Title: A review of i) the theoretical input into small scale structure formation, ii) what direct detection experiments need to know

Date: Jun 07, 2008 04:00 PM

URL: http://pirsa.org/08060010

Abstract:

A review of:

i) what direct detection experiments need to know

ii) the theoretical input into (very) small scale structure formation

Anne Green
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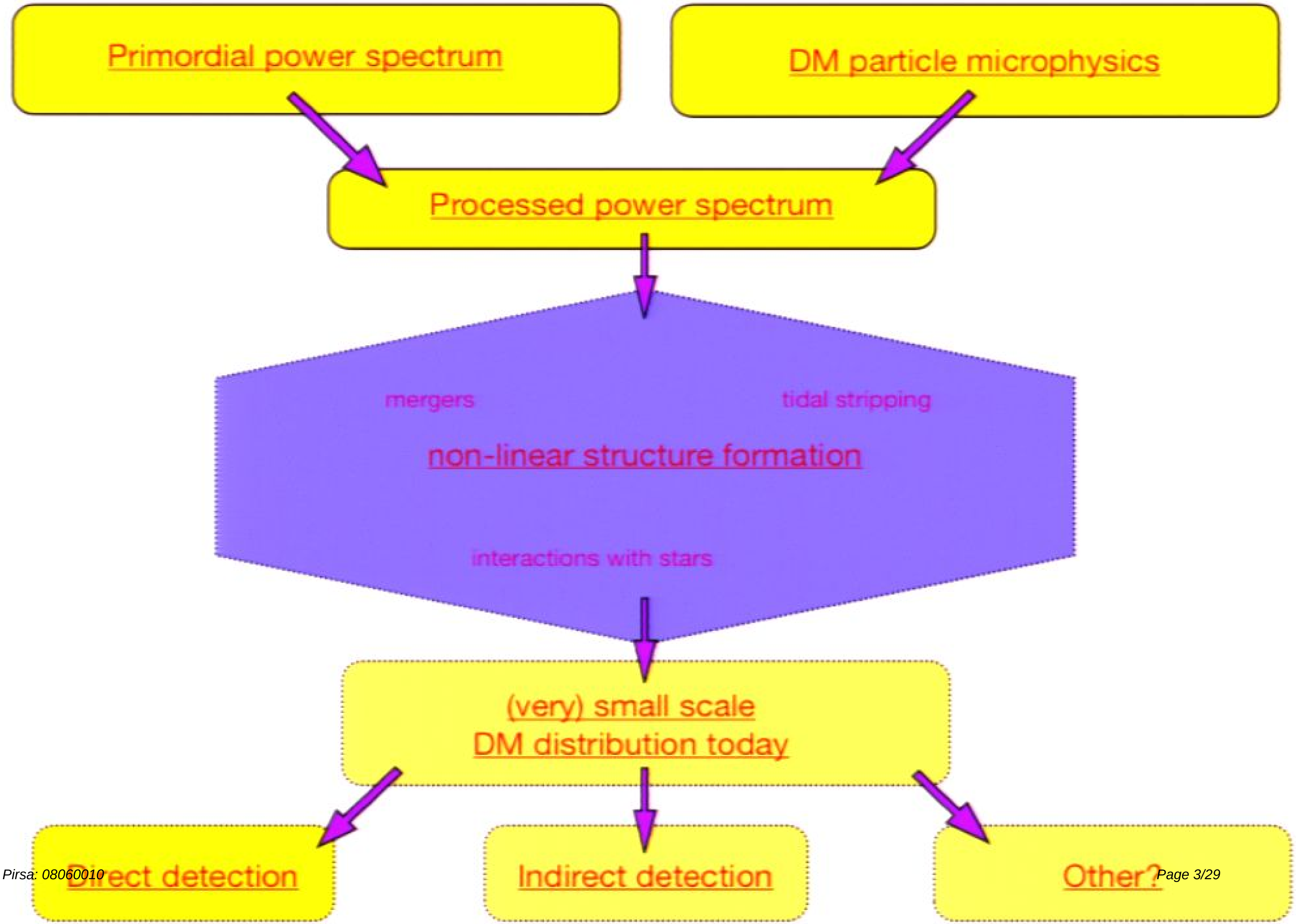
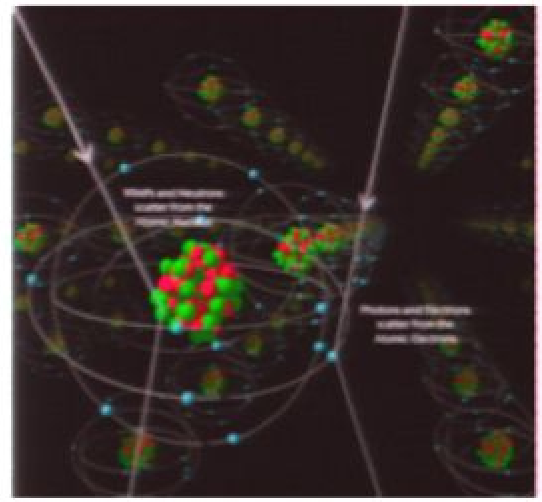

Primordial power spectrum
DM particle microphysics
Processed power spectrum
mergers
tidal stripping
non-linear structure formation
interactions with stars
(very) small scale
DM distribution today
Direct detection
Indirect detection
Other?

i) What do direct detection experiments need to know?

(specifically WIMP direct detection experiments)

Via elastic scattering on detector nuclei in the lab:

$$\chi + N \Rightarrow \chi + N$$



Detect recoil energy via ionisation, scintillation and/or heat (multiple channels help discriminate electron & nuclear recoils).

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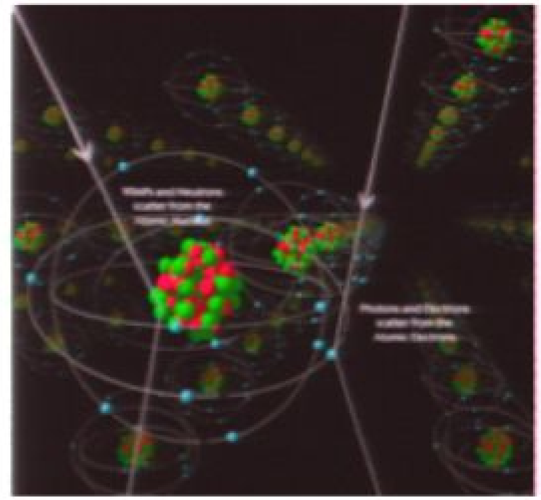


Detect recoil energy via ionisation, scintillation and/or heat
(multiple channels help discriminate electron & nuclear recoils).

Need to know energy spectrum of recoils to

i) constrain/measure the WIMP properties

ii) distinguish between neutron and WIMP induced nuclear recoils

Event rate:

(assuming spin-independent coupling)

WIMP speed distribution in rest frame of detector

target form factor

local WIMP density

$$\frac{dR}{dE} = \frac{(m_p + m_\chi)^2}{m_p^2 m_\chi^3} \sigma_p \rho_\chi A^2 F^2(E) \int_{v_{min}}^{\infty} \frac{f^E(v)}{v} dv$$

WIMP scattering cross-section on proton

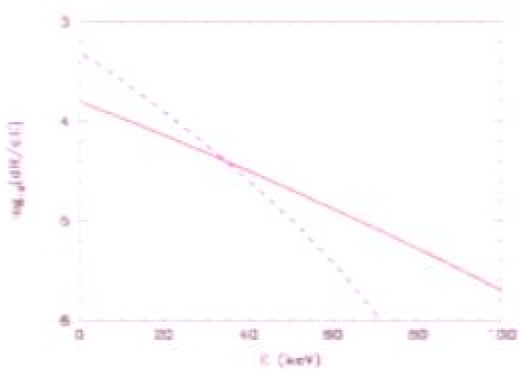
target mass number

minimum WIMP speed which can cause a recoil of energy E.

$$v_{min} = \left(\frac{E(m_A + m_\chi)^2}{m_A m_\chi^2} \right)^{1/2}$$

Potential signals:

i) Dependence of energy spectrum on target mass:

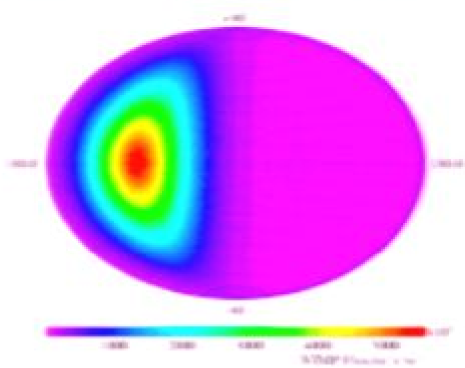


m_χ=100 GeV
Ge
Xe

ii) Annual modulation [Drukier, Freese &Spergel]

small effect, need large amount of data
(and stable operation of detector with time)

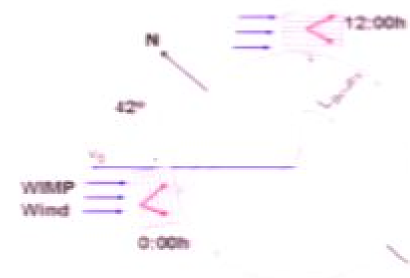
iii) Direction dependence [Spergel]



need detector which can measure
direction of recoils

iv) Diurnal modulation

of: peak recoil direction (in lab)
event rate



n.b. all these plots assume the WIMP velocity distribution is an isotropic gaussian

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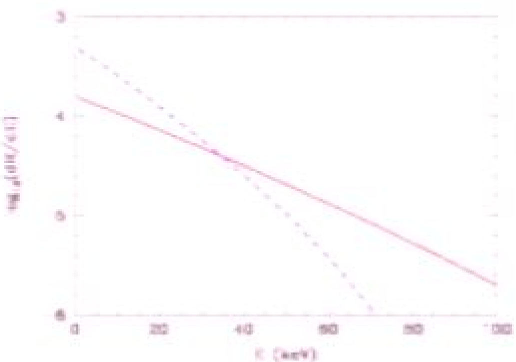

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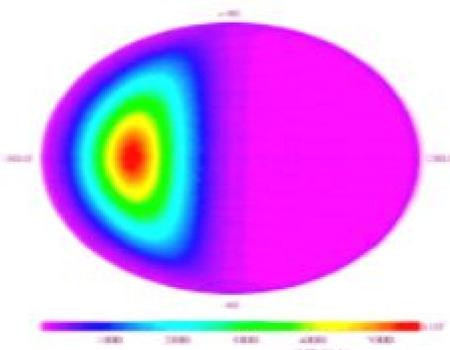

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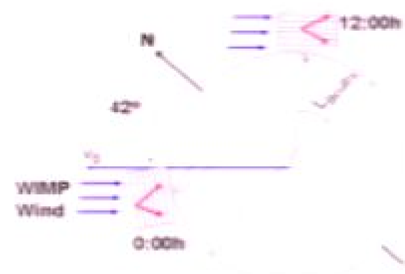

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(A selection of) current experiments

- **CDMS**
  Ge, 400 kg-days,
  E_R=5/10 keV
  ionisation & heat

- **Edelweiss**
  Ge, 60 kg-days,
  E_R=13 keV
  ionisation & heat

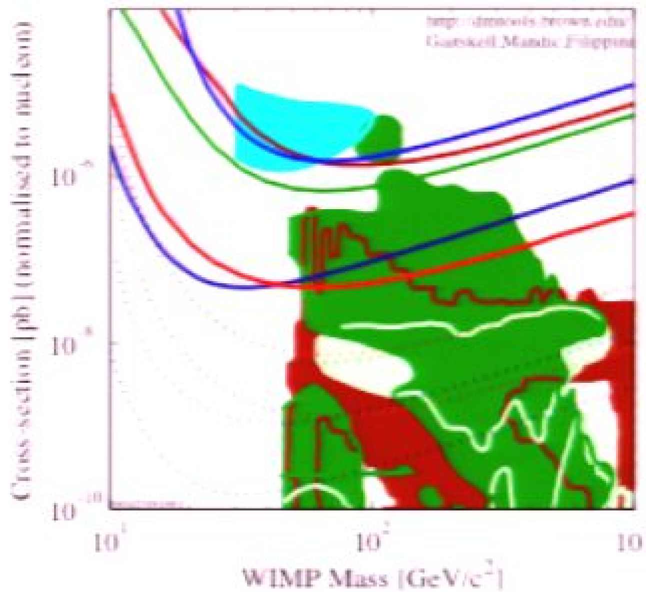
- **CRESST**
  $CaWO_4$, 20 kg-days,
  E_R=10 keV
  scintillation & heat

- **Xenon10**
  2-phase Xe, 300 kg-days,
  E_R = 4.5 keV
  scintillation & (indirectly) ionisation

- **DAMA**
  NaI, 200 00 kg-days,
  E_M = 2 keV
  scintillation

- **Zeplin**
  liquid Xe, 230 kg-days,
  E_M = 5 keV
  scintillation & (indirectly) ionisation

Constraints on the WIMP mass and spin-independent scattering cross-section on the proton



Assuming 'standard' halo model.

But:

All these signals depend on the (density and velocity) distribution of the WIMPs passing through the detector.

Relevant scale: speed of earth x duration of experiment
200 km/s x 1 year
0.1 milli-pc

Many orders of magnitude smaller than most astronomical problems.

And many many orders of magnitude than can be directly probed by numerical simulations.

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what happens to the material which is stripped from microhalos?

(probability of us being within a surviving microhalo/remnant is tiny, even if survival fraction is large)

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IF the ultra-local DM distribution consists of a small number of streams, then the direct detection differential event rate will consist of a series of steps:

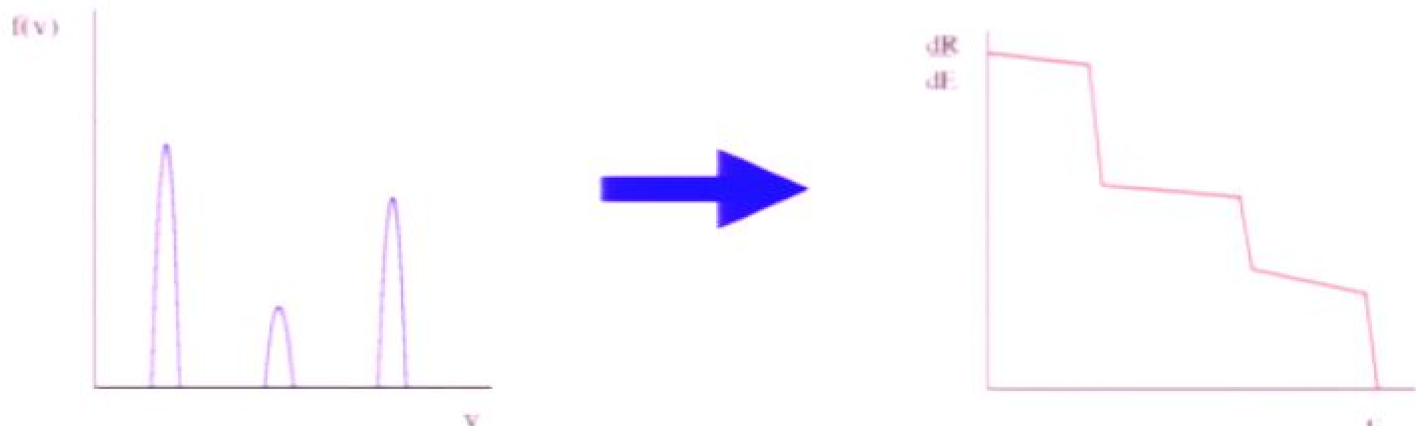
Positions of steps would depends on the WIMP mass, target nuclei mass and the (unknown) stream speeds.

Heights of steps would depends on the WIMP cross-section and the (unknown) stream densities.

A random example:

$$\frac{\mathrm{d}R}{\mathrm{d}E} \propto \int_{v_{\min}}^{\infty} \frac{f(v)}{v} \mathrm{d}v$$

$$v_{min} = \left(\frac{E m_A}{2\mu_{A\chi}} \right)^{1/2}$$



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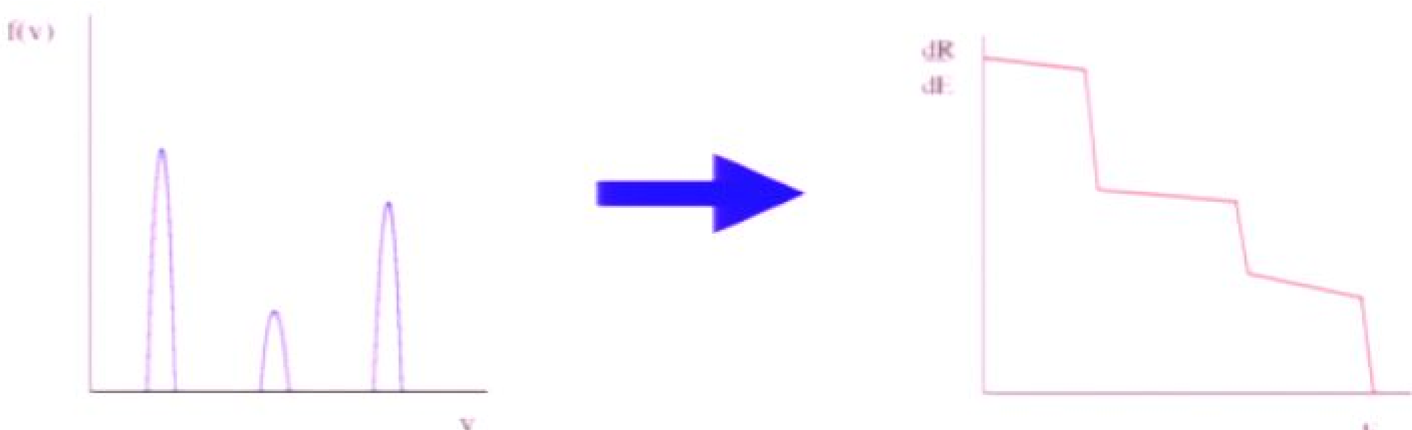

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ii) Theoretical input into (very) small scale structure formation

A) Particle microphysics

- i) basic calculation for generic WIMP
  - a) chemical decoupling
  - b) kinetic decoupling and damping
  - c) free-streaming
- ii) refinements of damping calculation
- iii) concrete WIMP candidates (neutralinos & UED models)

B) Primordial power spectrum

A) Particle microphysics

[Schmid, Schwarz & Widerin; Boehm, Fayet & Schaeffer; Chen, Kamionkowski & Zhang; **Hofmann, Schwarz & Stöcker**; Schwarz, Hofmann & Stöcker; Berezinsky, Dokuchaev & Eroshenko; Green, Hofmann & Schwarz; Loeb & Zaldarriaga; Bertschinger; Bringmann & Hofmann]

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i) basic calculation for generic WIMPs [Green, Hofmann & Schwarz]

Assume

- WIMPs are in thermal & chemical equilibrium with radiation in the early Universe *(not the case for WIMPzillas and DM axions)*.
- Standard cosmology (radiation domination from high T, followed by matter domination).
- WIMPs are the only CDM component
- no WIMP anti-WIMP asymmetry
- Thermal average of elastic scattering cross-section given by

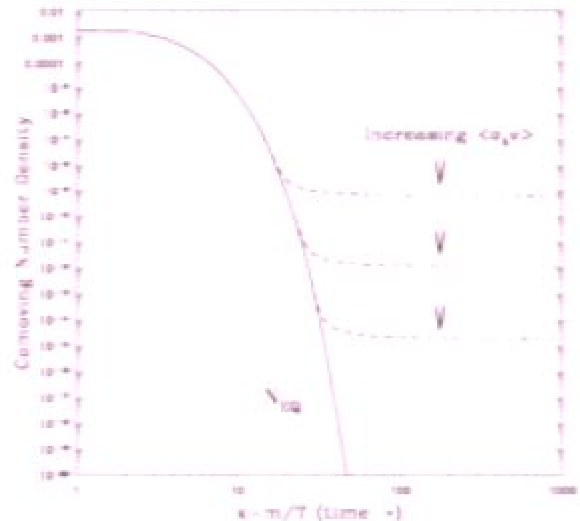
$$\langle \sigma_{el} \rangle = \sigma_0^{el} \left(\frac{T}{m} \right)^{l+1} \qquad \sigma_0^{el} \approx \frac{(G_F m_W^2)^2 m^2}{m_z^4}$$

in Standard Model scattering of light and heavy fermions mediated by Z0 exchange & l=0
in Supersymmetric extensions of the SM scattering mediated by sfermion exchange & l=1

a) Chemical decoupling ("freeze-out")

$$\chi + \chi \not\leftrightarrow X + X$$

As the Universe expands and the WIMP density drops, chemical interactions cease ($\Gamma < H$) and the comoving WIMP density becomes fixed.



$$x = \frac{m}{T}$$

Our generic WIMPs obey Boltzmann statistics, for T << m, and

$$\Omega_{wimp,0} = \frac{2}{3}\left(\frac{2^3}{\pi}\right)^{1/2}\left(\frac{g_s T^3}{(m_{Pl}H)^2}\right)_0 \frac{gm}{g_{s,cd}} x_{cd}^{3/2} \exp(-x_{cd}) cosh\left(\frac{x_{cd}}{m}\right)$$

Which can be solved iteratively for x_{cd}

$$x_{cd} \approx 23 + \ln\left(\frac{m}{100\,GeV}\right) - \ln(\Omega_{wimp}h^2) + \ln(g) + \ln\left(\frac{g_{s,0}}{g_{s,cd}}\right)$$

[For an accurate calculation: numerically solve the Boltzmann equation including all relevant annihilation channels.]

b) Kinetic decoupling

After freeze-out (chemical decoupling) WIMPS carry on interacting kinetically with other particles (specifically, after QCD phase transition, leptons):

$$\chi + X \Leftrightarrow \chi + X$$

WIMPs kinetically decouple when

$$\tau_{relax} \approx H^{-1}$$

n.b. the momentum transfer per scattering (~T) is small compared with the WIMP momentum (~M), therefore a very large number of collisions are required to keep or establish thermal equilibrium.

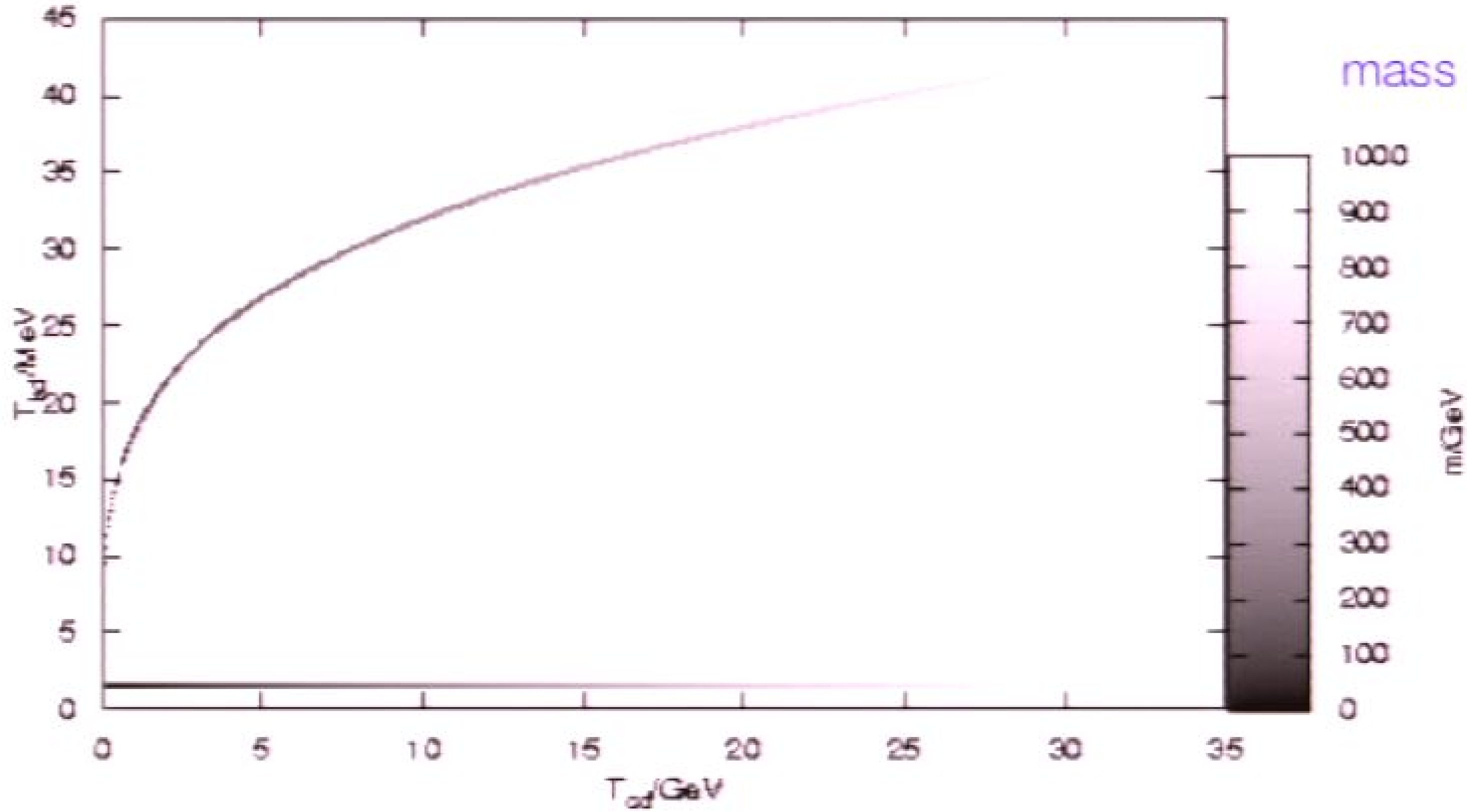
$$\tau_{relax} \sim \frac{m}{T}\tau_{col} \gg \tau_{col} \qquad \tau_{col} \equiv \frac{1}{\Gamma_{el}} = \frac{1}{\Sigma\langle v\sigma_{el}\rangle n}$$

For our generic WIMPs:

$T_{kd} = 2.4\ g^{-1/6}$ MeV l =0

$T_{kd} = 34\ g^{-1/8}\ (m/100\ \mathrm{GeV})^{1/4}$ MeV l =1

Kinetic decoupling temperature in MeV



Chemical decoupling temperature in GeV

Kinetic decoupling occurs at T~O(10 MeV) whereas chemical decoupling occurs at T~O(10 GeV).

Collisional damping

Energy transfer between radiation and WIMP fluids (due to bulk and shear viscosity) leads to collisional damping of density perturbations. [Hofmann, Schwarz & Stöcker]

$$\zeta_{\rm vis} \approx \frac{5}{3} n T \tau_{\rm relax} \qquad \eta_{\rm vis} \approx n T \tau_{\rm relax}$$

(simplified intutitive derivation) Linearised Navier-Stokes equation on sub-horizon scales + continuity equation:

$$\delta'' + \frac{\zeta + 4\eta_{vis}/3}{\rho_{wimp}} \frac{k^2}{a} \delta' + c_{wimp}^2 k^2 \delta = 0$$

$$' \equiv \frac{d}{d\eta}$$

$$c_{wimp}^2 = \frac{5}{3}\frac{T}{m}$$

Exponential damping with characteristic wavenumber k_d:

$$k_d = \left(\int_{\eta_i}^{\eta_{kd}} \frac{\zeta_{vis} + 4\eta_{vis}/3}{2\rho_{wimp} a} d\eta \right)^{-1/2}$$

$$k_d \approx \frac{40}{pc} \left(\frac{m}{100 GeV} \right)^{1/2} \left(\frac{T_{kd}}{30 MeV} \right)^{1/2}$$

c) Free-streaming

After kinetic decoupling WIMPs free-stream, leading to further (collision-less) exponential damping with characteristic wave-number:

$$\frac{1}{k_{fs}} \equiv l_{fs} \sim v_{kd} a_{kd} \int_{\eta_{kd}}^{\eta} \frac{d\eta'}{a(\eta')}$$

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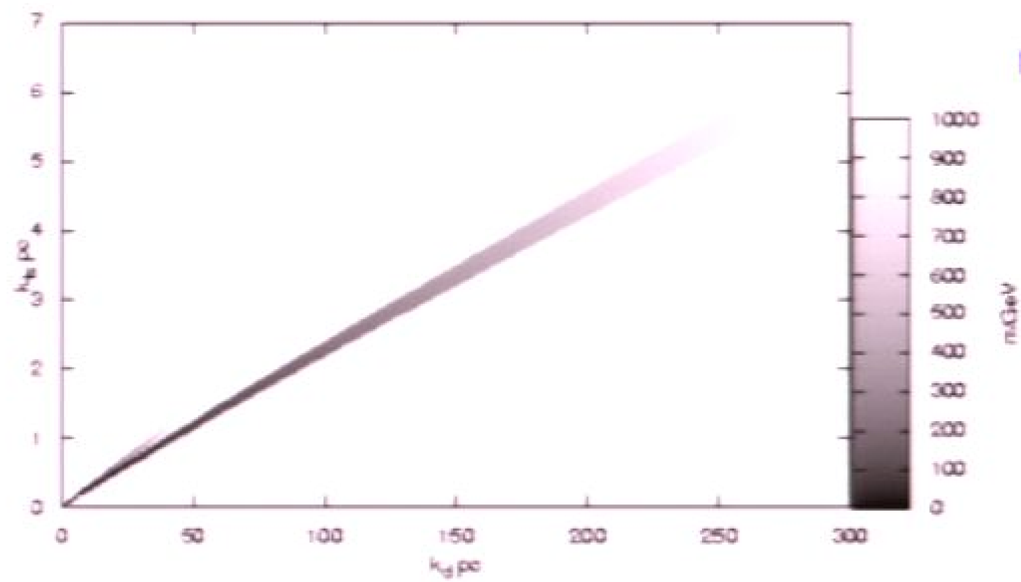
Free-streaming comoving wavenumber (pc)



mass in GeV

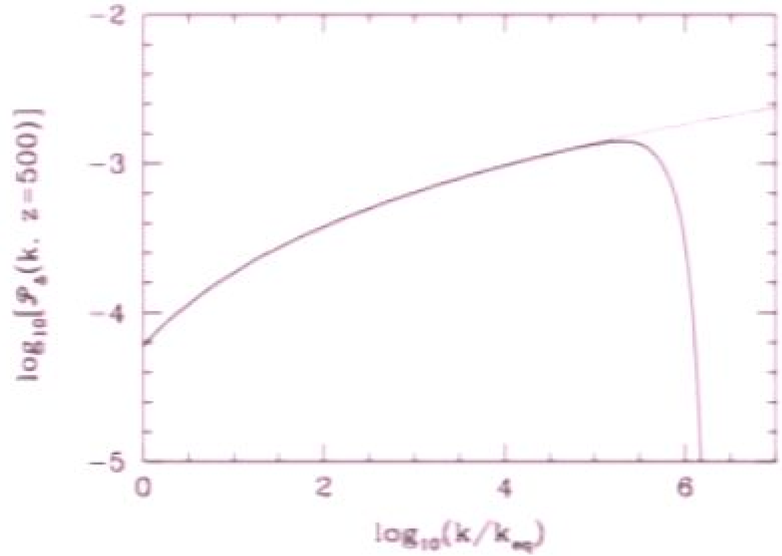
Collisional damping comoving wavenumber (pc)

Even for "generic WIMPs" there is roughly an order of magnitude variation in the free-streaming cut-off wavenumber.

Putting it all together:

Processed power spectrum at z=500

for a 100 GeV bino-like WIMP and a scale invariant, WMAP normalised, primordial power spectrum,

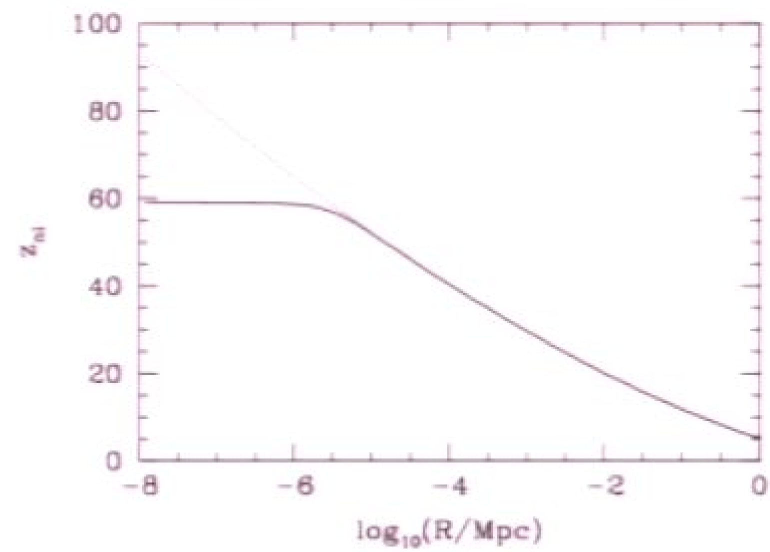


$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2}\langle|\delta_k^2|\rangle$$

z_{nl}, Red-shift at which typical, 1 sigma, fluctuations go non-linear

$$\sigma^2(R,z) = \int_0^\infty W^2(kR)\,\mathcal{P}_\delta(k,z)\frac{dk}{k}$$

$$\sigma(R, z_{nl}) = 1$$



estimate of mass & radius of smallest halos

(using spherical top-hat collapse)

$$M \approx 3\times10^{-7}\left(\frac{R}{pc}\right)^3 M_\odot \qquad r \approx \frac{1.05R}{2\,z_{coll}} \sim 0.02\,pc$$

(for a typical fluctuation with R ~1 pc)

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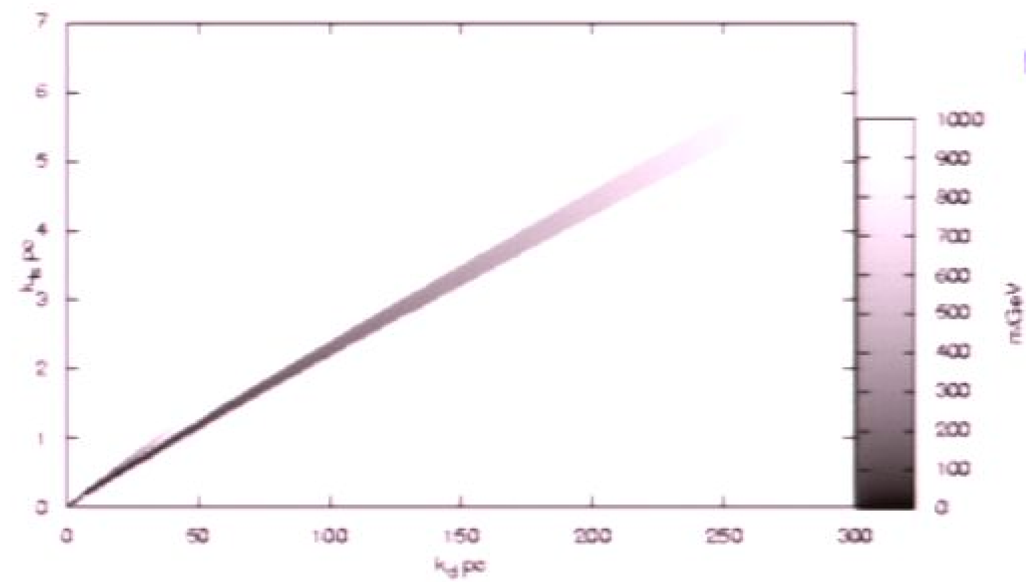

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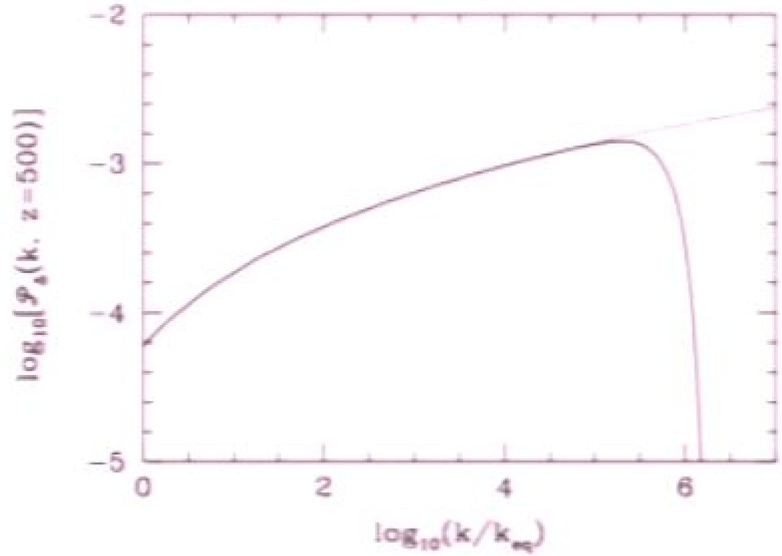

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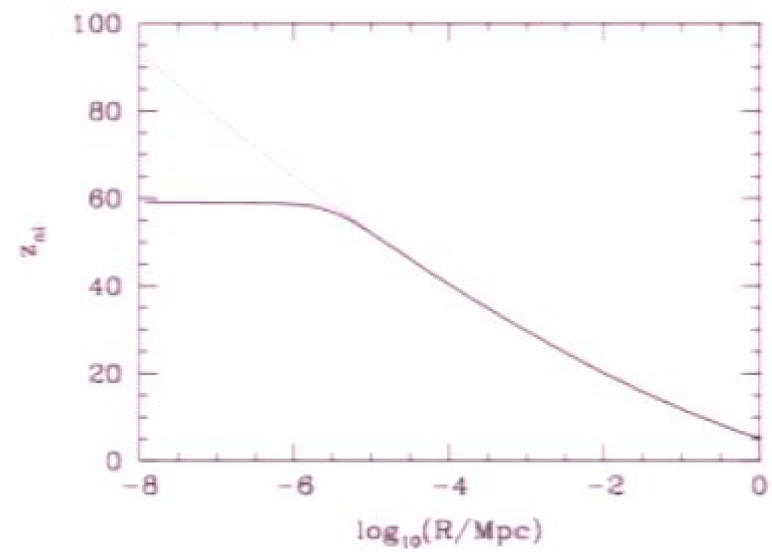

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(for a typical fluctuation with R ~1 pc)

ii) Refinements of damping calculation: [Loeb & Zaldarriaga; Bertschinger; Bringmann & Hofmann]

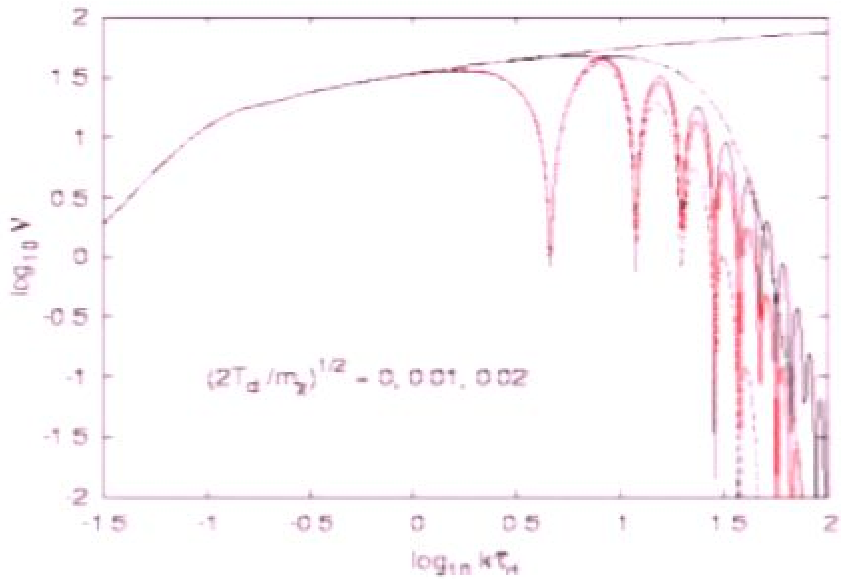
Loeb & Zaldarriaga:

Numerically treatment including effects that are important around horizon crossing (gravitational pull of oscillating radiation fluid).

Memory of coupling to radiation fluid leads to acoustic oscillations of CDM fluid and additional dampin

Bertschinger:

Numerically solved full Boltzmann equation describing WIMP-lepton scattering (Fokker-Planck equation describing diffuse in velocity space due to elastic scattering, advection and gravitational forces)].



Cut-off scale can be calculated fairly accurately with simplified analytic treatment. Detailed shape of power spectrum requires numerical calculation.

Bringmann & Hofmann:

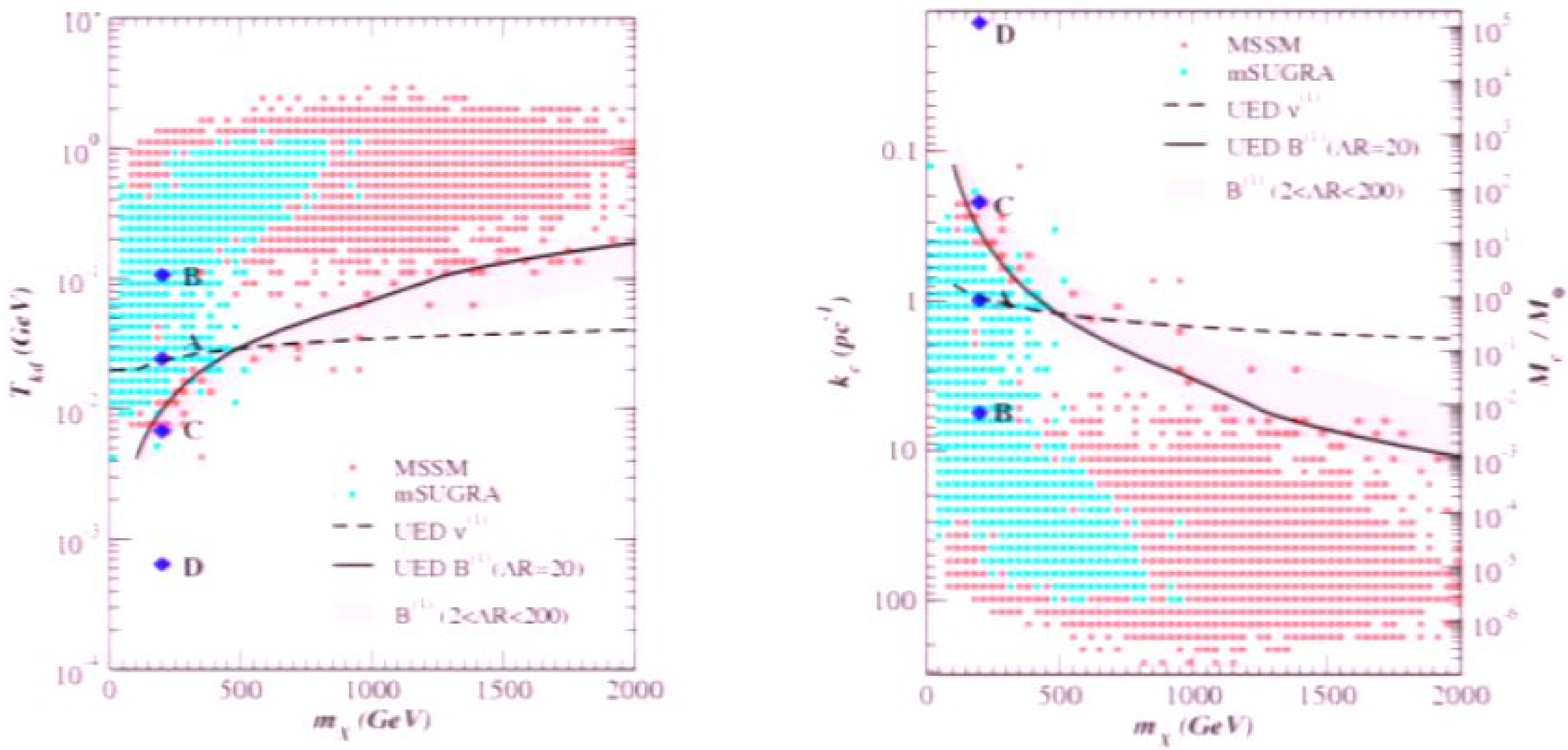
Full analytic treatment of the kinetic decoupling process from first principles.
(Showed can define a temperature for CDM, which characterizes the distribution function, even when out of thermal equilibrium.)

iii) Concrete WIMP candidates: [Profumo, Sigurdson & Kamionkowski]:

Scan MSSM and also consider Universal Extra Dimensions and heavy neutrino like dark matter.

Calculate WIMP abundance and kinetic decoupling temperature, T_{kd}, using detailed calculation of scattering cross-sections (including resonances & threshold effects).

Estimate cut-off mass as: $$M_c \approx 33.3 \left(\frac{T_{kd}}{10 MeV}\right)^{-3} M_{earth}$$ [Loeb & Zaldarriaga]



A: coannihilation region, light scalar sparticles, (quasi-degenerate) NLSP is stau

B: focus point region, heavy scalars, scattering from light fermions is via Z0 exchange

C: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 1\,\mathrm{GeV}$ } Sfermion resonances. At high T scattering

B) Primordial power spectrum

Processed power spectrum (which is input into numerical simulations, or any non-linear structure formation calculation) obviously depends on the primordial power spectrum.

Simplest assumption is a scale invariant power spectrum: n=1
(equal power on all scales, at horizon crossing)

But:

i) if inflation generates the primordial perturbations expect (small) deviations from scale-invariance

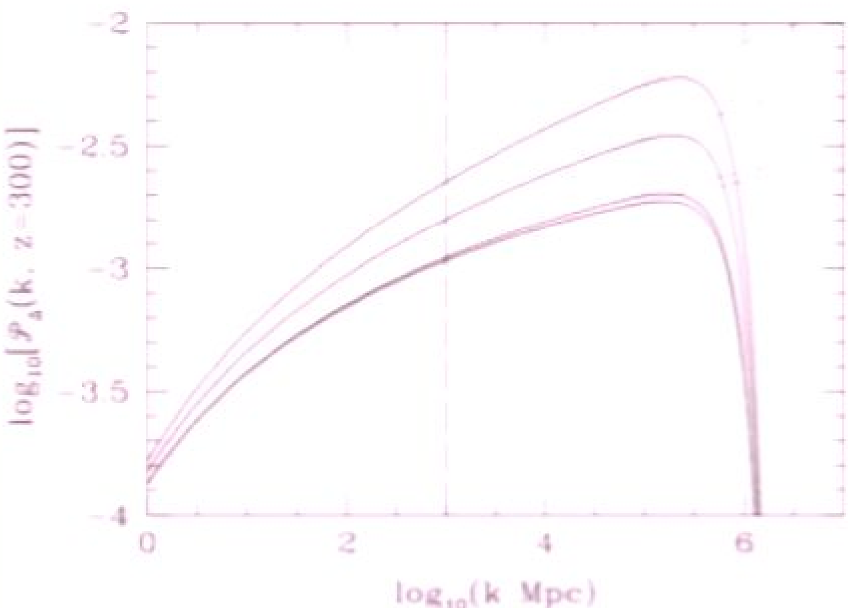
ii) latest WMAP analysis finds: $n = 0.960^{+0.014}_{-0.013}$ without running
(for a pivot scale of k_0 = 0.002/Mpc)

$n = 1.059^{+0.051}_{-0.049}$ $\alpha \equiv dn/d\ln k = -0.053 \pm 0.025$

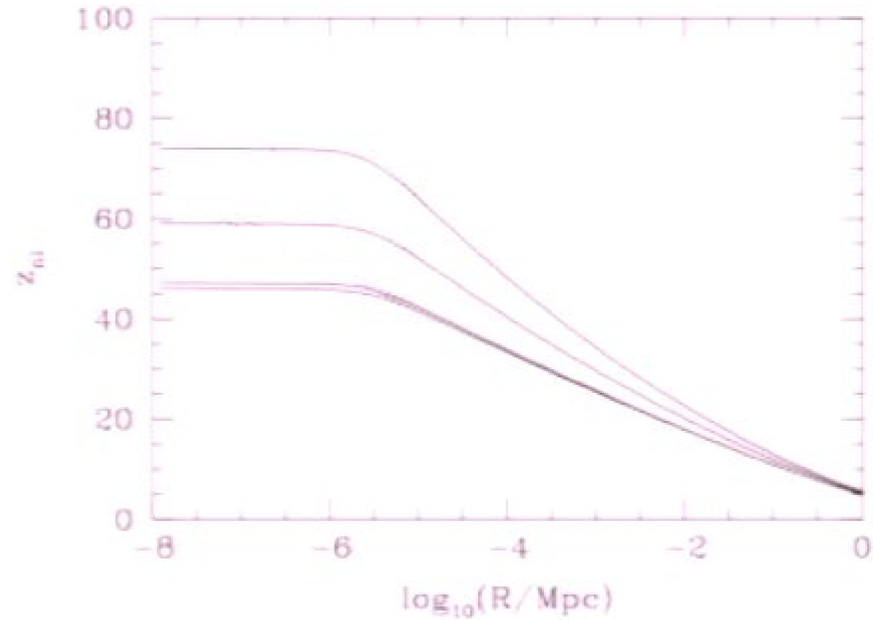
And

even a small scale dependence can lead to substantial variations in the size of the perturbations at the cut-off scale.

Processed power spectrum



z_{nl}



top to bottom:

false vacuum dominated hybrid inflation	n=1.036,	α=0
scale invariant	n=1.000,	α=0
power law inflation	n=0.964,	α=0
$m^2\Phi^2$ chaotic inflation	n=0.964,	α=-0.0006

$$\alpha \equiv \frac{dn}{d\ln k}$$

n.b.:

i) these simple models chosen to bracket range of n values compatible with WMAP first year results

ii) primordial power spectrum might not be scale free

Summary

i) WIMP direct detection experiments

Probe/need to know the WIMP velocity and density distribution on sub-milli-pc scales.

ii) theoretical input into small scale structure formation

Particle microphysics

Free-streaming erases density perturbations on scales smaller than O(1/pc). Consequently first and smallest WIMP halos to form typically have mass *roughly* of order the Earth mass.

Exact value of cut-off mass depends on the kinetic decoupling temperature (which depends on the WIMP elastic scattering cross-section) and can vary by many orders of magnitude.

Accurate calculation of detailed shape of processed power spectrum requires numerical treatment.

Primordial power spectrum

Cut-off scale is many orders of magnitude smaller than the scales probed by large scale structure observations.

Even small deviations from scale invariance (as found by WMAP CMB-LSS analysis) can lead to a factor of a few change in the amplitude of density perturbations on the smallest scales