Title: Soft wall dynamics in AdS/QCD

Date: Jun 06, 2008 09:00 AM

URL: http://pirsa.org/08060077

Abstract: Linear confinement in holographic QCD can be obtained with a soft-wall quadratic dilaton background. We present a dynamical five-dimensional model realizing this setup and discuss the implications for the hypothetical string theory dual to QCD.

AdS/CFT duality

Maldacena '97
Gubser, Klebanov, Polyakov '97
Witten '97

String/gravity theory in higher dimensions	dual $\Longleftrightarrow$	Strongly coupled gauge theory in 4D

Can we use it to understand real-world QCD?

Top-down AdS/CFT constructions:

- conformal
- nonconformal
- less or no SUSY
- confining
- flavor
- chiral symmetry breaking

but still no string/gravity dual to QCD

Work backwards: AdS/QCD

Erlich, Katz, Son, Stephanov '05
Da Rold, Pomarol '05

Basic idea: Use facts about QCD to model gravity dual

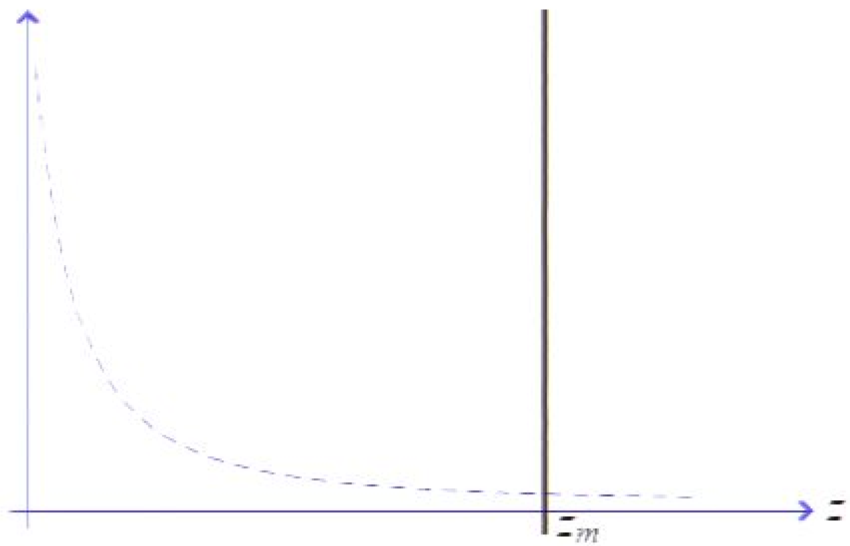
QCD ingredients:

- chiral symmetry — bulk gauge symmetry
- QCD operators:
  - $(\overline{q}\gamma^\mu t^a q)_{L,R} \to A^{Ma}_{L,R}(x,z)$
  - $\overline{q}q \to$ scalar $X(x,z)$ with $m_X^2 = -3$
- confinement?

Hard wall

- Confinement implemented with a cutoff to AdS spacetime in the IR: $z_m \sim \Lambda_{QCD}^{-1}$
- Predictions match QCD phenomenology with $O(10\%)$ error
- Highly excited states:

$$m_n^2 \sim n^2$$
$$m_S^2 \sim S^2$$



Hard wall model does not exhibit linear Regge behavior

Soft wall

Karch, Katz, Son, Stephanov '06

- Dilaton provides a smooth cutoff to spacetime

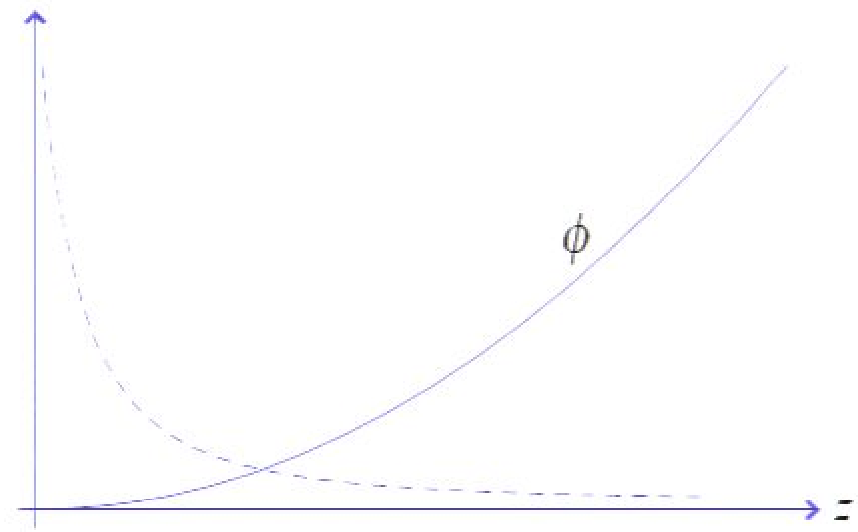
$$S = \int d^4x dz \sqrt{-g} e^{-\Phi} \mathcal{L}$$

- Simplest model:

$$g_{MN} = z^{-2} \eta_{MN}$$
$$\Phi(z) = z^2$$

- Regge trajectories:

$$m^2_{n,S} \sim (n + S)$$



Soft wall dynamics

How could this background arise dynamically?

- Factor $e^{-\Phi}$ in arises from open string/D-brane sector
- Start from string inspired action including closed string tachyon:

$$
\begin{aligned}
S \;=\; M^3 \int d^5x \sqrt{-g}\, e^{-2\Phi} \Big(& R + 4 g^{MN} \partial_M \Phi \partial_N \Phi \\
& - \frac{1}{2} g^{MN} \partial_M T \partial_N T - V(\Phi, T) \Big)
\end{aligned}
$$

- What about the potential $V(\Phi, T)$? Don't know *a priori* so work backwards

Search for solution

1) Move to Einstein frame

- Conformal transformation $g_{MN} = e^{4\Phi/3}\tilde{g}_{MN}; \quad \phi = \sqrt{8/3}\,\Phi$
- metric in Einstein frame: $\tilde{g}_{MN} = z^{-2}e^{-2az^2}\eta_{MN}$

2) Ansatz

$$
\begin{aligned}
ds^2 &= e^{-2A(y)}dx^2 + dy^2, \\
\phi &= \phi(y), \\
T &= T(y).
\end{aligned}
$$

3) "Superpotential method"

Skenderis, Townsend '99

DeWolfe, Freedman, Gubser, Karch '99

- converts second order system of equations into first order system
- introduce a "superpotential" $W(\phi, T)$

$$\begin{aligned} A' &= W, \\ \phi' &= 6\frac{\partial W}{\partial \phi}, \\ T' &= 6\frac{\partial W}{\partial T}. \end{aligned}$$

$$\widetilde{V}(\phi, T) = 18\left[\left(\frac{\partial W}{\partial \phi}\right)^2 + \left(\frac{\partial W}{\partial T}\right)^2\right] - 12W^2.$$

4) Convert between z and y coordinates:

$$y = \int dz \, \frac{e^{-az^2}}{z} = \frac{1}{2}\mathrm{Ei}\left(-az^2\right)$$

- Define the inverse function I through the relation

$$z^2 = -\frac{1}{a}\mathrm{Ei}^{(-1)}(2y) \equiv -\frac{1}{a}I(2y)$$

- Inverse function I follows the differentiation rule:

$$\frac{dI}{dy} = 2Ie^{-I}$$

5) Turn the crank

$$A(y) = az^2 + \frac{1}{2}\log z^2 = -I + \frac{1}{2}\log\left(-\frac{I}{a}\right)$$

- Take derivatives

$$W' = A'' = -6Ie^{-2I} + 4I^2e^{-2I} = \frac{\phi'^2}{6} + \frac{T'^2}{6}$$

- Identify

$$\begin{aligned} \phi' &= -2\sqrt{6}Ie^{-I} \\ T' &= 6\sqrt{-I}e^{-I} \end{aligned}$$

- Integrate

$$\begin{aligned} \phi &= -\sqrt{6}I = \sqrt{6}az^2 \\ T &= 6\sqrt{-I} = 6\sqrt{a}z \end{aligned}$$

Solution

$$
\begin{aligned}
\Phi(z) &= z^2 \\
T(z) &= \pm 2\sqrt{6} z \\
g_{MN} &= \frac{\eta_{MN}}{z^2}
\end{aligned}
$$

In the string frame, we have an AdS metric and quadratic dilaton!

Dilaton-tachyon potential

- Can invert solution to find superpotential $W(\phi, T)$
- One example:

$$W = -2\left(1 - \frac{\phi}{\sqrt{6}}\right) e^{\phi/\sqrt{6}} + 3e^{T^2/36}$$

- leads to the potential

$$\widetilde{V}(\phi, T) = \frac{T^2}{2} e^{T^2/18} + 2\phi^2 e^{2\phi/\sqrt{6}} - 12\left[3e^{T^2/36} - 2\left(1 - \frac{\phi}{\sqrt{6}}\right) e^{\phi/\sqrt{6}}\right]^2$$

- some terms in the potential are exotic; others are not e.g. $e^{2\phi/\sqrt{6}}$ matches 5D noncritical string theory

Operator/field dictionary

- Expand potential:

$$\widetilde{V}_{\pm}(\phi, T) \simeq -12 - 2\phi^2 - \frac{3}{2}T^2 + \cdots$$

- Match bulk masses to operator dimensions:

$$\begin{aligned} \Delta_\phi &= 2 + \sqrt{4 + m_\phi^2} = 2 \\ \Delta_T &= 2 + \sqrt{4 + m_T^2} = 3 \end{aligned}$$

Match to QCD operators?

$\Delta_\phi = 2$:

- No local, gauge invariant dimension 2 operator in QCD. So?
  - gluon mass?
  - SUSY QCD?
  - modified UV dynamics

$\Delta_T = 2 + \sqrt{4 + m_T^2} = 3$:

- Associate T with $\overline{q}q$ operator?
  - $T \sim z$: hard chiral symmetry breaking
  - constant mass splitting between chiral pairs
  - $\overline{q}q$ should originate from open string sector

Conclusions

- 5D Gravity-dilaton-tachyon solution
- Dynamical model of linear confinement
- Exotic potential, but certain terms found in noncritical string theory
- Simple dimension 2 and 3 operators, but relation to QCD not clear
- More work: Scalar perturbations, stability, connect to string theory . . .