Title: Constraining the inflationary action with cosmological observations

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Abstract: If inflation is to be considered in an unbiased way, as possibly originating from one of a wide range of underlying theories,then observations need not be simply applied to reconstructing the inflaton potential, V(phi), or a specific kinetic term, as in DBI inflation, but rather to reconstruct the inflationary action in its entirety. I will discuss the constraints that can be placed on a generalsingle field action from measurements of the primordial scalar and tensor fluctuation power spectra and non-Gaussianities. I will also briefly present the flow equation formalism for reconstructing a general inflationary Lagrangian, in a general gauge, that reduces to canonical and DBI inflation in a specific gauge.

CONSTRAINING THE INFLATIONARY ACTION WITH COSMOLOGICAL OBSERVATIONS

Ghazal Geshnizjani
(Perimeter Institute for Theoretical Physics)
Rachel Bean
(Cornell University)
Daniel Chung
(University of Wisconsin-Madison)

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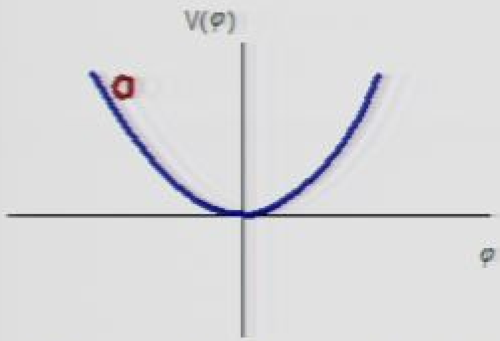
Minimal Kinetic Term Scalar Field

Choose 1 real field for minimality, simplicity, and its ability to capture the flavor of more complex dynamics.



$$S_\phi = \int dx^4 \sqrt{-g}\left[\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi) - V(\phi)\right]$$

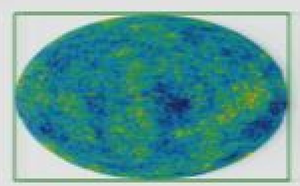
1D real manifold parameterized by

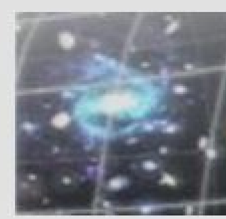
This potential results a long period of accelerated expansion (Inflation) to solve Horizon and flatness, etc. problems. $\rho \sim -p$

Most of the likely observables are controlled by this curve.

Observables traditionally consist of the following:

*Amplitude of the curvature perturbations as a function of k $\mathcal{P}_k^\zeta \Leftrightarrow k^{-1}|\delta\rho_k^2|$





*Amplitude of the tensor perturbation or primordial gravity waves as a function of k (future: Spider, CMBPol?, BBO?)

$\mathcal{P}_k^h$

Slow-Roll Inflationary Models

- Potential has to have small slope and curvature.

$$\epsilon \sim \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 \qquad 2\epsilon - \eta \equiv 2M_p^2\left(\frac{V''}{V}\right) \ll 1$$

- This leads to acceleration:

$$\frac{\ddot{a}}{a} = (1-\epsilon)H^2$$

- Can map the amplitude of curvature perturbations to the height and the shape of the potential

$$\mathcal{P}_k^\zeta \sim \left(\frac{1}{24\pi^2 M_p^4}\right)\frac{V}{\epsilon}$$

- Can map the amplitude of the gravity waves to the height of the potential

$$\mathcal{P}_k^h \simeq \frac{2}{3}\frac{V}{M_p^4}$$

Hopeful but Limited Potential Information

- 1 D manifold $V(\phi)$ constrained by 1D manifold of data (ideal).
- Bottom line: Ideal measurements may yield $V(\phi)$ over a range of ϕ if we assume single field minimal kinetic term models
- For sufficiently smooth potentials, the potential well can be captured by a Taylor expansion. (Analytic form.)

$$V(\phi) = \Lambda + b\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{3}A\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{\phi^{p+4}}{M^p}$$

- Well known consistency relationship can still rule out slow roll even with limited knowledge.

Non-minimal Kinetic Term Models

Scalar Fields may have non-canonical kinetic terms and they could also inflate the universe.

Minimal kinetic term is a symptom of a linear wave description of a particle but integrating out momentum shells to derive Wilsonian EFT generate higher powers

K-Inflation (C. Armendariz-Picon, T. Damour, V. Mukhanov)

$$S_{K(\phi)} = \int dx^4 \sqrt{-g}[X + f(\phi)X^2] \qquad X = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

"Non-particle" description for dynamics can also correspond to non-minimal kinetic terms

Warped D-Brane Inflation (Alishahiha, Silverstein, and Tong)

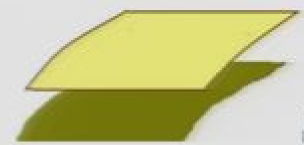
$$S_{DBI} = \int dx^4 \sqrt{-g}[-f(\phi)^{-1}\sqrt{1-2f(\phi)X} - (V(\phi) - f(\phi)^{-1})]$$

General framework to obtain Inflation:

$$\mathcal{L}(X,\phi) \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

Einstein Equations

1. Friedmann equations for homogenous background (Enough acceleration)



$$a(t), \quad H = \frac{\dot{a}}{a} \longleftrightarrow \rho(\phi(t))$$

2. Linear perturbation for quantum fluctuations (Power Spectrum: scalar, tensor)

$$\zeta(x,t) \longleftrightarrow \delta\phi(x,t) \Longrightarrow \delta\rho$$

CMB anisotropies and Large scale structure

3. Higher order perturbive calculation to obtain higher order correlation functions (Bispectrum,….)

Background evolution

single scalar field with a lagrangian of the form

$$\mathcal{L}(X,\phi) \longleftarrow X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

Similar to a hydrodynamical fluid:

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu},$$

$$\begin{aligned} p(X,\phi) &\equiv \mathcal{L}(X,\phi), \\ \rho(X,\phi) &\equiv 2X\mathcal{L}_X - \mathcal{L}(X,\phi) \\ u_\mu &\equiv \frac{\partial_\mu\phi}{\sqrt{2X}}, \end{aligned} \qquad L_X = \frac{\partial L}{\partial X}$$

Friedmann equations for the background :

$$X = \frac{1}{2}\dot{\phi}^2$$

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left[2X\frac{\partial p}{\partial X} - p(X,\varphi)\right] \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(2X\frac{\partial p}{\partial X} + 2p(X,\varphi)\right) \end{aligned}$$

Energy Conservation

$$\dot{\rho} = -3H(\rho+p)$$

The First Slow-roll Parameter:

Satisfying null energy condition: $\rho + p > 0$

Requiring Acceleration: $\ddot{a}(t) > 0 \longrightarrow \rho + 3p < 0$

Negative pressure

$$0 < \epsilon \equiv \frac{3(\rho+p)}{2\rho} = -\frac{\dot{H}}{H^2} < 1 \quad \text{OR} \quad 0 < \frac{-X\frac{\partial \mathcal{L}}{\partial X}}{\mathcal{L}} < 1$$

Efficient Inflation $\epsilon \ll 1 \longrightarrow \rho \sim -p$

Standard canonical scalar field $\mathcal{L} = X - V(\phi) \longrightarrow \frac{\partial \mathcal{L}}{\partial X} = 1 \longrightarrow \frac{X}{V(\phi)} \ll 1$

$$\epsilon \sim \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

Flat Potential

K-Inflation
(C. Armendariz-Picon, T. Damour, V. Mukhanov)

$\frac{\partial \mathcal{L}}{\partial X} = 0$ at $X = X_0$ $\xrightarrow{X \longrightarrow X_0}$ $\epsilon \ll 1$

Attractor solution

DBI INFLATIONARY MODEL (Alishahiha, Silverstein, and Tong)

$$-\frac{1}{f(\varphi)}\sqrt{1-2f(\varphi)X}-(V(\varphi)-f^{-1})$$

For small sound speed $c_s^2 = (\frac{\delta p}{\delta \rho})_\varphi$

Slow roll parameter:

$$\epsilon \sim \frac{M_p^2}{2} c_s (\frac{V'(\varphi)}{V(\varphi)})^2 \ll 1$$

Example:

$$V = V_2 \varphi^2 \quad + \quad f^{-1} \cong \frac{\varphi^4}{\lambda} \quad \Longrightarrow \quad \frac{\varphi^4}{V_2 \lambda} \ll M_p^2 \ll V_2 \lambda$$

$$\epsilon \sim \sqrt{3}\frac{M_p}{\sqrt{V_2 \lambda}}$$

Observational constraints on generalized slow-roll parameters from the primordial spectrum

- Considering small inhomogeneities for the scalar field

$$\phi(x,t) = \phi(t) + \delta\varphi(x,t)$$

- Metric in the longitudinal gauge

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a^2(t)\gamma_{ij}dx^i dx^k$$

Einstein equations linear order (Action up to second order) : Φ & $\delta\varphi$

Curvature fluctuations : $\zeta(x,t)$

$$\zeta \equiv \frac{5\rho+3p}{3(\rho+p)}\Phi + \frac{2\rho}{3(\rho+p)}\frac{\dot{\Phi}}{H} \qquad z \equiv \frac{a(\rho+p)^{1/2}}{c_s H}$$

$$u \equiv z\zeta$$

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} = \frac{1}{1+2\frac{X\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}}$$

in a flat universe after quantization one still gets an equation of motion similar to that for canonical actions (Garriga and Mukhanov)

$$\frac{d^2u_k}{d\tau^2} + \left(c_s^2k^2 - \frac{1}{z}\frac{d^2z}{d\tau^2}\right)u_k = 0$$

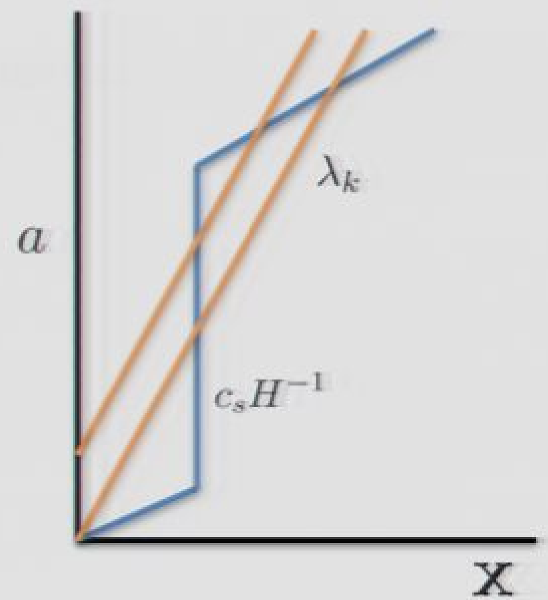
$$\frac{d^2u_k}{d\tau^2} + (c_s^2k^2 - 2(aH)^2\{(1+\frac{\eta}{2}+\kappa)(1-\frac{\epsilon}{2}+\frac{\eta}{4}+\frac{\kappa}{2}) + \frac{\dot\eta}{2H} + \frac{\dot\kappa}{H}\})u_k = 0$$

$$\eta \equiv \frac{\dot\epsilon}{H\epsilon}$$

$$\kappa \equiv \frac{\dot c_s}{Hc_s}$$

slow roll approximation : $\epsilon, \eta, \kappa, \eta_N, \kappa_N \ldots \ll 1$

To leading order a Bessel equation



Scalar Power Spectrum

$$\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z^2}\Big|_{c_sk=aH},$$
$$\sim \frac{1}{8\pi^2M_{pl}^2}\frac{H^2}{c_s\epsilon}\Big|_{c_sk=aH}$$

Scalar spectral index: $n_s - 1 \equiv \frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k}\Big|_{k=k_s},$

$$\approx -(2\epsilon+\eta+\kappa) + O(\epsilon^2, \epsilon\eta, \kappa_N, \ldots)$$

Quantum fluctuations are produced on scales ($\lambda = \frac{a}{k}$) much smaller than sound horizon (c_sH^{-1}) however their amplitude is set at sound horizon scale which is almost constant during Inflation. Sound Horizon reduces to Hubble radius (H^{-1}) if sound speed is one.

Alternative observations will result in getting complementary information:

Primordial Gravity Waves

Similar procedure for tensor power spectrum leave subdominant but distinct imprints in the CMB Polarization

$$\mathcal{P}_h = \left. \frac{2H^2}{M_{pl}^2 \pi^2} \right|_{k=aH}$$

Tensor spectral indexes:

$$n_t \equiv \left. \frac{d \ln \mathcal{P}_h}{d \ln k} \right|_{k=k_t} \approx -2\epsilon + O(\epsilon^2, ...)$$

What about Higher order correlation functions?

Three point function, Non-Gaussianity :

Parameterizing non-gaussianties as

$$\zeta = \zeta_G - \frac{3}{5} f_{NL} \zeta_G^2$$

(Chen, Huang, Kachru, and Shiu)

$$f_{NL}^{\mathrm{equil}} \approx (-0.26 + 0.12 c_s^2) \left(1 - \frac{1}{c_s^2}\right) - 0.08 \left(\frac{c_s^2}{\epsilon}\right) \frac{X^3 \mathcal{L}_{XXX}}{M_{pl}^2 H^2},$$

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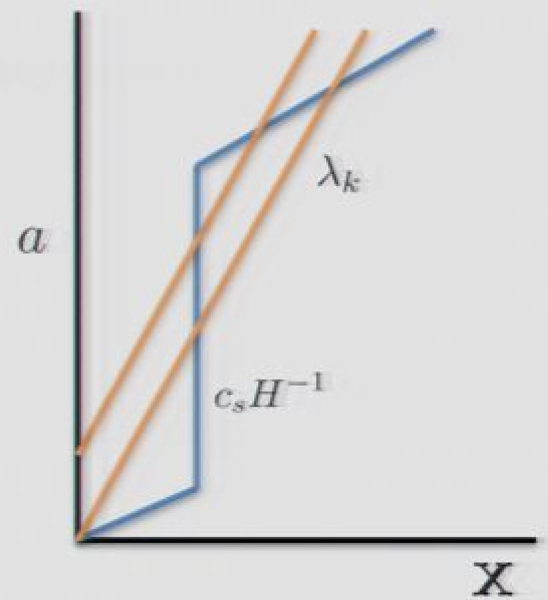
→ **To leading order a Bessel equation**



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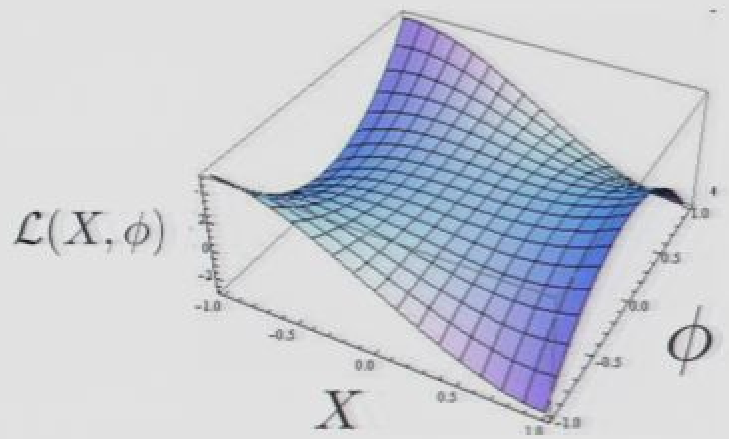
What does a general single field Lagrangian with non-minimal kinetic term look like to be consistent with inflationary phenomenology?

$$S = \int dx^4 \sqrt{-g}\mathcal{L}(X,\phi)$$

The general action is simply a 2-D manifold parameterized by (X,ϕ)



The question is to find the general form that satisfies the constraints from the data.

- Ease of constructing inflationary models numerically for fitting
- Parameterizing objects that encode data in a transparent way
- See if there are general theoretical restrictions

Gauge Ambiguity:

$$\{\phi, X_\phi\}$$

$$\phi = f(\varphi)$$

$$\{\phi, X_\phi\} \rightarrow \{\varphi, X_\varphi = X_\phi / [\partial_\varphi f]^2\}$$

$$\partial_\phi = \frac{\partial\varphi(\phi)}{\partial\phi}\partial_\varphi \qquad \partial_{X_\phi} = \frac{1}{[\partial_\varphi f]^2}\partial_{X_\varphi}$$

Observables should only constrain gauge independent combinations of X and derivatives of L. We can fix gauge by adjusting $f(\varphi)$ along a particular trajectory $\{\phi(N_e), X_\phi(N_e)\}$ to obtain desired $\partial_{X_\varphi}\mathcal{L}$ or $X_\varphi(N_e)$

e.g. $\partial_{X_\varphi}\mathcal{L}|_{\{\varphi(N_e), X_\varphi(N_e)\}} = \dfrac{1}{c_s(N_e)}$ or $X(N_e) = \dfrac{1}{2}$

Canonical Action $\mathcal{L}_X = 1 \longrightarrow X = \dfrac{1}{3}\epsilon(N_e)\rho(N_e)$ → $\phi(N)$

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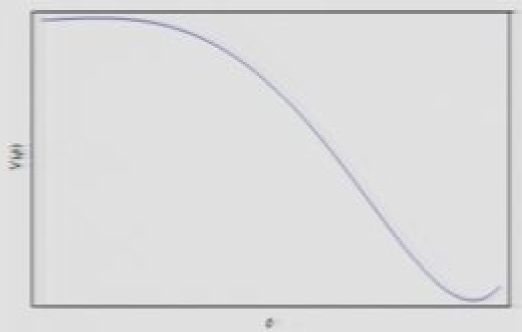
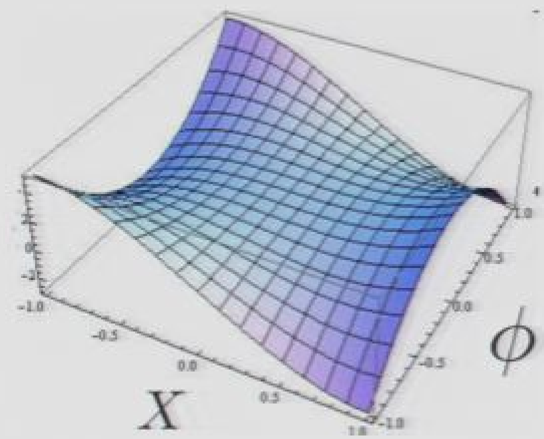


$\mathcal{L}(X,\phi)$



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Constructing Analytic Form

Assuming idealistic measurements of Curvature and Tensor perturbation we can break the degeneracy between H(Ne) and Cs(Ne).

Hierarchy of horizon flow functions
(Schwarz,Terrero-Escalante, Garc´ıa)

$$H \quad , \quad \epsilon = -\frac{d\ln H}{dN},$$

$$\eta \;=\; \frac{d\ln\epsilon}{dN} = \frac{1}{\epsilon}\frac{d^2}{dN}\ln H,\; \eta_N,\; \eta_{NN},\ldots$$

Hierarchy of sound speed flow functions

$$c_s,\; \kappa = \frac{1}{c_s}\frac{dc_s}{dN},\quad \kappa_N,\ldots$$

For on-shell trajectory:

$$H^2(N_e)$$

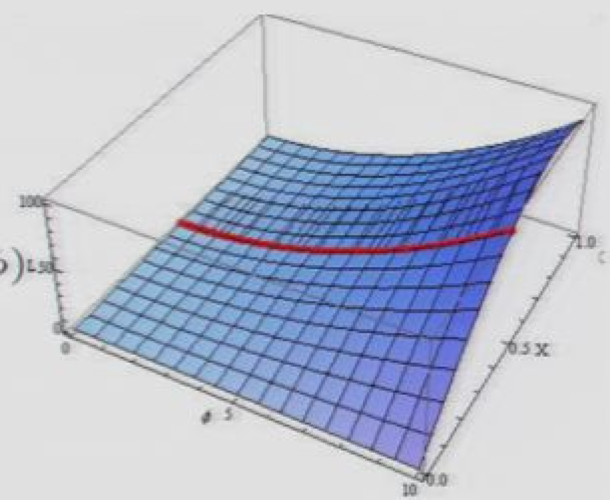
$$\mathcal{L}(N_e) \;=\; \rho(N_e)(-1+\frac{2\epsilon(N_e)}{3})$$

$$X\mathcal{L}_X(N_e) \;=\; \frac{1}{3}\epsilon(N_e)\rho(N_e)$$

$$X^2\mathcal{L}_{XX}(N_e) \;=\; \frac{1}{6}\left(\frac{1}{c_s^2(N_e)}-1\right)\epsilon(N_e)\rho(N_e)$$

Go to the gauge: $X_\varphi(N_e) = \frac{M_P^4}{2}$

$\mathcal{L}(X,\phi)$



The boundary condition:

$$\mathcal{L}|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$

$$\partial_{X_\varphi}\mathcal{L}|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$

$$\partial^2_{X_\varphi}\mathcal{L}|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$

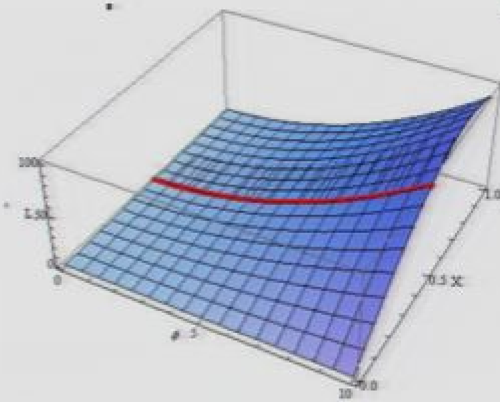
Analytic Form for Action Consistent with Data?

1) Extract

$$\mathcal{L}^{obs}\Big|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$

$$\partial_{X_\varphi}\mathcal{L}^{obs}\Big|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$

$$\partial^2_{X_\varphi}\mathcal{L}^{obs}\Big|_{\{\varphi(N_e),X(N_e)=\frac{M_P^4}{2}\}}$$



2) Choose an “arbitrary” function $q(\phi, X)$

Answer:

$$\begin{aligned}\mathcal{L}(X,\phi) &= q(X,\phi) + \mathcal{L}^{obs}\left(\frac{M_P^4}{2},\phi\right) - q\left(\frac{M_P^4}{2},\phi\right) \\ &+ \left[\mathcal{L}^{obs}_X\left(\frac{M_P^4}{2},\phi\right) - q_X\left(\frac{M_P^4}{2},\phi\right)\right]\left(X - \frac{M_P^4}{2}\right) \\ &+ \frac{1}{2}\left[\mathcal{L}^{obs}_{XX}\left(\frac{M_P^4}{2},\phi\right) - q_{XX}\left(\frac{M_P^4}{2},\phi\right)\right]\left(X - \frac{M_P^4}{2}\right)^2\end{aligned}$$

Remember we can extend this if measure

$$\begin{aligned}f_{NL}^{\rm equil} \approx\ & (-0.26 + 0.12c_s^2)\left(1 - \frac{1}{c_s^2}\right) \\ & -0.08\left(\frac{c_s^2}{\epsilon}\right)\frac{X^3\mathcal{L}_{XXX}}{M_{pl}^2H^2},\end{aligned}$$

Simple example:

Suppose measurements give $\epsilon \sim \frac{1}{2N_e} \ll 1 \quad c_s = 1 - \delta \quad \delta \ll 1$

$$H = H_1 \exp\left(\int_1^{N_e} \frac{dN}{2N}\right) = H_1 N_e^{1/2}$$

$$\begin{aligned} \mathcal{L}\left(\frac{1}{2}, \phi\right) &= M_{pl}^2 H_1^2 (1 - H_1^2 \phi^2) \\ \mathcal{L}_X\left(\frac{1}{2}, \phi\right) &= H_1^2 M_{pl}^2 \\ \mathcal{L}_{XX}\left(\frac{1}{2}, \phi\right) &\sim 2 M_{pl}^2 H_1^2 \delta \end{aligned}$$

$q = 0 \longrightarrow \tilde{\mathcal{L}}_1(X, \phi) \sim (M_{pl} H_1)^2 \left[-\frac{3}{4}(H_1 \phi)^2 + X + \delta\, X^2\right]$

$q = \lambda X^3 \longrightarrow$

$$\begin{aligned} \tilde{\mathcal{L}}_2(X, \phi) &= \tilde{\mathcal{L}}_1(X, \phi) \\ &+ \lambda \left[X^3 - \frac{1}{8} - \frac{3}{4}(X - \frac{1}{2}) - \frac{3}{2}(X - \frac{1}{2})^2\right] \end{aligned}$$

Both satisfy the equation of motion at X=1/2 and fit c_s an ϵ .
At this level (2X derivatives), the two are **observationally indistinguishable**.

Lesson: Need more observables!

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In retrospect obvious once one fixes the gauge, $(e.g.\ X = \frac{M_p^2}{2})$ the determination of all $[\frac{\partial}{\partial X}]^n \mathcal{L}(\phi, X)|_{X=\frac{M_p^4}{2}}$ from measurement is equivalent to specifying all Taylor expansion coefficients in the direction ***X***.

Possibilities:

1) tree-lev terms in 3-point func.
1) Higher order correlation functions.
2) Loop corrections (probably too small)

All these calculations can break down if the perturbative expansion around an inflating background does not remain valid (L. Leblond, S. Shandera)

Inflationary flow equations

How to construct inflationary models numerically for fitting data?

•Dirctly work in the slow-roll parameter space in terms of e-folding number:

$$H \;,\; \epsilon = -\frac{d\ln H}{dN}, \qquad c_s,\; \kappa = \frac{1}{c_s}\frac{dc_s}{dN},\;\; \kappa_N, \ldots$$

$$\eta = \frac{d\ln\epsilon}{dN} = \frac{1}{\epsilon}\frac{d^2}{dN}\ln H,\; \eta_N,\; \eta_{NN}, \ldots$$

•For a Canonical action if we assume that slow-roll parameters can be truncated at some order then we could estimate $H(\phi)$ and consequently $V(\phi)$ with a Taylor expansion (Kinney)- (Liddle, Parsons, and Barrow)

For a general action however:

$H(\phi)$ and $c_s(\phi)$ can only be estimated if the gauge is fixed.

$$H(\phi) = H_0 + M_{pl}H_0'\left(\frac{\Delta\phi}{M_{pl}}\right) + \ldots + \frac{1}{(l+1)!}M_{pl}^{l+1}H_0^{[l+1]}\left(\frac{\Delta\phi}{M_{pl}}\right)^{l+1} + \ldots$$

$$Q_l(H)|_{\phi_0} = \left[\left(\frac{dN_e}{d\phi}\frac{d}{dN_e}\right)^l H\right]_{\phi_0}$$

$$\frac{dN_e}{d\phi} = \pm\frac{H}{\sqrt{2X}} = \pm\left(\frac{\mathcal{L}_X}{2\epsilon}\right)^{1/2}$$

$$H \;,\; \epsilon = -\frac{d\ln H}{dN}, \qquad \eta = \frac{d\ln\epsilon}{dN} = \frac{1}{\epsilon}\frac{d^2}{dN}\ln H,\; \eta_N,\; \eta_{NN}, \ldots$$

Conclusion:

•The slow-roll approximation in its general form can be applied to non-canonical actions to obtain inflation.

•General single field actions (a 2D manifold parameterized by (X, ϕ)) can be written as Taylor expansion in the X direction which is orthogonal to the 1D manifold (parameterized by Φ) encoding the information from the data. (A surprisingly simple result for general parameterization.)

•Unlike in the case of minimal kinetic terms, even with ideal data for 3-point function for "all" N-efolds and fixed reheating scenarios, the set of models which are consistent with data forms an infinite set. Good news is that the infinite set organizes itself in a simple manner.

•Higher order correlation function measurements should narrow the set by constraining more terms for Taylor expansion in the X direction.