Title: Lattice Gauge Theory in the LHC Era

Date: Jun 06, 2008 04:45 PM

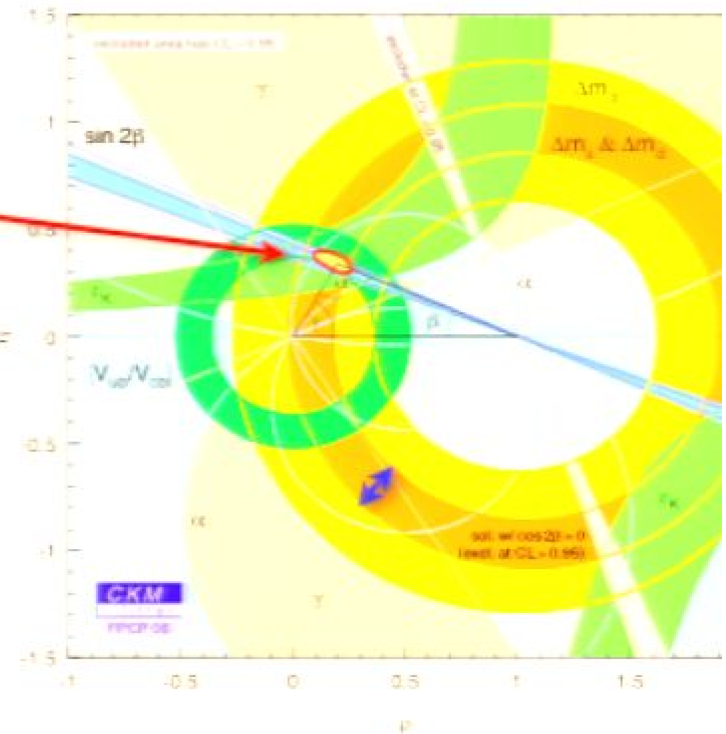
URL: http://pirsa.org/08060050

Abstract: Lattice QCD in this decade has succeeded in producing essential results for crucial components of Standard Model phenomenology such as constraints on the rho-eta plane. Much more will be required of lattice gauge theory in the LHC era: sub-per cent precision in QCD quantities and the ability to calculate in strongly interacting sectors of Beyond-the-Standard-Model theories such as SUSY or technicolor. I will review the status of current calculations and the prospects for accomplishing what needs to be done in the coming years.

Lattice QCD calculations

play an essential role in understanding the Standard Model.

The allowed region in the ρη plane depends heavily on the accuracy of the lattice calculations.
Improving them is a key goal for particle physics.

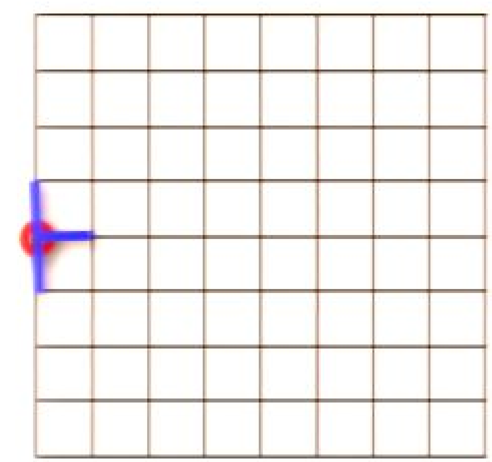


J. Charles, CKMfitter, FPCP 200

- What do we expect from lattice QCD in the LHC era?
- What can we hope for from BSM lattice gauge theory in the LHC era?

Lattice quantum field theories

Approximate the path integral by defining the fields on a four dimensional space-time lattice.



Quarks are defined on the sites of the lattice, and gluons are SU3 matrices on the links, U=exp(igA).

Continuum quantum field theory is obtained in the zero lattice spacing limit. This limit is computationally very expensive when Monte Carlo methods are used to solve the theory.

$$\mathcal{Z} = \int d[U, \psi, \overline{\psi}] \exp(-S) : it_M \to -t_E.$$

Theories with complex path integrals after Wick rotation can't be done with current metho

- Finite density.
- Complex fermion content.

In lattice theories, differential operators are replaced by discrete differences.

In simplest discretization of the Dirac equation

$$(i\gamma_\mu \partial_\mu - m)\psi = 0$$

the derivative is replaced by a simple discrete difference (naive and staggered fermions).

$$\partial\psi(x) \to \frac{\psi(x+a) - \psi(x-a)}{2a} + \mathcal{O}(a^2)$$

This produces a propagator with 16 poles; not only at the physical value p_μ =0, but also at $p_\mu = \pi/a$.

$$(\gamma_\mu p_\mu - m)^{-1} \to (\gamma_\mu \sin(ap_\mu)/a - m)^{-1}$$

→ Additional states: the "fermion doubling problem.

Three families of light lattice fermions:
Naive (and staggered) fermions

$$\mathcal{S} = \sum_x \overline{\psi}(x)\,(\gamma \cdot \Delta + m)\,\psi(x).$$

where $\Delta_\mu \psi(x) \equiv \frac{1}{2a}\left(U_\mu(x)\,\psi(x+a\hat{\mu}) - U^\dagger_\mu(x-a\hat{\mu})\,\psi(x-a\hat{\mu})\right)$

Doubling symmetry:

$$\begin{aligned} \psi(x) \quad \rightarrow \quad \tilde{\psi}(x) &\equiv i\gamma_5\gamma_\rho\,(-1)^{x_\rho/a}\,\psi(x) \\ &= i\gamma_5\gamma_\rho\,\exp(i\,x_\rho\pi/a)\,\psi(x) \end{aligned}$$

16 poles in propagator:

$$aG^{-1}(p) = \gamma \cdot \sin(ap) + am$$

Extra species removed in unquenched calculations by taking the root of the fermion determinant.

Wilson fermions

All but one pole removed by adding a Laplacian.

$$S = \sum_x \bar{\psi}(x)\,(\gamma \cdot \Delta + \frac{r}{2}\Delta'^2 + m)\,\psi(x)$$

where $\Delta'^2\,\psi(x) = U_\mu(x)\,\psi(x+a\hat{\mu}) + U^\dagger_\mu(x-a\hat{\mu})\,\psi(x-a\hat{\mu}) - 2\psi(x)$

Propagator pole only at p=0:

$$aG^{-1}(p) = \gamma \cdot \sin(ap) + \Sigma \sin^2(ap/2) + am$$

But

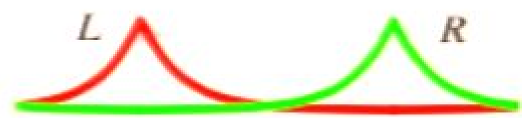
Strong breaking of chiral symmetry at the cut-off.
Additive mass renormalization.
Discretization errors introduce many spurious chiral symmetry breaking operators.

Domain wall/overlap/Ginsparg-Wilson fermions

Introduce fifth dimension s, s dependent mass m(s):

$$\mathcal{S} = \sum_x \overline{\psi}(x)\,(\gamma\cdot\Delta + \frac{r}{2}\Delta'^2 + m(s) + \gamma_5\partial_s)\,\psi(x)$$

Chiral fermions form on four-dimensional wall where *m(s)* = 0.



No species doubling, good chiral symmetry in limit $L_5 \to \infty$.

The three families of lattice fermions

have wildly incommensurate virtues and defects.

- Staggered/naive
  - Good chiral behavior (can get to light quark masses), but fermion doubling introduces theoretical complications. (Must take the root of the fermion determinant in numerical simulations.) Cheap.
- Wilson/clover
  - No fermion doubling but horrible chiral behavior. "Exceptional configurations". Messy collection of chiral symmetry breaking operators.
- Overlap/domain wall
  - Nice chiral behavior at the expense of adding a fifth space-time dimension. Expensive.

Staggered fermion calculations are the cheapest and currently most advanced phenomenologically.

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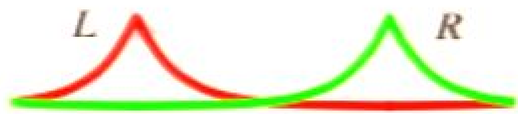

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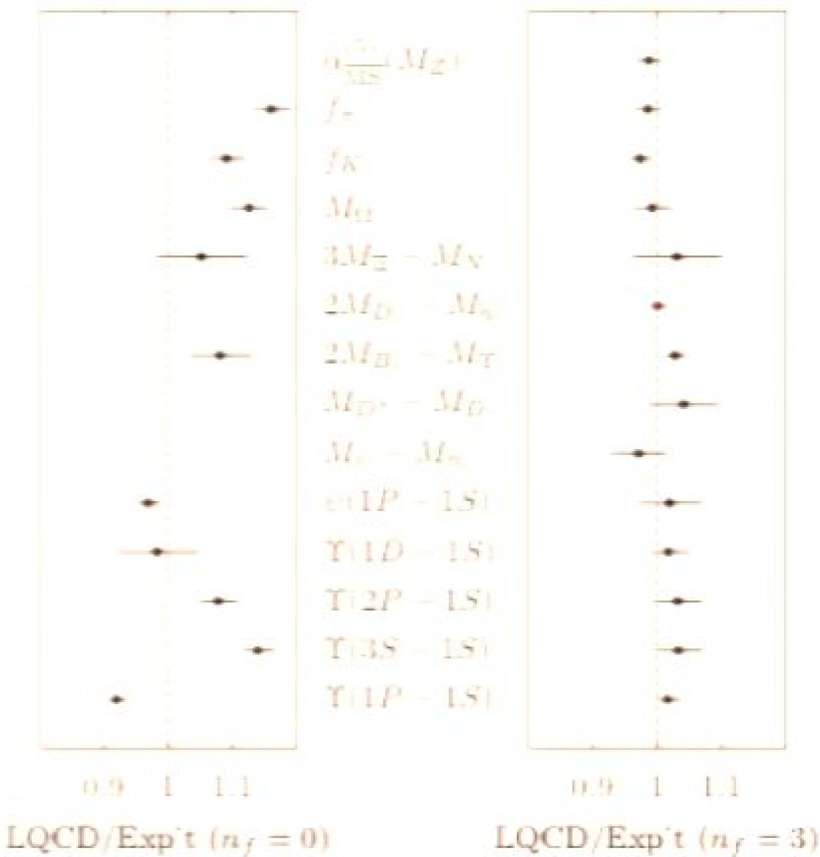
Lattice QCD confronts experiment

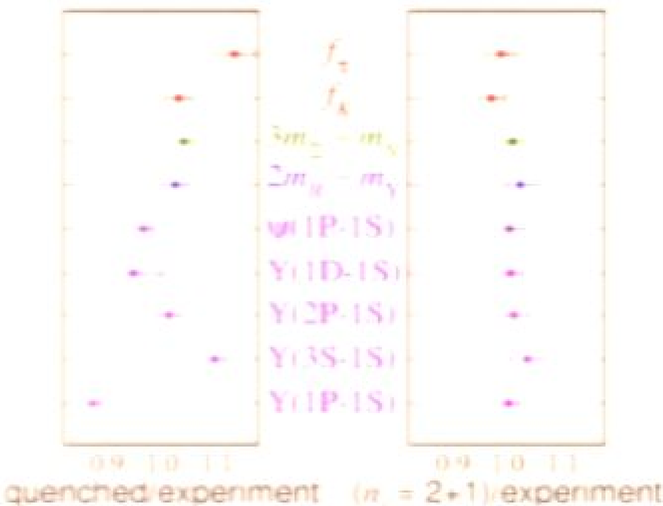
Great progress in the last five years:



For simple quantities, the 10%-ish errors visible in the "quenched approximation" are removed by including light quark-antiquark pairs.

Staggered fermions, Fermilab, HPQCD, MILC

The fundamental parameters: α_s and m_i

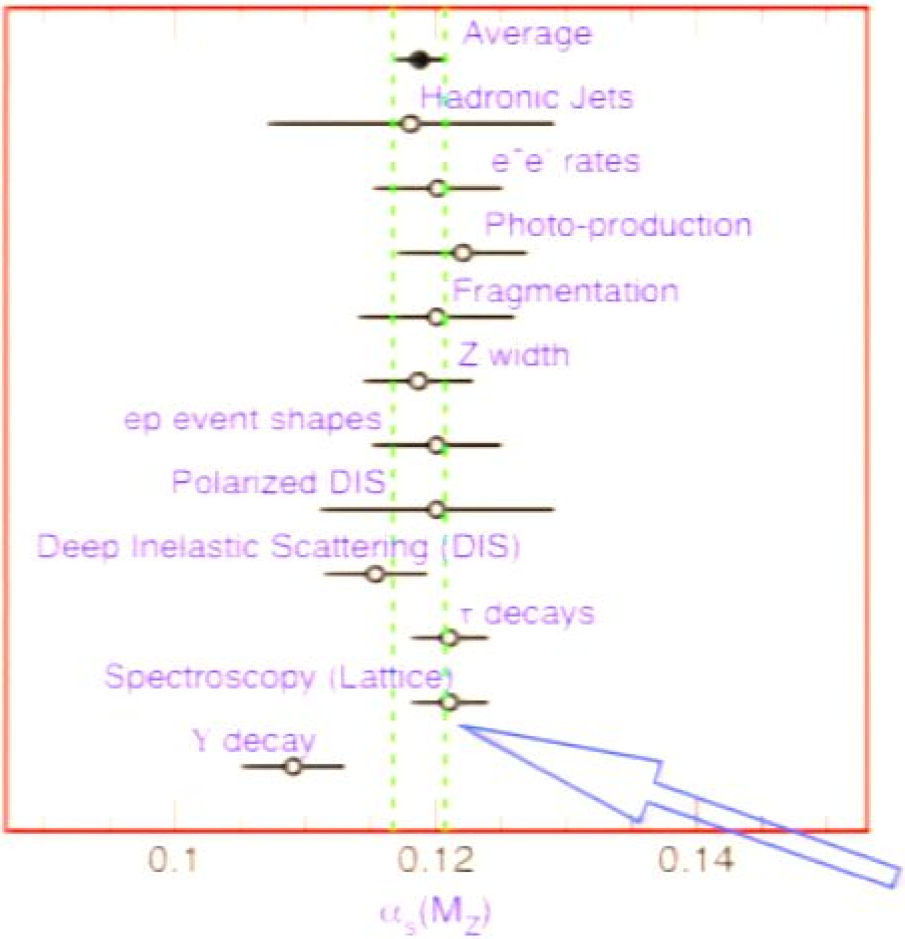


In lattice calculations, the fundamental lattice QCD parameters, α_s and m_i, are tuned so that the hadron spectrum reproduces experiment.

The $\overline{\text{MS}}$ parameters used by continuum particle physicists may be obtained from the lattice parameters by requiring that the two regulators produce the same short-distance physics. (Done either perturbatively or nonperturbatively.)

E. g.,

The strong coupling constant, α_s



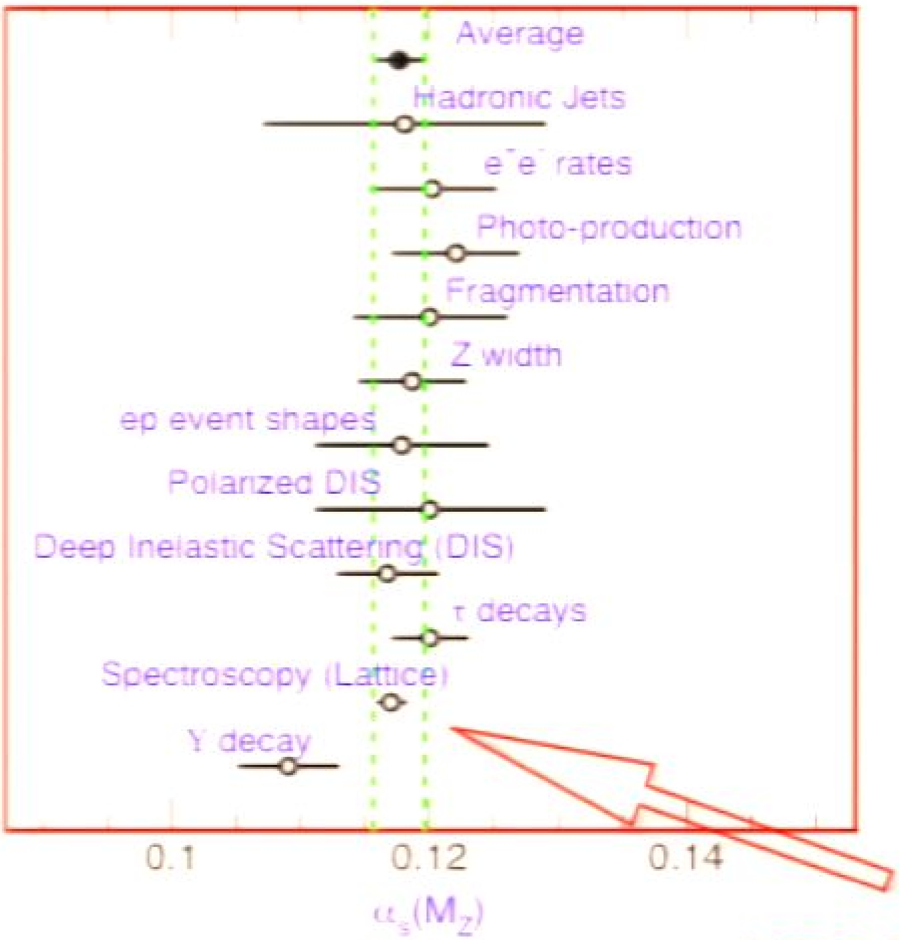
Review of Particle Properties. 2004.

Can be obtained from many high energy processes with perturbation theory.

On the lattice, tune the quark masses and strong coupling constant to reproduce observed hadron masses,
convert lattice coupling constant to continuum coupling constant.

Agrees! (Davies et al.)

The strong coupling constant, α_s



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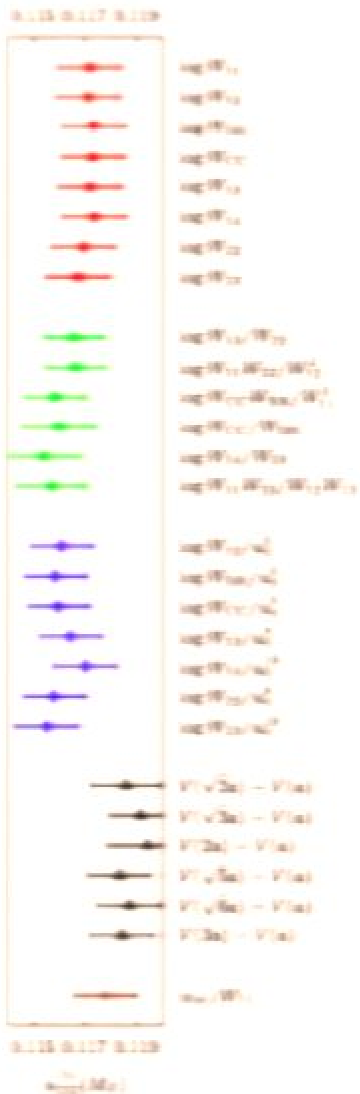
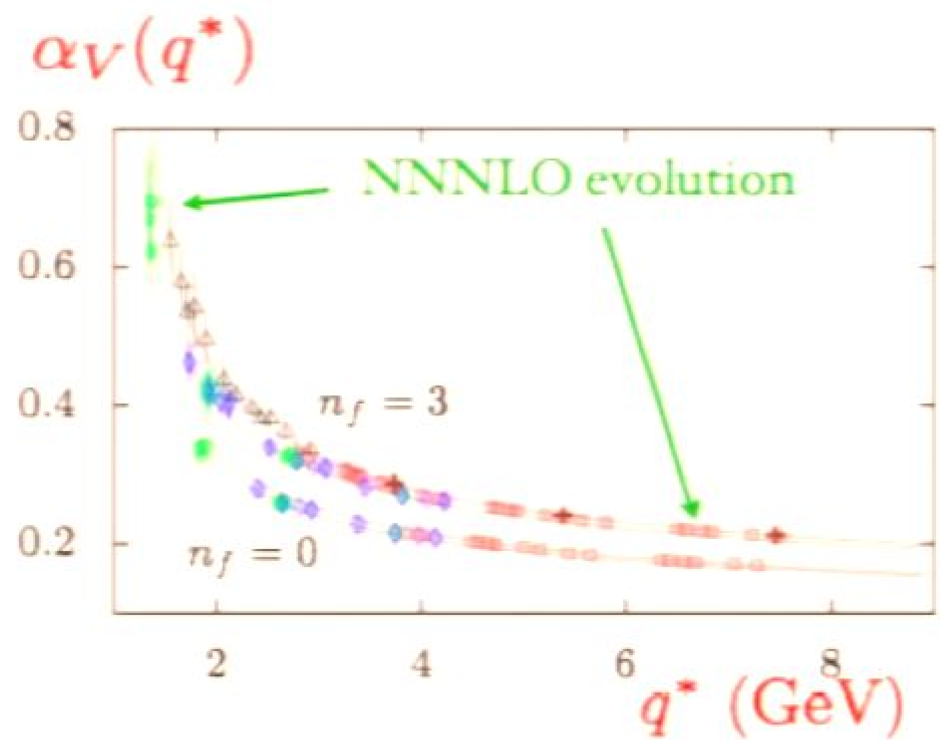
HPQCD, Mason et al., 2005.
Lattice QCD now gives the smallest errors.



The lattice α_s determination relies on results from > 25 lattice quantities of different sizes, sampling different moment scales. They show very good four-loop scaling among themselves, both quenched and unquenched.



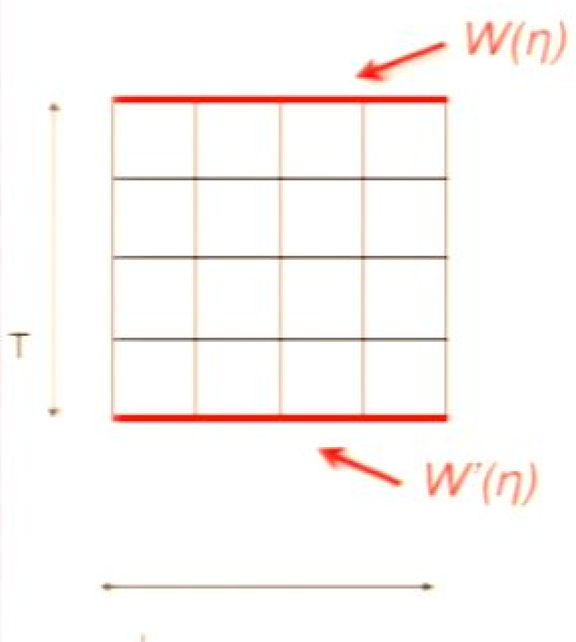
Current lattice result (HPQCD): $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1170(12).$

Nonperturbative short distance renormalization

Lattice perturbation theory works about as well as continuum QCD perturbation theory, very well in many cases (Lepage, Mackenzie).

But lattice theory at short distances is in more solid shape than continuum QCD because of the ability also to do nonperturbative short distance calculations.

E.G., nonperturbative determination of the β function (Lüscher, ...)

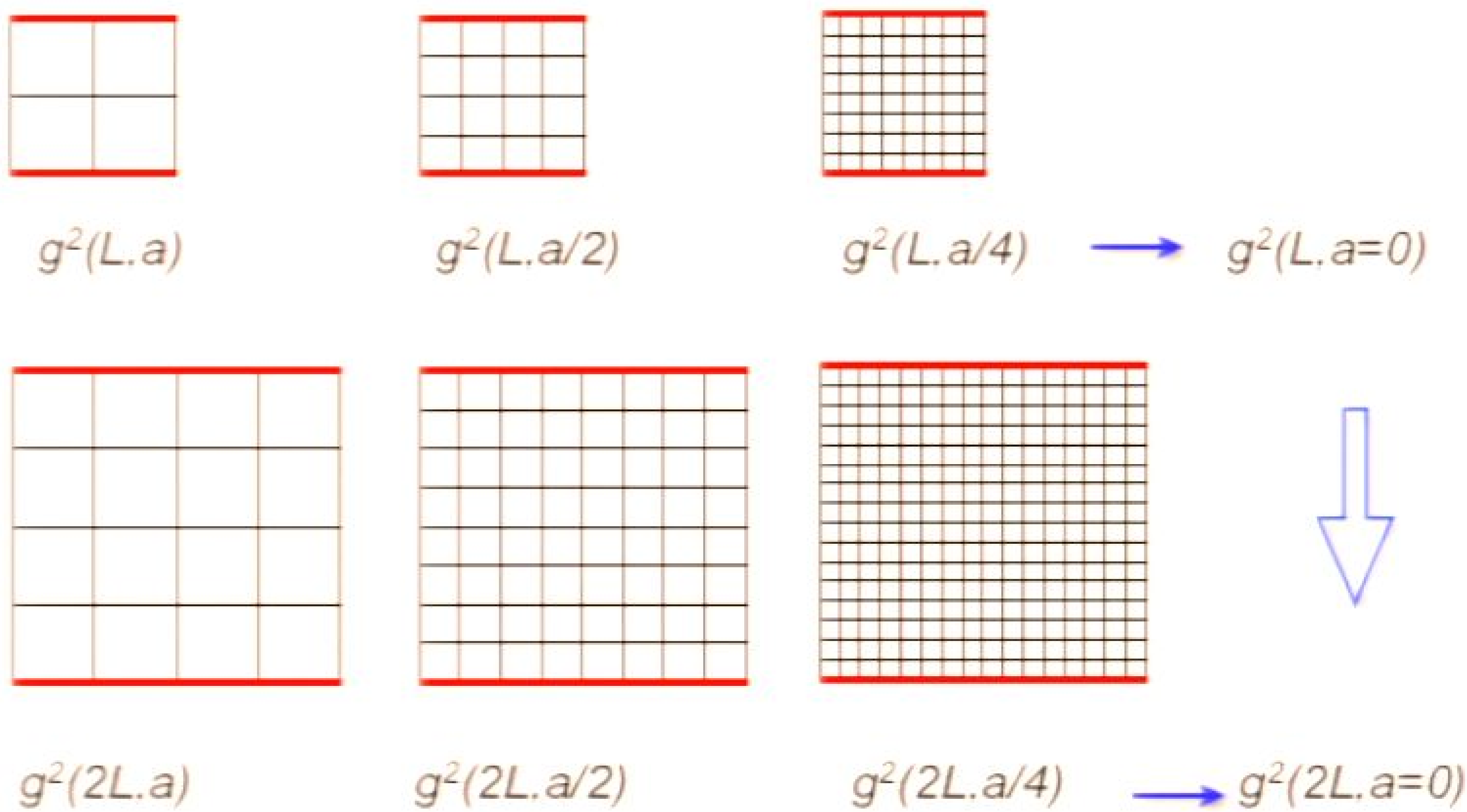


Apply external chromo-electric field $W(\eta)$, $W'(\eta)$ to QCD vacuum with Dirichlet boundary conditions. Response of vacuum energy to strength of external field is a perturbative quantity: defines a coupling constant that can be measured nonperturbatively.

$$\frac{k}{\overline{g}^2(L,T)} = -\frac{\partial}{\partial\eta}\log Z\Big|_{\eta=0}$$

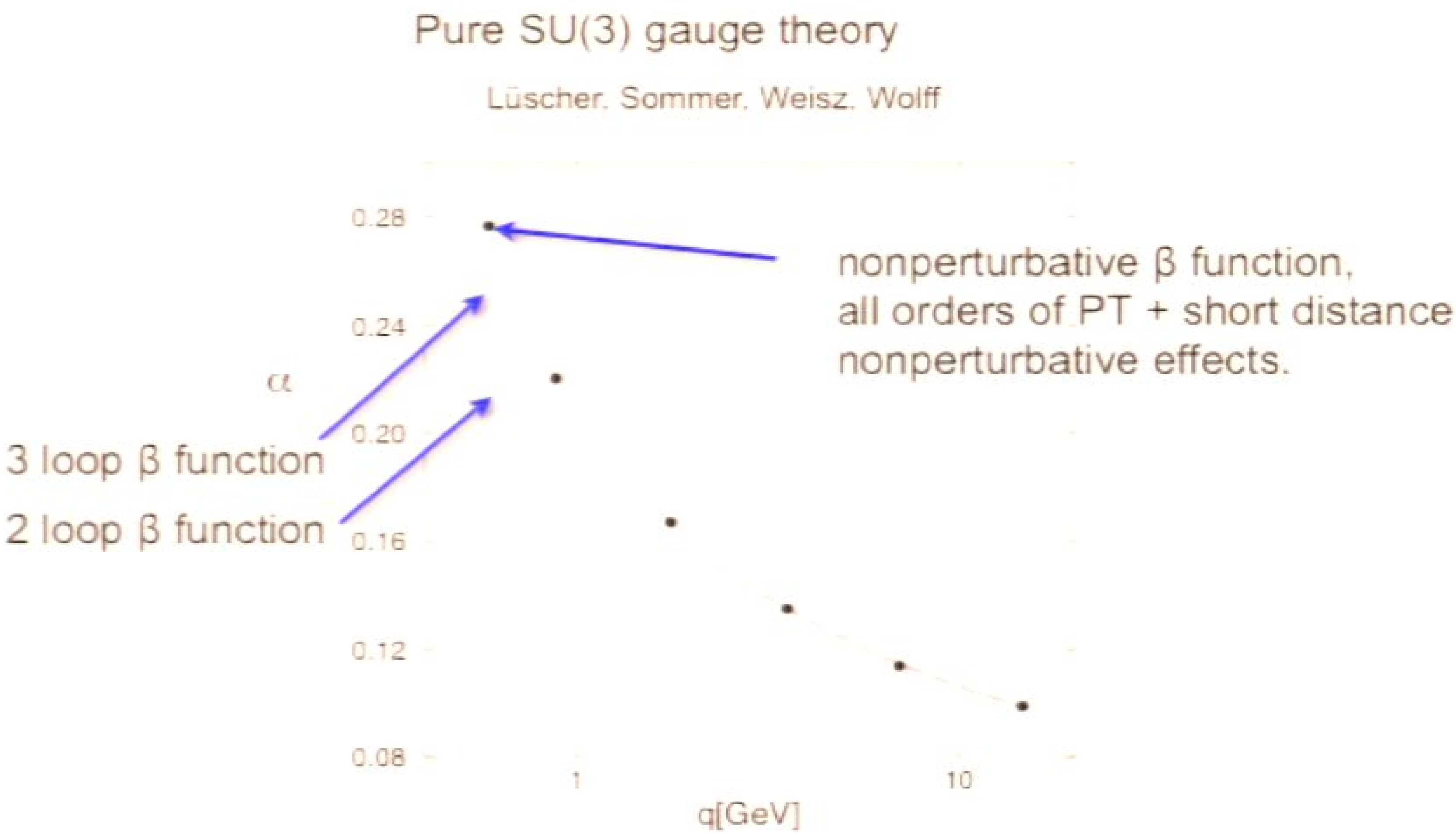
$$k = 12\left(\frac{L}{a}\right)^2\left[\sin\left(2\pi a^2/3LT\right) + \sin\left(\pi a^2/3LT\right)\right]$$

Step scaling method to obtain continuum nonperturbative running coupling.



Iterate to obtain running coupling.

In QCD, nonperturbative short distance calculations usually agree well with perturbation theory, as you would hope.



Light quark masses, m_s, m_u, and m_d

Only lattice QCD can obtain these from first principles in a systematically improvable way.

Old quark model folklore: m_s = 150 MeV.

Wrong!

Lattice results:

$$m_s = 88(0)(3)(4)(0)\ \text{MeV}$$
$$m_d = 4.6(0)(2)(2)(1)\ \text{MeV}$$
$$m_u = 1.9(0)(1)(1)(1)\ \text{MeV}$$

Obtain by matching lattice calculations of pion and kaon masses to experiment.

MILC, 2007, staggered.

Errors are (statistical)(systematic)(perturbative)(electromagnetic).

Best value and best method for obtaining m_s on the lattice are under vigorous discussion.

However, it is no longer controversial that the old "conventional" value of 150 MeV is wrong, a fact known only through lattice QCD.

Light quark masses, $m_{s,}$ $m_{u,}$ and m_d

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$m_u \neq 0!(?)$

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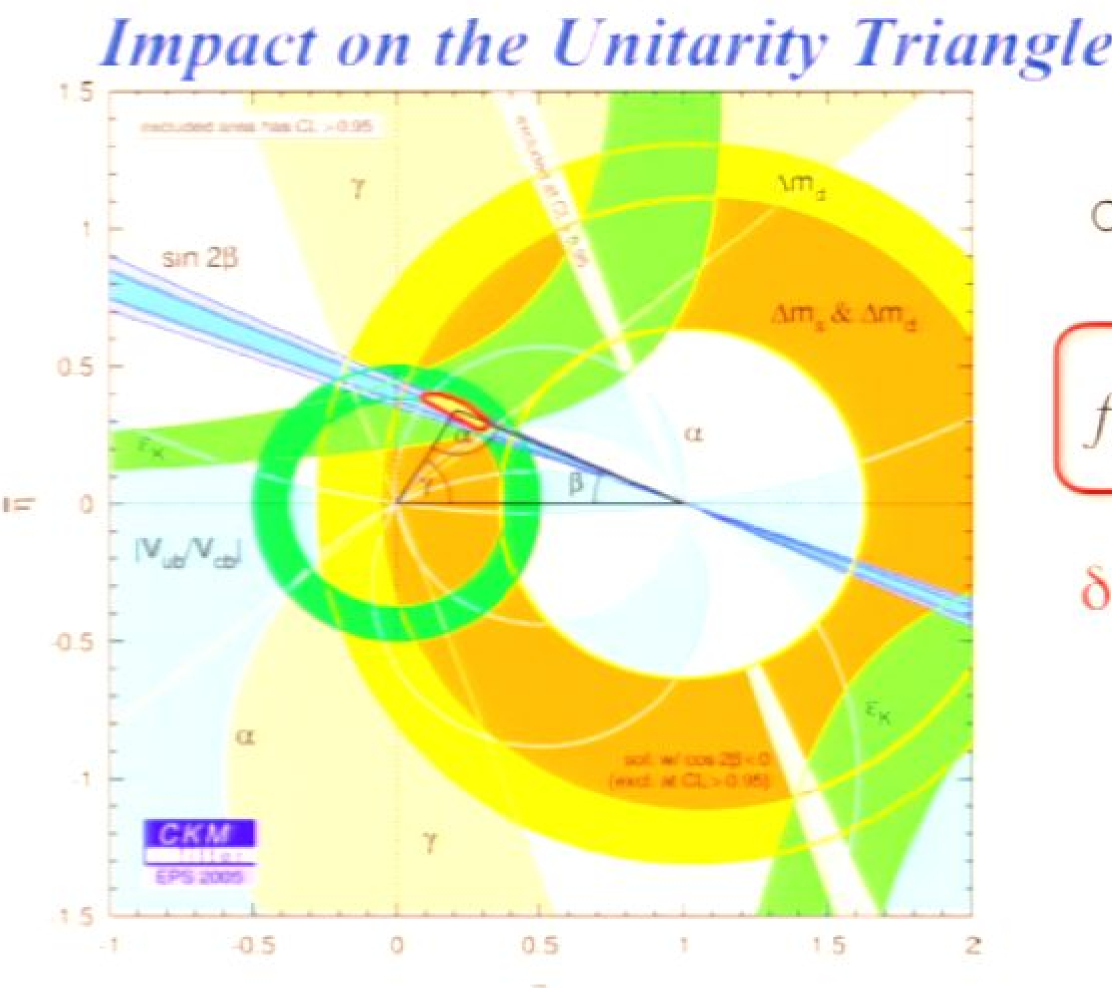
$B_s\bar{B}_s$ Mixing

Effect on CKM fits:



Okamoto Lattice 2005 review:

$$f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \;=\; 1.210(^{+47}_{-35}$$

$$\delta(|V_{td}|/|V_{ts}|) = 3-4\%$$

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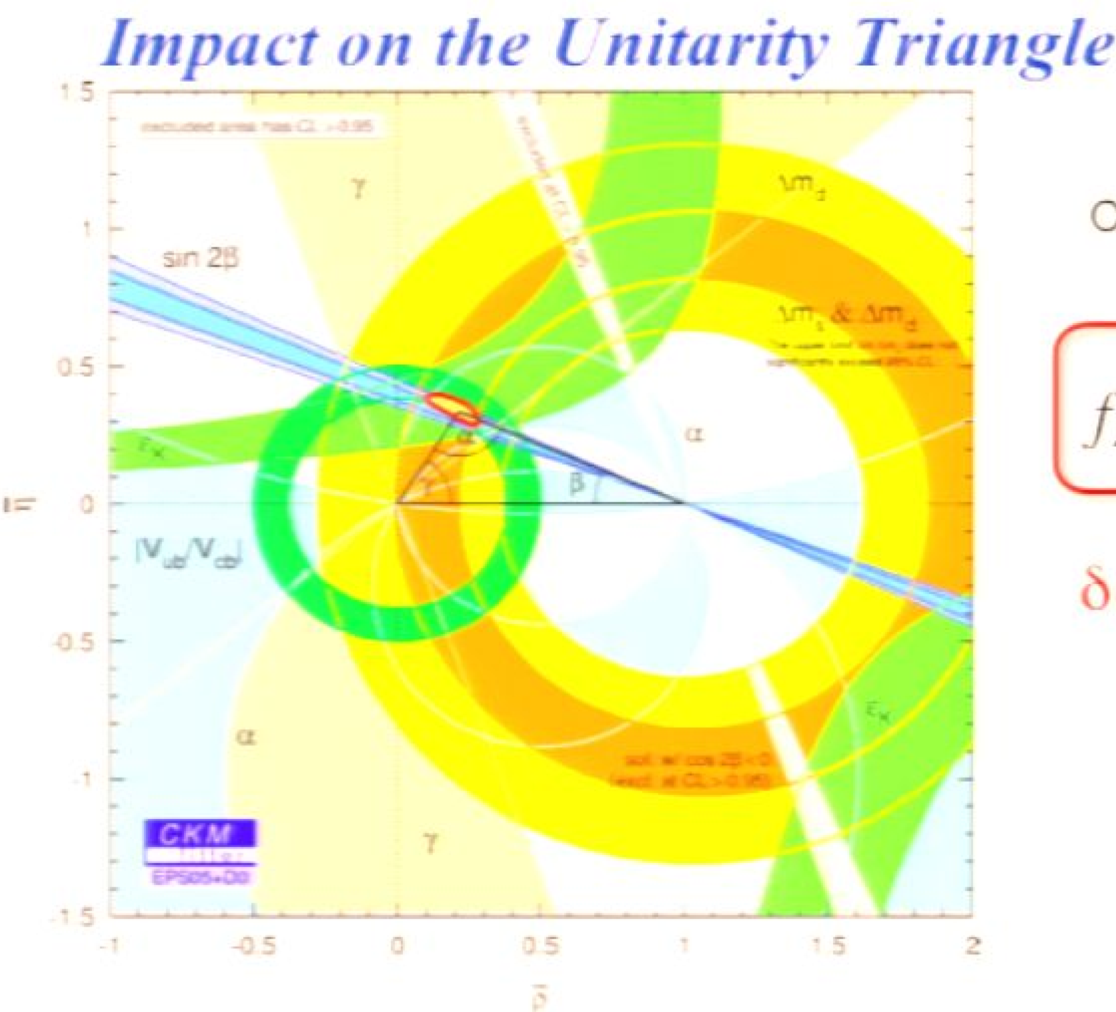

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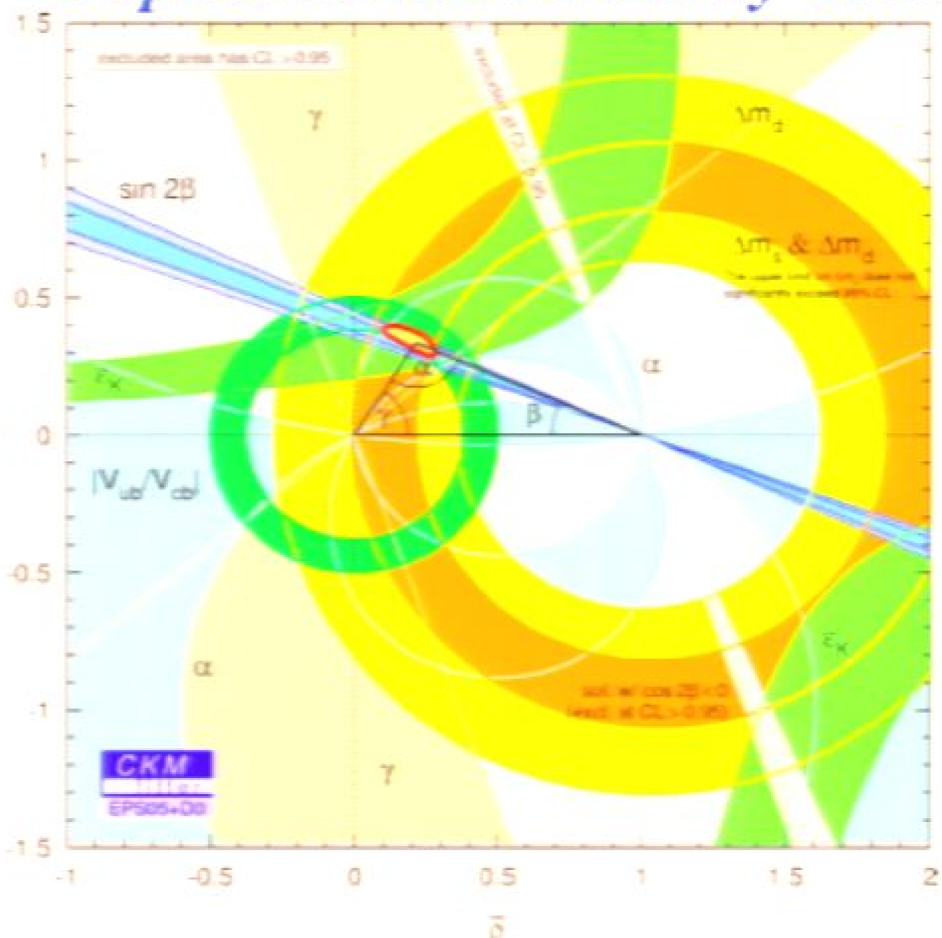
Effect on CKM fits:

Impact on the Unitarity Triangle



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$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta m_d M_{B_s^0}}{\Delta m_s M_{B_d^0}}}$$

$$= 0.208^{+0.001}_{-0.002}\,(\text{exp.})^{+0.008}_{-0.006}(\text{theo.})\,.$$

CDF

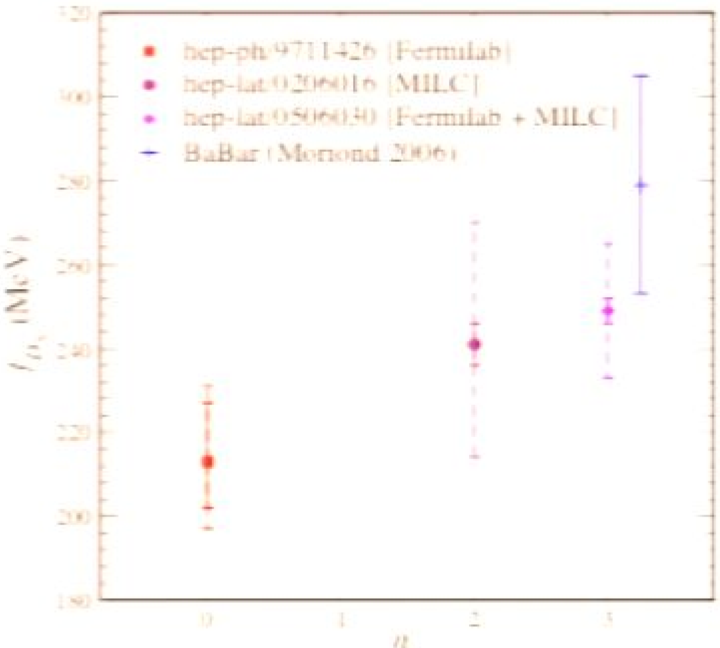
D. Buchholz

Uncertainty dominated by theory.

Predictions from lattice QCD:

- The D_s decay constant to 10%.
- Shape of D meson semileptonic decays.
- M_{Bc}

f_{Ds}



Fermilab/MILC. Phys. Rev. Lett. **95**: 122002, 2005.

Accuracies of D and D_s decay constants are being greatly improved by CLEO-c and the B factories.

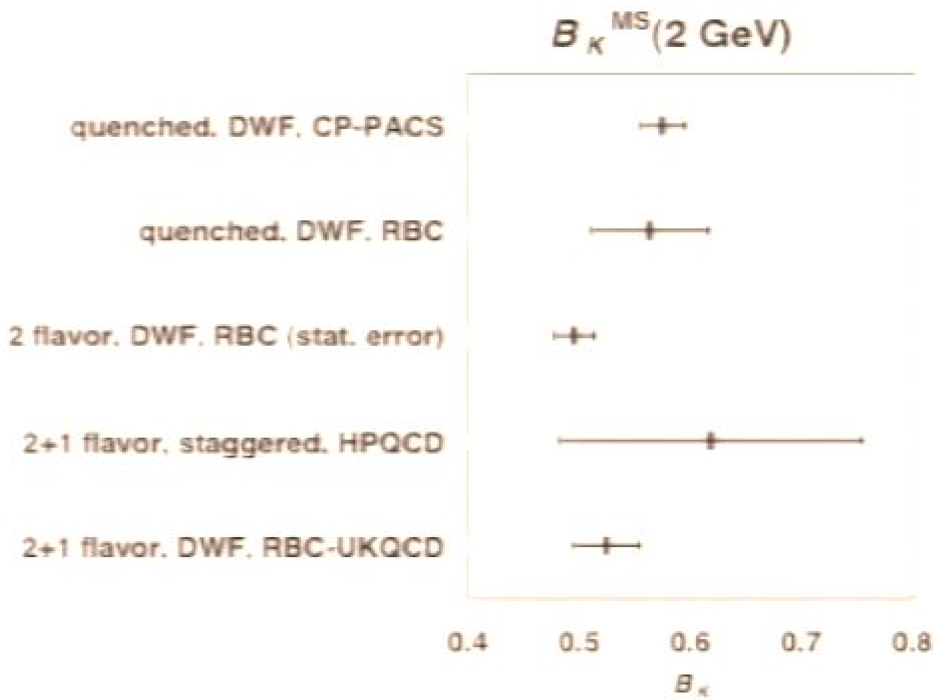
$$f_{D^+} = \begin{cases} 201 \pm 03 \pm 17 \text{ MeV [lattice]} \\ 223 \pm 17 \pm 03 \text{ MeV [CLEO]} \end{cases}$$

$$f_{D_s} = \begin{cases} 249 \pm 03 \pm 16 \text{ MeV [lattice]} \\ 279 \pm 17 \pm 20 \text{ MeV [BaBar]} \end{cases}$$

$$\frac{\sqrt{m_{D^+}} f_{D^+}}{\sqrt{m_{D_s}} f_{D_s}} = \begin{cases} 0.786 \pm 0.042 \text{ MeV [lattice]} \\ 0.779 \pm 0.093 \text{ MeV [expt]} \end{cases}$$

New in 2008: Kaon physics

RBC/UKQCD



B_K. Calculation of the $K\overline{K}$ mixing parameter B_K, which is used to determine the CP violating phase ir the CKM matrix from kaon decay measurements.

Most accurate unquenched calculation to date.

The clean chiral symmetry of doma wall fermions relative to staggered fermions simplifies this calculation.

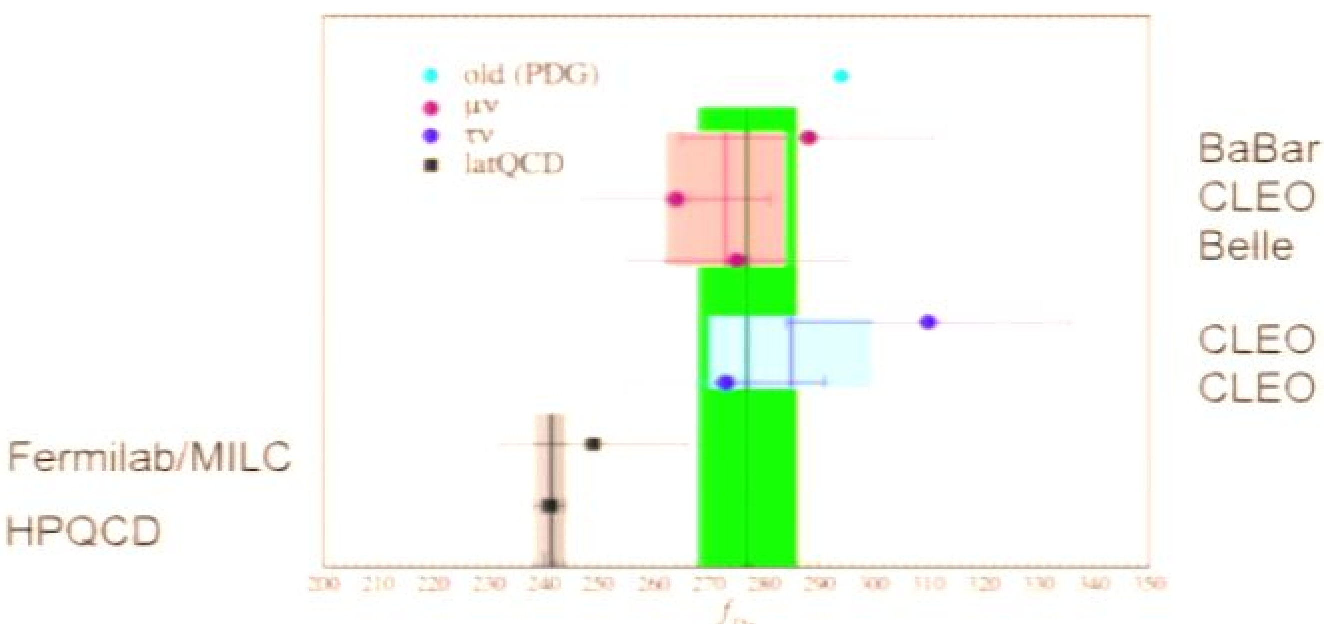
New in 2008: π, K, D, D_s decay constants

$$f_K = 157(2)\ \text{MeV} \qquad f_D = 207(4)\ \text{MeV}$$

HPQCD

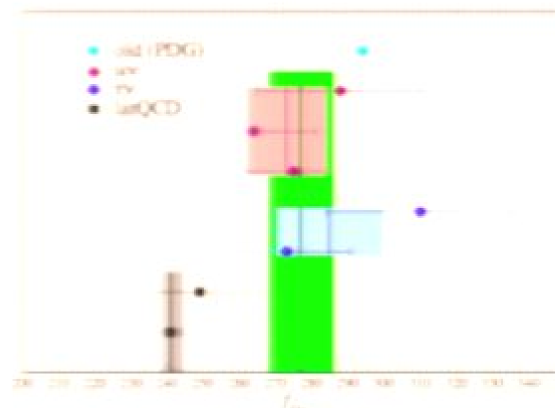
$$f_K/f_\pi = 1.189(7) \qquad f_{D_s} = 241(3)\ \text{MeV}$$

- Predicted of f_{Ds} to 10% was a success, but...
- Errors in f_{Ds} have been reduced, theory has stayed low and experiment has stayed high. (f_π, f_K, and f_D come out fine.)
- 3.8 σ



New in 2008: π, K, D, D_s decay constants

- 3.8 σ discrepancy is dominated by experimental *statistical* error(!).
  - Double theory error bar, discrepancy → 3.3σ.
  - Triple theory error bar (and include Fermilab/MILC 2005 value) → 3.1σ.
- Theory method is innovative and has not been confirmed yet, but...
- What if the discrepancy is real?
  - Kronfeld and Dobrescu, effect could be caused by:
    - Charged Higgs (in a new 2HDM)
    - Leptoquarks (of two ilks)



Current lattice program

$$\begin{pmatrix} \begin{array}{c} V_{ud} \\ f_\pi \\ \end{array} & \begin{array}{c} V_{us} \\ f_K \\ K \to \pi l \nu \end{array} & \begin{array}{c} V_{ub} \\ f_B \\ B \to \pi l \nu \end{array} \\ \begin{array}{c} V_{cd} \\ f_D \\ D \to \pi l \nu \end{array} & \begin{array}{c} V_{cs} \\ f_{D_s} \\ D \to K l \nu \end{array} & \begin{array}{c} V_{cb} \\ B \to D^* l \nu \\ B \to D l \nu \end{array} \\ \begin{array}{c} V_{td} \\ < B | \overline{B} > \end{array} & \begin{array}{c} V_{ts} \\ < B_s | \overline{B_s} > \end{array} & \begin{array}{c} V_{tb} \\ \end{array} \end{pmatrix}$$

Full CKM matrix can be attacked wit
lattice calculations, along with α_s, m_q,
B_K, ...

Current lattice program is much larger than I've discussed:.
Lots of activity in:

- baryons,
- thermodynamics: plasma T_c and viscosity,
- $\rho \to \pi\pi$, $K \to \pi\pi$,
- structure functions,
- ...

Where do we stand now?

- We now have a new tool in our tool set for quantum field theory.
  - Lattice field theory
  - Perturbation theory
  - Duality
  - ...
- We can calculate some nontrivial things, though nothing close to everything we would like.
- Lots of important applications await.

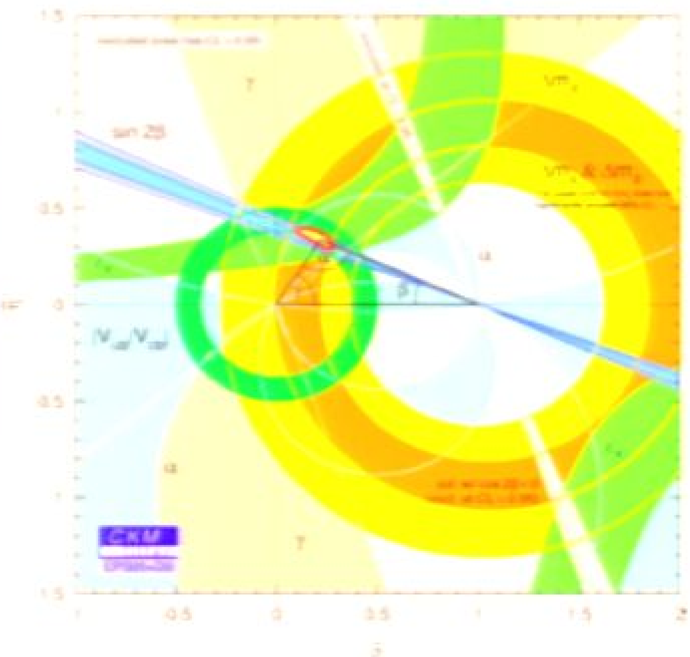
What is needed in the LHC era?

- In the quest to understand the physics beyond the SM'
  - Why is M_W/M_P small?
  - Why is M_ν/M_W small?
  - Why is non-SM flavor changing small?
- What is the role of the lattice?
  - 1) Search for BSM effects in flavor physics.
  - 2) Direct investigation of BSM theories
    - - SUSY
    - - Dynamical symmetry breaking
    - ...

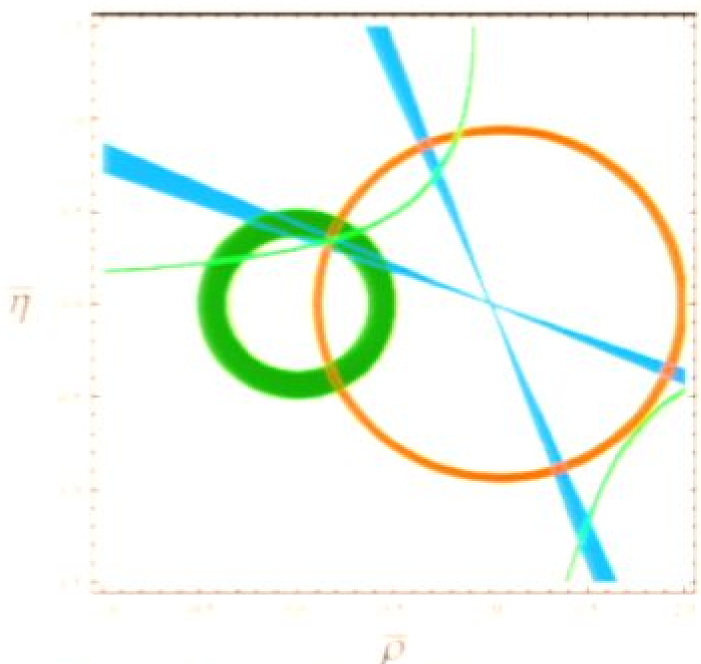
What is needed in the LHC era?

High precision in QCD!

What is needed in the LHC era?
High precision in QCD!



The $\rho\eta$ plane with today's experiment and theory errors.



The $\rho\eta$ plane with today's experimental errors and theory errors of 2014.

Experimental precision in *KK*, *BB*, and *BsBs* mixing is already at 1%!

Lattice calculations will get there in the coming years.

The LHC era: Beyond the Standard Model

Theory space is very big.

In the SM, we know the theory, in BSM we don't.
Can't investigate many BSM theories with the care invested in QCD calculations.

Serious programs in several places.

I will just give a couple of representative examples.

The LHC era: Beyond the Standard Model

Dynamical weak symmetry breaking

Search for a strongly coupled non-Abelian Coulomb phase in the "conformal window"

$$N_f^{\rm af} > N_f > N_f^{\rm c}$$

Such a theory could produce an evolution of the quark masses over a large momentum range with a large anomalous dimension. A confining theory very close to the conformal window could explain the absence of flavor changing neutral currents in the context of technicolor models: "Walking Technicolor".

Can be investigated nonperturbative step-scaling calculations which have been shown work in QCD.

The LHC era: Dynamical symmetry breaking

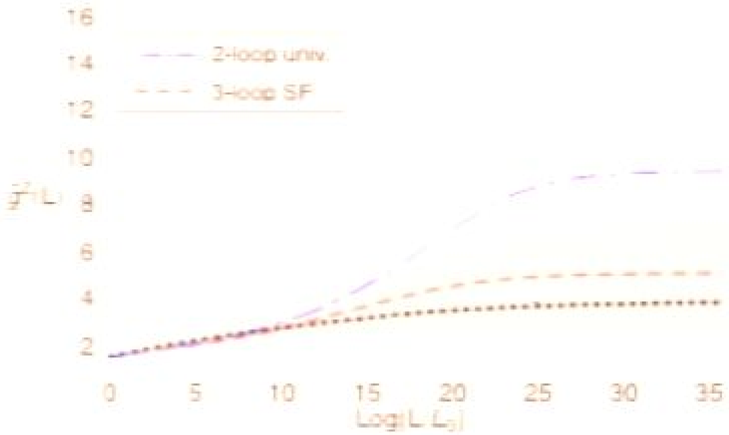


FIG. 1. Continuum running coupling from step scaling for N_f = 12. The statistical error on each point is smaller than the size of the symbol. Systematic error is shown in the shaded band.

N_f=12
Weak coupling fixed point
Agrees with 3-loop PT
(staggered fermions)

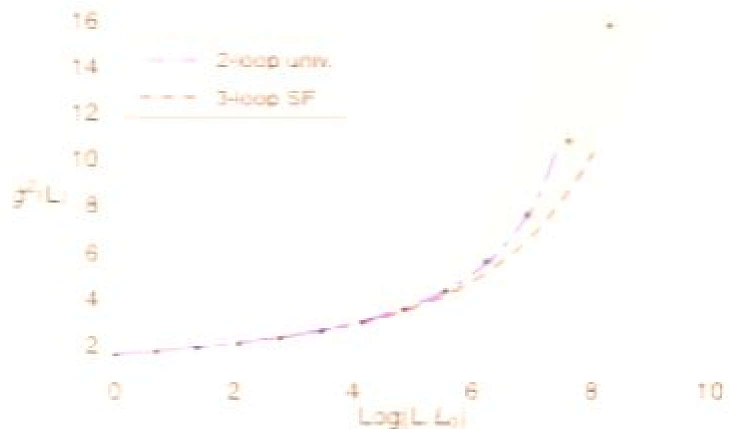


FIG. 2. Continuum running coupling from step scaling for N_f = 8. Errors are shown as in Fig. 1. Perturbation theory is shown up to only $\bar{g}^2(L) \sim 10$.

N_f=8
Confining
(staggered fermions)

Now under way:
What happens at N_f=10?
(Wilson fermions)

Appelquist, Fleming, and Neil

Use N_c=3 gauge for first serious tests. (Highly optimized codes and algorithms have been developed.)

Vary N_f to search nonperturbatively for strongly coupled non-Abelian fixed poin

The LHC era: SUSY

Discretization errors break SUSY
Simplest nontrivial 4 dimensional SUSY theory on the lattice:
Probably N=1 SUSY YM.
SUSY is recovered automatically in continuum limit.

Kaplan & Schmaltz, '99

Goal: Show SUSY is broken dynamically,
- already knew that

Numerical results (new):
- $\langle\lambda\lambda\rangle$/(string tension)**1.5
- Spectrum of fermionic and bosonic glueballs.

One large-scale project that is underway:
- Pure SUSY SU(3) YM, octet of domain-wall gluinos.
(SU(3) again because algorithms and codes have been highly optimized for QCD calculations.)

Next, could add matter fields.
- Squark action would have a Laplacian.
Would introduce a fine tuning in most straightforward approach.

Summary

- With lattice QCD, we now have a new tool for fundamental investigation of quantum field theory.
  - The era of precision lattice QCD with all errors controlled *has arrived*.
    - Many calculations of experimentally important quantities have been done entirely from first principles.
- We are beginning to scratch the surface of nonperturbative Beyond-the-Standard-Model physics directly.
- Much more is required of the lattice in the LHC era:
  - The existing successes must be pushed to <1% accuracy.
  - Many new calculations that are currently impossible are needed.
  - We need to learn to do equally accurate calculations in new strongly interacting theories.