Title: Higher Symmetry of Gravity and the Cosmological Constant Problem

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Abstract: According to Doering and Isham the spectral topos corresponds to any quantum system. The descriptions of the systems become similar to these given by classical theories. Topoi can also modify local smooth spacetime structure. Supposing that a quantum system modifies the local spacetime structure and interacts with a gravitational field via the spectral topos, a natural pattern for non-gravitating quantum zero-point modes of the system, appears. A way how to add gravity into the spectral topos of a system is presented. A theory of gravity and systems should be symmetric with respect to some 2-group derived from the category of systems. Hence, a fundamental symmetry of gravity is rather 2-group of automorphisms of the category of systems. This higher symmetry is responsible for the vanishing of the cotributions to the cosmological constant derived from zero-point modes of energy of quantum systems in spacetime. A connection with strings (via the coefficients of the 2-connection of some 2-bundle, with this 2group as the structure group) is also shown. Institute of Physics, University of Silesia

Higher Symmetry of Gravity and the Cosmological Constant Problem

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The Cosmological Constant

I. Why does the alsmost perfect cancellation of the CC term, Λ_0, and the contribution of the zero-point density of energy modes of quantum systems, ρ_{vac}, occur?

$$\Lambda = 8\pi G \rho_{vac} + \Lambda_0 \tag{1}$$

II. Why the value of the effective cc, i.e. Λ, is not exactly zero?

- Probably a new fundamental symmetry is needed?

We are categorifying:

- A symmetry group.

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  One consequence is the possible resolution of the cc problem...

1. Some Differential Geometry: 2-Groups

2. Gravitation and quantum systems in spacetime
   - Work of C. Isham on topoi
   - Quantum systems in 4-spacetime
   - Gravitational interactions with a quantum system

3. The Cosmological Constant problem I and II

Holonomies on loops

- The holonomy $\mathrm{hol}(\gamma)$ along a path γ in M assigns a group element $\mathrm{hol}(\gamma) \in \mathrm{G}$ to the path.

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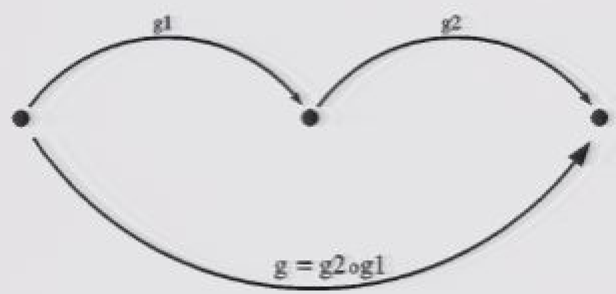

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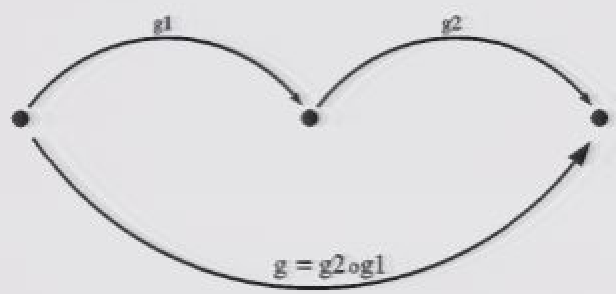

- Reversed paths have the reversed holonomies in G.

1. Holonomies as a functor

- The assignment $\mathrm{hol} : \mathcal{P}_1 \ni (\gamma) \to \mathrm{hol}(\gamma) \in \mathrm{G}$ defines a functor:
- $\mathrm{hol} : \mathcal{P}_1 \to \mathrm{G}$ where $\mathcal{P}_1$ is a path groupoid, i.e.
- objects in $\mathcal{P}_1$ are points in M and morphisms are „thin homotopy classes" of smooth paths $\gamma : [0,1] \to M$.

Holonomies by 1-forms

- The holonomy can be defined by $\mathfrak{g}$-valued 1-form A on M such that for any path $\gamma : [t_0, t_1] \to M$ we assign the group element $\mathrm{hol}(\gamma)$. One can compute this using a $\mathfrak{g}$-valued 1-form A on M (G is a smooth group) by solving the equation:
- $\frac{d}{dt} g(t) = A(\gamma'(t)) g(t)$ with the initial value $g(t_0) = 1$, then
- $\mathrm{hol}(\gamma) = \mathrm{g}(\mathrm{t}_1)$ (the group G is exponantiable, i.e. the equation always has a smooth solution)

Path holonomy

Theorem

There is a one-to-one correspondence between smooth functors

$$\mathrm{hol} : \mathcal{P}_1 \to \mathrm{G} \tag{2}$$

and $\mathfrak{g}$-valued 1-forms A on M, where $\mathcal{P}_1$ is the 1-groupoid.

Holonomies over surfaces

To define properly holonomies over surfaces we have to deal with 2-categories, 2-groups and 2-groupoids.

2-category

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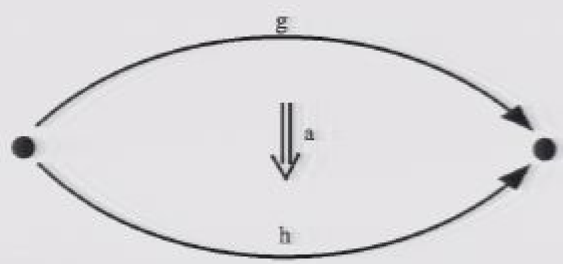
a set of 2-morphisms between 1-morphisms:



2-categories

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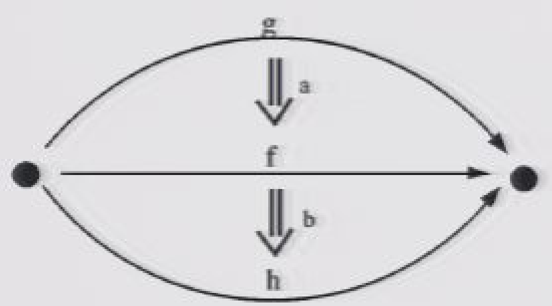
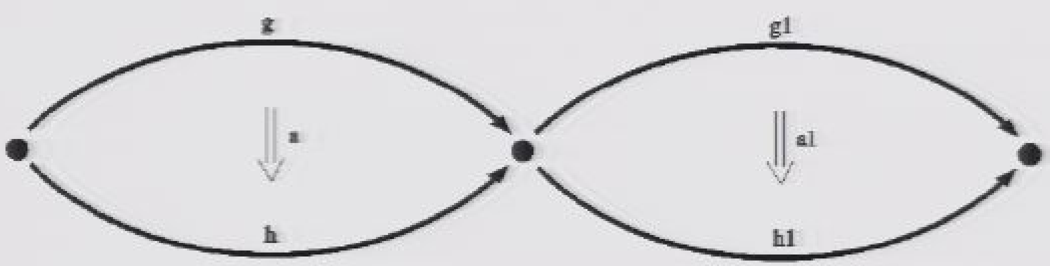
- and compose 2-morphisms vertically:

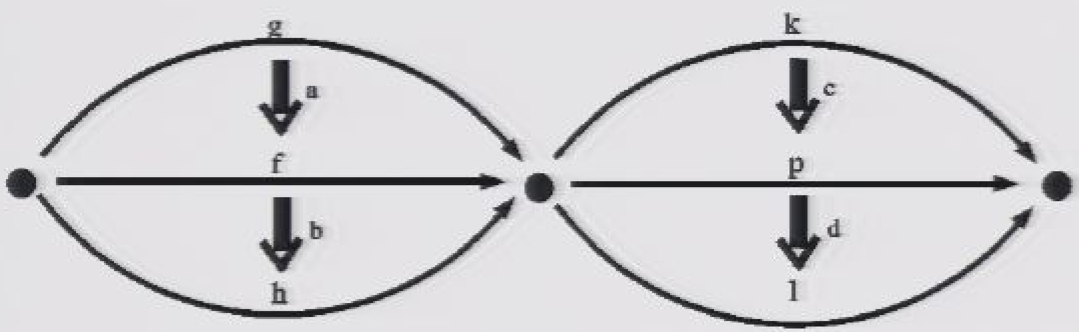


and horizontally



2-categories

The diagram:



gives a well defined 2-morphism. These are the unit low, associativity and the interchange low.

2-group

2-group $\mathcal{G}$ can be described as a category with

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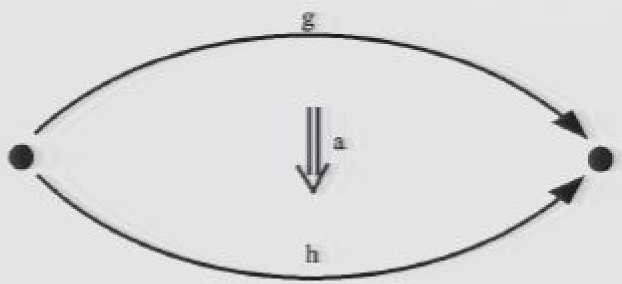

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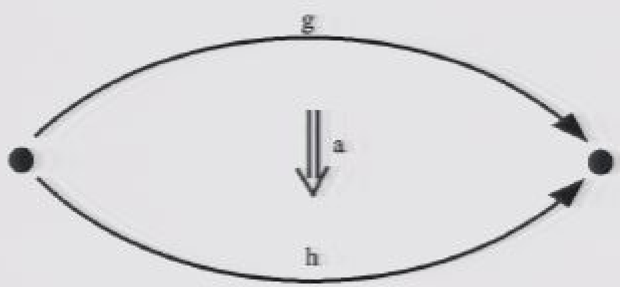

- Or: a 2-group is a 2-groupoid with just one object •.

2-groups

- The important example: 2-group of automorphisms $AUT(C)$ of any category C.
- The objects of $AUT(C)$ are invertible functors $g : C \to C$ and morphisms are natural isomorphisms $f : g \Rightarrow g'$.
- This 2-group is the higher analogue of the ordinary permutation group of sets.

2-groups

Equivalently, any 2-group $\mathcal{G}$ is determined by the system (G, H, t, α) i.e. a crossed module:

- the group G contains all objects of $\mathcal{G}$,
- the group H contains all morphisms of $\mathcal{G}$ which starts at 1,
- the morphism $t : H \to G$ assign the target to each morphism from H
- the action α of G on H is given by: $\alpha(g)h = 1_g h 1_g^{-1}$.

Crossed modules

- The system (G, H, t, α) is a crossed module provided:
- $t(\alpha(gh)) = gt(h)g^{-1}$
- $\alpha(t(h))h' = hh'h^{-1}$
- The differential form of a cross module is given by a system: $(\mathfrak{g}, \mathfrak{h}, dt, d\alpha)$ where $\mathfrak{g}, \mathfrak{h}$ are Lie algebras of G and H.
- Every 2-group (from some wide class) is determined by some crossed module, and conversely.

The path 2-groupoid

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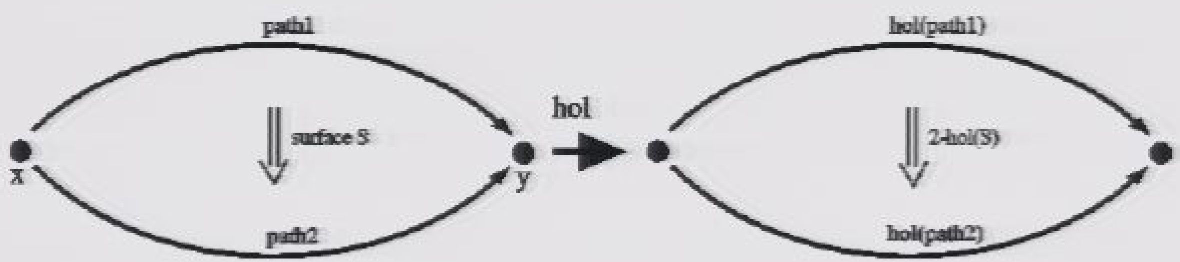
- 2-morphisms are thin homotopy classes of smooth maps $\Sigma : [0,1]^2 \to M$

Holonomy over surfaces as a 2-functor

If M is a smooth space, $\mathcal{G}$ a smooth 2-group, (G, H, t, α) its crossed module, $(\mathfrak{g}, \mathfrak{h}, dt, d\alpha)$ its differential crossed module. Supposing G and H are exponantiable, then
There is a one-to-one correspondence between

- smooth 2-functors: $\mathrm{hol} : \mathcal{P}_2(\mathrm{M}) \to \mathcal{G}$ i.e.:



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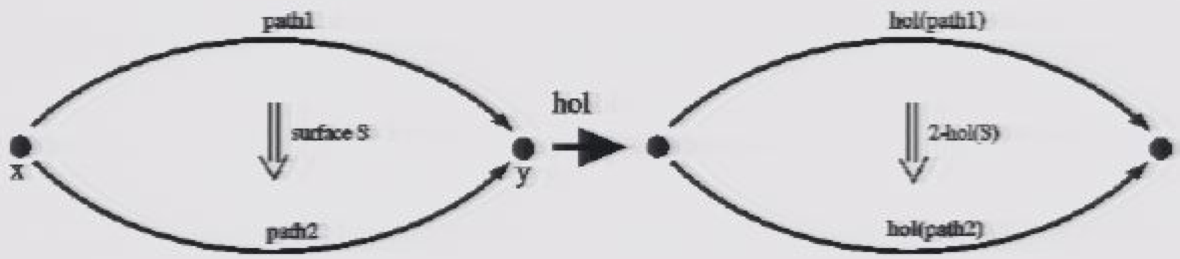

- and pairs (A, B) where A is a $\mathfrak{g}$-valued 1-form and B is a $\mathfrak{h}$-valued 2-form on M such that the condition is satisfied: $dA + A \wedge A + dt(B) = 0$

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- Moreover, when 2-group $\mathcal{G}$ has its morphisms given by $H = U(1)$, the connection coefficients refer to B-field 2-form and H-field 3-form appearing in the NS sector of IIB sustring theory.
- 2-bundless with connections correspond to nonabelian gerbes.

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- Thus, a fundamental symmetry of gravity would be one based on some 2-group.
- Let us see how to achieve this ...

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5. The 2-YM theory based on the „smooth" $AUT(TOP_s)$ gives the NS A, B and H fields of IIB string theory.

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- In the case of a classical theory, the topos is SET.
- The topos assigned to a quantum system S is $SET^{\mathcal{V}(\mathcal{H})^{op}}$, the category of presheaves on $\mathcal{V}(\mathcal{H})$ where $\mathcal{H}$ is the Hilbert space of states of a system S, $\mathcal{V}(\mathcal{H})$ consists of all unital boolean subalgebras of $\mathcal{B}(\mathcal{H})$ - the algebra of all bounded operators defined on $\mathcal{H}$.

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- Loosing non-distributivity in the topos means the uncertainty principle does not hold any longer!
- The intuitionistic logic is crucial. This agrees with a point of view in foundations of mathematics where topoi and intuitionism are more fundamental than SET.

Quantum systems in 4-spacetime

Suppose that at „sufficiently" small spatial distances the structure of spacetime manifold is modified by the topos $SET^{\mathcal{V}(\mathcal{H})^{op}}$. The local patch of such a modified spacetime can be chosen as the internal object R^4 and the change of local coordinates is also internalized.

Gravitational interactions with a quantum system

- Next we add classical gravitational field to the topos $SET^{\mathcal{V}(\mathcal{H})^{op}}$.
- GR can be formally formulated as some higher order formal theory (no use of the AC and middle third is made) (I. Moerdijk, 1991).
- A topos can model any formal intuitionistic theory of arbitrary high (language) order.

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- Gravity interacts with the *semi*-classical modes of the system in $SET^{\mathcal{V}(\mathcal{H})^{op}}$.
- For a quantum harmonic oscillator this means that the lowest energy is zero in the topos and the contribution to the vacuum energy vanishes.

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- The higher symmetry 2-group of gravity emerges as $AUT(TOP_s)$.
- When smoothed, this can give the sector of NS sustrings IIB compactified to 4-dim. (gerbes and 2-groups in string theory).

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- The exotic smooth structures can appear due to the modification of spacetime by topoi.
- The non-zero value of CC can be obtained respecting the exotic 4-smoothness of spacetime (T. Asselmeyer-Maluga and H. Rosé, 2006).

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4. New symmetry of gravity emerges as described rather by 2-group enlarging the group of diffeomorphisms. In a smooth limit, there can appear the A, B and H fields of the NS sector of IIB string theory.
5. The tools of higher groups and categories are intensively developed in the contemporary differential geometry, hence it is natural that these become more and more relevant to gravity and physics....

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