Title: Revenge of the S-Matrix; or, What is the Simplest QFT?

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Abstract:

Revenge of the S-Matrix

or

What is the simplest QFT?

N.A-H.

with Jared Kaplan

+ in progress with

Freddy Cachazo

Biggest Crises / Oppurtunity in fundamental physics:

- The Landscape
- Vacuum Selection
- Fundamental issues of
  - QM
  - Gravity
  - Cosmology

MUST BE DEALT WITH!

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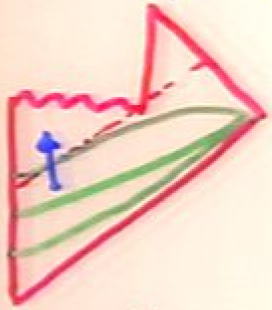
MUST BE DEALT WITH.

Something goes wrong with locality + gravity, not just @ l_{Pl}.



Information Paradox

Infinities in eternal inflation.

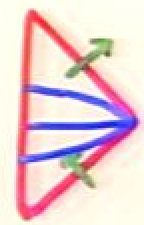
Can we talk about local things?

e.g. infalling observer into BH.

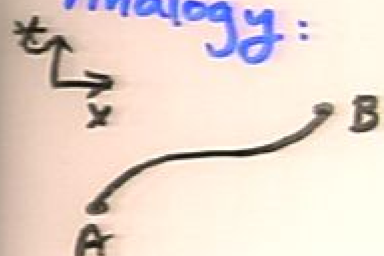
In cosmology, we are like these guys!

Local physics $\longrightarrow$ Flat Space

Can we talk about ordinary QFT in a different way, not manif. local?



Analogy:



$m\ddot{x} = -V'(x)$ manif. deterministic

S[A,B] extremized
not manif. det.

$\longrightarrow$ better jumping off point to QM

2000's "top down" attempt: Witten's twistor formulation of SYM

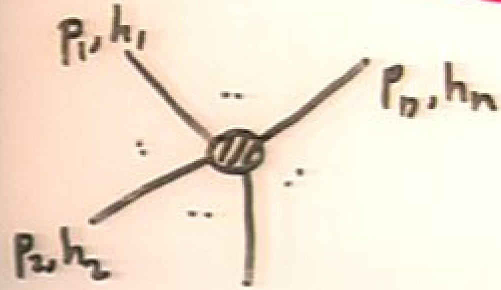
$\sim$ boundary theory for flat space.

CSW recursion relations

Very special to 4D, MHV amp. play special role, related to $F = \pm i\tilde{F}$ solutions of YM.

$\hookrightarrow$ led to BCF + W recursion relations, which can be described from "bottom up", and are much more general.

BCFW Redux



$\mathcal{M}(p_i, h_i)$

Complexify 2 momenta $p_{j,k}$, keeping them on shell:

$$p_j^\mu \longrightarrow p_j^\mu + q^\mu z \equiv p_j^\mu(z)$$

$$p_k^\mu \longrightarrow p_k^\mu - q^\mu z \equiv p_k^\mu(z)$$

$$0 = p_{\substack{j\\k}}^2(z) \implies \boxed{q \cdot p_{\substack{j\\k}} = 0\,,\ q^2 = 0}$$

$$q^2 = 0 \;,\; q \cdot p_{\substack{j\\k}} = 0$$

$$p_{\substack{j\\k}} = (1, \pm 1, 0, 0; 0, .. 0) \;,\; q = \tfrac{1}{\sqrt{2}}(0,0,1,i;0,..,0)$$

[Or keep momenta real, $(D-2,2)$ sig.].

Pol vectors:

$$z=0: \; \epsilon_j^- = \epsilon_k^+ = q \;,\; \epsilon_j^+ = \epsilon_k^- = \bar{q} \;,\; \epsilon_T = (0,0,0,0; .. 1 ..)$$

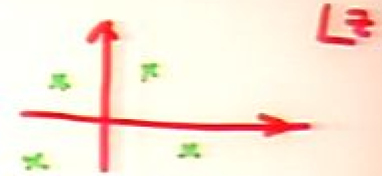
$$\epsilon_j^-(z) = \epsilon_k^+(z) = q \;,\; \epsilon_T(z) = (0,0,0,0; .. 1 ..)$$

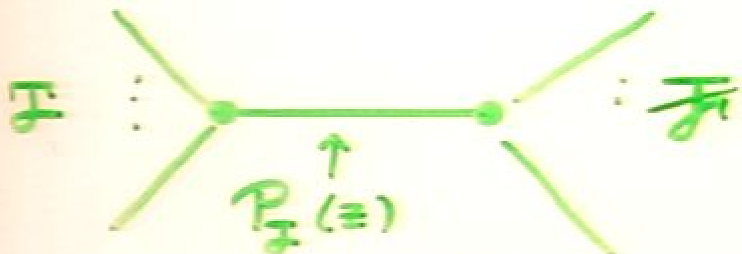
$$\epsilon_j^+(z) = \bar{q} - z\, p_k \;,\; \epsilon_k^- = \bar{q} + z\, p_j$$

$$\left[\, p_{\substack{j\\k}\,\mu}\, \epsilon_{\substack{j\\k}}^{\mu}(z) = 0, \;\; \epsilon_j^- \epsilon_j^+ = \epsilon_k^- \epsilon_k^+ = 1, .. \right].$$

$$\mathcal{M}(p_i, h_i) \longrightarrow \mathcal{M}^{h_j h_k}(z)$$

$\mathcal{M}(z)$: only simple poles





$$\mathcal{P}_{\mathcal{J}}(z) = \begin{cases} P_{\mathcal{J}} & j,k \in \mathcal{J} \\ P_{\mathcal{J}} + zq & j \in \mathcal{J} \\ P_{\mathcal{J}} - zq & k \in \mathcal{J} \end{cases}$$

$\mathcal{P}_{\mathcal{J}}^2(z) = P_{\mathcal{J}}^2 + 2P_{\mathcal{J}} \cdot q\, z$, poles at

$$z \to z_j = -\mathcal{P}_{\mathcal{J}}^2 / 2P_{\mathcal{J}} \cdot q$$

res $\mathcal{M}(z \to z_j)$

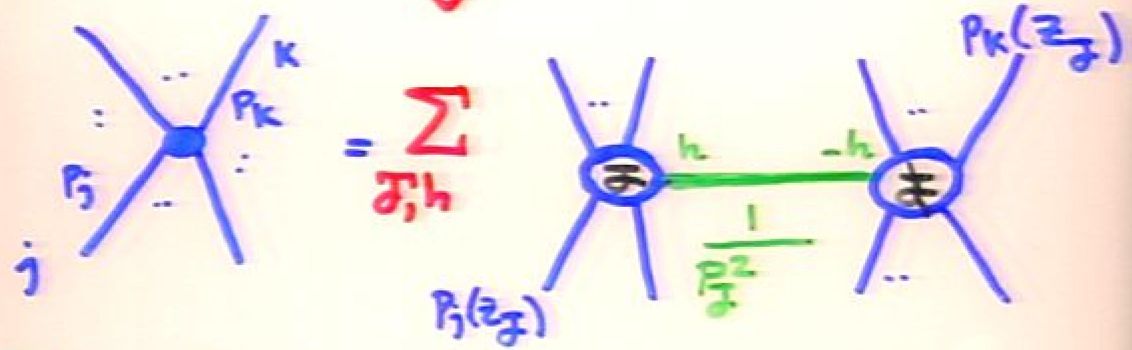
$= \sum_h \Big($  $\Big) \times \Big($  $\Big)$

Lower ... !) amp.

If $M(z \to \infty) \to 0$,

$$0 = \frac{1}{2\pi i}\oint \frac{dz}{z} M(z) = M(0) + \text{other res.}$$

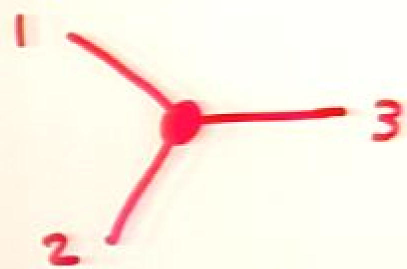
$$\Downarrow$$



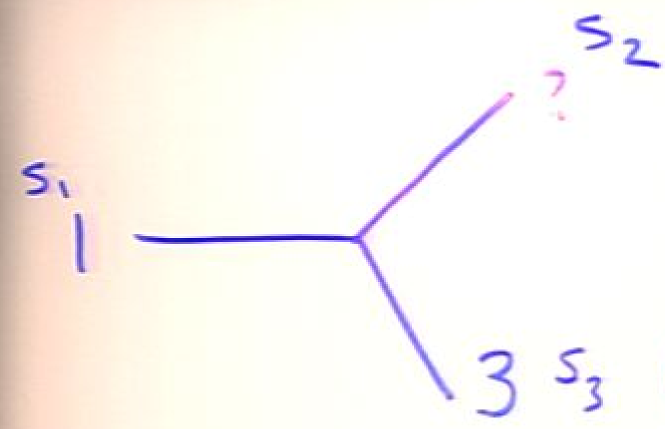
On-shell BCFW recursion relations.

[Sufficient $\mathcal{M}^{-,\text{any}}(z \to \infty) \to 0$ Gauge
$\mathcal{M}^{--,\text{any}}(z \to \infty) \to 0$ Grav]

Can recursively reduce all amplitudes to



which normally can't be on-shell, but can be for complex momenta (or in $(D-2,2)$ signature).



Remarkable Object [Cachazo, Benincasa]

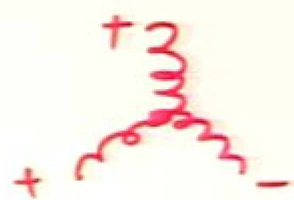
Completely determined by Lorentz Invariance.

In 4D, $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$,

$$\mathcal{M}^{123} = \langle 12\rangle^{\frac{s_1+s_2-s_3}{2}} \langle 23\rangle^{\cdot\cdot} \langle 31\rangle^{\cdot\cdot}$$

spin 1

$$f^{abc} \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}$$

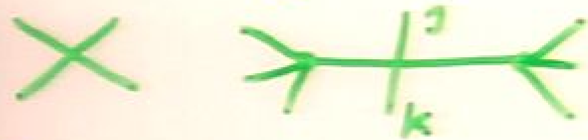


spin 2

$$\left[\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\right]^2$$

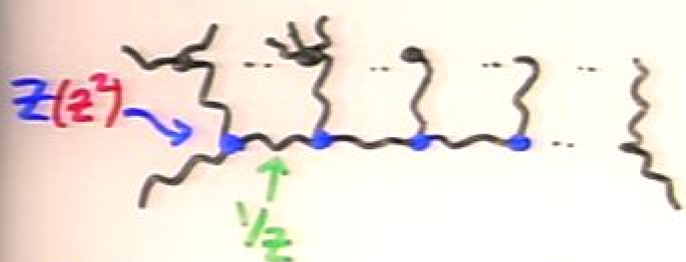
Behavior of $\mathcal{M}(z \to \infty)$ is Surprising

Naively, $\mathcal{M}(z \to \infty) \to 0$ is never true! e.g. ϕ^4 theory



$\mathcal{M}^{\phi^4}(z) \to z^0$

Gauge / Gravity is worse!



then fold in ϵ's, $\to z^0$ or z^1 ...

Gauge Na e

$-+ \quad z$

$--/++ \quad z^2$

$+- \quad z^3$

Grav.

$--,++ \quad z^{n-1}$

$\vdots$

$++,-- \quad z^{n+3}$

Gauge N ve Actual

$-+$	z	Easy $\rightarrow$	$\frac{1}{z}$
$--/++$	z^2	Needed CSW rules $\rightarrow$	$\frac{1}{z}$ BCFW
$+-$	z^3		z^3

Grav.

Cachazo, ...

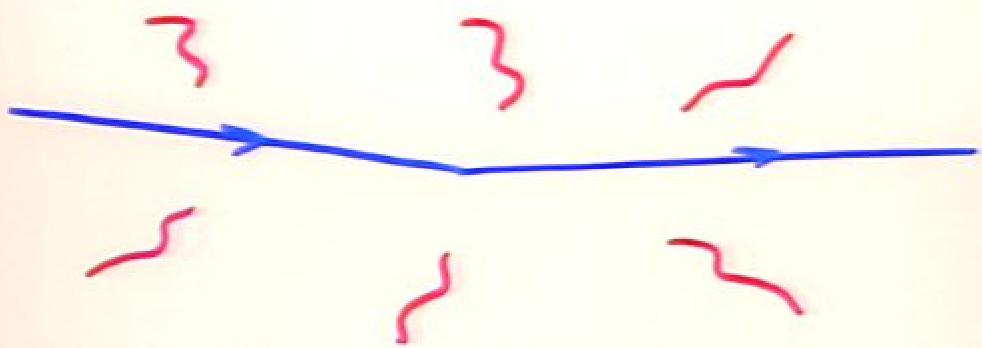
$--,++$	z^{n-1}	Grav $\sim$ Gauge^2	$\frac{1}{z^2}$
$\vdots$			$\vdots$
$++,--$	z^{n+3}		z^6

Unexpectedly good behavior of $\mathcal{M}(z\to\infty)$ encapsulates heavy cancellations in explicit diagram calculations

Understanding $\mathcal{M}(z \to \infty)$

$$p_{j}(z) = p_{j} \pm z\,q$$

$$p_{k}(z) = p_{k} \pm z\,q$$



$z \to \infty$: hard (complex) light-like particle blasting through soft background. Familiar for real momenta (eikonal). "Not much" scattering, "helicity conserved". We'll formalize + extend to [illegible] least.

Yang Mills

$\mathcal{A}_\mu = A_\mu + a_\mu$. Usual a_μ GFixing

$$\mathcal{L} = -\frac{1}{4} \operatorname{tr} D_\mu a_a D^\mu a_b \eta^{ab}$$
$$+ \frac{i}{2} \operatorname{tr} [a_a, a_b] F^{ab}$$

$z \to \infty$: "spin Lorentz invariance".

$$\boxed{\mathcal{M}^{ab} = (c z + \dots) \eta^{ab} + A^{ab} + \frac{1}{z} B^{ab} + \dots}$$

also Ward id. : $P_j(z)_a \mathcal{M}^{ab} \epsilon_b = 0$

$\Rightarrow q$ ... ^{ab} ... ^{ab}

Pure Gravity

$$\mathcal{L} = \sqrt{-g}\Big[\tfrac{1}{4} g^{\mu\nu} g^{\alpha\rho} g^{\beta\sigma} \nabla_\mu h_{\alpha\beta} \nabla_\nu h_{\rho\sigma} - \tfrac{1}{2} h_{\alpha\beta} h_{\mu\nu} R^{\beta\mu\alpha\nu}\Big]$$

(Bern – Grant trick used).

L, R $h_{\alpha\beta}$ indices separately contracted.

$\Rightarrow \underline{\underline{2}}$ copies of spin L.I.

$$h_{\mu\nu} = e^{a}_{\mu}\, \tilde{e}^{\tilde{a}}_{\nu}\, h_{a\tilde{a}}$$

Gauge $\omega^{+}_{ab} = \tilde{\omega}^{+}_{\tilde{a}\tilde{b}} = 0$

What is simplest QFT?

$$\begin{tikzcd} \phi^4 \arrow[r] & \text{YM} \arrow[r] \arrow[d] & \text{Gravity} \arrow[d] \\ & \mathcal{N}=4 \text{ SYM} & \mathcal{N}=8 \text{ SUGRA} \end{tikzcd}$$

Why? We've seen that amps of $s \geq 1$ particles are much nicer than scalars. But: fundamentally <u>discrete</u> objects $M^{++--+++\cdots-}$

	YM	Grav
$+2$		1
$+\frac{3}{2}$		
$+1$	1	
$+\frac{1}{2}$		
0		
$-\frac{1}{2}$		
-1	1	
$-\frac{3}{2}$		
-2		

Label states of mom. $(\lambda, \tilde{\lambda})$:

$$|\eta\rangle = e^{(\tilde{\omega}\tilde{Q}^I)\eta_I} |-\rangle \quad ([\omega\lambda]=1)$$

or

$$|\tilde{\eta}\rangle = e^{(\omega Q_I)\tilde{\eta}^I} |+\rangle \quad (\langle\omega\lambda\rangle=1)$$

$$Q_{\alpha I}|\eta\rangle = \lambda_\alpha \eta_I |\eta\rangle; \quad \tilde{Q}^{\dot{\alpha} I}|\tilde{\eta}\rangle = \tilde{\lambda}^{\dot{\alpha}}\tilde{\eta}^I |\tilde{\eta}\rangle$$

$|\eta\rangle \longleftrightarrow |\tilde{\eta}\rangle$ Complementary

MHV $\longleftrightarrow$ $\overline{\text{MHV}}$ Complementary

$|\tilde{\eta}\rangle$ ...

Label states of mom. $(\lambda, \tilde\lambda)$:

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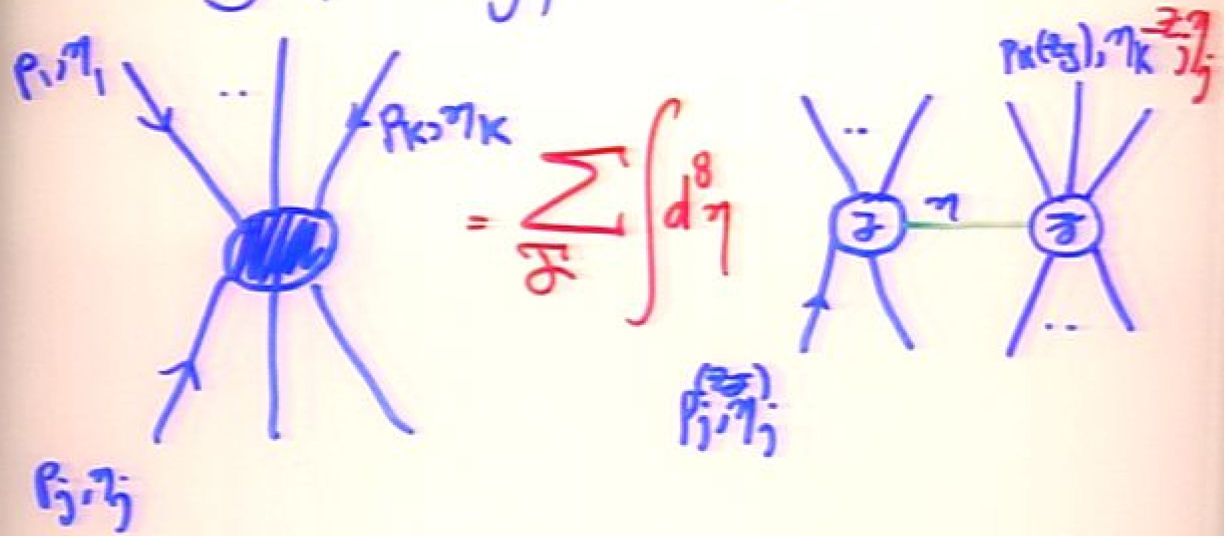
Using (trivially) SUSY, can show



Remarkable: SUSY transmits good properties of gravity amplitudes to lower spin (e.g. scalar) amps.

Fundamental Q:

* What is the dual theory (weak–weak!) that explains these amazing ~~properties~~ ties? Should exist + be nicest for $\mathcal{N}=4$, $\mathcal{N}=8$. But since BCFW is true in any D, it shouldn't crucially rely on 4D.

* To start with: is there an analog of twistor string theory that makes BCFW "obvious"?