Title: Nonlocal Inflation from String Theory

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Abstract: Many string theorists and cosmologists have recently turned their attention to building and testing string theory models of inflation. One of the main goals is to find novel features that could distinguish stringy models from their field theoretic counterparts. This is difficult because, in most examples, string theory is used to derived an effective theory operating at energies well below the string scale. However, since string theory provides a complete description of dynamics also at higher energies, it may be interesting to construct inflationary models which take advantage of this distinctive feature. I will discuss recent progress in this direction using p-adic string theory - a toy model of the bosonic string for which the full series of higher dimensional operators is known explicitly - as a playground for studying string cosmology to all order in $alpha\'$. The p-adic string is a nonlocal theory containing derivatives of all orders and this structure is also ubiquitous in string field theory. After discussing the difficulties (such as ghosts and classical instabilities) that arise in working with higher derivative theories I will show how to construct generic inflationary models with infinitely many derivatives. Novel features include the possibility of realizing slow roll inflation with a steep potential and large nongaussian signatures in the CMB.

References

1. p-adic Inflation, N. Barnaby, T. Biswas & J. Cline, JHEP **0704**, 056; arXiv:hep-th/0612230.
2. Large Nongaussianity from Nonlocal Inflation, N. Barnaby & J. Cline, JCAP **0707**, 017; arXiv:0704.3426.
3. Dynamics with Infinitely Many Derivatives: The Initial Value Problem, N. Barnaby & N. Kamran, JHEP **0802**, 008; arXiv:0709.3968.
4. Predictions for Nongaussianity from Nonlocal Inflation, N. Barnaby & J. Cline; arXiv:0802.3218.
5. Work in progress, N. Barnaby, T. Biswas & J. Cline.
6. Work in progress, N. Barnaby & P. Robinson.

Inflation from String Theory

- Construction of string theory models of inflation has attracted considerable interest.
- A number of interesting scenarios:
  - Brane-antibrane (KKLMMT, Baumann et al. (2006) Burgess et al. (2001), ...)
  - D3/D7 (Dasgupta et al. (2002), ...)
  - Moduli inflation (Racetrack inflation, Roulette inflation, ...)
  - DBI inflation (Silverstein & Tong (2004), ...)
  - Tachyonic inflation (Cremades et al. (2005), ...)
  - . . .
- Model building: crucial to look for distinctly stringy features or observational signatures...

Inflation from String Theory

- Can we distinguish between string theory models and field theoretic counterparts?
- Most examples use string theory to derive a low energy effective action describing dynamics well below m_s:

$$\mathcal{L}_{\mathrm{kklmmt}} \cong -\frac{1}{2}(\partial\phi)^2 - \left[V_0 + \frac{m^2}{2}\phi^2 - \frac{A}{\phi^4} + \cdots\right]$$

- String theory provides a complete description of dynamics also a higher energies.
  - Can we construct models which takes advantage of this?
- Daunting, since QFT description should be supplemented by infinitely many higher dimensional operators...

Example: p-adic String Theory

- Toy model of the bosonic string tachyon[a] for which the full series of higher dimension operators is known explicitly.
- World-sheet coordinates of the string are restricted to the field of p-adic numbers.
- All amplitudes can be computed exactly and are reproduced by the field-theoretic Lagrangian:

$$\mathcal{L} = \frac{m_s^4\, p^2}{g_s^2\,(p-1)}\left[-\frac{1}{2}\phi p^{-\frac{\Box}{2m_s^2}}\phi + \frac{1}{p+1}\phi^{p+1}\right]$$

- Contains infinitely many derivatives: $e^{-\Box} = 1 - \Box + \cdots$
- Playground to study string cosmology to all orders in α'!

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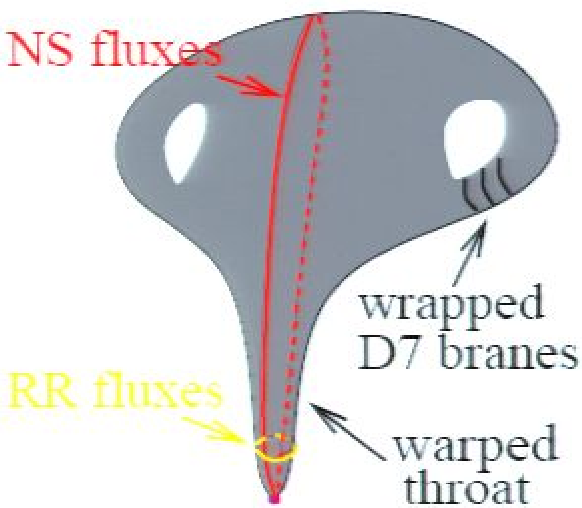
[a] Brekke, Freund, Olson & Witten (1987).

Towards UV Complete Inflation

- Embed inflation into general class of nonlocal theories:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi)$$

  - Includes: p-adic strings, SFT, $\cdots$
- Novelties:
  - Can realize slow roll inflation with a very steep potential.
  - Scenario is predictive, generically have $f_{NL} \gg 1$.
- Note: flat potentials surprisingly hard to obtain in realistic settings. (KKLMMT; Baumann et al. (2007); Burgess, Cline, Firouzjahi, Leblond, Shandera, Tye...)



Problems/Complications

- Difficulties of working with higher derivative theories are well known:[a]
  - Instabilities, ghosts, $\cdots$
  - Difficulties in setting up IVP.
- Any application to physics must address fundamental issues:
  - When can nonlocal theories be ghost-free?[b]
  - Can one make rigorous sense of the IVP in infinite order theories?[c]
- Before discussing cosmology need to make a detour to discuss formalism...

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[a] Woodard (1989).

[b] NB, Biswas & Cline (2008).

[c] NB, Kamran (2007).

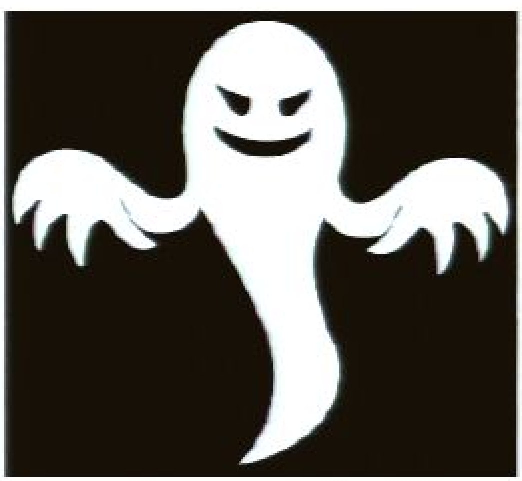
Finite High Derivative Corrections

- ⋆ Example: Lee-Wick model (1969):

$$\mathcal{L}_{LW} = \frac{1}{2}\phi\Box\phi - \frac{1}{2M^2}\phi\Box^2\phi - \frac{1}{2}m^2\phi^2 + \cdots$$

- ⋆ Propagator has two poles $\Rightarrow$ two physical states!

$$G(p^2) \propto \frac{1}{-p^2 - p^4/M^2 - m^2} \sim \underbrace{\frac{1}{-p^2 - m^2}}_{\text{reg. mass } m} - \underbrace{\frac{1}{-p^2 - M^2}}_{\text{ghost. mass } M}$$



- ⋆ Ghost = excitation with wrong-sign kinetic term
- ⋆ Hamiltonian is unbounded from below $\Rightarrow$ system is unstable!

Counting Initial Data

- Problem is VERY general:

$$S = S\left[\phi, \Box\phi, \cdots, \Box^N \phi\right]$$

$$\begin{aligned} N &= (\text{num poles in propagator}) \\ &= (\text{num physical states}) \\ &= \frac{1}{2}(\text{num initial data}) \\ &= \frac{1}{2}(\text{dim phase space}) \end{aligned}$$

- Ostrogradksi Theorem: Hamiltonian is always unstable, except in the local case $N = 1$.
- What about $N = \infty$?

Infinite Order Theories

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi)$$

$$F(z) = \sum_{n=0}^{\infty} a_n z^n$$

- EOM is infinite order. How many initial data?

$$F(\Box)\phi = V'(\phi)$$

- Stability intimitely related to initial data counting.
  - Infinite order PDEs fundamentally different from $N \gg 1$, more like integral eqns.
- Formal treatment of IVP infinite order PDEs.[a]
  - Systematic prescription for counting data.
  - Rigorous study of $f(\partial_t)$ operators.

[a] NB & Kamran (2007).

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Pole Counting in Nonlocal Theories

- Consider nonlocal theory:

$$\mathcal{L} = \frac{1}{2}\phi\,\Gamma(\Box)(\Box - m^2)\,\phi$$

with $\Gamma(z)$ having no zeroes (eg $\Gamma(z) \sim e^{-z}$).

- EOM:

$$\Gamma(\Box)\left(\Box - m^2\right)\phi = 0$$

- Propagator has only one pole:

$$G(p^2) \sim \frac{1}{\Gamma(-p^2)}\frac{1}{-p^2 - m^2}$$

- Only a single physical excitation $\Rightarrow$ only 2 initial data!
- Ostrogradski construction doesn't apply.

Infinite Derivative Dynamics

- Theorem: pole counting exhausts all allowed initial data.[a]
  - Single pole infinite order theories can be ghost-free!
  - Even some multi-pole theories are okay.[b]
  - Formalism gives prescription to project ghosts out of otherwise pathological theories.

[a] NB & Kamran (2007).

[b] NB, Biswas, Cline, Prokushin (in prog).

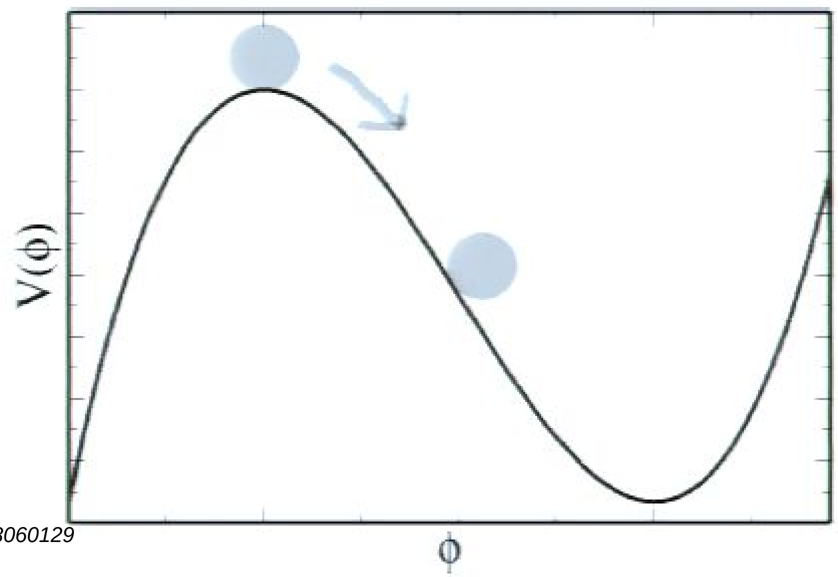
Nonlocal Hill-Top Inflation

- Seek inflation in theories of the form:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - U(\phi)$$

$$F(z) = \sum_{n=1}^{\infty} c_n z^n$$

$$U(\phi) = U_0 - \frac{\mu^2}{2}\phi^2 + \frac{g}{3}\phi^3 + \cdots$$



- Seek inflationary solution rolling away from $\phi = 0$.
- In string theory examples corresponds to inflation during brane decay.

Naive Derivative Truncation

- Naively expect that during slow roll high derivative corrections are negligible:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\phi F(\Box)\phi - U(\phi) \\ &\cong -\frac{1}{2}(\partial\phi)^2 - U_0 + \frac{\mu^2}{2}\phi^2 + \mathcal{O}(\Box^2) + \cdots \end{aligned}$$

- Expect that inflation is only possible when $|\eta| \sim M_p^2|U''/U| \ll 1 \Rightarrow \mu^2 \ll H^2$.
- Naive picture is not always correct: can still obtain slow roll even when $M_p^2|U''/U| \gg 1$!
- Most models of string cosmology follow this approach...

Nonlocal Dynamics

Near the top of the potential ($\phi = 0$) have:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \left(U_0 - \frac{\mu^2}{2}\phi^2 + \cdots\right)$$

- Equation of motion:

$$F(\Box)\phi = -\mu^2\phi$$

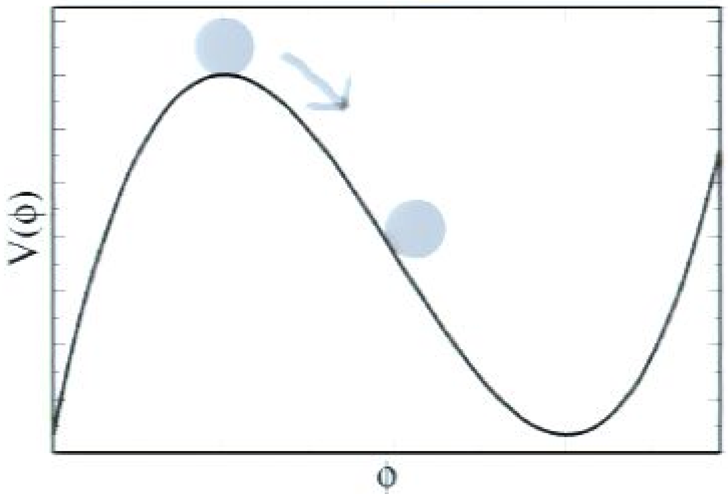
- Can obtain solution by taking:

$$\Box\phi = -\omega^2\phi \quad \text{if} \quad F(-\omega^2) = -\mu^2$$

- Dual to a local theory with mass ω.
- The effective mass, ω^2, can be small even naive mass μ^2 is large!

Stretching the Inflaton Potential

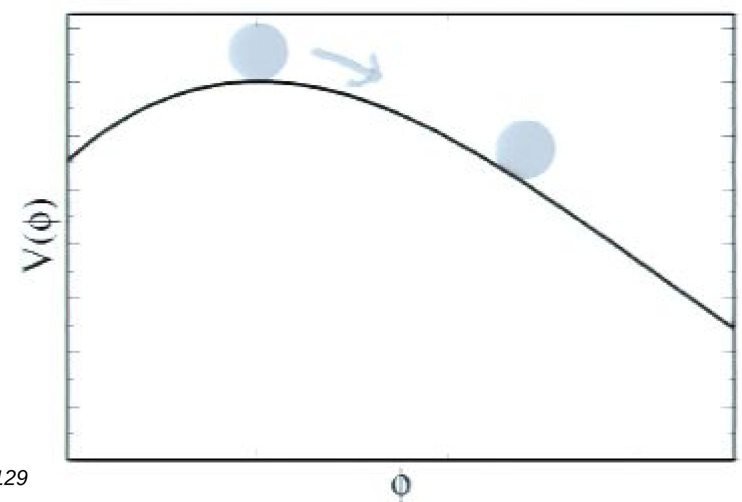
$$\mathcal{L} = \left[\frac{1}{2}\phi F\left(\Box\right)\phi - \left(U_0 - \frac{\mu^2}{2}\phi^2 + \cdots\right)\right]$$



$$\mathcal{L} = \frac{1}{2}\phi\Box\phi - U(\phi) + \mathcal{O}(\Box^2)$$

$$U(\phi) = U_0 - \frac{\mu^2}{2}\phi^2 + \cdots$$

Steep potential, higher derivative terms slow the rolling.



$$\mathcal{L}_{\rm dual} = \frac{1}{2}\varphi\Box\varphi - V(\varphi)$$

$$V(\varphi) = U_0 - \frac{\omega^2}{2}\varphi^2 + \cdots$$

Effective potential in dual local theory is stretched.[a]

[a] Lidsey (2007).

Example: p-adic Inflation

Explicit example in p-adic string theory:[a]

$$\mathcal{L} = \frac{m_s^4}{g_p^2}\left[\frac{1}{2}\phi\left(1-p^{-\frac{\Box}{2m_s^2}}\right)\phi - U(\phi)\right]$$

$$U(\phi) = \underbrace{\frac{p-1}{2(p+1)}}_{\equiv U_0} - \underbrace{\frac{p-1}{2}\phi^2}_{\mu^2 \equiv p-1 \gg 1} + \cdots$$

- ⋆ Inflation proceeds in the strongly nonlocal regime $p \gg 1$, $H > m_s$.
- ⋆ Slow roll dynamics persists even for $|\eta| \sim 10^{11}$!
- ⋆ Predictions: $n_s < 1$, $r < 0.006$, $m_s < 10^{-6} M_p$, nongaussianity$\cdots$

[a] NB, Biswas & Cline (2006).

Including Interactions

Include the cubic term in the action:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - U(\phi)$$

$$U(\phi) = U_0 - \frac{\mu^2}{2!}\phi^2 + \frac{g}{3!}\phi^3 + \cdots$$

- ⋆ For $g \neq 0$ the correspondence between local and nonlocal theories breaks down.
- ⋆ Expect $\langle\phi^3\rangle \propto f_{NL} \propto g$ so for large g the nongaussianity could be large.
- ⋆ In conventional models $g \gg 1$ would spoil inflaton but this need not be true in nonlocal theories!
- ⋆ In p-adic inflation:

$$|g| \sim p^2 \gg 1 \quad \text{for} \quad p \lesssim 10^{13}$$

Field Redefinitions

For ghost-free theory:

$$\mathcal{L} = \frac{1}{2}\phi\Gamma(\Box)(\Box + \omega^2)\phi - U_0 - \frac{g}{3!}\phi^3 + \cdots$$

(where $\Gamma(z)$ has no zeroes).

- Nonlocal field redef $\varphi = \Gamma(\Box)^{1/2}\phi$ gives

$$\mathcal{L} = \frac{1}{2}\varphi(\Box + \omega^2)\varphi - U_0 - \frac{g}{3!}\left(\Gamma(\Box)^{-1/2}\varphi\right)^3 + \cdots$$

- Canonical kinetic structure, nonlocality in the interactions.
- Appropriate starting point to match onto standard perturbation theory calculation.
- Use Seery, Malik & Lyth (2008) formalism for computing f_{NL} from field eqns.

Perturbed Field Equations

Canonical field equation:

$$(\Box + \omega^2)\varphi = \frac{g}{2}\Gamma(\Box)^{-1/2}\left[\Gamma(\Box)^{-1/2}\varphi\right]^2$$

- Gaussian perturbations:

$$(\Box + \omega^2)\delta_1\varphi \cong 0$$

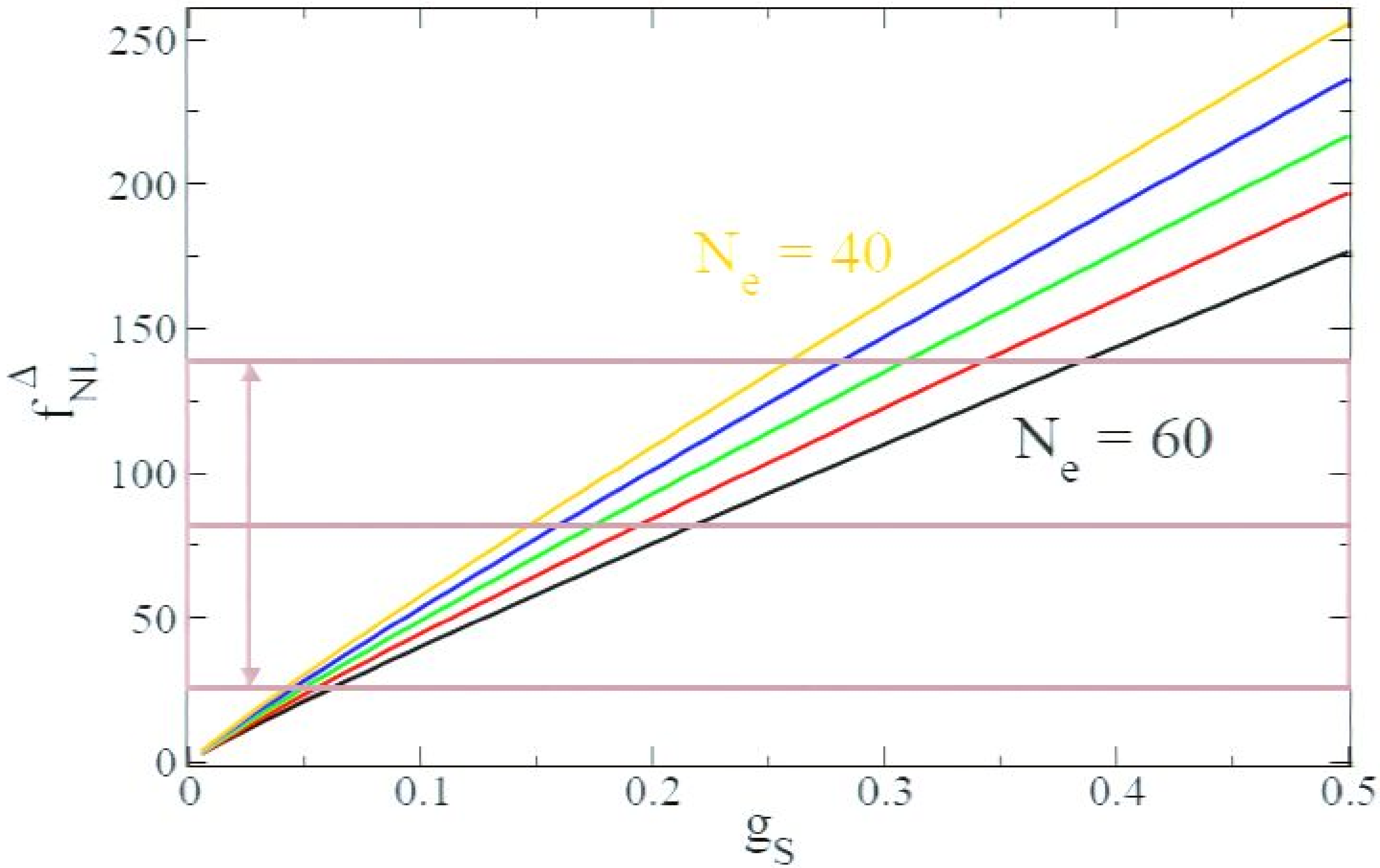
  insensitive to nonlocality.

- Second order:

$$(\Box + \omega^2)\delta_2\varphi \cong \frac{g}{2}\Gamma(\Box)^{-1/2}\left[\Gamma(\Box)^{-1/2}\delta_1\varphi\right]^2 + \cdots$$

- Nonlocal structure in source term mimics a large cubic coupling V''', leads to $f_{NL} \sim 10^2 \gg 1$!

p-adic Inflation



- ⋆ For natural values $g_s \sim 0.1 - 0.3$ reproduce central value for Yadav & Wandelt detection.

Conclusions

- ⋆ Study of high derivative theories is well-motivated.
- ⋆ Provides a playground for studying string cosmology to all orders in α'.
- ⋆ Novelties:
  - Nonlocal inflation procees even when the potential is steep.
  - Large nongaussianity in the CMB.
- ⋆ Rich mathematical structure...
- ⋆ Possibility of realizing similar phenomena in more realistic string theories.
  - Effect relies on UV completion: CMB as a probe of distinctly stringy phenomena!

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