Title: Conformal Collider Physics

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Abstract: We study observables in a conformal field theory which are very closely related to the ones used to describe hadronic events at colliders. We focus on the correlation functions of the energies deposited on calorimeters placed at a large distance from the collision. We consider initial states produced by an operator insertion and we study some general properties of the energy correlation functions for conformal field theories. We argue that the small angle singularities of energy correlation functions are controlled by the twist of non-local light-ray operators with a definite spin. We relate the charge two point function to a particular moment of the parton distribution functions appearing in deep inelastic scattering. The one point energy correlation functions are characterized by a few numbers. For ${cal N}=1$ superconformal theories the one point function for states created by the R-current or the stress tensor are determined by the two parameters a and c charac terizing the conformal anomaly. Demanding that the measured energies are positive we get bounds on a/c. We also give a prescription for computing the energy and charge correlation functions in theories that have a gravity dual. The prescription amounts to probing the falling string state as it crosses the AdS horizon with gravitational shock waves. In the leading, two derivative, gravity approximation the energy is uniformly distributed on the sphere at infinity, with no fluctuations. We compute the stringy corrections and we show that they lead to small, non-gaussian, fluctuations in the energy distribution. Corrections to the one point functions or antenna patterns are related to higher derivative corrections in the bulk.

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1. Opening Remarks

- Gauge Theory/String Theory duality. Can we build a strong coupling description of QCD out of gravity? This is difficult, but there has been recent progress.
- String Theory on $AdS_5 \times S^5$ —— $\mathcal{N} = 4$ SYM
- Several limits in which $\mathcal{N} = 4$ is not that different from QCD: Quark-gluon plasma, transcendentality hypothesis [Lipatov; Beisert, Eden, Staudacher], etc.
- Can we use AdS/CFT to predict properties of real world gauge theories?
- Recent examples of this type of applications are the study of the viscosity to entropy ratio as well as superconductivity.
- What about collider physics? With the LHC coming up it is interesting to try to understand both how new physics and QCD will look inside this new accelerator.

2. Conformal Collider Physics 101

2.1. What can/should we calculate?

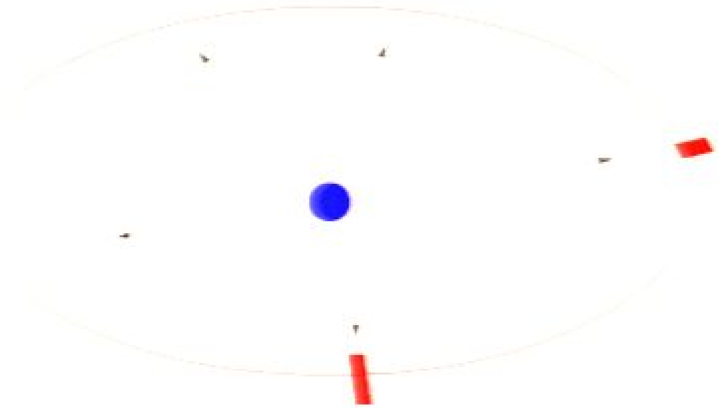
- But the theory is conformal!

- Many properties of QCD depend strongly on the UV physics. We can expand in the coupling constant (beta function) and treat non-conformality perturbatively.
- There are phenomenological models in which there are conformal hidden sectors.
- We can treat this as a warm up for QCD.
- Understand better the Ads/CFT correspondence.

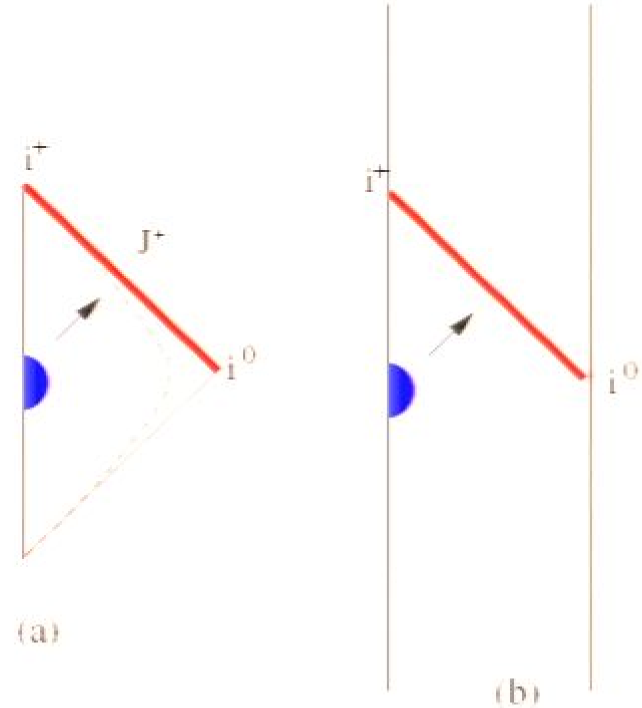
• What do we calculate then?

- There has been recent progress in the understanding of gauge theory amplitudes in AdS/CFT [Alday, Maldacena].
- The problem with these observables is that they are not IR safe (regularization dependent).
- In QCD a natural IR safe observable is the energy (or charge) correlator.
- We usually think of these observables as being related to cross sections, parton models and the S-matrix. But it is just UV physics!
- Is there a better way to think about these quantities such that we are not tied to perturbation theory?

2.2. The energy and charge correlators



- We will be calculating $\langle \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\rangle \equiv \frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle}$ with $\mathcal{E}(\theta) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T^0_i(t, r\vec{n}^i)$
- Notice that our idealized calorimeters can interact directly with the CFT.
- Now use the conformal symmetry to map the future boundary of Minkowski space to a a finite position. y coordinates.



$$\mathcal{E}(y_1, y_2) \sim \int_{-\infty}^{\infty} dy^- T_{--}(y^-, y^+ = 0, y^1, y^2) \quad (1)$$

- We only need one component of the energy momentum tensor T_{--}.
- It can be shown that all components of the four momentum depend on only T_{--}. This is what we would expect from massless particles and it is valid for any CFT (even interacting ones!).

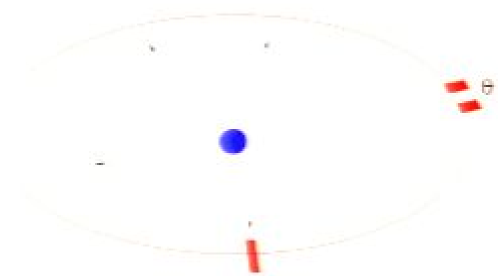
- Before we start doing calculation we will make one assumption

$$\int dy^- \langle T_{--} \rangle \geq 0 \tag{2}$$

- We can also define $\mathcal{Q}(\vec{n}) = \lim_{r \to \infty} r^2 \int_{-\infty}^{\infty} dt\, n^i j_i(t, r\vec{n})$

3. CFT general results

3.1. Small angle behavior and OPE



(a) (b)

- It is known that the two point function has a small angle divergence perturbatively. Colinear radiation.

- $\mathcal{E}(y^1, y^2)\mathcal{E}(0, 0) \sim \int dy^- T_{--}(y^-, y^+ = 0, \vec{y}) \int dy'^- T_{--}(y'^-, y^+ = 0, \vec{0})$

- We are looking for operators that are: integrals of spin 3 operators and of leading twist.
- At zero coupling, family of local operators of spin j is $\mathcal{U}_j = Tr[\phi \overleftrightarrow{\partial}_-^j \phi]$.
- These are only primary for even j.
- A non local extension of these operators for any complex j is $\mathcal{U}(y^-, y'^-) = Tr[\phi(y^-)W(y^-, y'^-)\phi(y'^-)] = Tr[\phi(y^-)Pe^{\int_{y^-}^{y'^-} A}\phi(y'^-)]$. Light ray operators.
- The operators we are looking for are

$$\mathcal{U}_{j-1} = \int_{-\infty}^{\infty} dy^- \int_0^{\infty} \frac{du}{u^{j+1}} Tr[\phi(y^- + u)W(y^- + u, y^- - u)\phi(y^- - u)] \quad (3)$$

- The precise form of these operators can be found by diagonalizing the matrix of anomalous dimensions.

3.2. One point functions

- $\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0|\mathcal{O}_q^\dagger \mathcal{E}(\vec{n})\mathcal{O}_q|0\rangle}{\langle 0|\mathcal{O}_q^\dagger \mathcal{O}_q|0\rangle}$

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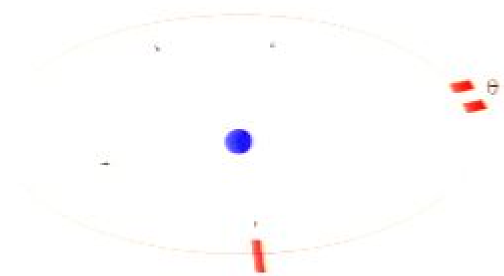

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- 3 point functions. Unusual time ordering.
- 1 point functions determined by conformal symmetry. In many cases $O(3)$ will be enough.
- Scalar source — uniform function $\langle \mathcal{E}(\vec{n}) \rangle = \frac{q}{4\pi}$
- Fixed by Ward identities.
- Current sources $\langle \mathcal{E}(\vec{n}) \rangle = \frac{q}{4\pi} \left[1 + a_2 (\cos^2\theta - \frac{1}{3}) \right]$.
- Energy positivity — $3 \geq a_2 \geq -\frac{3}{2}$.
- This is true in perturbative QCD.
- For $\mathcal{N} = 1$ theories this is zero for any non R current. Fermions and bosons cancel. For the R current it can only depend on a and c.
- Using free theories — general result $\langle \mathcal{E}(\theta) \rangle = 1 + 3\frac{c-a}{c}(\cos^2\theta - \frac{1}{3})$.
- Uniform for $\mathcal{N} = 4$.
- We can use the energy momentum tensor as a source as well.
- $\langle \mathcal{E}(\theta) \rangle = \frac{q^0}{4\pi} \left[1 + t_2 \left(\frac{\epsilon_{ij}^* \epsilon_{il} n_i n_j}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{2}{15} \right) \right]$.

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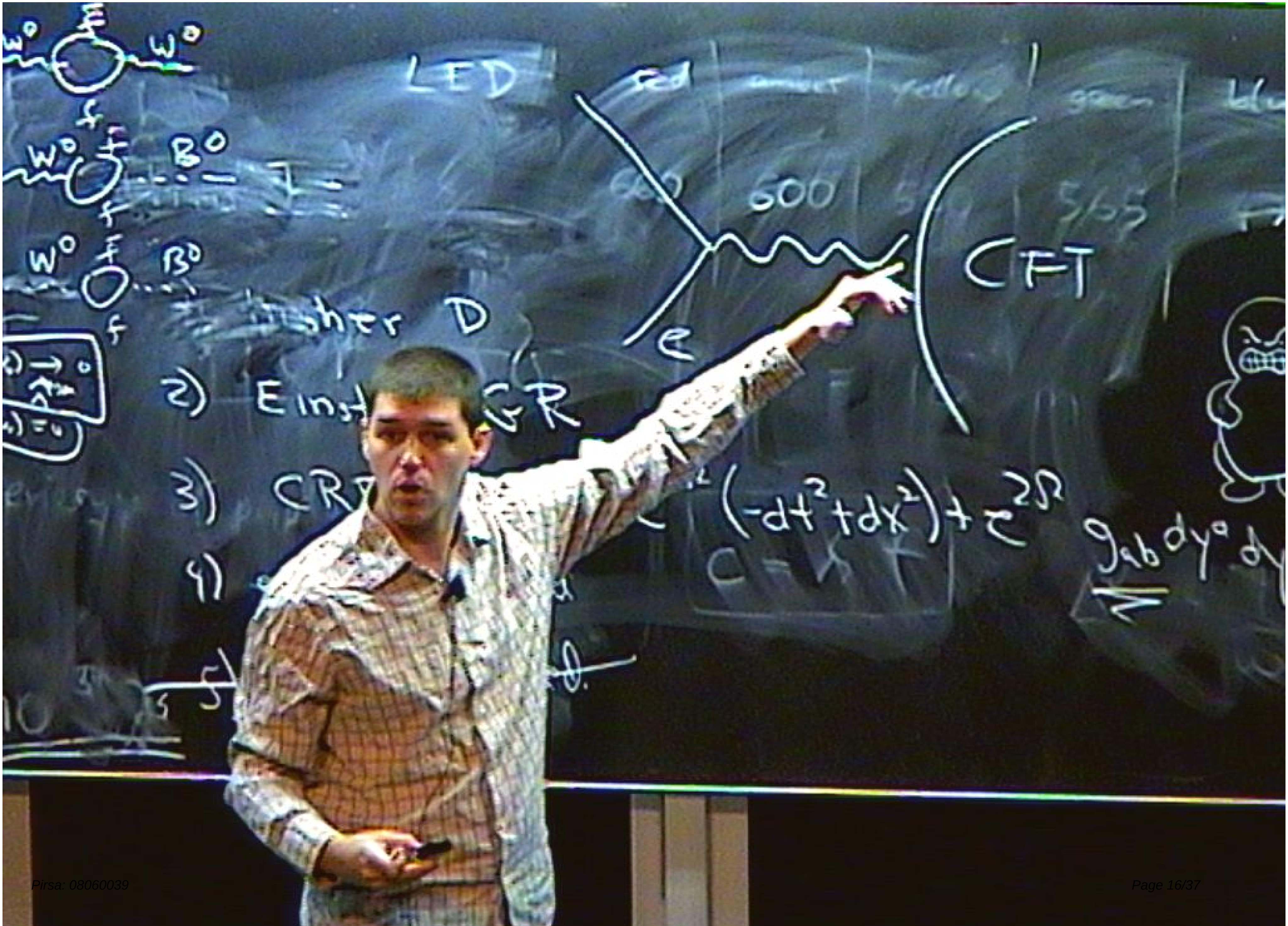
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2) Einst GR
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- $\mathcal{N} = 1 - t_2 = 6(c-a)/c\,.\qquad t_4 = 0$
- Positivity — $\frac{3}{2}c \geq a \geq \frac{c}{2}$. Similar bounds for non susy and $\mathcal{N} = 2$.
- Detour: The lower bound agree with results in GB gravity in [Brigante, Liu, Myers, Shenker, Yaida]. Work in progress: We have been able to reproduce the higher bound as well in this setup. Understanding this calculation allows for an understanding of the positivity of energy condition.
- If we include parity odd terms for the charge correlators we see charge asymmetries related to anomalies. Here $O(3)$ is not enough.

4. CFTs with gravity duals

4.1. Coordinates and classical configurations

- Coordinates for AdS_5:

$$ds^2 = -dW^+dW^- - \frac{1}{4}\frac{(W^-dW^+ + W^+dW^-)^2}{1 - W^+W^-} + (1 - W^+W^-)ds^2_{H_3} \quad (4)$$

- Lorentz = ismotries.

- We insert calorimeters at $W^+ = 0$ on the boundary and integrate over the $-$ direction. The fields in the bulk can be calculated.
- Prescription: $\mathcal{E}(\vec{n}')$ — $h_{MN}^{\mathcal{E}(\vec{n}')} dX^N dX^M \sim \delta(W^+)(dW^+)^2 \frac{1}{(W^0 - W^i n_i')^3}$
- $\mathcal{Q}(\vec{n}')$ — $A_M dx^M \sim dW^+ \delta(W^+) \frac{1}{(W^0 - W^i n_i')^2}$.
- Source fields have definite momentum on the boundary. $P_x^\mu|_{W^+=0} = -2iW^\mu \partial_{W^-}$
- $O_q(W^+ = 0, W^-, W^\mu) \sim (q^0)^{\Delta-4} e^{iq^0 W^-/2} \delta^3(\vec{W})$
- To sum up: We take snap shots of the infalling string state at the horizon of the usual Poincare coordinates.

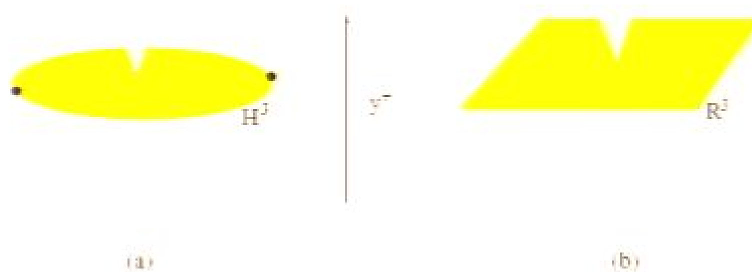
4.2. Actions and higher derivatives corrections

- We know that for $\mathcal{N} = 4$ all correlators are uniform.
- We need higher derivatives to calculate non trivial angle dependence.
- Current sources: $S = -\frac{1}{4g^2} \int d^5x \sqrt{g} F^2 + \frac{\alpha_1}{g^2 M_*^2} \int d^5x \sqrt{g} W^{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho}$.
- These are the only relevant terms for the 3 point function. There are only 2 gauge invariant vertices in flat space.

- Calculation: $a_2 = -\frac{48\alpha_1}{R^2_{AdS}M_*^2}$.
- Tensor: $S = \frac{M_{pl}^3}{2}\left[\int d^5x\sqrt{g}R + \frac{\gamma_1}{M_{pl}^2}W_{\mu\nu\delta\sigma}W^{\mu\nu\delta\sigma} + \frac{\gamma_2}{M_{pl}^4}W_{\mu\nu\delta\sigma}W^{\delta\sigma\rho\gamma}W_{\rho\gamma}{}^{\mu\nu}\right]$
- One R^2 term up to field redefinitions. There's one more R^3 term but does not contribute to 3pt functions. 3 invariant vertices in this case.
- $t_2 = \frac{48\gamma_1}{R^2_{AdS}M^2_{pl}} + o(\frac{\gamma_2}{R^4_{AdS}M^4_{pl}})$ $t_4 = \frac{4320\gamma_2}{R^4_{AdS}M^4_{pl}}$.
- Both corrections vanish for type II ST. t_4 vanishes for heterotic ST.
- Flat space — AdS.
- n point functions can be calculated. Distribution functions.

5. Stringy calculations

5.1. Energy correlators



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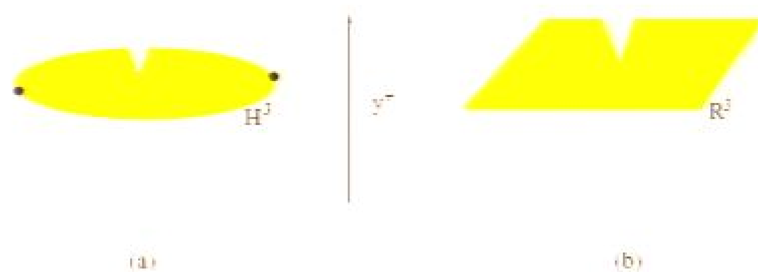
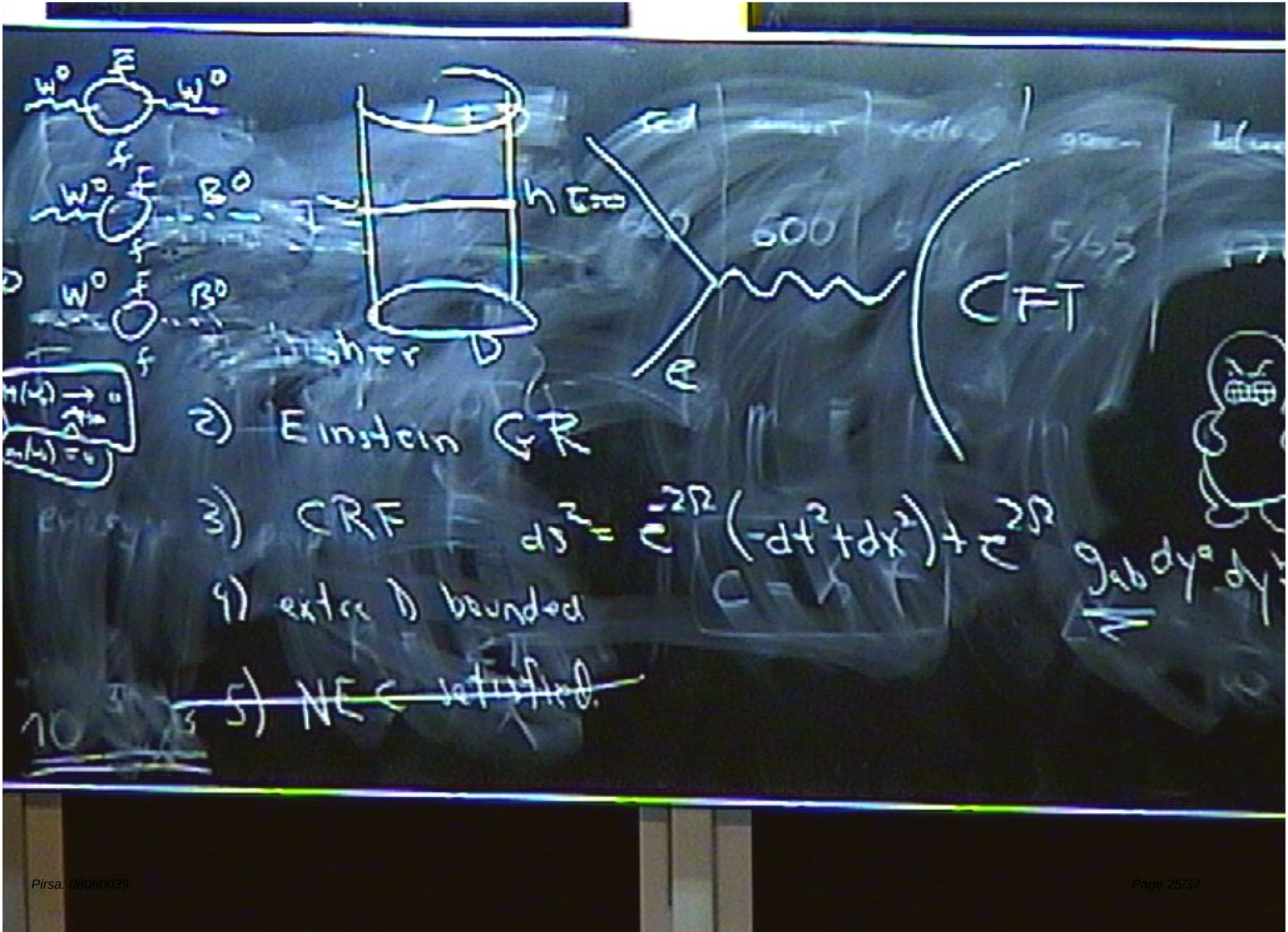


- Strategy: Calculate in light cone ST in flat space and then translate to AdS. We use leading results and normalizations.
- We evaluate: $\langle \Psi | e^{-ip_- \int_0^{2\pi} \frac{d\sigma}{2\pi} h(\vec{y}(\sigma))|_{\tau=0}} | \Psi \rangle$.
- Assume initial state does not have bosonic oscillator excitations (massless graviton in IIB).

$$(-ip_-)^n \langle v_{cm} | \prod_j e^{i\vec{k}_j \vec{y}} | v_{cm} \rangle \langle 0 | \prod_j \int \frac{d\sigma_j}{2\pi} e^{i\vec{k}_j \vec{y}_{osc}(\sigma)} | 0 \rangle \quad (5)$$

$$\sim (-ip_-)^n \langle v_{cm} | \prod_j e^{i\vec{k}_j \vec{y}} | v_{cm} \rangle \prod_j \int \frac{d\sigma_j}{2\pi} \prod_{j<i} |2 \sin \frac{\sigma_i - \sigma_j}{2}|^{\alpha' \vec{k}_i . \vec{k}_j} \quad (6)$$

- CM mode is trivial. 2pt function: $\int_0^{2\pi} \frac{d\sigma}{2\pi} |2 \sin \frac{\sigma}{2}|^{\alpha' k_1 . k_2} = \frac{2^{\alpha' k_1 . k_2}}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{2} + \frac{\alpha' k_1 . k_2}{2})}{\Gamma(1 + \frac{\alpha' k_1 . k_2}{2})} = 1 + \frac{\pi^2}{24} (\alpha' k_1 . k_2)^2 + \cdots$.
- Notice that first term is of order α'^2. This is related to the derivative corrections for supersymmetric string theory.
- The result is finite, in spite of the δ function.



2) Einstein GR
3) CRF
4) extra D bounded
5) NEC satisfied.
CFT
ds² = e^{-2Ω}(-dt² + dx²) + e^{2Ω} g_{ab} dy^a dy^b
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$$\sim(-ip_-)^n\langle\psi_{cm}|\prod_j e^{i\vec{k}_j\vec{y}}|\psi_{cm}\rangle\prod_j\int\frac{d\sigma_j}{2\pi}\prod_{j<i}|2\sin\frac{\sigma_i-\sigma_j}{2}|^{\alpha'\vec{k}_i.\vec{k}_j} \tag{6}$$

- CM mode is trivial. 2pt function: $\int_0^{2\pi}\frac{d\sigma}{2\pi}|2\sin\frac{\sigma}{2}|^{\alpha' k_1.k_2}=\frac{2^{\alpha' k_1.k_2}}{\sqrt{\pi}}\frac{\Gamma(\frac{1}{2}+\frac{\alpha' k_1.k_2}{2})}{\Gamma(1+\frac{\alpha' k_1.k_2}{2})}=1+\frac{\pi^2}{24}(\alpha' k_1.k_2)^2+\cdots$.
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- Strategy: Calculate in light cone ST in flat space and then translate to AdS. We use leading results and normalizations.
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- Now translate to position space, use the wave function of the calorimeter and translate to AdS: $\langle \mathcal{E}(\vec{n}'_1)\mathcal{E}(\vec{n}'_2)\rangle = (\frac{q^0}{4\pi})^2 \left[1 + \frac{6\pi^2}{\lambda}(\cos^2\theta_{12} - \frac{1}{3}) + \cdots\right]$.
- At this order the distribution rises for forward and backward regions as we expect.
- In the same way we can calculate n-point functions. Fluctuations are not gaussian.

5.2. Charge correlators

- Charge correlators present divergencies in field theory. A similar thing happens with the OPE in CFTs.

- We insert calorimeters at $W^+ = 0$ on the boundary and integrate over the $-$ direction. The fields in the bulk can be calculated.
- Prescription: $\mathcal{E}(\vec{n}')$ — $h^{\mathcal{E}(\vec{n}')}_{MN} dX^N dX^M \sim \delta(W^+)(dW^+)^2 \frac{1}{(W^0 - W_i n_i')^3}$
- $\mathcal{Q}(\vec{n}')$ — $A_M dx^M \sim dW^+ \delta(W^+) \frac{1}{(W^0 - W^i n_i')^2}$.
- Source fields have definite momentum on the boundary. $P^\mu_x|_{W^+=0} = -2iW^\mu \partial_{W^-}$
- $O_q(W^+ = 0, W^-, W^\mu) \sim (q^0)^{\Delta - 4}\, e^{iq^0 W^-/2} \delta^3(\vec{W})$
- To sum up: We take snap shots of the infalling string state at the horizon of the usual Poincare coordinates.

4.2. Actions and higher derivatives corrections

- We know that for $\mathcal{N} = 4$ all correlators are uniform.
- We need higher derivatives to calculate non trivial angle dependence.
- Current sources: $S = -\frac{1}{4g^2} \int d^5x \sqrt{g} F^2 + \frac{\alpha_1}{g^2 M_*^2} \int d^5x \sqrt{g} W^{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho}$.
- These are the only relevant terms for the 3 point function. There are only 2 gauge invariant vertices in flat space.

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- We evaluate: $\langle\Psi|e^{-ip_- \int_0^{2\pi}\frac{d\sigma}{2\pi}h(\vec{y}(\sigma))|_{\tau=0}}|\Psi\rangle$.
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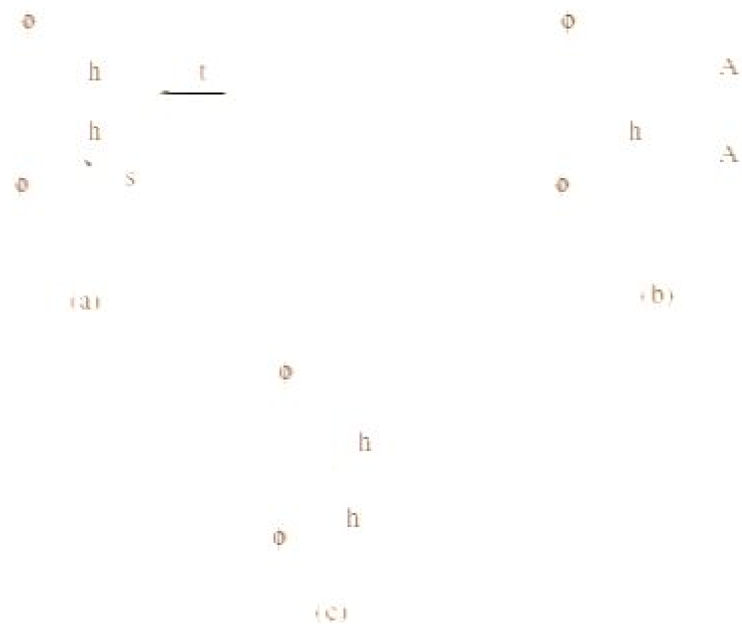


- In string theory the result is finite and goes to zero in the low energy limit.
- $\int_0^{2\pi} \frac{d\sigma}{(2\pi)} |2 \sin \frac{\sigma}{2}|^{\alpha' k_1.k_2 - 2} = \frac{2^{\alpha' k_1.k_2 - 2}}{\sqrt{\pi}} \frac{\Gamma(-\frac{1}{2} + \frac{\alpha' k_1.k_2}{2})}{\Gamma(\frac{\alpha' k_1.k_2}{2})} \sim -\frac{\alpha' k_1.k_2}{4} + \cdots$
- Translation to AdS: $\langle \mathcal{Q}(\vec{n}_1) \mathcal{Q}(\vec{n}_2) \rangle = \frac{2}{\sqrt{\lambda}} \vec{n}_1.\vec{n}_2 = \frac{2}{\sqrt{\lambda}} \cos \theta_{12}$
- Oppositely charged particles go in opposite directions.
- Low energy must be taken after doing the calculation. The δ function forces string theory on us.
- This story can be generalized to flavor charges associated to open strings.

5.3. Small Angle behavior

- $$\int_0^{2\pi} \frac{d\sigma}{2\pi} |2\sin\frac{\sigma}{2}|^{\alpha' k_1.k_2} = \frac{2^{\alpha' k_1.k_2}}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{2} + \frac{\alpha' k_1.k_2}{2})}{\Gamma(1 + \frac{\alpha' k_1.k_2}{2})} = 1 + \frac{\pi^2}{24}(\alpha' k_1.k_2)^2 + \cdots \quad (7)$$

- Singularities: $t \equiv -(k_1 + k_2)^2 = \frac{2+4n}{\alpha'}$, $\quad n = 0, 1, 2, \cdots$
- This is not where usual closed string theory poles are!
- The difference comes from

$$\text{Usual Case :} \quad \int dz^2 |z|^{\alpha' k_1.k_2} \sim \frac{1}{\alpha' k_1.k_2 + 2} \quad (8)$$

$$\text{Our Case :} \quad \int d\sigma |\sigma|^{\alpha' k_1.k_2} \sim \frac{1}{\alpha' k_1.k_2 + 1} \quad (9)$$

- Level matching.
- Why is this different from calculating standard amplitudes?

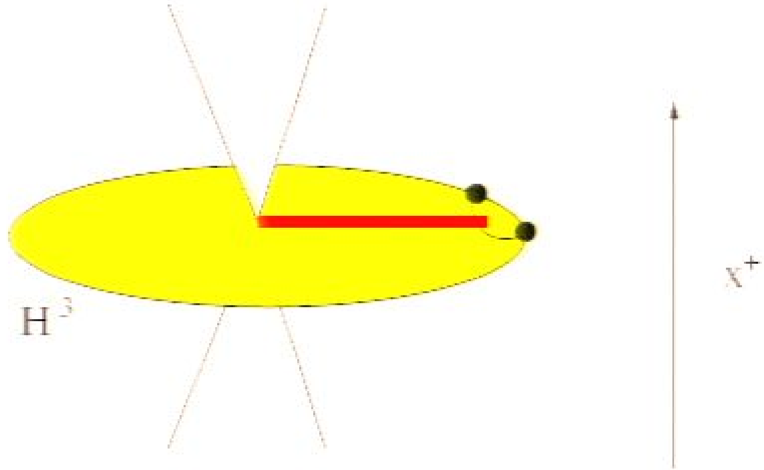
- What we do is equivalent to integrating the usual amplitudes over s.
- It is the Regge behavior of the amplitudes $\mathcal{A}_4 \sim s^{-2+\frac{\alpha' t}{2}}$ that makes them converge.
- The same can be understood using worldheet OPEs.

$$p_- e^{ik_1 \cdot y(\tau=0,\sigma)} p_- e^{ik_2 \cdot y(0,0)} \sim p_-^2 |\sigma|^{\alpha' k_1 \cdot k_2} [e^{i(k_1+k_2) y(0,0)} + \dots] \qquad (10)$$

- This yields the first pole in the above discussion.
- In conformal gauge this should look like

$$(\partial_\alpha y^+ \partial_\alpha y^+)^{\frac{3}{2}} \delta(y^+) e^{ik \cdot y} \qquad (11)$$

- This is a non local string state. This is the string dual to the non local operators with spin 3 we discussed in the CFT.



- To leading order in λ we can calculate from their flat space mass, their conformal weight. $\Delta \sim mR_{AdS} \sim \sqrt{2}\lambda^{1/4} + \cdots$
- This can be generalized to arbitrary spin j ($j << \lambda^{\frac{1}{2}}$).

$$\Delta(j) \sim \sqrt{2}\sqrt{j-2}\lambda^{1/4} + \cdots \qquad (12)$$

- We see that all these operators acquire large anomalous dimensions (and twist) at strong coupling.

6. Conclusions

- IR safe observables in a conformal theory through correlation functions.

- This description of the observables does not rely on a partonic description and is therefore suitable for finite coupling.
- Interesting small angle behavior in terms of non local operators, compatible with results from perturbation theory.
- This behavior has a clear dual interpretation in terms of non local string states.
- Correlators in the gravity theory is just a snapshot of the states as they cross the AdS horizon.
- Bounds on $\frac{a}{c}$ for different types of theories based on an energy positivity condition.

6.1. Future directions and improvements

- Come up with a robust argument for the energy positivity condition. This must be related to causality as in [Brigante, Liu, Myers, Shenker, Yaida]. Study GB in different backgrounds.
- Other dimensions. Condensed matter type applications? IR free theories?
- $\frac{1}{N}$ corrections.

- Hadronization. Nonconformal theories. Coupling to nonconformal theories.
- Study more complicated initial states. Collision of closed strings in the bulk? pp collisions.