Title: Conformal Gravity Challenges String Theory

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Abstract: The cosmological constant problem and the compatibility of gravity with quantum mechanics are the two most pressing problems in all of gravitational theory. While string theory nicely addresses the latter, it has so far failed to provide any compelling solution to the former. On the other hand, while conformal gravity nicely addresses the cosmological constant problem [by naturally quenching the amount by which the cosmological constant gravitates rather than by quenching the cosmological constant itself -- Mannheim, Prog. Part. Nuc. Phys. 56, 340 (2006)], the fourth order derivative conformal theory has long been thought to possess a ghost when quantized. However, it has recently been shown by Bender and Mannheim [Phys. Rev. Lett. 100, 110402 (2008)] that not only do theories based on fourth order derivative equations of motion not have ghosts, they actually never had any to begin with, with the apparent presence of ghosts being due entirely to treating operators which were not Hermitian on the real axis as though they were. When this is taken care of via an underlying PT symmetry that such theories are found to possess, there are then no ghosts at all and the theory is fully unitary. Conformal gravity is thus advanced as a fully consistent four-dimensional alternative to ten-dimensional string theory.

CONFORMAL GRAVITY CHALLENGES STRING THEORY –

UNITARITY OF FOURTH ORDER DERIVATIVE THEORIES

Philip D. Mannheim

Presentation at PASCOS-08. June 2008

1. C. M. Bender and P. D. Mannheim. *No-ghost theorem for the fourth-order derivative Pais-Uhlenbeck oscillator model*. June 2007 (0706.0207 [hep-th]). Phys. Rev. Lett. **100**. 110402 (2008).

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5. C. M. Bender. *Making Sense of Non-Hermitian Hamiltonians*. March 2007 (arXiv:hep-th/070309). Rep. Prog. Phys. **70**. 947 (2007).

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1 FOURTH ORDER UNITARITY PROBLEM

$$I = \frac{1}{2}\int d^4x \left[\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi - M^2\partial_\mu\phi\partial^\mu\phi\right] \tag{1}$$

$$(\partial_t^2 - \nabla^2)(\partial_t^2 - \nabla^2 + M^2)\phi(\bar{x}, t) = 0 \tag{2}$$

$$D^{(4)}(k^2) = \frac{1}{k^2(k^2 - M^2)} = \frac{1}{M^2}\left(\frac{1}{k^2 - M^2} - \frac{1}{k^2}\right) \tag{3}$$

$$\Delta_{\mathrm{F}}^{\mathrm{int}}(x - y) \quad = i\langle\Omega|T[\phi(x)\phi(y)]|\Omega\rangle \tag{4}$$

$$\sum_n |n\rangle\langle n| = \mathbf{1} \tag{5}$$

$$\rho(q^2) = (2\pi)^3 \sum_n \delta^4(k_\mu^n - q_\mu)|\langle\Omega|\phi(0)|k_\mu^n\rangle|^2\theta(q_0) \tag{6}$$

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$$D(\bar{x}, \bar{x}', E) = \sum_n \frac{\psi_n(\bar{x})\psi_n^*(\bar{x}')}{E - E_n} \tag{8}$$

$$\sum_n |n\rangle\langle n| - \sum_m |m\rangle\langle m| = \mathbf{1} \tag{9}$$

2 THE PAIS-UHLENBECK OSCILLATOR

$$\phi(\bar{x},t) \sim z(t)e^{i\bar{k}\cdot\bar{x}}, \quad \omega_1 = (\bar{k}^2 + M^2)^{1/2}, \quad \omega_2 = |\bar{k}| \tag{10}$$

$$\frac{d^4 z}{dt^4} + (\omega_1^2 + \omega_2^2)\frac{d^2 z}{dt^2} + \omega_1^2\omega_2^2 z = 0 \tag{11}$$

$$I_{\rm PU} = \frac{\gamma}{2}\int dt[\ddot{z}^2 - (\omega_1^2 + \omega_2^2)\dot{z}^2 + \omega_1^2\omega_2^2 z^2] \tag{12}$$

$$H_{\rm PU} = \frac{p_x^2}{2\gamma} + p_z x + \frac{\gamma}{2}\left(\omega_1^2 + \omega_2^2\right)x^2 - \frac{\gamma}{2}\omega_1^2\omega_2^2 z^2, \quad x = \dot{z} \tag{13}$$

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$$p_x = -i\frac{\partial}{\partial x}, \quad p_z = -i\frac{\partial}{\partial z} \tag{19}$$

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$$y = e^{-\pi p_z z/2} z e^{\pi p_z z/2} = -iz, \; q = e^{-\pi p_z z/2} p_z e^{\pi p_z z/2} = ip_z \tag{22}$$

$$H = \frac{p^2}{2\gamma} - iqx + \frac{\gamma}{2}\left(\omega_1^2+\omega_2^2\right)x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2, \quad p = p_x \tag{23}$$

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$$\begin{aligned} z &= a_1 + a_1^\dagger + a_2 + a_2^\dagger, \\ p_z &= i\gamma\omega_1\omega_2^2(a_1 - a_1^\dagger) + i\gamma\omega_1^2\omega_2(a_2 - a_2^\dagger), \\ x &= -i\omega_1(a_1 - a_1^\dagger) - i\omega_2(a_2 - a_2^\dagger), \\ p_x &= -\gamma\omega_1^2(a_1 + a_1^\dagger) - \gamma\omega_2^2(a_2 + a_2^\dagger) \end{aligned} \tag{15}$$

$$H_{\rm PU} = 2\gamma(\omega_1^2 - \omega_2^2)(\omega_1^2 a_1^\dagger a_1 - \omega_2^2 a_2^\dagger a_2) + (\omega_1 + \omega_2)/2 \tag{16}$$

$$[a_1, a_1^\dagger] = \frac{1}{2\gamma\omega_1(\omega_1^2 - \omega_2^2)}, \quad [a_2, a_2^\dagger] = -\frac{1}{2\gamma\omega_2(\omega_1^2 - \omega_2^2)} \tag{17}$$

$$\begin{aligned} a_1|\Omega\rangle &= a_2|\Omega\rangle = 0, E(|\Omega\rangle) = \frac{1}{2}(\omega_1 + \omega_2), \ \langle\Omega|a_2 a_2^\dagger|\Omega\rangle < 0, \\ a_1|\hat{\Omega}\rangle &= a_2^\dagger|\hat{\Omega}\rangle = 0, E(|\hat{\Omega}\rangle) = \frac{1}{2}(\omega_1 - \omega_2), \ \langle\hat{\Omega}|a_2^\dagger a_2|\hat{\Omega}\rangle > 0 \end{aligned} \tag{18}$$

3 SOLUTION: THE HAMILTONIAN IS NOT HERMITIAN – BUT IT IS PT SYMMETRIC

$$p_x = -i\frac{\partial}{\partial x}, \quad p_z = -i\frac{\partial}{\partial z} \tag{19}$$

$$\psi_0(z,x) = \exp\left[\frac{\gamma}{2}(\omega_1+\omega_2)\omega_1\omega_2 z^2 + i\gamma\omega_1\omega_2 zx - \frac{\gamma}{2}(\omega_1+\omega_2)x^2\right] \tag{20}$$

$$[z,p_z] = i, \quad \left[e^{i\theta}z, -\frac{i}{e^{i\theta}}\frac{\partial}{\partial z}\right]\psi(e^{i\theta}z) = i\psi(e^{i\theta}z) \tag{21}$$

$$y = e^{-\pi p_z z/2} z e^{\pi p_z z/2} = -iz, \; q = e^{-\pi p_z z/2} p_z e^{\pi p_z z/2} = ip_z \tag{22}$$

$$H = \frac{p^2}{2\gamma} - iqx + \frac{\gamma}{2}\left(\omega_1^2+\omega_2^2\right)x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2, \quad p = p_x \tag{23}$$

$$[p,x] = i, \quad [q,y] = i \tag{24}$$

$$C^2 = 1, \quad [C,PT] = 0, \quad [C,H] = 0, \quad C = e^Q P \tag{25}$$

$$Q = \alpha[pq + \gamma^2\omega_1^2\omega_2^2 xy], \quad \alpha = \frac{1}{\gamma\omega_1\omega_2}\log\left(\frac{\omega_1+\omega_2}{\omega_1-\omega_2}\right) \tag{26}$$

$$\tilde{H} = e^{-Q/2}He^{Q/2} = \frac{p^2}{2\gamma} + \frac{q^2}{2\gamma\omega_1^2} + \frac{\gamma}{2}\omega_1^2 x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \tag{27}$$

4 THE NORM IS NOT THE DIRAC NORM – IT IS THE PT NORM

$$\tilde{H} = e^{-Q/2}He^{Q/2} = \frac{p^2}{2\gamma} + \frac{q^2}{2\gamma\omega_1^2} + \frac{\gamma}{2}\omega_1^2 x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \quad (28)$$

$$\tilde{H}|\tilde{n}\rangle = E_n|\tilde{n}\rangle, \quad H|n\rangle = E_n|n\rangle, \quad |n\rangle = e^{Q/2}|\tilde{n}\rangle \quad (29)$$

$$\langle\tilde{n}|\tilde{H} = E_n\langle\tilde{n}|, \quad \langle n| \equiv \langle\tilde{n}|e^{Q/2}, \quad \langle n|e^{-Q}H = \langle n|e^{-Q}E_n \quad (30)$$

$$\langle\tilde{n}|\tilde{m}\rangle = \delta_{m,n}, \quad \sum_n |\tilde{n}\rangle\langle\tilde{n}| = \mathbf{1}, \quad \tilde{H} = \sum_n |\tilde{n}\rangle E_n\langle\tilde{n}| \quad (31)$$

$$\langle n|e^{-Q}|m\rangle = \delta_{m,n}, \quad \sum_n |n\rangle\langle n|e^{-Q} = \mathbf{1}, \quad H = \sum_n |n\rangle E_n\langle n|e^{-Q} \quad (32)$$

$$\sum_n |n\rangle\langle n|PC = \mathbf{1}, \quad H = \sum_n |n\rangle E_n\langle n|PC, \quad C_n = \pm 1 \quad (33)$$

$$\langle x,y|e^{-iHt}|x',y'\rangle = \sum_n \psi_n(x,y)C_n e^{-iE_nt}\psi_n^c(x',y') \quad (34)$$

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5 NON-HERMITICITY AND UNITARITY

$$i\frac{d}{dt}|\alpha_S(t)\rangle = H|\alpha_S(t)\rangle, \qquad -i\frac{d}{dt}\langle\alpha_S(t)| = \langle\alpha_S(t)|H^\dagger \tag{35}$$

$$|\alpha_S(t)\rangle = e^{-iHt}|\alpha_S(0)\rangle, \qquad \langle\alpha_S(t)| = \langle\alpha_S(0)|e^{iH^\dagger t} \tag{36}$$

$$\langle\alpha_S(t)|\alpha_S(t)\rangle = \langle\alpha_S(0)|e^{iH^\dagger t}e^{-iHt}|\alpha_S(0)\rangle \neq \langle\alpha_S(0)|\alpha_S(0)\rangle \tag{37}$$

$$\langle\alpha_S(t)|A_S|\alpha_S(t)\rangle = \langle\alpha_S(0)|e^{iH^\dagger t}A_S e^{-iHt}|\alpha_S(0)\rangle \tag{38}$$

$$A_H(t) = e^{iH^\dagger t}A_S e^{-iHt} \tag{39}$$

$$i\frac{d}{dt}A_H(t) = A_H(t)H - H^\dagger A_H(t) \tag{40}$$

$$i\frac{d}{dt}A_H(t) = A_H(t)H - HA_H(t) \tag{41}$$

$$i\frac{d}{dt}|\alpha_S(t)\rangle = H|\alpha_S(t)\rangle, \qquad -i\frac{d}{dt}\langle\hat{\alpha}_S(t)| = \langle\hat{\alpha}_S(t)|H \tag{42}$$

$$|\alpha_S(t)\rangle = e^{-iHt}|\alpha_S(0)\rangle, \qquad \langle\hat{\alpha}_S(t)| = \langle\hat{\alpha}_S(0)|e^{iHt} \tag{43}$$

$$\langle\hat{\alpha}_S(t)|\alpha_S(t)\rangle = \langle\hat{\alpha}_S(0)|e^{iHt}e^{-iHt}|\alpha_S(0)\rangle = \langle\hat{\alpha}_S(0)|\alpha_S(0)\rangle \tag{44}$$

$$\langle\hat{\alpha}_S(t)| = \langle\alpha_S(t)|e^{-Q}, \quad H^\dagger = e^{-Q}He^{Q} \tag{45}$$

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1 FOURTH ORDER UNITARITY PROBLEM

$$I = \frac{1}{2}\int d^4x \left[\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi - M^2\partial_\mu\phi\partial^\mu\phi\right] \tag{1}$$

$$(\partial_t^2 - \nabla^2)(\partial_t^2 - \nabla^2 + M^2)\phi(\bar{x}, t) = 0 \tag{2}$$

$$D^{(4)}(k^2) = \frac{1}{k^2(k^2 - M^2)} = \frac{1}{M^2}\left(\frac{1}{k^2 - M^2} - \frac{1}{k^2}\right) \tag{3}$$

$$\Delta_{\mathrm{F}}^{\mathrm{int}}(x - y) \quad = i\langle\Omega|T[\phi(x)\phi(y)]|\Omega\rangle \tag{4}$$

$$\sum_n |n\rangle\langle n| = \mathbf{1} \tag{5}$$

$$\rho(q^2) = (2\pi)^3 \sum_n \delta^4(k_\mu^n - q_\mu)|\langle\Omega|\phi(0)|k_\mu^n\rangle|^2\theta(q_0) \tag{6}$$

$$\Delta_{\mathrm{F}}^{\mathrm{int}}(x - y) = \int_0^\infty dm^2\, \rho(m^2)\Delta_{(\mathrm{F},2)}^{\mathrm{free}}(x - y; m^2) \tag{7}$$

$$D(\bar{x}, \bar{x}', E) = \sum_n \frac{\psi_n(\bar{x})\psi_n^*(\bar{x}')}{E - E_n} \tag{8}$$

$$\sum_n |n\rangle\langle n| - \sum_m |m\rangle\langle m| = \mathbf{1} \tag{9}$$

5 NON-HERMITICITY AND UNITARITY

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$$|\alpha_S(t)\rangle = e^{-iHt}|\alpha_S(0)\rangle, \qquad \langle\alpha_S(t)| = \langle\alpha_S(0)|e^{iH^\dagger t} \quad (36)$$

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$$\langle\hat{\alpha}_S(t)| = \langle\alpha_S(t)|e^{-Q}, \quad H^\dagger = e^{-Q}He^{Q} \quad (45)$$

6 NON-DIAGONALIZABILITY AND UNITARITY– THE SINGULAR EQUAL-FREQUENCY LIMIT

$$\psi_0(x,y) = \exp\left[-\frac{\gamma}{2}(\omega_1+\omega_2)(x^2+\omega_1\omega_2 y^2) - \gamma\omega_1\omega_2 yx\right] \quad (46)$$

$$\psi_0(x,y,t) = \psi_0(x,y)\exp(-iE_0 t), \quad E_0 = (\omega_1+\omega_2)/2 \quad (47)$$

$$\begin{aligned} \psi_1(x,y,t) &= (x+\omega_2 y)\psi_0(x,y,t)e^{-i\omega_1 t}, \quad E_1 = E_0+\omega_1 \\ \psi_2(x,y,t) &= (x+\omega_1 y)\psi_0(x,y,t)e^{-i\omega_2 t}, \quad E_2 = E_0+\omega_2 \end{aligned} \quad (48)$$

$$\begin{aligned} \psi_3(x,y,t) &= \left[(x+\omega_2 y)^2 - 1/2\gamma\omega_1\right]\psi_0(x,y,t)e^{-2i\omega_1 t}, \quad E_3 = E_0+2\omega_1 \\ \psi_4(x,y,t) &= \left[(x+\omega_1 y)(x+\omega_2 y) - 1/\gamma(\omega_1+\omega_2)\right]\psi_0^{\mathrm{R}}(x,y,t) \\ &\quad \times e^{-i(\omega_1+\omega_2)t}, \quad E_4 = E_0+\omega_1+\omega_2 \\ \psi_5(x,y,t) &= \left[(x+\omega_1 y)^2 - 1/2\gamma\omega_2\right]\psi_0^{\mathrm{R}}(x,y,t)e^{-2i\omega_2 t}, \quad E_5 = E_0+2\omega_2 \end{aligned} \quad (49)$$

$$\omega_1 \equiv \omega+\epsilon, \quad \omega_2 \equiv \omega-\epsilon, \quad \epsilon \to 0 \quad (50)$$

$$\hat{\psi}_0(x,y,t) = \exp\left[-\gamma\omega^3 y^2 - \gamma\omega^2 yx - \gamma\omega x^2 - i\omega t\right], \hat{E}_0 = \omega \quad (51)$$

$$\hat{\psi}_1(x,y,t) = (x+\omega y)\hat{\psi}_0(x,y,t)e^{-i\omega t}, \quad \hat{E}_1 = \hat{E}_0+\omega \quad (52)$$

$$\hat{\psi}_2(x,y,t) = \left[(x+\omega y)^2 - 1/2\gamma\omega\right]\hat{\psi}_0(x,y,t)e^{-2i\omega t}, \quad \hat{E}_2 = \hat{E}_0+2\omega \quad (53)$$

7 THE MISSING ENERGY EIGENSTATES....

$$H_{1P}(\epsilon) = \frac{1}{2\omega}\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} \tag{54}$$

$$S^{-1}\left(\frac{1}{2\omega}\right)\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} S = \begin{pmatrix} 2\omega+\epsilon & 0 \\ 0 & 2\omega-\epsilon \end{pmatrix} \tag{55}$$

$$\begin{aligned} S &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} 2\omega+\epsilon & -(4\omega^2-\epsilon^2)\epsilon \\ \epsilon & (2\omega+\epsilon)\epsilon^2 \end{pmatrix}, \\ S^{-1} &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} (2\omega+\epsilon)\epsilon^2 & (4\omega^2-\epsilon^2)\epsilon \\ -\epsilon & 2\omega+\epsilon \end{pmatrix} \end{aligned} \tag{56}$$

$$\begin{aligned} |2\omega+\epsilon\rangle &\equiv \begin{pmatrix} 2\omega+\epsilon \\ \epsilon \end{pmatrix}, \\ |2\omega-\epsilon\rangle &\equiv \begin{pmatrix} 2\omega-\epsilon \\ -\epsilon \end{pmatrix} \end{aligned} \tag{57}$$

$$H_{1P}(\epsilon=0) = 2\omega\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{58}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} c+d \\ d \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} \tag{59}$$

8 ... BECAME NONSTATIONARY

$$\begin{aligned}\psi_{1a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_2(x,y,t)-\psi_1(x,y,t)}{2\epsilon}\\ &= [(x+\omega y)it+y]\,\psi_0(x,y,t)e^{-i\omega t} \qquad (60)\end{aligned}$$

$$\begin{aligned}\psi_{2a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_5(x,y,t)-\psi_3(x,y,t)}{2\epsilon}\\ &= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2it+2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right]\\ &\quad\times\psi_0(x,y,t)e^{-2i\omega t} \qquad (61)\end{aligned}$$

$$\begin{aligned}\psi_{2b}(x,y,t) &= \lim_{\epsilon\to 0}\frac{2\psi_4(x,y,t)-\psi_3(x,y,t)-\psi_5(x,y,t)}{2\epsilon^2}\\ &= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2t^2-\left(2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right)2it\right.\\ &\quad\left.-2y^2+\frac{1}{2\gamma\omega^3}\right]\psi_0(x,y,t)e^{-2i\omega t} \qquad (62)\end{aligned}$$

$$i\frac{\partial}{\partial t}\psi(x,y,t)=\left(-\frac{1}{2\gamma}\frac{\partial^2}{\partial x^2}-x\frac{\partial}{\partial y}+\gamma\omega^2x^2+\frac{\gamma}{2}\omega^4y^2\right)\psi(x,y,t) \qquad (63)$$

$$i\frac{\partial}{\partial t}\int dx\,dy\,\psi_B^c(x,y,t)\psi_A(x,y,t)=-\int dx\,dy\,x\frac{\partial}{\partial y}\left[\psi_B^c(x,y,t)\psi_A(x,y,t)\right] \qquad (64)$$

$$i\frac{\partial}{\partial t}\int dx\,dy\,\psi_B^c(x,y,t)\psi_A(x,y,t)=0 \qquad (65)$$

Conformal Supergravity in Twistor-String Theory

N. Berkovits and E. Witten

June 2004 (arXiv:hep-th/040605). JHEP 0408 (2004) 009

"The net effect is that the translation generator D acts as

$$\begin{pmatrix} P & * \\ 0 & P \end{pmatrix}$$

where P would represent ordinary translations and the off-diagonal $*$ arises from $[D, \partial_I] \neq 0$.

This matrix is not diagonalizable. This clashes with our usual experience. We are accustomed to the idea that the translation generators are Hermitian operators and so can be diagonalized. However, conformal supergravity is not a unitary theory, and one symptom of this is that the translation generators are un-diagonalizable."

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$$S^{-1}\left(\frac{1}{2\omega}\right)\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} S = \begin{pmatrix} 2\omega+\epsilon & 0 \\ 0 & 2\omega-\epsilon \end{pmatrix} \qquad (55)$$

$$\begin{aligned} S &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} 2\omega+\epsilon & -(4\omega^2-\epsilon^2)\epsilon \\ \epsilon & (2\omega+\epsilon)\epsilon^2 \end{pmatrix}, \\ S^{-1} &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} (2\omega+\epsilon)\epsilon^2 & (4\omega^2-\epsilon^2)\epsilon \\ -\epsilon & 2\omega+\epsilon \end{pmatrix} \end{aligned} \qquad (56)$$

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$$H_{1P}(\epsilon=0) = 2\omega\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad (58)$$

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$$\begin{aligned}\psi_{1a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_2(x,y,t)-\psi_1(x,y,t)}{2\epsilon}\\ &= [(x+\omega y)it+y]\,\psi_0(x,y,t)e^{-i\omega t} \qquad (60)\end{aligned}$$

$$\begin{aligned}\psi_{2a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_5(x,y,t)-\psi_3(x,y,t)}{2\epsilon}\\ &= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2it+2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right]\\ &\quad \times\psi_0(x,y,t)e^{-2i\omega t} \qquad (61)\end{aligned}$$

$$\begin{aligned}\psi_{2b}(x,y,t) &= \lim_{\epsilon\to 0}\frac{2\psi_4(x,y,t)-\psi_3(x,y,t)-\psi_5(x,y,t)}{2\epsilon^2}\\ &= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2t^2-\left(2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right)2it\right.\\ &\quad \left.-2y^2+\frac{1}{2\gamma\omega^3}\right]\psi_0(x,y,t)e^{-2i\omega t} \qquad (62)\end{aligned}$$

$$i\frac{\partial}{\partial t}\psi(x,y,t)=\left(-\frac{1}{2\gamma}\frac{\partial^2}{\partial x^2}-x\frac{\partial}{\partial y}+\gamma\omega^2x^2+\frac{\gamma}{2}\omega^4y^2\right)\psi(x,y,t) \qquad (63)$$

$$i\frac{\partial}{\partial t}\int dx\,dy\,\psi_B^c(x,y,t)\psi_A(x,y,t)=-\int dx\,dy\,x\frac{\partial}{\partial y}\left[\psi_B^c(x,y,t)\psi_A(x,y,t)\right] \qquad (64)$$

$$i\frac{\partial}{\partial t}\int dx\,dy\,\psi_B^c(x,y,t)\psi_A(x,y,t)=0 \qquad (65)$$

Conformal Supergravity in Twistor-String Theory

N. Berkovits and E. Witten

June 2004 (arXiv:hep-th/040605), JHEP 0408 (2004) 009

"The net effect is that the translation generator D acts as

$$\begin{pmatrix} P & * \\ 0 & P \end{pmatrix}$$

where P would represent ordinary translations and the off-diagonal $*$ arises from $[D, \partial_I] \neq 0$.

This matrix is not diagonalizable. This clashes with our usual experience. We are accustomed to the idea that the translation generators are Hermitian operators and so can be diagonalized. However, conformal supergravity is not a unitary theory, and one symptom of this is that the translation generators are un-diagonalizable."

7 THE MISSING ENERGY EIGENSTATES....

$$H_{1P}(\epsilon) = \frac{1}{2\omega}\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} \tag{54}$$

$$S^{-1}\left(\frac{1}{2\omega}\right)\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} S = \begin{pmatrix} 2\omega+\epsilon & 0 \\ 0 & 2\omega-\epsilon \end{pmatrix} \tag{55}$$

$$\begin{aligned} S &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} 2\omega+\epsilon & -(4\omega^2-\epsilon^2)\epsilon \\ \epsilon & (2\omega+\epsilon)\epsilon^2 \end{pmatrix}, \\ S^{-1} &= \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} (2\omega+\epsilon)\epsilon^2 & (4\omega^2-\epsilon^2)\epsilon \\ -\epsilon & 2\omega+\epsilon \end{pmatrix} \end{aligned} \tag{56}$$

$$\begin{aligned} |2\omega+\epsilon\rangle &\equiv \begin{pmatrix} 2\omega+\epsilon \\ \epsilon \end{pmatrix}, \\ |2\omega-\epsilon\rangle &\equiv \begin{pmatrix} 2\omega-\epsilon \\ -\epsilon \end{pmatrix} \end{aligned} \tag{57}$$

$$H_{1P}(\epsilon=0) = 2\omega\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{58}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} c+d \\ d \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} \tag{59}$$

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2 THE PAIS-UHLENBECK OSCILLATOR

$$\phi(\bar{x},t) \sim z(t)e^{i\bar{k}\cdot\bar{x}}, \quad \omega_1 = (\bar{k}^2 + M^2)^{1/2}, \quad \omega_2 = |\bar{k}| \tag{10}$$

$$\frac{d^4 z}{dt^4} + (\omega_1^2 + \omega_2^2)\frac{d^2 z}{dt^2} + \omega_1^2\omega_2^2 z = 0 \tag{11}$$

$$I_{\rm PU} = \frac{\gamma}{2}\int dt[\ddot{z}^2 - (\omega_1^2 + \omega_2^2)\dot{z}^2 + \omega_1^2\omega_2^2 z^2] \tag{12}$$

$$H_{\rm PU} = \frac{p_x^2}{2\gamma} + p_z x + \frac{\gamma}{2}\left(\omega_1^2 + \omega_2^2\right)x^2 - \frac{\gamma}{2}\omega_1^2\omega_2^2 z^2, \quad x = \dot{z} \tag{13}$$

$$[x, p_x] = i, \quad [z, p_z] = i \tag{14}$$

$$\begin{aligned} z &= a_1 + a_1^\dagger + a_2 + a_2^\dagger, \\ p_z &= i\gamma\omega_1\omega_2^2(a_1 - a_1^\dagger) + i\gamma\omega_1^2\omega_2(a_2 - a_2^\dagger), \\ x &= -i\omega_1(a_1 - a_1^\dagger) - i\omega_2(a_2 - a_2^\dagger) \end{aligned}$$

$$\rho(q^2) = (2\pi)^3\sum_n \delta^4(k_\mu^n - q_\mu)|\langle\Omega|\phi(0)|k_\mu^n\rangle|^2\theta(q_0) \tag{6}$$

$$\Delta_{\rm F}^{\rm int}(x-y) = \int_0^\infty dm^2\,\rho(m^2)\Delta_{\rm (F,2)}^{\rm free}(x-y;m^2) \tag{7}$$

$$D(\bar{x},\bar{x}',E) = \sum_n \frac{\psi_n(\bar{x})\psi_n^*(\bar{x}')}{E - E_n} \tag{8}$$

$$\sum_n |n\rangle\langle n| - \sum_m |m\rangle\langle m| = \mathbf{1} \tag{9}$$

1 FOURTH ORDER UNITARITY PROBLEM

$$I = \frac{1}{2}\int d^4x \left[\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi - M^2\partial_\mu\phi\partial^\mu\phi\right] \qquad (1)$$

$$(\partial_t^2 - \nabla^2)(\partial_t^2 - \nabla^2 + M^2)\phi(\bar{x}, t) = 0 \qquad (2)$$

$$D^{(4)}(k^2) = \frac{1}{k^2(k^2 - M^2)} = \frac{1}{M^2}\left(\frac{1}{k^2 - M^2} - \frac{1}{k^2}\right) \qquad (3)$$

$$\Delta_{\mathrm{F}}^{\mathrm{int}}(x-y) \quad = i\langle\Omega|T[\phi(x)\phi(y)]|\Omega\rangle \qquad (4)$$

$$\sum_n |n\rangle\langle n| = \mathbf{1} \qquad (5)$$

$$\rho(q^2) = (2\pi)^3 \sum_n \delta^4(k_\mu^n - q_\mu)|\langle\Omega|\phi(0)|k_\mu^n\rangle|^2\theta(q_0) \qquad (6)$$

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$$H_{\rm PU} = 2\gamma(\omega_1^2 - \omega_2^2)(\omega_1^2 a_1^\dagger a_1 - \omega_2^2 a_2^\dagger a_2) + (\omega_1 + \omega_2)/2 \tag{16}$$

$$[a_1, a_1^\dagger] = \frac{1}{2\gamma\omega_1(\omega_1^2 - \omega_2^2)}, \quad [a_2, a_2^\dagger] = -\frac{1}{2\gamma\omega_2(\omega_1^2 - \omega_2^2)} \tag{17}$$

$$\begin{aligned} a_1|\Omega\rangle &= a_2|\Omega\rangle = 0, \; E(|\Omega\rangle) = \frac{1}{2}(\omega_1 + \omega_2), \; \langle\Omega|a_2 a_2^\dagger|\Omega\rangle < 0, \\ a_1|\hat{\Omega}\rangle &= a_2^\dagger|\hat{\Omega}\rangle = 0, \; E(|\hat{\Omega}\rangle) = \frac{1}{2}(\omega_1 - \omega_2), \; \langle\hat{\Omega}|a_2^\dagger a_2|\hat{\Omega}\rangle > 0 \end{aligned} \tag{18}$$

CONFORMAL GRAVITY CHALLENGES STRING THEORY – UNITARITY OF FOURTH ORDER DERIVATIVE THEORIES

Philip D. Mannheim

Presentation at PASCOS-08. June 2008

1. C. M. Bender and P. D. Mannheim. *No-ghost theorem for the fourth-order derivative Pais-Uhlenbeck oscillator model*. June 2007 (0706.0207 [hep-th]). Phys. Rev. Lett. **100**. 110402 (2008).

2. P. D. Mannheim. *Conformal Gravity Challenges String Theory*. Pascos-07. July 2007 (0707.2283 [hep-th]).

3. C. M. Bender and P. D. Mannheim. *Giving up the ghost*. July 2007. Jour. Phys. A. in press.

4. C. M. Bender and P. D. Mannheim. *Exactly solvable PT-symmetric Hamiltonian having no Hermitian counterpart*. April 2008 (0804.4190 [hep-th]).

5. C. M. Bender. *Making Sense of Non-Hermitian Hamiltonians*. March 2007 (arXiv:hep-th/070309). Rep. Prog. Phys. **70**. 947 (2007).

6. P. D. Mannheim and A. Davidson. *Fourth order theories without ghosts*. January 2000 (hep-th/0001115).

7. P. D. Mannheim and A. Davidson. *Dirac quantization of the Pais-Uhlenbeck fourth order oscillator*. August 2004 (hep-th/0408104). Phys. Rev. **A71**. 042110 (2005).

8. P. D. Mannheim. *Solution to the ghost problem in fourth order derivative theories*. August 2006 (hep-th/0608154). Found. Phys. **37**. 532 (2007).

9. P. D. Mannheim. *Alternatives to dark matter and dark energy*. May 2005 (astro-ph/0505266). Progress in Particle and Nuclear Physics **56**. 340 (2006).