Title: Unitary Representations of the Conformal Group for pedestrians

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Abstract: We comment on several points concerning unparticles which have been overlooked in the literature. One regards Mack\'s unitarity constraint lower bounds on CFT operator dimensions,e.g,. d>= 3 for primary, gauge invariant, vector unparticle operators. We correct the results in the literature to account for this, and also for a needed correction in the form of the propagator for vector and tensor unparticles. We show that the unitarity constraints can be directly related to unitarity requirements on scattering amplitudes of particles, e.g., those of the standard model, coupled to the CFT operators. We also stress the existence of explicit standard model contact terms, which are generically induced by the coupling to the CFT (or any other hidden sector), and are subject to LEP bounds. Barring an unknown mechanism to tune away these contact interactions, they can swamp interference effects generated by the CFT.

Unitary Representations of the Conformal Group for Pedestrians

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with Kenneth Intriligator and Ira Rothstein

PASCOS, Perimeter Institute
June, 2008

arXiv: 0801.1140

Outline

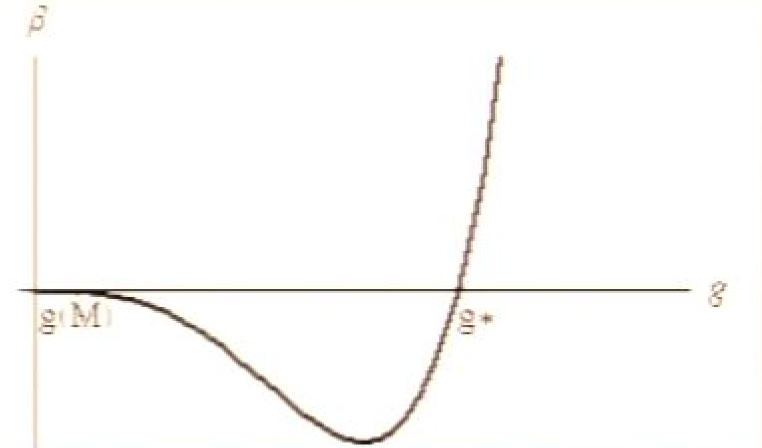
- Motivation: unparticles
- Unitary Representations of the Conformal Group
  - Recover Mack's results from optical theorem
    Note: no new results; rather, simple physical understanding, intuition

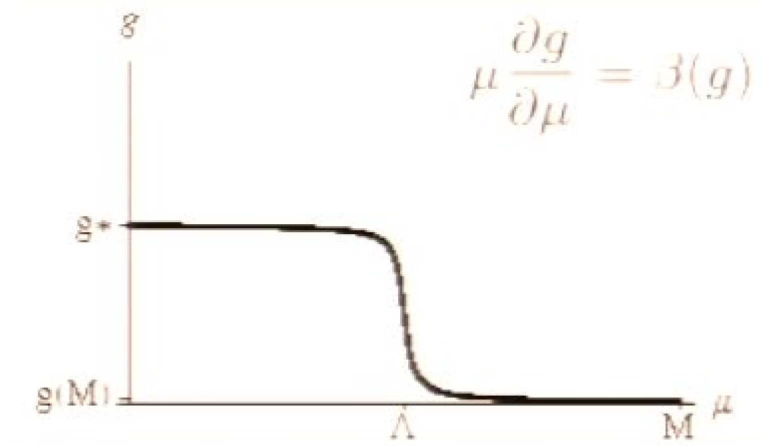
G. Mack, Commun. math. Phys. 55, 1–28 (

- Comments on Unparticles
  - Simplest observations: what d?
  - Scale invariance alone does not help
  - Contact terms
    - induced by messengers
    - induced by CFT exchange
  - Effects of corrected propagator (vectors non-transverse)

By way of motivation: unparticles

IR fixed point:





- Bellow Λ:
  - β-function vanishes
  - $g(\mu) = g^*$ is constant
  - scale invariance
  - non-zero anomalous dimensions

Banks-Zaks (BZ):
- perturbative g*
- $\beta_1 = \beta_2 \gg \beta_3$

$\beta(g) = \sum_n \beta_n$

T. Banks and A. Zaks, Nucl. Phys. B196, 189 (19

- Conformal Field Theories (CFTs) do not have a traditional S-matrix description: they do not have free, asymptotically separated in and out states

- Georgi: can one see effects of a CFT with normal particle detectors?
  - Probes: normal (SM) particles
  - Couple normal particles (SM) weakly to a CFT
  - Use scattering of normal particles to investigate CFT effects:
    - Virtual effects of CFT on normal to normal scattering (or normal particle decay)
    - Production of CFT stuff, that is, normal to CFT scattering

H. Georgi, PRL 98 (2007) 2216
Phys. Lett. B 650 (2007) 275

- Findings:
  - Production: is as if the final state phase space is that of a fractional number of particles. Hence "unparticles."
  - Momentum dependence and angular distribution of scattering cross sections is different from normal

- Weak coupling of SM to CFT:
  - Interaction is only through irrelevant operators (conformal invariance safe)
  - UV completion: mediated by new heavy particles ("messengers"), mass M_U $(M_U \gg \Lambda$
  - Computability: CFT is Banks-Zaks IR fixed point, but perturbative (asymptotically free) in UV where it couples to SM

$$\mathcal{H}' = C(M)\frac{1}{M_U^d}\mathcal{O}_{\rm BZ}(M_U)\mathcal{O}_{\rm SM} = C(\Lambda)\frac{\Lambda^{\gamma_*}}{M_U^d}\hat{\mathcal{O}}\mathcal{O}_{\rm SM}$$

where $\quad d = d_{\rm BZ} + d_{\rm SM} - 4 \qquad \gamma_* = \gamma(g_*)$

$$\mu\frac{d}{d\mu}\mathcal{O}_{\rm BZ} = \gamma\mathcal{O}_{\rm BZ}$$

$$\hat{\mathcal{O}} \equiv \mu^{-\gamma_*}\mathcal{O}_{\rm BZ}(\mu)$$

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2 to 2 scattering of normal particles:

need $\langle T[\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0)]\rangle$

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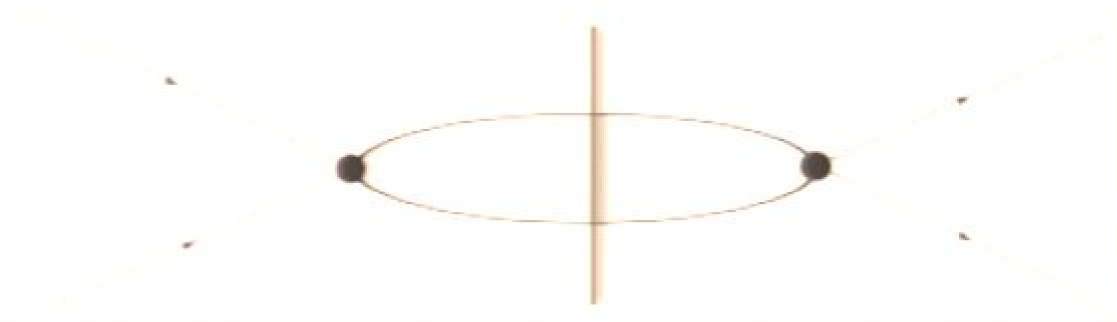
unparticle production by scattering of normal particles:



forward scattering amplitude

need $\text{Im}\,\langle T[\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0)]\rangle$

G. Mack, Commun. math. Phys. 55, 1–28 (1

Unitary Representations

The conditions for unitary representations of the conformal group follow from unitarity of the S-matrix from the Field Theory

(actually, from the weaker condition that the imaginary part of the forward scattering amplitude be non-negative).

Program, in four steps:

- 2-point Green's functions are determined (up to a constant) by conformal invariance
- Compute propagators, ie, Fourier Transform of 2-point function
- Use these propagators in forward scattering amplitude
- Optical theorem: imaginary part of forward scattering amplitude is non-negative

Step 1: Operator 2-point functions in CFT

Do only primaries (operator that is not the derivative of another operator).
Non-vanishing only if two operators have the same dimension.

Scalar propagator

$$\langle \hat{\mathcal{O}}(x)^\dagger \hat{\mathcal{O}}(0) \rangle = C_S \frac{1}{(2\pi)^2} \frac{1}{(x^2)^d}$$

Fixed up to normalization by scale invariance (Poincare always assumed).
C_S is a normalization constant.

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Vector propagator

$$\langle \hat{\mathcal{O}}_\mu(x)^\dagger \hat{\mathcal{O}}_\nu(0) \rangle = C_V \frac{1}{(2\pi)^2} \frac{I_{\mu\nu}(x)}{(x^2)^d}$$

where $$I_{\mu\nu} \equiv g_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}$$

Fixed by requiring, in addition, invariance under special conformal transformations.
C_V is a normalization constant.

Note that the divergence vanishes (it is "conserved"). $\partial^\mu \frac{I_{\mu\nu}(x)}{(x^2)^d} = 0$, only for $d = 3$

Clearly any higher tensor can be expressed in terms of this. For example

symmetric, traceless 2-index tensor propagator

$$\langle \hat{\mathcal{O}}_{\mu\nu}(x)^\dagger \hat{\mathcal{O}}_{\lambda\sigma}(0)\rangle = C_T \frac{1}{(2\pi)^2} \frac{\left(I_{\mu\lambda}(x) I_{\nu\sigma}(x) - \frac{1}{4} g_{\mu\nu} g_{\lambda\sigma}\right) \pm \mu \leftrightarrow \nu}{(x^2)^d}$$

Its divergence vanishes (it is "conserved") only for $d = 4$

A similar construction for spinors uses gamma matrices

Step 2: 2-point functions in momentum space, "unparticle propagators"

Start from straightforward integration in euclidean space:

scalar:

$$\frac{1}{(2\pi)^2}\frac{1}{(x^2)^d} = \frac{\Gamma(2-d)}{4^{d-1}\Gamma(d)}\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}(k^2)^{d-2}$$

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Analytically continue in d and take derivatives to get other 2-point functions (euclidean), for example:

vector:

$$\frac{1}{(2\pi)^2}\frac{g_{\mu\nu} - 2x_\mu x_\nu/x^2}{(x^2)^d} = \frac{(d-1)\Gamma(2-d)}{4^{d-1}\Gamma(d+1)}\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}(k^2)^{d-2}\left[g_{\mu\nu} - \frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right]$$

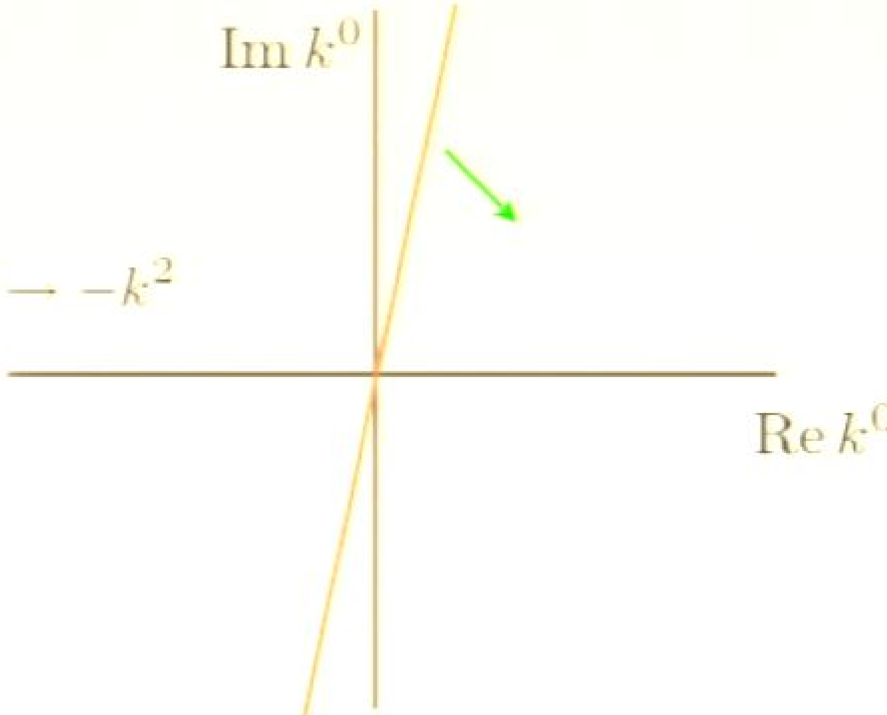
tensor:

$$\frac{1}{(2\pi)^2}\frac{(I_{\mu\lambda}(x)I_{\nu\sigma}(x) + \mu \leftrightarrow \nu) - \frac{1}{2}g_{\mu\nu}g_{\lambda\sigma}}{(x^2)^d} = \frac{\Gamma(2-d)}{4^{d-1}\Gamma(d+2)}\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}(k^2)^{d-2}\Big[d(d-1)(g_{\mu\lambda}g_{\nu\sigma} + \mu \leftrightarrow \nu)$$

$$-\frac{1}{2}[4-d(d-1)]g_{\mu\nu}g_{\lambda\sigma} - 2(d-1)(d-2)\left(g_{\mu\lambda}\frac{k_\nu k_\sigma}{k^2} + g_{\mu\sigma}\frac{k_\nu k_\lambda}{k^2} + \mu \leftrightarrow \nu\right)$$

$$+ 4(d-2)\left(g_{\mu\nu}\frac{k_\lambda k_\sigma}{k^2} + g_{\lambda\sigma}\frac{k_\mu k_\nu}{k^2}\right) + 8(d-2)(d-3)\frac{k_\mu k_\nu k_\lambda k_\sigma}{(k^2)^2}\Big]$$

Rotate to Minkowski



take $k^4 = -ik^0$ so $k^2 \to -k^2$

(metric: $(+ - - -)$)

and get propagators, *e.g.*, vector

$$\int d^4x e^{-ik\cdot x} \langle 0|T(O_\mu(x)O_\nu(0))|0\rangle = -iC(-k^2 - i\epsilon)^{d-3}\left[k^2 g_{\mu\nu} - \frac{2(d-2)}{d-1}k_\mu k_\nu\right]$$

Steps 3&4: Conditions on *d* from optical theorem

Strategy: Probe the CFT with external particles (sources)
Require that optical theorem holds for total cross section of external particles

Im () ≥ 0

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Strategy: Probe the CFT with external particles (sources)
Require that optical theorem holds for total cross section of external particles

$$\mathrm{Im}\left(\right.$$  $$\left. \right) \geq 0$$

1. Scalar operator

Interaction with source χ: $\mathcal{L} \supset g\chi\hat{\mathcal{O}} + \text{h.c.}$

Amplitude follows from propagator in Minkowski space:

$$\mathcal{A} = g^2 C_S |\chi|^2 \frac{\Gamma(2-d)}{4^{d-1}\Gamma(d)}(-k^2 - i\epsilon)^{d-2} \quad \Rightarrow \quad \mathrm{Im}\,\mathcal{A}_{\text{fwd}} = \frac{C_S \pi g^2 (d-1)}{4^{d-1}\Gamma(d)^2}|\chi|^2\theta(k^0)\theta(k^2)(k^2)^{d}$$

$$\Rightarrow \quad C_S(d-1) \geq 0$$

It is easy to see that $C_S > 0$. Recall

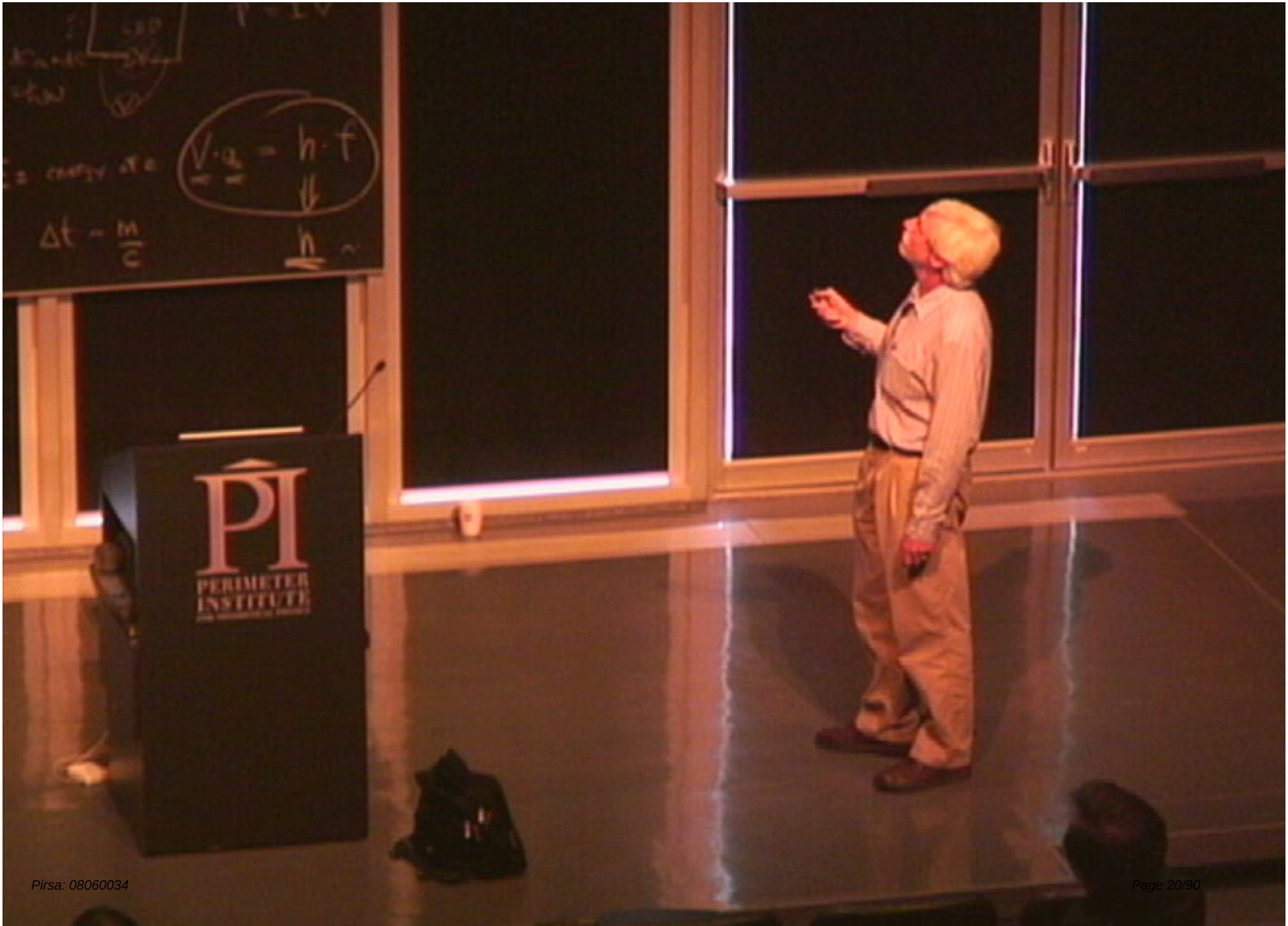
$$\langle \hat{\mathcal{O}}(x)^\dagger \hat{\mathcal{O}}(0) \rangle = C_S \, \frac{1}{(2\pi)^2} \, \frac{1}{(x^2)^d}$$

and note that

$$\langle \bar{\mathcal{O}}^\dagger \bar{\mathcal{O}} \rangle > 0 \qquad \text{where} \qquad \bar{\mathcal{O}} \equiv \int_{\mathcal{R}} d^4x \, \hat{\mathcal{O}}$$

(compact support)

Note: This argument works for the normalization constant for any operator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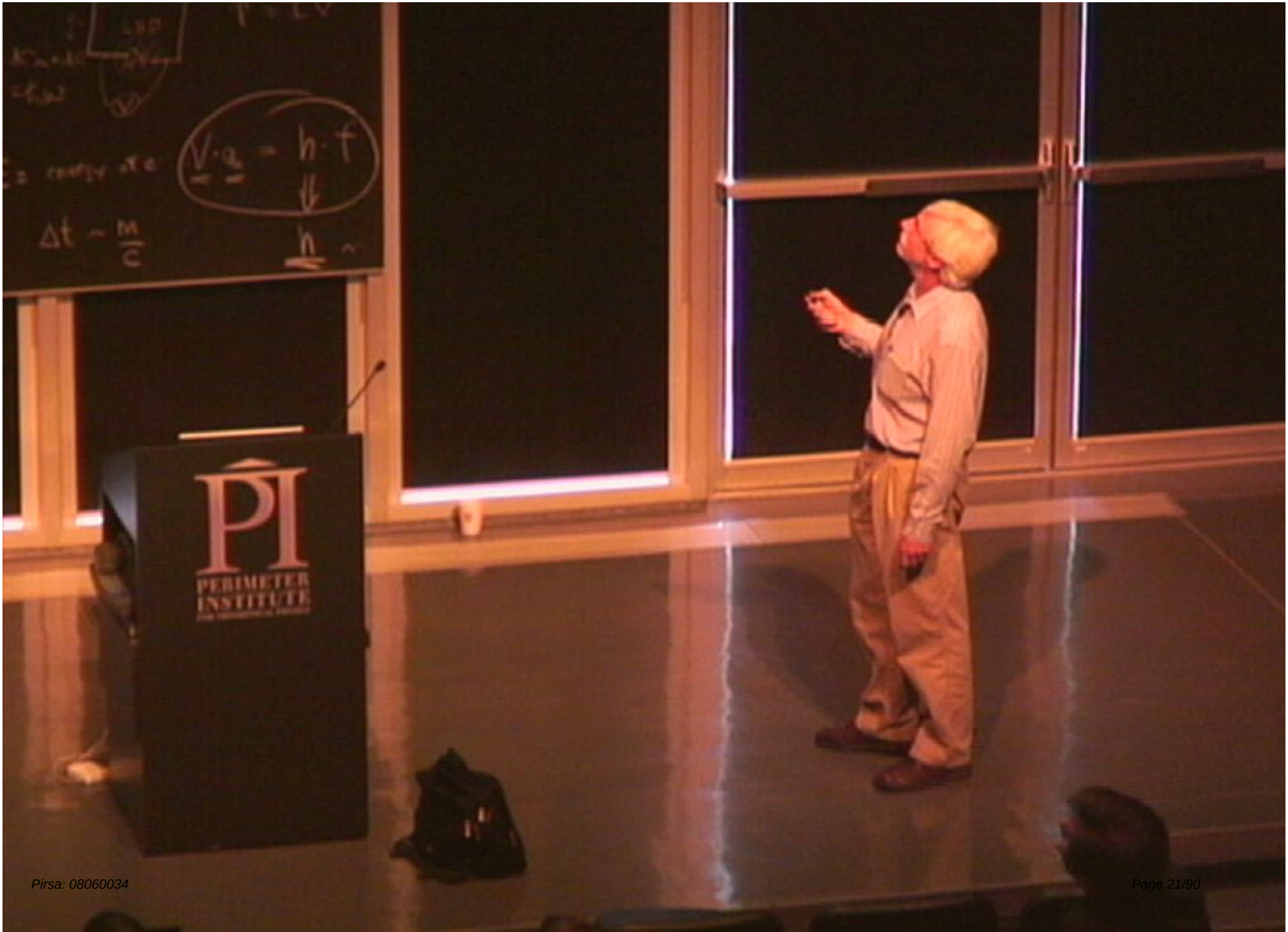
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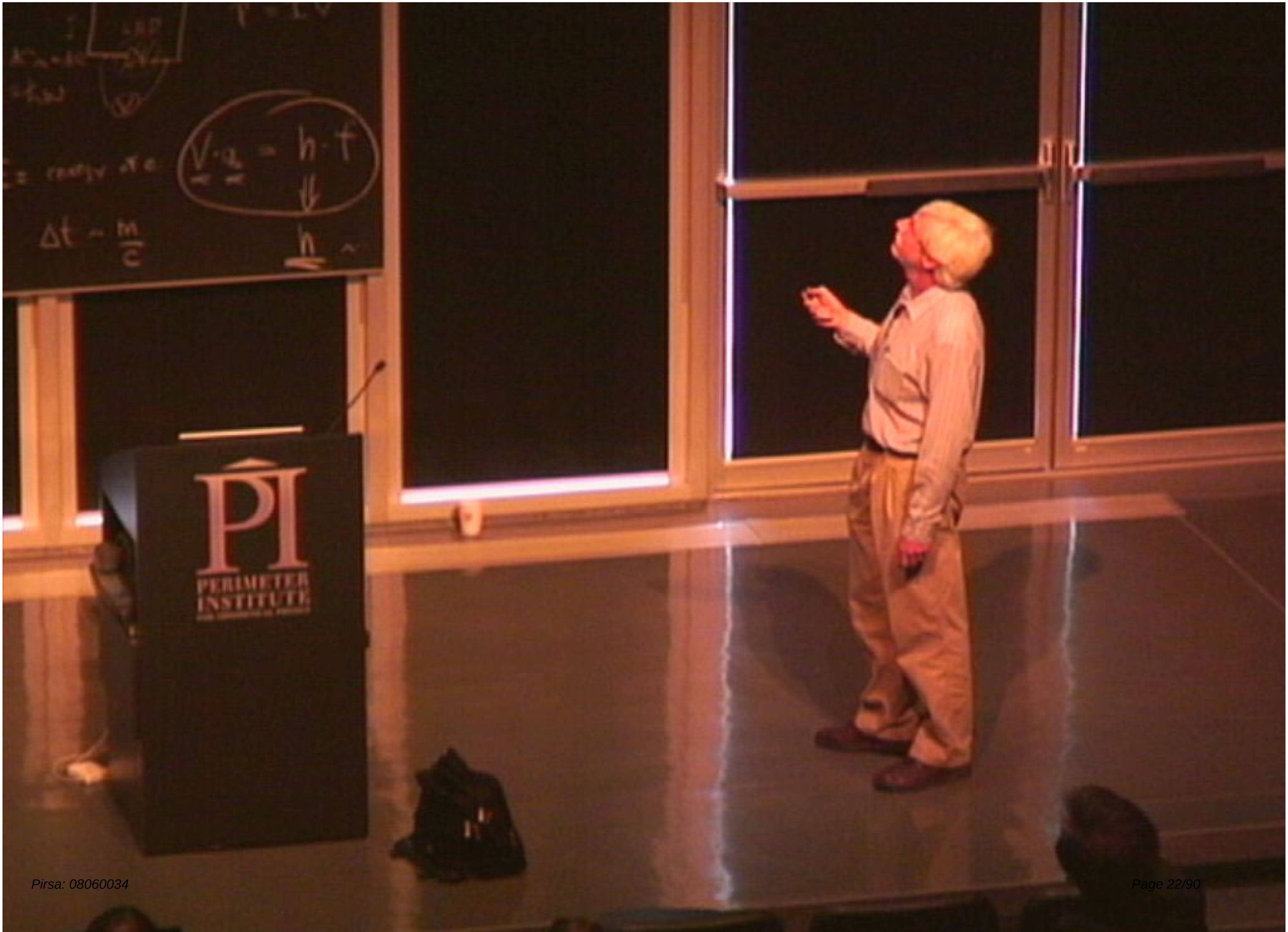
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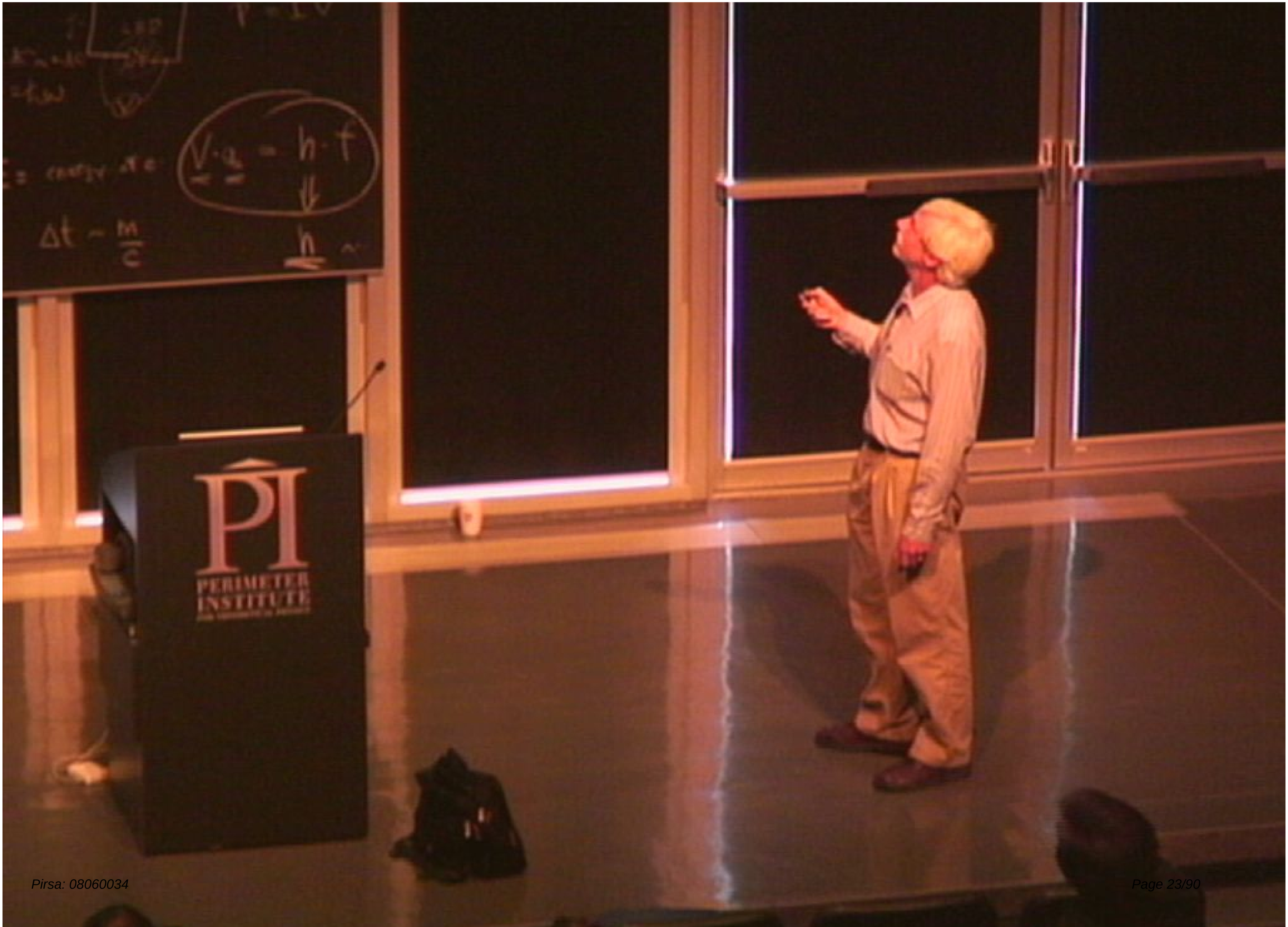
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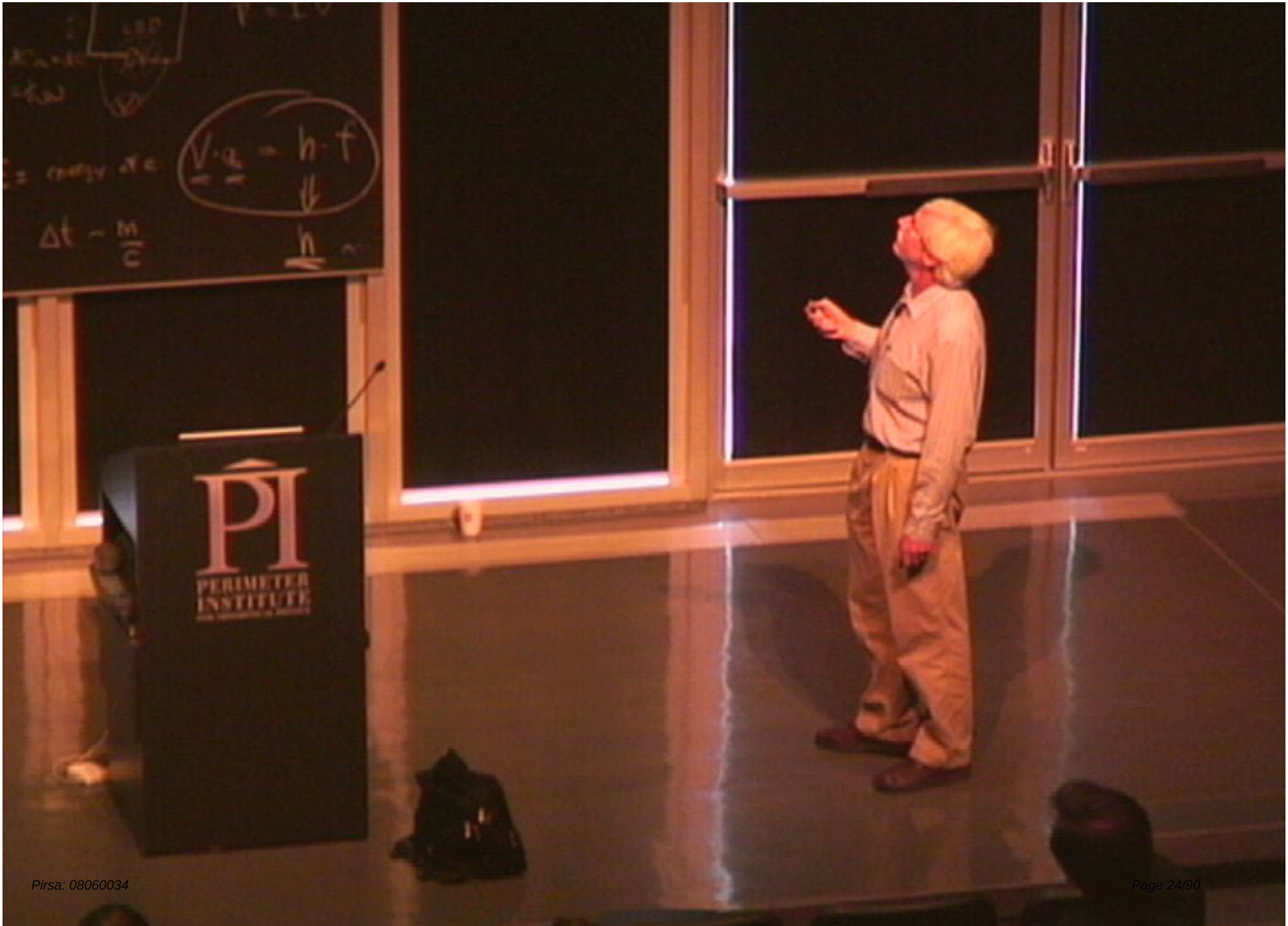
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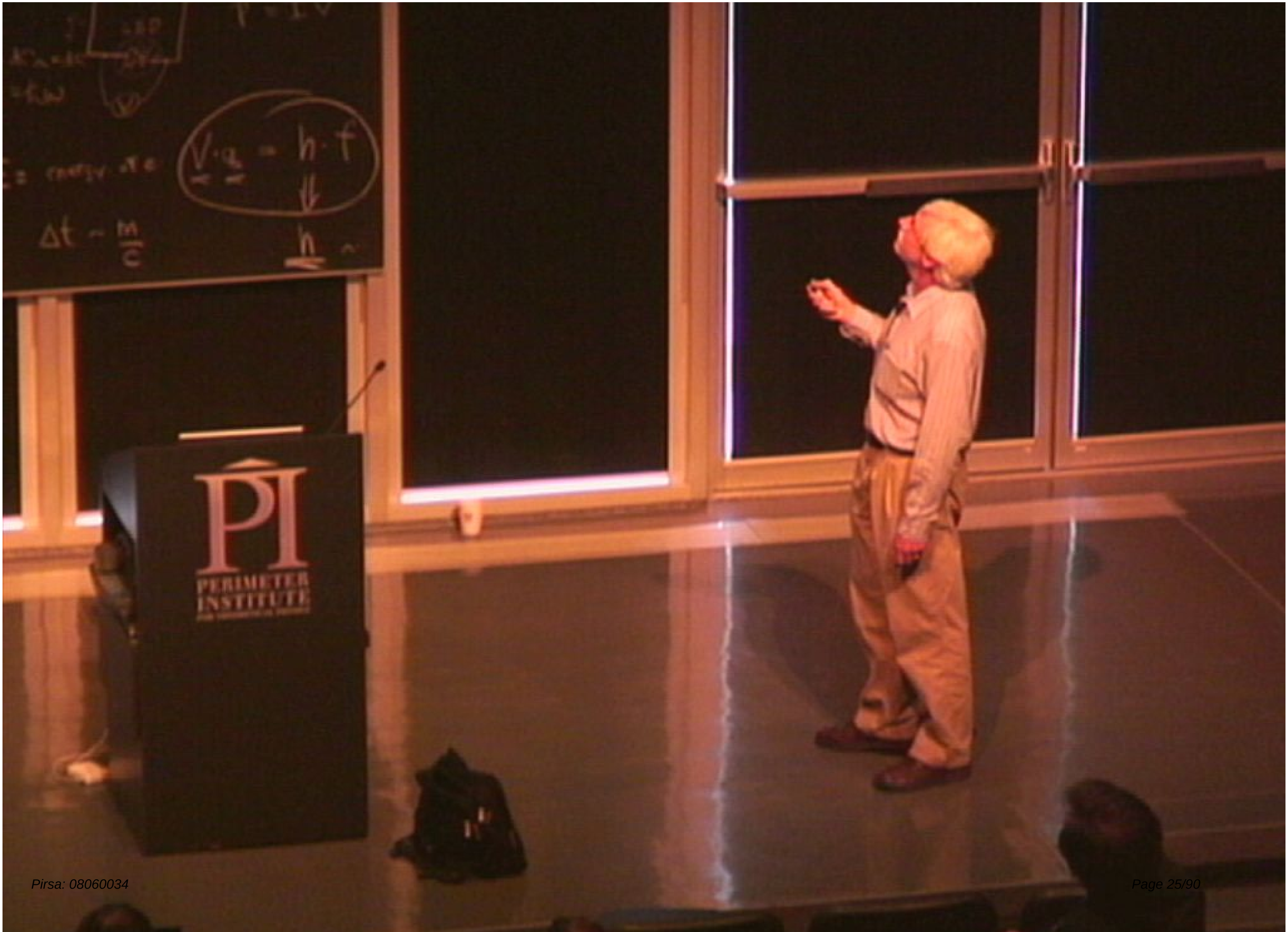
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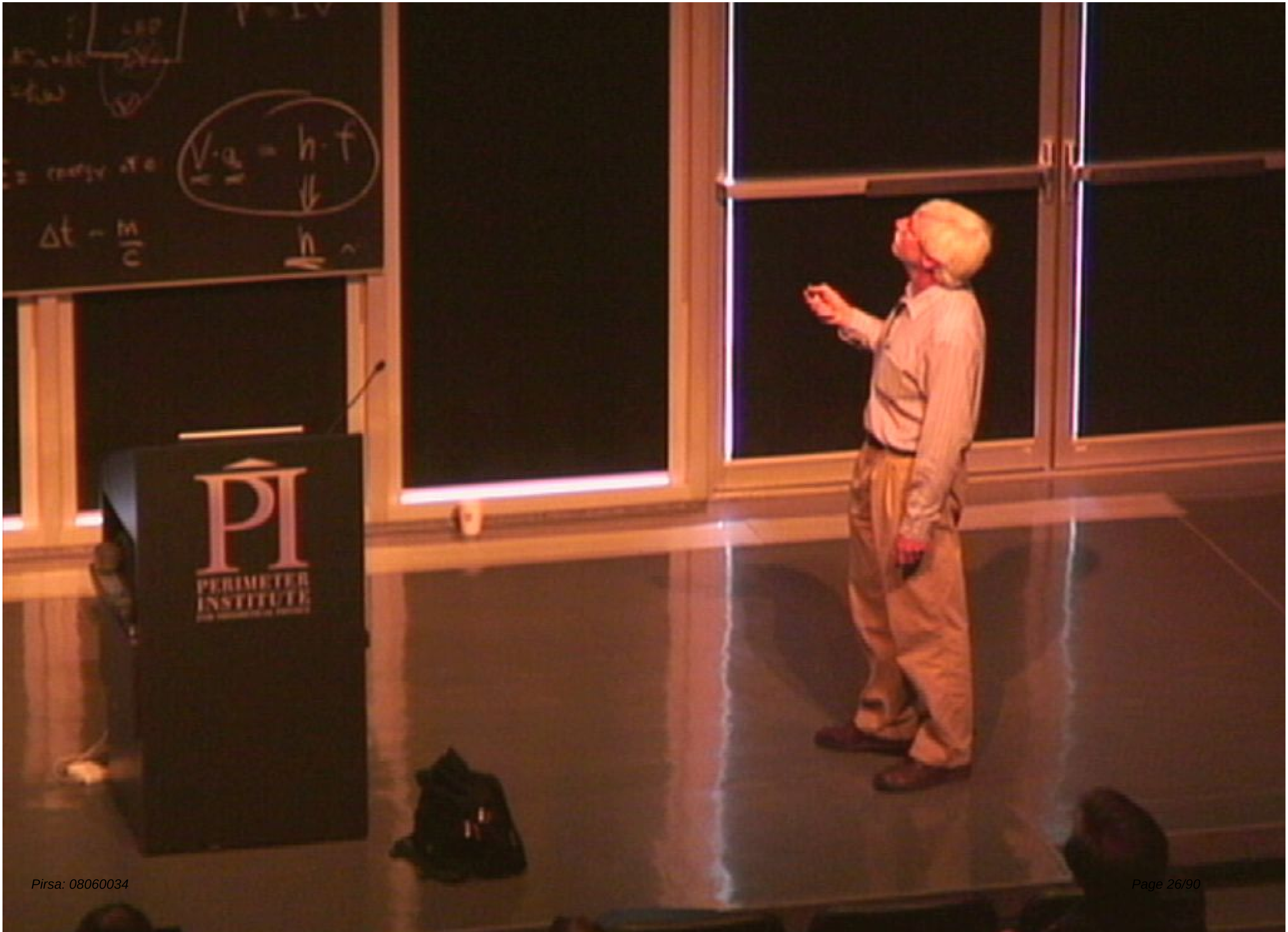
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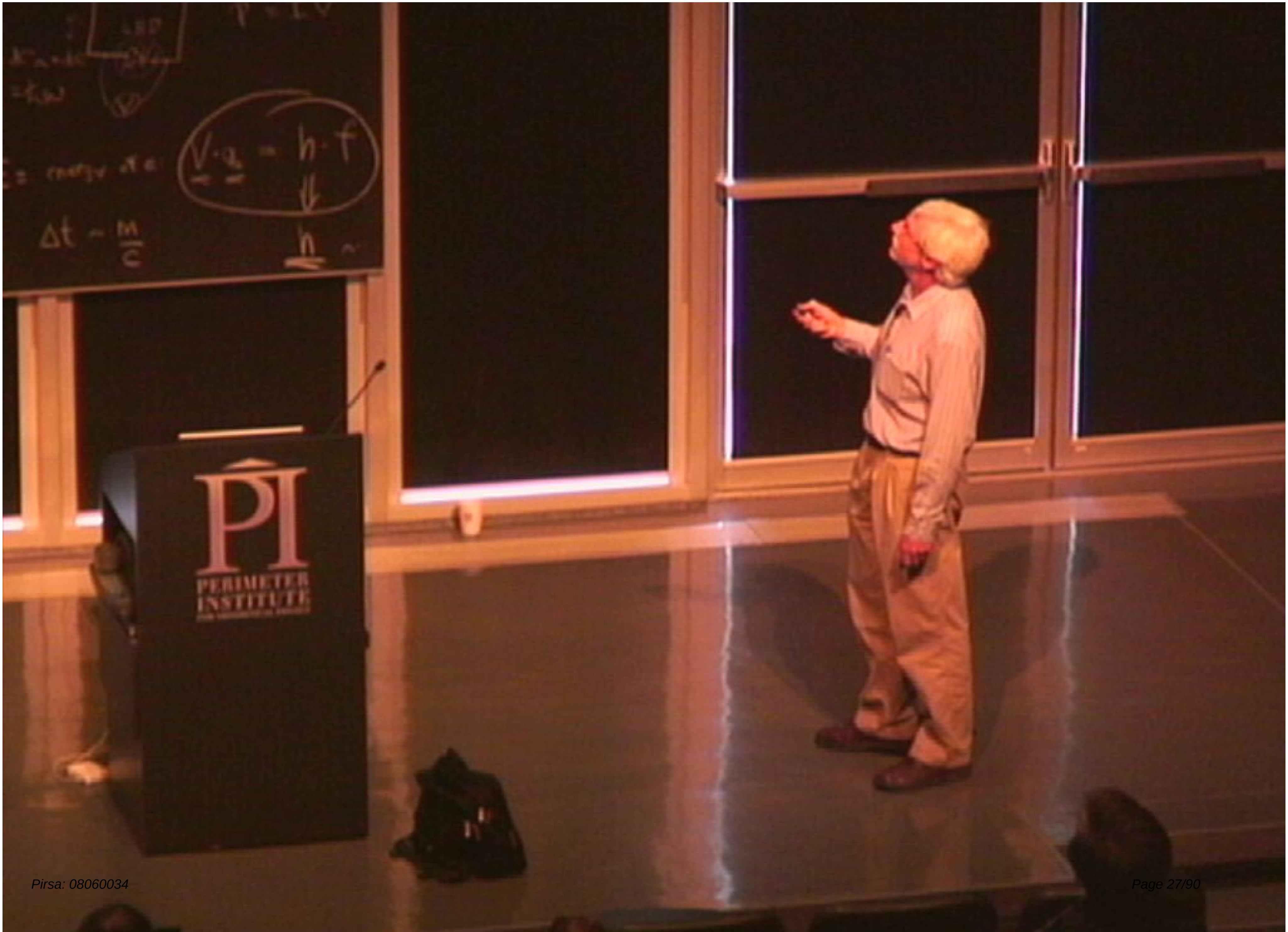
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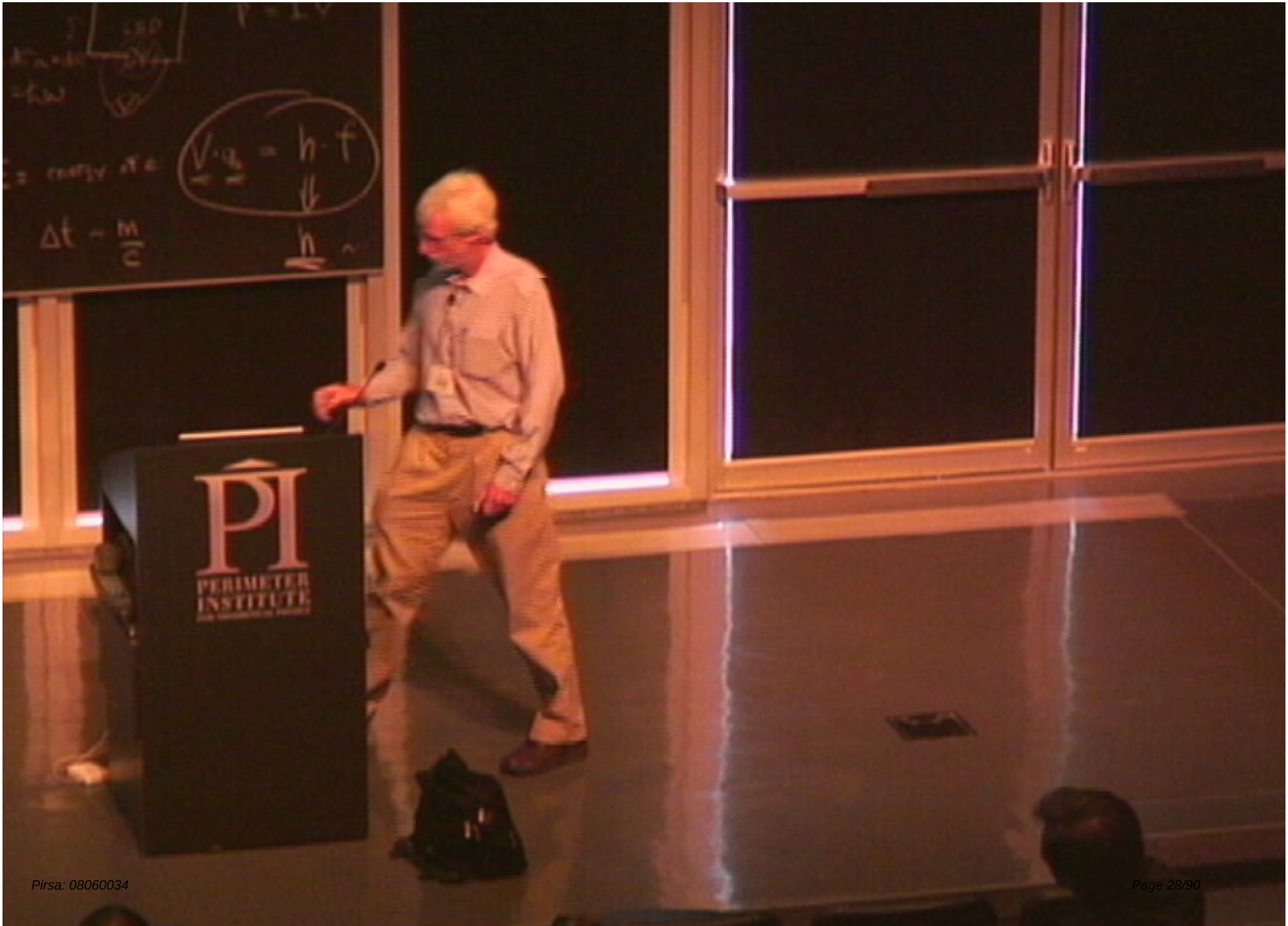
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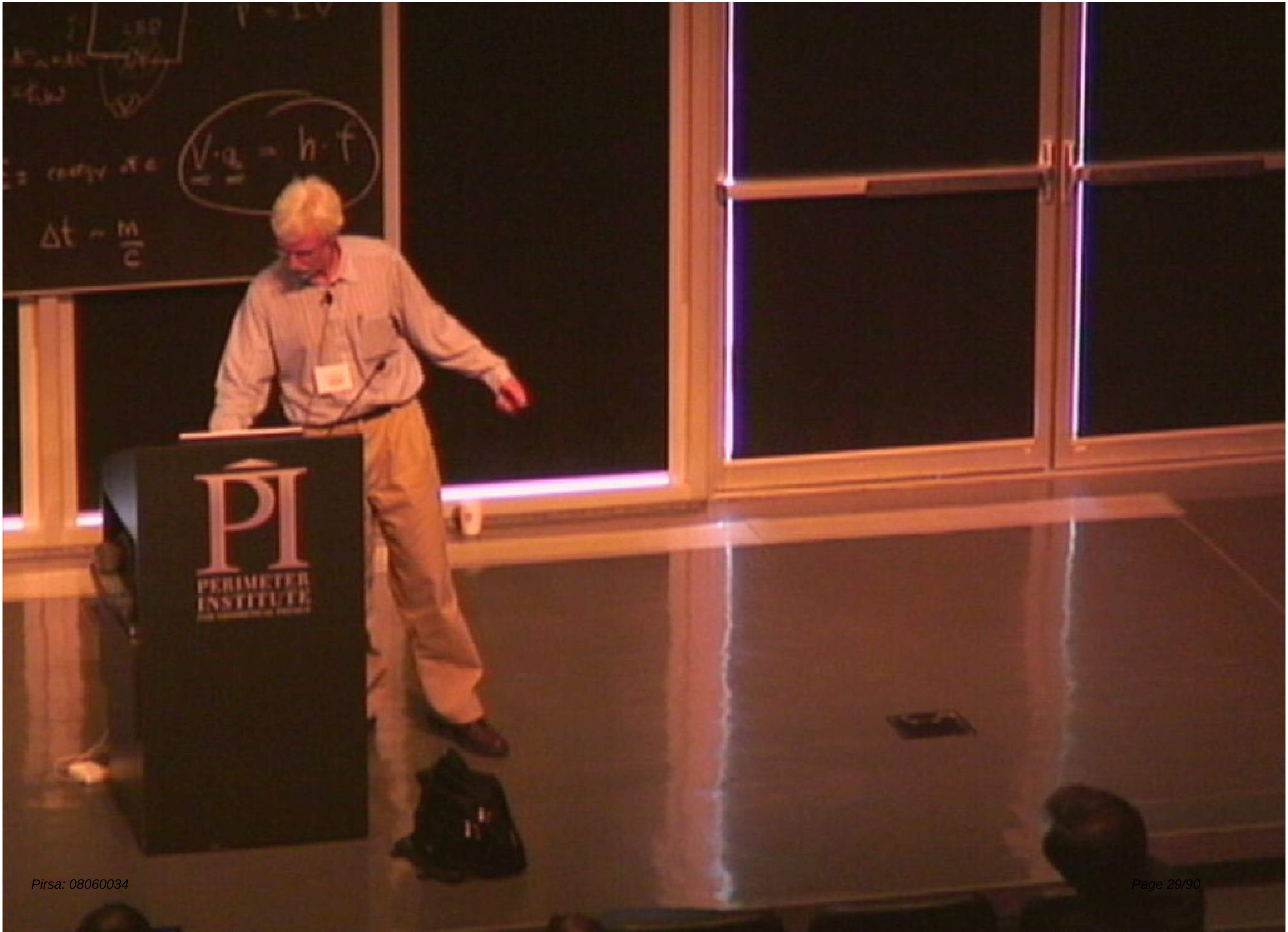
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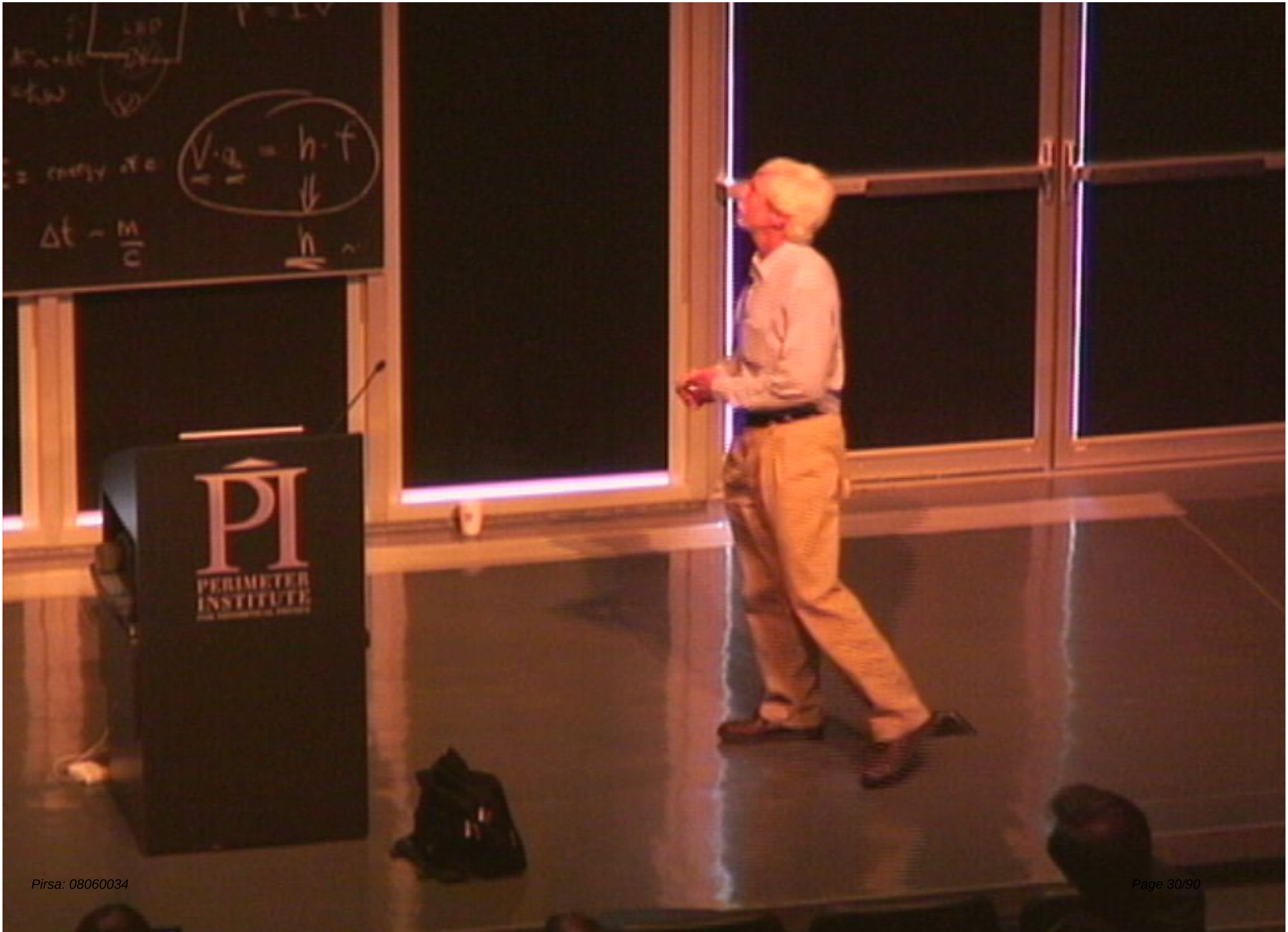
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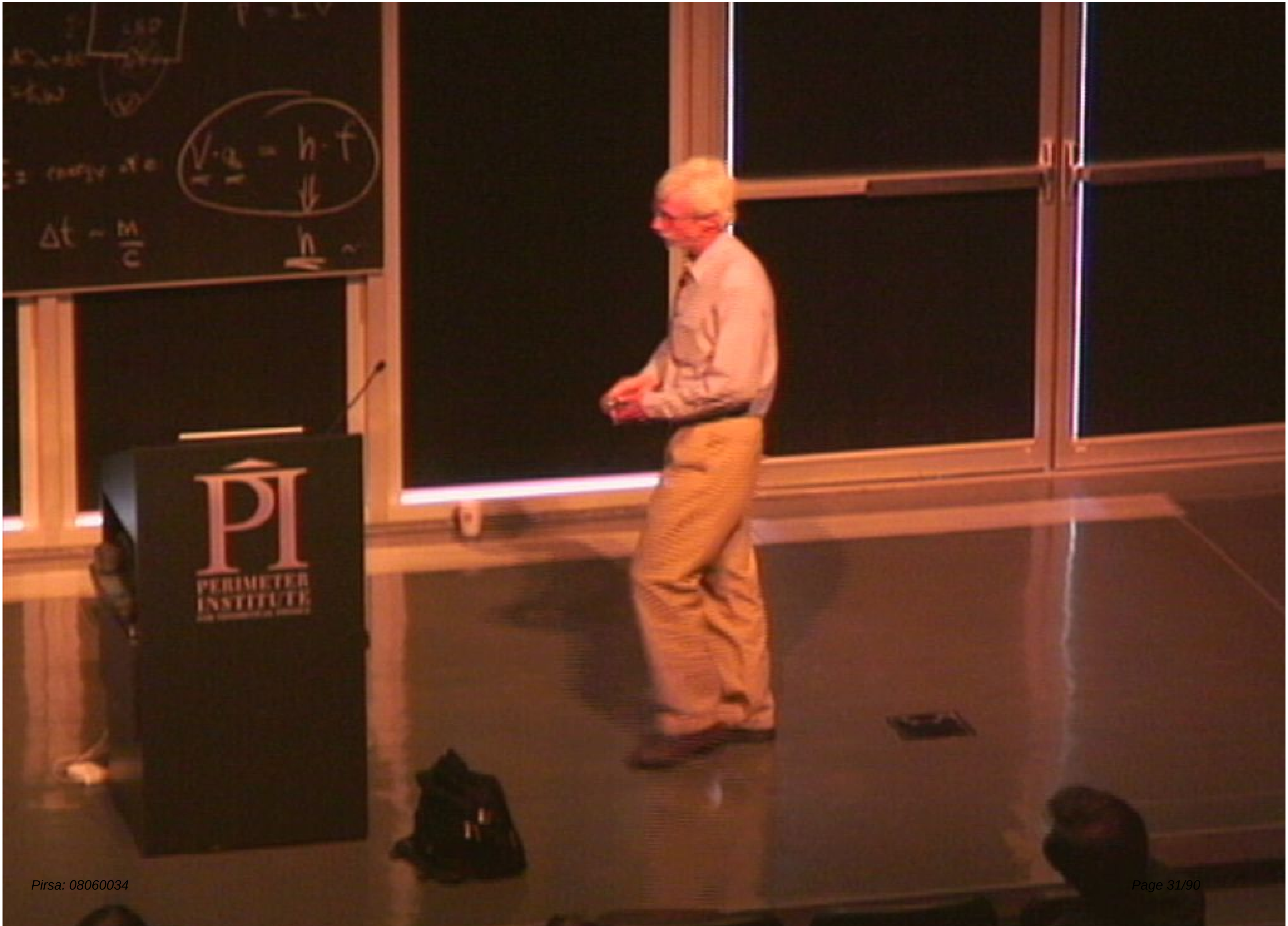
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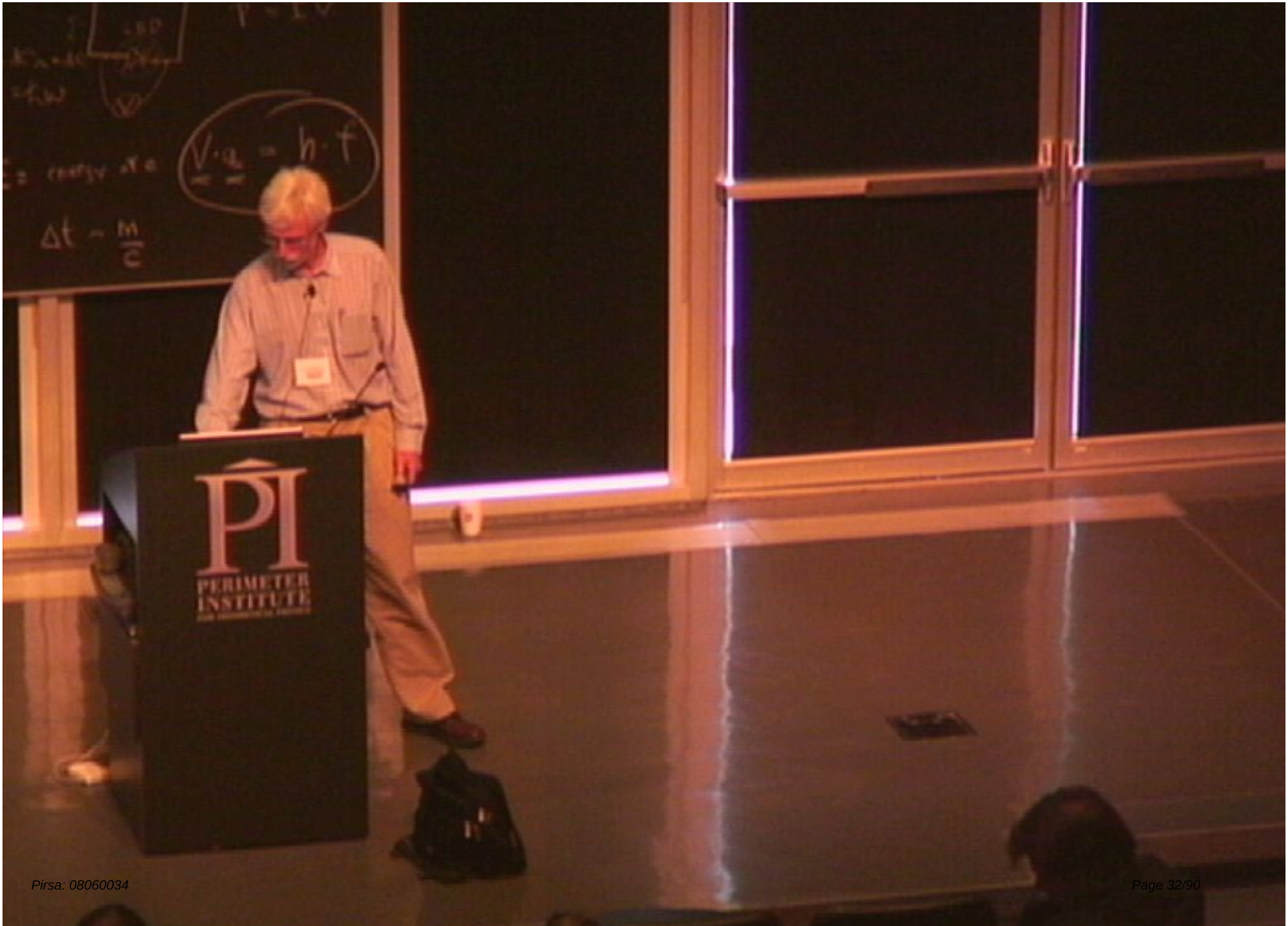
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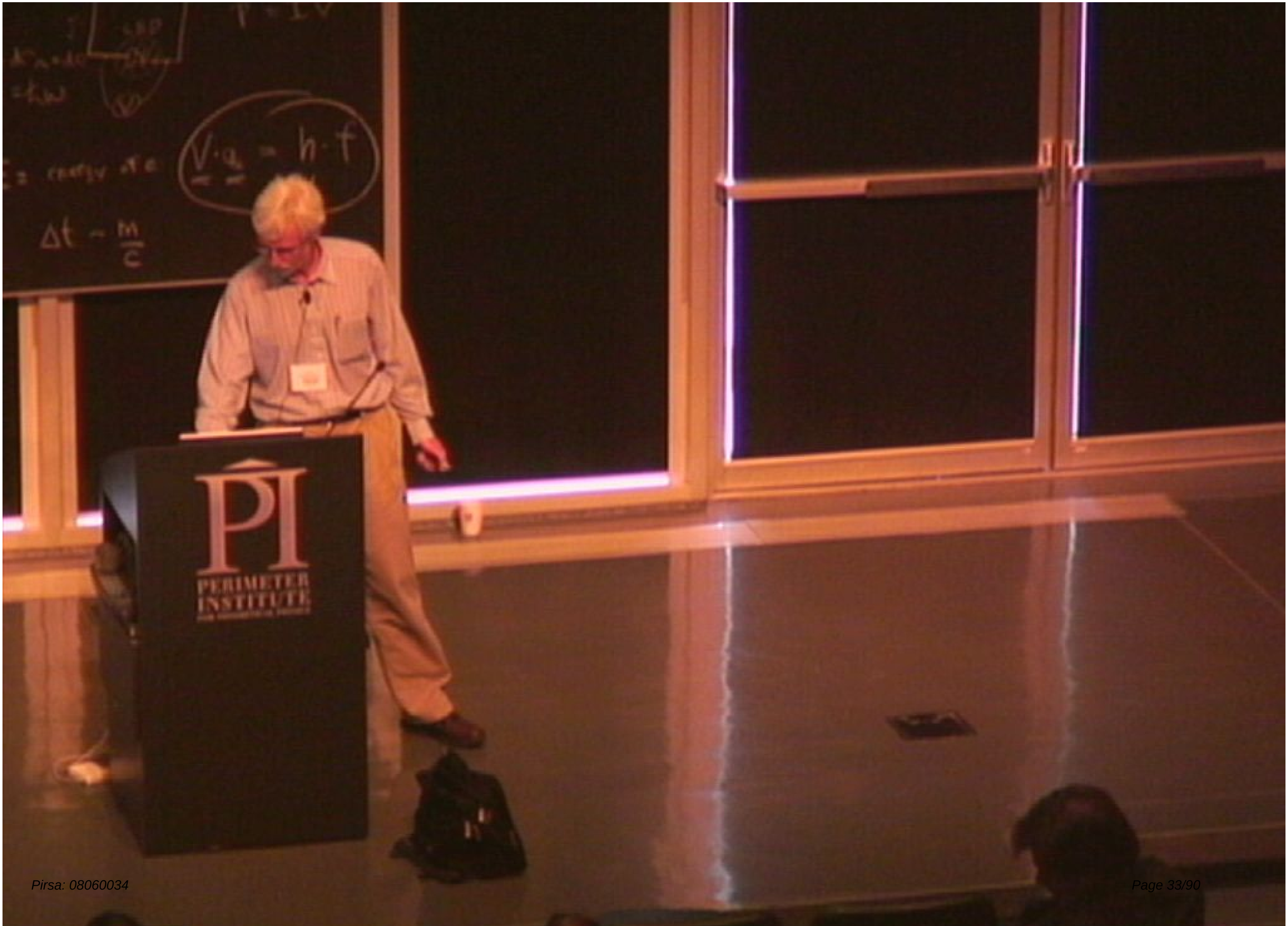
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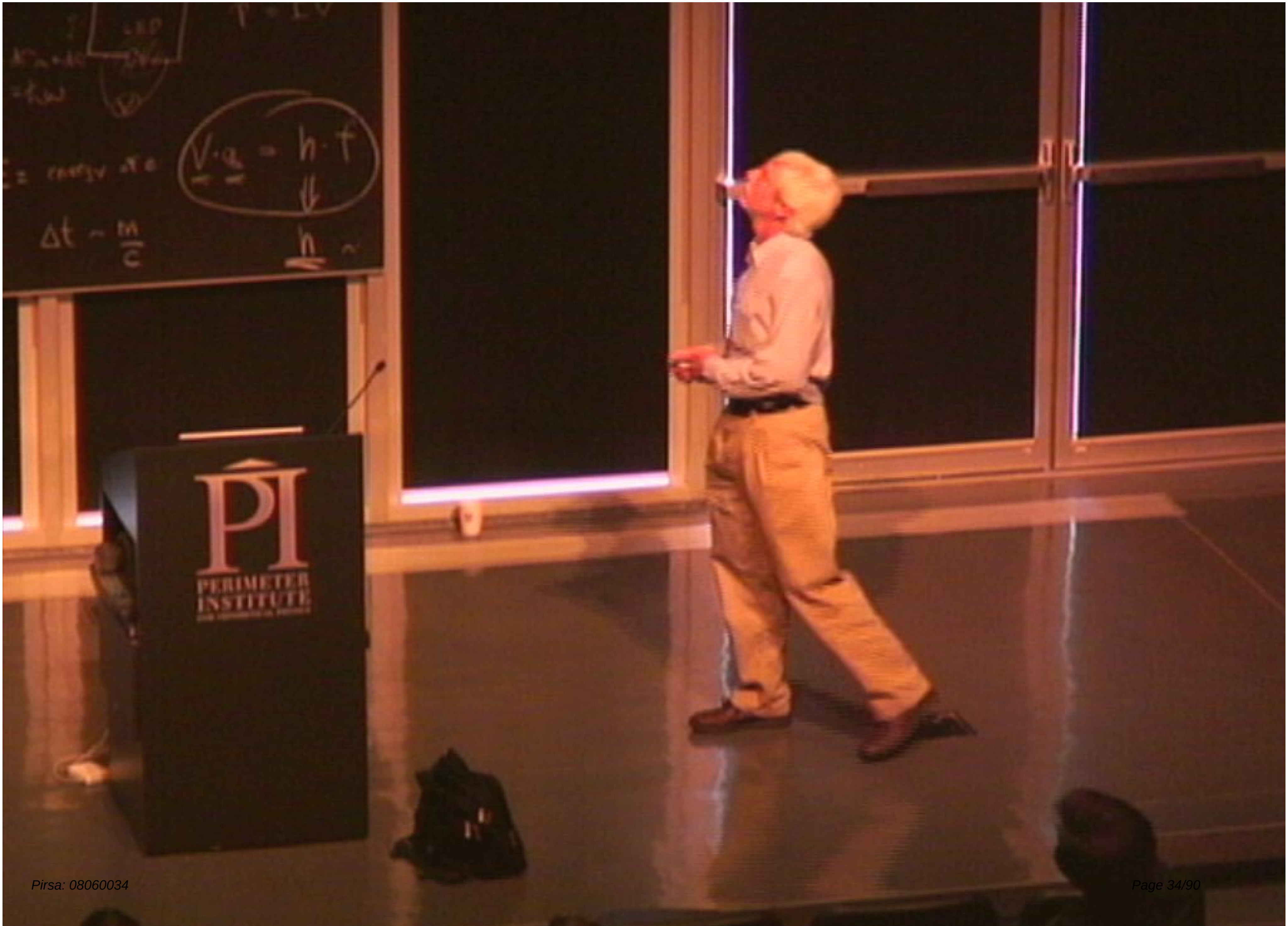
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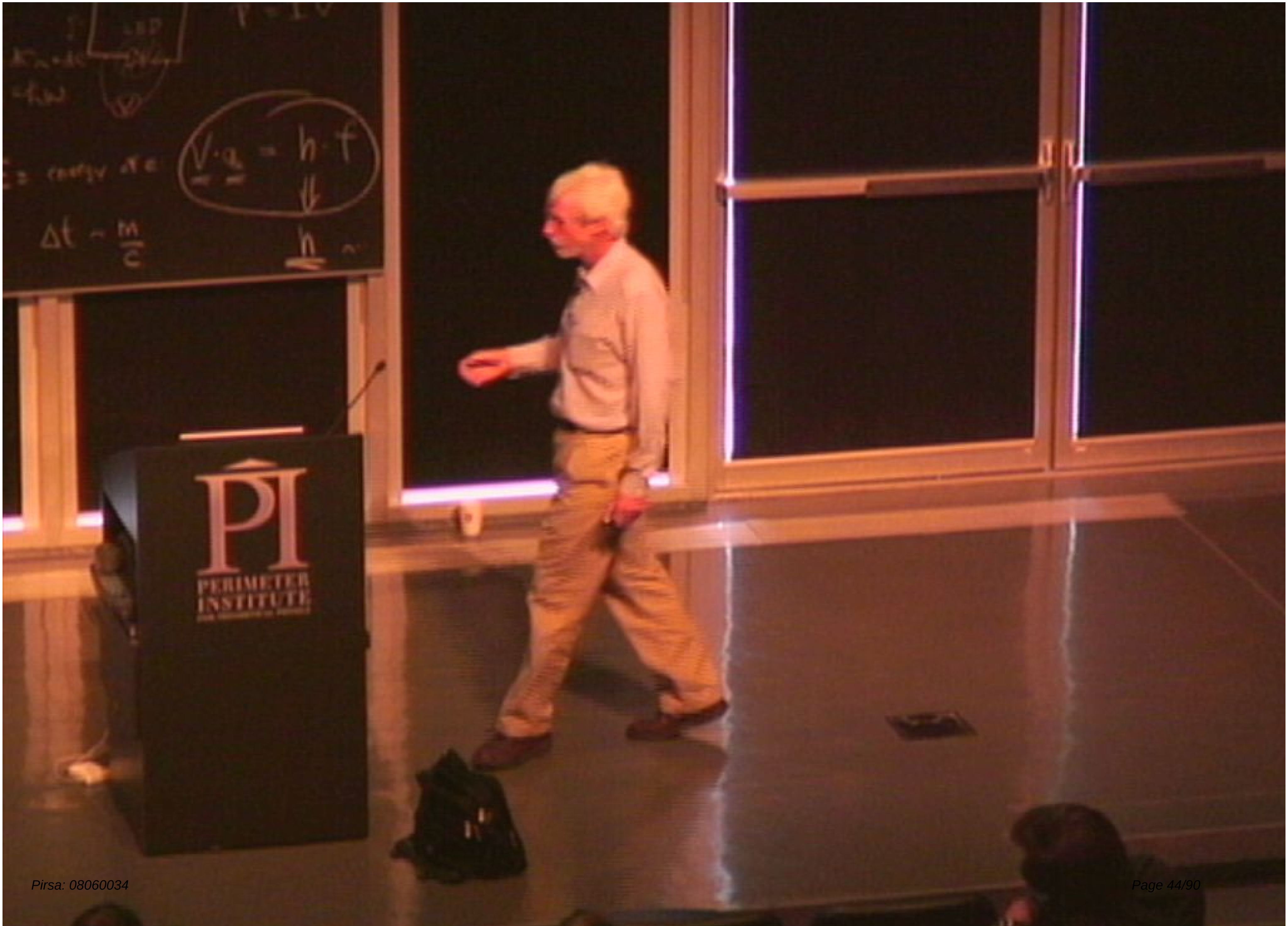
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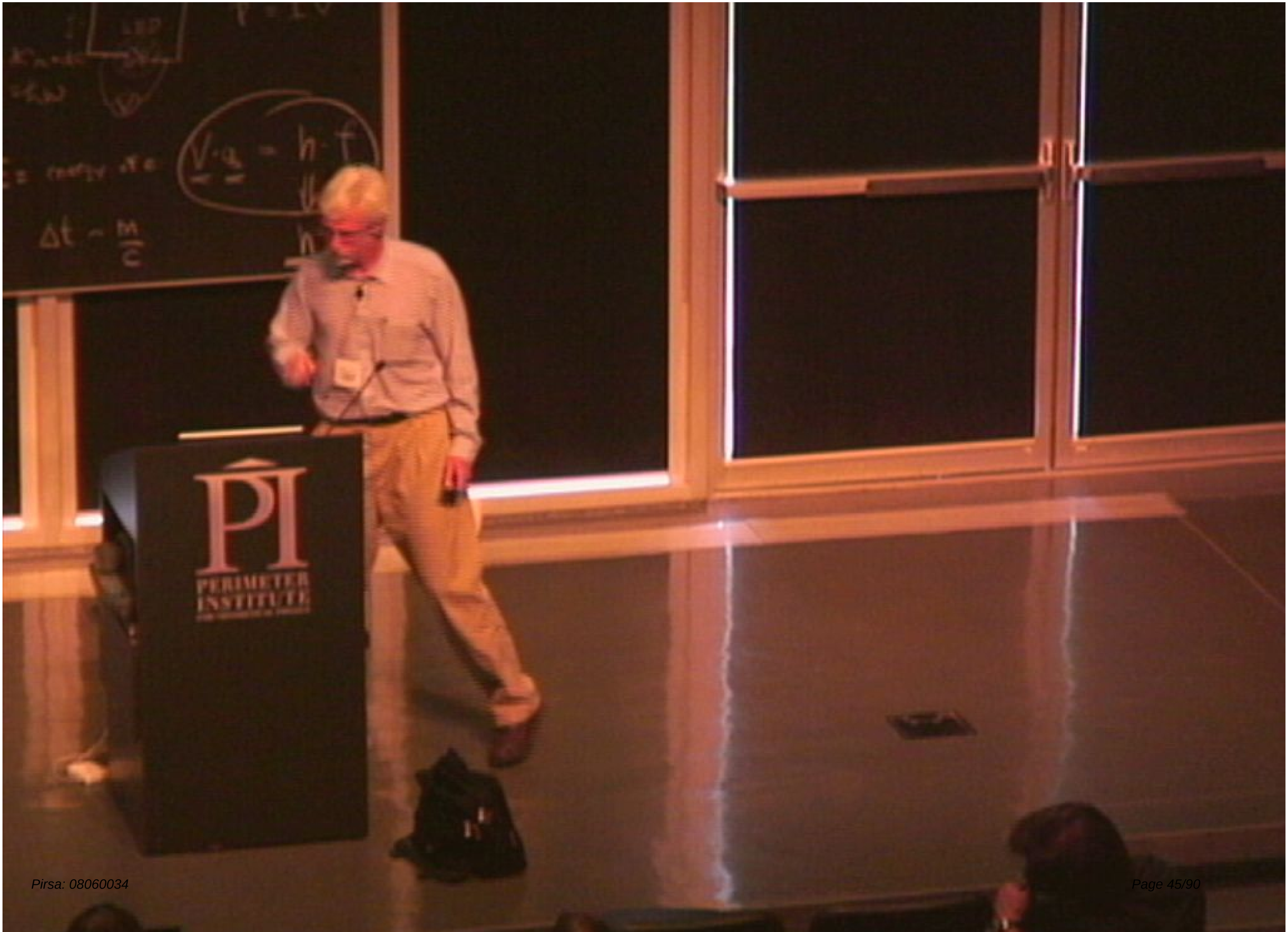


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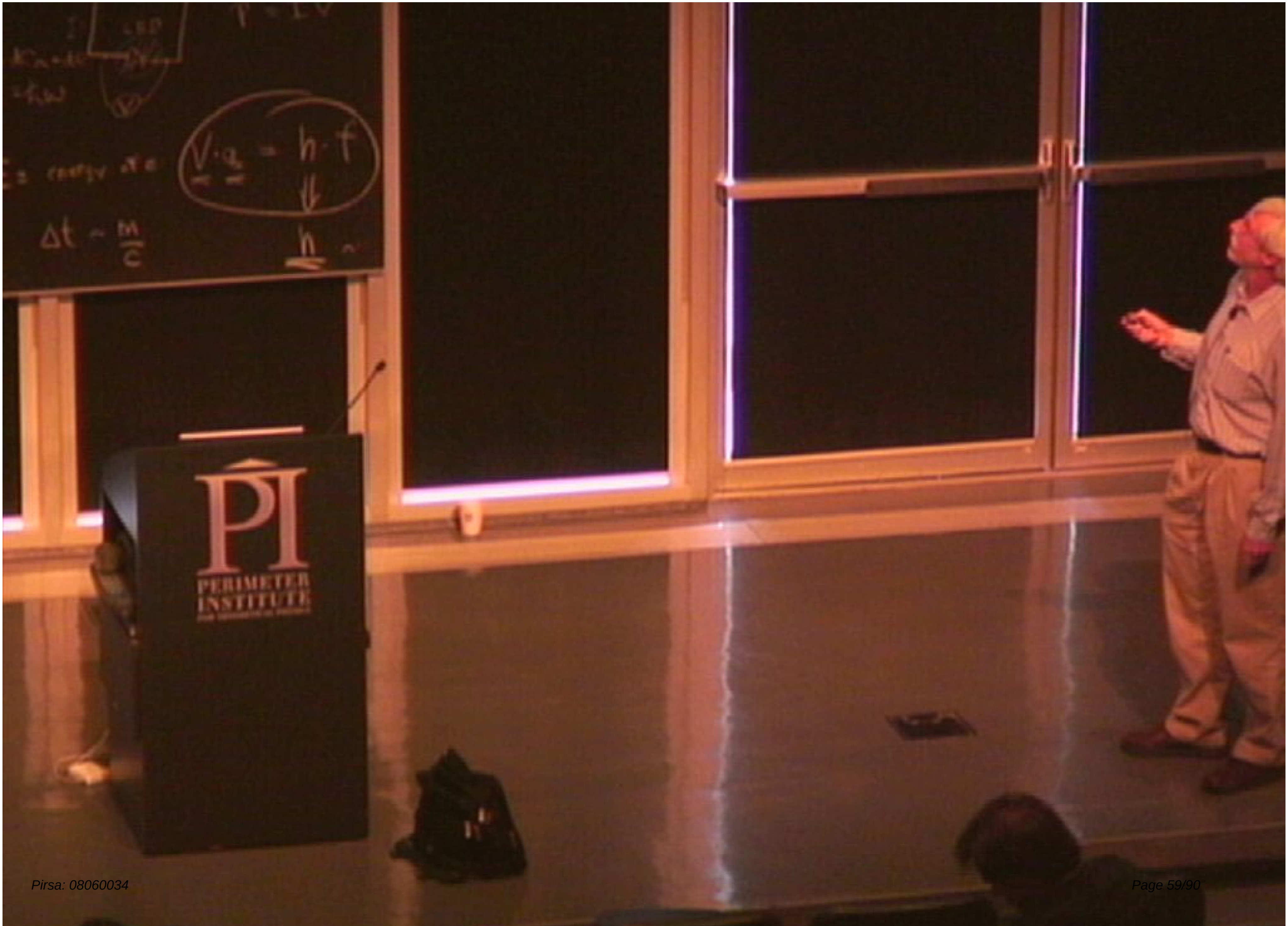
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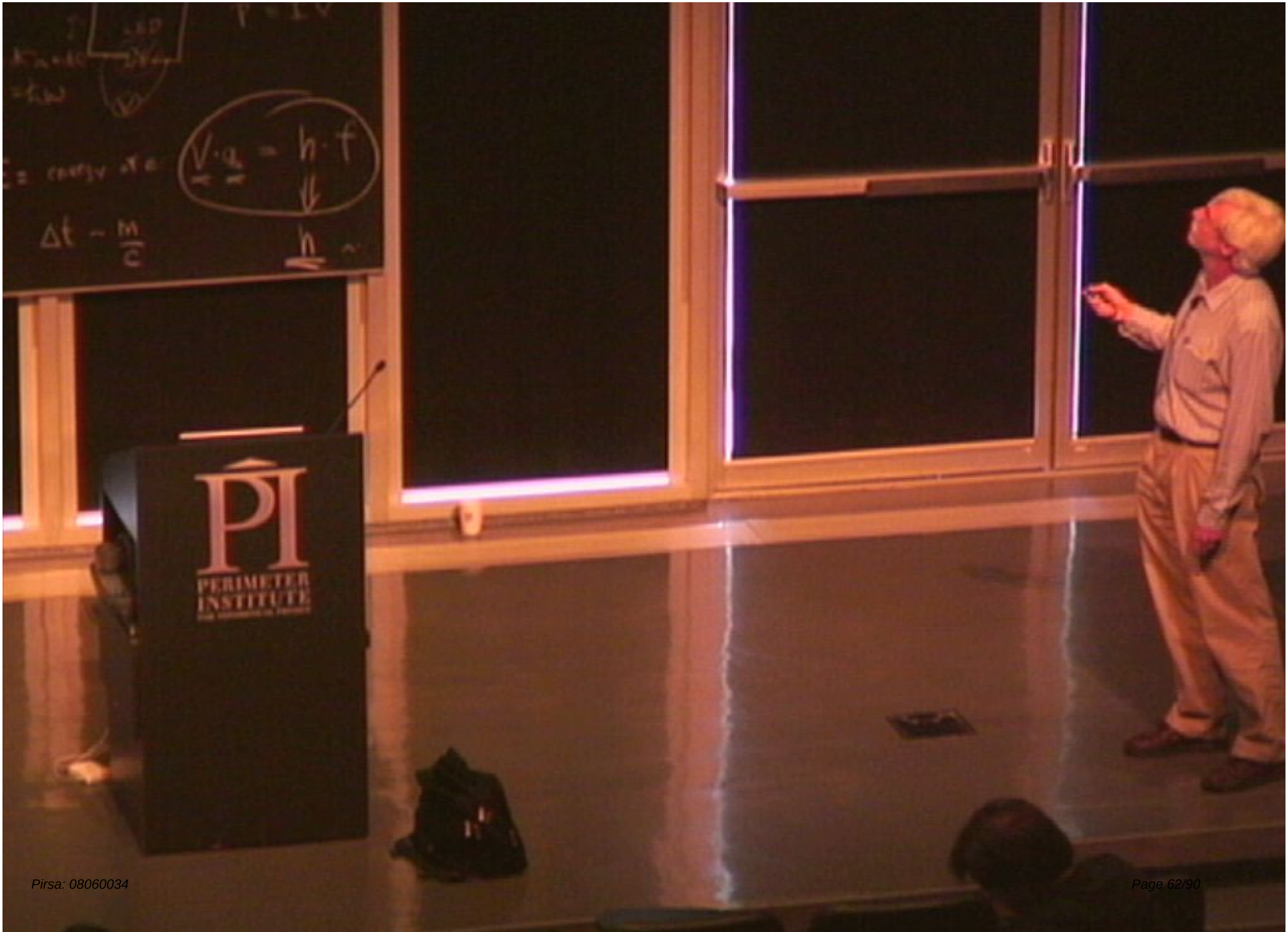
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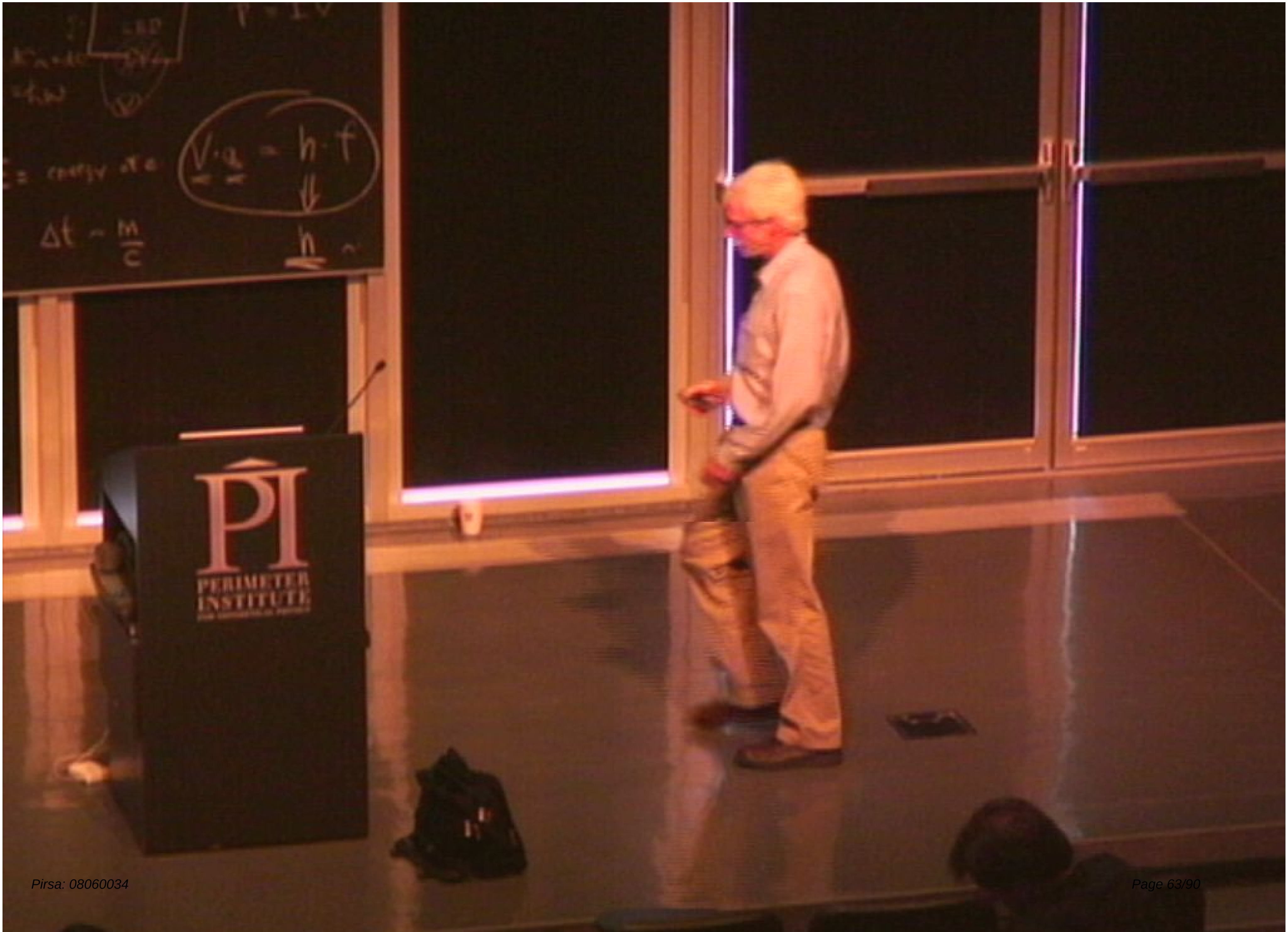

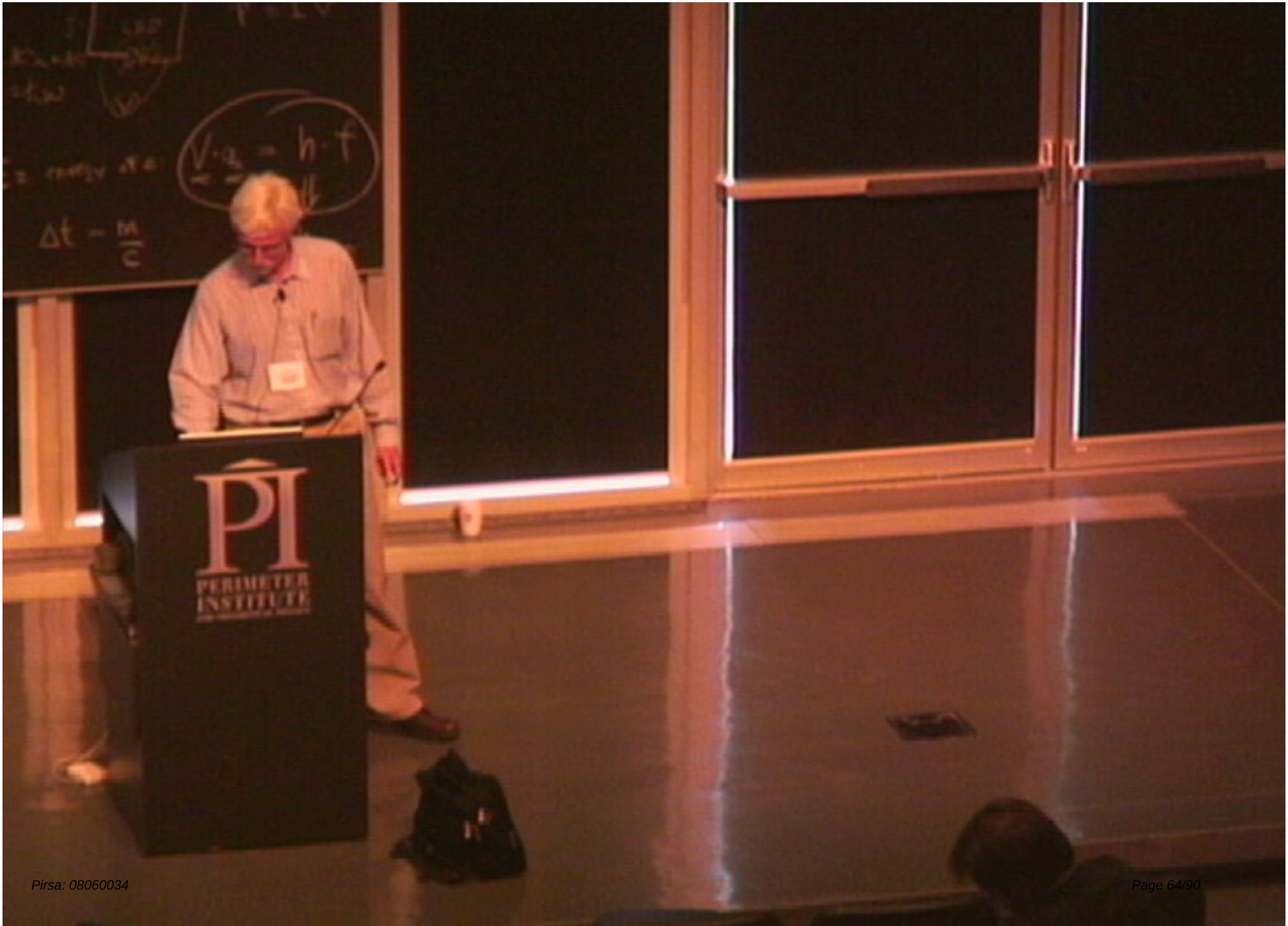

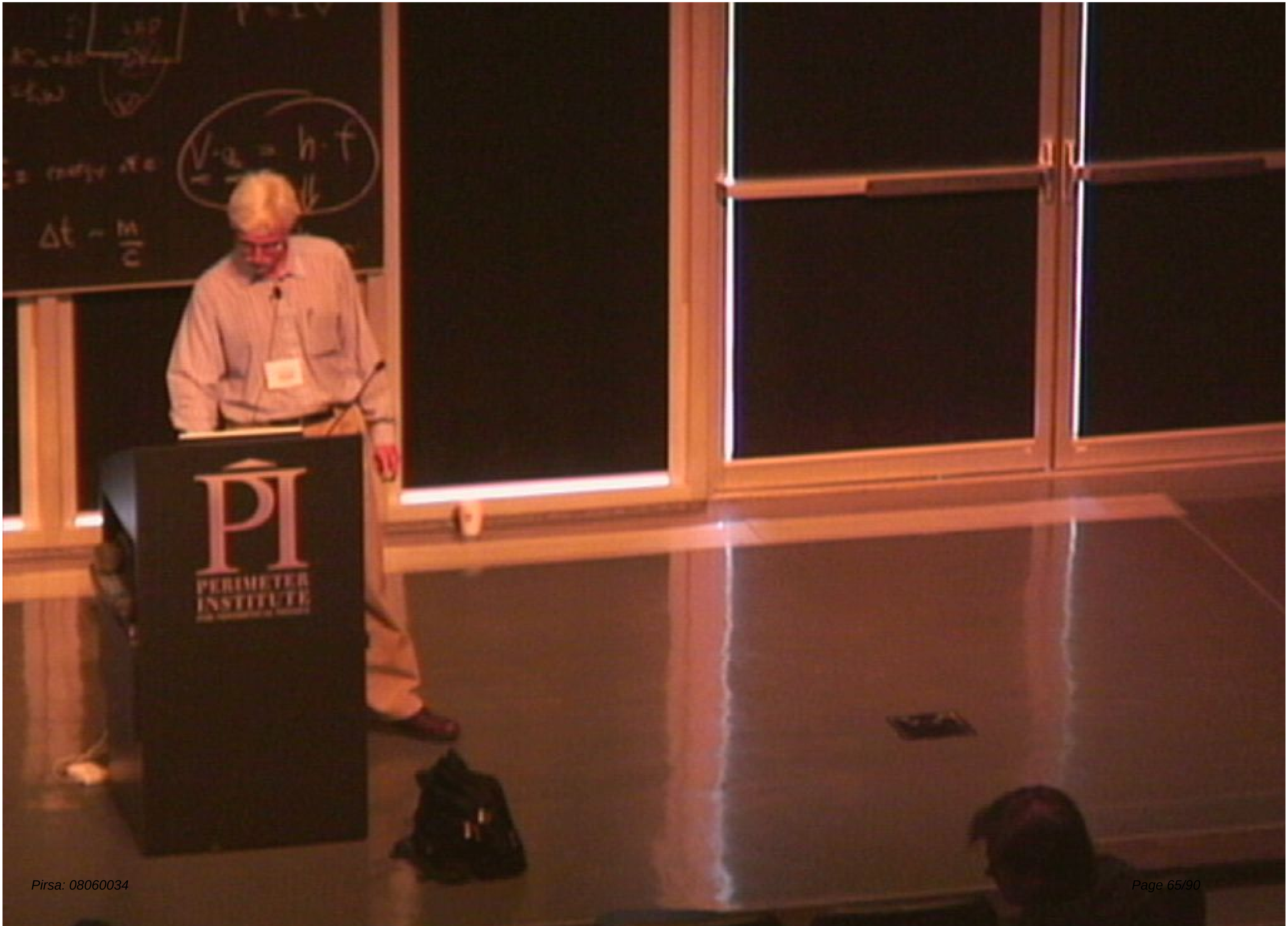
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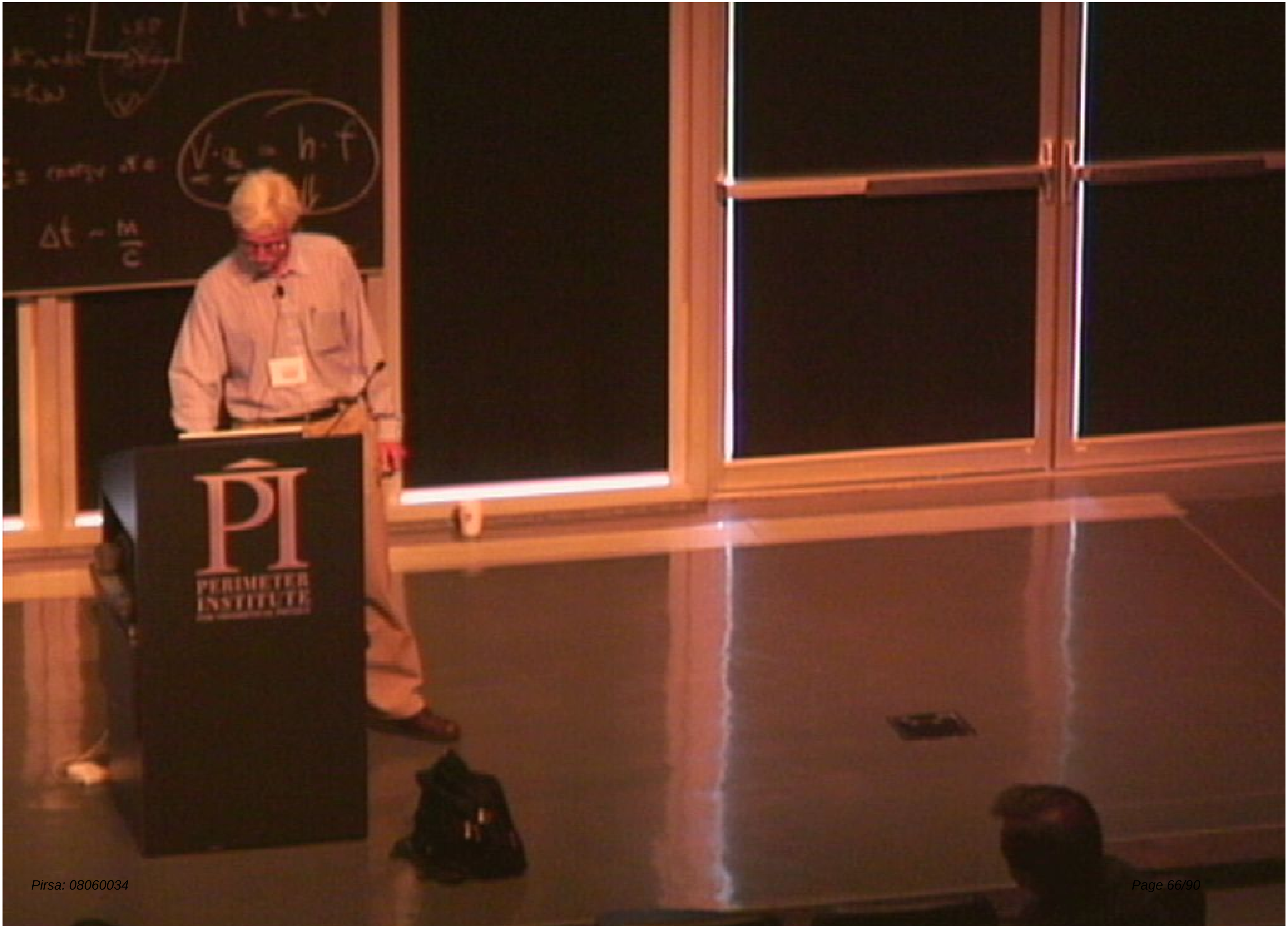
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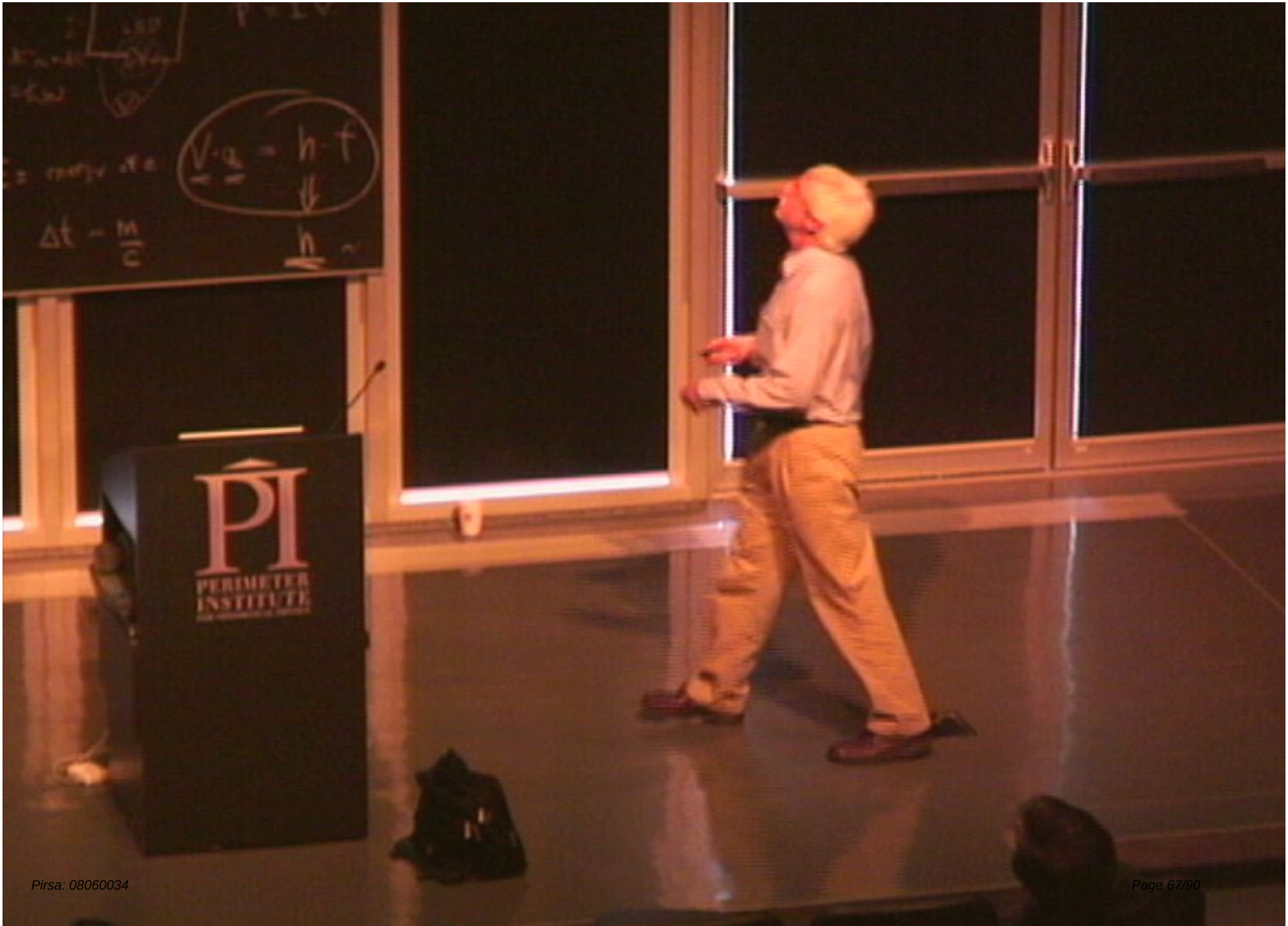
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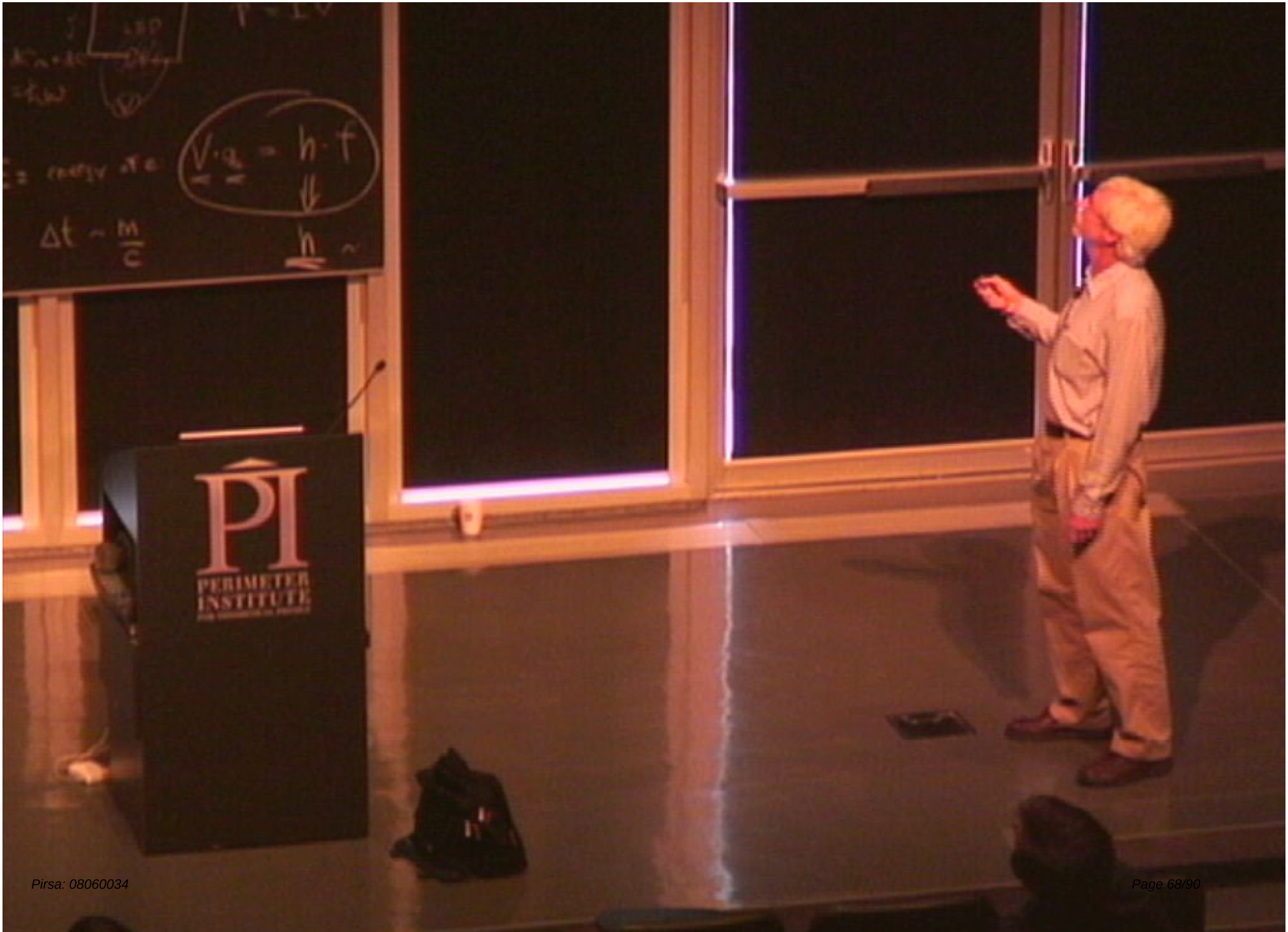
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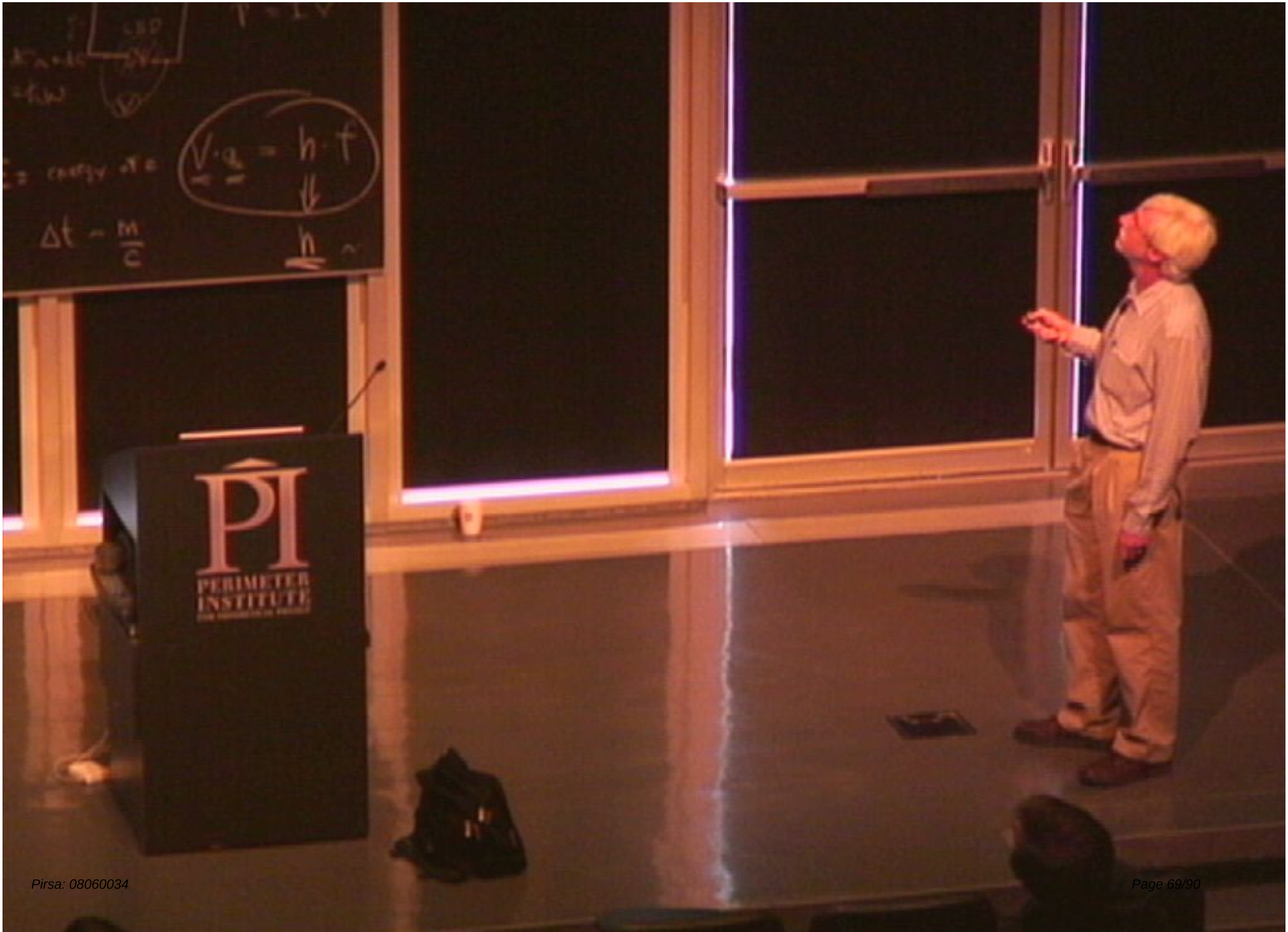
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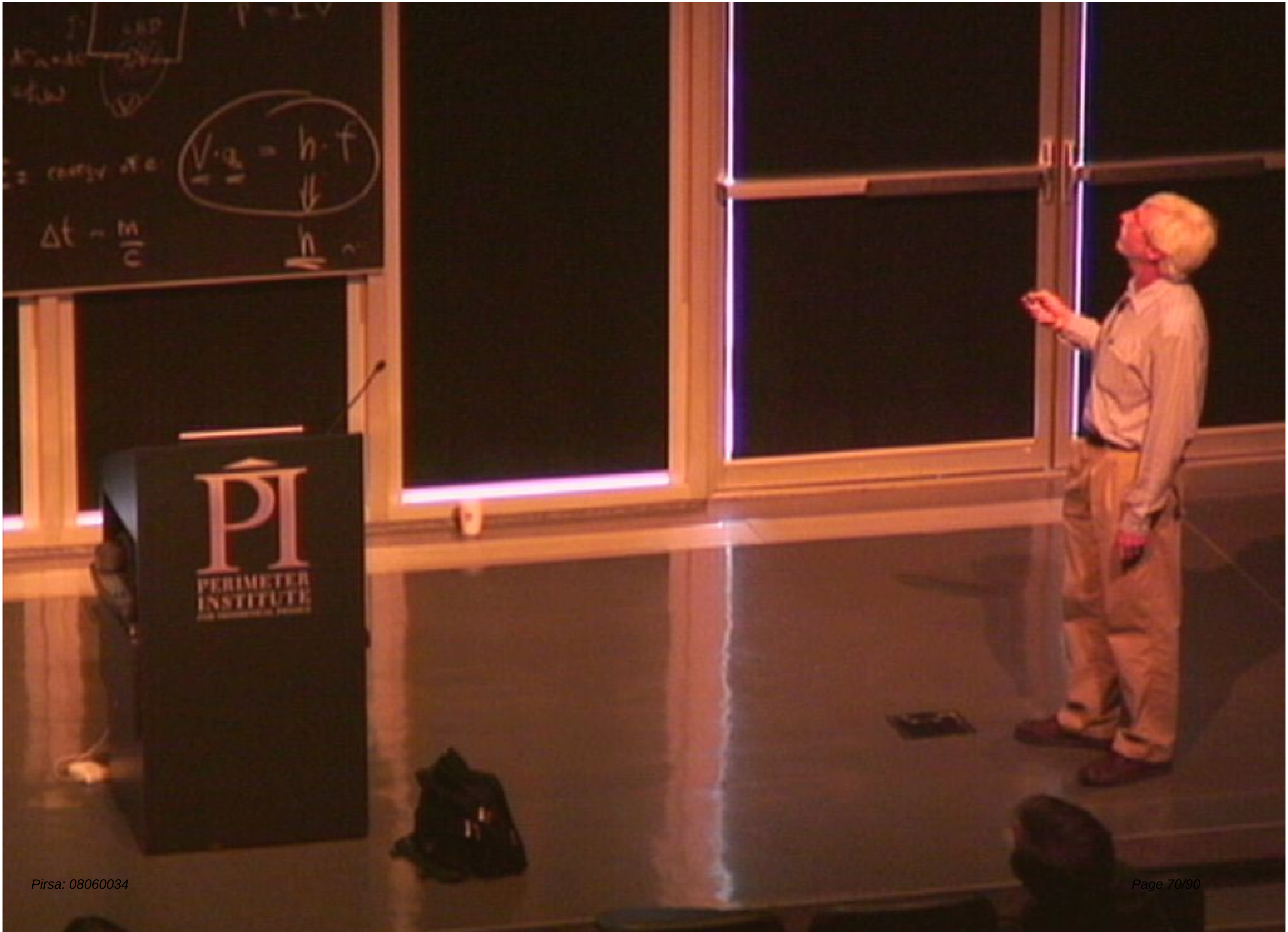
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2. A bit more interesting ... vectors. Take $\mathcal{L} \supset g\chi_\mu \hat{O}^\mu + \text{h.c}$

Amplitude follows from propagator in Minkowski space:

$$\text{Im}\,\mathcal{A}_{\text{fwd}} = -\frac{g^2 \pi C_V (d-1)^2}{4^{d-1} d\,\Gamma^2(d)} \left[\chi \cdot \chi^\dagger - \frac{2(d-2)}{d-1}\frac{|\chi \cdot k|^2}{k^2}\right] \theta(k^2)\theta(k^0)(k^2)^{d-2}$$

In the CM-frame, this is positive if

$$\frac{C_V}{d}\left[|\vec{\chi}|^2 + \frac{d-3}{d-1}|\chi_0|^2\right] \geq 0$$

Again we can show $C_V > 0$. So we have $d \geq 0$ and $(d-3)/(d-1) \geq 0$.

But $d \leq 1$ is excluded. The amplitude (not just forward) must exist for arbitrary (but nice) in/out wave-functions. Hence, require

$$\int d^4k\, \chi_1^{\mu\dagger}(-k) D_{\mu\nu}(k) \chi_2^\nu(k) < \infty$$

The most dangerous term in the propagator is $(k^2)^{d-2}(k_\mu k_\nu / k^2)$ giving $d \geq 2$. Hence

$$\boxed{d \geq 3}$$

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$$\int d^4k\, \chi_1^{\mu\dagger}(-k) D_{\mu\nu}(k) \chi_2^\nu(k) < \infty$$

The most dangerous term in the propagator is $(k^2)^{d-2}(k_\mu k_\nu / k^2)$ giving $d \geq 2$. Hence

$$\boxed{d \geq 3}$$

Small Representations

Saturate the bound. Different behavior depending on whether $j_1 j_2 = 0$ or $j_1 j_2 \neq 0$

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$$\text{Im}\,\mathcal{A}_{\text{fwd}} = \frac{C_S \pi g^2 (d-1)}{4^{d-1}\Gamma(d)^2} |\chi|^2 \theta(k^0)\theta(k^2)(k^2)^{d-2} \to C_S \pi g^2 |\chi|^2 \theta(k^0)\delta(k^2) \qquad \text{as } d \to 1$$

so:

- $d = 1$ corresponds to a free massless scalar field.
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- $m = 0$ (delta, not theta)
- scalar descendent has zero norm: $\partial^2 \hat{\mathcal{O}} = 0$

For vectors ($j_1 j_2 \neq 0$) at $d = 3$, limit is smooth, but tensor becomes transverse:

$$-i(-k^2 - i\epsilon)^{d-3}\left[k^2 g_{\mu\nu} - \frac{2(d-2)}{d-1} k_\mu k_\nu\right] \to -i\left[k^2 g_{\mu\nu} - k_\mu k_\nu\right]$$

so:

- $d = 3$ corresponds to a conserved current (we already knew)
- $m > 0$ (theta, not delta)
- scalar first descendent, $\partial^\mu \hat{\mathcal{O}}_\mu$, has zero norm: $\partial^\mu \hat{\mathcal{O}}_\mu = 0$

The results generalize:

- $j_1 j_2 = 0$ primaries. $d \to j_1 + j_2 + 1$
  - tensor is independent of d, except for overall factor
  - overall factor vanishes in limit giving delta function

$$\mathrm{Im}\,\mathcal{A}_{\mathrm{fwd}} \propto \epsilon \frac{\theta(k^2)}{(k^2)^{1-\epsilon}} \to \delta(k^2) \qquad \text{as} \quad \epsilon = d - (j_1 + j_2 + 1) \to 0$$

  - it's a free field (second descendent vanishes)

$$k^2 \delta(k^2) = 0 \quad \Rightarrow \quad \partial^2 \hat{\mathcal{O}}_{(j_1, j_2)} = 0$$

  - for $j_1 \neq 0$ or $j_2 \neq 0$ a first descendent has zero norm (follows from $k^2 = 0$)

$$\partial_\mu \hat{\mathcal{O}}^{\mu\ldots}_{(j_1, j_2)} = 0$$

- $j_1 j_2 \neq 0$ primaries. $d \to j_1 + j_2 + 2$
  - limit is smooth, keep theta function
  - tensor depends on d, becomes transverse in limit
  - *i.e.*, some first descendants have zero norm

$$\partial_\mu \hat{\mathcal{O}}^{\mu\ldots}_{(j_1, j_2)} = 0$$

(additional examples upon request
backup slides

Comments on Unparticles

Simplest observations

In the unparticle literature

- it is customary to study only $1 < d < 2$
- integral dimensions are avoided
- all fields are studies in $1 < d < 2$ regardless of spin

As you know

- There is no upper bound on d
- Nothing special at integral d (but some subtleties, see bellow)
- Lower bound depends on (j_1, j_2)

Scale but not conformal: way out?

- No known examples of 4d unitary QFTs that are scale invariant but not conformal invariant
- Easy to construct examples of 4d non-unitary scale invariant but not conformal invariant theories.
- In 2d Polchinski has proved scale invariance + unitarity implies CFT
- Riva and Cardy's counter-example is non-unitary

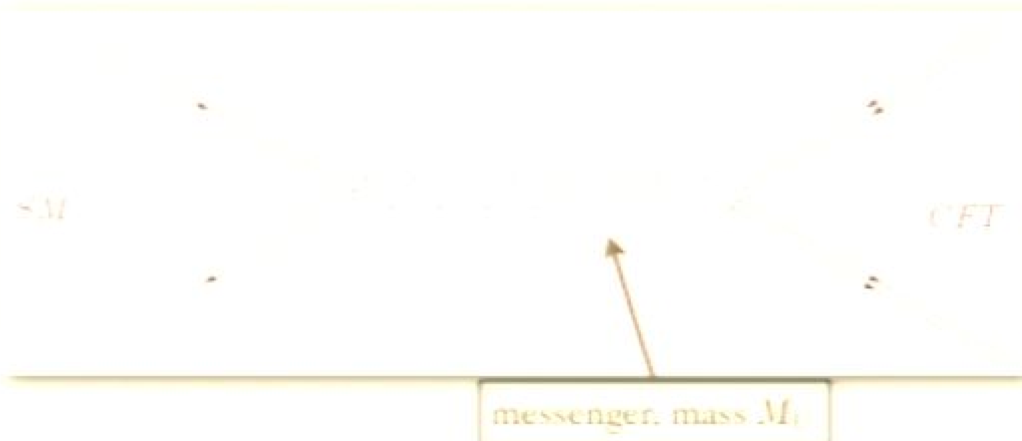
- Using only scale invariance, convergence of the amplitudes give, generally

$$d \geq j_1 + j_2 + 1$$

Conformal invariance tightens this for the case $j_1 j_2 \neq 0$, and requires infinite towers of states and, when d saturates the bound, puts conditions on first descendants.

Contact Terms I:
Trivial but most dangerous

UV completion of weak SM-CFT coupling through messenger:



$$\mathcal{L}_{\rm eff} \supset D_1(M_U)\frac{1}{M_U^2}\mathcal{Q}_1$$

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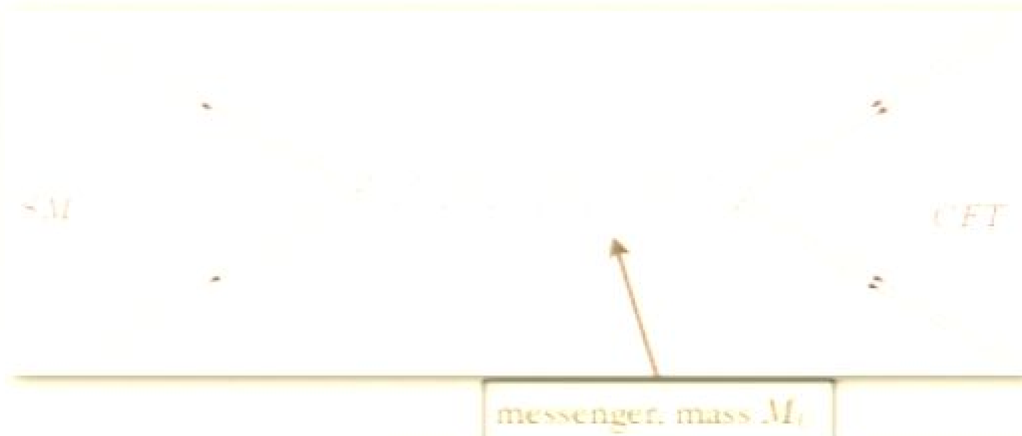

$$\mathcal{L}_{\rm eff} \supset D_1(M_U)\,\frac{1}{M_U^2}\,\mathcal{Q}_1$$

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SM CFT

Then *almost* unavoidably:

$$\mathcal{L}_{\rm eff} \supset D_2(M_U)\,\frac{1}{M_U^2}\,\mathcal{Q}_2 - D_2(M_U)\,\frac{1}{M_U^4}\,j_\mu \partial^2 j^\mu + \ldots \qquad \mathcal{Q}_2 \equiv j_\mu j^\mu$$

For an exclusive process, *e.g.* $ee \to ee$, or $ee \to \mu\mu$

vs

$$\sim \frac{1}{M_U^4} \qquad\qquad \sim \frac{1}{M_U^2} + \frac{E^2}{M_U^4}$$

$$\frac{\mathcal{A}_{\text{unparticle}}}{\mathcal{A}_{\text{contact}}} \sim \frac{D_1^2}{D_2}\left(\frac{E}{M_U}\right)^2\left(\frac{E}{\Lambda_U}\right)^{2(d-3)}$$

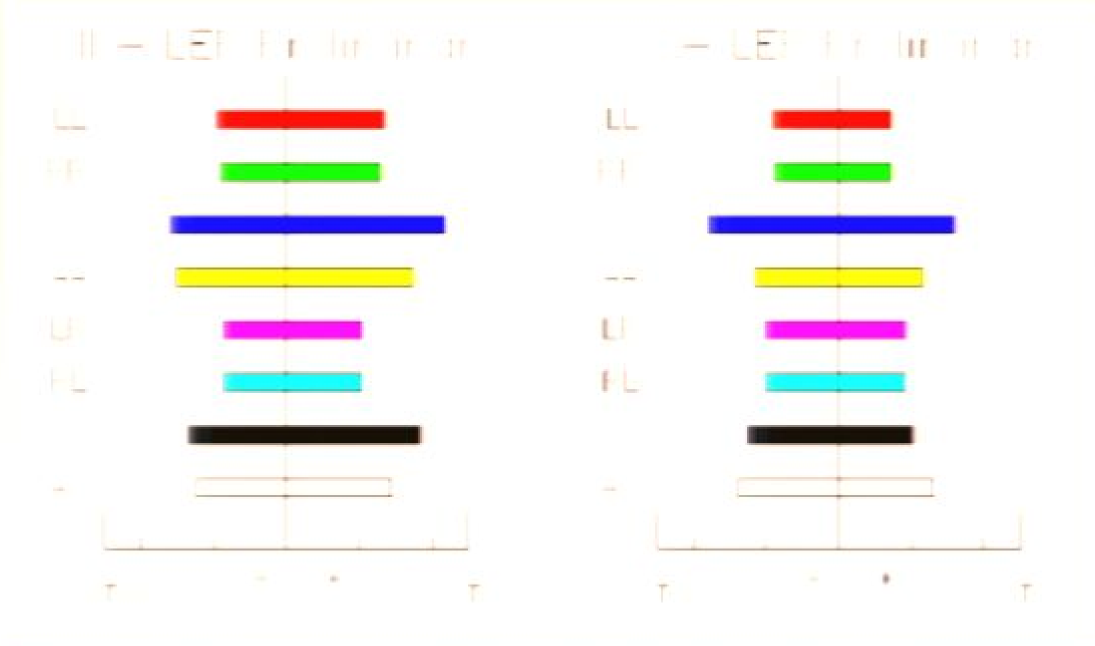
E = CM energy

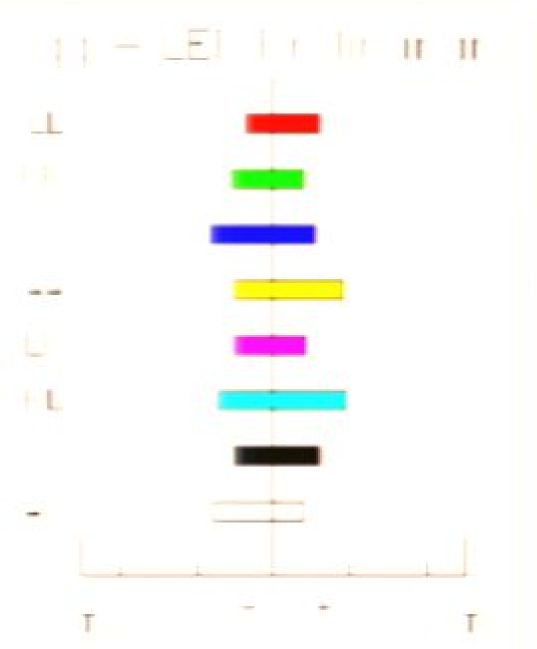
And the contact interactions are already tightly constrained:

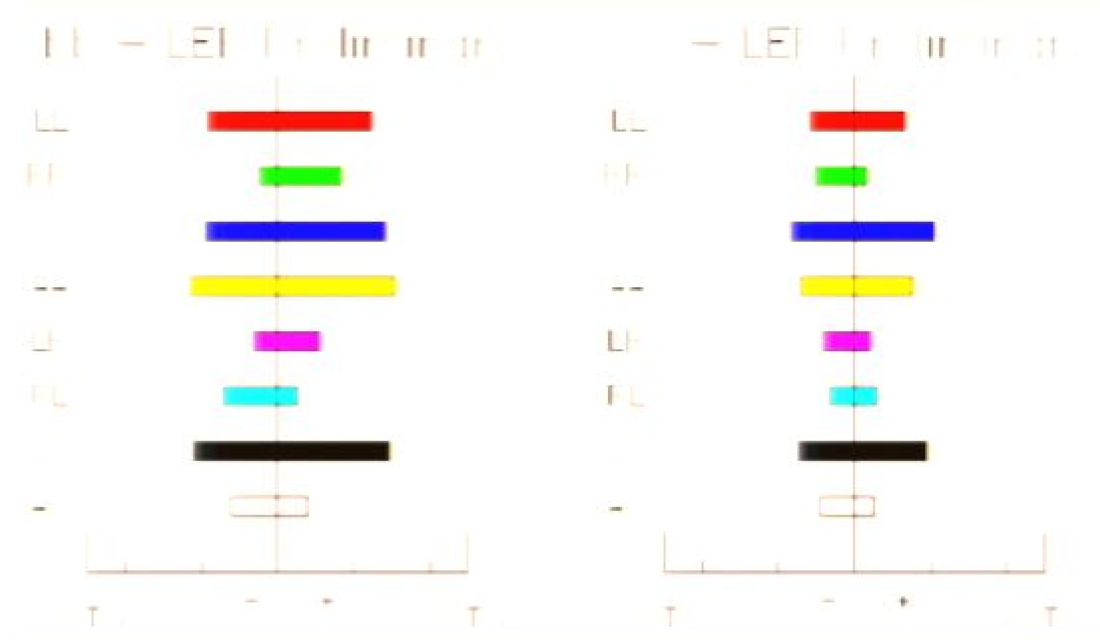
$$\mathcal{L}_{\text{eff}} = \frac{g^2}{(1+\delta)\Lambda^2}\sum_{i,j=L,R}\eta_{ij}\overline{e}_i\gamma_\mu e_i \overline{f}_j\gamma^\mu f_j,$$

J. Alcaraz et al. [ALEPH Collaboration] arXiv:hep-ex/0612034.

where $g^2/4\pi$ is taken to be 1 by convention. $\delta = 1(0)$ for $f = e$ $(f \neq e)$. $\eta_{ij} = \pm 1$ or 0 for different interaction types. Λ is the scale of the contact interactions. e_i and f_j are left or right-handed spinors.







Contact Terms II: more subtle, still dangerous, unavoidable

Recall, in CFT:

vector: $$\frac{1}{(2\pi)^2}\frac{g_{\mu\nu}-2x_\mu x_\nu/x^2}{(x^2)^d} = \frac{(d-1)\Gamma(2-d)}{4^{d-1}\Gamma(d+1)}\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}(k^2)^{d-2}\left[g_{\mu\nu}-\frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right]$$

For d = integer on RHS? Take, *e.g.*, $d \rightarrow 3$

- pole $1/(d-3)$ multiplies local term (concentrated at $x = 0$) → counterterm
- limit of rest gives a non-local propagator

$$\frac{1}{(2\pi)^2}\frac{g_{\mu\nu}-2x_\mu x_\nu/x^2}{(x^2)^3} = \frac{1}{48}\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}\,(k^2 g_{\mu\nu}-k_\mu k_\nu)\ln(k^2)$$

- need to introduce a scale in log, but conformal invariance OK since log(μ^2) gives contact term (exactly like infinite pole): renormalization
- contact term required to get RGE-invariance (*i.e.*, μ-independent results)
- exactly as expected from free field theory with $\mathcal{O}^\mu = \bar{\psi}\gamma^\mu\psi$

Counterterms are needed even for non-integral d

They are required for RGE invariant amplitudes!

For example, for $ee \to \mu\mu$

$$\mathcal{O}_1(0) = \int d^4x\, T(\mathcal{Q}_1(x)\mathcal{Q}_1($$

$$\langle \mu\mu | \mathcal{H}_{\rm eff} | ee \rangle. \qquad \mathcal{H}_{\rm eff} = \frac{1}{M_U^4} \sum_{i=1,2} C_i \mathcal{O}_i$$

$$\mathcal{O}_2(0) = -j^\mu(0)\partial^2 j_\mu(0)$$

is RGE invariant provided

$$C_1(\mu) = \left(\frac{\mu}{\Lambda_U}\right)^{\gamma_{11}(g_*)} C_1(\Lambda_U)$$

$$C_2(\mu) = C_2(\Lambda_U) + \frac{\gamma_{12}(g_*)}{\gamma_{11}(g_*)}\left[\left(\frac{\mu}{\Lambda_U}\right)^{\gamma_{11}(g_*)} - 1\right] C_1(\Lambda_U)$$

Non-transverse propagators

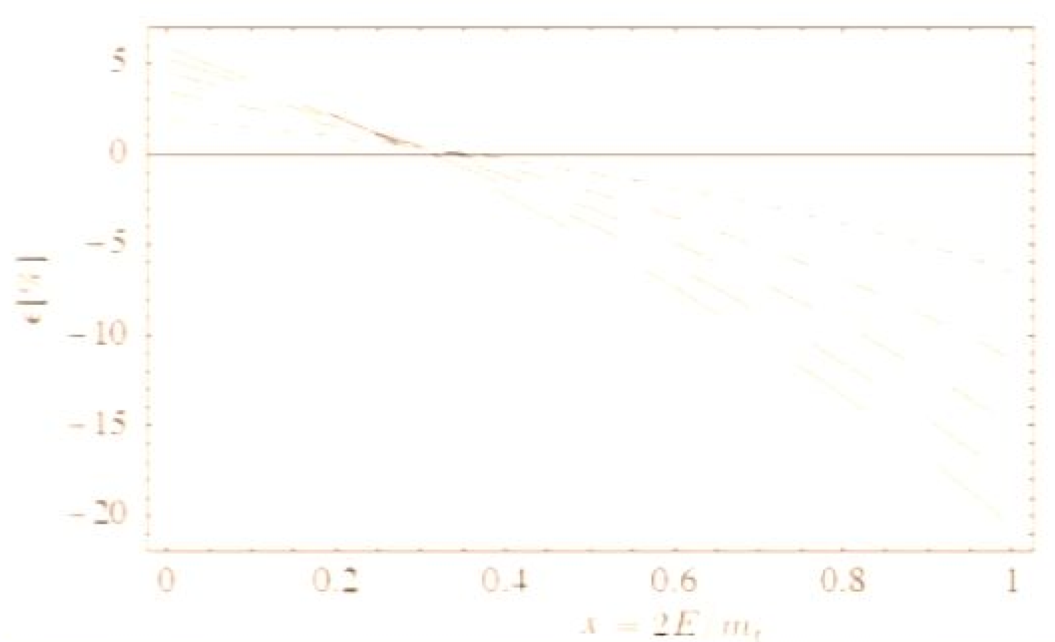
Consider vector unparticles (for definiteness, but tensors, etc. are similar).
Compare transverse propagator vs CFT's propagator, same scaling dimension.

Is there a physical distinction?

- Not for SM=conserved current.
- Yes for non-conserved current.

Example: $t \to q + U$ $\qquad \mathcal{H}_{\text{eff}} = \frac{\lambda}{\Lambda^{d-1}} \bar{q}\gamma_\mu(1-\gamma_5) t \hat{\mathcal{O}}^\mu + \text{h.c.}$

$$\epsilon = \frac{\Delta\left(\frac{1}{\Gamma}\frac{d\Gamma}{dx}\right)}{\frac{1}{\Gamma}\frac{d\Gamma}{dx}}$$



$d = 3.2, 3.4, 3.6, 3.8, 4.$

Summary

- Unitary representations of the conformal group:
  - Poincare content of unitary representations of the Conformal Group can be understood simply from unitarity of S-matrix
  - Analysis yields additional insights:
    - tensor structure of propagators
    - renormalization

- Unparticles
  - Lower bounds on d, in particular $d \geq 3$ (4) for vectors (tensors) greatly suppress effects
  - Neither vector nor tensor propagators are transverse: only when the unitarity bound on the dimension is saturated does the operator satisfy conservation laws (or free field equations of motion for $j_1 j_2 = 0$)
  - Ultra-heavy particle exchange necessarily introduces SM contact interactions, which generally dominate over other unparticle interference effects.
  - CFT exchange induces additional SM contact interactions (which cure the apparent problems with integer dimensions).

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