Title: The lessons from the running of the tensor-to-scalar ratio

Date: Jun 06, 2008 11:15 AM

URL: http://pirsa.org/08060098

Abstract: We derive a simple consistency relation from the running of the tensor-to-scalar ratio. This new relation is first order in the slow-roll approximation. While for single field models we can obtain what can be found by using other observables, multi-field cases in general give non-trivial contributions dependent on the geometry of the field space and the inflationary dynamics, which can be probed observationally from this relation.

Outline

1. Introduction
2. Running of the tensor-to-scalar ratio
3. Implications
   - Single field models
   - Multi field models
     - Curved field space
     - Non-trivial field dynamics
4. Conclusions

Why gravitational waves (GWs)?

During inflation

$$\begin{cases} \delta\phi & \rightarrow & \text{scalar perturbations} \\ \delta g_{ij} & \rightarrow & \text{tensor perturbations, which appears as GWs} \end{cases}$$

GWs: CMB up to $l \lesssim 100$, B mode polarization

Why gravitational waves (GWs)?

During inflation

$$\begin{cases} \delta\phi & \rightarrow \quad \text{scalar perturbations} \\ \delta g_{ij} & \rightarrow \quad \text{tensor perturbations, which appears as GWs} \end{cases}$$

GWs: CMB up to $l \lesssim 100$, B mode polarization

Q1: Why are GWs increasingly important?

Q2: How can we make a complete use of r?

Why gravitational waves (GWs)?

During inflation

$$\begin{cases} \delta\phi & \rightarrow \quad \text{scalar perturbations} \\ \delta g_{ij} & \rightarrow \quad \text{tensor perturbations, which appears as GWs} \end{cases}$$

GWs: CMB up to $l \lesssim 100$, B mode polarization

Q1: Why are GWs increasingly important?

- $\mathscr{P}_T$, or $r \equiv \mathscr{P}_T / \mathscr{P}_S$, **directly** determines $V_{\rm inf}$
- Crucial test for many inflation models
  (current observations, e.g. WMAP5: $r < 0.2$ with 95% C.L.)

Q2: How can we make a complete use of r?

- A new consistency relation
- Important information

A new consistency relation

We have the following set of results:

		power spectrum	spectral index
tensor		$\mathscr{P}_{\rm T} = (H/2\pi)^2 8/m_{\rm Pl}^2$	$\mathscr{P}_{\rm T} \propto k^{n_{\rm T}}$
scalar	single	$\mathscr{P}_{\rm S} = (H/2\pi)^2 (H/\dot{\phi})^2$	$\mathscr{P}_{\rm S} \propto k^{n_{\rm S}-1}$
	multi	$\mathscr{P}_{\rm S} = (H/2\pi)^2 h^{ij} N_{,i} N_{,j}$	

A new consistency relation

We have the following set of results:

		power spectrum	spectral index
tensor		$\mathscr{P}_{\rm T} = (H/2\pi)^2 8/m_{\rm Pl}^2$	$\mathscr{P}_{\rm T} \propto k^{n_{\rm T}}$
scalar	single	$\mathscr{P}_{\rm S} = (H/2\pi)^2 (H/\dot{\phi})^2$	$\mathscr{P}_{\rm S} \propto k^{n_{\rm S}-1}$
	multi	$\mathscr{P}_{\rm S} = (H/2\pi)^2 h^{ij} N_{,i} N_{,j}$	

$$r \equiv \frac{\mathscr{P}_{\rm T}}{\mathscr{P}_{\rm S}} \propto k^{1-n_{\rm S}+n_{\rm T}}$$

A new consistency relation

We have the following set of results:

		power spectrum	spectral index
tensor		$\mathscr{P}_{\rm T} = (H/2\pi)^2 8/m_{\rm Pl}^2$	$\mathscr{P}_{\rm T} \propto k^{n_{\rm T}}$
scalar	single	$\mathscr{P}_{\rm S} = (H/2\pi)^2 (H/\dot{\phi})^2$	$\mathscr{P}_{\rm S} \propto k^{n_{\rm S}-1}$
	multi	$\mathscr{P}_{\rm S} = (H/2\pi)^2 h^{ij} N_{,i} N_{,j}$	

$$r \equiv \frac{\mathscr{P}_{\rm T}}{\mathscr{P}_{\rm S}} \propto k^{1-n_{\rm S}+n_{\rm T}}$$

We can obtain **a new consistency relation**

$$\frac{d\log r}{d\log k} = 1 - n_{\rm S} + n_{\rm T}$$

First order quantity in the slow-roll parameters: **observable**

Running of r in single field models

- For single field inflation models

$$n_S - 1 = -6\epsilon + 2\eta$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

A new consistency relation

We have the following set of results:

		power spectrum	spectral index
tensor		$\mathcal{P}_{\rm T} = (H/2\pi)^2 8/m_{\rm Pl}^2$	$\mathcal{P}_{\rm T} \propto k^{n_{\rm T}}$
scalar	single	$\mathcal{P}_{\rm S} = (H/2\pi)^2 (H/\dot{\phi})^2$	$\mathcal{P}_{\rm S} \propto k^{n_{\rm S}-1}$
	multi	$\mathcal{P}_{\rm S} = (H/2\pi)^2 h^{ij} N_{,i} N_{,j}$	

$$r \equiv \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_{\rm S}} \propto k^{1-n_{\rm S}+n_{\rm T}}$$

We can obtain a new consistency relation

We can obtain **a new consistency relation**

$$\frac{d\log r}{d\log k} = 1 - n_{\rm S} + n_{\rm T}$$

First order quantity in the slow-roll parameters: **observable**

Running of r in single field models

- For single field inflation models

$$n_S - 1 = -6\epsilon + 2\eta$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

Running of r in single field models

- For single field inflation models

$$n_S - 1 = -6\epsilon + 2\eta$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

- We can read

$$\frac{d \log r}{d \log k} = 2(2\epsilon - \eta)$$

Running of r in single field models

- For single field inflation models

$$n_S - 1 = -6\epsilon + 2\eta$$

where

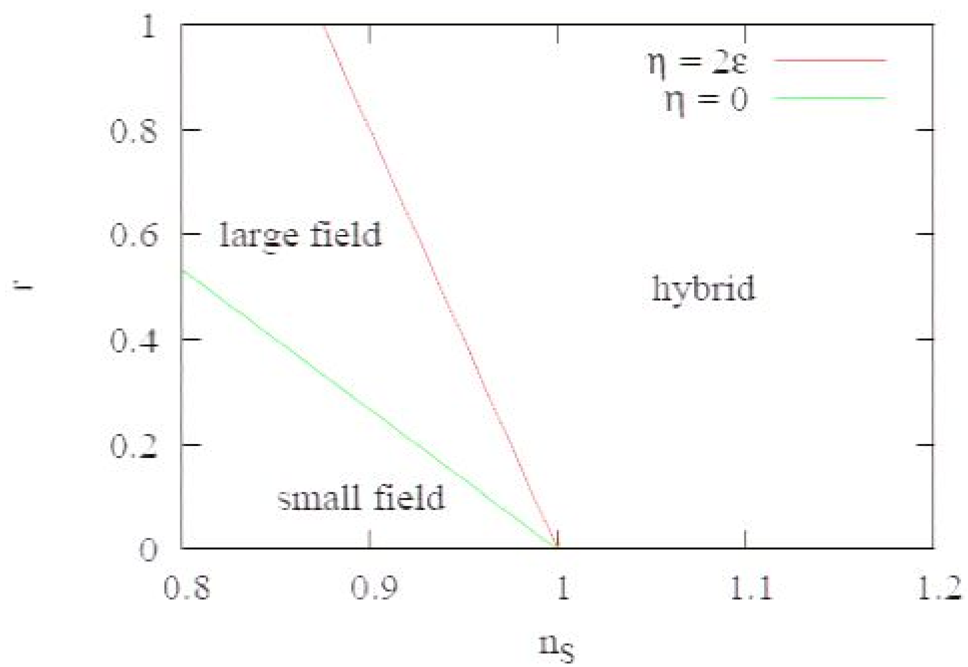
$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

- We can read

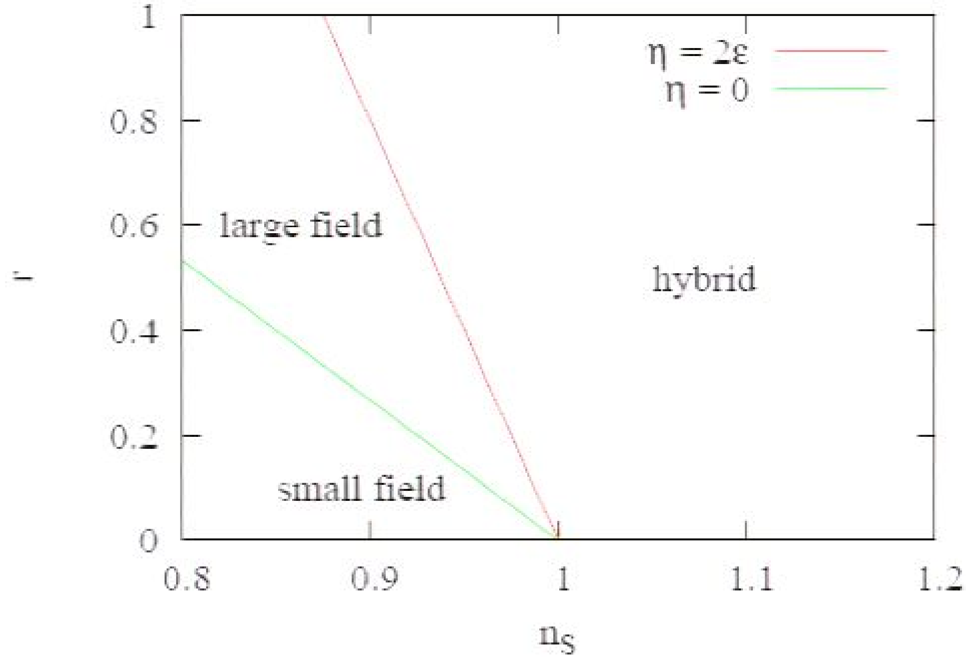
$$\frac{d\log r}{d\log k} = 2(2\epsilon - \eta)$$

- Zoology of inflation models: positive or negative

Zoology of inflation



Zoology of inflation



Exactly the same information by measuring $\mathscr{P}_S$, $\mathscr{P}_T$ (or r) and n_S $\rightarrow$ consistency check, but **NOTHING NEW**

Spectral index n_S in multi field models

$$n_S - 1 = -2\epsilon - \frac{r}{4} + 2\eta_{\rm multi} - 2\frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{R^{i}{}_{ab}{}^{j}}{3m_{\rm Pl}^2}\frac{\dot\phi^a\dot\phi^b}{H^2}$$

$$N \equiv \int H dt$$

$$N_{,i} \equiv \frac{\partial N}{\partial \phi^i}$$

$$R^{i}{}_{jkl} \equiv \Gamma^{i}_{jk,l} - \Gamma^{i}_{jl,k} + \Gamma^{m}_{jk}\Gamma^{i}_{lm} - \Gamma^{m}_{jl}\Gamma^{i}_{km}$$

$$\Gamma^{i}_{jk} \equiv \frac{1}{2}h^{il}\left(h_{jl,k} + h_{kl,j} - h_{jk,l}\right)$$

$$\eta_{\rm multi} \equiv m_{\rm Pl}^2 \frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{V^{;ij}}{V}$$

Running of r in multi field models

$$\frac{d\log r}{d\log k} - \frac{r}{4} = -2\eta_{\rm multi} + 2\frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{R^{i}{}_{ab}{}^{j}}{3m_{\rm Pl}^2}\frac{\dot\phi^a\dot\phi^b}{H^2}$$

Running of r in multi field models

$$\frac{d\log r}{d\log k} - \frac{r}{4} = -2\eta_{\text{multi}} + 2\frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{R^{i}{}_{ab}{}^{j}}{3m_{\text{Pl}}^{2}}\frac{\dot\phi^{a}\dot\phi^{b}}{H^{2}}$$

1. We **CANNOT** extract the same information with only $\mathscr{P}_{\text{S}}$, $\mathscr{P}_{\text{T}}$ and n_{S}
2. On the LHS are only observable quantities: can be determined by near future experiments
3. Contributions from curved field space and/or from non-trivial dynamics

When the field space is curved···

$$R^{i}{}_{ab}{}^{j} \neq 0 : \text{curved field space}$$

When the field space is curved$\cdots$

$$R^{i}{}_{ab}{}^{j} \neq 0 : \text{curved field space}$$

Consider the Kähler potential

$$\begin{aligned} m_{\rm Pl}^{-2} K &= -3 \log(T + T^*) \\ &= -3 \log\left[(X + iY) + (X + iY)^*\right] \end{aligned}$$

$$\text{Curved: } R^{X}{}_{YXY} = R^{Y}{}_{XYX} = \frac{1}{X^2} \neq 0$$

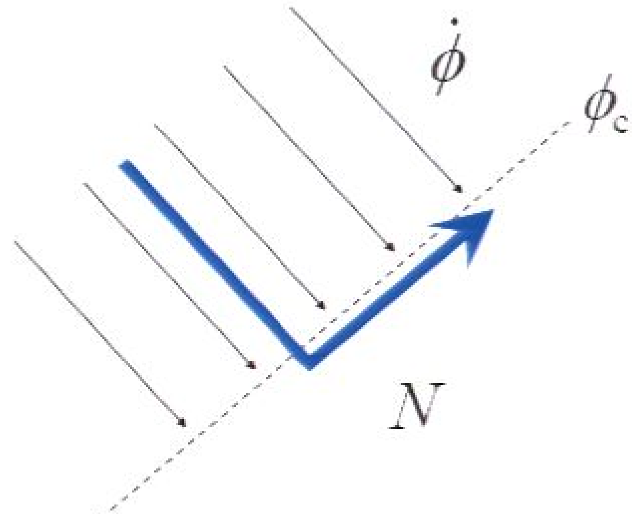
cf. Typically $X \gtrsim \mathcal{O}(10)$: observable?

When the field space is flat$\cdots$

Non-trivial contribution even if $h_{ij} = \delta_{ij}$

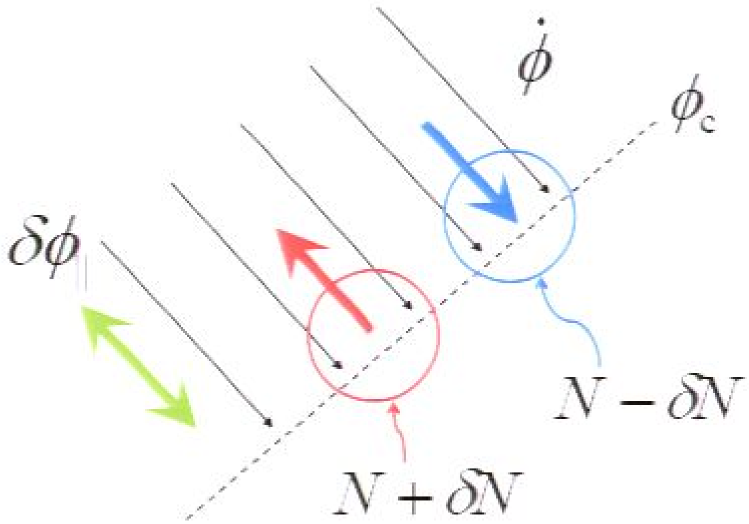
When the field space is flat···

Non-trivial contribution even if $h_{ij} = \delta_{ij}$



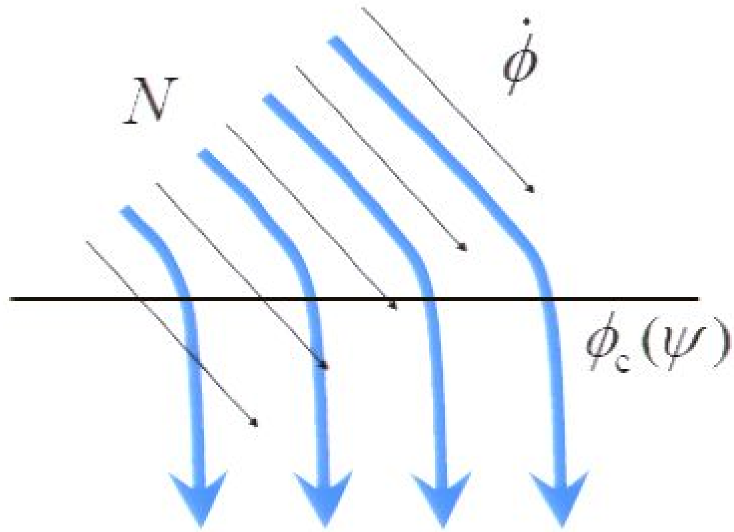
When the field space is flat···

Non-trivial contribution even if $h_{ij} = \delta_{ij}$



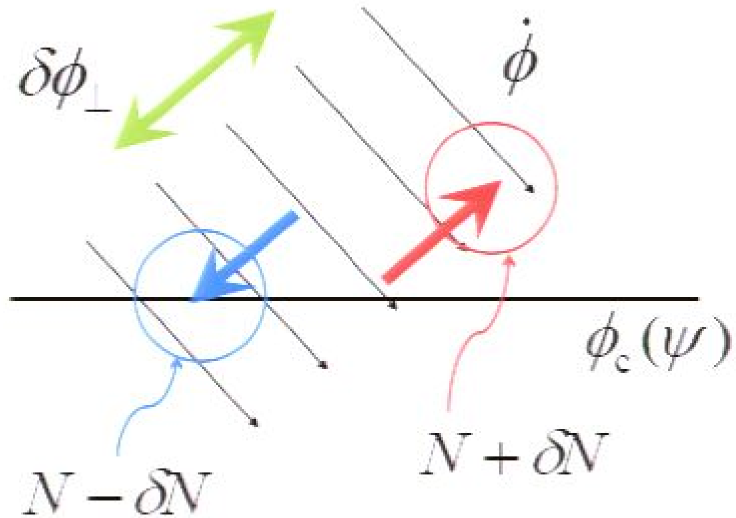
When the field space is flat···

Non-trivial contribution even if $h_{ij} = \delta_{ij}$



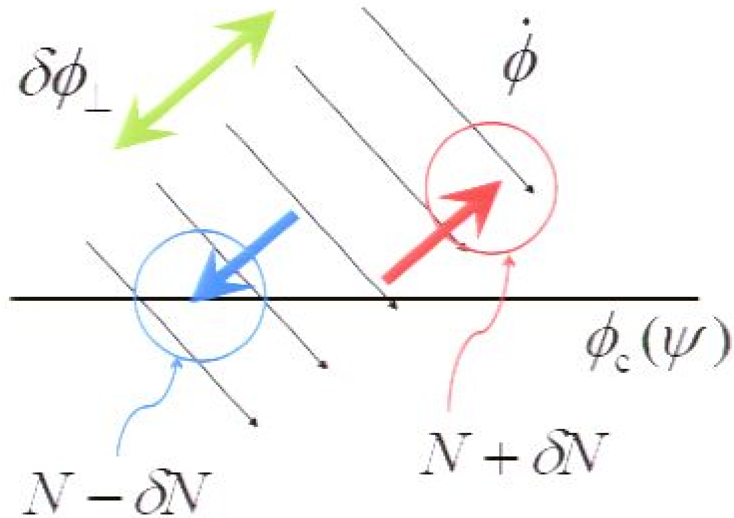
When the field space is flat···

Non-trivial contribution even if $h_{ij} = \delta_{ij}$



When the field space is flat···

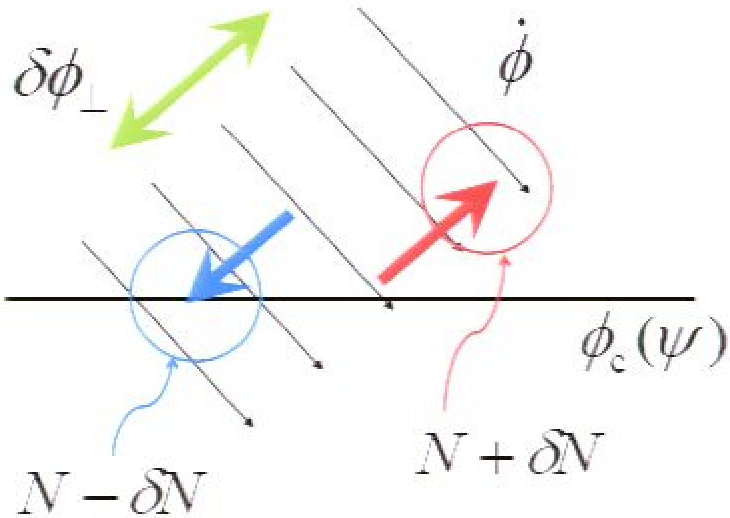
Non-trivial contribution even if $h_{ij} = \delta_{ij}$



$V^{;ij}$ is contracted with $N_{,i}N_{,j} \rightarrow$ *projection* operator

When the field space is flat···

Non-trivial contribution even if $h_{ij} = \delta_{ij}$



$V^{,ij}$ is contracted with $N_{,i}N_{,j}$ → *projection* operator

$$dN = Hdt \implies N_{,i}\dot{\phi}^i = \nabla N \cdot \dot{\Phi} = H$$

This **NEVER** necessarily means that the gradient of N is aligned in a specific manner with respect to the velocity of the field

Conclusions

- A new consistency relation from the running of r

- For single field models

- For multi field models

Conclusions

- A new consistency relation from the running of r

$$\frac{d\log r}{d\log k} = 1 - n_{\mathrm{S}} + n_{\mathrm{T}}$$

- For single field models

- For multi field models

Conclusions

- A new consistency relation from the running of r

$$\frac{d\log r}{d\log k} = 1 - n_S + n_T$$

- For single field models

$$\frac{d\log r}{d\log k} = 2(2\epsilon - \eta)$$

  Hybrid type model or not: NO NEW INFORMATION
- For multi field models

Conclusions

- A new consistency relation from the running of r

$$\frac{d\log r}{d\log k} = 1 - n_S + n_T$$

- For single field models

$$\frac{d\log r}{d\log k} = 2(2\epsilon - \eta)$$

Hybrid type model or not: NO NEW INFORMATION

- For multi field models

$$\frac{d\log r}{d\log k} - \frac{r}{4} = -2\eta_{\text{multi}} + 2\frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{R^{i}{}_{ab}{}^{j}}{3m_{\text{Pl}}^2}\frac{\dot{\phi}^a\dot{\phi}^b}{H^2}$$

We can obtain **COMPLETELY NEW INFORMATION** on the field space and/or inflationary dynamics

Conclusions

- A new consistency relation from the running of r

$$\frac{d\log r}{d\log k} = 1 - n_S + n_T$$

- For single field models

$$\frac{d\log r}{d\log k} = 2(2\epsilon - \eta)$$

Hybrid type model or not: NO NEW INFORMATION

- For multi field models

$$\frac{d\log r}{d\log k} - \frac{r}{4} = -2\eta_{\text{multi}} + 2\frac{N_{,i}N_{,j}}{h^{kl}N_{,k}N_{,l}}\frac{R^{i}{}_{ab}{}^{j}}{3m_{\text{Pl}}^2}\frac{\dot\phi^a\dot\phi^b}{H^2}$$

We can obtain **COMPLETELY NEW INFORMATION** on the field space and/or inflationary dynamics