

Title: Exploring the bottom of the CDM hierarchy using scale-free simulations

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URL: <http://pirsa.org/08060020>

Abstract: Many numerical studies show that dark matter halos have a plethora of substructure, down to the smallest resolved scales. However, the very bottom of the Cold Dark Matter (CDM) hierarchy at a few earth masses, where the spectral index  $n$  approaches  $-3$  and structure begins to form simultaneously on a variety of scales, remains relatively unexplored. It is possible that the subhalo mass distribution, which appears to be described by a simple power-law down to mass scales  $10^6$  solar masses, remains unchanged and independent of scale and  $n$ . A few studies have indicated that this appears to be the case, which is surprising considering all other statistical indicators, such as the halo mass function, as well as the internal properties of halos, such as concentration, show a dependence on  $n$ . To explore the effect of the spectral index on the subhalo mass function we ran two large, scale-free simulations,  $P(k)=Ak^n$  with  $n=-1$  and  $-2.5$ . We find that the subhalo mass function does depend on the spectral index, with the power-law becoming shallower as  $n \rightarrow -3$ .

# Subhalo Mass Function in Scale-Free Cosmologies at Steep Spectral Indices



Pascal Elahi

PASCOS 2008

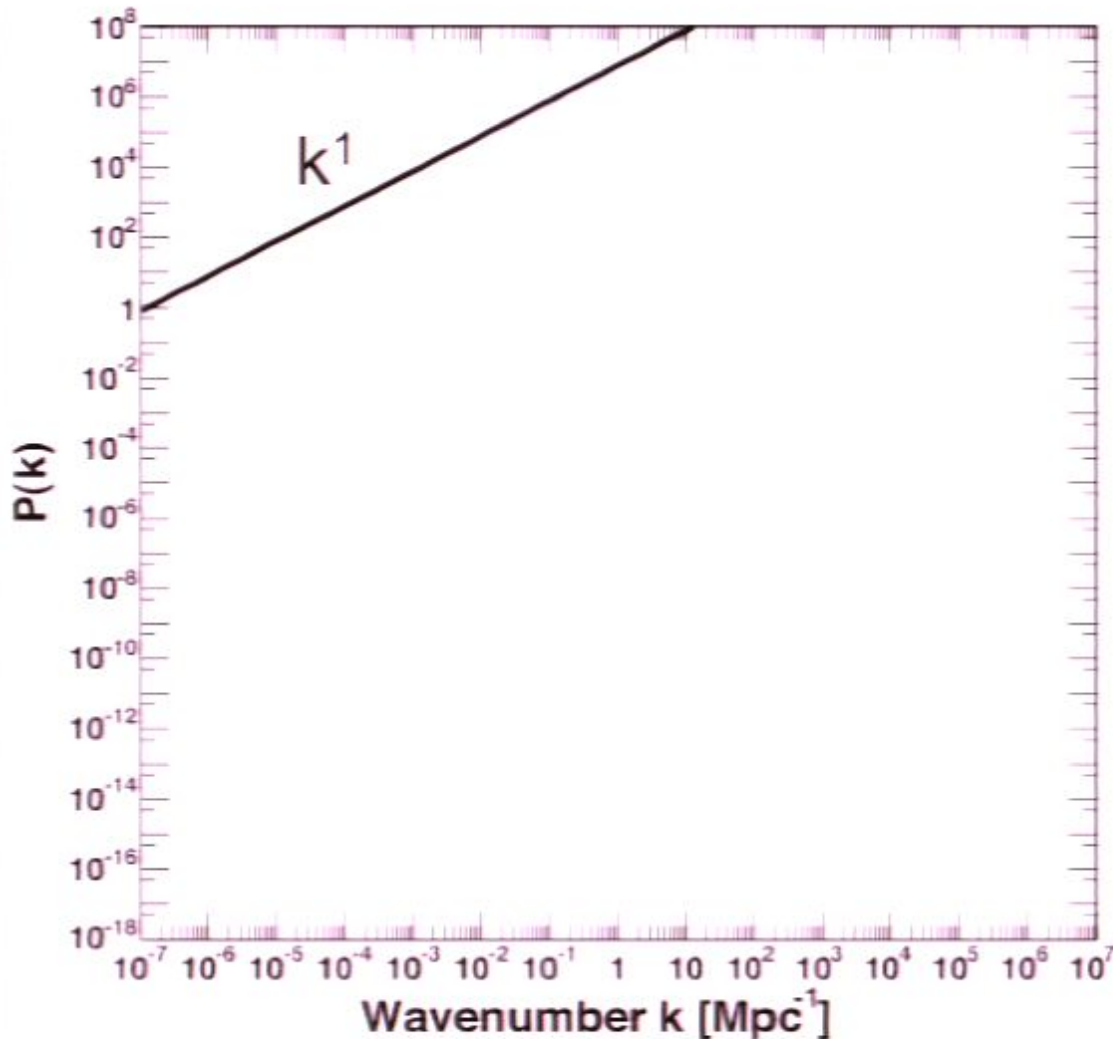
L. M. Widrow, R. J. Thacker,  
E. Scannapieco



# Outline

1. Structure Formation,  $\Lambda$ CDM & SUSY
2. Numerical Experiments: Why Scale-Free?
3. Results: Subhalo Mass Function & Self-Similarity

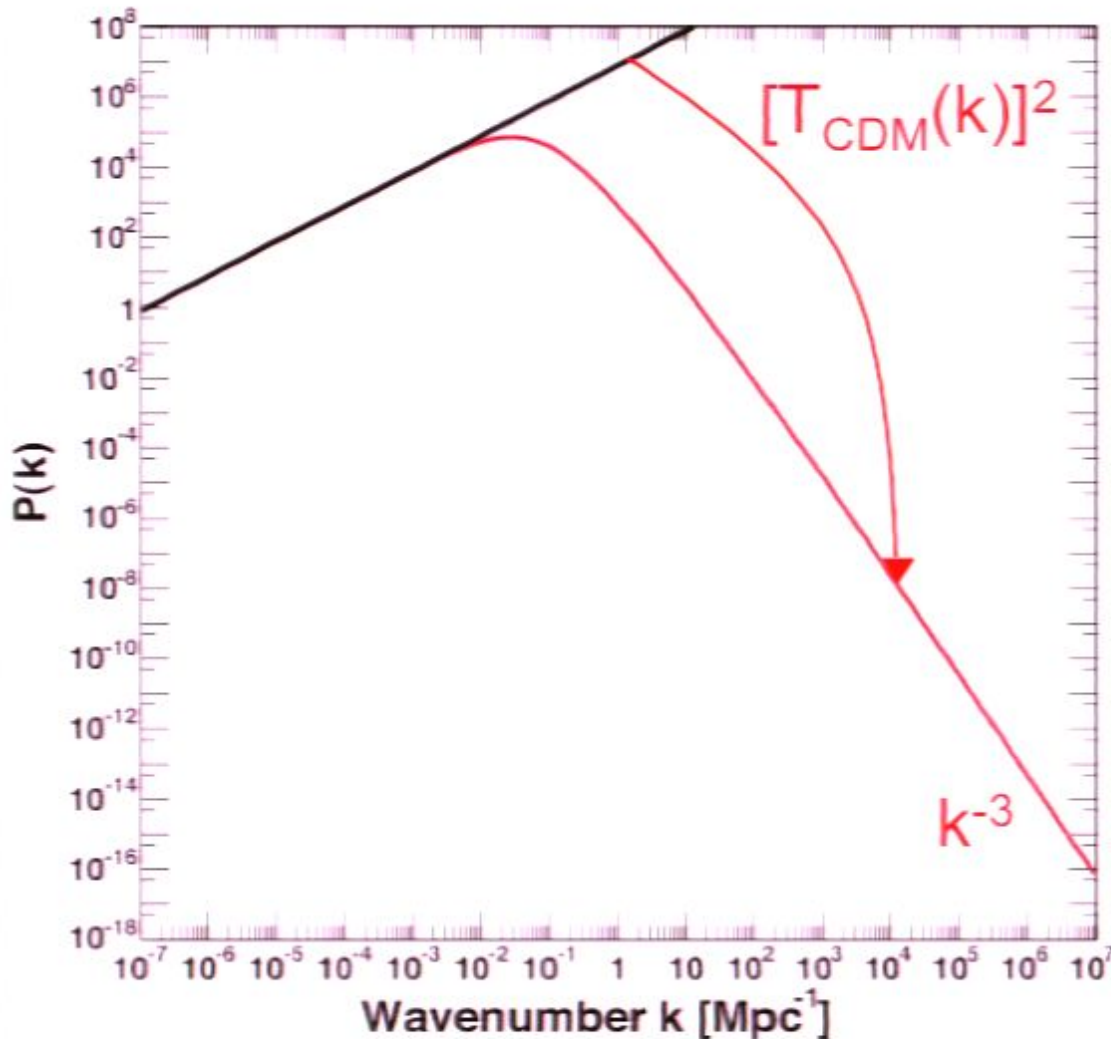
# Density Perturbations



- Inflation gives scale-free Gaussian density perturbations. Various models.
  - e.g. Guth (1998)

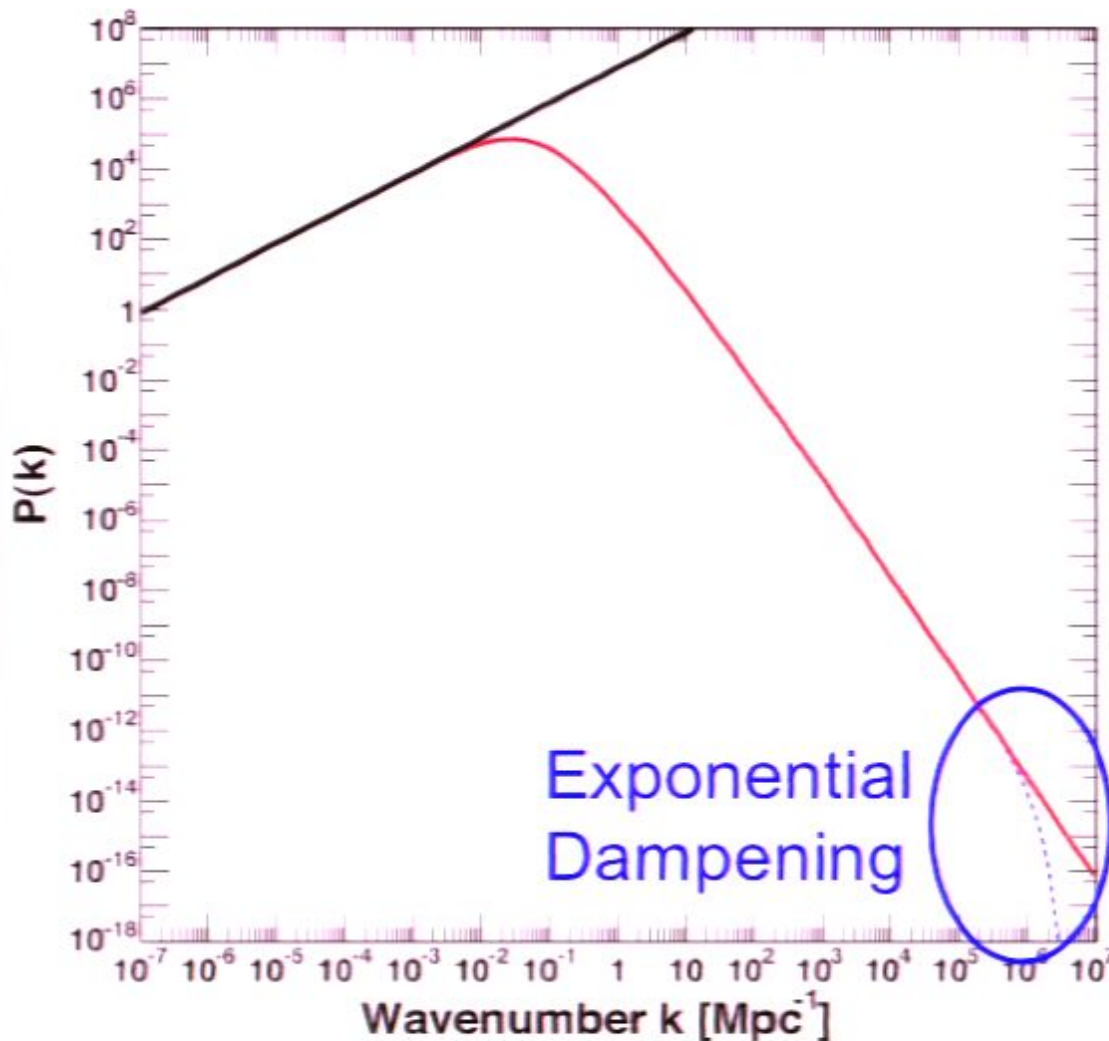
$$P(k) \propto \langle |\delta(k)|^2 \rangle$$

# Density Perturbations



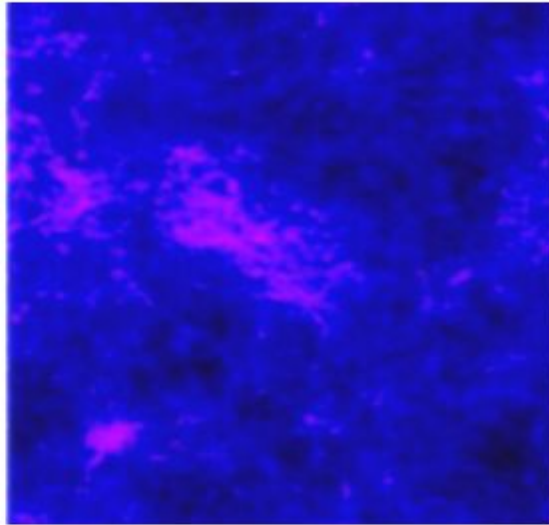
- Modes evolve upon entering horizon, dependent on density make up of universe. CDM dominated mode evolution given by transfer function  $T_{\text{cdm}}(k)$ .
  - e.g. Eisenstien & Hu (1998), (1999)

# Density Perturbations

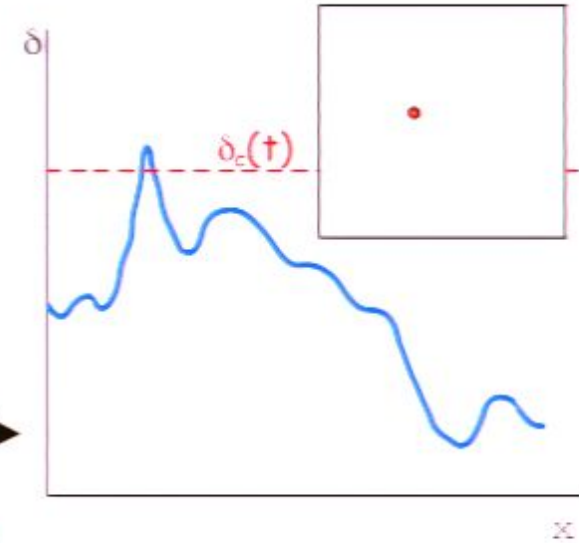


- CDM:  $\sigma_{\text{cross}}$  &  $m_{\text{WIMP}}$  govern scale of free-streaming/collisional-dampening, below which  $\delta$  washed out.
- **Bottom of CDM hierarchy.**
  - SUSY Neutralino  $m \sim 100$  GeV  $\rightarrow$  parsec ( $k \sim 10^6$  h/Mpc), earth mass ( $\sim 10^{-6} M_{\odot}$ ). **So galaxy is  $10^{18}$  times more massive!**

# & Structures



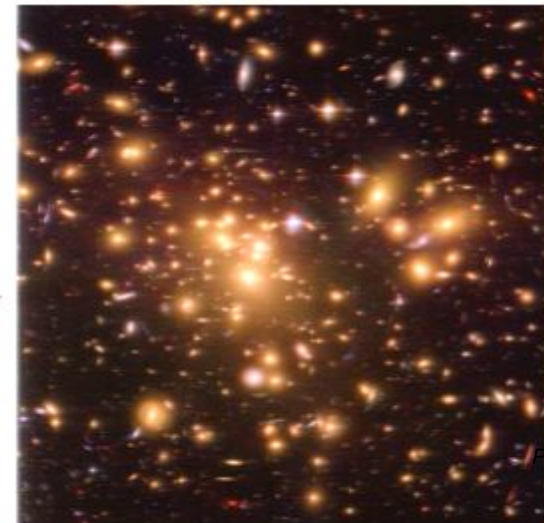
Density perturbations grow and collapse



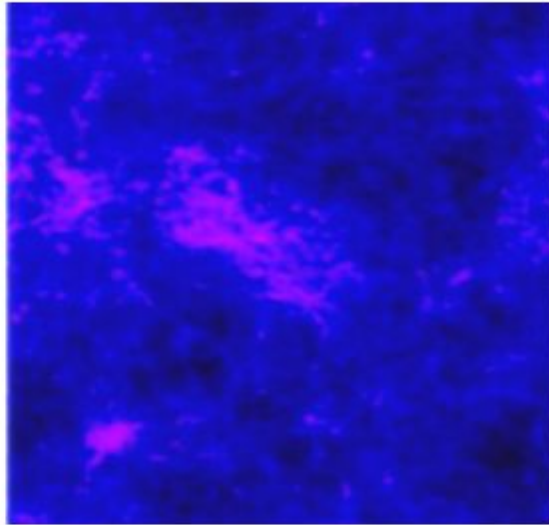
Form hierarchy of objects



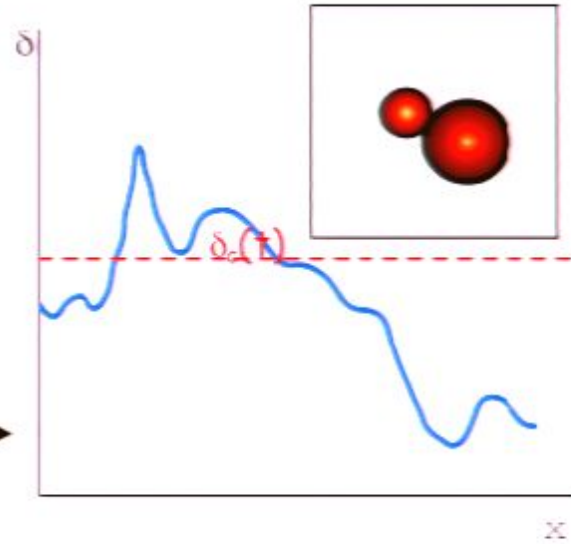
Add baryonic physics



# & Structures



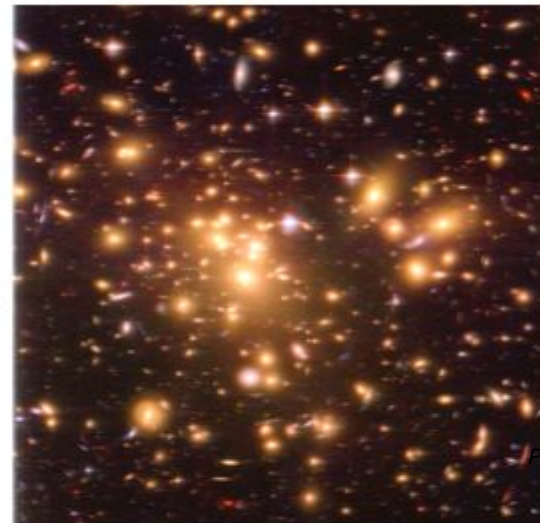
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Form hierarchy of objects

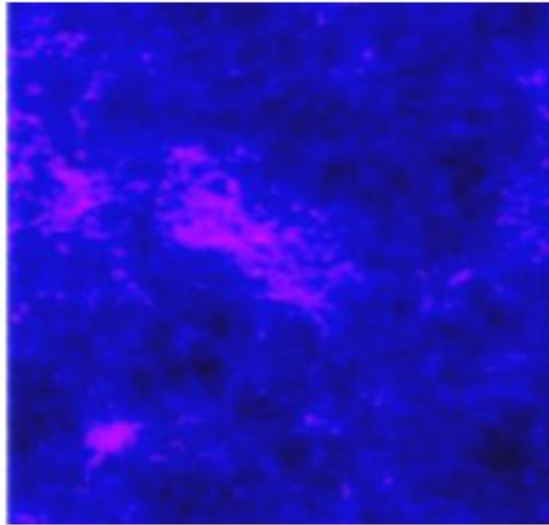


Add baryonic physics

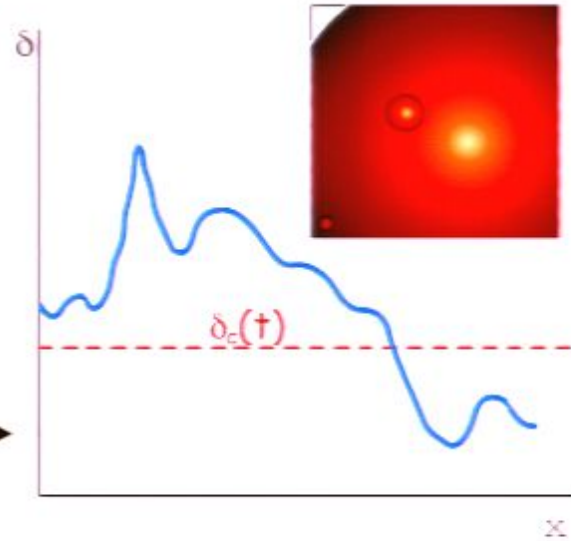




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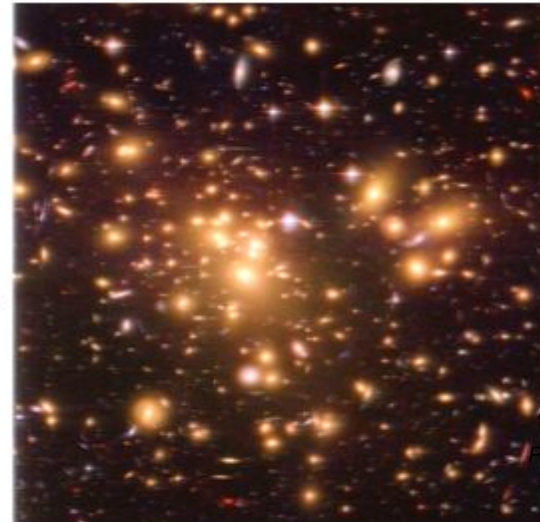
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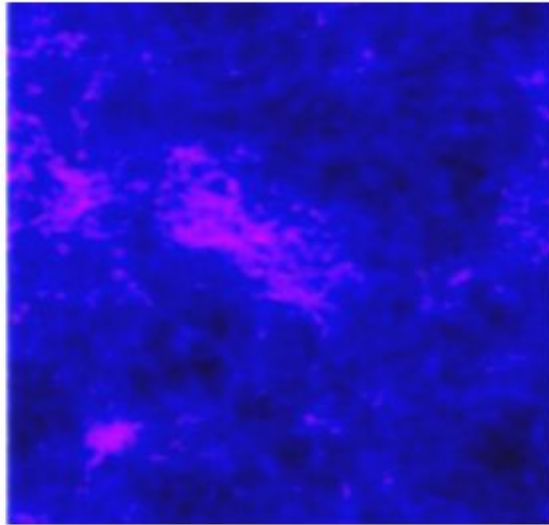
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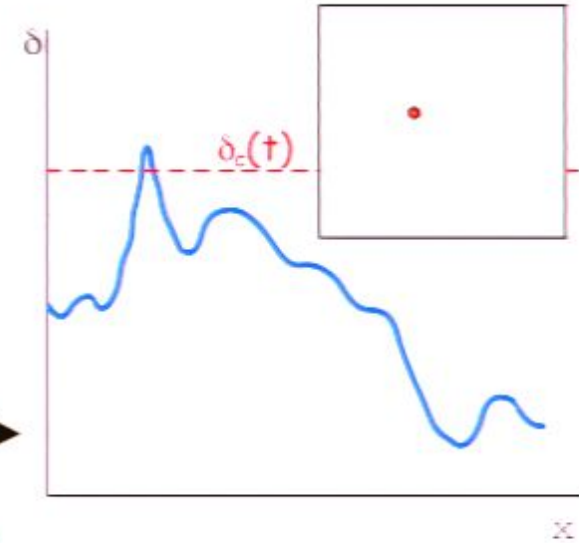
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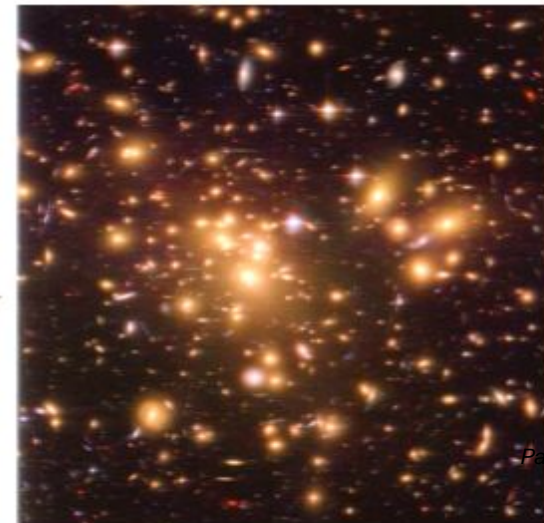
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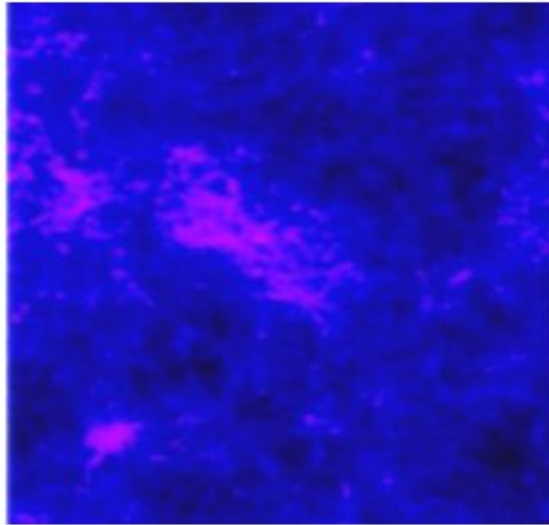
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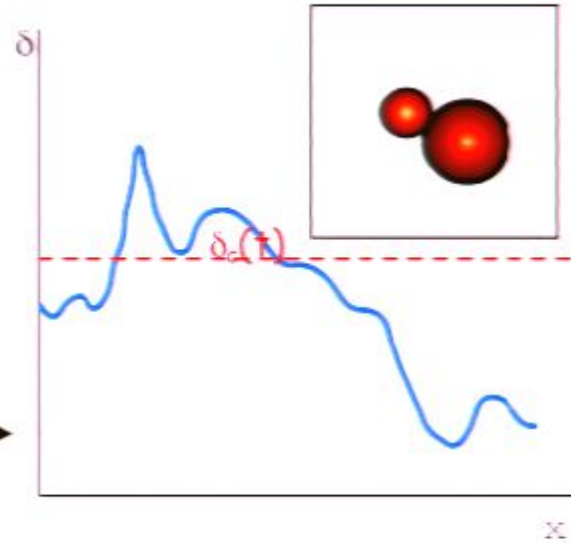
Add baryonic physics



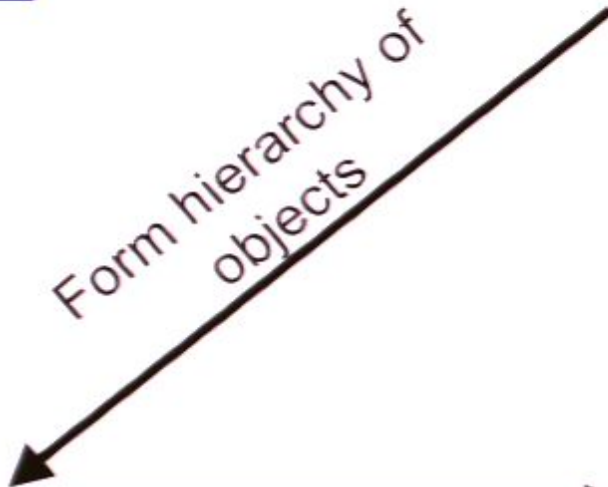
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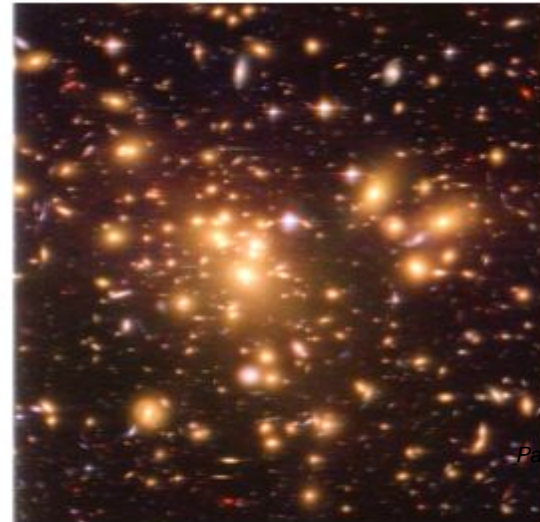
Density perturbations grow and collapse



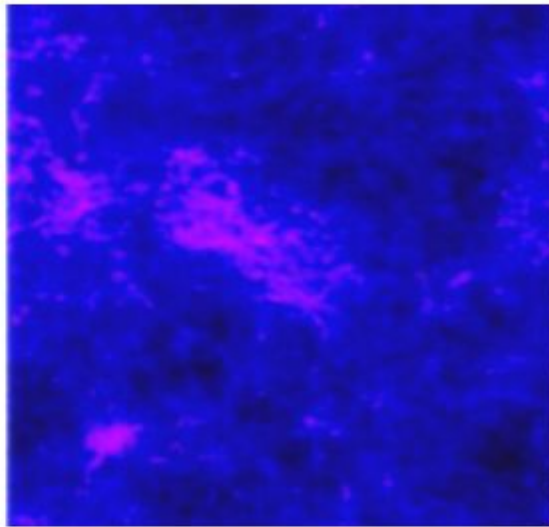
Form hierarchy of objects



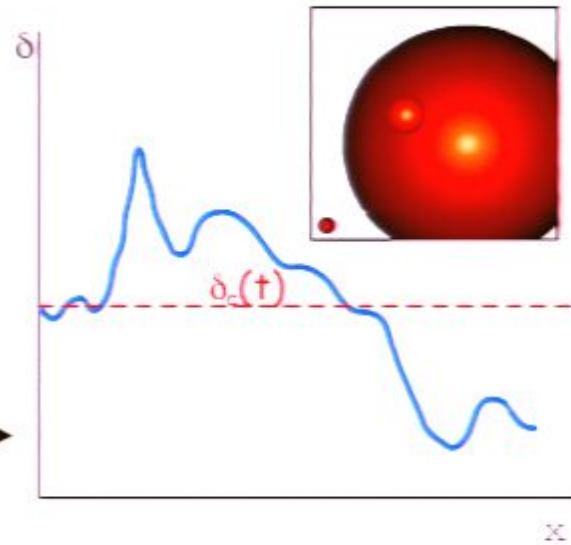
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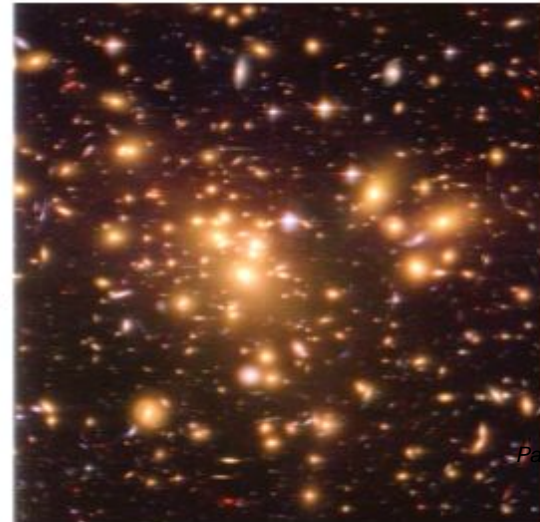
Density perturbations grow and collapse



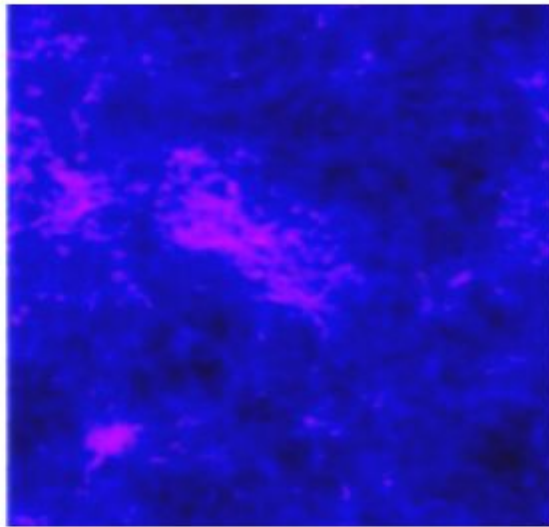
Form hierarchy of objects



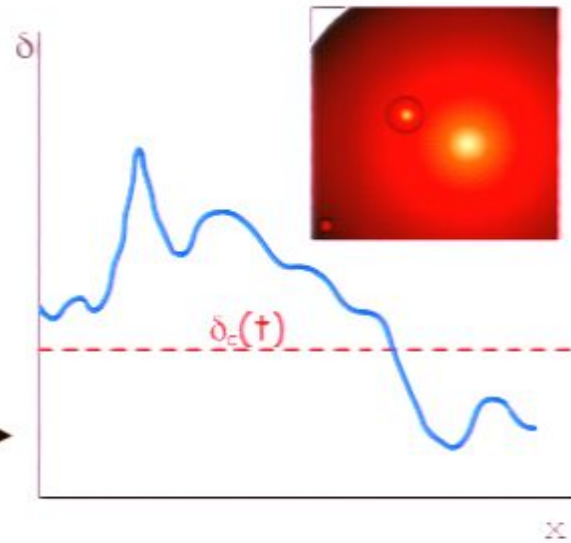
Add baryonic physics



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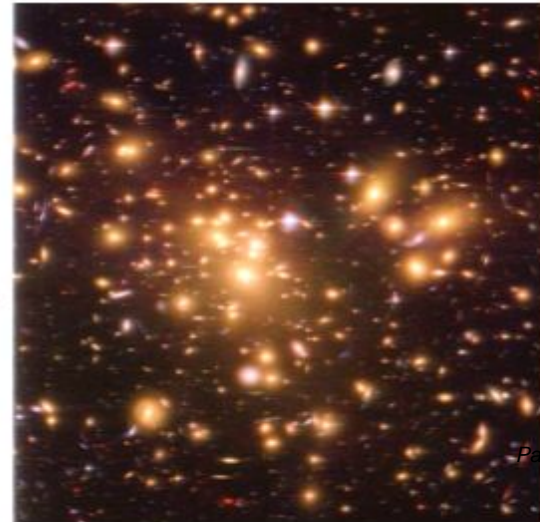
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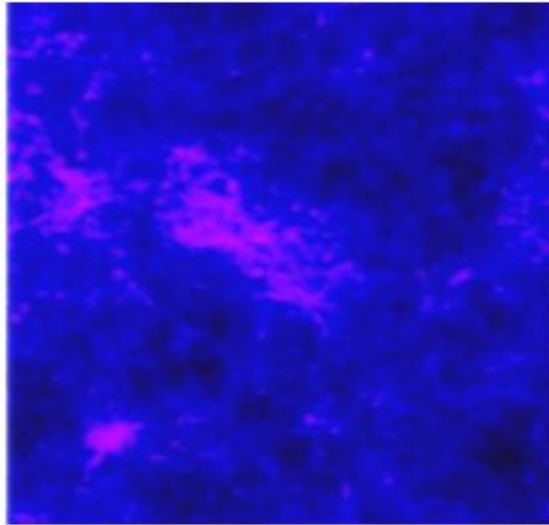
Form hierarchy of objects



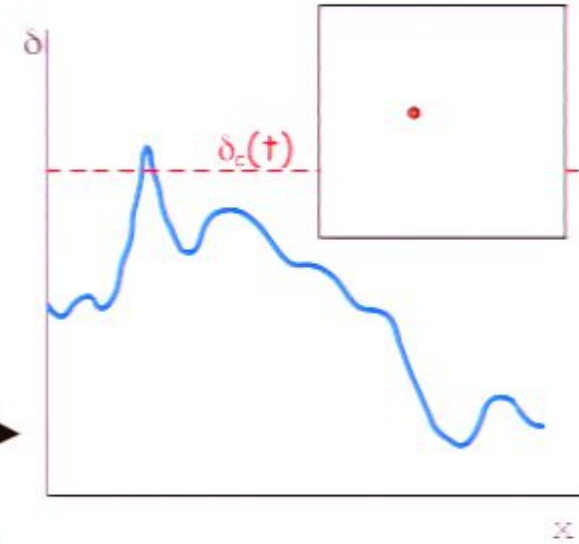
Add baryonic physics



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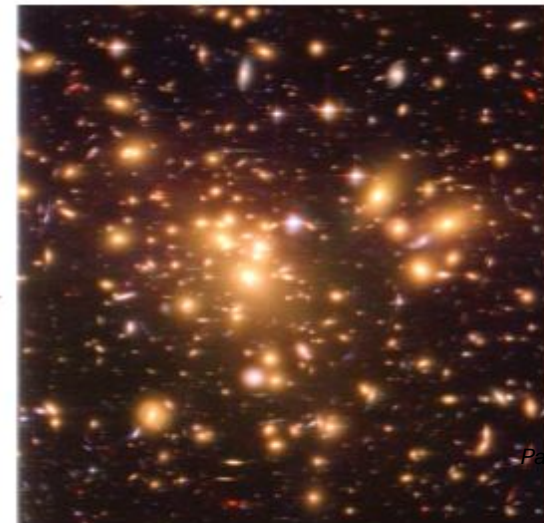
Density  
perturbations grow  
and collapse



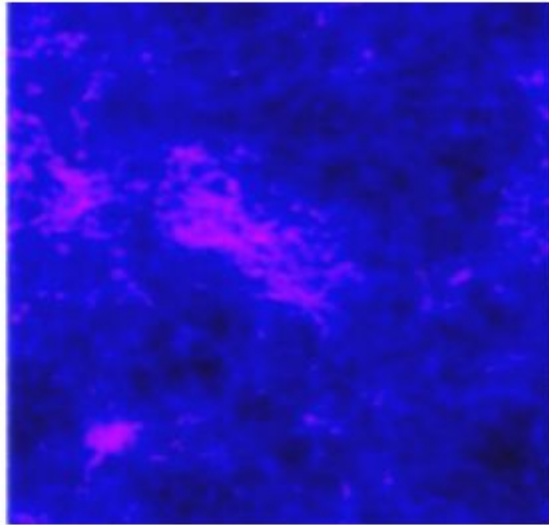
Form hierarchy of  
objects



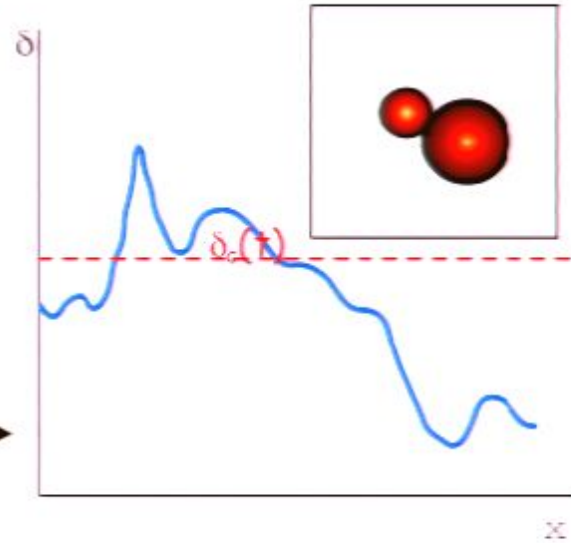
Add baryonic physics



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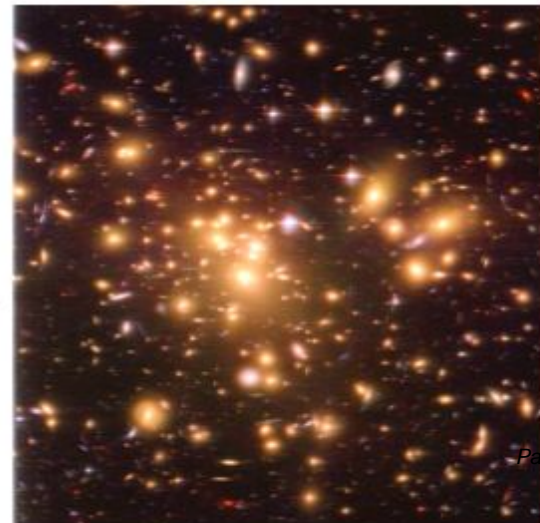
Density perturbations grow and collapse



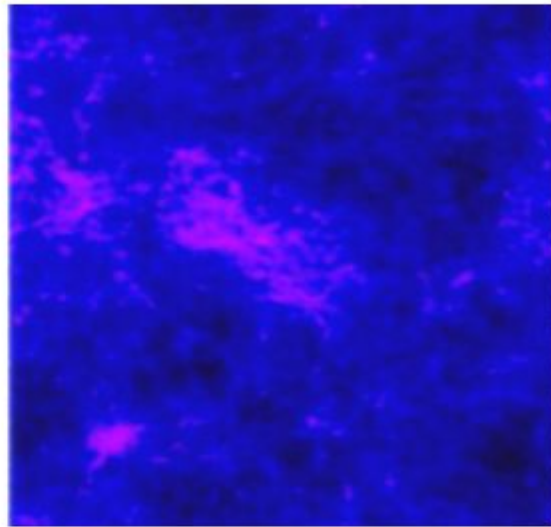
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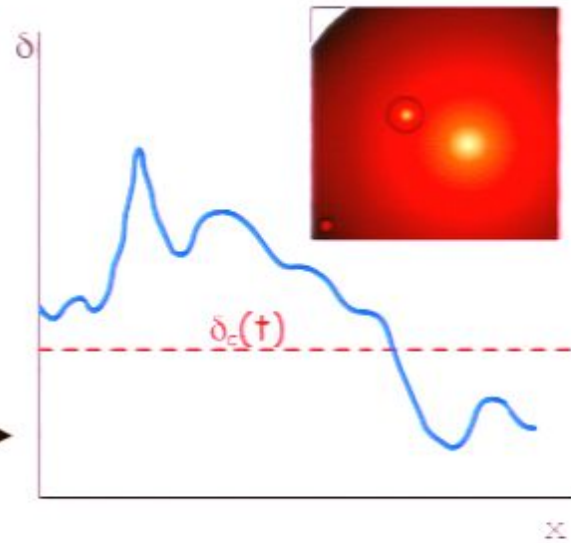
Add baryonic physics



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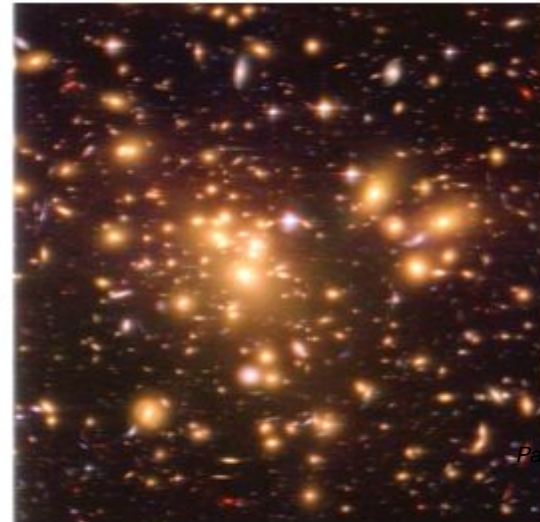
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Form hierarchy of objects

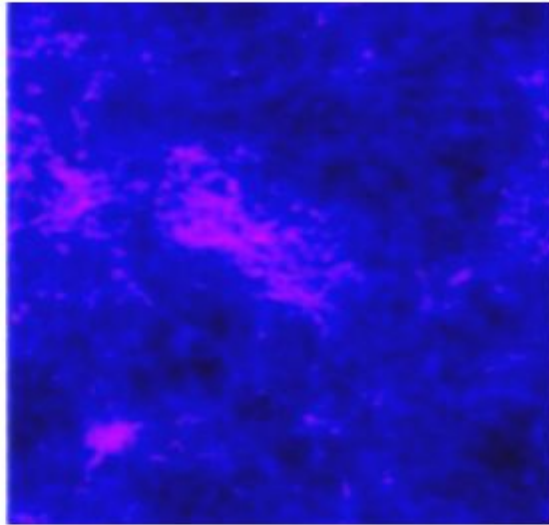


Add baryonic physics

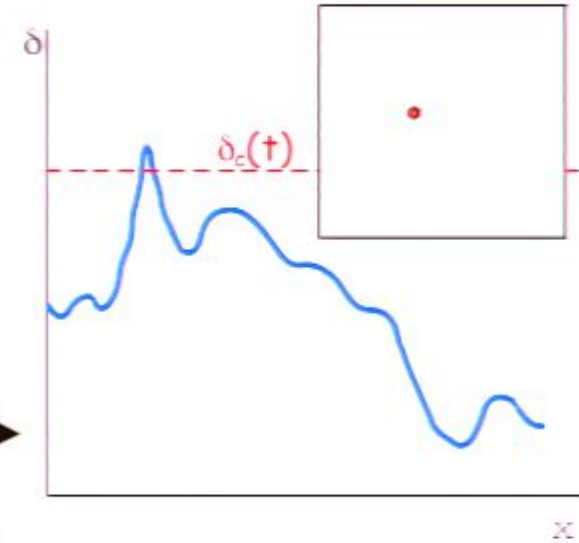




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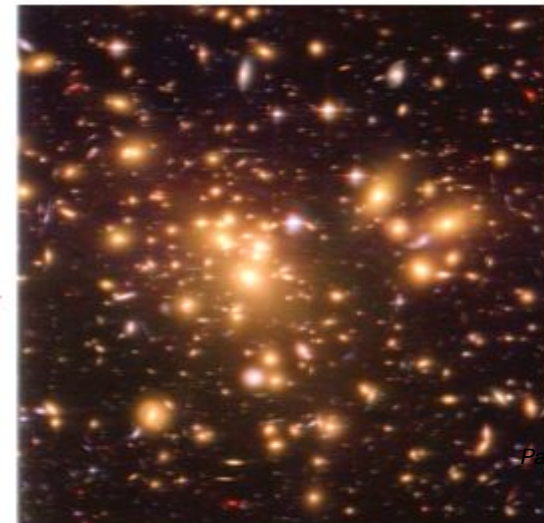
Density perturbations grow and collapse



Form hierarchy of objects



Add baryonic physics



# Why do we care about the bottom of the CDM hierarchy?

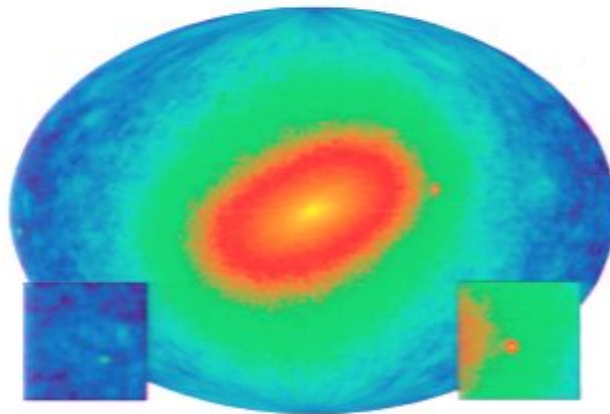
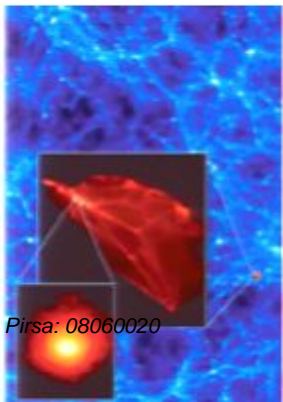
## DETECTION

### Clumpy → Indirect

- IF CDM self-annihilate, detect  $\gamma$ -rays or other decay products. Signal  $\propto \rho^2$ . More subhaloes, better signal. Density/energy distribution at small scales?

### GLAST, HESS, Icecube

- [e.g. Diemand, Khulen & Madau (2007); Bertone, Hooper & Silk (2005)]

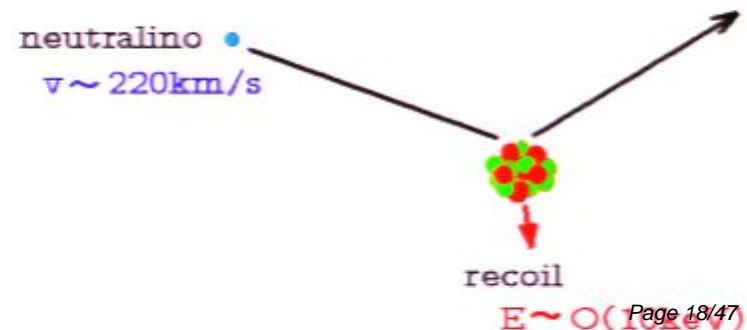


### Smooth → Direct

- IF SUSY WIMPS, nuclear recoil experiments could detect it. Remnants/tidal streams leave energy signature.

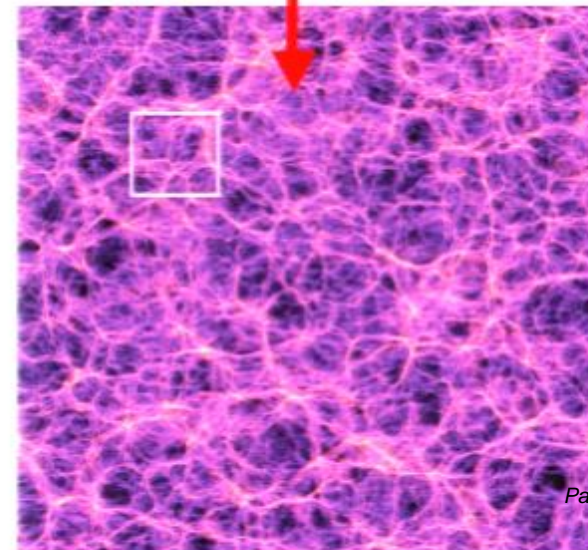
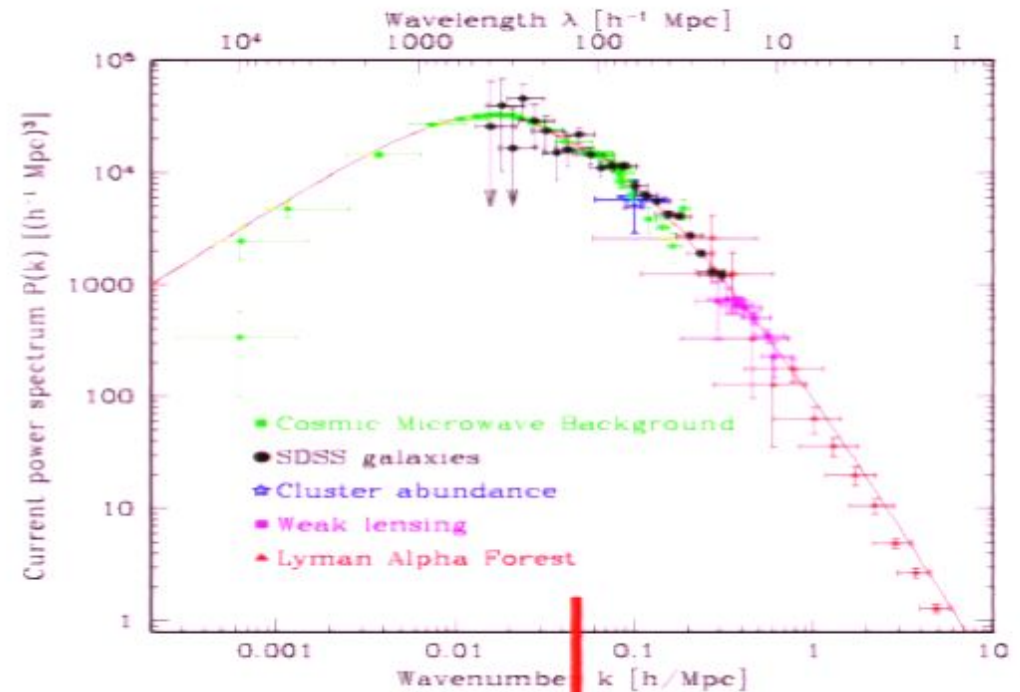
### DAMA, PICASO, etc

- [e.g. Stiff, Widrow & Frieman (2001) Green (2007), Kamionkowski & Koushiappas (2008)]



# Exploring Structure Formation

1. Inflation (or some IC), along with CDM physics gives  $P(k)$
2. “Quasilinear” regime: Zeldovich (ZA), second order perturbation theory (2LPT). Use to start simulation.
3. Nonlinear gravitational clustering: **Simulations**. BUT dynamic range issue. How to model  $10^8$  factors of scale,  $10^{24}$  factors of mass? What can save us? **Self-similarity**.



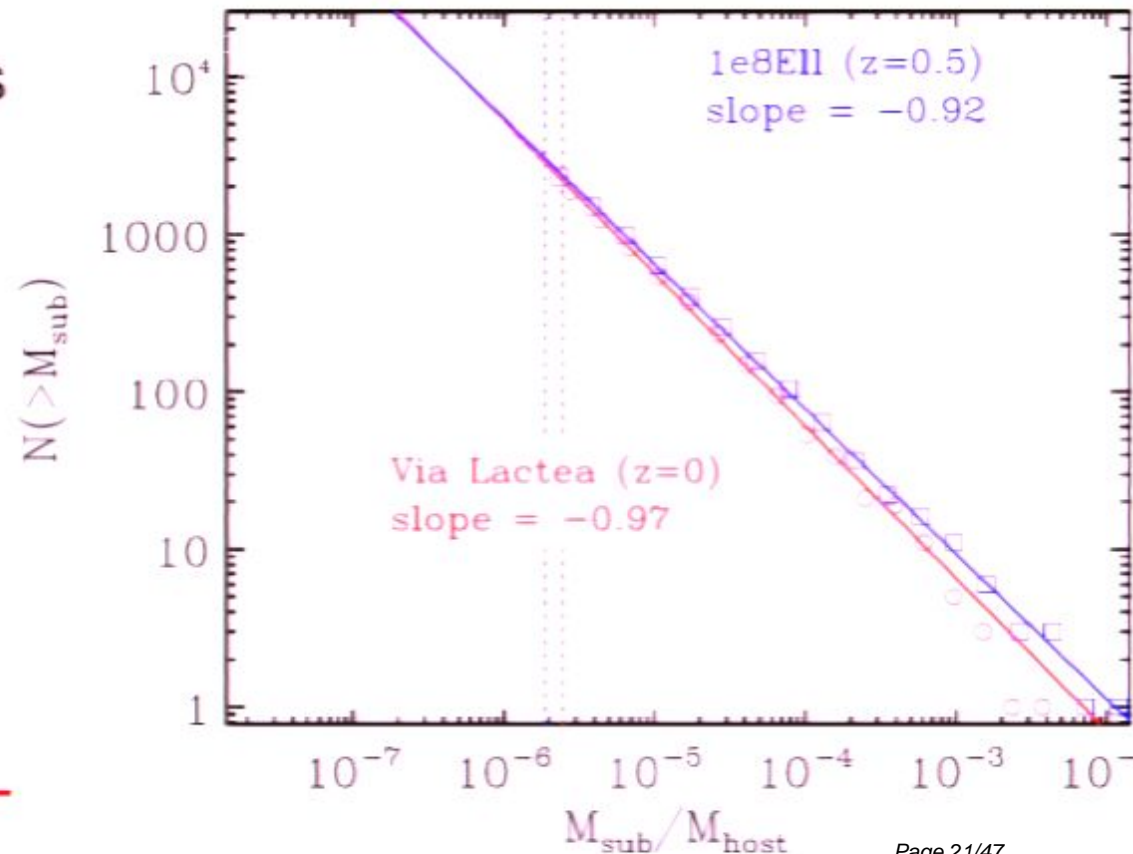
Cluster, Millennium Run  
(Springel et al. 2005)

MW, Via Lactea  
(Diemand, Kuhlen  
& Madau 2007)

3.9 Mpc/h

# Substructure: Self-Similar?

- Simulations show clusters like galaxies, subhaloes smaller versions of haloes
- cumulative subhalo mass function  
 $n(>M) \propto M^\alpha$ , ( $\alpha \approx -1$ )
  - (e.g. Moore *et al* 1999, Gao *et al* 2004, Diemand *et al* 2007, Madau *et al* 2008)
- Amplitude known to vary with peak height, so is mass function truly scale-free?



MW, Via Lactea  
(Diemand, Kuhlen  
& Madau 2007)

$0.1 M_{\odot}$ , SUSY  
(Diemand, Kuhlen  
& Madau 2006)

$z=75$   
 $r=0.10$  pc

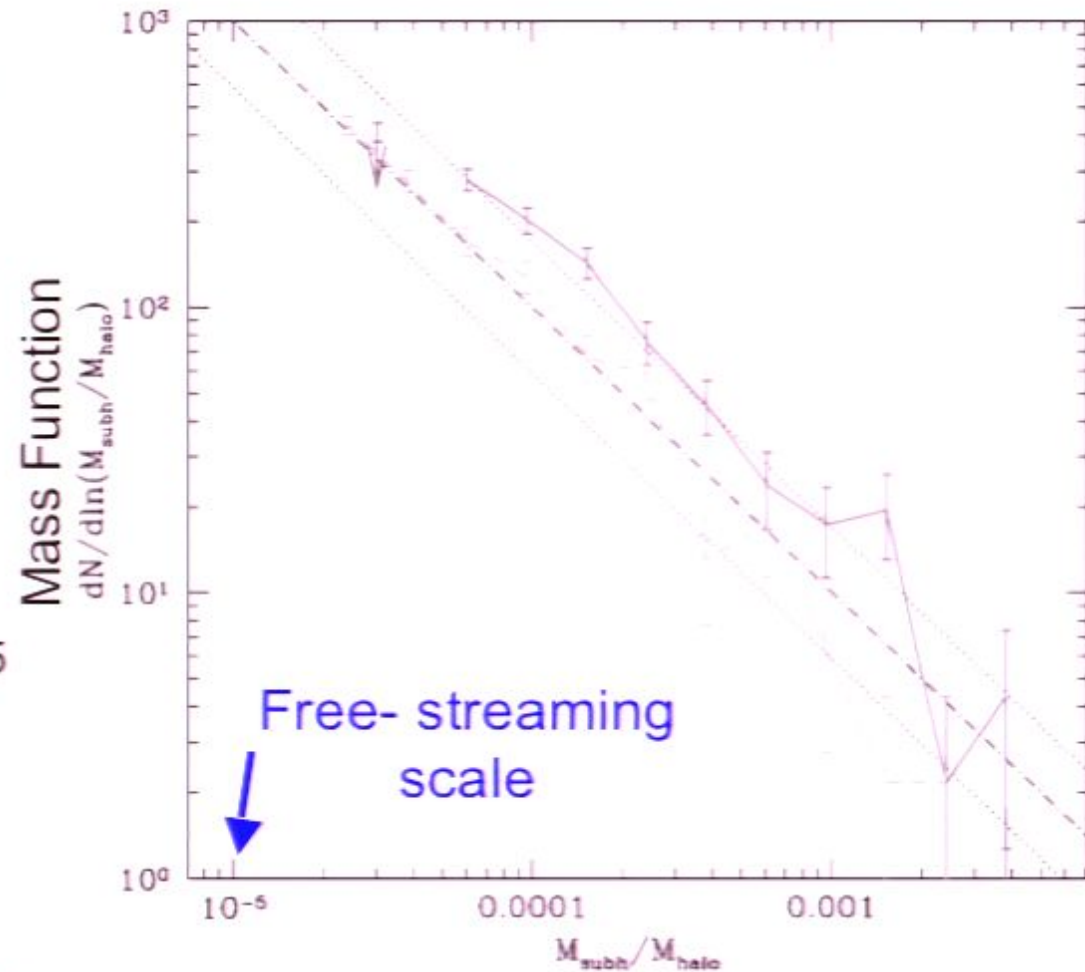
Still Self-Similar?



# High Redshift SUSY Sims

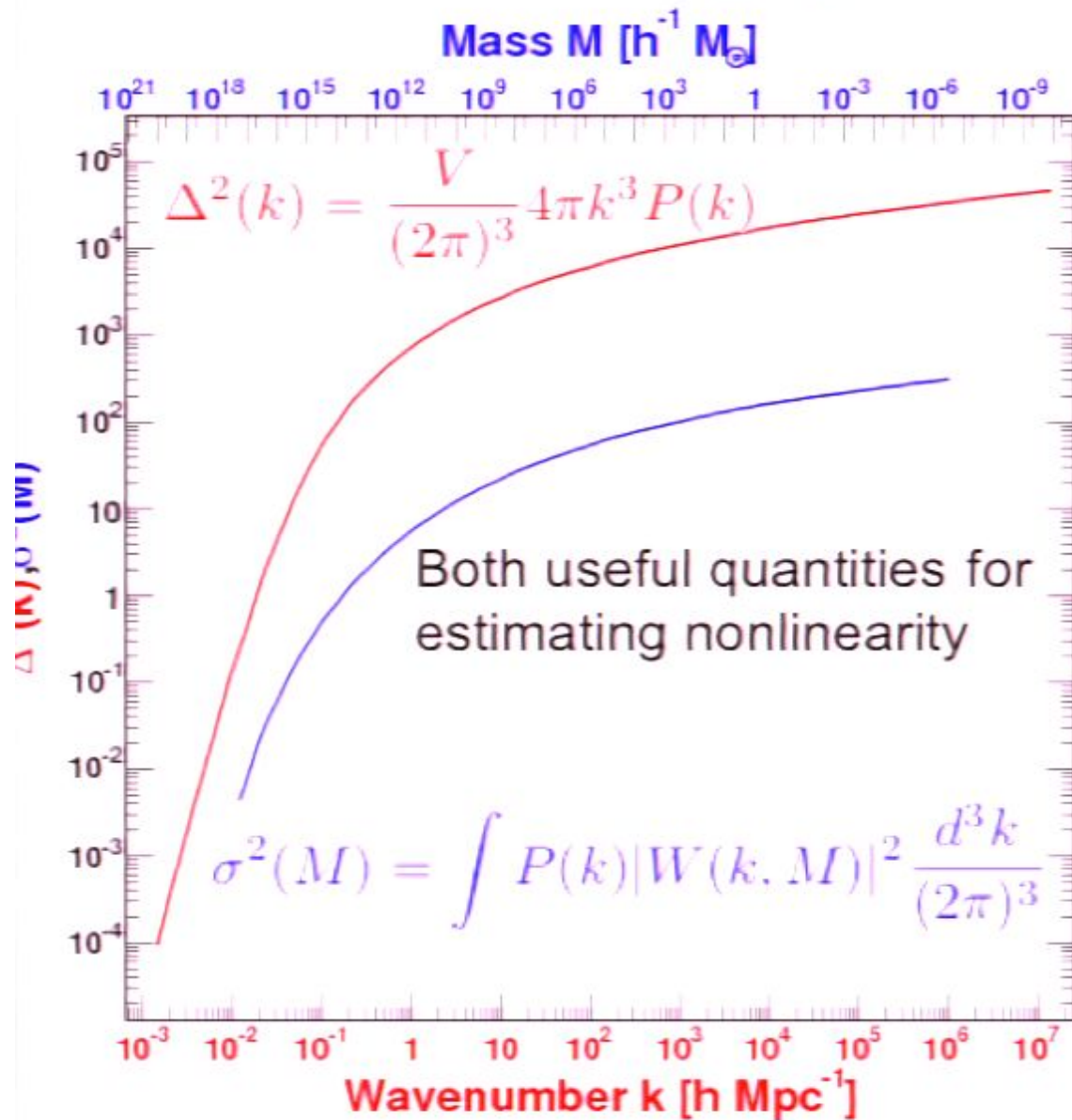
- Pure DM simulations,  $\Lambda$ CDM cosmology, high  $z$ , 3 kpc box size.
- Shows similar distribution, normalization uncertain. **BUT** sims look different and small volume so can we trust it?
- Can get estimates of  $\sim 10^{15}$  SUSY objects.

Side note: SUSY subhaloes susceptible to disruption by stars (?)  
e.g. Zhao et al (2007)

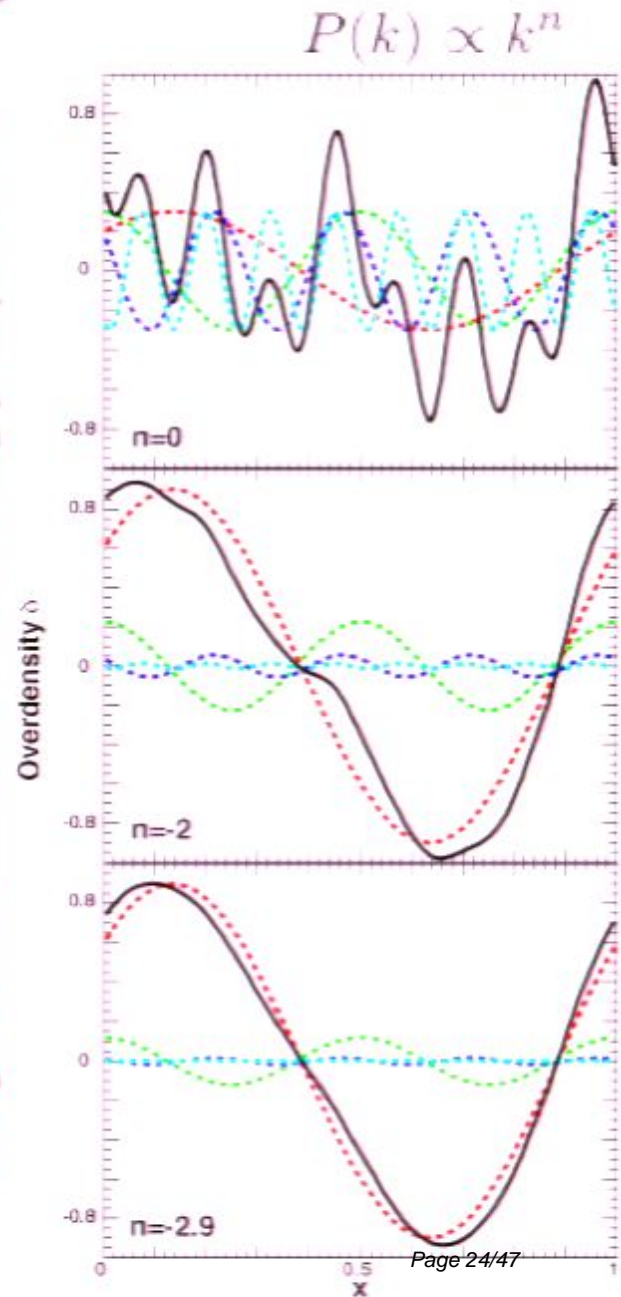


Diemand, Moore & Stadel (2005),  
Diemand, Khulen & Madau (2006)

# P(k) Revisited

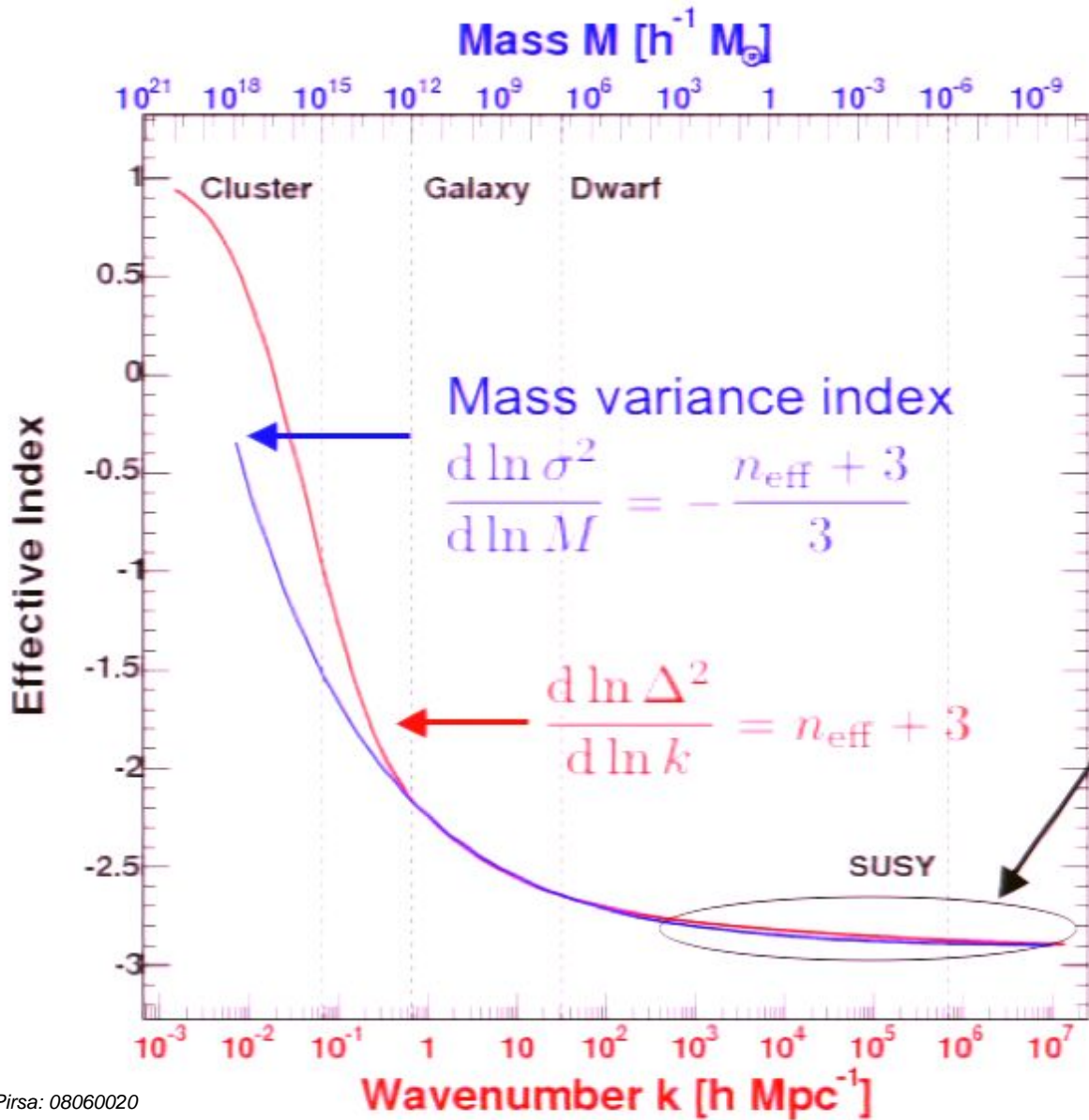


Large-scale modes increasingly important.





# Effective Index



Relative formation time:

$$a_1(t)/a_2(t) = (M_1/M_2)^{(n_{\text{eff}}+3)/3}$$

Small scales,  $n \rightarrow -3$ ,  
simultaneous collapse.  
Haloes: no time to virialize  
before accreted.

Would this leave imprint on  
subhalo mass distribution?

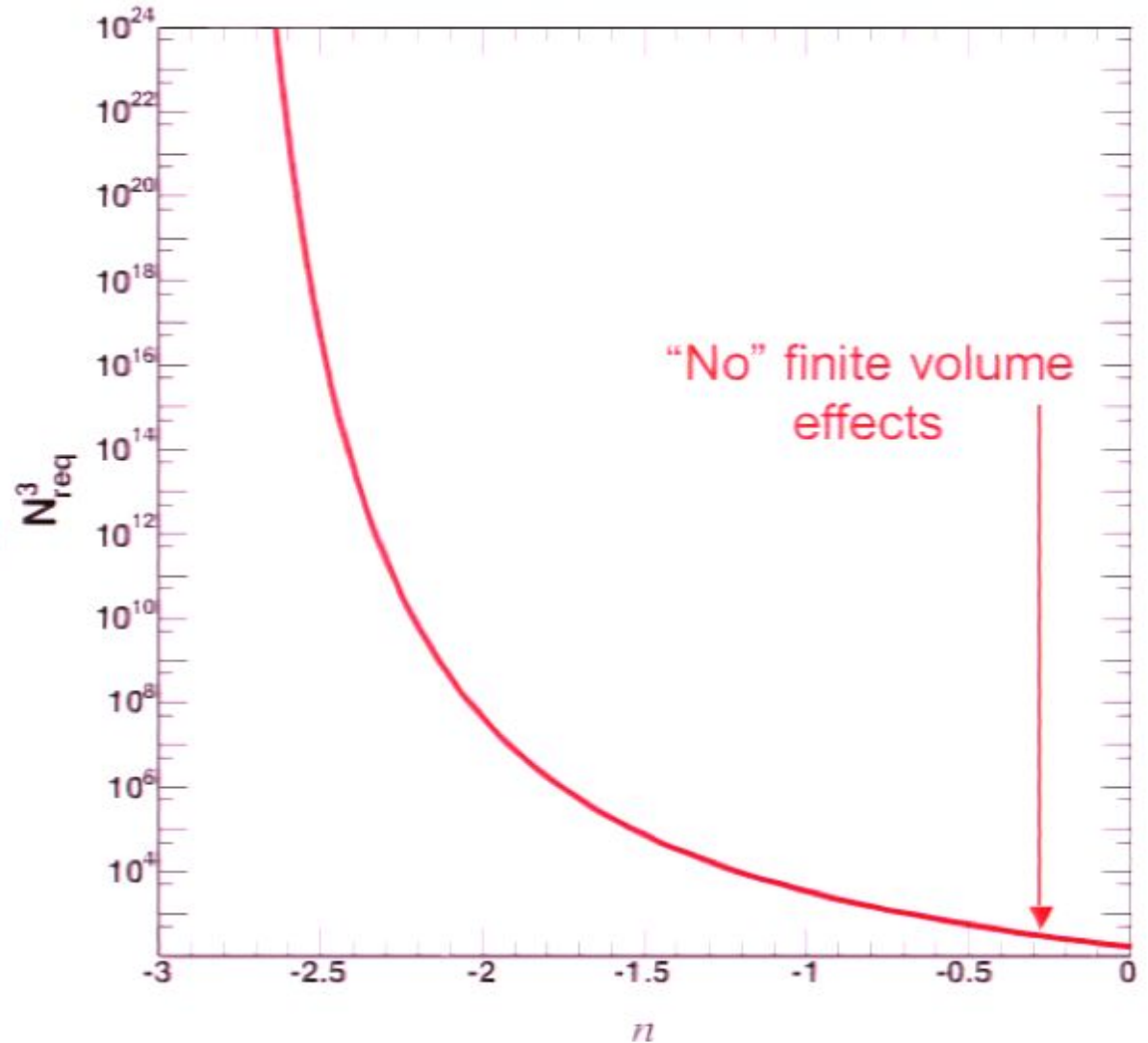
# Slippery Slope to $n \rightarrow -3$

Finite volume effects play an important role at steep indices.

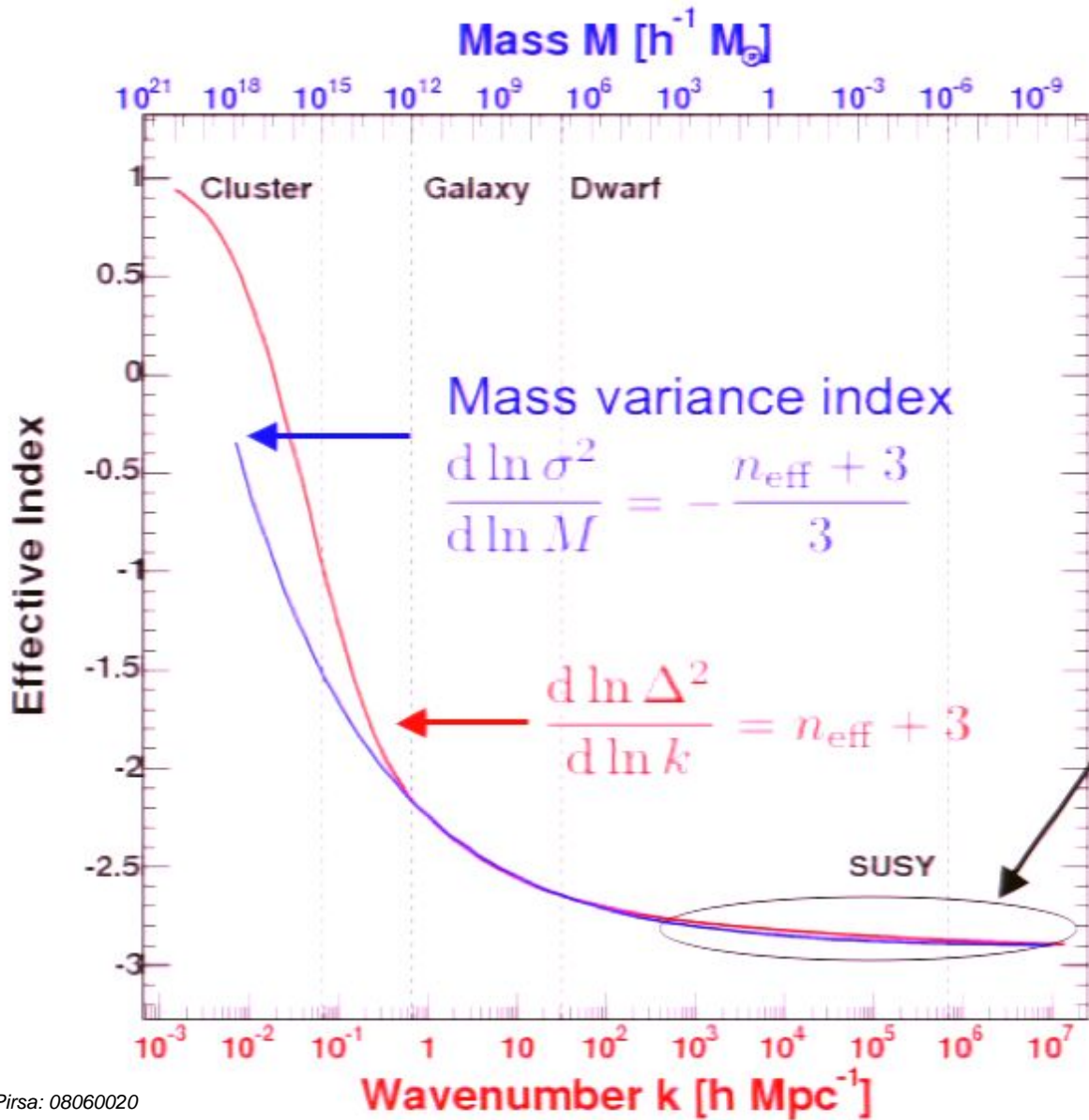
$$\sigma_{\text{miss}}^2 \approx \frac{\Delta^2(k_b)}{n+3} = 0.04$$

Minimizing **finite volume** effects gives

$$N_{\text{req}}^3 \geq 2^3 \left[ \frac{\Delta^2(k_{Ny})}{0.04(n+3)} \right]^{3/(n+3)}$$



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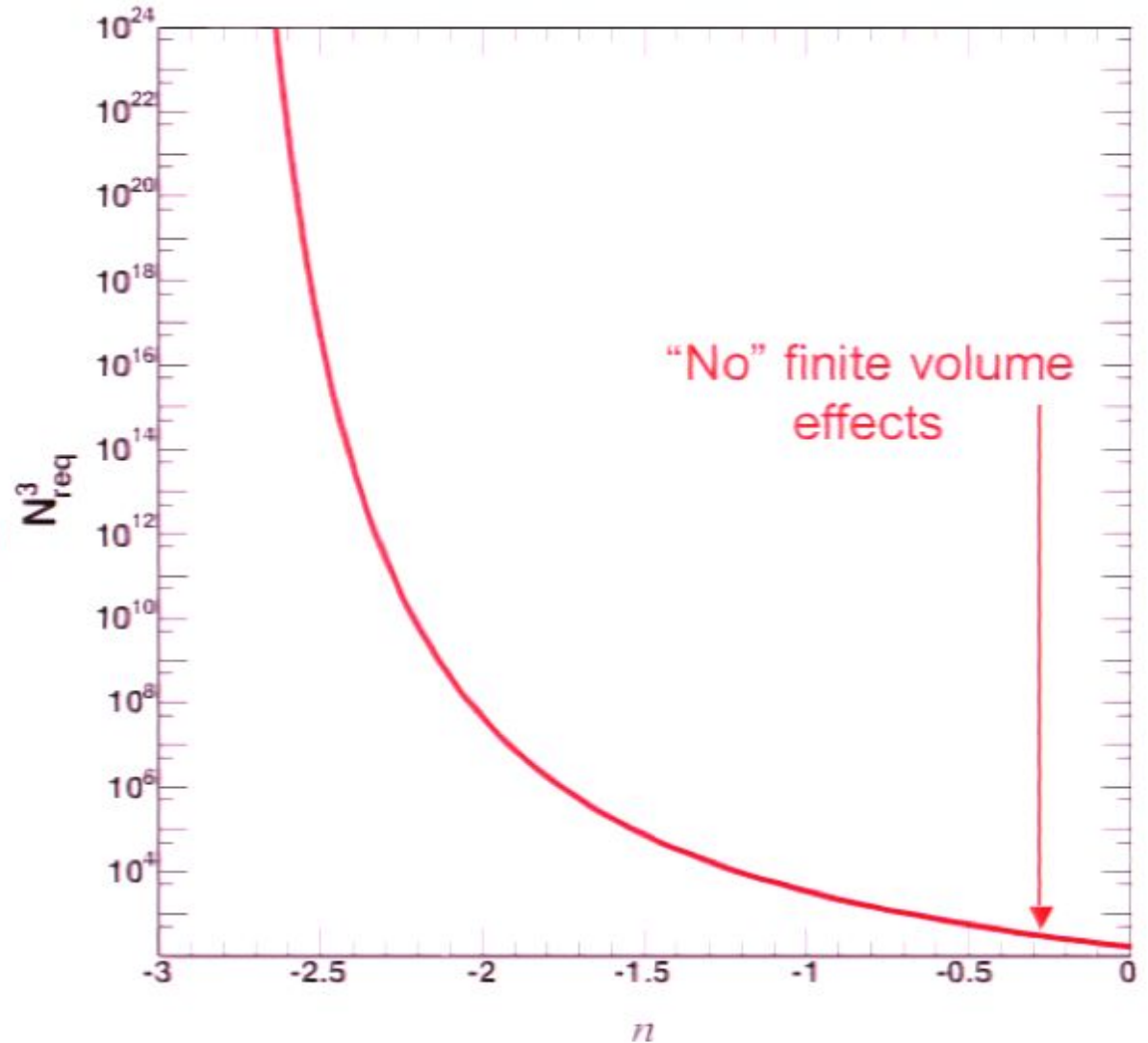
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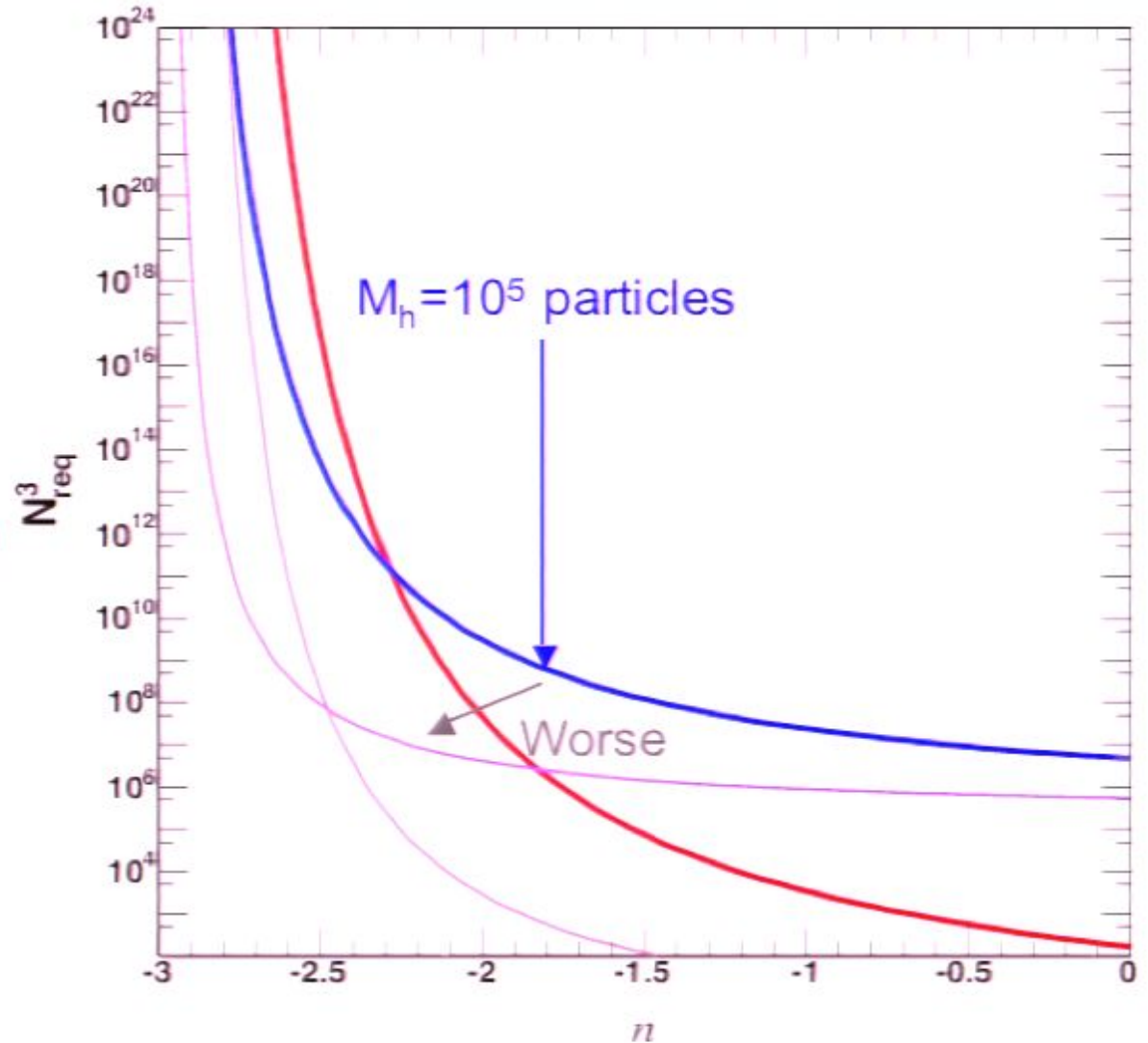
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**Minimum resolution requirements for haloes**

$$N_{\text{req}}^3 \geq \frac{6M_h}{\pi m_p} \left[ \frac{\sigma^2(M_h)}{\sigma^2(M_b)} \right]^{3/(n+3)}$$



# Slippery Slope to $n \rightarrow -3$

Finite volume effects play an important role at steep indices.

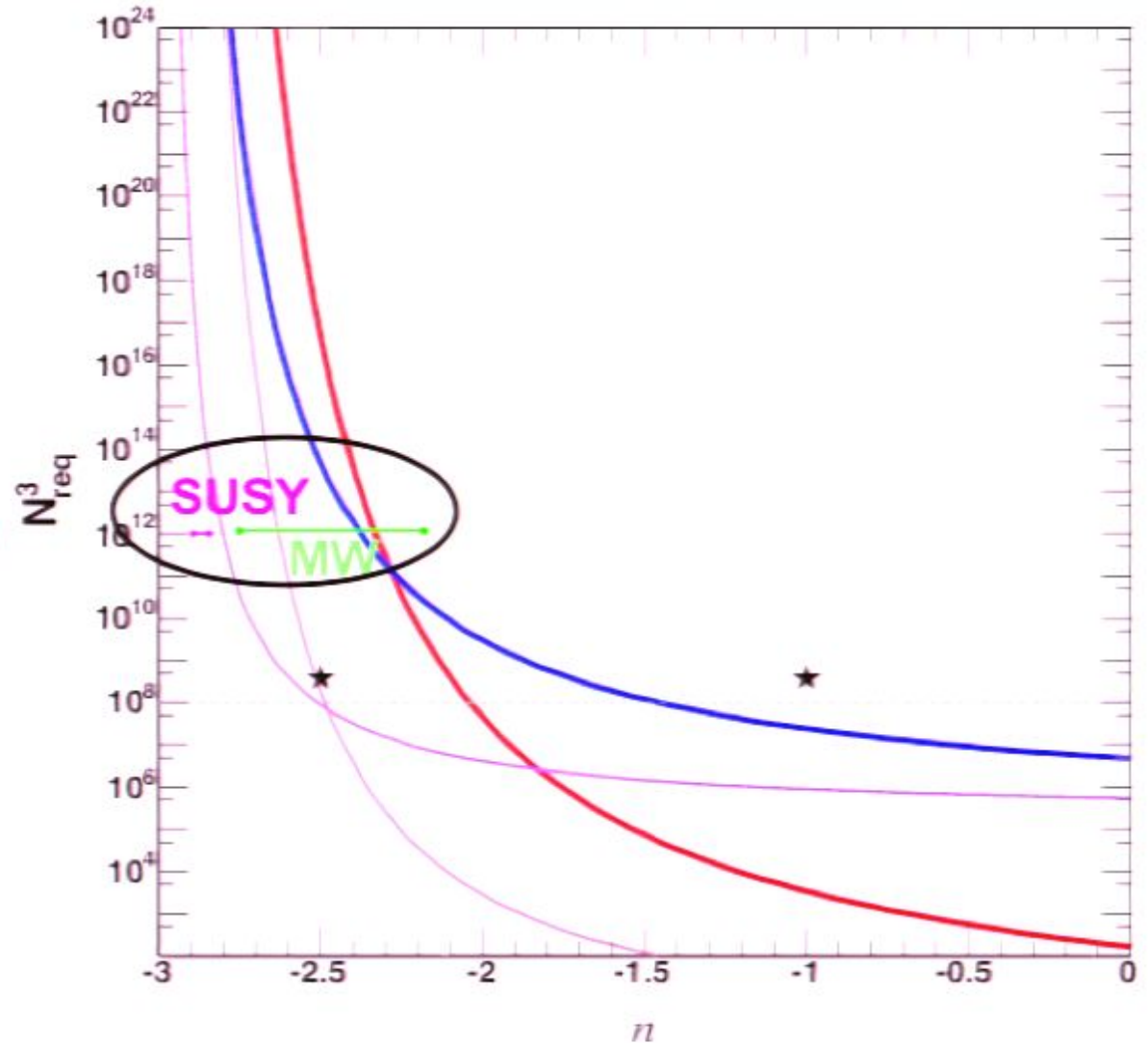
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Minimizing finite volume effects gives

$$N_{\text{req}}^3 \geq 2^3 \left[ \frac{\Delta^2(k_{N_y})}{0.04(n+3)} \right]^{3/(n+3)}$$

Ensuring haloes with enough particles not too big

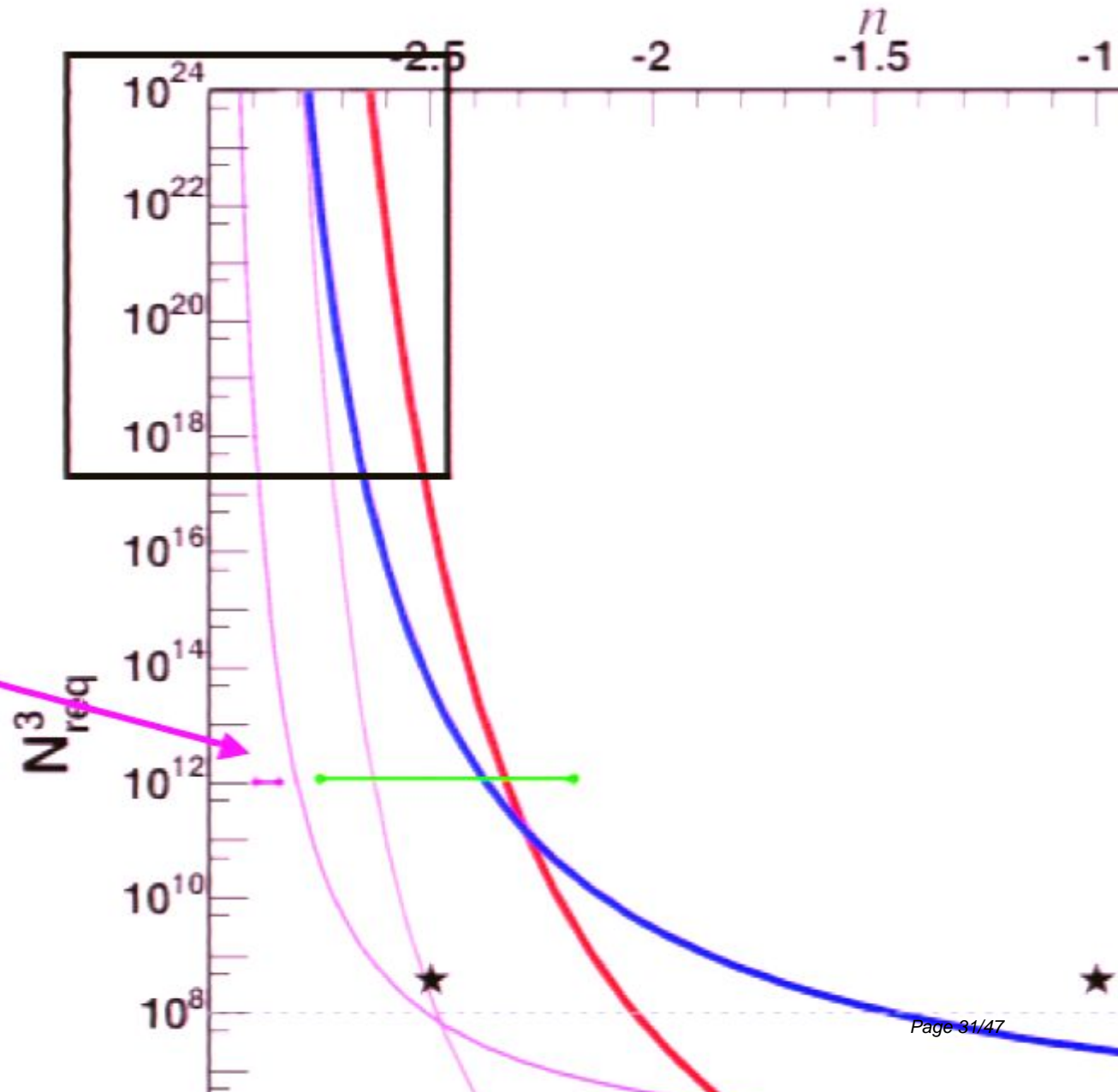
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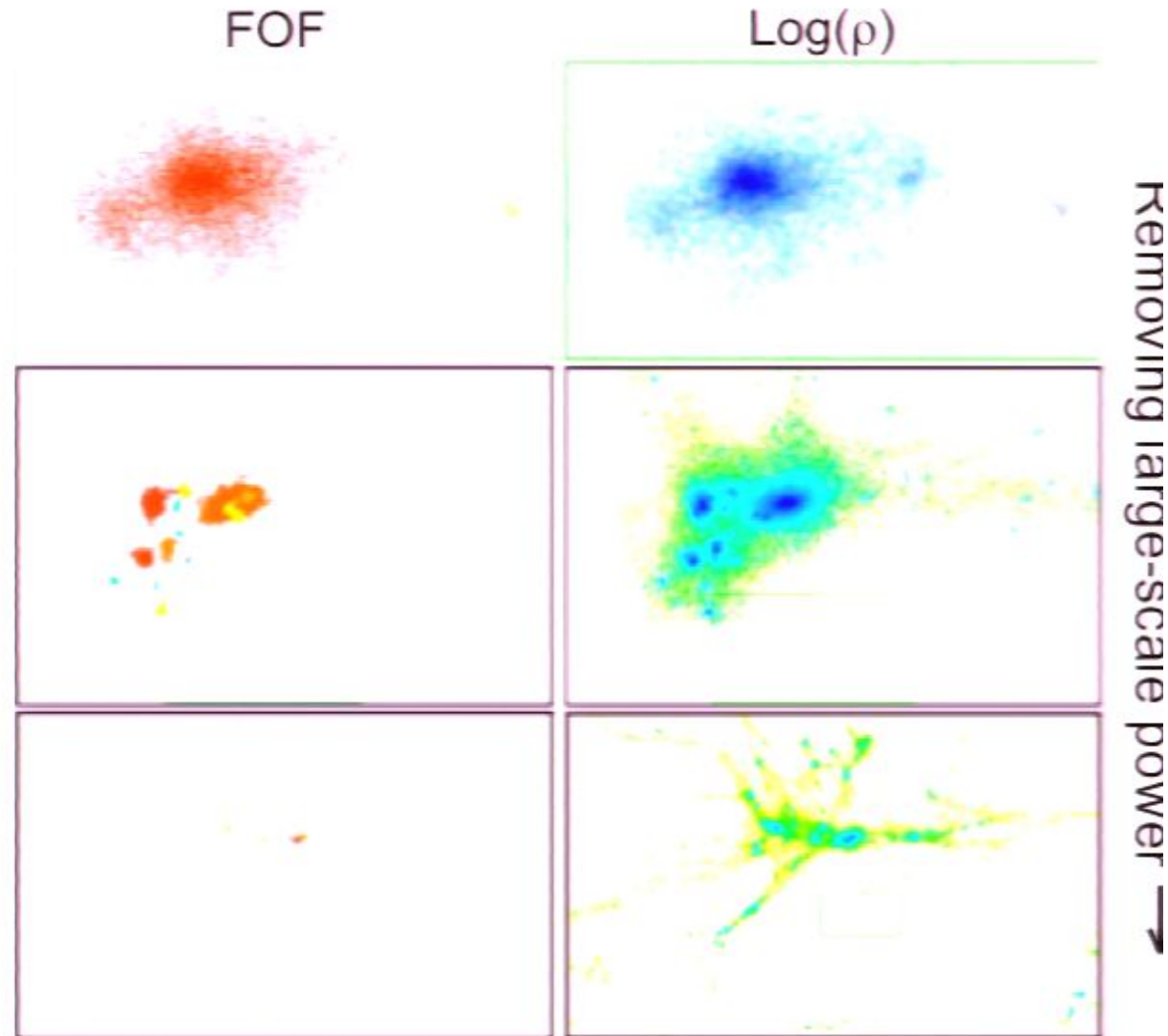
Very Tricky to do small steep simulations

SUSY simulation does not meet even poor criteria



# How wrong can it be? Case of $n=-2.9$

- $n=-2.9$ ,  $N^3=512^3$ , 3 sims with cutoff in  $P(k)$  at  $L_b$ ,  $L_b/2$ ,  $L_b/4$ 
  - Note sims already “wrong”, too much missing variance ( $N_{\text{req}}$  enormous)
- Increasing missing variance
  1. Pushes formation time forward
  2. More haloes  $\rightarrow$  more subhaloes(?)



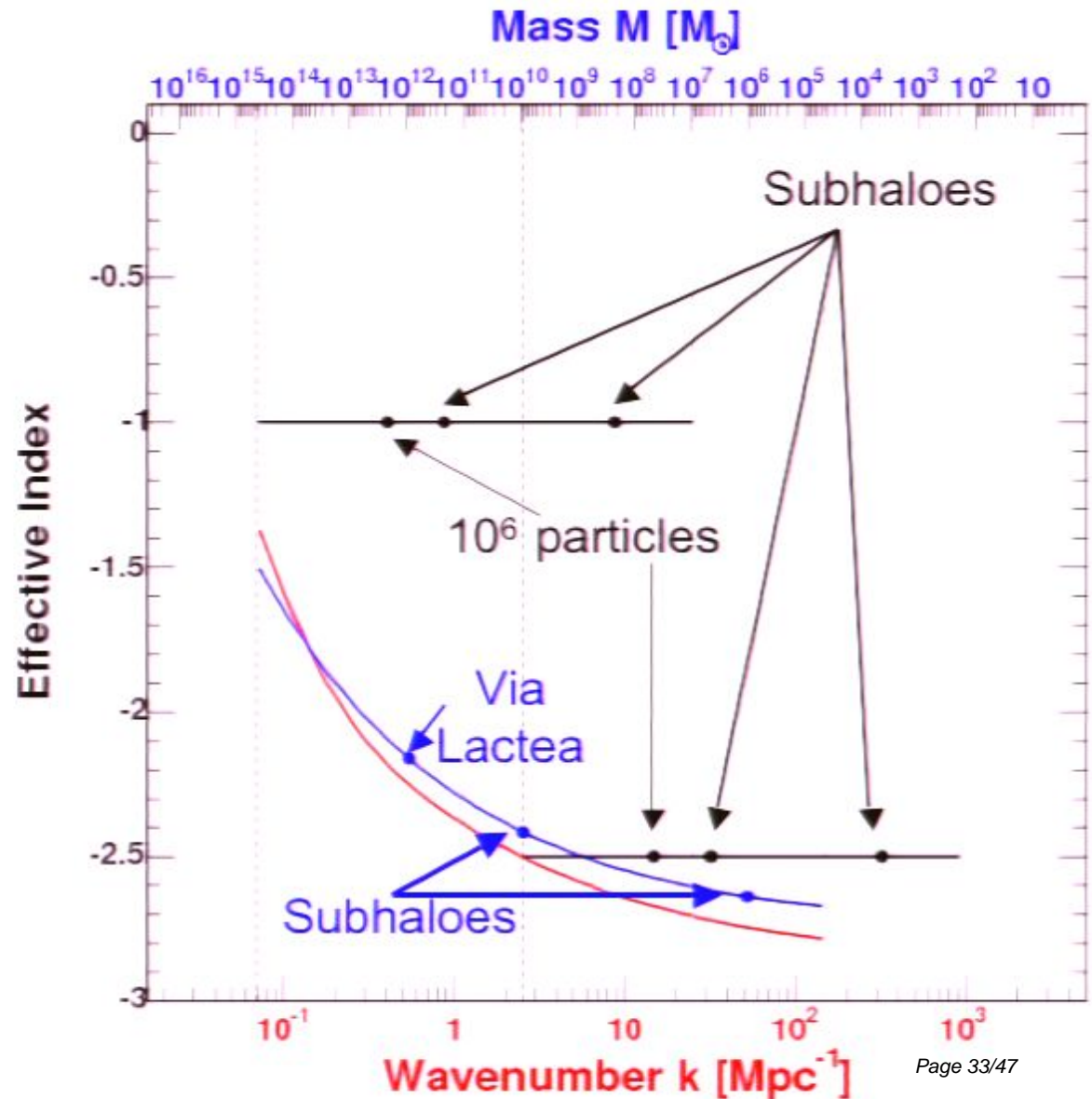


# Numerical Experiments

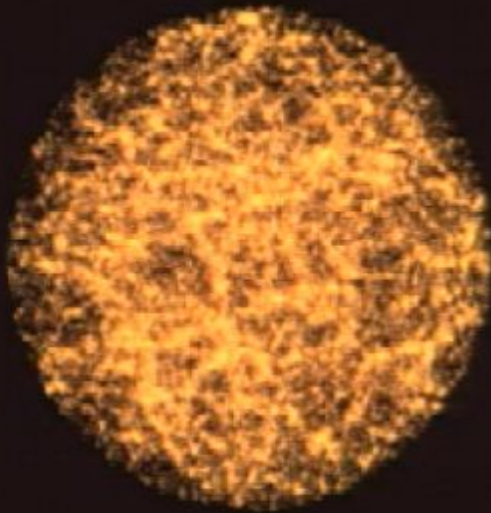
Scale-Free, Einstein-de Sitter ( $P(k)=Ak^n$ ),  
 $n=[-1, -2.5]$ ,  $N^3=720^3$

Why Scale-Free?

1. Simple, “know” form of answer ahead of time (self-similar scaling).
2. Separate merger/accretion physics from IC.
3. Model specific scales in CDM hierarchy. (e.g. Knollmann, Power & Knebe 2008)

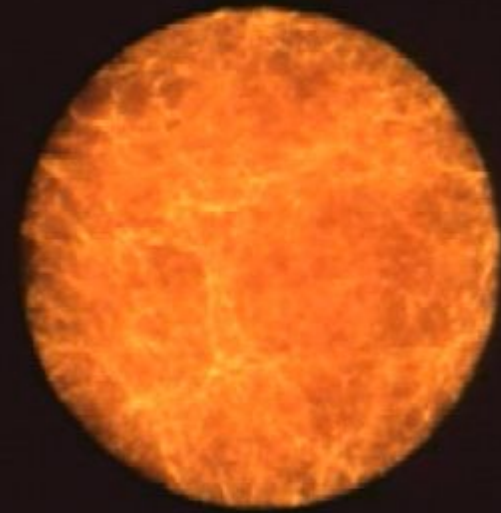


# Scale-Free Simulations



**$n=-1$** : index corresponds  
> galaxy cluster masses

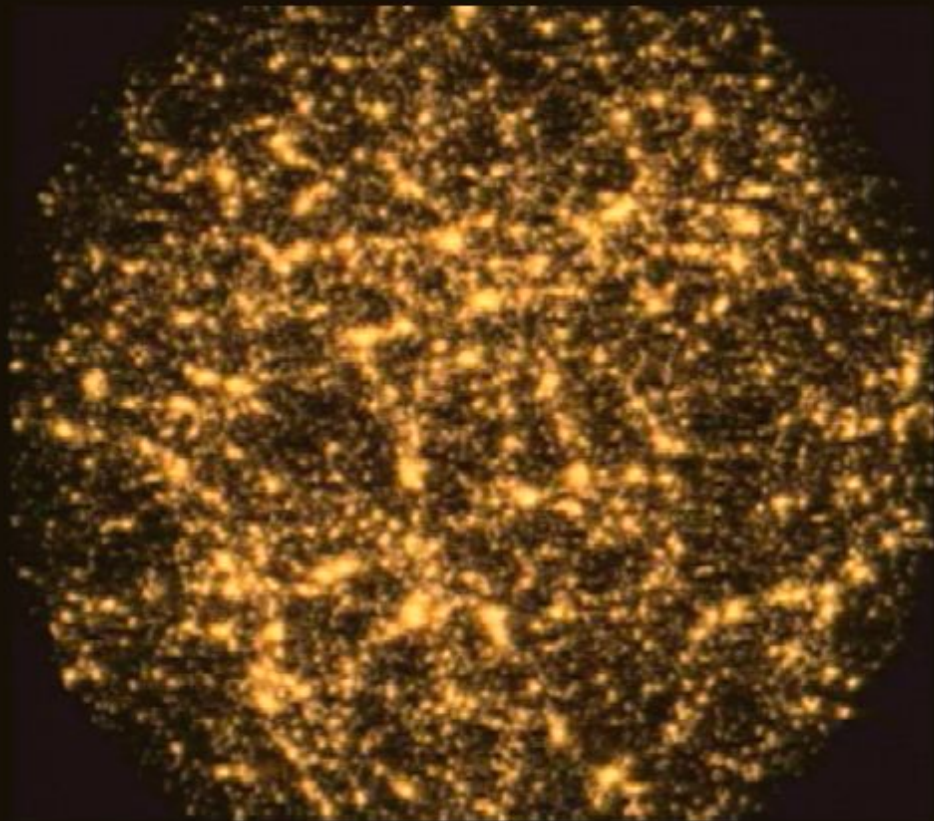
Pirsa: 20060020



**$n=-2.5$** : corresponds to dwarf  
galaxy or smaller

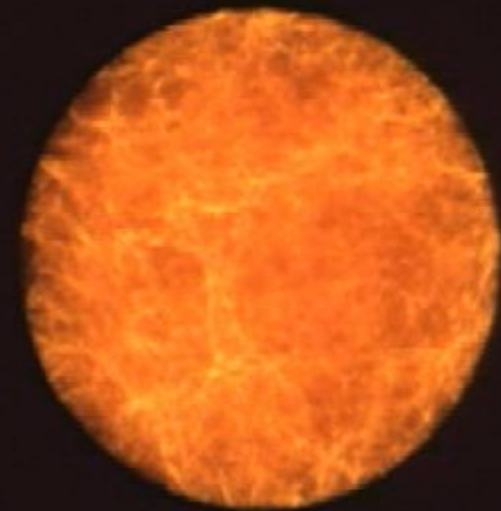
Page 34/47

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Pirsa: 20060020



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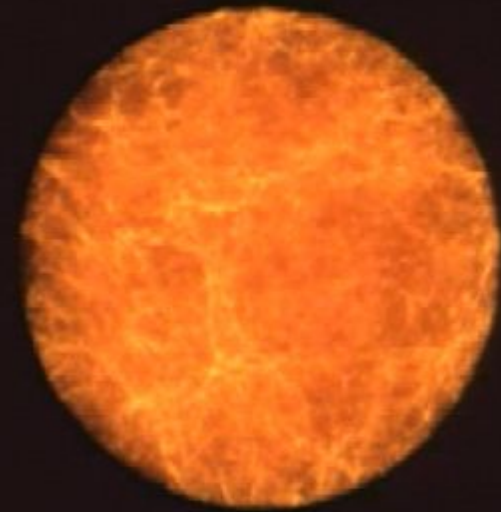
Page 35/47

# Scale-Free Simulations



**$n = -1$** : index corresponds  
> galaxy cluster masses

Pirsa: 20060020



**$n = -2.5$** : corresponds to dwarf  
galaxy or smaller

Page 36/47

# Scale-Free Simulations



**$n=-1$** : index corresponds  
> galaxy cluster masses

Pirsa: 20060020



**$n=-2.5$** : corresponds to dwarf  
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Page 37/47

# Searching for (Sub)Haloes

- Search for candidate (sub)haloes using Friends-of-Friends (**3DFOF**) and **6DFOF**.

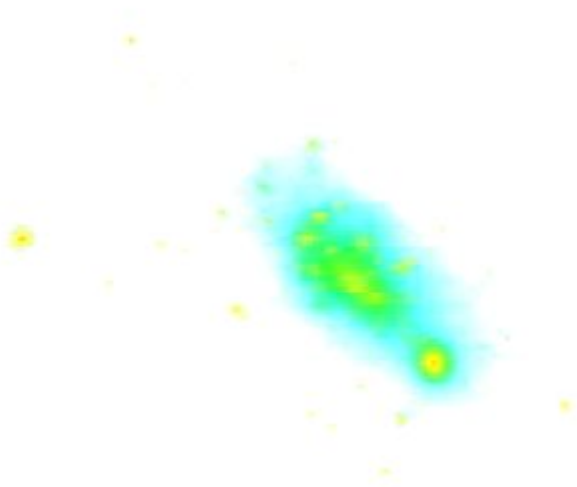
- **3DFOF**:  $\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{l_h^2 (L_b/N)^2} < 1$

- **6DFOF**:  $\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{l_s^2 (L_b/N)^2} + \frac{(\mathbf{v}_1 - \mathbf{v}_2)^2}{b_v^2 v_h^2} < 1, v_h^2 = \frac{GM}{R}$

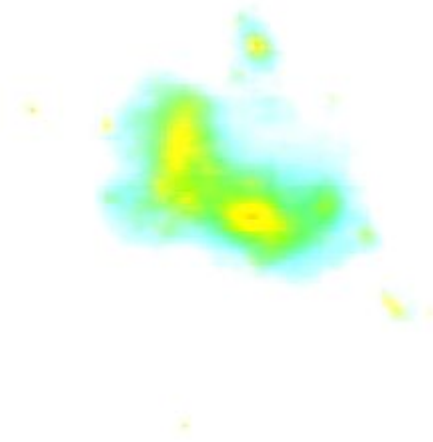
- Halo:  $l_h = 0.2$
- Subhalo: **3DFOF**  $l_s < 0.2$ .  
**6DFOF** uses halos to calculate  $v_h$ ,  
 $l_s = 0.1$  &  $b_v = [0.05, 0.10, 0.20]$ .

- UNBIND**

# Halos & Subhalos

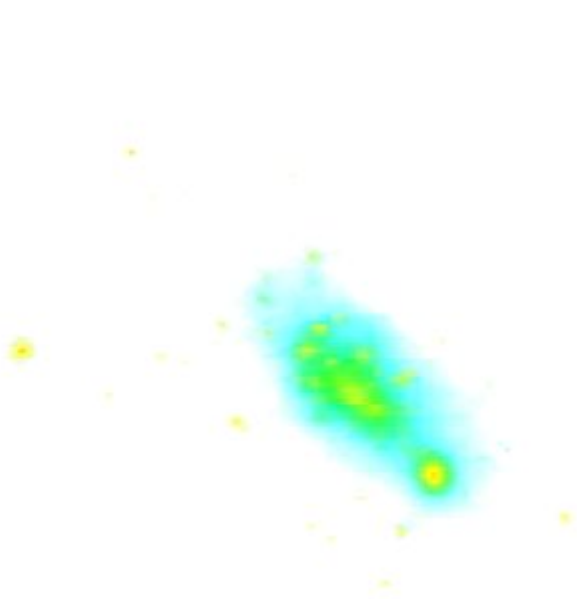


$n=-1$



$n=-2.5$

# Halos & Subhalos



$n=-1$



$n=-2.5$



# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.10



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.



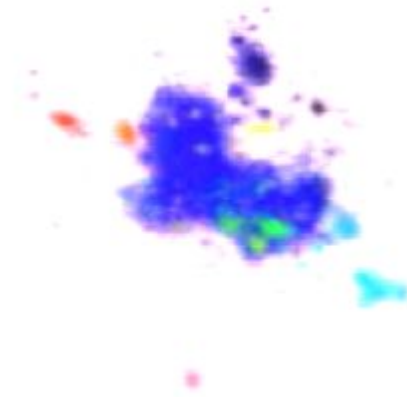
# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.10



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.



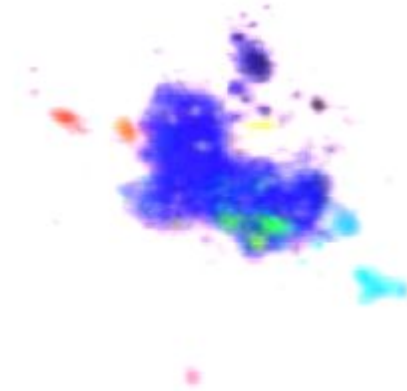
# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.10



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.



# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.10



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.

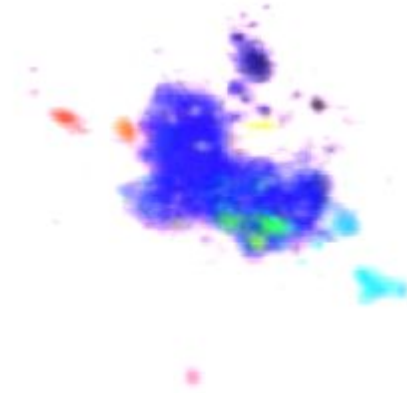
# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.07



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.

# 3DFOF vs 6DFOF

3DFOF=0.10

**n=-1:** 3DFOF/density  
info ok.



3DFOF=0.07



**n=-2.5:** Substructure  
not as noticeable  
from density, need  
full phase-space.

# Summary

**Very Tricky** to get the simulations right at steep indices (high redshifts).

- Worry about missing power
- Must unbind.

Sims suggest **self-similarity cannot naively save us**, as  $n \rightarrow -3$ :

- Less substructure
- Morphologically different, poorly defined boundary. Loosely bound streams, better for direct detection.
- Subhalo mass function logarithmic slope shallower,  $\alpha \sim -0.8$  compared to  $-0.9$ . How sensitive is it?