Title: F-theory and GUTs: Experimental Predictions

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Abstract:

F-theory and GUTs:

Experimental Predictions

based on joint work with
Jonathan Heckman and Chris Beasley

Cumrun Vafa

String theory is yet to make a verifiable quantitative prediction for the real world. It is about time we change that! In this talk I would like to present some work we have done which takes a modest step in that direction.

We study realization of GUTs in the context of F-theory and find novel ways, not available in the context of heterotic strings, or perturbative type II strings to realize GUTs. Moreover these new ingredients allow us to solve some of the most difficult puzzles for a GUT phenomenology. Typically one ingredient solves a few problems at once! Our main assumption is that M_GUT and M_Planck dynamics decouple. Technically this means that we can in principle take M_Planck to infinity, while fixing M_GUT. This dramatically cuts down the allowed possibilities to a finite set!

Given the limited time available, let me first present the main results before getting into details:

1-The minimal F-theory GUT comes from a unique class of models, which although it has some discrete choices and continuous parameters to adjust, it is sufficiently robust that it leads to specific predictions. This proceeds through an SU(5) GUT in 8d, broken to MSSM group in 4d, by internal U(1) hypercharge flux (a mechanism not available to heterotic strings). The non-minimal GUTs have to go through a flipped GUT scenario. In particular we also study a flipped SU(5) GUT in 4d arising from SO(10) gauge symmetry in 8d.

2-Matter and Higgs arise on Riemann surfaces in the internal geometry. They are distinguished by the condition of whether the net U(1) flux through them vanish or not. This can be used to solve the doublet-triplet splitting problem.

3- μ problem solved by having H_u and H_d on distinct matter curves. If they do not intersect we find μ =0. If they do intersect in one scenario we find a hierachically small μ ;

$$\mu = M_{GUT} \cdot \exp(-1/\varepsilon^{2\gamma})$$

$$\varepsilon = \frac{M_{GUT}}{\alpha_{GUT} M_{Planck}} = 0.07 \; ; \; \frac{1}{3} < \gamma < 1$$

4- Naturally light neutrinos via two distinct mechanisms--

A) Dirac neutrino masses, with $m_\nu^D \cong .5 \times 10^{-2 \pm .5}$ eV
(same mechanism which solves the μ problem).

B) Modified seesaw like mechanism with light neutrinos $m_\nu^{light} = .2 \times 10^{\pm 1.}$ e
and heavy neutrinos at about $m_\nu^{heavy} = 3 \times 10^{12 \pm 1.5}$ GeV

5-Dangerous proton violating terms suppressed through a related mecahanism which solves the doublet-triplet splitting problem.

6-Explanation of why mass relations within a generation are not exact but work best for the massive generation. This mechanism uses the Aharanov-Bohm effect in the internal geometry.

7- Supersymmetry can be communicated through gauge mediation mechanism
The same mechanism which solves the μ problem, and provides light neutrino masses, can also lead to small mass for messenger fields.

8-Various mechanism can in principle be used to produce texture for Yukawa couplings. Most natural is to use discrete symmetries in the internal geometry which is automatically available for us. In fact the maximal discrete symmetry group available is the Weyl group of E8.

The organization for the rest of this talk is as follows:

- Review F-theory compactifications leading to N=1 supersymmetry in 4d

- The condition for decoupling gravity dynamics from GUT dynamics

- Gauge symmetry breaking mechanisms and the U(1) hypercharge flux

- The minimal SU(5)/MSSM GUT model

- Solution to doublet triplet splitting problem, μ problem, dangerous Baryon violating operators and violation of mass relations for light generations

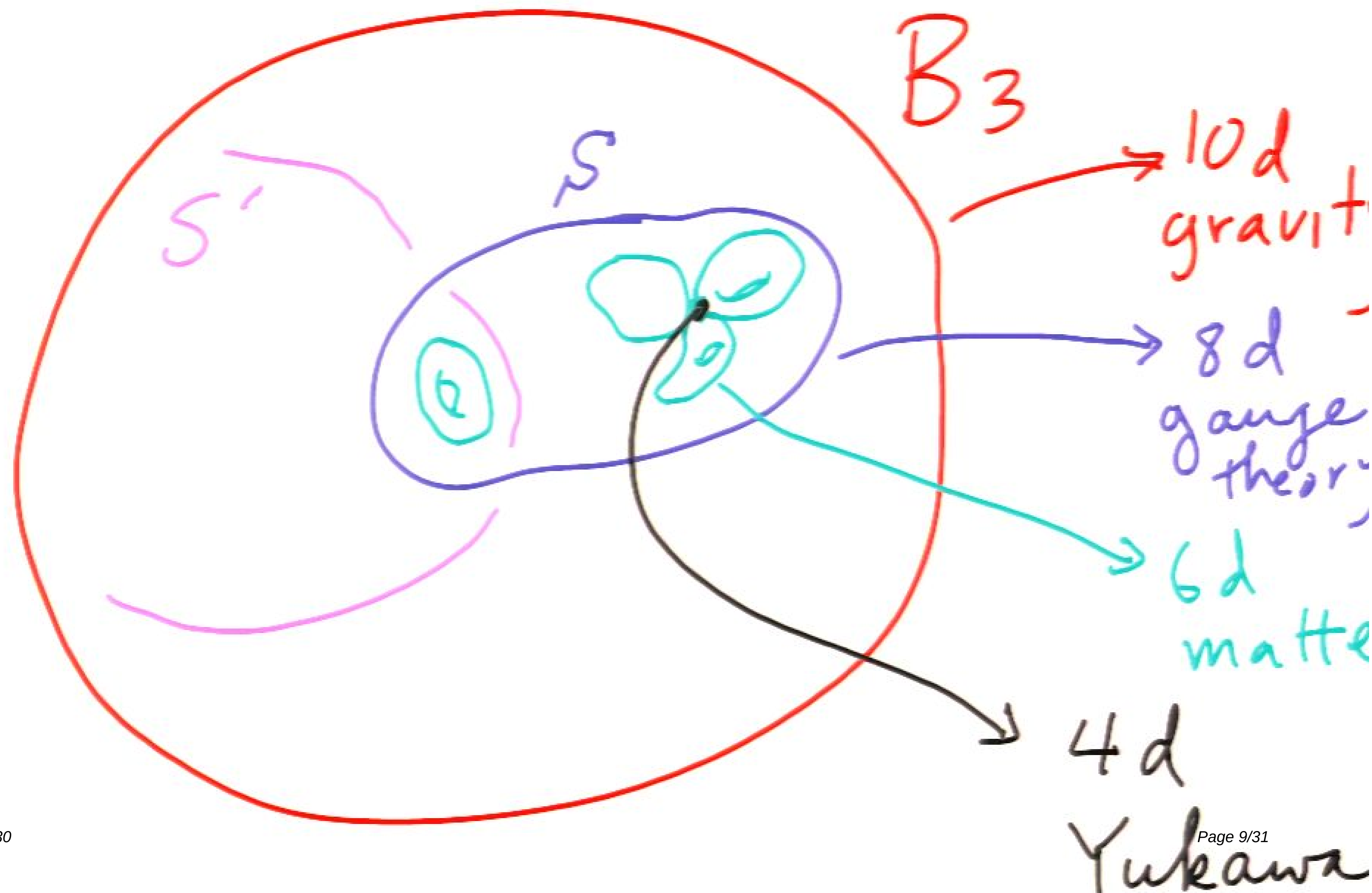
-Neutrino masses

Review of N=1 Compactifications of F-theory

Recall that F-theory is a strong coupling completion of type IIB superstrings. In order to obtain N=1 supersymmetry in 4 dimensions we need to consider elliptic Calabi-Yau 4-folds (with a section) where the 6 dimensional base of elliptic fibration is the visible geometry to IIB. Moreover on codimension 2 subspaces, where the elliptic fibration degenerates, i.e. 8 dimensional supspaces of spacetime, we get the 7-branes. The type of the gauge group they lead to depends on the nature of the elliptic singularity and in this way we can naturally obtain A-D-E gauge symmetries in 8 dimensions (4d internal geometry). On loci where two 7-branes intersect, i.e. in 6 dimensions, (Riemann surfaces in the internal geomtry) we get matter fields. On loci where triple 7-branes intersect, i.e. points in the internal geometry, we get Yukawa couplings: So we obtain a dimensional hierarchy of structure:

	10d	→	Gravity	(6d internal)
7-brane	8d	→	Gauge Theory	(4d internal)
2x7-brane	6d	→	Matter Fields	(2d internal)
3x7-brane	4d	→	Yukawa Couplings	(0d internal)

INTERNAL GEOMETRY FOR F-THEORY:



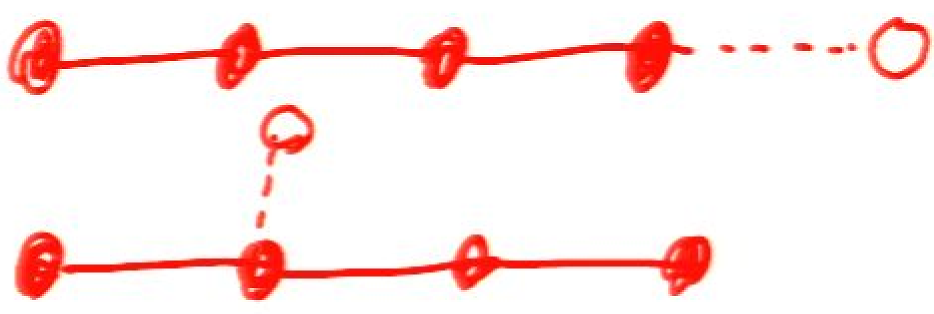
1-The type of gauge symmetry is, as we have already noted, A-D-E (we can in principle get non-simply laced, but we would not need them)

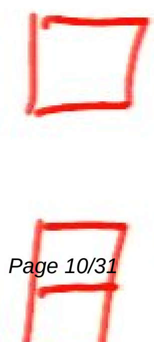
2-The matter arises by local Higgsing: When two 7-Branes meet on a Riemann surface, there is a local enhancement of gauge symmetry on the Riemann surface which dictates the matter structure: Typically if the 7-brane meets one additional 7-brane, we obtain an enhancement

G (rank r) → G' (rank r+1) on Riemann surface

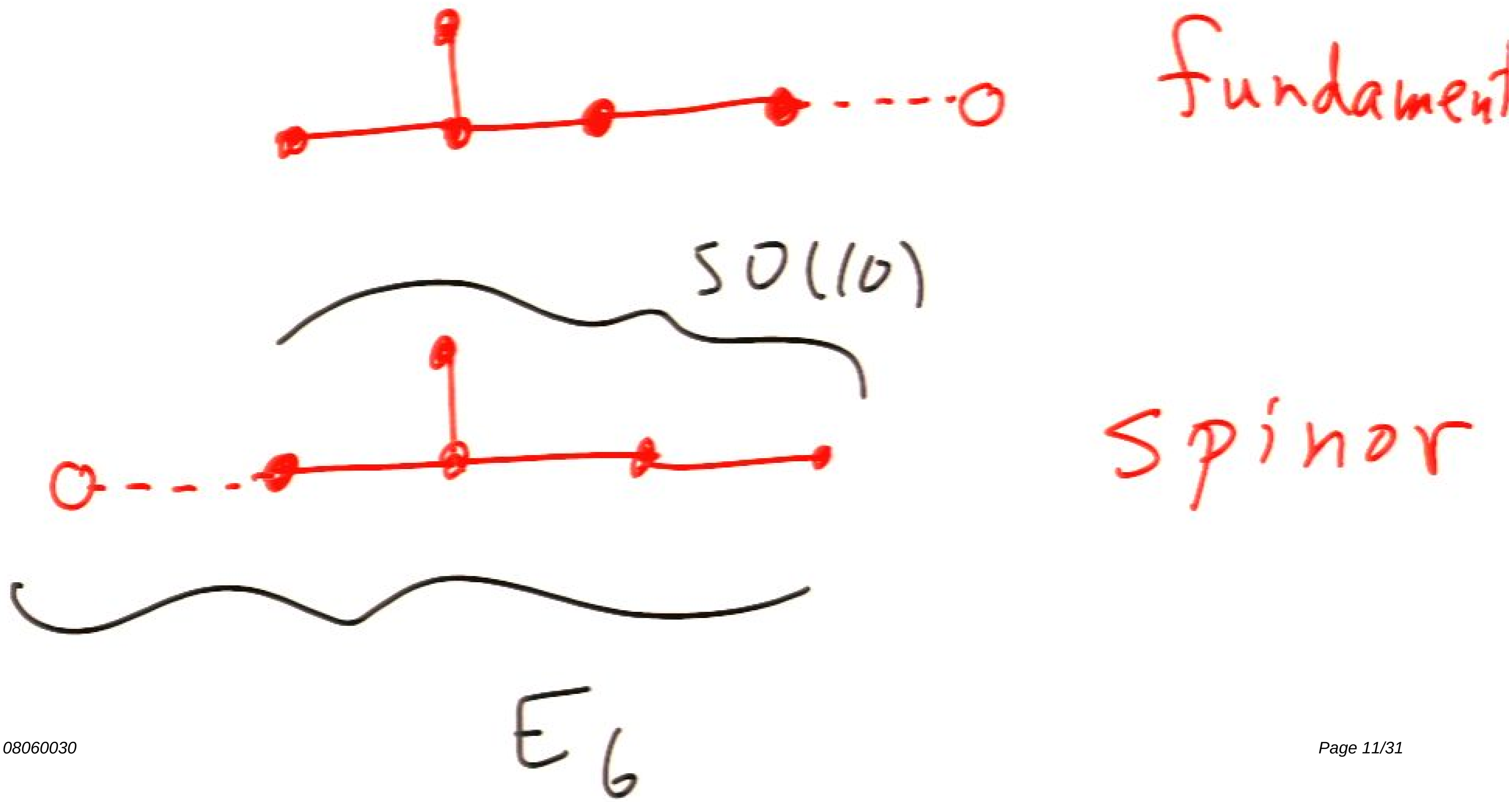
Moreover the matter that arises on the Riemann surface corresponds to the adjoint decomposition of G' in G. This means we only get very restrictive type of matter:

For example, for SU(N) we can only get matter in fundamental and rank 2 antisymmetric tenstor matter:





For SO(2N) we get matter in fundamentals only.   For a few exceptional SO(2N) which embed in E_k we can also get matter in the spinor (this includes SO(10) spinor 16).



Yukawa couplings, arise from one further enhancement in rank:
Moreover the matter curves will be on the same brane or different ones, depending on whether the matter is neutral or not under the gauge symmetry the brane carries.

For SU(N) in general we get interactions involving:

$(\square, \overline{\square}, 1)$ or $(\square, \square, \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}})$

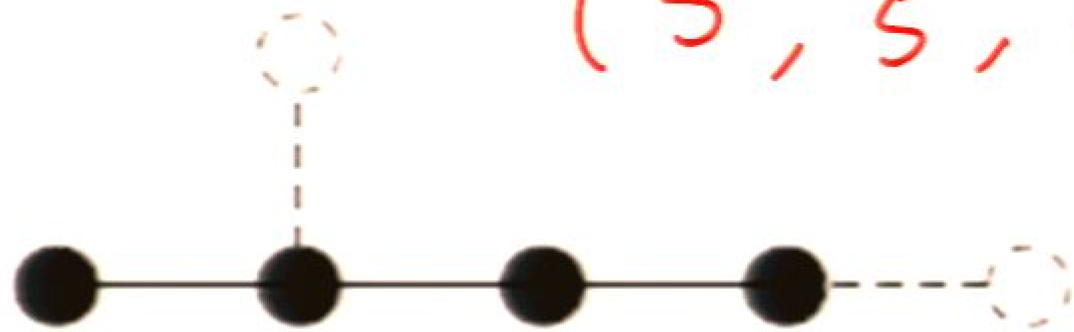
These are familiar from perturbative branes and orientifold construction,
For the special case of SU(5) we get another interaction not available for perturbative D-branes:

$(\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, \begin{smallmatrix}\square\\\square\end{smallmatrix})$

Through E6 enhancement – not available in pert. D-brane constructions double enhancements lead to Yukaw couplings

$(5, 5, \overline{10})$

i)



$(5, 10, 10)$

ii)

$\underbrace{\hspace{6em}}_{E_6}$

necessa

for GUTs

Without making any further assumptions, the landscape is unwieldy. We make one assumption, which we think is reasonable and cuts down the allowed possibilities to a finite manageable set: The assumption is that GUT dynamics is decoupled from that of Planck. What we take this to mean is that we are in a compactification class where at least in principle we could take M_Planck to infinity keeping M_GUT fixed. The fact that this is natural is reinforced by the fact that typical GUT models are asymptotically free, and can be decoupled from gravity.

This implies that the 7-brane which carries the GUT group must be wrapped on a 4-cycle of the Calabi-Yau that can be shrunk. This can only happen if the this is a complex surface with positive first Chern class, leading to positive curvature. In particular this means that the 7-brane is on a del Pezzo. All del Pezzo's are connected (by changing sizes of spheres) to the highest rank del Pezzo, which is dP8 (P^2 blown up at 8 points). Thus we have narrowed the geometry of the 7-brane to a single geometry! There are only a finite number of parameters dictating its shape and size.

How to Higgs the GUT Group to MSSM: U(1) Hypercharge Flux

It turns out that Higgsing the GUT group to MSSM is a major challenge in string theory. This is because the most typical scenario, namely the adjoint representation Higgs field is not available. The way Higgsing is achieved typically in heterotic strings is by turning on discrete Wilson lines. However this is not generically allowed, as the relavent Calabi-Yau may not have a non-trivial fundamental group.

In our case, i.e. the relevant part of the gauge geometry being del Pezzo, this is not allowed, as the fundamental group is trivial! So how can we Higgs say SU(5) $\rightarrow$ SU(3) x SU(2) x U(1)? The most obvious answer is: Use an internal flux in the U(1) hypercharge subgroup of SU(5)! This simple idea is not allowed in heterotic string! In heterotic string if one uses this U(1) flux, it higgses the hypercharge, due to Green-Schwarz mechanism. However it turns out this is allowed in the context of F-theory without Higgsing the U(1)! (similar to a mechanism uncovered by Buican et.al.): The U(1) flux which is non-trivial on the 7-brane worldvolume can be trivial in the base of the F-theory (and in some sense in many cases it automatically is!). Thus we have a new simple way to Higgs the GUT group: The internal magnetic flux of hypercharge higgses GUT to MSSM!

In particular if we have a U(1) flux on the 7-brane S, as long as the flux is trivial in the background base B, then U(1) is not Higgsed. Moreover, this is very easy to satisfy because supersymmetry requires the U(1) flux to be orthogonal to Kahler class,

$$F \wedge k = 0$$

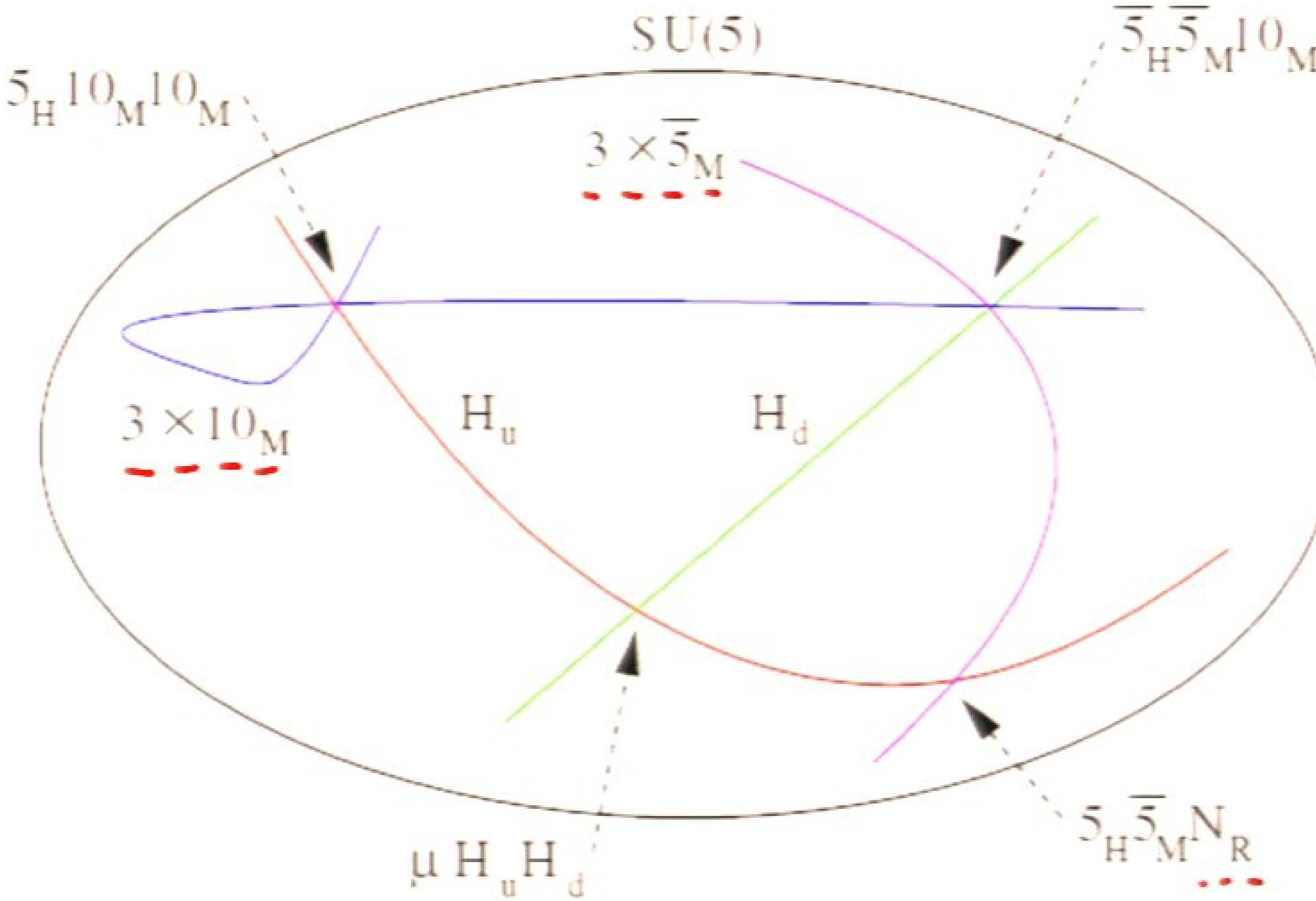
And this makes this mechanism very natural.

Note that if the Base of the F-theory is of the form

$$B = \mathbb{P}^1 \times S$$

then this is not allowed: The flux in S is not trivial in B.
Indeed this is exactly why this does not work for heterotic string:
The F-theory geometries dual to heterotic string require such a geomet
(because F-theory on elliptic K3 fibered over $\mathbb{P}^1$ is dual to heterotic strings on T^2).

A Minimal SU(5) GUT in F-thoery



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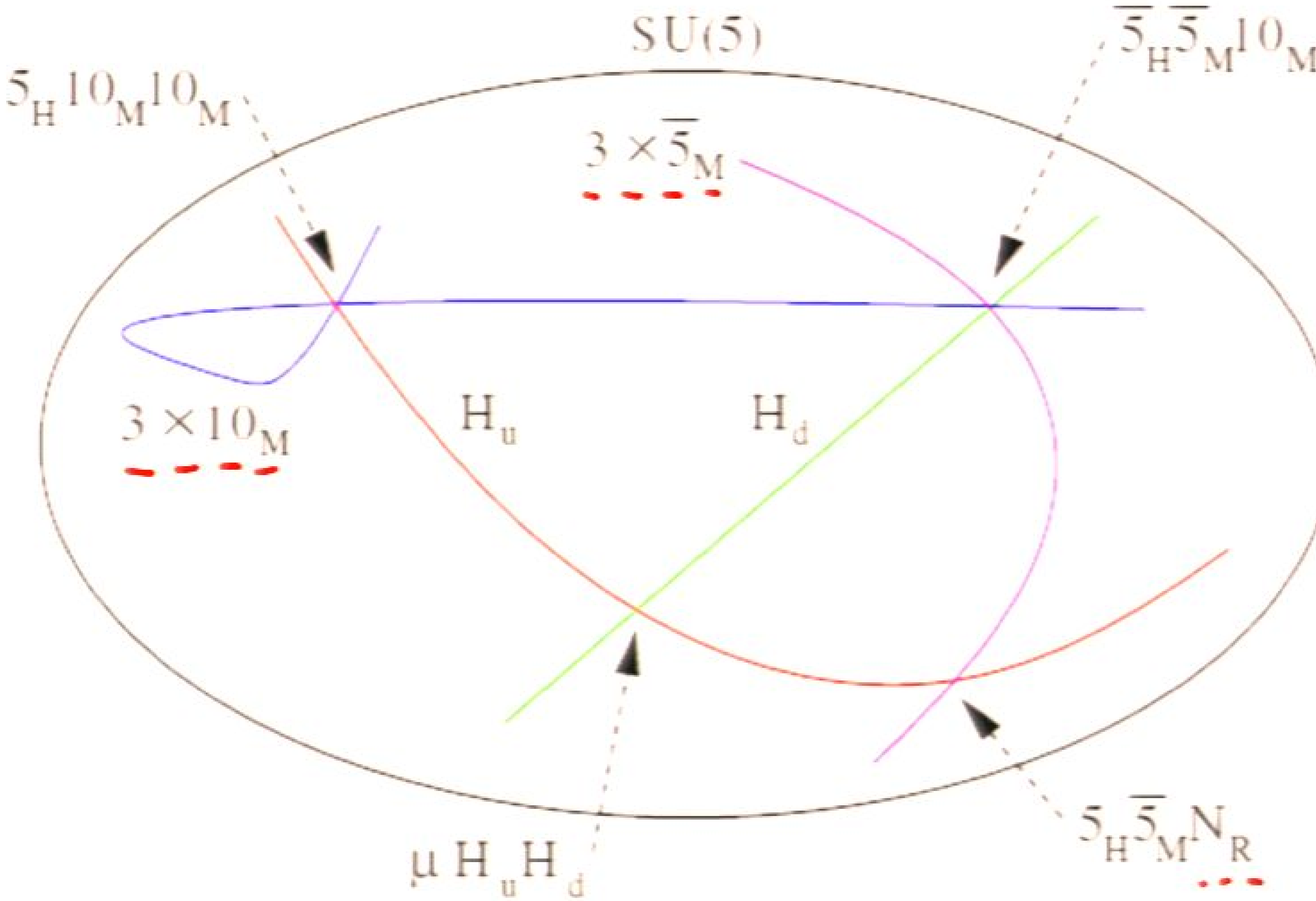
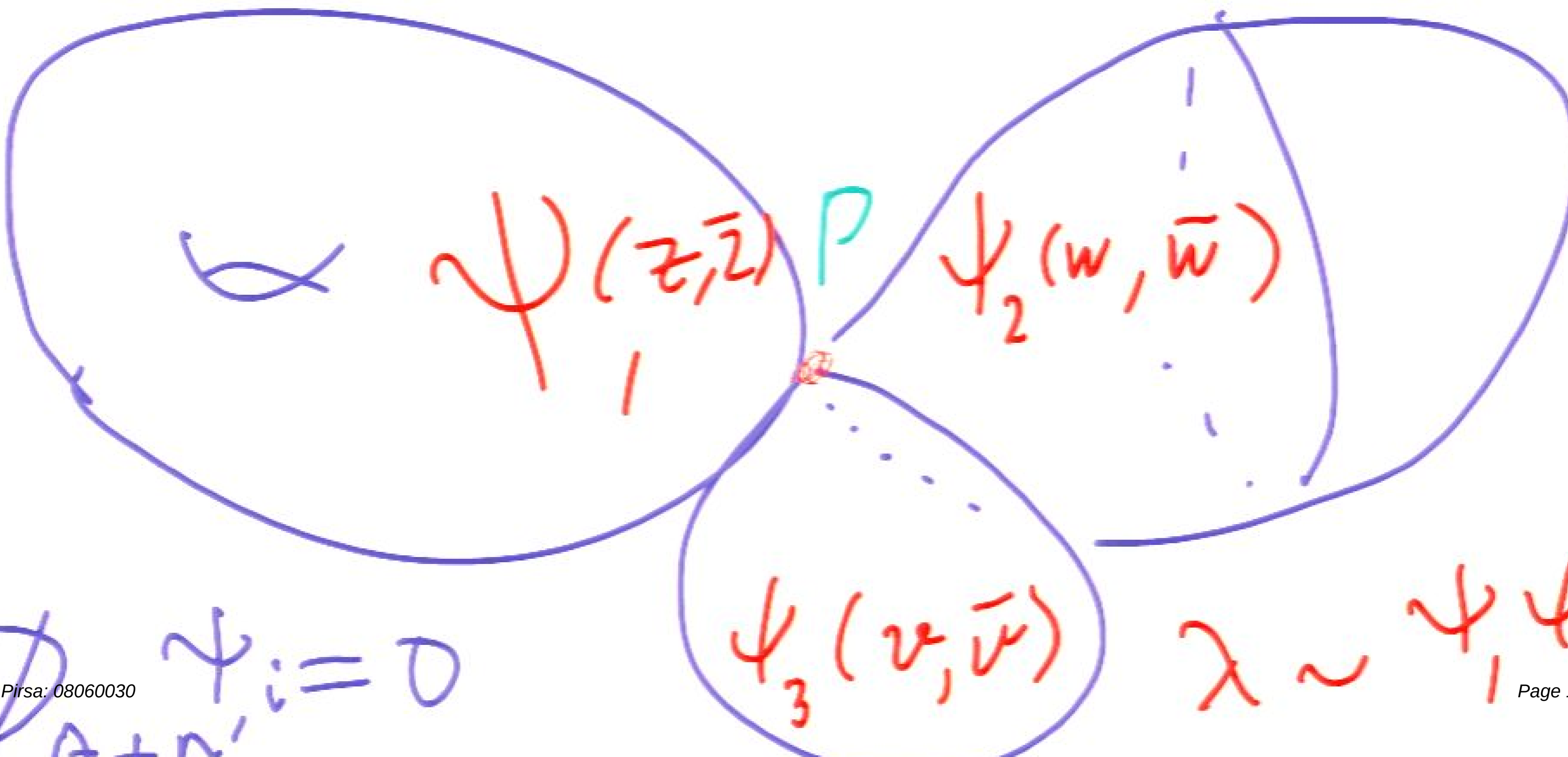


Note that the number of 4d matters in a given representation coming from a matter curve is dictated by the zero modes of the Dirac operator coupled to gauge fields living on the branes. Moeover the corresponding Yukawa couplings come from the product of the zero mode wave functions on the intersection point:



$$D_{A+A'}\psi_i = 0$$

$$\lambda \sim \psi_1\psi_2\psi_3|$$

Resolution of Puzzles

How could it be that light Higgs multiplets in 4d have no light triplet partners
How could the 5→2 without a trace for the light triplet 3, where at the same time the matter multiplets come in complete representations?

Answer: The observed 4d light modes correspond to zero modes of the Dirac operator coupled to internal gauge fields. If the net hypercharge flux on the matter curves do not vanish, the zero modes (the light particles) do not form complete multiplets. Thus the Higgs is coming from a curve where there is non-trivial hypercharge flux, and the matter multiplets come from curves with zero net flux.

Why is it that the mass of the particles in a given generation do not satisfy the relations obtained from GUT, except for the heavy third generation (leading to $m_b = m_\tau$ at the GUT scale)?

Answer: The matter multiplet couples to U(1) hypercharge flux. Even though the net U(1) flux on the curve vanishes, there is no reason (and one can show it is not true) that the B-field of the U(1) hypercharge flux is identically zero on the curve. Therefore different elements of the multiplet which have differenet hypercharges will experience different amounts of decoherence due to Aharanov-Bohm phase shifts and this affects the mass relations, because the Yukawa couplings come from the product of the wave functions which now have no reason to be the same in a given multiplet. The smaller the size of th matter curve, one can show the smaller the B-field has to be, and thus the mass relations would be better satisfied for smaller curves. On the other hand the Yukawa couplings which dictate the masses are inversely proportional to the size of the curve, and thus the smaller the curve the more massive. Thus we expect the most massive generation to best satisfy the mass relation anticipated in the naïve GUT models.

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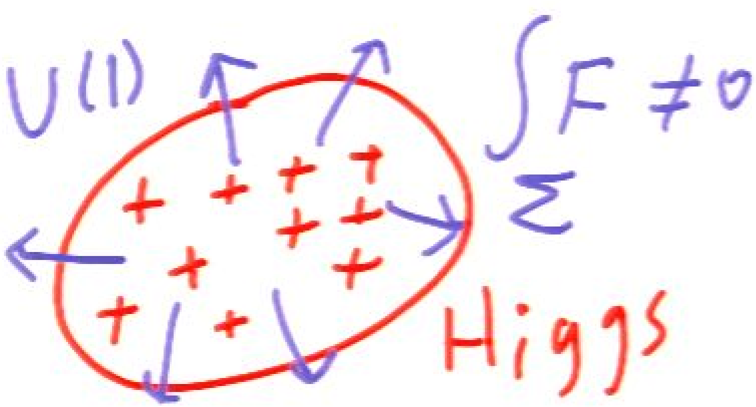

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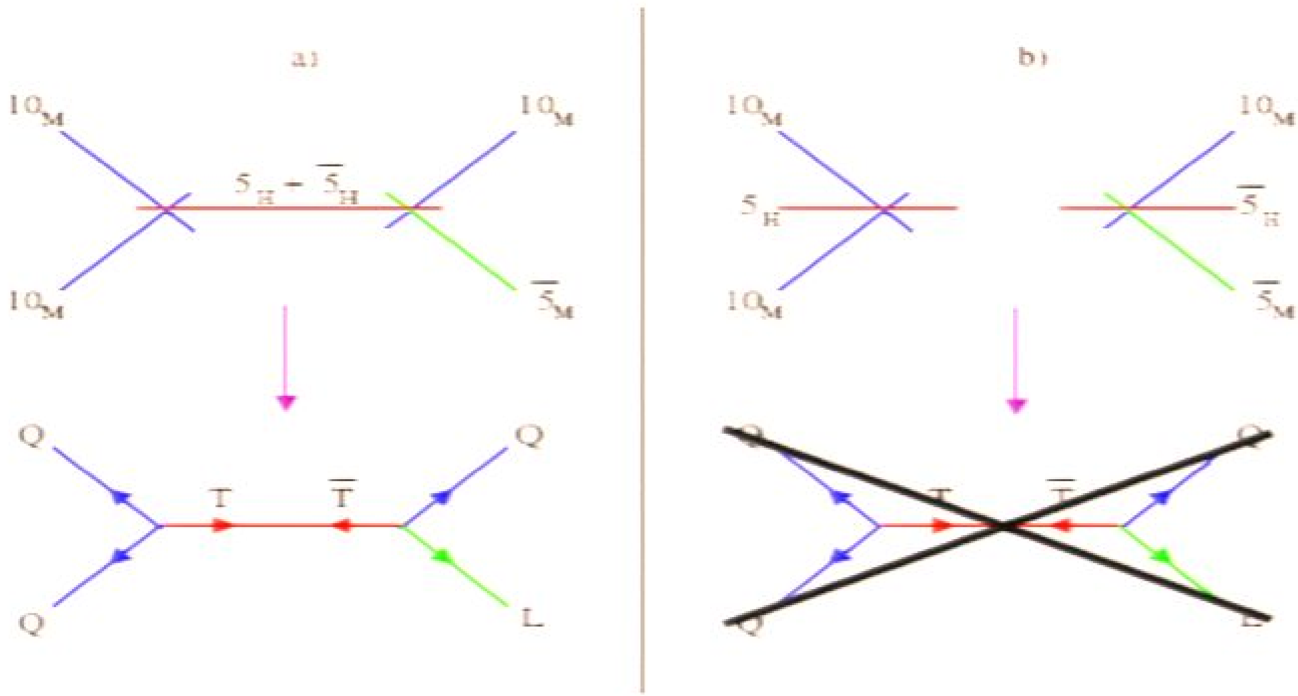
How is the μ problem solved, in other words, why is the Higgs field so light compared to the GUT scale? How to avoid the dangerous Baryon violating terms?

Answer: If the H_u and H_d matter come from distinct curves the dangerous dimension 5 baryon violating terms can be avoided.



This separation can also be used to solve the μ problem in two potential ways:

A) The H_u and H_d curves simply do not meet and thus there is no Yukawa coupling that could give it mass, and thus they are massless. One can show this is protected from receiving quantum corrections. In such a scenario supersymmetry breaking would have to induce the μ term

B) The second scenario is more attractive because it can predict the weak scale due to the hierarchy between M_GUT and M_Planck! Suppose the H_u and H_d matter curves are on distinct curves that do meet. Then automatically there is a Yukawa coupling with a matter curve which is neutral relative to the GUT group and is normal to the del Pezzo. Thus the μ term would be related to the vev of the field it intersects and the Yukawa coupling.

$$\underset{}{\lambda} \; \underset{1}{\langle x \rangle} \; \underset{5}{H_u} \; \underset{\bar{5}}{H_d}$$

The mass scale for the vev $\langle x \rangle$ is naturally of the order of GUT-- one can show that it is of the order of

$$\langle x \rangle \sim M_{GUT} \cdot \left(\frac{M_{GUT}}{\alpha_{GUT} M_{P}} \right.$$

which is just slightly below the GUT scale. The question then is what the Yukawa coupling is. This is proportional to the product of the wave function at the intersection point. But this can be naturally small by the following reason:

Since the del Pezzo is positively curved (with curvature $R \sim (M^2_{GU}$ then the Ricci flatness of the Calabi-Yau demands that the normal Directions to it be highly negatively curved $R \sim -M^2_{G.U.T.}$ Thus when we solve for all the singlet matter wave functions on the normal Matter curves, they hare naturally suppressed:

$$\partial \bar{\partial} \psi \pm c R \psi = 0$$

where both signs can occur depending on the details of the model. This leads to solutions of the form

$$\psi = e^{\pm c R |z|^2} \longleftarrow \text{Landau}$$

So the wave function is attracted or repelled from our brane

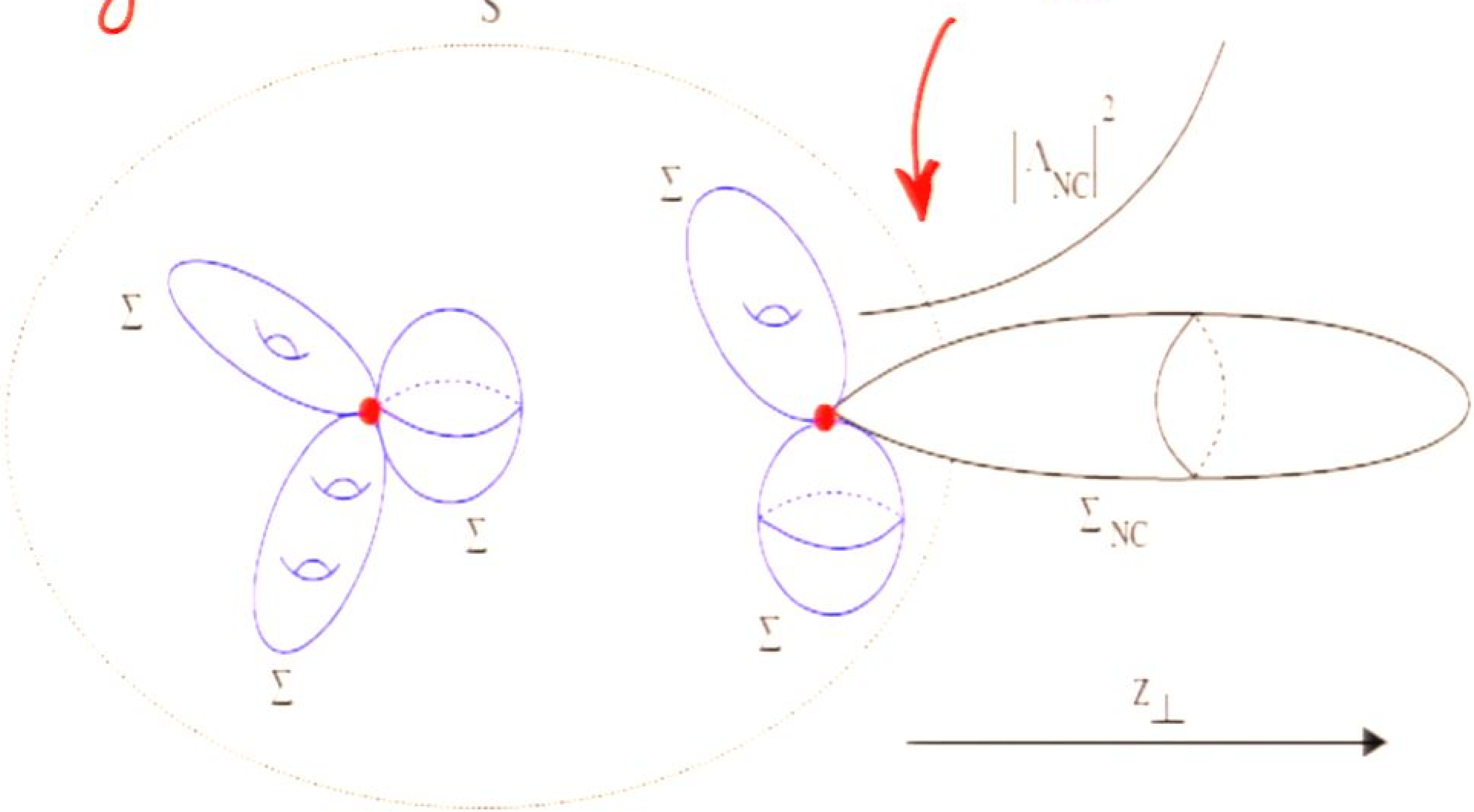
Note that this attraction/repulsion uses the fact that our brane is very special: The very assumption that it is decoupled from the rest of the bulk implies that it is highly positively curved. In the case of repelling this naturally leads to hierarchy because the wave function is repelled from our brane, leading to very small Yukawa couplings:

$$\psi \sim \exp(c M_{GUT}^2 R^2$$

$$\mu \sim M_{GUT} \cdot \varepsilon^{2\gamma} \cdot \exp\left(-1/\varepsilon^{2\gamma}\right)$$

$$\varepsilon \sim M_{GUT} / M_{Pl} \sim 0.07; \quad \frac{1}{3} < \gamma < 1$$

singlet wave function suppression



What masses does one get for neutrinos?

Answer: There are two possible mechanisms, One is based on the Same mechanism employed to explain the hierarchically small μ term: Namely since the right handed neutrinos are on the normal matter curve we have an exponential suppression term for the wave function leading to a small Yukawa coupling for

Dirac mass $\quad \epsilon$ H_u L N $\longrightarrow m_\nu^D = \epsilon \langle H_u$

Note that this leads to a Dirac mass for the neutrino which can be estimated as follows: Assuming the suppression is the same as that for the μ term

ϵ <X> H_u H_d $\longrightarrow \mu = \epsilon \langle X$

naive seesaw

$$m_\nu^D = \mu \cdot \frac{\langle H_u \rangle}{\langle X \rangle}$$

$$m_\nu^D = \frac{\mu \langle H_u \rangle}{M_{GUT}} \cdot \alpha$$

$$\langle X \rangle = M_{GUT}^2 / \ldots M_{Pl}$$

In the second scenario, we can assume that the wave function for the right handed neutrino is not repelled, but attracted to our brane. In this case, we get a modified seesaw type mechanism:

$$m_\nu = \alpha_{GUT}^{3/4} \begin{pmatrix} 0 & M_W \\ M_W & M_{GUT} \cdot \varepsilon^{4\gamma} \end{pmatrix}$$

$\frac{1}{3} < \gamma < 1$

$\varepsilon = \frac{M_{GUT}}{\alpha_{GUT} M_p} \sim 0.07$

$$\Rightarrow \begin{cases} m_\nu^{light} \simeq 0.2 \times 10^{\pm 1.5}\ eV \\ m_\nu^{heavy} \simeq 3 \times 10^{12 \pm 1.5}\ GeV \end{cases}$$

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