Title: Non-Gaussianities in extended D-term inflation

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Abstract: I will present extensions of hybrid inflationary models in the context of supersymmetric D-term in- flation. I will show that there exists a large domain of parameters in which significant primordial non-Gaussianities can be produced while preserving a scale free power spectrum for the metric fluctuations. In particular I will explicitly present the expected bi- and trispectrum for such models and compared the results to the current and expected observational constraints. It is to be noticed that it is necessary to use both the bi- and tri-spectra of CMB anisotropies to efficiently reduce the parameter space of such models.

Francis Bernardeau
IPhT Saclay and CITA

Pascos 2008

Non-gaussianities in extended D-term inflationary models

Based on works in collaboration with
Tristan Brunier (IPhT Saclay)
Jean-Philippe Uzan (IAP)

PRD**67** 121301, PRD**69** 063520, PRD**71** 063529, PRD**73** 083524, arXiv:0705.250

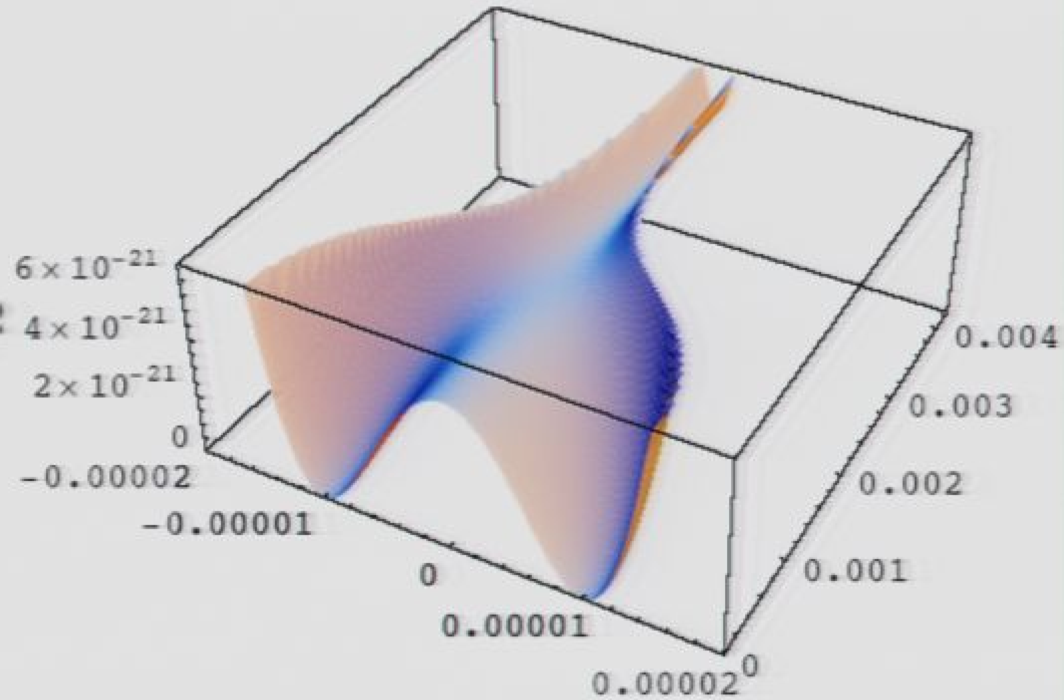
A conservative point of view for NG in CMB observations …

Generic inflation predicts (almost) exact Gaussian metric fluctuations

- Departures are possible,
- What kind of "reasonable" NG can be expected (in "simple" extensions of standard inflation) ?
  - Curvaton
  - DBI type kinetic term
  - ✓ **Multiple field inflation: high energy physics construction**

An interesting framework : hybrid inflation

$$V(\varphi,\sigma) = V(\varphi) + \frac{\mu}{2}\left(\sigma^2 - \sigma_0^2\right)^2 + \frac{g}{2}\sigma^2\varphi^2$$



- For vev of fieds: vev of φ can be much smaller than Planck mass.
- From high energy physics BSMs, global and local susy, and superstrings (brane/antibrane collisions)
- **What is happening if you add light fields ?**

Inflation with global susy : F- and D-term hybrid models

- SUSY model from F-term (Dvali, Shafi Schaefer '94)

Superpotential: $$W = -\lambda \mu^2 S + \lambda S \Phi_+ \Phi_-$$

mass parameter — Dimensionless parameter — Chiral superfields

- SUSY model from D-term with nonzero g and ξ (Binétruy et Dvali '96)

Superpotential: $$W = \lambda S \Phi_+ \Phi_-$$

Dimensionless parameter — Chiral superfield with no charge — Superfields with charges +1 and -1

- F term inflation potentials (from Coleman-Weinberg formula)

$$V = \lambda^2 \mathcal{S}^2 \left(|\eta|^2 + |\overline{\eta}|^2\right) + |\frac{1}{2}\lambda(\eta^2 - \overline{\eta}^2) - \mu^2|^2$$

$$V_{\text{infla.}} = \lambda^2 \mathcal{S}_c^4 \left(1 + \frac{\lambda^2}{16\pi^2} \log \frac{\mathcal{S}}{\Lambda}\right) \qquad \mu^2 \leq 10^{-6} \left(\frac{50}{N_e}\right)^{1/2}$$

- D-term inflation potentials

$$V = V_F + V_D = \lambda^2 |\mathcal{S}|^2 \left(|\overline{\phi}|^2 + |\phi|^2\right) + \lambda^2 |\overline{\phi}\phi|^2 + \frac{g^2}{2}\left(|\phi|^2 - |\overline{\phi}|^2 + \xi\right)^2$$

$$V_{\text{infla.}} = \frac{\lambda^4 \mathcal{S}_c^4}{2g^2} \left(1 + \frac{g^2}{8\pi^2} \log \frac{\mathcal{S}}{\Lambda}\right) \qquad \xi \leq 10^{-6} \left(\frac{50}{N_e}\right)^{1/2}$$

- End of inflation leads to cosmological defects (strings) of linear energy density μ or ξ

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More fields in the superpotential...

- What is happening if field content is extended?

$$W = -\mu_i^2 S_i + \nu_i S_i^3 + \lambda_j \left(\alpha_i S_i\right)\left(\overline{\Phi}_j \Phi_j\right)$$

with only cubic terms

- Interesting cases are (in context of D-term inflation)
  - $W = \lambda S \overline{\Phi} \Phi + \mu^2 C$ curvaton model
  - $W = \nu_i S_i^3 + \lambda \left(\alpha_i S_i\right) \overline{\Phi} \Phi$ multiple-field inflation

The resulting effective potential

- Model: 2-field inflation,

  $\nu_1 \approx 0,\ \nu_2 \ll 1$ (and $\nu_2 \gg \lambda^2$)

- the effective potential is then the following,

$$V = V_{\text{1-loop}} + \nu_2^2 |\mathcal{S}_2|^4 + \lambda^2 \left|\cos\theta \mathcal{S}_1 + \sin\theta \mathcal{S}_2\right|^2 |\overline{\phi}|^2$$
$$+ \frac{g^2}{2}\left(-|\overline{\phi}|^2 + \xi\right)^2$$

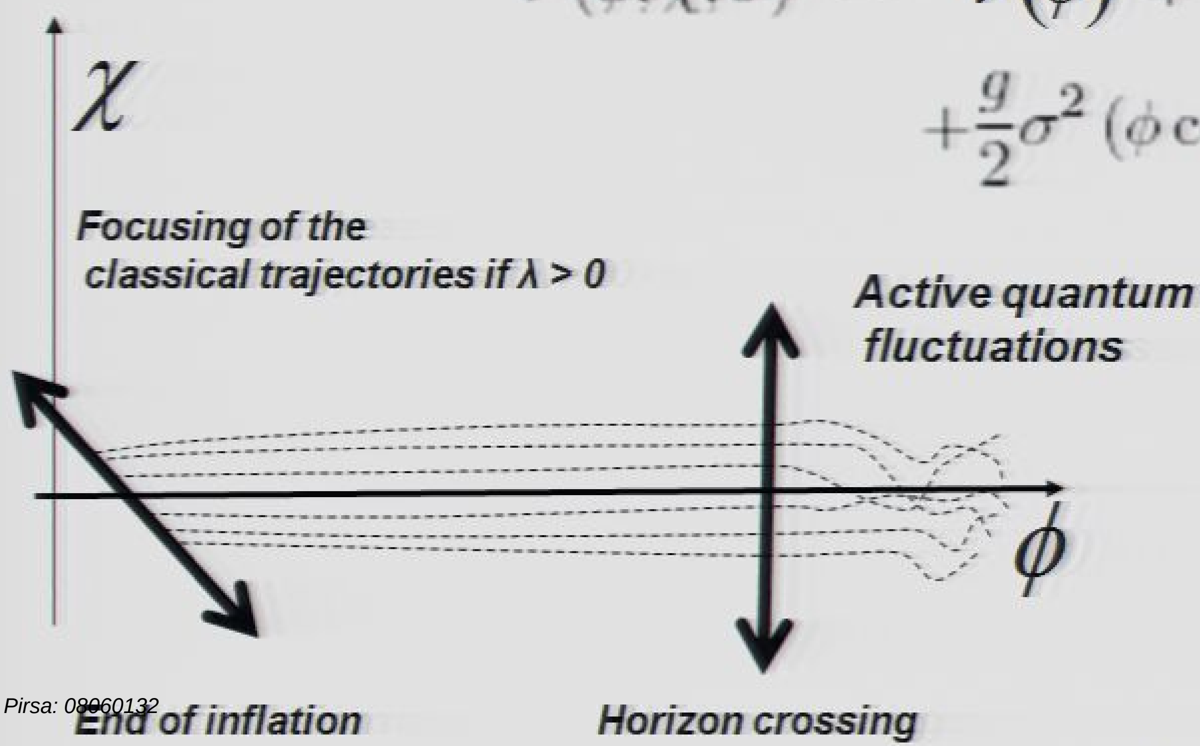
We are left with a model of the form *(FB & Uzan '03)*

The inflaton

A light transverse field

A massive transverse field that eventually undergoes a phase transition

$$V(\phi,\chi,\sigma) = V(\phi) + \frac{\lambda}{4!}\chi^4 + \frac{\mu}{2}\left(\sigma^2 - \sigma_0^2\right)^2 + \frac{g}{2}\sigma^2\left(\phi\cos\alpha + \chi\sin\alpha\right)^2$$



Inflationary period stops at a time that depends on both the φ and χ values

Mode transfers, δN formalism

- At linear order:

$$\delta N = -\left.\frac{3H^2}{V_{,\varphi}}\right|_{\text{Horizon crossing}} \delta\varphi + \left.\frac{3H^2}{V_{,\varphi}}\right|_{\text{end of inflation}} \tan\theta\, \delta\chi_1$$

Standard adiabatic fluctuations

Transfer of isocurvature modes

Coef are assumed to be roughly equal (OK with cosmic string constraints)

evolution of statistical properties of these modes ?

Test (complex) scalar field in (quasi) de Sitter space

- In curvaton model, a free field...
- Impact of significant self coupling ?
  - Is the model viable ? E.g., is the mass protected against radiative corrections ? (Brunier, FB, Uzan, PRD, hep-ph/0412186)
  - What are the effects of quartic self-coupling on the resulting statistical properties of the metric perturbations ?

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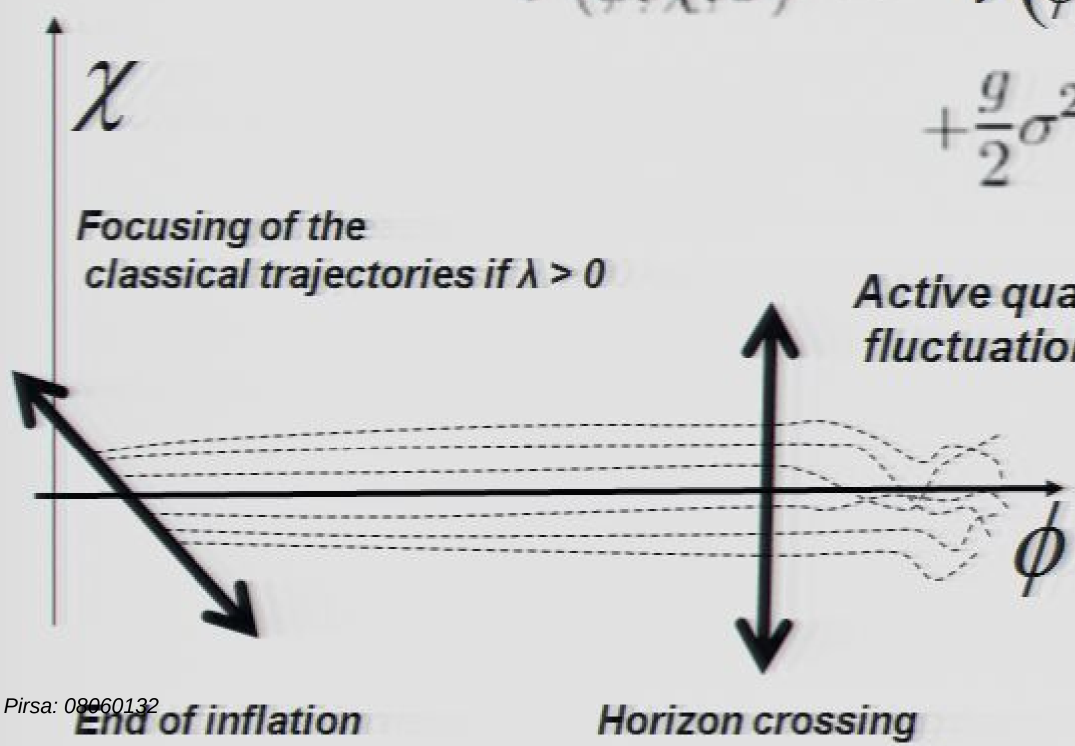

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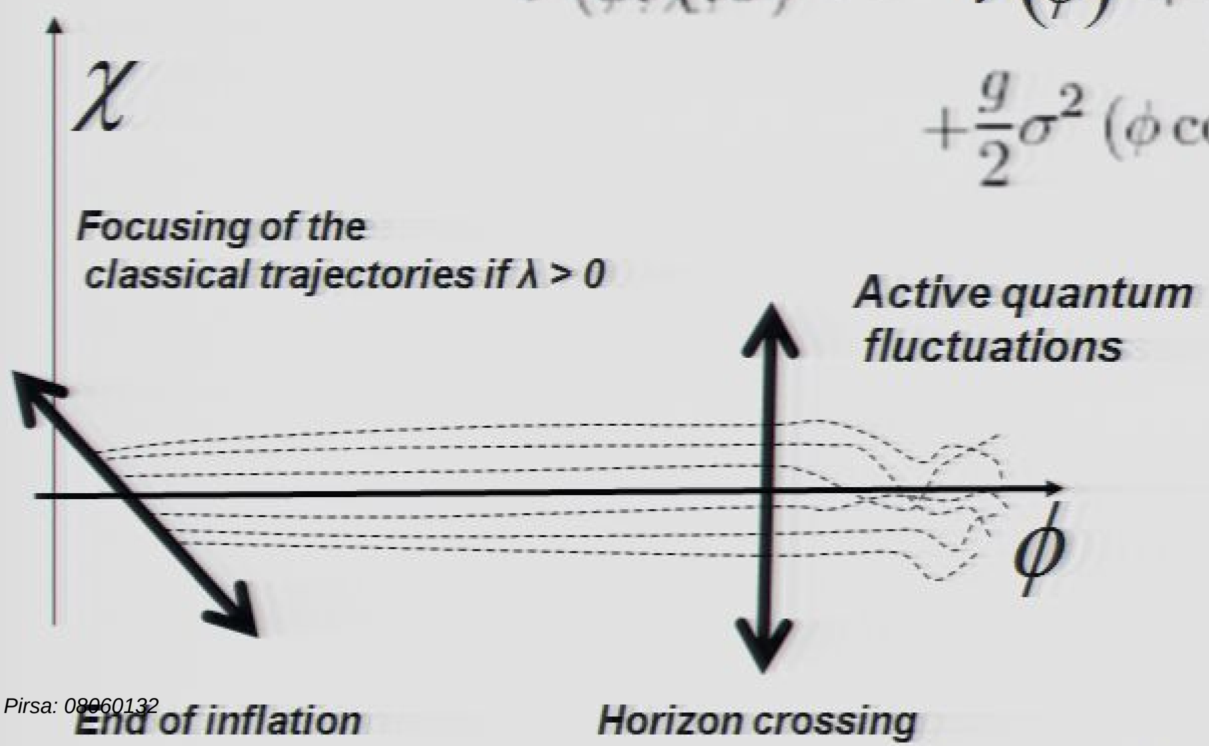

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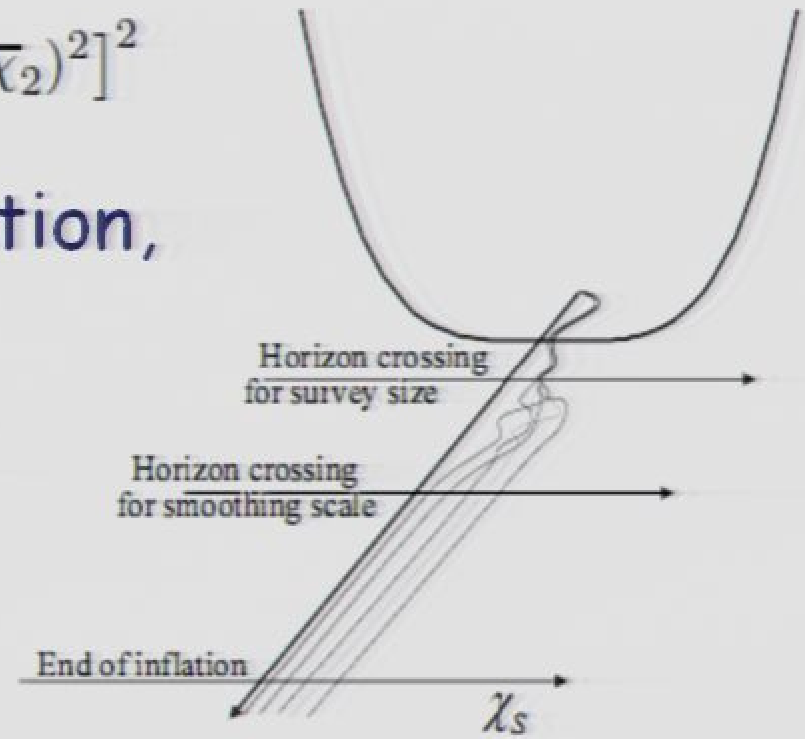
Finite volume effect: an effective potential

- Super-Hubble modes cannot be observed = extra parameters

$$V(\chi_1,\chi_2)=\frac{\nu^2}{4}\left[(\chi_1+\overline{\chi}_1)^2+(\chi_2+\overline{\chi}_2)^2\right]^2$$

- From Fokker-Planck equation,

$$\mathcal{P}(\overline{\chi}_1,\overline{\chi}_2)=\frac{\sqrt{2}\nu}{H^2\sqrt{3\pi}}\exp\left[-\frac{2\pi^2\nu^2\left(\overline{\chi}_1^2+\overline{\chi}_2^2\right)^2}{3H^4}\right]$$



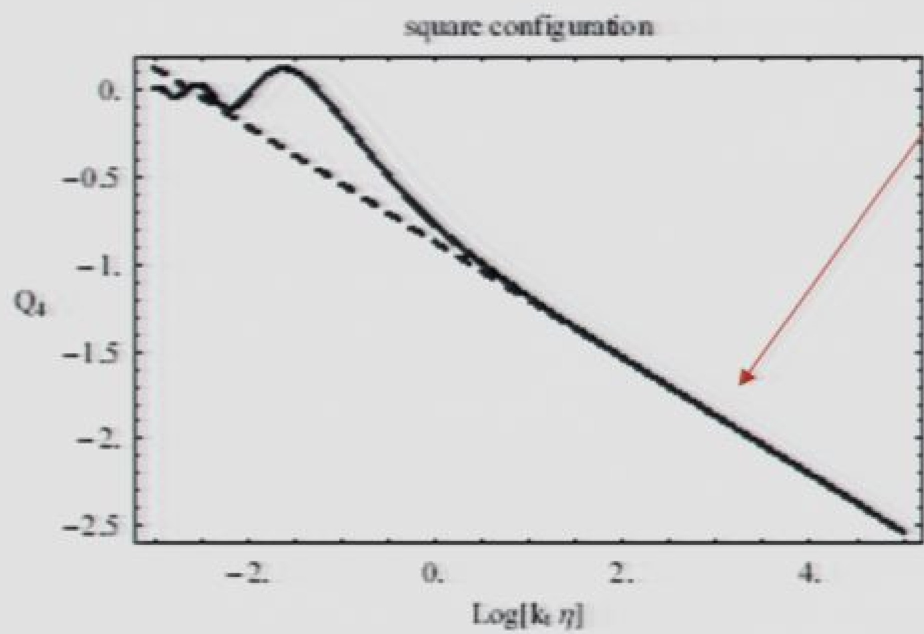
Exact results from quantum theory

For a quartic coupling and in the super-horizon limit
(FB, Brunier & Uzan '03)

$$\langle \chi(\mathbf{k}_1)\dots\chi(\mathbf{k}_4)\rangle_c = -\frac{\lambda H^4}{24}\frac{\delta_{\rm Dirac}(\sum \mathbf{k}_i)}{\prod k_i^3}\left[-\sum k_i^3\left(\gamma+\zeta(\{k_i\})+\log\left[-\eta\sum k_i\right]\right)\right]$$

$= N_{efolds}$

$$Q_4(k_1,k_2,k_3,k_4) = \frac{P_4(k_1,k_2,k_3,k_4)}{P_2(k_1)P_2(k_2)P_2(k_3)+\text{sym.}}$$



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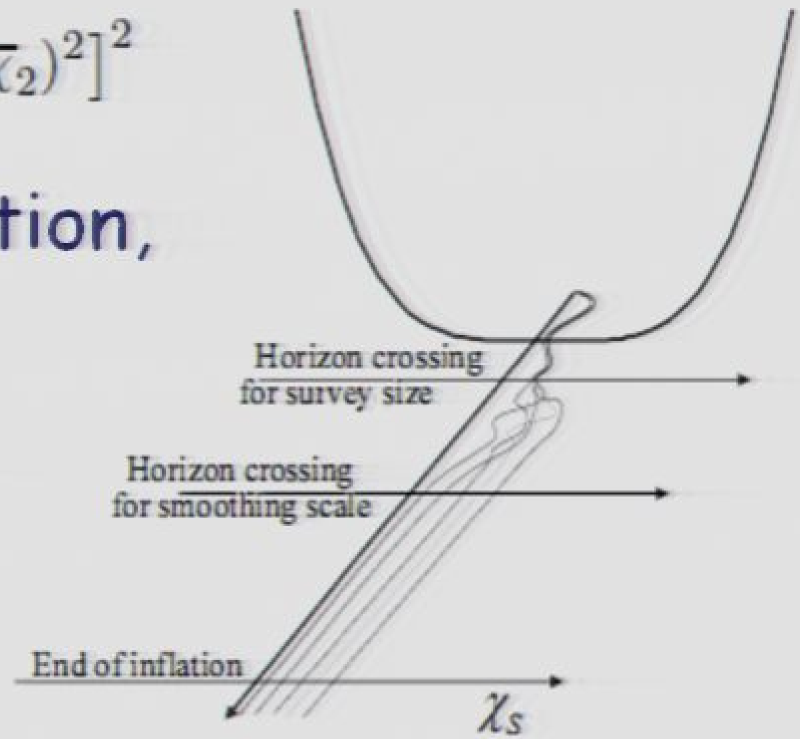

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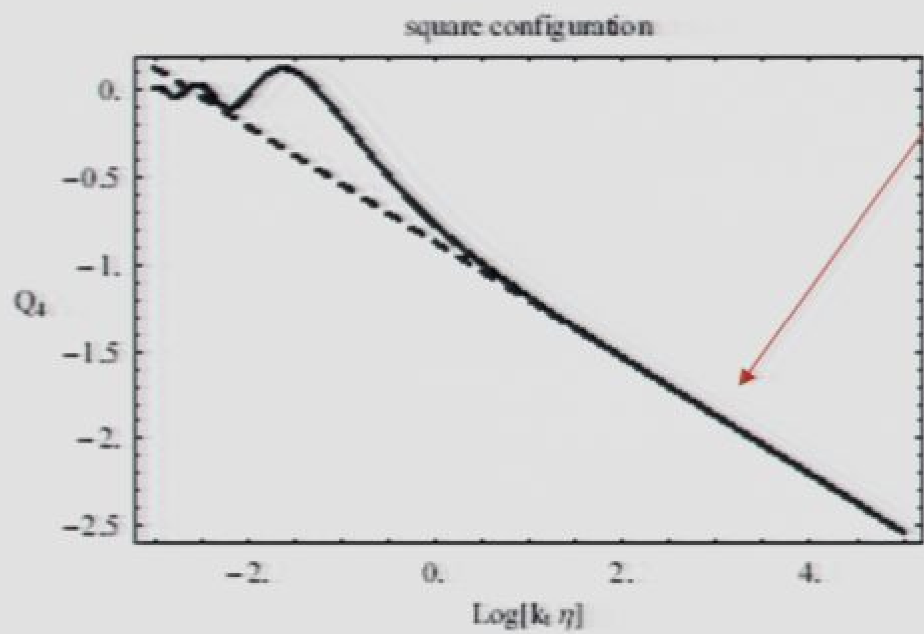

The three- and four-point functions

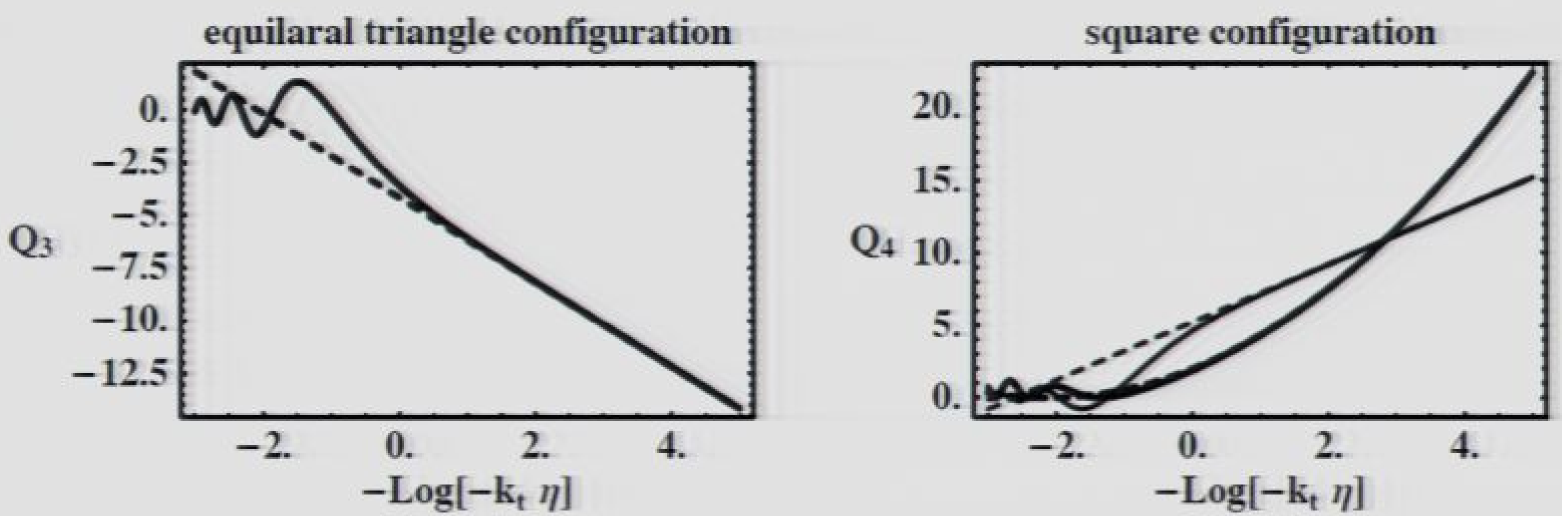

FIG. 5: Behavior of the reduced correlateors, Q_3 and Q_4 of the field χ_1, as a function of $N_e \equiv \log(k_t\eta)$. The function Q_3 (left panel) is to be multiplied by $\nu^2\overline{\chi}_1/H^2$; the function $Q_4^{\rm star}$ (thick lines of right panel) is to be multiplied by $-\nu^2/H^2$ and the function $Q_4^{\rm line}$ (thin lines) is to be multiplied by $2\nu^4(9\overline{\chi}_1^2+\overline{\chi}_2^2)/H^4$. The dashed lines correspond to the corresponding asymptotic behahiors.

Consequences : Bispectrum and trispectrum

- Definition : $\langle \sigma(\mathbf{k}_1)\dots\sigma(\mathbf{k}_n)\rangle_c = \delta_{\rm Dirac}(\mathbf{k}_1+\cdots+\mathbf{k}_n)P_n^\sigma$
- $$P_3^{\delta N}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2f_{\rm NL}\left[P(k_1)P(k_2)+{\rm sym.}\right]$$

$$f_{\rm NL} = -\nu^2 N_e \frac{\overline{\chi}_1}{H}\frac{\sin^3\theta}{2\mathcal{P}_0}$$

$\sim 10^{-4}$

This is to be contrasted with generic inflation : f_{NL} is of order unity

- $$P_4^{\delta N}(\mathbf{k}_1,\dots,\mathbf{k}_4) = 4f_{\rm NL}^2\left(\frac{9\overline{\chi}_1^2+\overline{\chi}_2^2}{9\overline{\chi}_1^2\sin^2\theta}\right)\left[P(k_1)P(|\mathbf{k}_1+\mathbf{k}_2|)P(k_3)+{\rm sym.}\right] + 6g_{\rm NL}\left[P(k_1)P(k_2)P(k_3)+{\rm sym.}\right]$$

$$g_{\rm NL} = -\nu^2 N_e \frac{\sin^4\theta}{12\mathcal{P}_0^2}$$

Exclusion diagrams (FB, Brunier '07)

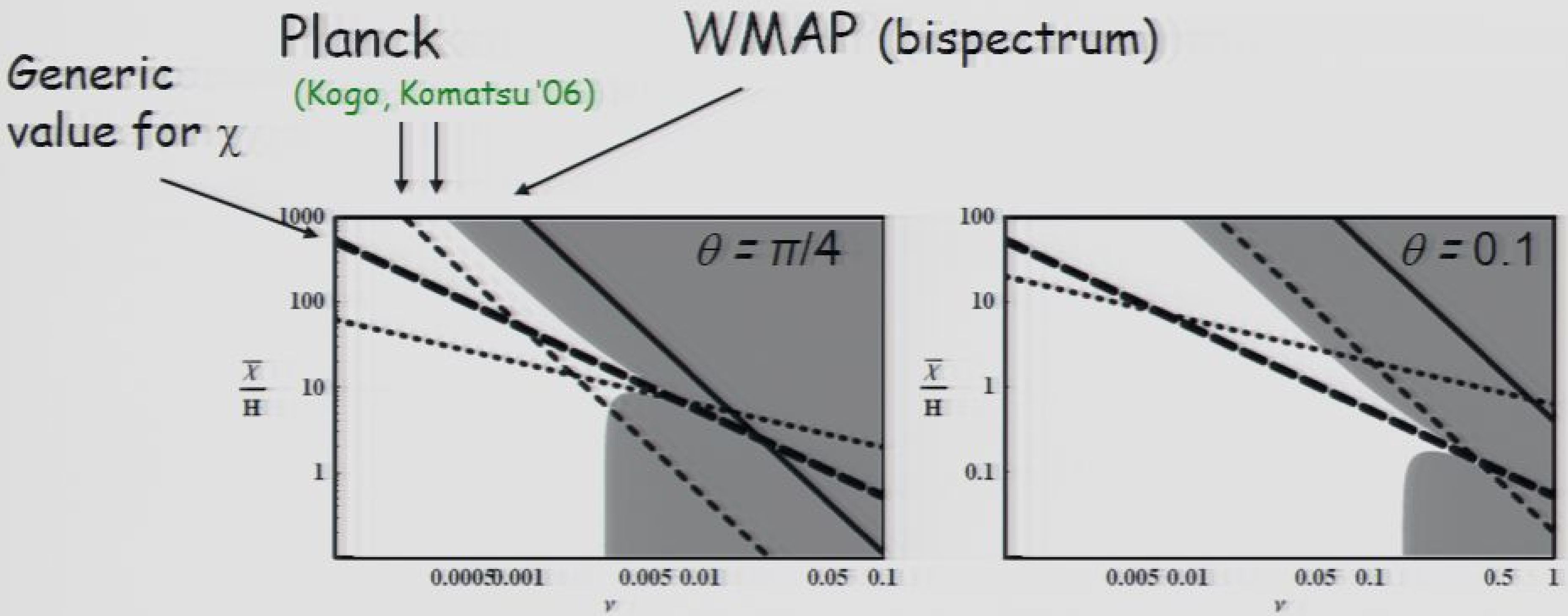

FIG. 3: Exclusion diagrams for parameters ν and $\overline{\chi}$ for $\theta = \pi/4$ (left panel) and for $\theta = 0.1$ (right panel). The locations of the dotted lines where $\overline{\chi}$ is equal to its expected one σ fluctuation. The gray areas and solid or short dashed lines correspond to the exclusion zones, obtained by WMAP (solid line) or expected by Planck (short dashed for bispectrum, gray areas for tri-spectrum). The bispectrum constraint corresponds to a straight line (of slope -2); the trispectrum is more complicated due to two competing terms in the trispectrum. The long dashed is the location where the terms cancel. We adopted the results of [29] on the upperbounds the Planck mission is expected to provide, $f_{NL} = 5$ and $\tau_{NL} = 560$.

Conclusions

- Extended models of hybrid inflation - even with standard kinetic term - can lead to a rich phenomenology ...
  - Breaking of the relation between tensor and scalar metric fluctuations
  - Possibility of having large Non-Gaussian adiabatic fluctuations
  - Specific signatures (SH local couplings) <> prediction with DBI action
- Parameter space is largely open
  - Somewhat different phenomenology when $N_e \nu^2 \gg 1$ and $\nu^2 \ll 1$.

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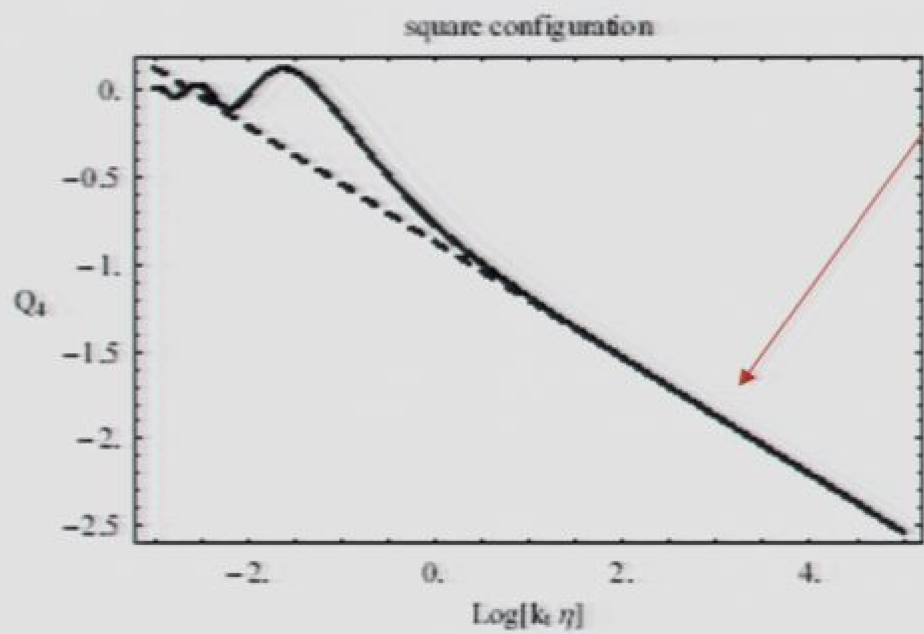

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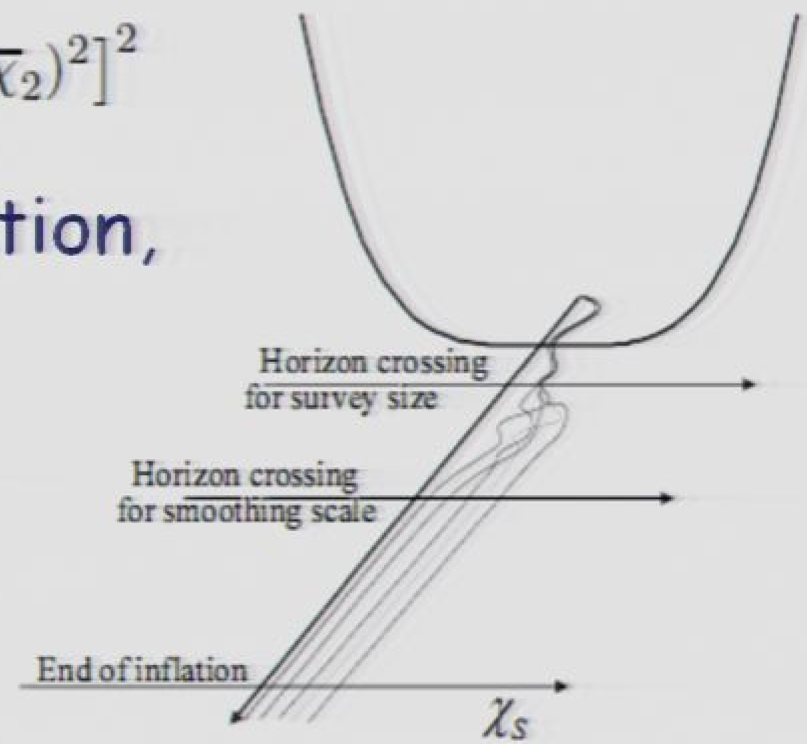

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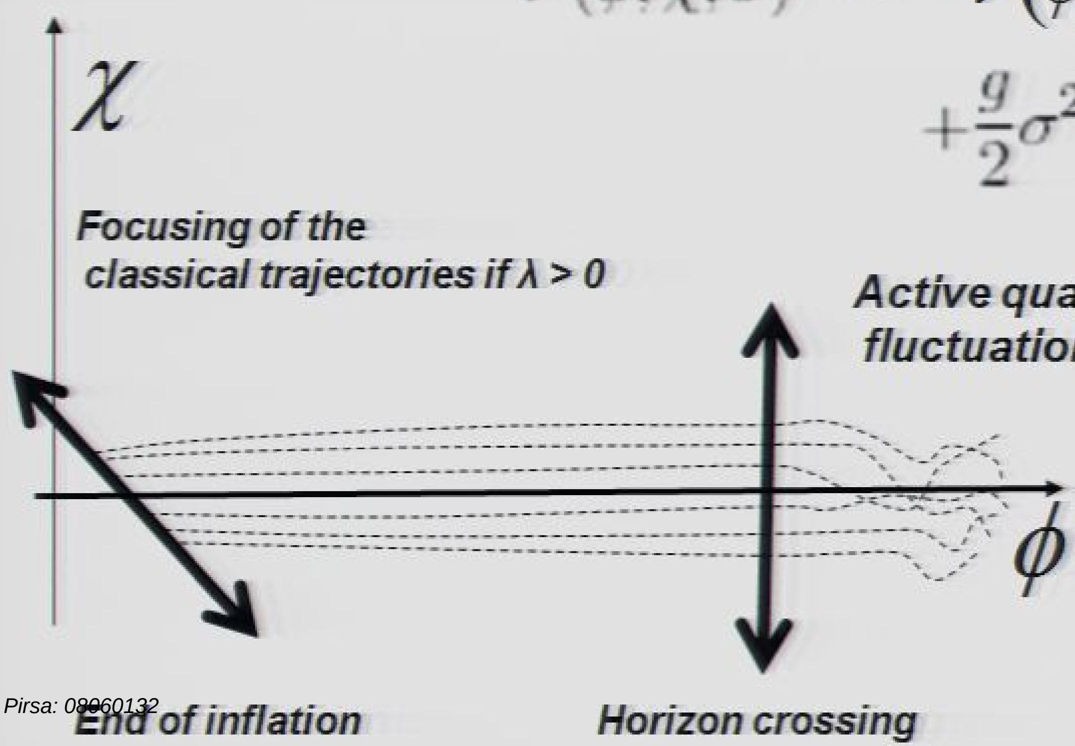

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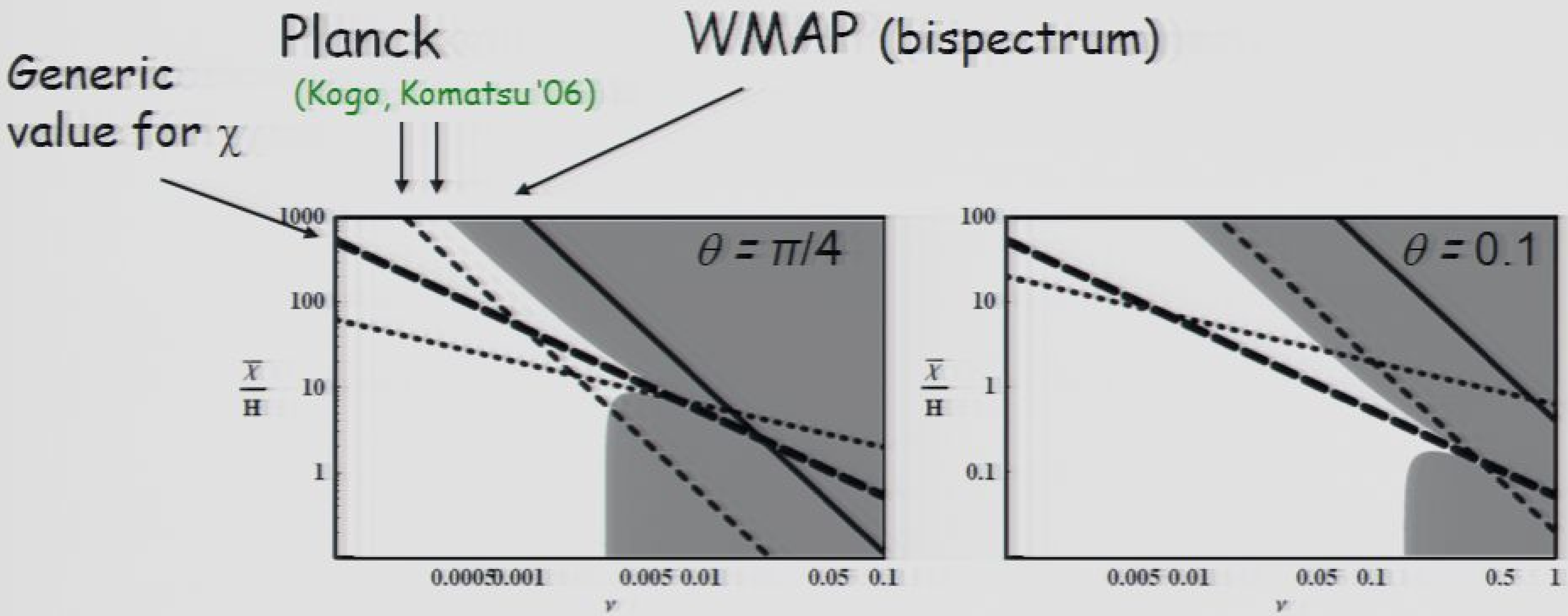

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