Title: U(1)_R mediation in 6D brane-world gauged supergravity

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Abstract: We construct a consistent supersymmetric action for chiral and vector multiplets living on codimension-two branes in a six-dimensional chiral gauged supergravity. A nonzero brane tension can be compatible with the bulk supersymmetry by introducing a brane-localized Fayet-Iliopoulos term proportional to the brane tension. Moreover, we show that a brane chiral multiplet with nonzero R charge has a nontrivial coupling to the extra component of the U(1)_R gauge field strength and a singular scalar self-interaction term. We find that a unwarped football solution with 4D Minkowski vacuum exists for arbitrary brane tension and it preserves 4D N=1 supersymmetry. Dimensionally reducing to 4D for the football solution, we obtain the low energy action for moduli and brane multiplets in 4D effective supergravity with gauged U(1)_R. By assuming the bulk gaugino condensates as well as nonzero brane F- and/or D-term for the uplifting potential, we have all the moduli stabilized with a vanishing cosmological constant. The brane scalar with nonzero R charge then gets a soft mass of order the gravitino mass, the sign of which depends on the sign of the R charge as well as whether the brane F- or D-term dominates.

Outline

1. Introduction
2. 6D gauged supergravity
3. SUSY conical branes with matter multiplets
4. Modulus stabilization and SUSY breaking

SUSY flavor problem

- Weak-scale supersymmetry, solving the hierarchy problem, has been considered as one of the most promising candidate beyond the Standard Model.
- However, generic weak-scale soft mass parameters given by

$$\mathcal{L}_{\rm soft} = -m^2_{ij}\phi^{*i}\phi^j - A_{ijk}\phi^i\phi^j\phi^k - B_{ij}\phi^i\phi^j - \frac{1}{2}M_a\lambda^a\lambda^a, \quad (1)$$

  would lead to unacceptably large flavor and/or CP violations, e.g., $m^2_{ij} \neq m^2_0\delta_{ij}$ typically in gravity mediation scenarios.
- When the hidden sector is separated from the visible sector in extra dimensions, SUSY breaking in the visible sector may be dominated by flavor-independent bulk interactions: "Sequestering mechanism". However, the situation is subtle in higher dimensions $(D > 5)$. [Anisimov et al(2001); Falkowski,Lee,Lüdeling(2005)]

- The Bianchi identities are

$$\partial_{[Q} F_{MN]} = 0, \tag{4}$$

$$\partial_{[Q} G_{MNP]} = \frac{3}{4} F_{[MN} F_{QP]}. \tag{5}$$

- Gauge invariance of the field strength G_{MNP} requires the Kalb-Ramond field B_{MN} to transform under $\delta A_M = \partial_M \Lambda$ as

$$\delta_\Lambda B_{MN} = -\frac{1}{2} \Lambda F_{MN}. \tag{6}$$

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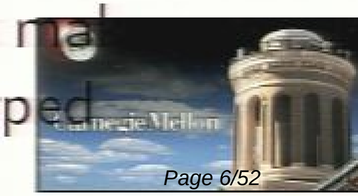
Flux compactifications in string theory

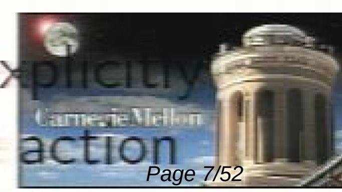
- Recently, KKLT flux compactifications of Type IIB string theory have drawn a lot of interest because all the moduli of dilaton and geometric moduli can be stabilized by the fluxes combined with non-perturbative effects.
- But the vacuum energy is typically negative after the moduli stabilization. When the vacuum is uplifted by an anti-D brane located at the warped throat on CY, the SUSY breaking is transmitted to the visible brane by a volume modulus so that the modulus mediation is in comparable size to the anomaly mediation: "modulus-anomaly mediation". [Choi et al(2005)]
- It may be a flavor symmetry in the hidden sector that m the sequestering working, e.g. the isometry of the warped throat. [Choi et al(2006); Kachru,Sundrum(2007)]



Flux compactifications in 6D supergravity

- 6D chiral gauged supergravity is a simple setup where the important aspects of the flux compactification can be studied analytically.
- By turning on a $U(1)_R$ gauge flux in the internal dimensions, Salam-Sezgin obtained a flux compactification on $M_4 \times S^2$ where 4D $\mathcal{N} = 1$ SUSY remains and one of moduli is stabilized by the flux. [Salam,Sezgin(1984)]
- General warped compactifications with tensionful codimension-two branes have drawn attention towards the self-tuning solution for the cosmological constant problem.

  [Carroll,Guica(2003); Navarro(2003)]
- However, the branes have been taken to break SUSY explicitly at the action level. We first construct the SUSY brane action and discuss about the $U(1)_R$ mediation of SUSY breakdown.



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Salam-Sezgin Supergravity

[Nishino,Sezgin(1984)]

- 6D chiral gauged supergravity is composed of a gravity multiplet(e_M^A, ψ_M, B_{MN}^+), and a tensor multiplet(ϕ, χ, B_{MN}^-) as well as a vector multiplet(A_M, λ), which gauges the $U(1)_R$ symmetry.
- The bosonic Lagrangian of the Salam-Sezgin supergravity is

$$e_6^{-1}\mathcal{L}_b = R - \frac{1}{4}(\partial_M\phi)^2 - \frac{e^{\phi}}{12}G_{MNP}G^{MNP} - \frac{e^{\frac{1}{2}\phi}}{4}F_{MN}F^{MN} - 8g^2e^{-\frac{1}{2}\phi}$$

where the field strength tensors are

$$F_{MN} = \partial_M A_N - \partial_N A_M, \quad (2)$$

$$G_{MNP} = 3\partial_{[M}B_{NP]} + \frac{3}{2}F_{[MN}A_{P]}. \quad (3)$$

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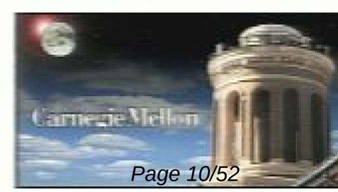

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- The fermionic Langrangian is given by

$$e_6^{-1}\mathcal{L}_f = \bar{\psi}_M \Gamma^{MNP} \mathcal{D}_N \psi_P + \bar{\chi}\Gamma^M \mathcal{D}_M \chi + \bar{\lambda}\Gamma^M \mathcal{D}_M \lambda$$

$$+\frac{1}{4}(\partial_M \phi)(\bar{\psi}_N \Gamma^M \Gamma^N \chi + \bar{\chi}\Gamma^N \Gamma^M \psi_N)$$
$$+\frac{1}{24} e^{\frac{1}{2}\phi} G_{MNP}(\bar{\psi}^R \Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R \Gamma^{MNP}\Gamma^R \chi$$
$$-\bar{\chi}\Gamma^R\Gamma^{MNP}\psi_R - \bar{\chi}\Gamma^{MNP}\chi + \bar{\lambda}\Gamma^{MNP}\lambda)$$
$$-\frac{1}{4\sqrt{2}} e^{\frac{1}{4}\phi} F_{MN}(\bar{\psi}_Q \Gamma^{MN}\Gamma^Q \lambda + \bar{\lambda}\Gamma^Q \Gamma^{MN}\psi_Q + \bar{\chi}\Gamma^{MN}\lambda - \bar{\lambda}\Gamma^{MN}\chi)$$
$$+i\sqrt{2}g e^{-\frac{1}{4}\phi}(\bar{\psi}_M \Gamma^M \lambda + \bar{\lambda}\Gamma^M \psi_M - \bar{\chi}\lambda + \bar{\lambda}\chi). \quad (7)$$

- All the spinors have the same R charge $+1$, e.g. $\mathcal{D}_M \psi_N = (\partial_M + \frac{1}{4}\omega_{MAB}\Gamma^{AB} - igA_M)\psi_N$.

- The SUSY transformations (up to the trilinear fermion terms) are

$$
\begin{aligned}
\delta e_M^A &= -\frac{1}{4}\bar{\varepsilon}\Gamma^A\psi_M + \text{h.c.}, \quad \delta\phi = \frac{1}{2}\bar{\varepsilon}\chi + \text{h.c.}, \\
\delta B_{MN} &= A_{[M}\delta A_{N]} + \frac{e^{-\frac{1}{2}\phi}}{4}(\bar{\varepsilon}\Gamma_M\psi_N - \bar{\varepsilon}\Gamma_N\psi_M + \bar{\varepsilon}\Gamma_{MN}\chi + \text{h.c.}), \\
\delta\chi &= -\frac{1}{4}(\partial_M\phi)\Gamma^M\varepsilon + \frac{1}{24}e^{\frac{1}{2}\phi}G_{MNP}\Gamma^{MNP}\varepsilon, \\
\delta\psi_M &= \mathcal{D}_M\varepsilon + \frac{1}{48}e^{\frac{1}{2}\phi}G_{PQR}\Gamma^{PQR}\Gamma_M\varepsilon, \\
\delta A_M &= \frac{1}{2\sqrt{2}}e^{-\frac{1}{4}\phi}(\bar{\varepsilon}\Gamma_M\lambda + \text{h.c.}), \\
\delta\lambda &= \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{MN}\Gamma^{MN}\varepsilon - i\sqrt{2}g\, e^{-\frac{1}{4}\phi}\varepsilon.
\end{aligned}
$$

- The bulk action ($\mathcal{L}_{\rm bulk} = \mathcal{L}_b + \mathcal{L}_f$) is invariant up to the Bianchi identities as follows,

$$\delta\mathcal{L}_{\rm bulk} = e_6\Big[-\frac{1}{24}e^{\frac{1}{2}\phi}\Big(\partial_S G_{MNP} - \frac{3}{4}F_{MN}F_{SP}\Big) \times\Big(\bar{\psi}^R\Gamma_{RMNPS}\varepsilon - \bar{\chi}\Gamma^{SMNP}\varepsilon + {\rm h.c.}\Big) + \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\Big(\partial_Q F_{MN}\bar{\lambda}\Gamma^{QMN}\varepsilon + {\rm h.c.}\Big)\Big]. \quad (8)$$

- For the modified Bianchi identities, the above variation can be used to cancel the variation of the brane matter action.

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The brane chiral multiplet

- The Z_2 orbifold symmetry is imposed to keep only $\mathcal{N}=1$ SUSY (i.e. $P_L\varepsilon \equiv \varepsilon_+$) at the branes.
- The SUSY action for the brane chiral multiplet (ψ_Q, Q) is

$$\begin{aligned}\mathcal{L}_{\rm chiral} \;=\;& e_4\Big[e^{\frac{1}{2}\phi}\Big(-(D^\mu Q)^\dagger D_\mu Q+\frac{1}{2}\bar{\psi}_Q\gamma^\mu D_\mu\psi_Q+\text{h.c.}\Big)\\ &+\sqrt{2}irge^{\frac{1}{4}\phi}\bar{\psi}_Q\lambda Q+\text{h.c.}-4rg^2|Q|^2-T_0\\ &+e^{\frac{1}{2}\phi}\Big(\frac{1}{2}\bar{\psi}_{\mu+}\gamma^\nu\gamma^\mu\psi_Q(D_\nu Q)^\dagger+\frac{1}{2}\bar{\psi}_Q\gamma^\mu\chi_+D_\mu Q+\text{h.c.}\Big)\Big]\end{aligned}$$

where $D_\mu Q=(\partial_\mu+irgA_\mu)Q$, $D_\mu\psi_Q=(\partial_\mu+i(r-1)gA_\mu+\frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta})\psi_Q$, and SUSY transformations are $\delta Q=\frac{1}{2}\bar{\varepsilon}_+\psi_Q$, $\delta\psi_Q=-\frac{1}{2}\gamma^\mu\varepsilon_+D_\mu Q$.

- The bulk action and the SUSY transformations are modified by replacing G_{MNP} and F_{MN} with the hatted ones (keeping A_M as it is):

$$\hat{G}_{\mu mn} = G_{\mu mn} + (j_\mu - \xi_0 A_\mu)\delta_{mn}, \quad (9)$$

$$\hat{G}_{\tau\rho\sigma} = G_{\tau\rho\sigma} + \frac{\delta^2(y)}{e_2} j_{\tau\rho\sigma}, \quad (10)$$

$$\hat{F}_{mn} = F_{mn} - (rg|Q|^2 + \xi_0)\delta_{mn} \quad (11)$$

with the Fayet-Iliopoulos term being $\xi_0 = \frac{T_0}{4g}$, $\delta_{mn} \equiv \epsilon_{mn}\frac{\delta^2(y)}{e_2}$ and

$$j_\mu = \frac{1}{2} i\Big[Q^\dagger D_\mu Q - (D_\mu Q)^\dagger Q + \frac{1}{2}\bar{\psi}_Q \gamma_\mu \psi_Q\Big], \quad (12)$$

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- The modified Bianchi identities are given by

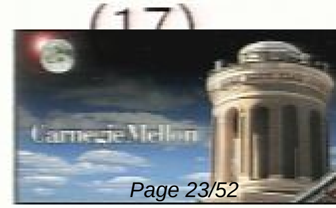
$$\partial_{[\mu}\hat{G}_{\nu mn]} = \frac{3}{4}\hat{F}_{[\mu\nu}\hat{F}_{mn]} + \frac{i}{2}(D_{[\mu}Q)^{\dagger}(D_{\nu]}Q)\delta_{mn}, \quad (14)$$

$$\partial_{[\mu}\hat{F}_{mn]} = -\frac{1}{3}rg\partial_{\mu}|Q|^2\delta_{mn}. \quad (15)$$

- The gauge and SUSY transformations of the KR field get additional terms as

$$\delta_{\Lambda}B_{mn} = \Lambda\Big(-\frac{1}{2}F_{mn} + \xi_0\delta_{mn}\Big), \quad (16)$$

$$\delta B_{mn} = \cdots + \frac{1}{4}i\bar{\psi}_Q\varepsilon Q\delta_{mn} + \mathrm{h.c.}. \quad (17)$$



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The brane vector multiplet

- For a brane vector multiplet, (W_μ, Λ), we need to add

$$\begin{aligned}
\mathcal{L}_{\rm vector} &= e_4\Big[-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}\bar{\Lambda}\gamma^\mu D_\mu\Lambda + {\rm h.c.} \\
&\quad -ie\sqrt{2}e^{\frac{1}{2}\phi}Q\bar{\psi}_Q\Lambda + {\rm h.c.} - \frac{1}{2}e^2|Q|^4 e^\phi \\
&\quad -\frac{1}{4\sqrt{2}}\bar{\Lambda}\gamma^\mu\gamma^{\nu\rho}\psi_{\mu+}W_{\nu\rho} - \frac{i}{2\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\bar{\Lambda}\gamma^\mu\psi_{\mu+} + {\rm h.c.} \\
&\quad -\frac{i}{\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\bar{\chi}_+\Lambda + {\rm h.c.}\Big]
\end{aligned}$$

where $D_\mu\Lambda = (\partial_\mu - igA_\mu + \frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta})\Lambda$ and SUSY transformations are $\delta\Lambda = \frac{1}{4\sqrt{2}}\gamma^{\mu\nu}\varepsilon_+ W_{\mu\nu} + \frac{i}{2\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\varepsilon_+$, $\delta W_\mu = \frac{1}{2\sqrt{2}}\bar{\varepsilon}_+\gamma_\mu\Lambda + {\rm h.c.}$.

- While $\hat{F}_{mn}$ is the same, the modified field strength $\hat{G}_{MNP}$ gets additional terms as

$$\hat{G}_{\mu mn} = G_{\mu mn} + (J_\mu - \xi_0 A_\mu)\delta_{mn}, \quad (18)$$

$$\hat{G}_{\tau\rho\sigma} = G_{\tau\rho\sigma} + \frac{\delta^2(y)}{e_2} J_{\tau\rho\sigma}, \quad (19)$$

with

$$J_\mu = j_\mu - \frac{1}{4} i e^{-\frac{1}{2}\phi} \bar{\Lambda}\gamma_\mu \Lambda,$$

$$J_{\tau\rho\sigma} = j_{\tau\rho\sigma} - \frac{1}{8} e^{-\frac{1}{2}\phi} \bar{\Lambda}\gamma_{\tau\rho\sigma}\Lambda.$$

- The Bianchi identity for $\hat{G}_{MNP}$ gets an additional term as

$$\partial_{[\mu} \hat{G}_{\nu mn]} - \frac{3}{4} \hat{F}_{[\mu\nu} \hat{F}_{mn]} = \cdots + \frac{1}{4} e|Q|^2 W_{\mu\nu}\delta_{mn}.$$

The brane potentials

- The supersymmetric brane-localized gravitino mass term is

$$\mathcal{L}_{\rm gmass} = -e_4 \frac{1}{2} W_0 e^{\frac{1}{2}\psi} (\bar{\psi}_{\mu+}\gamma^{\mu\nu} C \bar{\psi}^T_{\nu+} + \bar{\psi}_1 \gamma^\mu C \bar{\psi}^T_{\mu+} + \bar{\psi}_2 \gamma^\mu C \bar{\psi}^T_{\mu+} + \bar{\lambda}_+ C \bar{\lambda}^T_+) + {\rm h.c.} \quad (20)$$

where W_0 is a constant parameter, e^ψ is the volume modulus and

$$\psi_1 = \psi_{5+} + i\psi_{6+}, \quad \psi_2 = \psi_{5+} - i\psi_{6+}. \quad (21)$$

- From this, the brane F-term is inferred to be

$$\mathcal{L}_F = -e_4 e^{\psi - \frac{1}{2}\phi} |F_Q|^2 \quad (22)$$

with $F_Q = \frac{\partial W}{\partial Q}$.

- The brane D-term takes the form, $\mathcal{L}_D = -e_4 \frac{1}{2} e^\phi D^2$.

Flux compactifications with SUSY branes

- Consider the two branes case and take $\hat{F}_{mn}$ with two localized FI terms on the branes, $\xi_i = \frac{T_i}{4g}(i = 1, 2)$.
- Turning on the $U(1)_R$ flux, the general regular solution keeps the warped product of the 4D Minkowski space with two compact dimensions, [Gibbons et al(2003); Aghababaie et al(2003)]

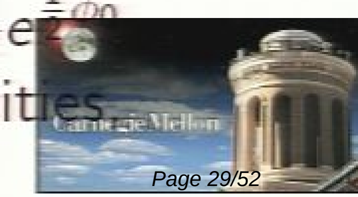
$$ds^2 = W^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + R^2(r)(dr^2 + \lambda^2\Theta^2(r)d\theta^2), \quad (23)$$

$$\hat{F}_{r\theta} = \lambda q e^{-\frac{1}{2}\phi_0}\frac{\Theta R^2}{W^6}, \quad \phi = \phi_0 + 4\ln W, \quad (24)$$

with $R = \frac{W}{f_0}, \Theta = \frac{r}{W^4}, W^4 = \frac{f_1}{f_0}$ and $f_0 = 1 + \frac{r^2}{r_0^2}, f_1 = 1 + \frac{r^2}{r_1^2}$.

Here λ, q, ϕ_0 are constants, $r_0^2 = \frac{1}{2g^2}e^{\frac{1}{2}\phi_0}$ and $r_1^2 = \frac{8}{q^2}e^{\frac{1}{2}\phi_0}$.

- Two brane tensions are located at the conical singularities $r = 0$ and $r = \infty$: $\frac{T_1}{4\pi} = 1 - \lambda$ and $\frac{T_2}{4\pi} = 1 - \lambda\frac{r_1^2}{r_0^2}$.



- From the gauge equation,

$$\hat{F}_{r\theta} = F_{r\theta} - \frac{\xi_1}{2\pi}\delta(r) = \lambda e^{-\frac{1}{2}\phi_0} q \frac{\Theta R^2}{W^6}, \tag{25}$$

the gauge potential become nonzero at $r=0$ and $r=\infty$:

$$A_\theta = -\frac{4\lambda}{q}\left(\frac{1}{f_1} - 1\right) + \frac{\xi_1}{2\pi}; \quad A_\theta = -\frac{4\lambda}{q}\frac{1}{f_1} - \frac{\xi_2}{2\pi}. \tag{26}$$

- The quantization condition is modified to $\frac{4\lambda g}{q} = n - \frac{g}{2\pi}(\xi_1 + \xi_2)$ with $n \in \mathbf{Z}$. After the brane conditions, it becomes

$$\left(1 - \frac{T_0}{4\pi}\right)\left(1 - \frac{T_\infty}{4\pi}\right) = \left[n - \frac{g}{2\pi}(\xi_1 + \xi_2)\right]^2. \tag{27}$$

- The warped solutions break the bulk SUSY completely: e.g.

$$\delta\chi = -\frac{W'}{W}[\cos\theta\,\sigma^1 \otimes \gamma^5 + \sin\theta\,\sigma^2 \otimes \mathbf{1}]\varepsilon \neq 0. \qquad (28)$$

- In the case of the football solution with a constant warp factor, from $q = 4g$ and $T_0 = T_\infty = 4\pi(1-\lambda)$, we obtain $n = 1$ and arbitrary λ. Moreover, the nontrivial fermionic SUSY transformations are

$$\begin{aligned} \delta\lambda &= i2\sqrt{2}g(P_R\varepsilon), \qquad (29) \\ \delta\psi_\theta &= \Big[\partial_\theta + \frac{i}{2}\Big\{1 + \lambda\Big(1 - \frac{2}{f_0}\Big)\Big\}\gamma^5 + i\lambda\Big(\frac{1}{f_0} - 1\Big) - i\frac{g\xi_0}{2\pi}\Big]\varepsilon \\ &= \partial_\theta(P_L\varepsilon). \end{aligned}$$

For a constant $P_L\varepsilon$, there is a 4D $\mathcal{N} = 1$ SUSY for any λ.

4D effective supergravity

- Consider the low energy effective action for light bulk and brane fields for the supersymmetric football.
- We take the ansatz for the 6D solution as

$$ds^2 = e^{-\psi(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{\psi(x)} ds_2^2, \quad (31)$$

$$\phi = f(x), \quad (32)$$

$$\hat{F}_{MN} = \langle \hat{F}_{MN} \rangle + \mathcal{F}_{MN}, \quad (33)$$

where the background VEV of the gauge field strength is $\langle \hat{F}_{mn} \rangle = q\epsilon_{mn}$, ds_2^2 and ϵ_{mn} are the 2D metric and the 2D volume form of the static solution, and ψ is the volume modulus.

- The 6D equations,

$$\partial_M(\sqrt{-g_6}e^{\phi}\hat{G}^{MNP}) = 0, \quad (34)$$

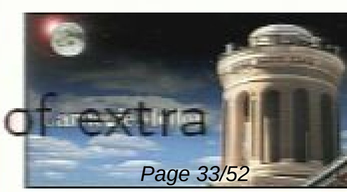
$$\partial_M(\sqrt{-g_6}e^{\frac{1}{2}\phi}\hat{F}^{MN}) = \frac{1}{2}\sqrt{-g_6}e^{\phi}\hat{G}^{PQN}\hat{F}_{PQ}, \quad (35)$$

and the Bianchi identities, are solved by the modified field strengths,

$$\hat{G}_{\mu mn} = \Big(-b + qA_\mu + \frac{J_\mu}{V}\Big)\epsilon_{mn}, \quad (36)$$

$$\hat{F}_{mn} = \Big(q - \frac{rg|Q|^2}{V}\Big)\epsilon_{mn}, \quad (37)$$

where $b = -\frac{1}{2}\mathcal{B}_{mn}\epsilon^{mn}$ for the globally well-defined $\mathcal{B} = B - \frac{1}{2}\langle A\rangle \wedge \mathcal{A}$ that satisfies $\delta_{\Lambda_0}(d\mathcal{B}){=}0$ for the background gauge transform Λ_0, and V is the volume of extra dimensions for the football solution.



- Plugging the solutions into the bulk/brane action and using the duality $e^f G_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\tau}\partial^\tau \sigma$, the bosonic effective action is

$$\begin{aligned}
\mathcal{L}_{\rm boson} \;=\;& M_P^2\sqrt{-g}\Big\{\frac{1}{2}R(g) - \frac{(\partial_\mu s)^2}{4s^2} - \frac{(\partial_\mu t)^2}{4t^2} - \frac{(\partial_\mu \sigma)^2}{4s^2} \\
& -\frac{1}{4M_P^2} sF_{\mu\nu}F^{\mu\nu} - \frac{1}{M_P^2 t}(D^\mu Q)^\dagger (D_\mu Q) - \frac{1}{4M_P^2}W_{\mu\nu}W^{\mu\nu} \\
& -\frac{1}{4t^2}\Big(\partial_\mu b - 4g_R A_\mu - \frac{i}{M_P^2}(Q^\dagger D_\mu Q - (D_\mu Q)^\dagger Q)\Big)^2 \\
& -\frac{2g_R^2 M_P^2}{s}\Big[1 - \frac{1}{t}\Big(1 - \frac{r}{2M_P^2}|Q|^2\Big)\Big]^2\Big\} \qquad (38)
\end{aligned}$$

where $s = e^{\psi + \frac{1}{2}f}$, $t = e^{\psi - \frac{1}{2}f}$, $M_P^2 = M_*^4 V$ with $V = \lambda r_0^2$ and $g_R = g/\sqrt{V}$.

- The Kähler potential reads

$$K = -\ln\left(\frac{1}{2}(S+S^\dagger)\right) - \ln\left(\frac{1}{2}(T+T^\dagger-\delta_{GS}V_R) - \frac{1}{M_P^2}\tilde{Q}^\dagger e^{-2rg_R V_R}\tilde{Q}\right) - \frac{2\xi_R}{M_P^2}V_R$$

where $\delta_{GS}=8g_R$ and $\xi_R = 2g_R M_P^2$ and the scalar components of the moduli superfields S, T are given by

$$S = s + i\sigma, \qquad T = t + \frac{1}{M_P^2}|Q|^2 + ib.$$

V_R: $U(1)_R$ vector superfield,
$\tilde{Q}$: a chiral superfield containing (Q^*, ψ_Q^c).

- The T modulus is fixed due to the interplay of the constant FI term with the field-dependent FI term in the 4D effective supergravity.
- The remaining S modulus can be also fixed by introducing bulk gaugino condensates. A negative vacuum energy generated by the bulk gaugino condensates needs to be fine-tuned to zero by brane F- and/or D-term uplifting potentials.
- The $U(1)_R$ D-term leads to a tree-level soft mass for the brane scalar with nonzero R charge. So, for nonzero appropriate R charges of sleptons, we can cure the problem of the negative slepton mass squared in anomaly mediation.
- SUSY phenomenology in a realistic model is work in progress.

Conclusion

- We constructed a SUSY action for brane matter multiplets on the codimension-two brane in a 6D gauged supergravity.
- A nonzero tension of the supersymmetric brane is accompanied by the corresponding magnetic charge or the localized Fayet-Iliopoulos term of the $U(1)_R$ gauge field proportional to the tension.
- Thanks to the localized FI terms, the football solution keeps 4D $\mathcal{N} = 1$ SUSY. In this case, since there is no quantization condition for the deficit angle, the brane tension can be arbitrary.

- The $U(1)_R$ mediation can dominate over the anomaly mediation, particularly for solving the negative lepton mass squared problem:

$$m^2_{Q_i} = |r_i|\left(m^2_{3/2} - \frac{1}{2}\frac{|F_S|^2}{M_P^2}\right) + m^2_{\mathrm{anom},i} \tag{54}$$

where $m^2_{\mathrm{anom},i} = \frac{c_i b_a}{8\pi^2}\alpha_a^2|F_C|^2$ with $c_i > 0$ the quadratic Casimir invariant, $b_a = (-\frac{33}{5}, -1, 3)$ and $F_C = m_{3/2} + \frac{1}{3}K_i F^i$ the auxiliary field of the conformal compensator. But we would need universal R charges at least for the first two generations for no SUSY flavor problem.

- When the SM gauge fields are localized on the brane, there is no tree-level gaugino mass due to $f_W = 1$. Then, the gaugino masses are given by anomaly mediation: $m_{\lambda_a} = -\frac{b_a g_a^2}{16\pi^2}F_C$.

- In the F-term domination with $D=0$, since $|F_{Q'}|^2 \simeq -stV_1 = st(2m_{3/2}^2 M_P^2 - |F_S|^2)$, the brane scalar mass becomes

$$m_Q^2 \simeq -r\Big(m_{3/2}^2 - \frac{1}{2}\frac{|F_S|^2}{M_P^2}\Big). \tag{52}$$

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V_R: $U(1)_R$ vector superfield,
$\tilde{Q}$: a chiral superfield containing (Q^*, ψ_Q^c).

- The gauge kinetic functions for the bulk and brane vector multiplets are

$$f_R = S, \qquad f_W = 1 \tag{39}$$

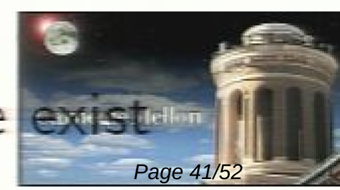
- For the 4D reduction, the brane-localized gravitino mass term becomes

$$\mathcal{L}_{\rm gmass} = -e_4 \frac{1}{2} W_0 e^{-\psi} \bar{\psi}_{\mu+} \gamma^{\mu\nu} C \bar{\psi}^T_{\nu+} + {\rm h.c.}. \tag{40}$$

By the comparison to the gravitino mass in 4D supergravity, $\mathcal{L}_m = -e_4 \frac{1}{2} e^{K/2} W \bar{\psi}_{\mu+} \gamma^{\mu\nu} C \bar{\psi}^T_{\nu+} + {\rm h.c.}$, we find the effective superpotential is independent of the moduli:

$$W = W_0. \tag{41}$$

- The result is easily generalized to the case where the superpotential depends on the brane matters and there exist brane matters at the other brane.



Modulus stabilization

- The 4D effective scalar potential is

$$V_0 = \frac{2g_R^2 M_P^4}{s}\left[1 - \frac{1}{t}\left(1 - \frac{r}{2M_P^2}|Q|^2\right)\right]^2. \quad (42)$$

So, $t = 1$ and $|Q| = 0$ at the SUSY minimum with a zero vacuum energy while s is undetermined. The effective brane scalar mass vanishes.

- We assume that the bulk non-perturbative dynamics generates a modulus potential from an S-dependent superpotential $W(S)$:

$$V_1 = \frac{e^K}{M_P^2}\left[\left|\frac{\partial W}{\partial S} + \frac{\partial K}{\partial S}W\right|^2 K^{-1}_{SS^\dagger} - 2|W|^2\right]. \quad (43)$$

- Including the non-perturbative correction and the uplifting potentials, the 4D scalar potential becomes

$$V_{\rm tot} = V_0 + V_1 + V_2 + V_3 \tag{44}$$

with $V_2 = \frac{1}{s}|F_{Q'}|^2$, $V_3 = \frac{1}{2t^2}D^2$.

- Then, $Q = 0$ is still the minimum for $r(t-1) > 0$, while the T modulus is shifted to

$$t = \frac{1 + \frac{1}{2}\alpha D^2}{1 - \frac{1}{2}\alpha t V_1}; \quad \alpha \equiv \frac{s}{2g_R^2 M_P^4}. \tag{45}$$

- The S modulus is determined approximately by $F_S = 0$ but it is shifted a bit by the F-term uplifting.
- After eliminating the t dependence, the zero vacuum energy condition becomes

$$(2 + \alpha D^2)\frac{1}{s}|F_{Q'}|^2 = -2tV_1\left(1 - \frac{1}{4}\alpha t V_1\right) - D^2. \tag{46}$$

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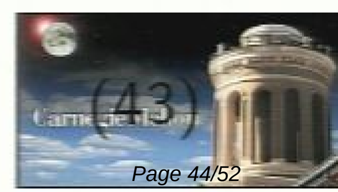

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- For instance, the double gaugino condensates would lead to a racetrack form,

$$W(S) = \Lambda_1 e^{-\beta_1 S} + \Lambda_2 e^{-\beta_2 S}. \qquad (47)$$

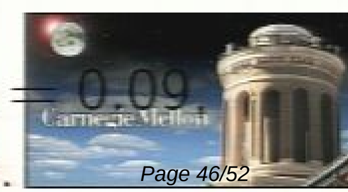
- Then the $F_S = 0$ condition fixes both $\mathrm{Re}\, S$ and $\mathrm{Im}\, S$ as

$$\mathrm{Im}\, S = \frac{\pi(2n+1)}{\beta_1 - \beta_2}, \qquad (48)$$

$$\mathrm{Re}\, S = \frac{1}{\beta_1 - \beta_2} \ln \frac{\Lambda_1(2\beta_1 \mathrm{Re}\, S + 1)}{\Lambda_2(2\beta_2 \mathrm{Re}\, S + 1)}. \qquad (49)$$

  For $|\beta_1 - \beta_2| \ll \beta_1$, the potential is minimized at a large $\mathrm{Re}\, S$ for which the superpotential description in the 4D effective supergravity is reliable.

- Choosing $\Lambda_1/M_P^3 = 1$, $\Lambda_2/M_P^3 = 0.9$, $\beta_1 = 0.1$ and $\beta_2 = 0.09$ we get $\mathrm{Re}\, S \simeq 18$ and $m_s \sim 3m_{3/2}$, while $m_t \sim g_R M_P$.



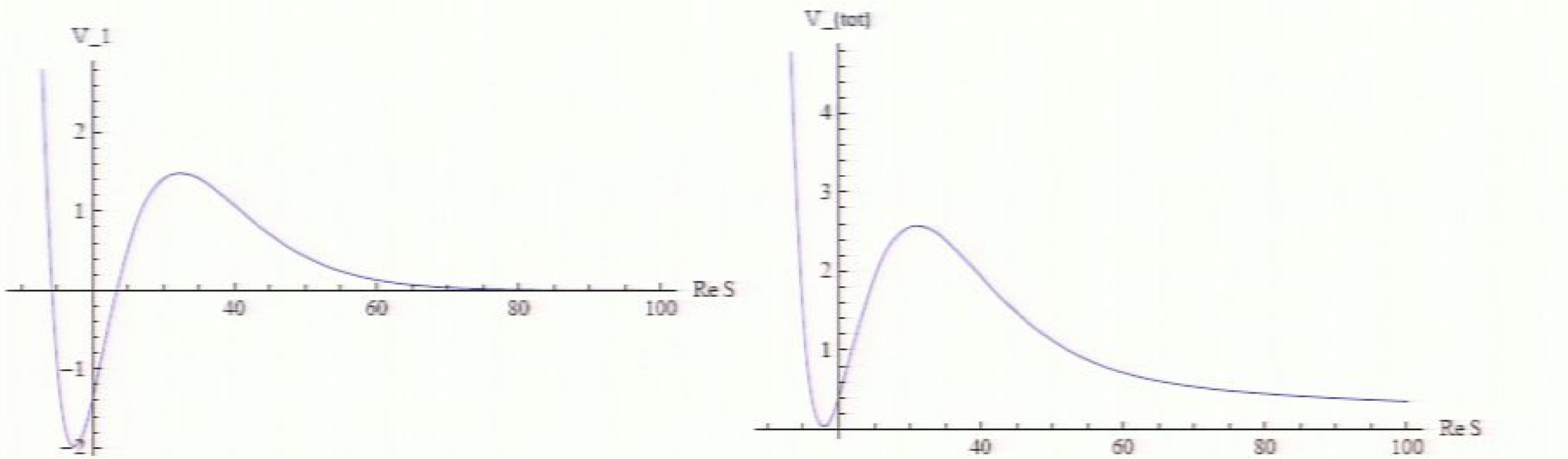


Figure: The modulus potential: the bulk non-perturbative correction only on the left, and the total correction including the F-term uplifting potential on the right. The potential value is normalized by $m_{3/2}^2 M_P^2$ and the parameters of the superpotential are chosen as $\Lambda_1/M_P^3 = 1$, $\Lambda_2/M_P^3 = 1.1$, $\beta_1 = 0.1$, $\beta_2 = 0.11$. On the right, the uplifting potential is given by $V_2/(m_{3/2}^2 M_P^2) = 3.2 \times 10^{-4}/\mathrm{Re}\, S$.

Soft masses

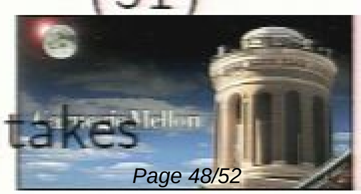
- After fixing all the moduli at the zero vacuum energy, we find that the $U(1)_R$ D-term leads to a soft mass for the brane scalar with nonzero R charge as

$$\begin{aligned} m_Q^2 &= r\, g_R D_R|_{Q=0} \\ &= \frac{D^2 + tV_1}{1-\frac{1}{2}\alpha t V_1}\frac{\frac{1}{2}r}{tM_P^2}. \end{aligned} \quad (50)$$

- By using $D^2 \simeq -\frac{2}{s}|F_{Q'}|^2 - 2tV_1$ for $\alpha r V_1 \ll 1$ and $\alpha D^2 \ll 1$, the scalar soft mass becomes

$$m_Q^2 \simeq r\Big(m_{3/2}^2 - \frac{1}{2}\frac{|F_S|^2}{M_P^2} - \frac{|F_{Q'}|^2}{stM_P^2}\Big). \quad (51)$$

- For zero R charge, the sequestering of SUSY breaking takes place at tree level.



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