Title: dS Minimum without anti-D3 branes and Large Volume Axionic Swiss-Cheese Inflation

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Abstract: We first discuss the possibility of getting a non-supersymmetric dS minimum with the inclusion of perturbative and non-perturbative alpha\' corrections and instanton contributions in the large volume limit of certain Swiss Cheese Calabi Yau orientifold type IIB compactifications. We then discuss axionic slow roll inflation with the NS-NS axions providing a flat direction for slow-roll inflation to proceed from a saddle point to the nearest dS minimum.

"SWISS-CHEESE" CALABI-YAU (CONLON + QUEVEDO et al '05)

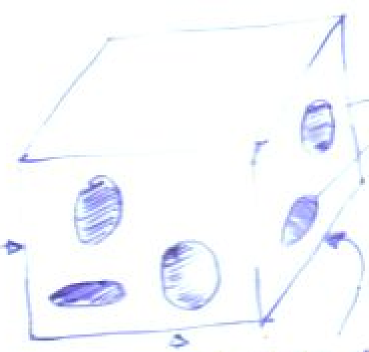
1) • A 3-MANIFOLD WITH 2-CYCLES: ONE $t_{B(IG)}$ IS LARGE AND THE OTHER $t_{S(MALL)}$'s ARE SMALL

• $\mathcal{V} = t_B^{3/2} - \sum_i t_{S,i}^{3/2}$ $\left[\frac{\partial^2 \mathcal{V}}{\partial t_i \partial t_j} \text{ HAS SIGNATURE } (1, h^2-1) \right]$

• INCREASING $\begin{pmatrix} t_{S,i} \\ t_B \end{pmatrix}$ $\begin{pmatrix} \text{DECREASES} \\ \text{INCREASES} \end{pmatrix}$ $\mathcal{V}$

(t_B) → CONTROLS OVERALL VOLUME

2) •



PICTURE OF A "SWISS CHEESE" CALABI-YAU

• $\mathcal{V} = \left(\tau_B + \sum_i a_i \tau_{S,i}\right)^{3/2} - \left(\sum_i b_i \tau_{S,i}\right)^{3/2} - \dots$,

τ_j : CY$_3$ DIVISOR$_j$ VOLUME

• $\mathbb{P}^4_{[1,1,1,6,9]}$: $\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_B^{3/2} - \tau_s^{3/2}\right)$,

$\tau_s = \frac{t_s^2}{2}$, $\tau_B = \frac{(t_s + 6 t_B)^2}{2}$

APPLICATION OF SWISS-CHEESE CY_3's TO LARGE VOLUME SCENARIOS (BALASUBRAMANIAN et al. 05)

FOR CY_3 ($h^{2,1} < h^{1,2}$) COMPACTIFICATIONS, IN THE LARGE VOLUME LIMIT, COMBINATION OF $(\alpha')^3$ CORRECTIONS AND NON-PERTURBATIVE CONTRIBUTIONS TO THE SUPERPOTENTIAL, GENERICALLY GIVES RISE TO A LARGE VOLUME NON-SUPERSYMMETRIC AdS MINIMUM (KKLT: AdS VACUUM IS SUPERSYMMETRIC).

- SAME SHOWN FOR ... :
  $V'(\tau_B \sim \mathcal{V}^{2/3}, \tau_S \sim \ln \mathcal{V} \;\&\; \mathcal{V} \to \infty) \longrightarrow 0^-$
- THE AdS MINIMUM IS INDEPENDENT OF THE FLUX SUPERPOTENTIAL
- THE NON-PERTURBATIVE (INSTANTON) EFFECTS DO NOT DESTABILIZE COMPLEX STRUCTURE MODULI AND τ
- SUPERSYMMETRY IS BROKEN BY KÄHLER MODULI ONLY

PERTURBATIVE α' - CORRECTIONS (TYPE IIB on CY_3 + 3 FORM FLUXES)

[BECKERS et al, '02]

1) THE $(\alpha')^3$ CORRECTIONS CONTRIBUTING TO THE KÄHLER MODULI (HYPERMULTIPLETS) SPACE METRIC ARE OBTAINED FROM:

$$\int d^{10}x \sqrt{g}\, e^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{(\alpha')^3 \zeta(3)}{3 \cdot 2^{11}} J_0 + (\alpha')^3 (\nabla^2\phi)\, Q\right)$$

$$J_0 = t^{M_1N_1 \cdots M_4N_4}\, t_{M_1'N_1' \cdots M_4'N_4'}\, R^{M_1'N_1'}{}_{M_1N_1} \cdots R^{M_4'N_4'}{}_{M_4N_4}$$

$$+ \frac{1}{4}\epsilon^{ABM_1N_1 \cdots M_4N_4}\, \epsilon_{ABM_1'N_1' \cdots M_4'N_4'}\, R^{M_1'N_1'}{}_{M_1N_1} \cdots R^{M_4'N_4'}{}_{M_4N_4}$$

TEN-DIMENSIONAL GENERALIZATION OF EIGHT-DIMENSIONAL EULER DENSITY

[SCHWARZ] $\rightarrow -\frac{1}{2}\epsilon^{M_1N_1 \cdots M_4N_4}$

$$-\frac{1}{2}\Big[\left(\delta^{M_1M_2}\delta^{N_1N_2} - \delta^{M_1N_2}\delta^{N_1M_2}\right)\left(\delta^{M_3M_4}\delta^{N_3N_4} \cdots\right)$$
$$+\left(\delta^{M_1[M_3}\delta^{N_1|N_3]}\right)\left(\delta^{M_2[M_4}\delta^{N_2|N_4]}\right)$$
$$+\left(\delta^{M_1[M_3}\delta^{N_1|N_3]}\right)\left(\delta^{M_2[M_4}\delta^{N_2|N_4]}\right)\Big]$$

$$+\frac{1}{2}\Big[\delta^{M_1M_2}\delta^{N_1M_3}\delta^{N_3M_4}\delta^{N_4M_1} + \delta^{M_1M_2}\delta^{M_3M_4}\delta^{N_2N_4}\delta^{N_1 \cdots}$$
$$+\delta^{M_1M_3}\delta^{N_3M_4}\delta^{M_2N_4}\delta^{N_2M_1}\Big]$$

$$Q = \frac{1}{12(2\pi)^3}\left(R_{IJ}{}^{KL}R_{KL}{}^{MN}R_{MN}{}^{IJ} - 2R_I{}^K{}_J{}^L R_K{}^M{}_L{}^N R_M{}^I{}_N{}^J\right)$$

(+ TERMS ": ANTISYMMETRIZATION w.r.t. (M_1N_1) (M_2M_3) (M_3N_3) (M_4N_4))

2) THE HYPERMULTIPLET MODULI SPACE METRIC:

$$G_{a\bar{b}} = -\frac{\partial^2}{\partial z^a \partial \bar{z}^b}\left[\ln\left(X^i \bar{F}_i(\bar{X}) + \bar{X}^i F_i(X)\right)\right]$$

$i \to (0,a)$

$z^a = \frac{X^a}{X^0}$, $a \to h^{1,1}(CY_3)$

(PREPOTENTIAL) RECEIVES PERTURBATIVE AND NON-PERTURBATIVE WORLD-SHEET CORRECTIONS:

NON-PERTURBATIVE → DISCUSSED LATER

$$F(X) = \frac{i}{3} K_{abc} \frac{X^a X^b X^c}{X^0} + (X^0)^2 \xi$$

K_{abc} → CLASSICAL INTERSECTION #s

$\xi \to -\frac{\zeta(3)}{2}\chi(CY_3)(\alpha')^3$

$$K = -\ln\left[X^i \bar{F}_i + \bar{X}^i F_i\right]$$

$$= -\ln\left[-\frac{i}{6} K_{abc}(z^a - \bar{z}^a)(z^b - \bar{z}^b)(z^c - \bar{z}^c) + 4\xi\right]$$

IN EINSTEIN FRAME

$$K_T = -\ln\left[-i(\tau - \bar{\tau})\right] - 2\ln\left(\mathcal{V} + \frac{1}{2}\xi e^{-\frac{3\phi}{2}}\right) - \ln\left(-i\int_{CY_3} \Omega \wedge \bar{\Omega}\right)$$

[K_{np} TO BE DISCUSSED LATER]

3) THE CORRECTED SUPERGRAVITY POTENTIAL:

(I → C.S + KÄHLER + τ)

$$V = e^{K}\left[(G^{-1})^{I\bar{J}} D_I W \, \bar{D}_{\bar{J}} \bar{W} - 3|W|^2 \right]_{D_{c.s.}W=0}$$

$$= e^{K}\left[(G^{-1})^{\alpha\bar{\beta}} D_\alpha W \, \bar{D}_{\bar{\beta}} \bar{W} + (G^{-1})^{\tau\bar{\tau}} D_\tau W \, \bar{D}_{\bar{\tau}} \bar{W} - \frac{9\xi \mathcal{V} e^{-\phi}}{(\xi-\mathcal{V})(\xi+2\mathcal{V})} \left(W \bar{D}_{\bar{\tau}} \bar{W} + \bar{W} D_\tau W \right) - \frac{3\xi(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2)}{(\xi-\mathcal{V})(\xi+2\mathcal{V})} |W|^2 \right]$$

- "NO-SCALE" STRUCTURE IS NOT PRESERVED ($\because \mathcal{V}$-DEPENDENCE [OF V])
- $|W|^2$-TERM IS NOT CANCELLED

NON-PERTURBATIVE α'- AND INSTANTON CORRECTIONS IN $\mathcal{N}=1$ TYPE IIB COMPACTIFICATIONS

1) SOME NON-PERTURBATIVE (α'- CORRECTIONS (INHERITED FROM THE PARENT $\mathcal{N}=2$ THEORY) SURVIVE IN THE LARGE VOLUME LIMIT OF THE ORIENTIFOLD, AND CORRECT THE KÄHLER POTENTIAL

→ DEPEND ON $B_2^{NS-NS} \in \mathbb{R}$, $C_2^{RR} \in \mathbb{R}$, $\tau \in \mathbb{C}$

\+ D-INSTANTON CONTRIBUTIONS

[TO TRANSFORM COVARIANTLY UNDER SYMMETRIES OF THE TYPE IIB ORIENTIFOLD]

ORIENTIFOLD: $(-)^{F_L} \Omega \sigma$ TO PRESERVE $\mathcal{N}=1$ → σ: HOLOMORPHIC ISOMETRIC INVOLUTION

2) $\mathcal{N}=2$ TYPE IIB : $SL(2,\mathbb{Z})$ SYMMETRY

UNDER ORIENTIFOLDING, WHERE τ AND G^a DO NOT VARY OVER THE CY_3

G^a: SCALARS FORMED BY COMPLEXIFYING NS-NS & RR SCALARS VIA τ

$\mathcal{N}=1$ TYPE IIB : $(SL(2,\mathbb{Z}) \supset) T_S$ → REQUIRES CONSIDER CONTRIBUTIONS FROM D1- & D(-1)-BRANES; BUT, STILL, NOT GUARANTEED TO GET THE COMPLETE ANSWER

MODULARITY ARGUMENTS: BETTER FOR W (PROTECTED BY HOLOMORPHICITY) THAN K; PERHAPS HETEROTIC-F THEORY DUALITY NEEDS TO BE USED

3) $\mathcal{N}=1$ TYPE IIB ORIENTIFOLDS

(i) $\sigma: \sigma^* J = J, \; \sigma^* \Omega = -\Omega$

$$\Updownarrow$$

$$\sigma^*(\phi, C_0, C_4) = (\phi, C_0, C_4);$$

C_0 = axion, $C_4 = \rho_\alpha \tilde{\omega}^\alpha$ → integral basis of $H^4_+(CY_3, \mathbb{Z})$: $*_6 \tilde{\omega}^\alpha = \omega_\alpha$

$$\sigma^*(B_2^{NS\text{-}NS}, C_2^{RR}) = -(B_2^{NS\text{-}NS}, C_2^{RR})$$

$B_2 = b^a \omega_a$, $C_2 = c^a \omega_a$ → integral basis of $H^2_-(CY_3, \mathbb{Z})$

(ii) $\mathcal{M} = \tilde{\mathcal{M}}_{SK} \times \tilde{\mathcal{M}}_Q$

$\tilde{\mathcal{M}}_{SK} \subset \mathcal{M}_{SK}(N=2)$ | $\tilde{\mathcal{M}}_Q \subset \mathcal{M}_Q(N=2)$

- $\dim(\tilde{\mathcal{M}}_{SK}) = h^{2,1}_-$
- $K_{CS}(z, \bar{z})$ → COMPLEX SCALARS
  $= -\ln\left(i \int_{CY_3} \Omega(z) \wedge \bar{\Omega}(\bar{z})\right)$

- $J = v^\alpha \omega_\alpha$, $\alpha = 1, \ldots, h^{1,1}_+(CY_3)$ → integral basis for $H^2_+(CY_3, \mathbb{Z})$
  & $B_2 = b^a \omega_a$, $a = 1, \ldots, h^{1,1}_-(CY_3)$
- $K_Q(\tau, G, T) = -2 \ln\left(i \int_{CY_3} \langle e^{-\phi}\rho, e^{-\phi}\bar{\rho}\rangle_{\text{Mukai}}\right)$

[WHERE $\rho = (1, t^A \omega_A, -\mathcal{F}_A \tilde{\omega}^A, (2\mathcal{F} - t^A \mathcal{F}_A)\epsilon)$, $t^A = -B_2 + iJ$, $\mathcal{F}_A = \frac{\partial \mathcal{F}}{\partial t^A}$]

$= K_Q(\tau - \bar{\tau}, G^a - \bar{G}^a, T_\alpha - \bar{T}_\alpha)$ → $\mathcal{N}=1$ COORDINATES

$[\varphi_c \equiv e^{-B_2} \wedge C_{RR} + i\,\mathrm{Re}(e^{-\phi}\rho) = \tau + G^a \omega_a - T_\alpha \tilde{\omega}^\alpha]$

f) (NON-) PERTURBATIVE α'-CORRECTIONS IN THE LARGE VOLUME LIMIT OF CY_3/σ

(i) v^α IS LARGE (CONTRIBUT'S: SUPPRESSED), BUT CONTRIBUTIONS DEPENDING ON $t^a = -b^a$ ARE NOT SUPPRESSED

$\Downarrow$

$\exists$ NON-PERTURBATIVE α'-CORRECTIONS INHERITED FROM THE PARENT $\mathcal{N}=2$ THEORY

$\exists$ EXAMPLES $\{n^0_\beta\}$ TRUNCATES FOR $k_A = (\beta, k_a)$

$\Downarrow$

$$\mathcal{F} = \mathcal{F}_{classical} + \mathcal{F}_{Pert} + \mathcal{F}_{np} = i \sum_{\beta \in H_2(CY_3, \mathbb{Z})} n^0_\beta \, Li_3\left(e^{2\pi i k_a t^a}\right)$$

$\mathcal{F}_{classical} = -\frac{1}{3!} K_{ABC} t^A t^B t^C$ — under orientifolding $K_{\alpha\beta a} = K_{abc} = 0$

$\mathcal{F}_{Pert} = -\frac{i}{2}\zeta(3)\chi(CY_3)$ — $(\alpha')^3$-CORRECTION [BECKERS et al 02]

n^0_β: genus-0 Gopakumar-Vafa invariants for β; $k_a = \int_\beta \omega_a$; $t^a = -b^a$

[THIRD POLYLOGARITHM: $Li_3(x) \equiv \sum_{n>0} \frac{x^n}{n^3}$]

(ii) By substituting $\mathcal{F}$ into K^{Q}_{K}, one identifies the $\mathcal{N}=1$ coordinates: [GRIMM]

$$\tau \equiv C_0 + i e^{-\phi}$$

$$G^a \equiv c^a - \tau b^a$$

$$T_\alpha \equiv \frac{i}{2} e^{-\phi} K_{\alpha\beta\gamma} v^\beta v^\gamma - \left(\tilde{\rho}_\alpha - \frac{1}{2} K_{\alpha a b} c^a b^b\right) - \frac{1}{2(\tau-\bar{\tau})} K_{\alpha a b} G^a (G^b - \bar{G}^b)$$

5) SYMMETRIES OF THE KÄHLER POTENTIAL INHERITED FROM TYPE IIB ($SL(2,\mathbb{Z})$ + SHIFTS IN $B_2^{a s}$) [GRIMM, O?]

(i) $B_2 \to B_2 + 2\pi \chi_2$, $\chi_2 = n^a \omega_a$ [SHIFT SYMMETRY]

- $\mathcal{V}_E$ BEING GEOMETRICAL, IS INVARIANT
- F_{pert} IS INDEPENDENT OF B_2 AND HENCE IS INVARIANT
- $\mathcal{F}_{np} = \mathcal{F}_{np}\left(e^{-i\int_\beta B_2}\right) \to$ INVARIANT

(ii) $SL(2,\mathbb{Z})$ [INHERITED FROM $N=2$, TYPE IIB]:

$$\tau \to \left(\frac{a\tau + b}{c\tau + d}\right), \quad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}$$

- $\Rightarrow G^a \to \frac{G^a}{(c\tau + d)}$, $T_\alpha \to T_\alpha + \frac{1}{2} c\, \kappa_{\alpha a b} \frac{G^a G^b}{(c\tau + d)}$ } $T_S \hat{\subset} SL(2,\mathbb{Z})$
- $\because (\tau - \bar{\tau}) \to (\tau - \bar{\tau}) / |c\tau + d|^2$, $\mathcal{V}_E$ (BEING GEOMETRICAL) IS T_S-INVARIANT,

$\therefore F_{pert}$ IS NOT INVARIANT UNDER T_S AS ONE NEEDS TO ADDITIONALL CONTRIBUTIONS FROM D(-1)-BRANES AND D1-INSTANTONS

(5) (ii) THE PROPOSED [GRIMM, '07]

MODULAR-INVARIANT COMPLETION:

$$\frac{\chi(CY_3)}{2} f(\tau,\bar\tau) - 4g(\tau,\bar\tau; G,\bar G) \; : \; e^K \to e^K |c\tau+d|^2$$

$f(\tau,\bar\tau)$: Based on R^4-corrections to $S^{N=2,\,IIB,\,D=10}_{SUGRA}$ (Green+Gutperle, '97)

$$\sum_{(m,n\in\mathbb{Z}^2/(0,0))} \frac{\left(\frac{\tau-\bar\tau}{2i}\right)^{3/2}}{|m+n\tau|^3}$$

EISENSTEIN SERIES

For $N=1$: Perhaps, restrict to orbits of T_S

$g(\tau,\bar\tau;G,\bar G)$:

→ one needs to consider web of (p,q)-strings to restore T_S

→ Based on Modular completion of $N=2$ quaternionic geometry by summation $\forall\, SL(2,\mathbb{Z})$-images of world sheet corrections [Llama et al, '06]:

$$\sum_{\beta\in H_2^-(CY_3,\mathbb{Z})} n^0_\beta \sum_{(m,n\in\mathbb{Z}^2/(0,0))} \frac{\left(\frac{\tau-\bar\tau}{2i}\right)^{3/2}}{\ldots} \cos\left[(n+m\tau)\frac{k_\beta\cdot(G-\bar G)}{(\tau-\bar\tau)} - m\, k_\beta\cdot G\right]$$

6) D-INSTANTON W IN TYPE IIB ORIENTIFOLDS

(i) CONSIDER A D3-BRANE WRAPPED AROUND A DIVISOR Σ IN CY_3/σ: IT CONTRIBUTES TO W (WITTEN)

$\rightarrow W = f(\{X^I\})\, e^{-V_\Sigma + i\phi_\Sigma}$, $\phi_\Sigma = \int_\Sigma C_4$

- $\{X^I\}$: moduli of CY_4 or equivalently CY_3
- V_Σ: volume of Σ in Einstein's frame

Moduli:

- C.S. DEFORMA'S OF CY_4 — ORIENTIFOLD LIMIT: τ (C.S. OF T^2); C.S. DEFORMA'S OF CY_3/σ & D7-BRANE MODULI
- $h^{2,1}(CY_4)$ COMPLEX SCALARS FROM $C_3^{M\text{-THEORY}} \ni G^a$, WILSON LINES OF D7-BRANES
- COORDINATES LABELING THE POSITION OF THE D3-BRANES IN CY_4 OR CY_3/σ

$\rightarrow$ NOT THE COMPLETE FORM AS (field strength on D3-BRANE) & COUPLINGS TO C_2 & B_2

$\rightarrow$ USING $S^{ED3\ w.v.} = T_{D3}\int_{\Sigma_4} e^{-\phi}\sqrt{\det(g - B_2 + F)} + T_{D3}\int_{\Sigma_4} e^{C} \wedge e^{-B_2 + F}$,

in the string frame:

$$e^{-\frac{1}{2}\int_\Sigma (e^{-\phi})(J^2 - B^2) - i\int_\Sigma (C_4 - C_2\wedge B_2 + \frac{1}{2}C_0 B_2^2)}$$

($\sim (-B_2 + iJ)^2$)

$$= e^{-i\int_{\Sigma \in H_4(CY_3, \mathbb{Z})} e_c} = e^{i n_\Sigma^\alpha T_\alpha}$$

$\rightarrow$ (ii) ASSUME $f(X) = A_0\, \Theta(\tau, G^a)$ — ENCODES DEPENDENCE ON OTHER MODULI

(iii) AS $e^K \rightarrow e^K|c\tau + d|^2$, HENCE $W \rightarrow W(c\tau + d)^{-1}$ [ASSUMING ABSENCE OF PHASE] ($\because e^K|W|^2$ = GRAVITINO MASS)

$\Rightarrow$ W MUST HAVE MODULAR WEIGHT -1 UNDER T_S

(iii) Under $b^a \to b^a + 2\pi n^a$:

- $G^a \to G^a - 2\pi \tau n^a$; $T_\alpha \to T_\alpha - 2\pi K_{\alpha ab} n^a G^b + 2\pi^2 \tau K_{\alpha ab} n^a n^b$
- ∵ K IS INVARIANT & $e^K |W|^2$ MUST BE INVARIANT
- ∴ W CAN TRANSFORM, AT BEST, BY A PHASE FACTOR → DROP
- TO SIMPLIFY, ASSUME ONLY $T_{\alpha=\alpha'}$ TRANSFORMS NON-TRIVIALLY

$\Updownarrow$

ONLY NON-ZERO INTERSECTION #s $K_{\alpha ab}$: $K_{\alpha' ab} = -C_{ab}; n^{\alpha'}_2 = n$

- $\Theta_\Sigma(\tau, G)$: HOLOMORPHIC JACOBI FORM OF WEIGHT -1 AND INDEX n: $\Theta_n(\tau, G)$

UNDER $G^a \to G^a - 2\pi n^a$, $T_\alpha \to T_\alpha - 2\pi K_{\alpha ab} n^a G^b + 2\pi^2 \tau K_{\alpha ab} n^a n^b$:

$$\Theta_n(\tau, G) \to e^{-2i\pi n C_{ab} n^a G^b + 2\pi^2 n \tau C_{ab} n^a n^b} \Theta_n(\tau, G)$$

UNDER $G^a \to \frac{G^a}{(c\tau+d)}$, $T_\alpha \to T_\alpha + \frac{1}{2} c \frac{K_{\alpha ab} G^a G^b}{(c\tau+d)}$:

$$\Theta_n(\tau, G) \to (c\tau+d)^{-1} e^{\frac{inc}{2} \frac{C_{ab} G^a G^b}{(c\tau+d)}} \Theta_n(\tau, G)$$

- [GRIMM, '07] $W_{np}(\tau, G) = \sum_{n^\alpha} \frac{\Theta_{(n^\alpha)} e^{i n^\alpha T_\alpha}}{f(\eta(\tau))}$

$$\sum_{m^a \in \text{a suitable +ive definite rational lattice}} e^{\frac{i\tau m^2}{2}} e^{i n^\alpha m_a G^a}$$

($C_{ab} m^a m^b$ written above m^2)

- "a" indexes the real subspace (of $H^{1,1}$) of real dimensionality $h^{1,1}_- = 2$; "α" indexes the real subspace (of $H^{1,1}$) of real dimensionality $h^{1,1}_+ = 2$ $\left(\Rightarrow h^{1,1}_{in} = \frac{1}{2} \times (h^{1,1}_+ + h^{1,1}_-) \right.$ complex dimensionality of $\left. H^{1,1} \right)$

- Example of σ: $z_1 \leftrightarrow z_2$, $z_3 \to z_3$

$$\to \omega^{(-)} = \left\{ \sum (dz^1 \wedge d\bar{z}^{\bar{2}} - dz^2 \wedge d\bar{z}^{\bar{1}}),\ i(dz^1 \wedge d\bar{z}^{\bar{1}} - dz^2 \wedge d\bar{z}^{\bar{2}}) \right\}$$

$$\underset{\mathbb{R}}{\omega^{(+)}} = \left\{ \sum i(dz^1 \wedge d\bar{z}^{\bar{2}} + dz^2 \wedge d\bar{z}^{\bar{1}}),\ \sum i\, dz^1 \wedge d\bar{z}^{\bar{1}} \right\}$$

$$\to B_2 = B_{1\bar{2}} dz^1 \wedge d\bar{z}^{\bar{2}} + B_{2\bar{3}} dz^2 \wedge d\bar{z}^{\bar{3}} + B_{3\bar{1}} dz^3 \wedge d\bar{z}^{\bar{1}}$$
$$+ B_{2\bar{1}} dz^2 \wedge d\bar{z}^{\bar{1}} + B_{3\bar{2}} dz^3 \wedge d\bar{z}^{\bar{2}} + B_{1\bar{3}} dz^1 \wedge d\bar{z}^{\bar{3}}$$
$$+ B_{1\bar{1}} dz^1 \wedge d\bar{z}^{\bar{1}} + B_{2\bar{2}} dz^2 \wedge d\bar{z}^{\bar{2}} + B_{3\bar{3}} dz^3 \wedge d\bar{z}^{\bar{3}}$$

$$\downarrow \quad B_{1\bar{2}} = B_{2\bar{3}} = B_{3\bar{1}} = b^1$$
$$\& \; B_{1\bar{1}} = -B_{2\bar{2}} = i b^2,\ B_{3\bar{3}} = 0$$

$$b^1 \omega_1^{(-)} + b^2 \omega_2^{(-)} = \sum^{h^{1,1}_-(CY_3)} b^i \omega_i^{(-)}$$

Kähler Potential after inclusion of Perturbative- (Beckers et al, '02) and Non-Perturbative α' (Grimm, '07) and String Loop Corrections (Berg et al, '05, '07; Cicoli et al, '07)

$$K = -\ln\left(-i(\tau-\bar\tau)\right) - \ln\left(-i\int_{CY_3}\Omega\wedge\bar\Omega\right)$$

$$-2\ln\Bigg[\mathcal{V} + \frac{\chi(CY_3)}{2}\sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\bar\tau-\tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}$$

$$-4\sum_{\beta\in H_2^-(CY_3,\mathbf{Z})} n^0_\beta \sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\bar\tau-\tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}$$

$$\times\cos\left((n+m\tau)k_a\frac{(G^a-\bar G^a)}{\tau-\bar\tau} - mk_aG^a\right)\Bigg]$$

$$+\frac{C_s^{KK(1)}(U_\alpha,\bar U_{\bar\alpha})\sqrt{\tau_s}}{\mathcal{V}\left(\sum_{(m,n)\in\mathbf{Z}^2/(0,0)}\frac{\frac{(\tau-\bar\tau)}{2i}}{|m+n\tau|^2}\right)} + \frac{C_b^{KK(1)}(U_\alpha,\bar U_{\bar\alpha})\sqrt{\tau_b}}{\mathcal{V}\left(\sum_{(m,n)\in\mathbf{Z}^2/(0,0)}\frac{\frac{(\tau-\bar\tau)}{2i}}{|m+n\tau|^2}\right)}$$

$$\Rightarrow V \sim \frac{Y\sqrt{\ln \mathcal{V}}}{\mathcal{V}^{2n'+2}} (n')^2 \frac{\left(\sum_{m^a} e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n'}{g_s} + \frac{n' \kappa_{1ab} b^a b^b}{2g_s}}\right)^2}{|f(\eta(\tau))|^2} \quad (I)$$

$$+ \frac{W \ln \mathcal{V}}{\mathcal{V}^{n'+2}} \left(\frac{\Theta_{n'}(\bar\tau, \bar G)}{\bar f(\bar\eta(\bar\tau))}\right) e^{-in'\left(-\tilde\rho_1 + \frac{1}{2}\kappa_{1ab}\frac{(\tau G^a - \bar\tau \bar G^a)}{(\bar\tau - \tau)}\frac{(G^b - \bar G^b)}{(\bar\tau - \tau)} - \frac{1}{2}\kappa_{1ab} G^a \frac{(G^b - \bar G^b)}{(\tau - \bar\tau)}\right)} + c.c. \quad (II)$$

$$+ \frac{|W|^2}{\mathcal{V}^3}\left(\frac{3k_2^2 + k_1^2}{k_1^2 - k_2^2}\right) \frac{\left|\sum_c \sum_{n,m \in \mathbb{Z}^2/(0,0)} A_{n,m,n_k c}(\tau) \sin(nk \cdot b + mk \cdot c)\right|^2}{\sum_{c'} \sum_{n',m' \in \mathbb{Z}^2/(0,0)} |n + m\tau|^3 |A_{n',m',n_k c}(\tau)|^2 \cos(nk \cdot b + mk \cdot c)}$$

$$+ \xi \frac{|W|^2}{\mathcal{V}^3}$$

- $V_{(II)}$ is the most dominant for $n' = 1$ & generic values of ρ_1, G^a, $k^{1,2}$ and $\mathcal{O}(1)$ W_{cs} ~ Balasubramanian et al
- As in KKLT scenarios, $|W_{cs}| << 1$ & further, assume, $W \sim W_{np}$; for $n' > 1$, $V_{(I)}$ is the most dominant
  $|\partial_{\rho_1} W_{np}|^2 >> 0$
- $\partial_{c^a} V_I = 0 \Rightarrow nk \cdot b + mk \cdot c = N\pi,\ N \in \mathbb{Z}$

Saddle Point

Kähler Potential after inclusion of Perturbative- (Beckers et al, '02) and Non-Perturbative α' (Grimm, '07) and String Loop Corrections (Berg et al, '05, '07; Cicoli et al, '07)

$$K = -\ln\left(-i(\tau-\bar{\tau})\right) - \ln\left(-i\int_{CY_3}\Omega\wedge\bar{\Omega}\right)$$

$$-2\ln\Bigg[\mathcal{V} + \frac{\chi(CY_3)}{2}\sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\bar{\tau}-\tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}$$

$$-4\sum_{\beta\in H_2^-(CY_3,\mathbf{Z})} n^0_\beta \sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\bar{\tau}-\tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}$$

$$\times\cos\left((n+m\tau)k_a\frac{(G^a-\bar{G}^a)}{\tau-\bar{\tau}} - mk_aG^a\right)\Bigg]$$

$$+\frac{C_s^{KK(1)}(U_\alpha,\bar{U}_{\bar{\alpha}})\sqrt{\tau_s}}{\mathcal{V}\left(\sum_{(m,n)\in\mathbf{Z}^2/(0,0)}\frac{\frac{(\tau-\bar{\tau})}{2i}}{|m+n\tau|^2}\right)} + \frac{C_b^{KK(1)}(U_\alpha,\bar{U}_{\bar{\alpha}})\sqrt{\tau_b}}{\mathcal{V}\left(\sum_{(m,n)\in\mathbf{Z}^2/(0,0)}\frac{\frac{(\tau-\bar{\tau})}{2i}}{|m+n\tau|^2}\right)}$$

$$\Rightarrow V \sim \frac{Y\sqrt{\ln \mathcal{V}}}{\mathcal{V}^{2n'+2}} (n')^2 \overset{(I)}{\frac{\left(\sum_{m^a} e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n'}{g_s} + \frac{n' k_{1ab} b^a b^b}{2g_s}}\right)^2}{|f(\eta(\tau))|^2}}$$

$$+ \frac{W(\ln \mathcal{V})}{\mathcal{V}^{n'+2}} \left(\frac{\Theta_{n'}(\bar{\tau}, \bar{G})}{\bar{f}(\bar{\eta}(\bar{\tau}))}\right) \overset{(II)}{e^{-in'\left(-\tilde{\rho}_1 + \frac{1}{2}k_{1ab}\frac{(\bar{\tau}G^a - \tau\bar{G}^a)(G^b - \bar{G}^b)}{(\bar{\tau}-\tau)(\bar{\tau}-\tau)} - \frac{1}{2}k_{1ab}\frac{G^a(G^b - \bar{G}^b)}{(\tau-\bar{\tau})}\right)}} + c.c.$$

$$+ \frac{|W|^2}{\mathcal{V}^3}\left(\frac{3k_2^2 + k_1^2}{k_1^2 - k_2^2}\right) \frac{\left|\sum_c \sum_{n,m \in \mathbb{Z}^2/(0,0)} A_{n,m,n_k^c}(\tau) \sin(nk\cdot b + mk\cdot c)\right|^2}{\sum_{c'} \sum_{n,m \in \mathbb{Z}^2/(0,0)} |n+m\tau|^3 |A_{n,m,n_k^c}(\tau)|^2 \cos(nk\cdot b + mk\cdot c)}$$

$$+ \frac{\xi |W|^2}{\mathcal{V}^3}$$

- $V_{(II)}$ is the most dominant for $n'=1$ & generic values of ρ_1, G^a, $k^{1,2}$ and $\mathcal{O}(1)$ W_{cs} ~ Balasubramanian et al
- As in KKLT scenarios, $|W_{cs}| << 1$ & further, assume, $W \sim W_{np}$; for $n' > 1$, $V_{(I)}$ is the most dominant
  $|\partial_{\rho_1} W_{np}|^2 \geq 0$
- $\partial_{c^a} V_I = 0 \Rightarrow nk\cdot b + mk\cdot c = N\pi,\ N \in \mathbb{Z}$

Saddle Point

- Possible Minimum (in the LVS limit) at:

$$\sqrt{\ln(\mathcal{Y})}\sum_{m_{a'},\ \text{no sum w.r.t. } a'} e^{-\frac{m_{a'}^2}{2g_s}+\frac{m_{a'}b^{a'}n^1}{g_s}+\frac{n^1\kappa_{1a'b}b^{a'}b^b}{2g_s}}\left(\frac{m_{a'}n^1}{g_s}+\frac{\kappa_{1a'b}n^1b^b}{g_s}\right)=0$$

where

$$\mathcal{Y}\equiv\mathcal{V}+\frac{\lambda}{2}\sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\tau-\bar{\tau})^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}-4\sum_{\beta\in H_2(CY_3,\mathbf{Z})}n^0_\beta\sum_{m,n\in\mathbf{Z}^2/(0,0)}\frac{(\tau-\bar{\tau})^{\frac{3}{2}}}{(2i)^{\frac{3}{2}}|m+n\tau|^3}cos(nk.b+mk.c)$$

- For all directions in moduli space: $\mathcal{O}(1)\, W_{cs}$ and away from $D_{cs} W_{cs+\tau} = D_\tau W_{cs+\tau} = 0$, one gets an $\mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$ - contribution from $\sum_{\alpha,\beta \in cs} e^K (G^{-1})^{\alpha\bar{\beta}} (D_\alpha W_{cs})(\overline{D_{\beta}} \overline{W}_{cs}) > 0$

↓ Most dominant

- This suggests very strongly $\exists$ dS minimum

↓ No $\overline{D3}$-branes / uplifting terms were added as in KKLT scenarios.

• ~~NECESSARY~~ SUFFICIENT CONDITIONS FOR MULTI MODULI INFLATION:

$(M_p \equiv 1)$ $\left| \epsilon \equiv \frac{1}{2} \frac{G^{ab} \partial_a V \partial_b V}{V^2} \right| \ll 1,$

$$\left| \eta \equiv \text{MOST -IVE E.V.} \left[N^a{}_b \equiv \frac{G^{ac} \partial_b V_{,c}}{V} \right] \right| \ll 1$$

(QUEVEDO; LINDE; KALLOSH; BURGESS: '02 – '04)

• IN A COMPLEXIFIED BASIS:

$\epsilon \equiv \frac{G^{A\bar{B}} \partial_A V \partial_{\bar{B}} V}{V^2}$; $N^A{}_B = G^{A\bar{C}} \partial_{\bar{C}} \partial_B V$

and $N^{\bar{A}}{}_B = G^{\bar{A}C} (\partial_C \partial_B V - \Gamma^D_{BC} \partial_D V)$

where $\Gamma^D_{BC} = \frac{1}{2} G^{D\bar{F}} \partial_B \partial_C \partial_{\bar{F}} K \equiv \frac{1}{2} G^{D\bar{F}} \partial_B G_{C\bar{F}}$
$= \frac{1}{2} G^{D\bar{F}} \partial_C G_{B\bar{F}}$

ε << 1 CONDITION (LVS LIMIT)

- $$\partial_{G^a} V \Big|_{D_{cs} W = D_\tau W = 0;\ D_a W \neq 0}$$

POTENTIALLY PROBLEMATIC

$$e^K (\partial_{G^a} \mathcal{G}^{\rho_s \bar{\rho}_s})(\partial_{\rho_s} W_{np})(\bar{\partial}_{\bar{\rho}_s} \bar{W}_{np})$$

$$e^K \mathcal{G}^{\rho_s \bar{\rho}_s}(\partial_{G^a} \partial_{\rho_s} W_{np})(\bar{\partial}_{\bar{\rho}_s} \bar{W}_{np}).$$

Near $nk \cdot b + mk \cdot c = N\pi$

→ 0

$$\frac{\mathcal{V}}{\sum_{\beta \in H_2^-} (n^0_\beta)^2}.$$

Using Castelnuovo's theory (Klemm et al, '06), Gopakumar-Vafa invariants for compact CYs expressed as projective varieties in WCPs, are very large: ~$10^{15\text{-}20}$

→ **<< 1**

$\eta << 1$ CONDITION (LVS LIMIT)

$$\bar{\partial}_{\bar{G}^d}\,\partial_{G^a} V$$

↓ **Most dominant term**

$$\mathcal{G}^{\rho_s\bar{\rho}_s}\,\partial_{G^a}\partial_{\rho_s}W_{np}\,\bar{\partial}_{\bar{G}^b}\bar{\partial}_{\bar{\rho}_s}\bar{W}_{np}\,e^K$$

↓ **Contribution to η**

$$\frac{\mathcal{V}}{\sum_{\beta\in H_2^-}(n^0_\beta)^2} \quad \mathbf{<<1}$$

- $\Gamma^{G^a}_{G^bG^c} = g^{G^a\bar{G}^d}\,\partial_{G^a}\,g_{G^c\bar{G}^d}$

↓

$$N^{\bar{G}^a}{}_{G^b} \ni \frac{g^{G^c\bar{G}^a}\,\Gamma^{G^d}_{G^cG^b}\,\partial_{G^d}V}{V}$$

↓ $\because \Gamma^{G^d}_{G^cG^b}\Big|_{n\theta\cdot l + m\theta\cdot c = N\pi} = 0$

0

η PROBLEM IS SOLVED!

- By constructing the Hessian, and looking at its eigenvalues, one can show that $b^2 - \frac{k_2}{k_1} b^1$ provides a flat slow-roll direction for inflation to commence at a saddle point and proceed to the nearest dS minimum, if $\frac{k_2}{k_1} \square\ 1$

NUMBER OF e-FOLDINGS

- $I \equiv b^2 - \frac{k_2}{k_1} b^1$ = INFLATON

$$\rightarrow \quad N_e = - \int_{\text{SADDLE POINT}}^{\text{dS MINIMUM}} \frac{V}{\partial_I V} \, dI$$

$$\sim \frac{\sqrt{\sum_{\beta \in H_2^-} (n_\beta^0)^2}}{\mathbf{n}^1 \sqrt{\mathcal{V}}}$$

Hence, $n^0{}_\beta \sim 60 n^1 \sqrt{Vol}$ there is the possibility of getting 60 e-folds

Summary

- Showed the positivity of the potential in the LVS limit and for D3-instanton number > 1, suggesting strongly the existence of a dS minimum without the addition of anti-D3 branes
- Showed axionic slow-roll inflation from a saddle point and proceeding towards dS minimum