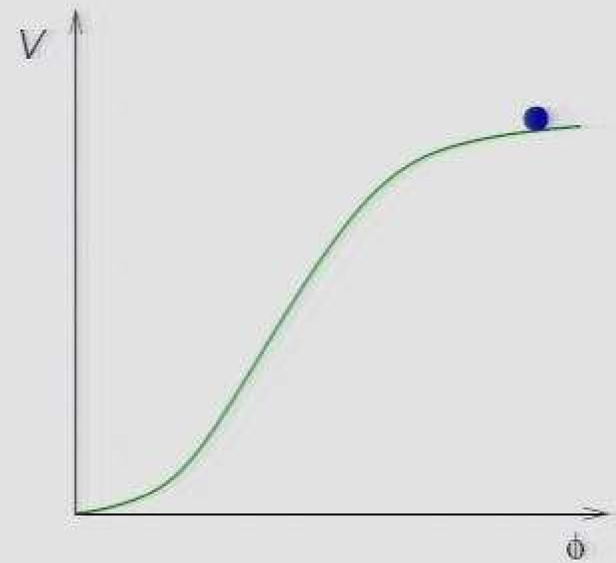
Inflation

- Scalar inflaton field ϕ, with $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$
- Potential $V(\phi)$
  - Dominates $\rho \approx V(\phi)$
  - Flat $\epsilon = \frac{1}{2}M_{\rm Pl}^2(V'/V)^2 \ll 1$, $|\eta| = M_{\rm Pl}^2|V''/V| \ll 1$

  $\Rightarrow$ Slow roll



$$H^2 = \frac{\rho}{3M_{\rm Pl}^2} \sim \text{constant} \quad \Rightarrow \quad a(t) \sim e^{Ht}$$

$\Rightarrow$ Solves horizon, flatness problems

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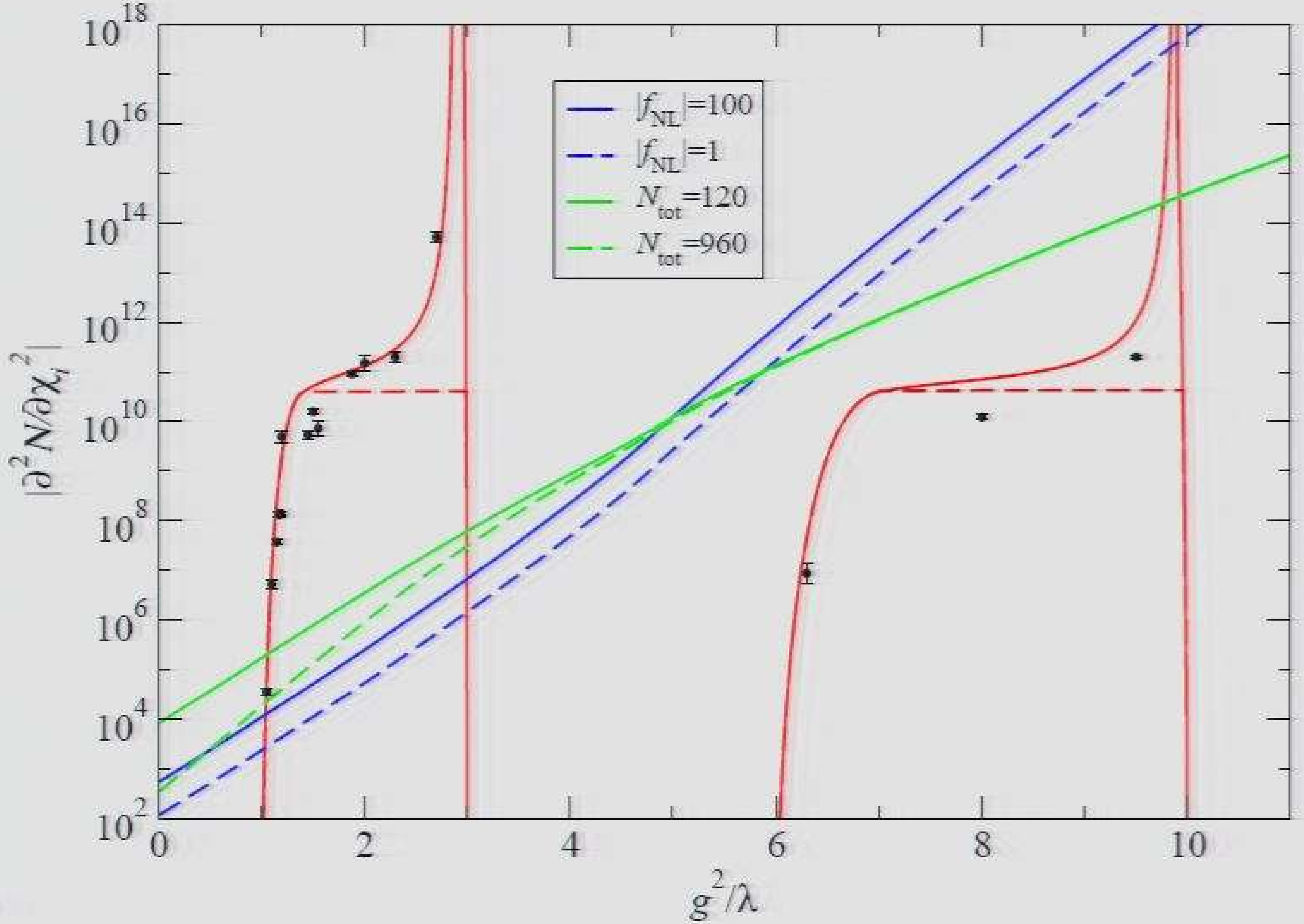

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