Title: On the epistemic view: Strengths and weakneses of Spekkensâ€™ toy theory

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Abstract: We investigate the strengths and weaknesses of the Spekkens toy model for quantum states. We axiomatize the Spekkens toy model into a set of five axioms, regarding valid states, transformations, measurements and composition of systems. We present two relaxations of the Spekkens toy model, giving rise to two variant toy theories. By relaxing the axiom regarding valid transformations a group of toy operations is obtained that is equivalent to the projective extended Clifford Group for one and two qubits. However, the physical state of affairs resulting from this relaxation is undesirable, violating the desideratum that single toy bit operations must compose under the tensor product. The second variant toy theory is obtained by relaxing the axioms regarding valid states and measurements, resulting in a toy model that exhibits the Kochen-Specker property. Like the previous toy model, the relaxation renders the toy model physically undesirable. Therefore, we claim that the Spekkens toy model is optimal; altering its axioms does not yield a better epistemic description of quantum theory. This work is a collaboration with Gilad Gour, Aidan Roy and Barry C. Sanders.

Toy models based on the epistemic view for quantum states

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1. Background and Motivation
2. The Spekkens Toy Model
3. Relaxing STM
   - Relaxing axiom regarding valid operations
   - The CTM
4. Summary

Background

- Zeilinger's Foundational Principle—Simplest system holds one bit of information
- Caves, Fuchs, and Shack—Wavefunction represents subjective information
- Clifton, Bub, and Halvorson—Three information constraints out of which quantum theory arises
- Kirkpatrick—Quantumness in a
- Spekkens—Exact knowledge is incomplete knowledge

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Motivation

- What classical principles can we employ in order to shed some light into quantum theory
- What is really “quantum“ about quantum theory
- Can we extend Spekkens’ toy model to capture more quantum phenomena?
- How robust are Spekkens axioms?

Principles and Definitions

KBP: For any system, at any time, one can know only half the information regarding the ontic state of the system

Ontic state: A state of reality

Canonical Set: Minimum number of binary questions that completely specify the system

Epistemic State: A state of knowledge

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States

STM 0: All systems obey the KBP.

STM 1: The simplest toy system (toy bit) is described by a single hidden parameter $X := \{1, 2, 3, 4\}$

KBP dictates that X is known to be in a subset of 2 or 4 of the ontic states.

$$|14) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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X	1	2	3	4

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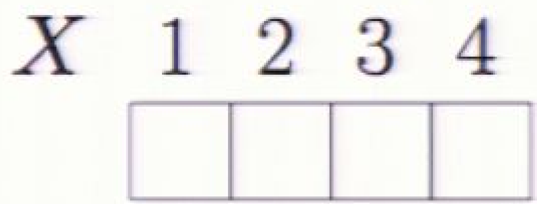

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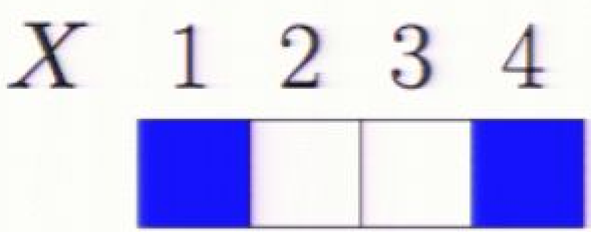

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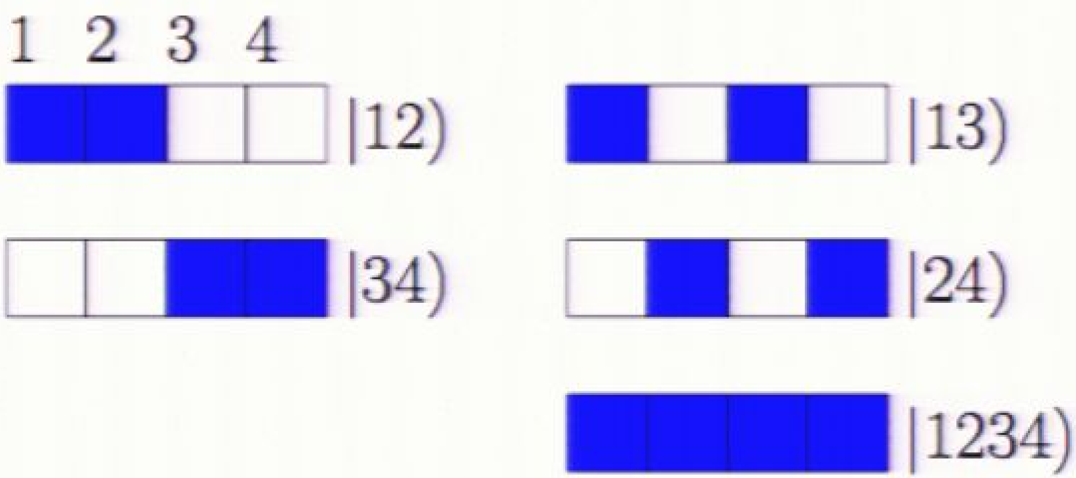
States

Representations

Box representation



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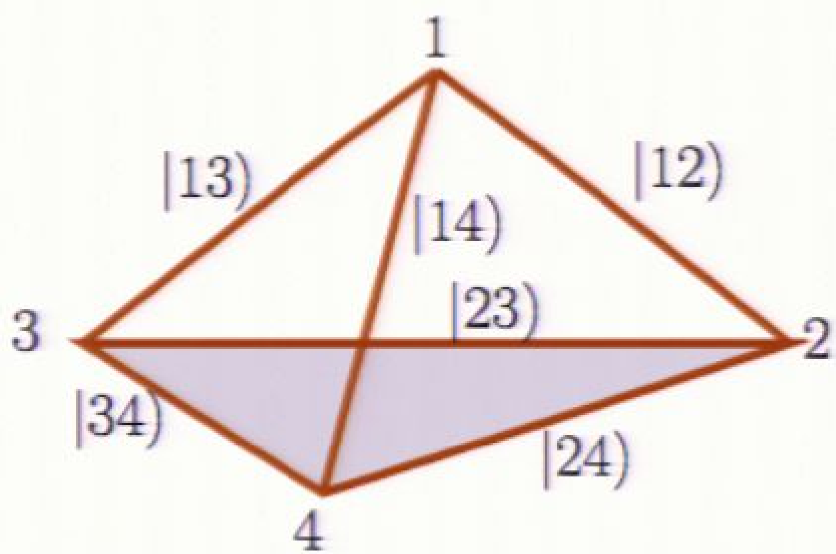
Tetrahedron Representation



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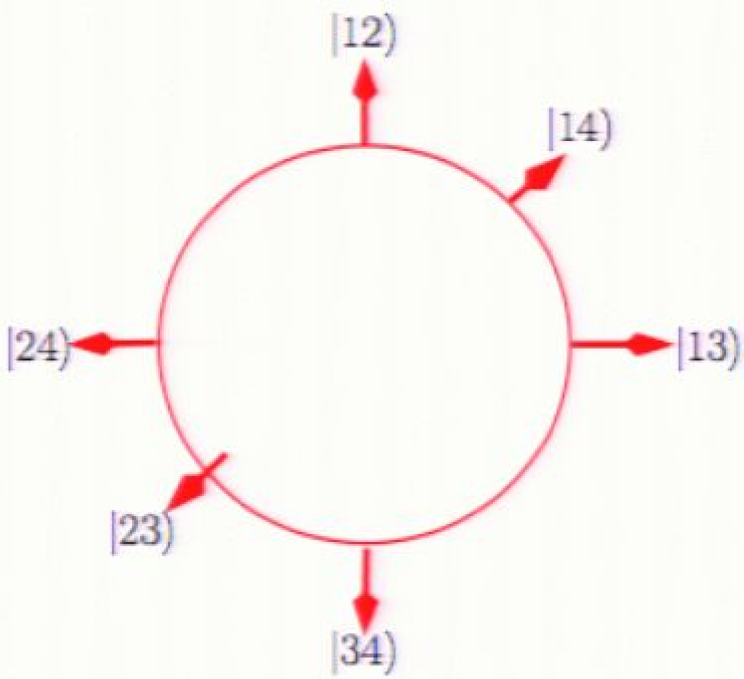
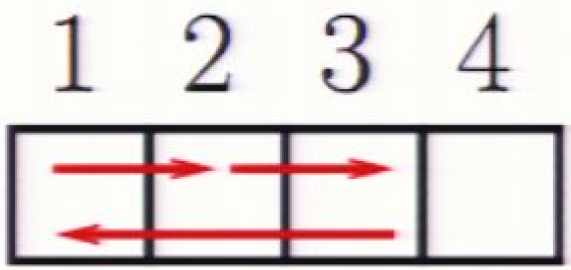
Sphere Representation



Transformations

STM 2: A valid reversible operation takes epistemic states to epistemic states by permuting ontic states.



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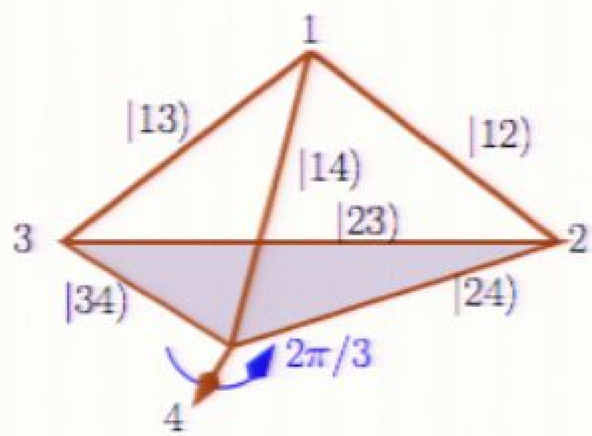

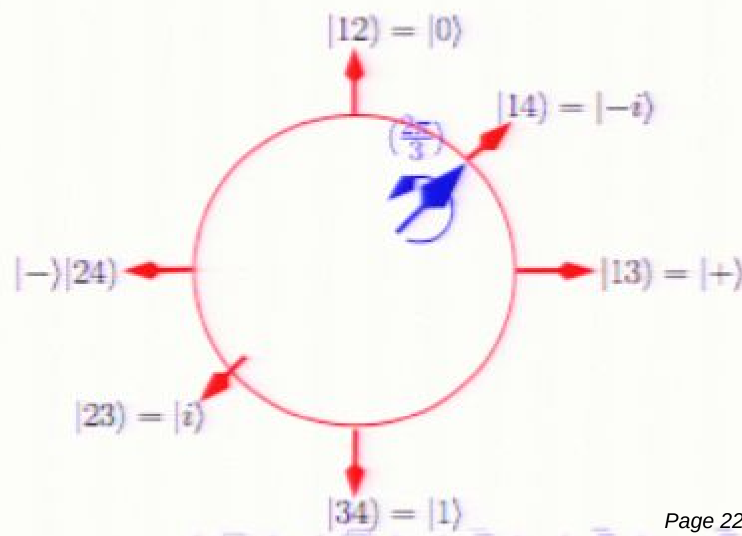

Measurements

STM 3: A reproducible measurement is a partition of the ontic states into a disjoint set of epistemic states.

After the measurement the epistemic state of the system is updated to the outcome of the measurement.

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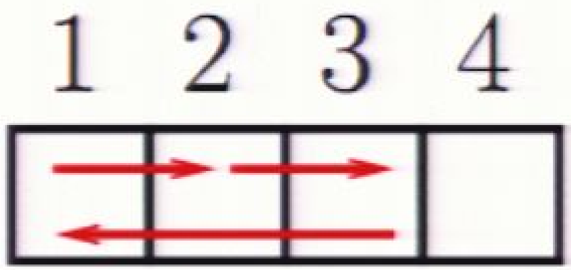

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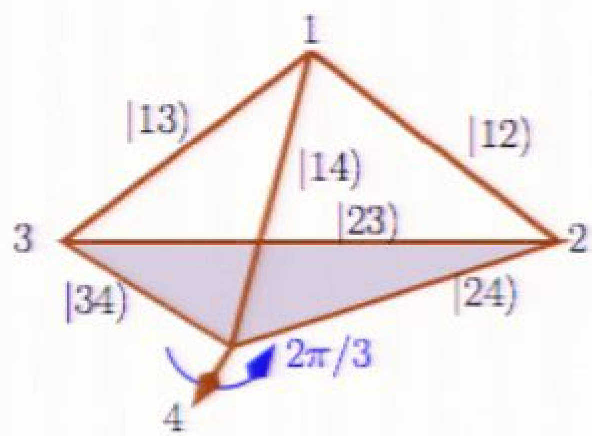

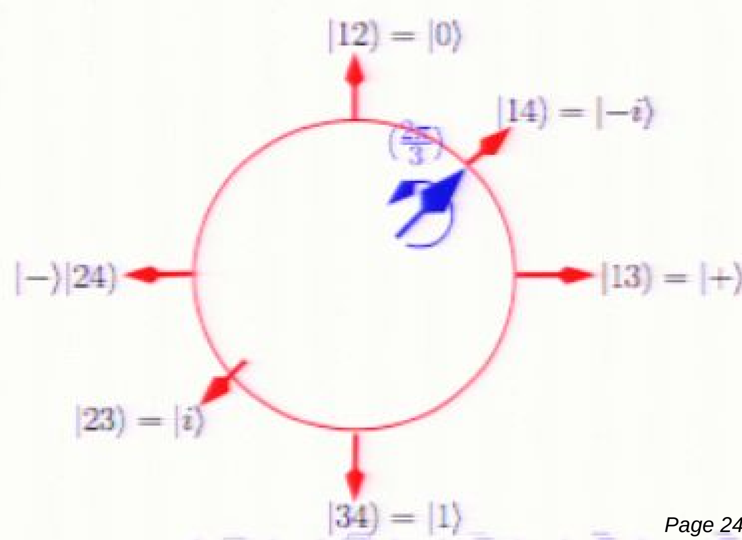

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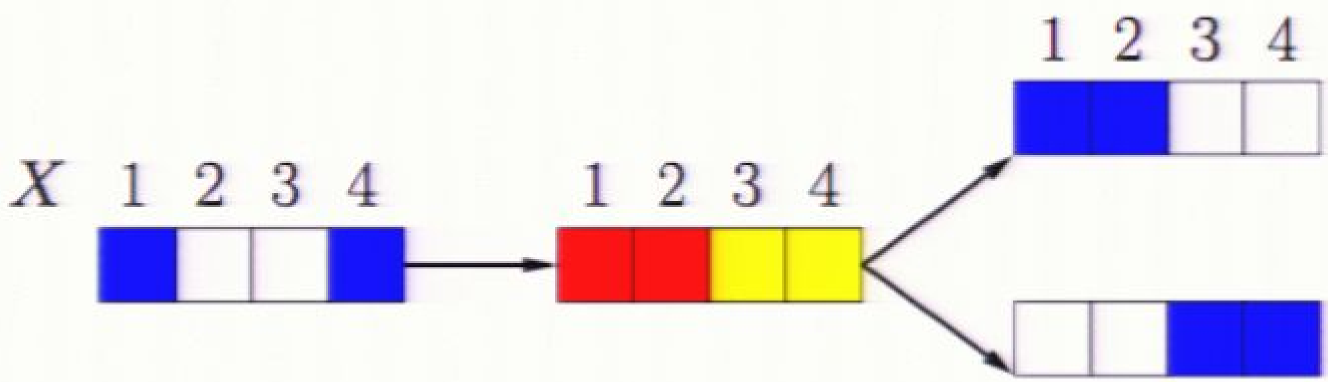

Composition of systems

STM 4: Elementary systems compose under the tensor product. KBP applies to the composite system as well as to the parts.

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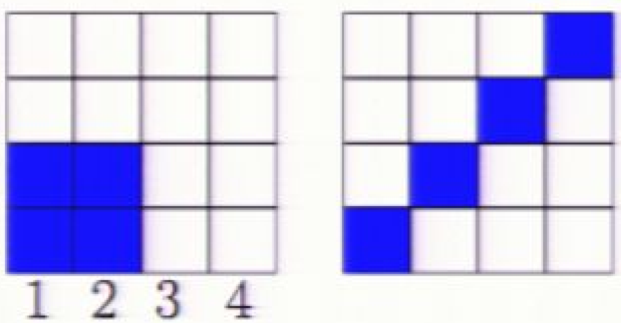

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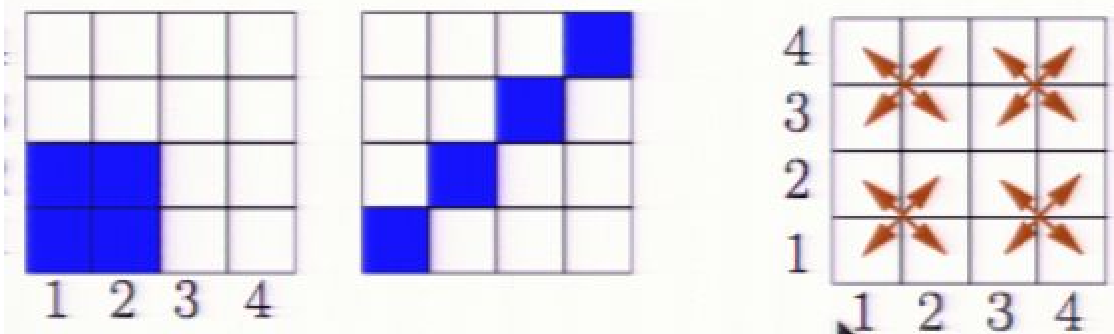

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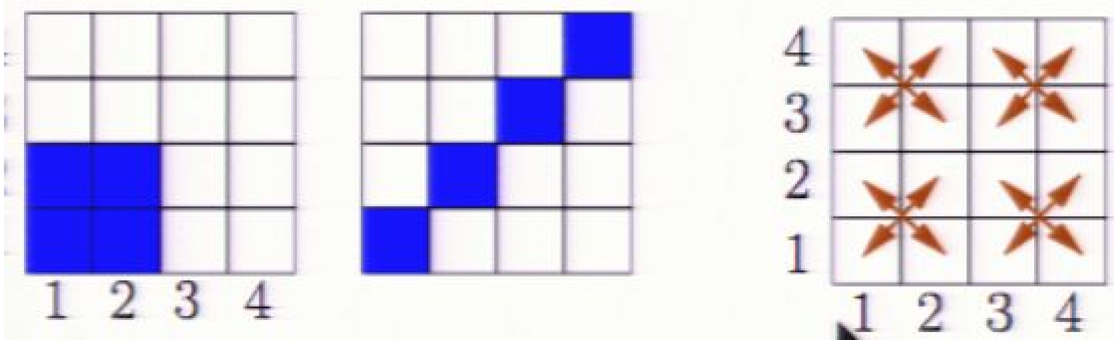

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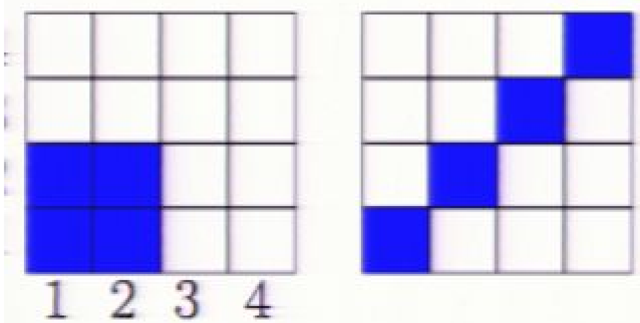

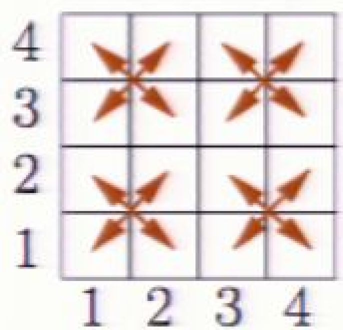

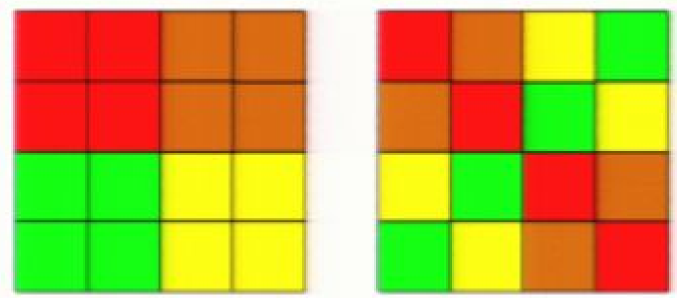

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Dos and Don'ts

Dos

- Non-commutativity of measurement
- No-cloning and no-broadcasting
- Remote Steering
- Teleportation

Dont's

- Violation of Bell-CHSH
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The epistemic view

- STM's valid operations are deterministic on the ontic scale
- The toy universe is epistemic
- Empiricism—knowledge from experience
- Valid operations map epistemic states to epistemic states

Loosening constraints

STM 2′: A valid reversible operation is a linear transformation that permutes the epistemic states of the system.

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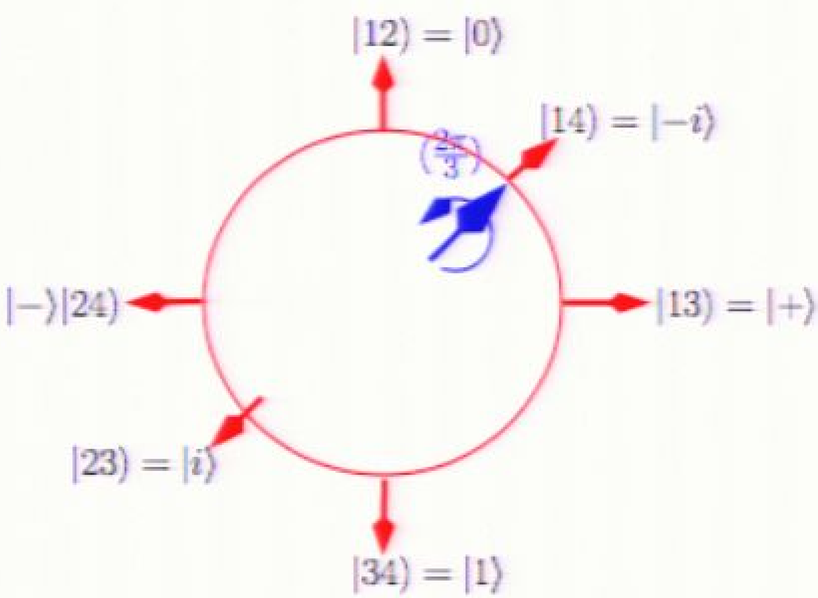

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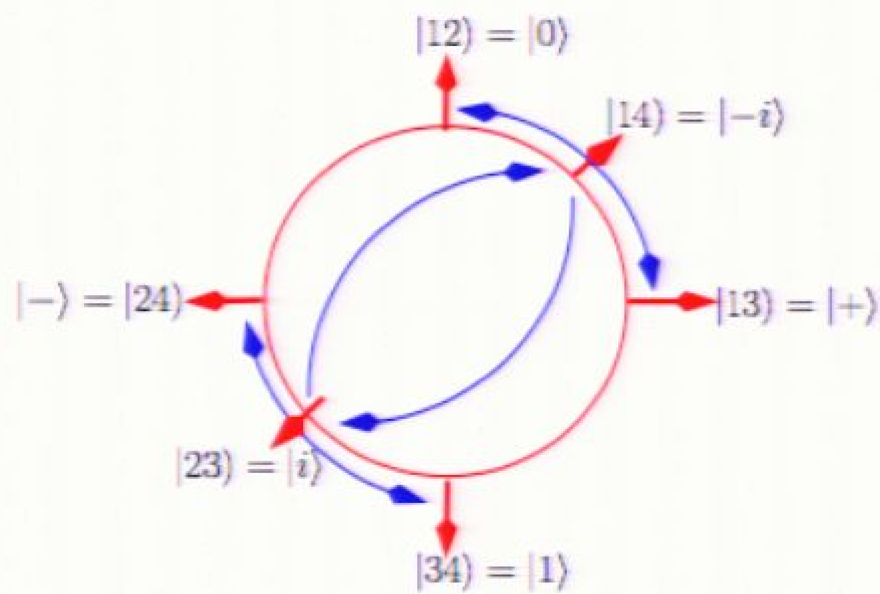

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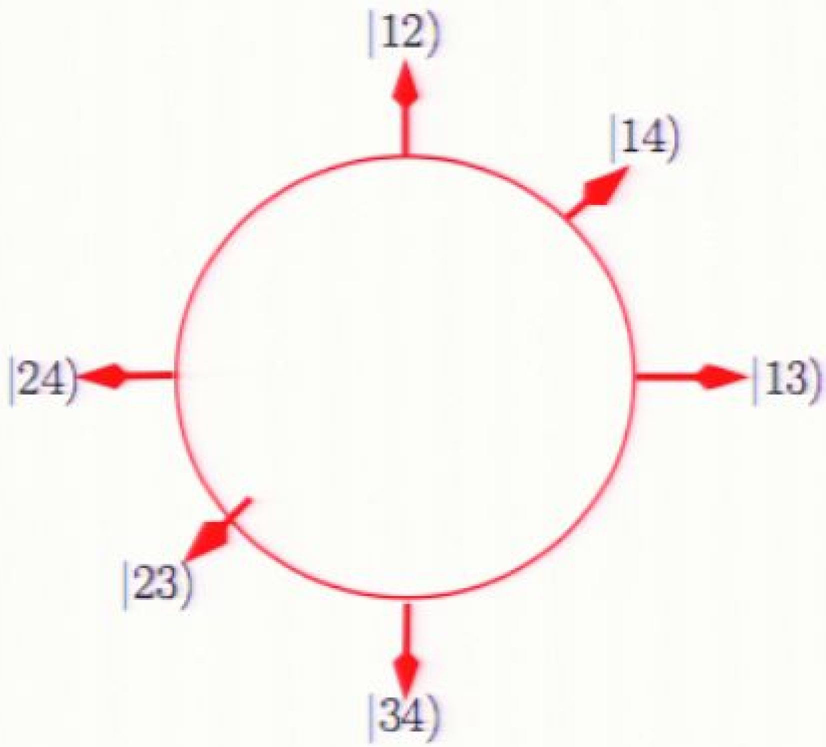

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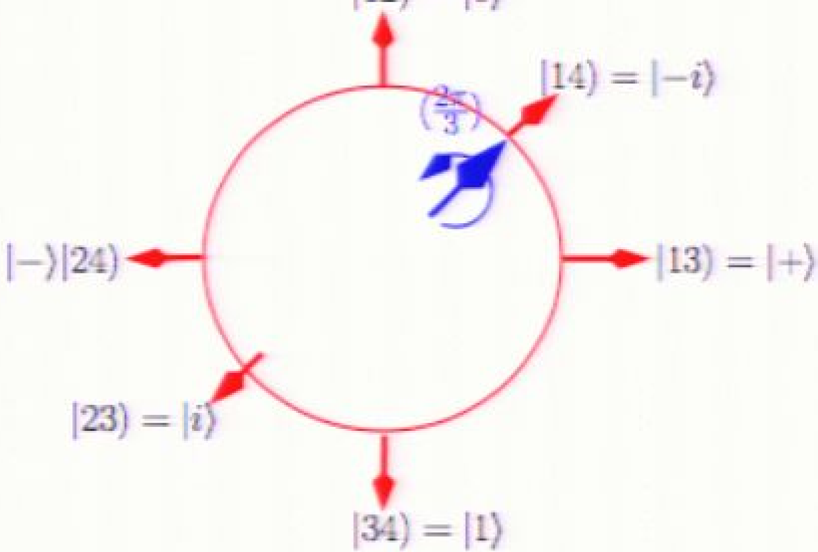

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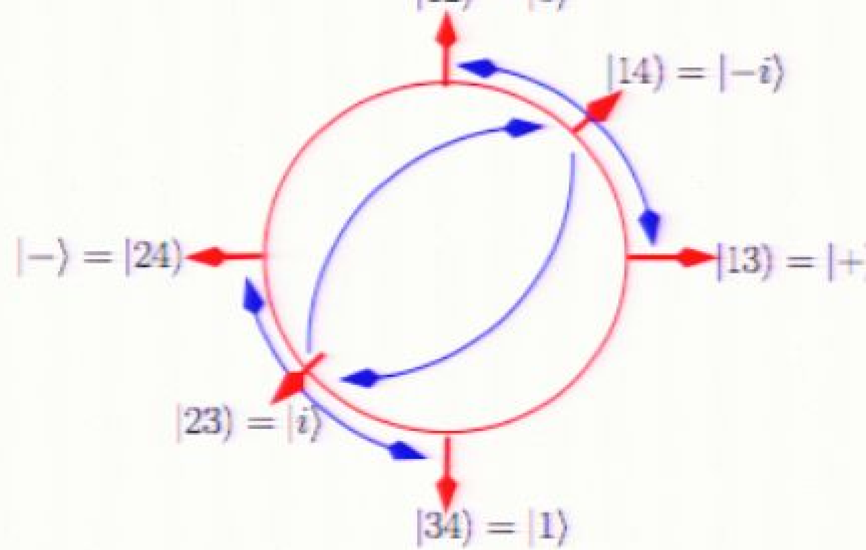

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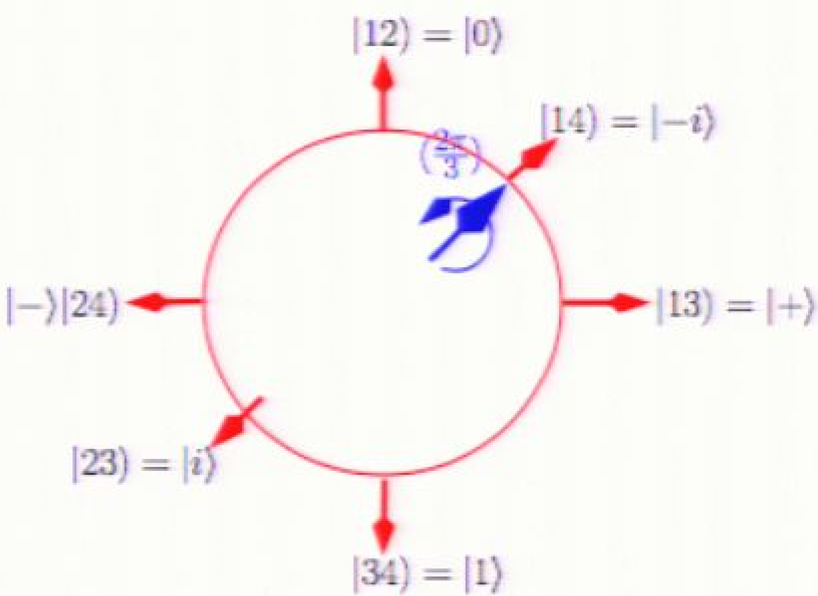

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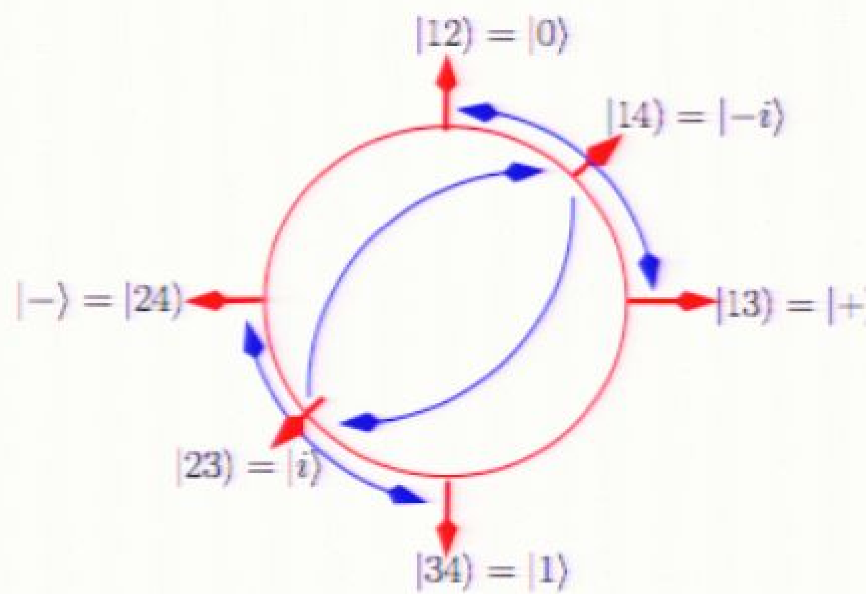

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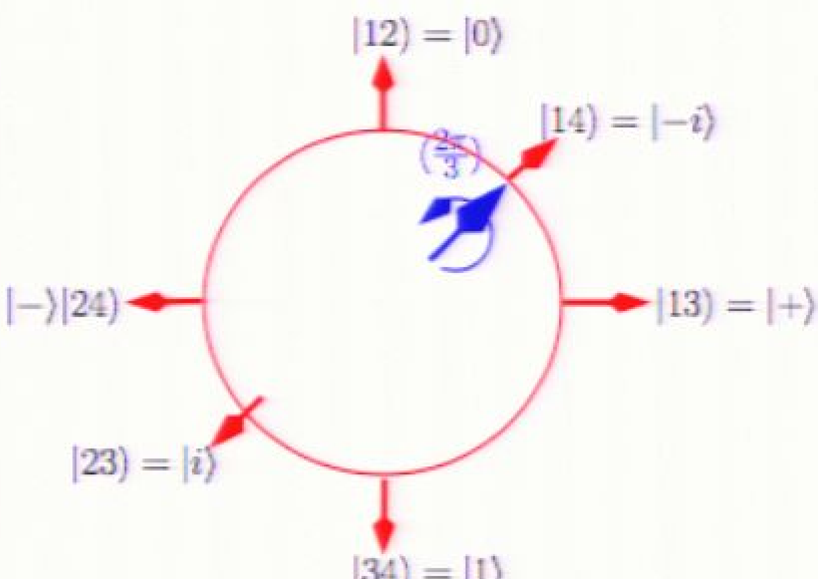

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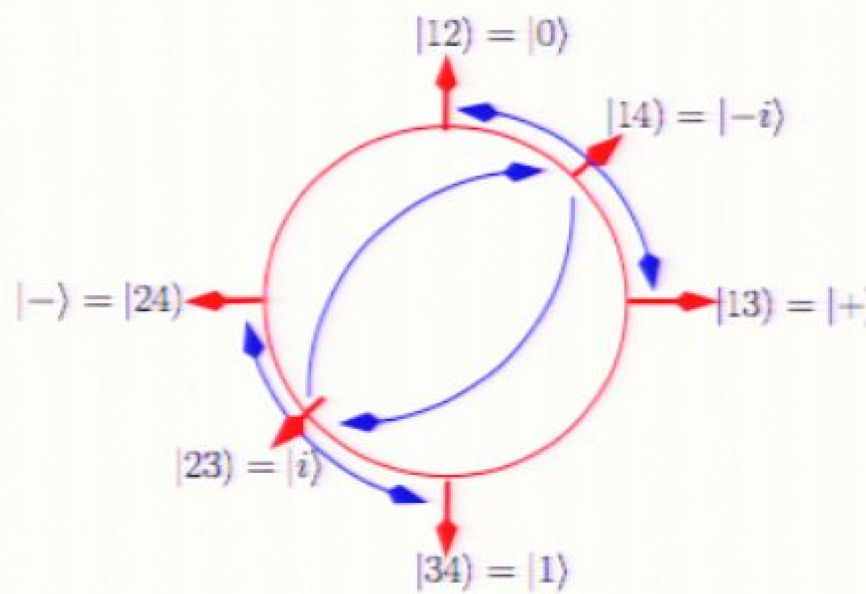

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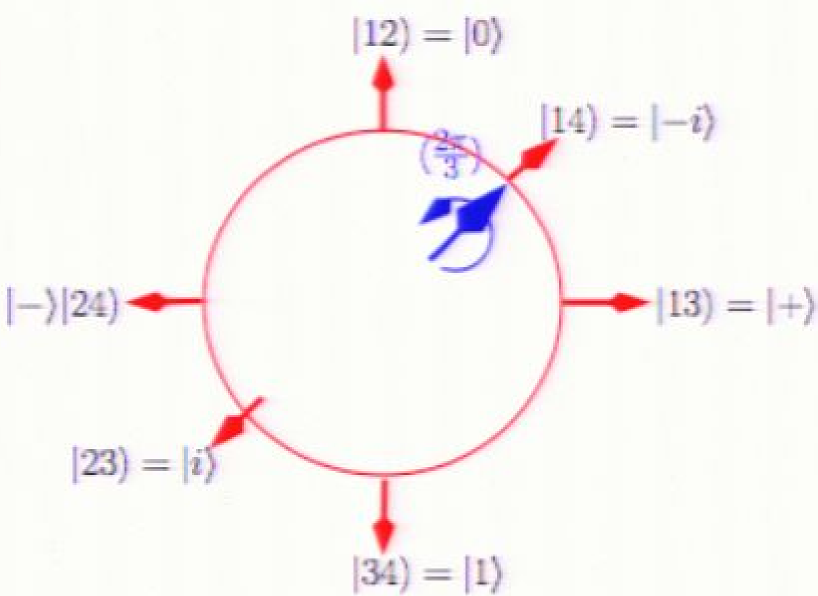

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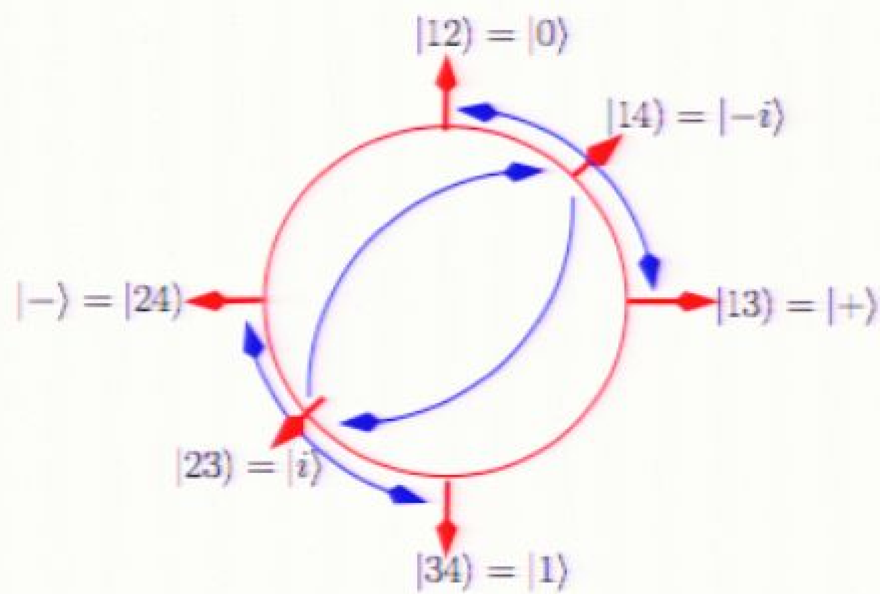

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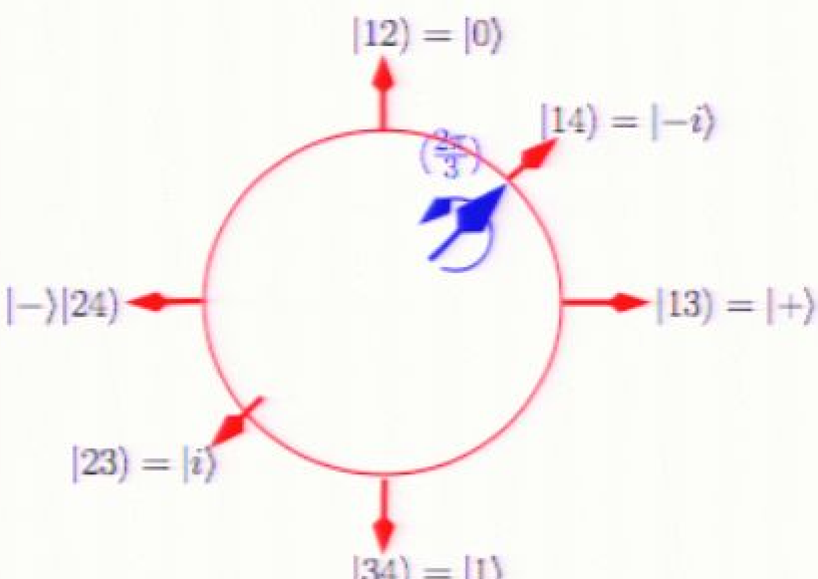

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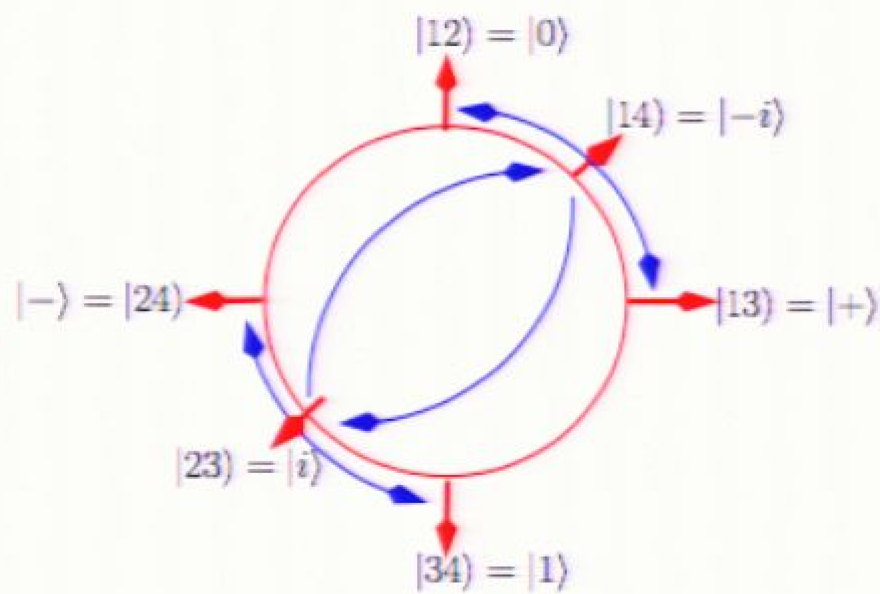

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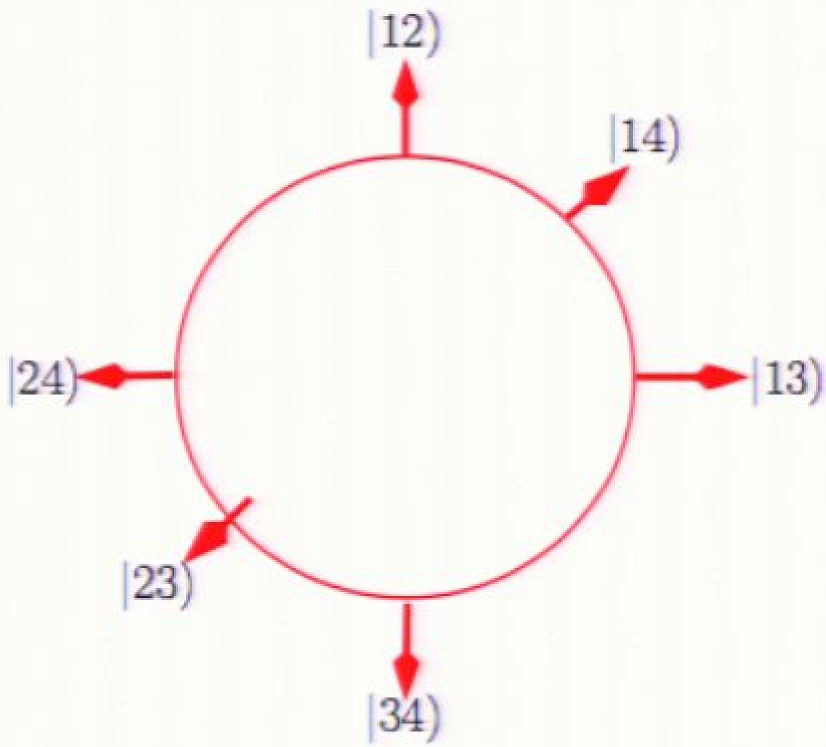

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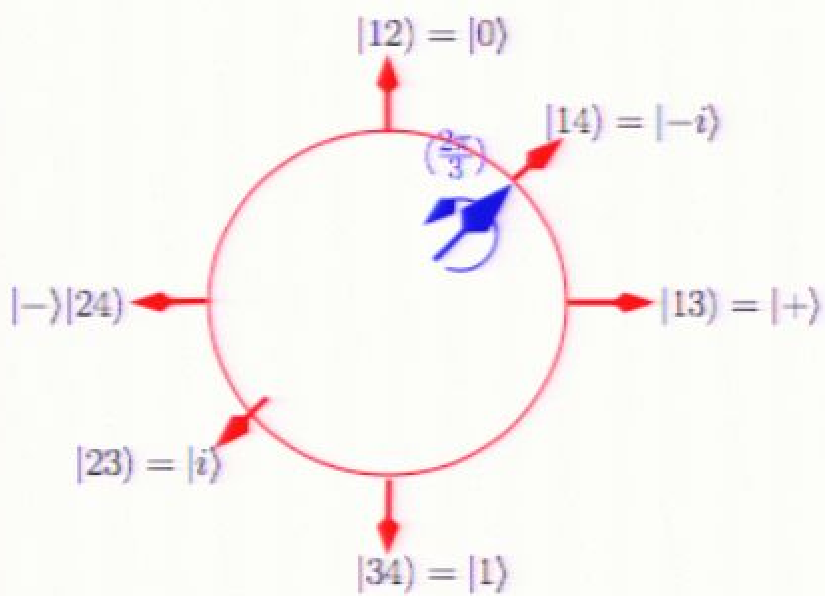

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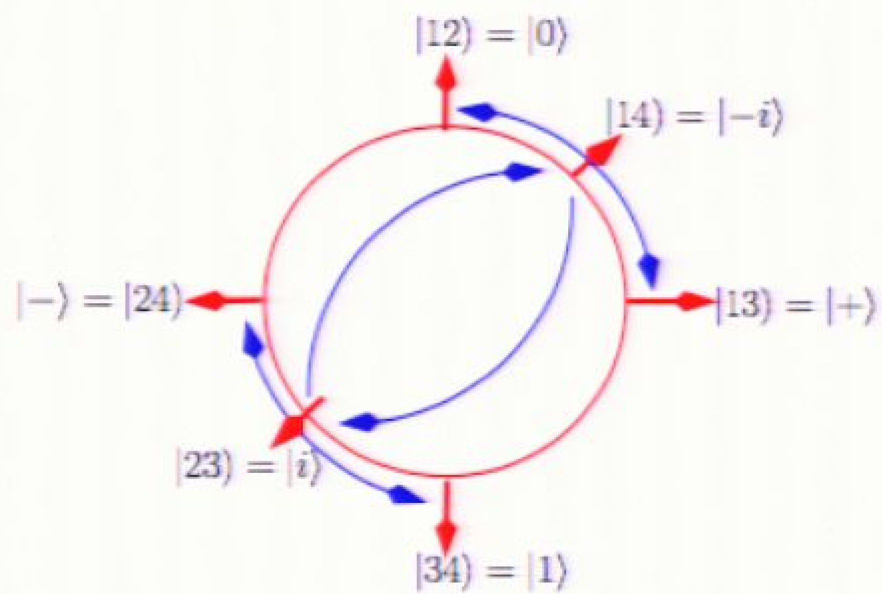

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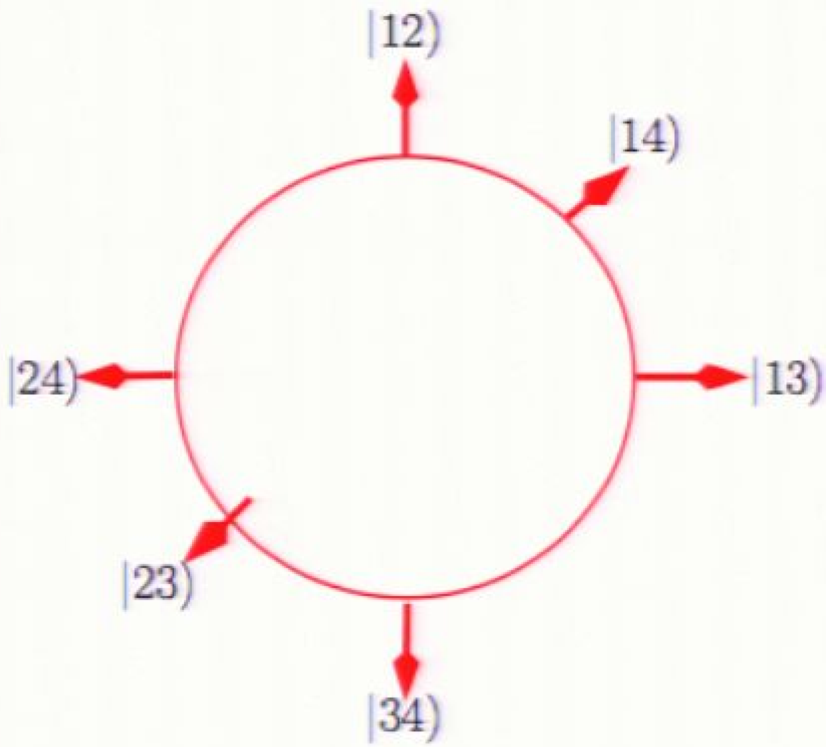

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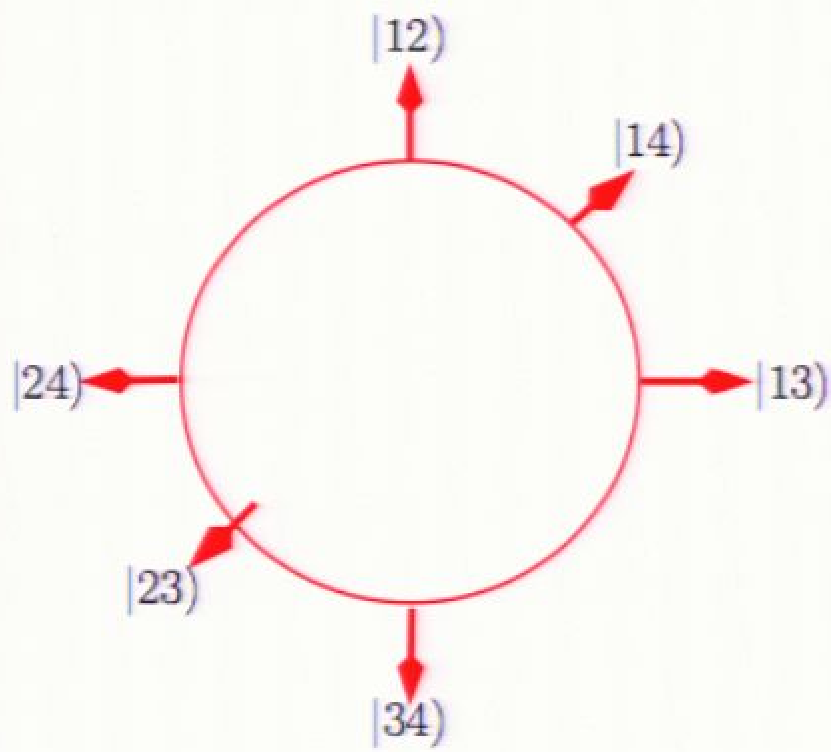

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- If $A \in TG(1)$ and $A \notin S_4$ then $I \otimes A$ is not a valid operation.

- Group of valid two toy bit operations, $|TG(2)| = 23040$

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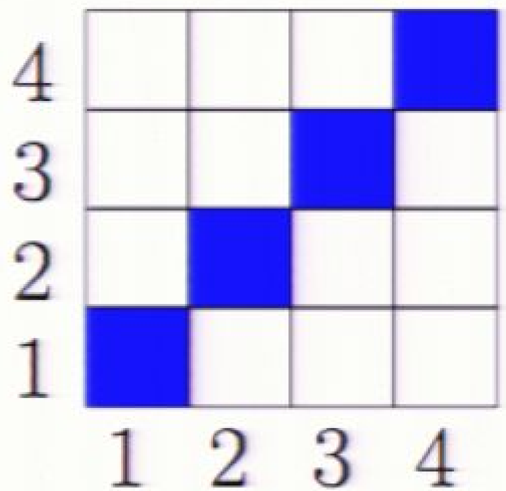

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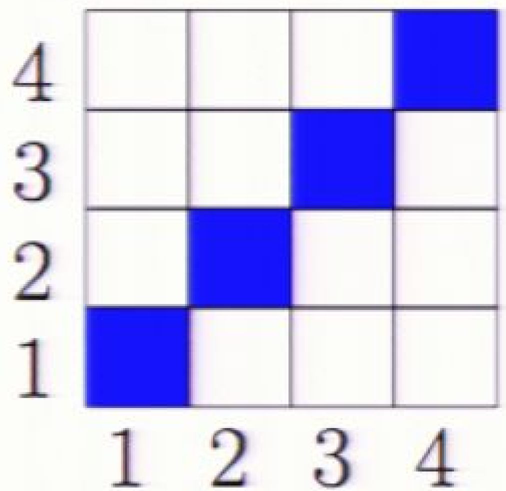

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Clifford Group $\mathcal{C}$, all rotations that map $\mathcal{P}$ to itself under conjugation.

Projective Clifford Group $\mathcal{C}/U(1)$, all rotations that map $\mathcal{P}$ to itself under conjugation, where $U(1) := \{e^{i\theta}U \sim U, U \in U(2)\}$.

Extended Clifford Group $\mathcal{EC}$, all rotations and reflections that map $\mathcal{P}$ to itself under conjugation.

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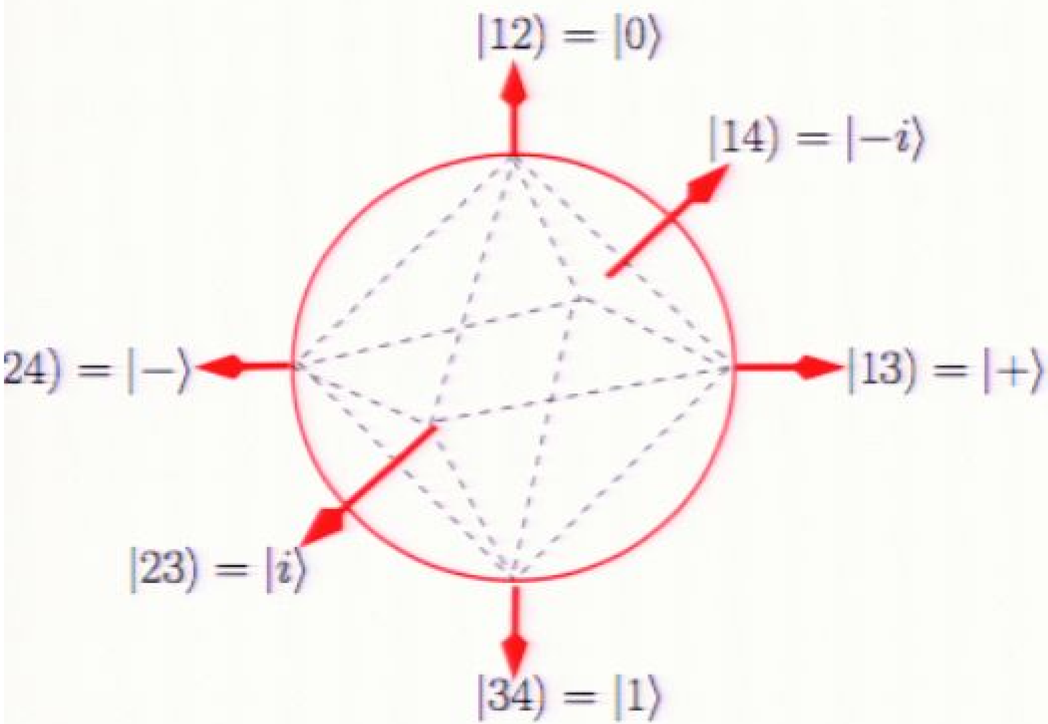
Isomorphisms

Single Toy bit



- $|\mathcal{C}(1)/U(1)| = 24|$ is the symmetry group of the octahedron under rotations.
- Isomorphic to S_4
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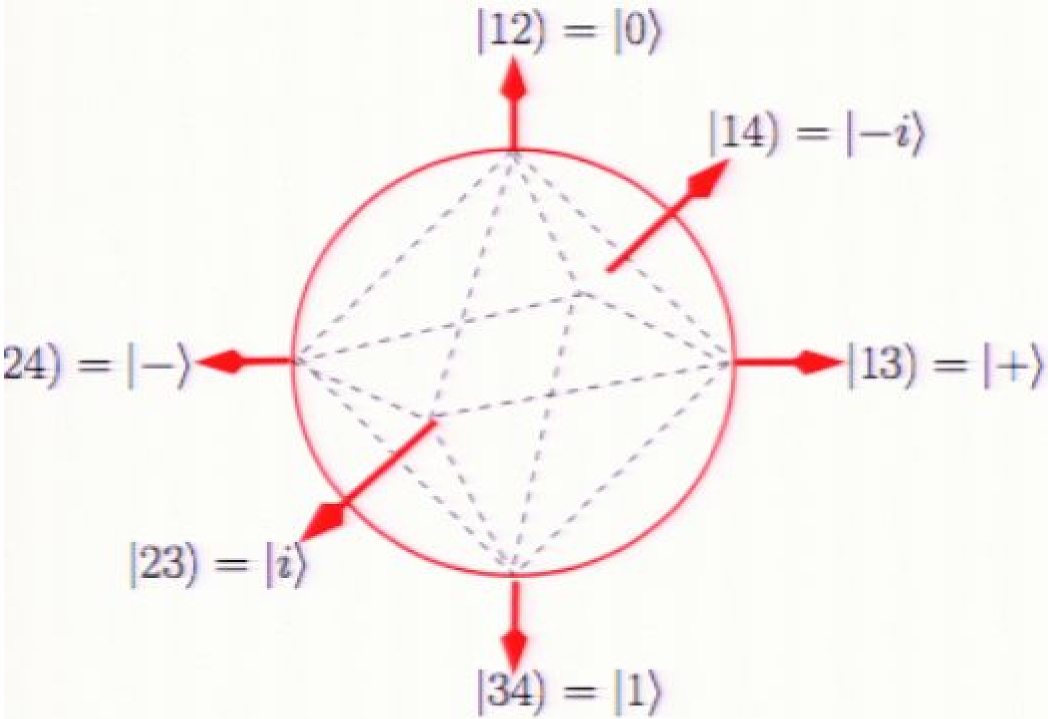

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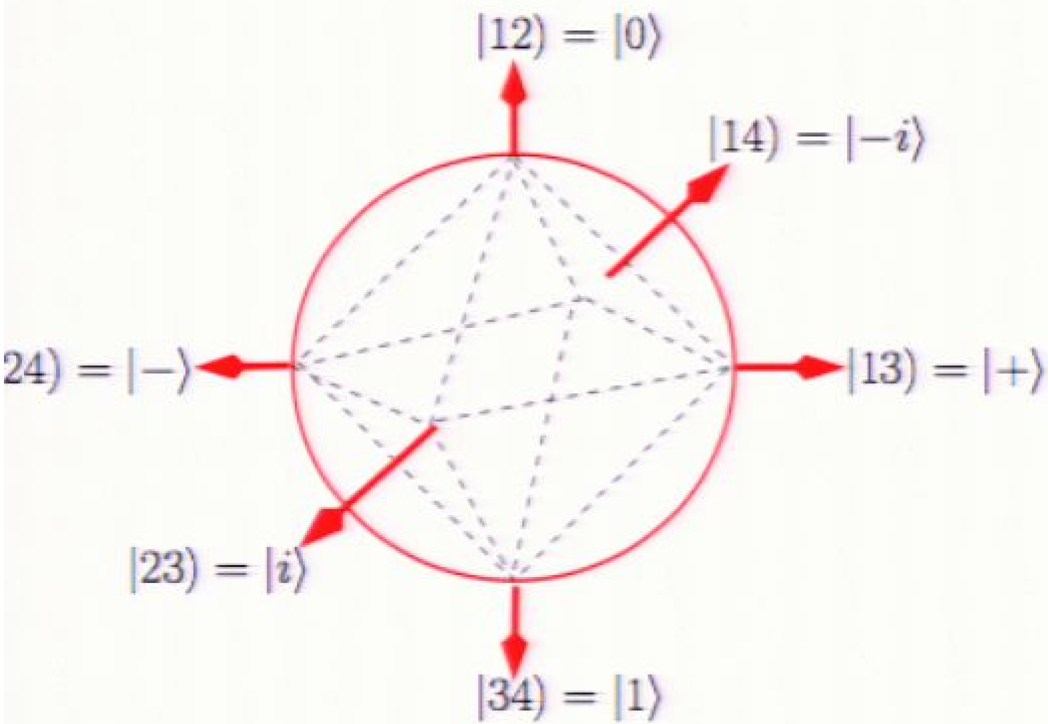

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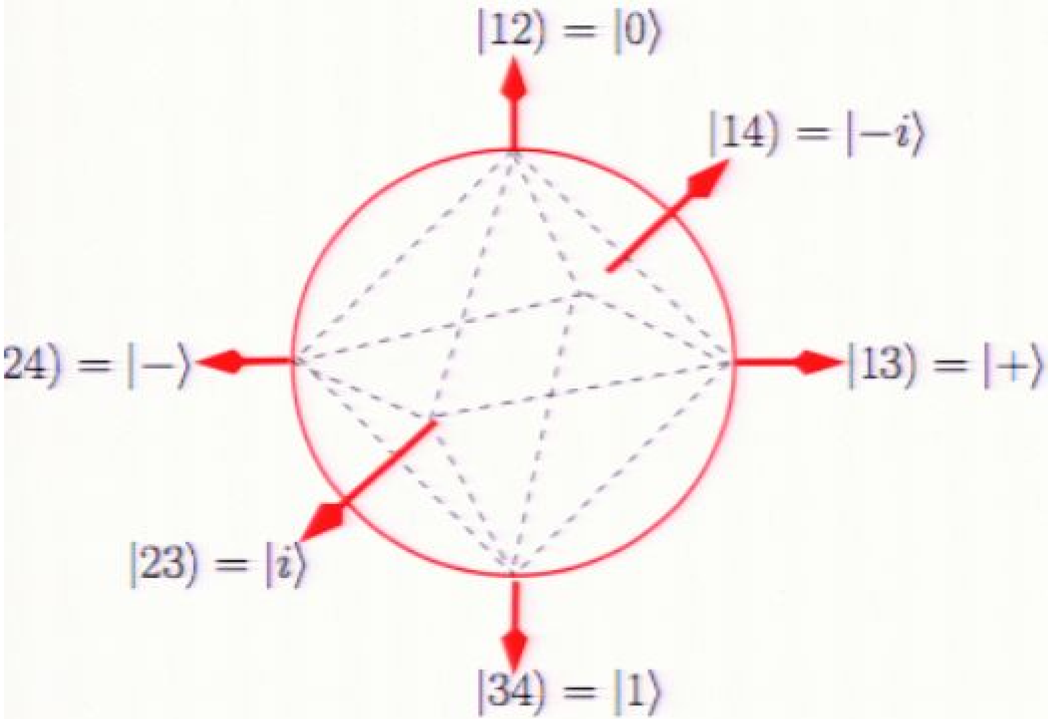

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- $|\mathcal{C}(2)/U(1)| = 11520|$, but it is not isomorphic to $SG(2)$
- $SG(2)$ contains a maximal subgroup of order 720, $\mathcal{C}(2)/U(1)$ does not.
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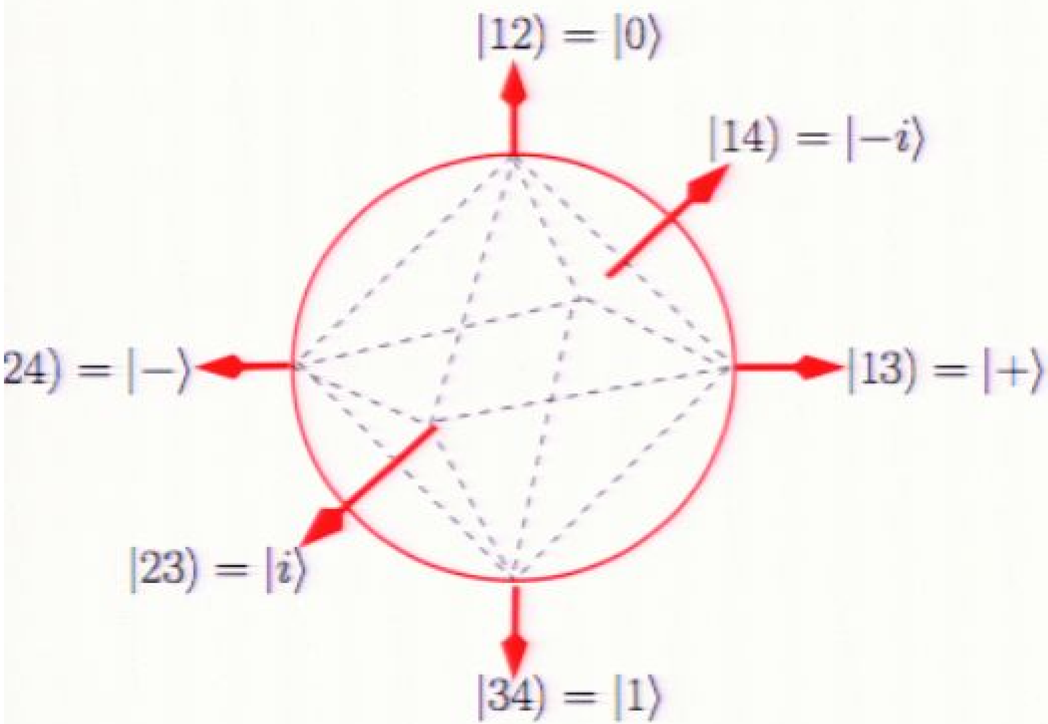

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Positive and Completely Positive Maps

Positive Maps: A map Δ is positive if it takes $\rho \geq 0$ to $\Delta\rho \geq 0$

Completely Positive Maps: Δ is completely positive if for $\rho \geq 0$, then $(\Delta \otimes I)\rho \otimes |0\rangle\langle 0| \geq 0$

Separable Operators: $\varrho \in \mathcal{H}_1 \otimes \mathcal{H}_2$ is separable if it can be written in the form
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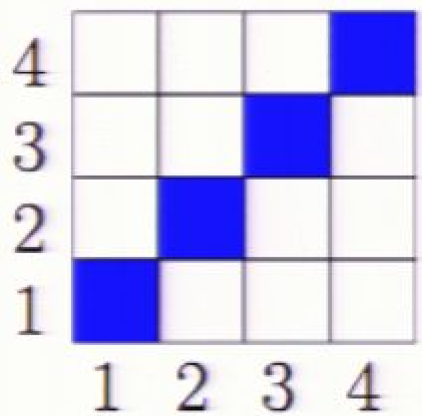

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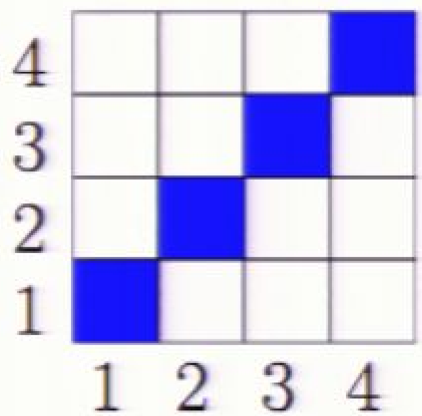

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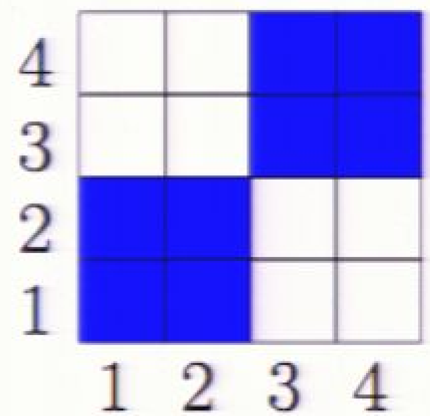

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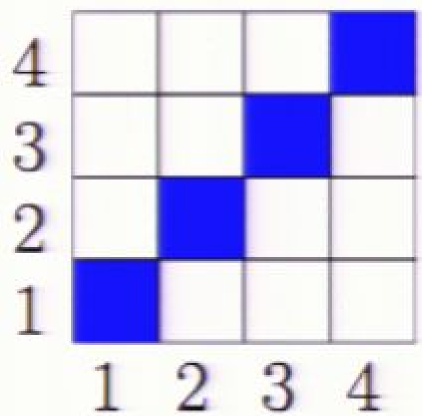

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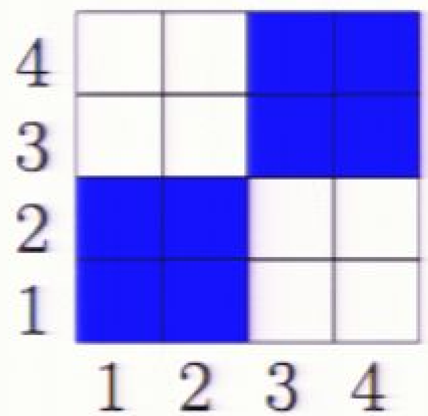

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Results

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Toy model with STM 2′

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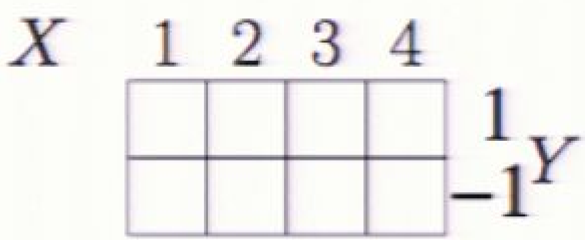

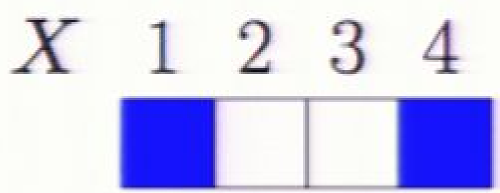

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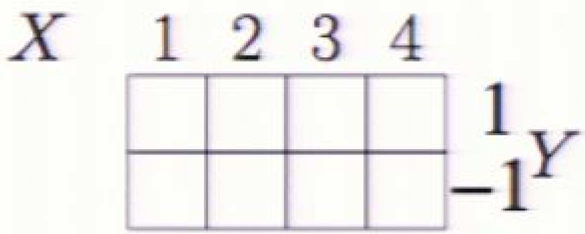

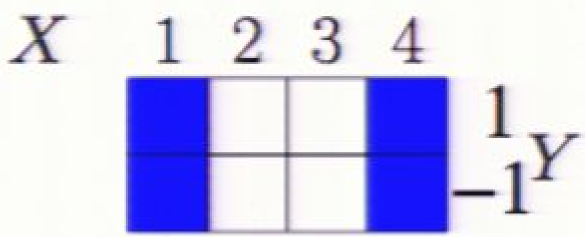

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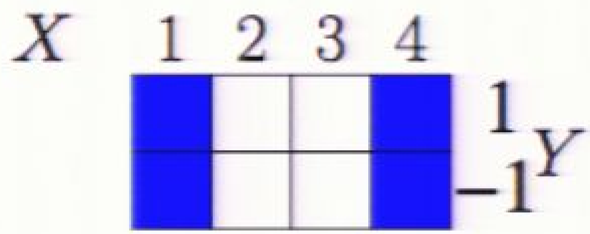

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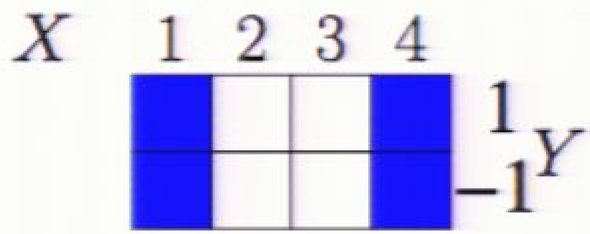
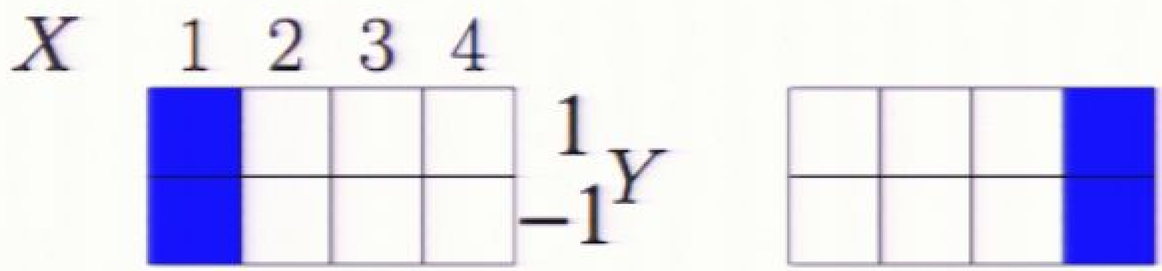


Not possible



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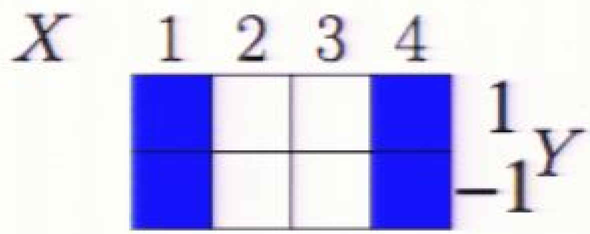

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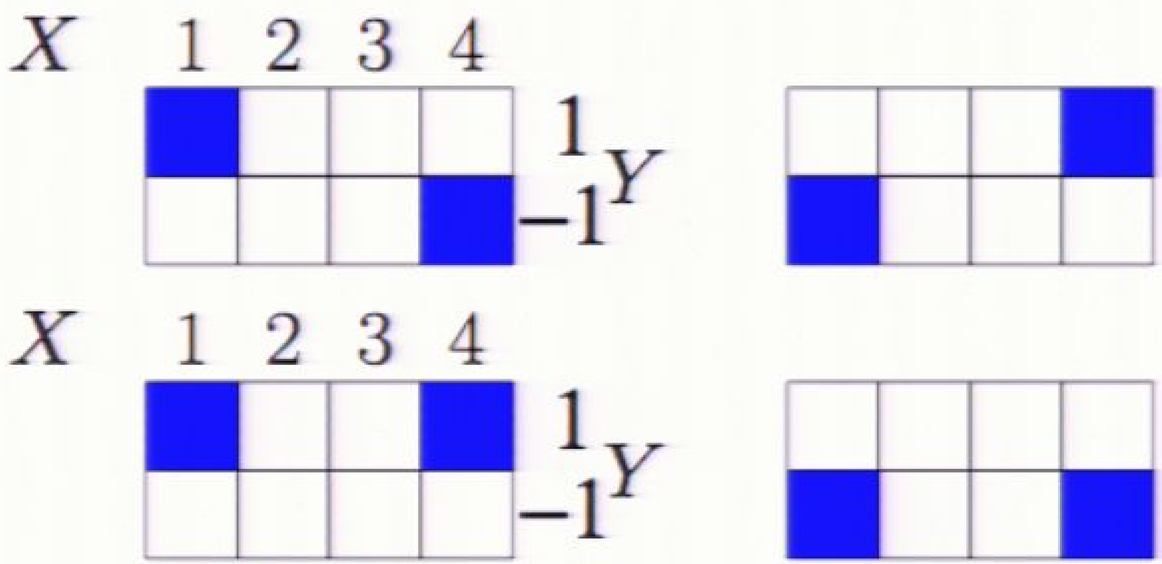

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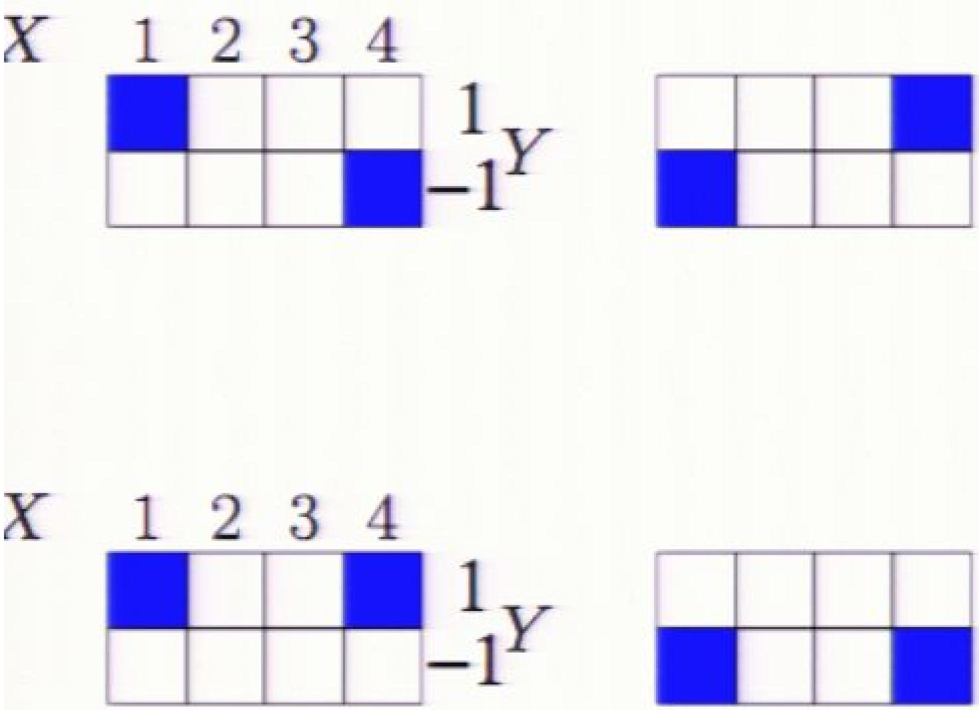

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- We can know how the state was prepared.
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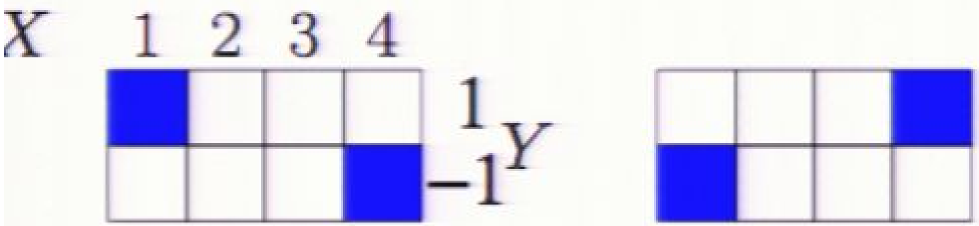

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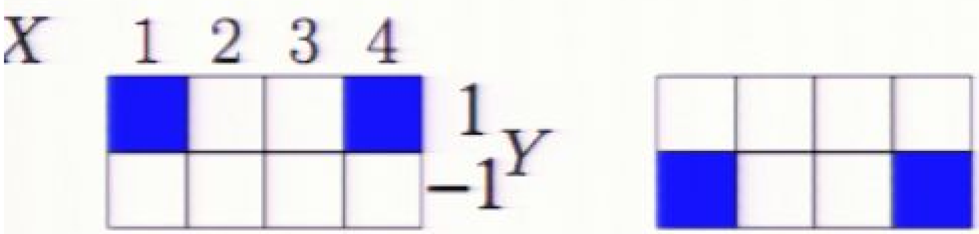

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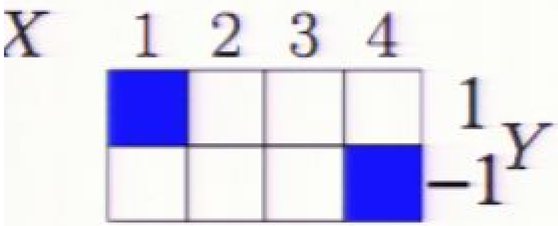

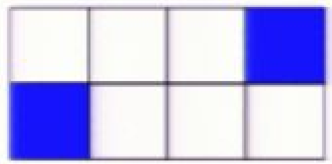

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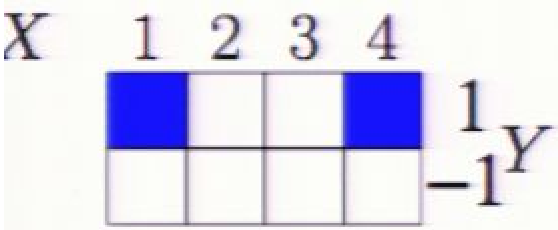

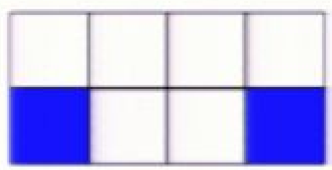

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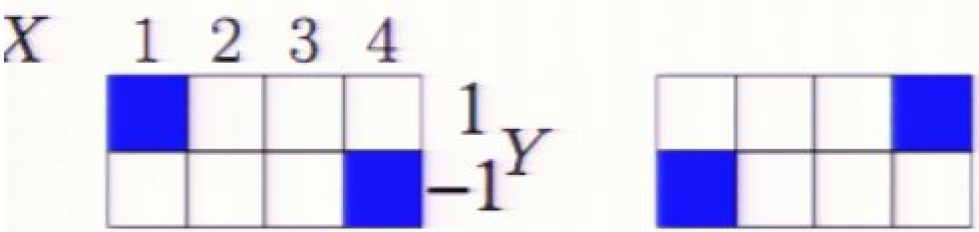

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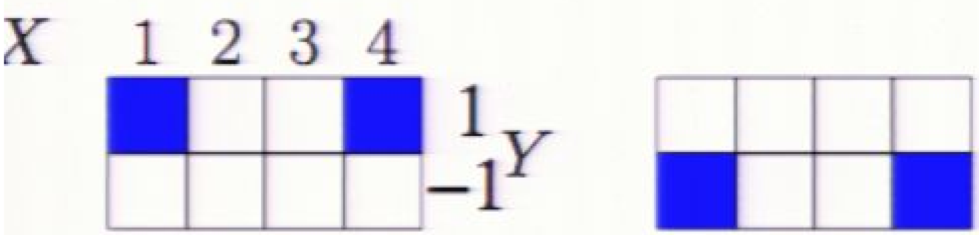

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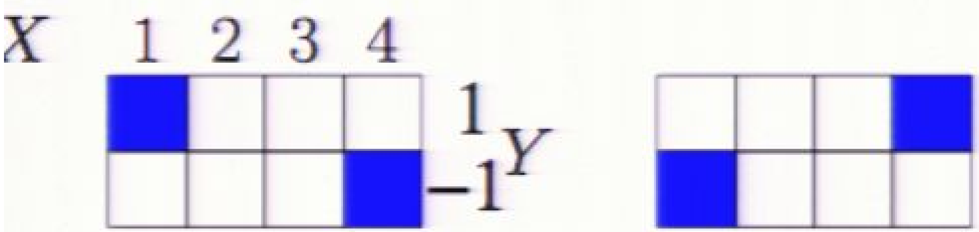

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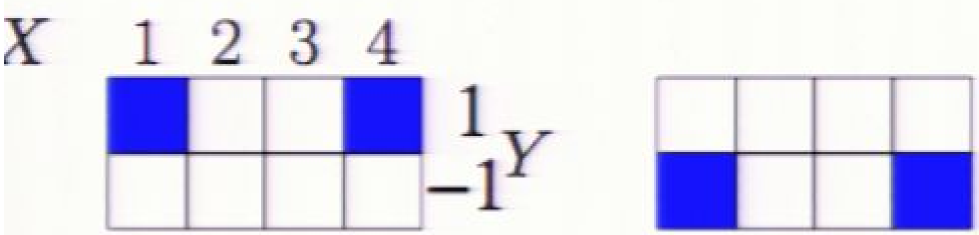

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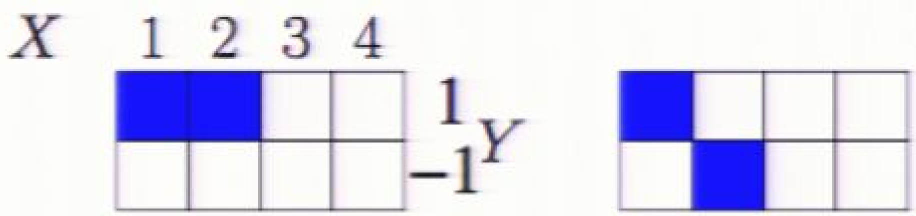

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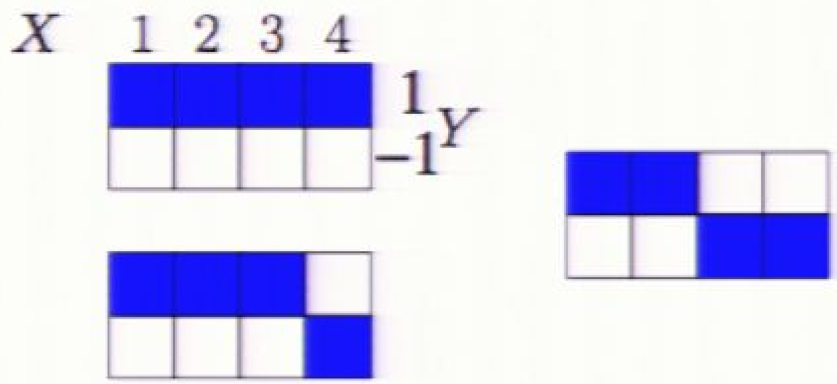

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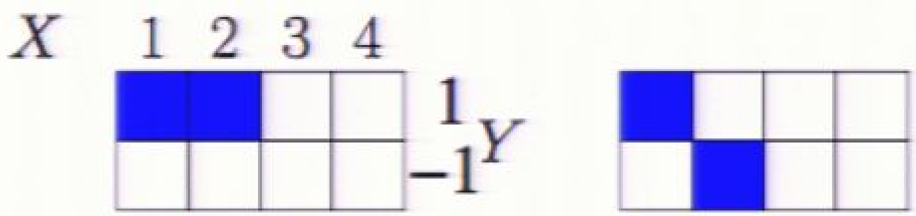

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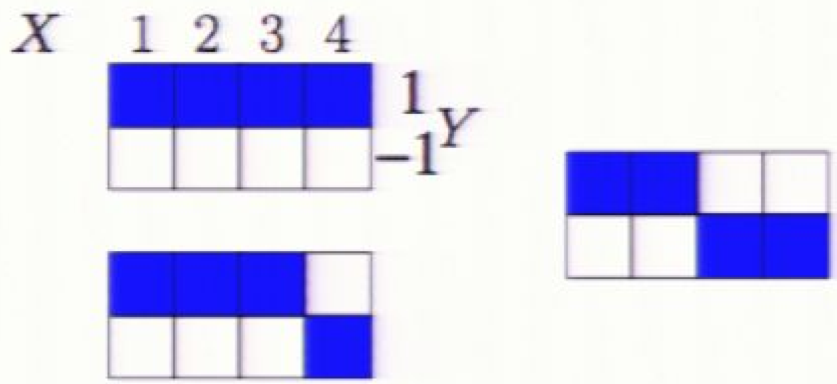

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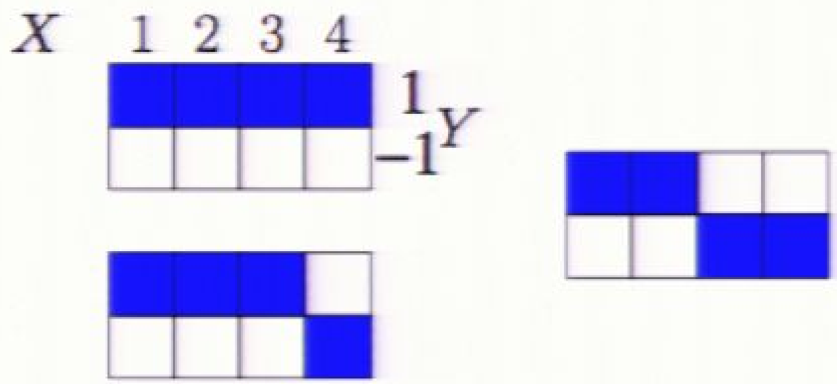

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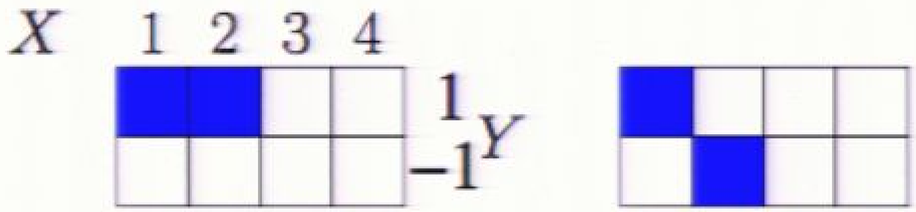

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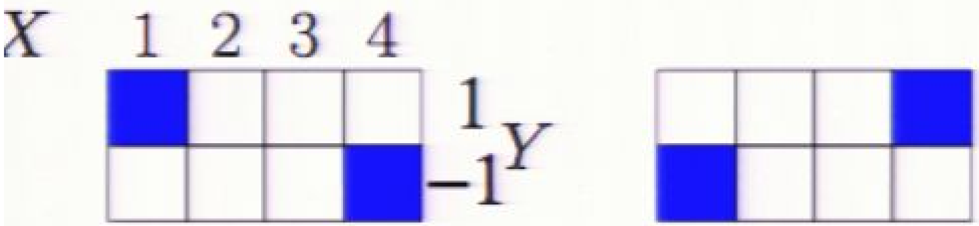

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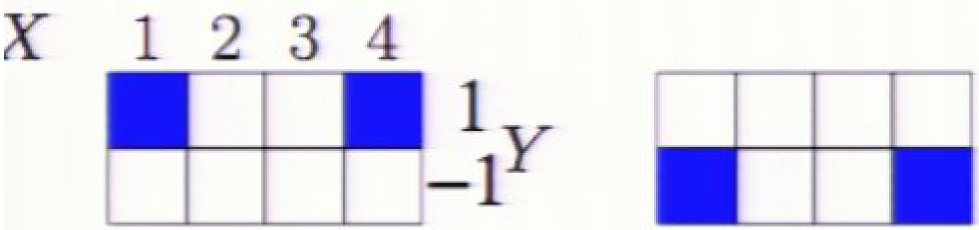

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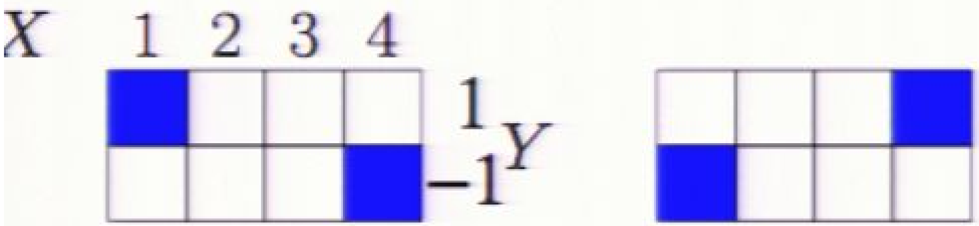

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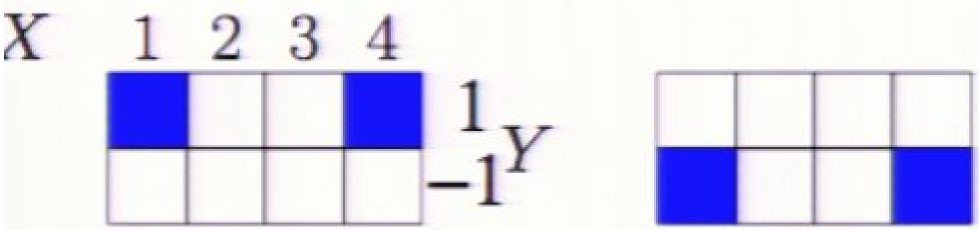

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CT 4: Elementary systems compose under the tensor product. KBP on X and the indeterminacy of Y apply to the composite system as well as to the parts.

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	1	2	3	4
4				
3				
2	+	−		
1	+	−		

			−
		−	
	+		
+			

- Product states either have all, or 2 positive Y values
- Correlated states have either all, 1, or 2 positive Y values
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- Two types; tensor products of $A \in CG(1)$, or 16×16 matrices
- Two toy bit group of operations, $|CG(2)| = 23,592,960$

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$$
\begin{matrix}
I \otimes \sigma_z & \sigma_z \otimes I & \sigma_z \otimes \sigma_z \\
\sigma_x \otimes I & I \otimes \sigma_x & \sigma_x \otimes \sigma_x \\
\sigma_x \otimes \sigma_z & \sigma_z \otimes \sigma_x & \sigma_y \otimes \sigma_y
\end{matrix}
$$

- the operators of every row or column commute;
- each of the nine operators has eigenvalues ± 1;
- each operator in a given row or column is the product of the other two, except in the third column

$$(\sigma_z \otimes \sigma_z)(\sigma_x \otimes \sigma_x) = -(\sigma_y \otimes \sigma_y).$$

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Results II

STM

- All variables governed by KBP
- 6 single, 60 two toy bit states
- 24 single, 11520 two toy bit operations
- No Bell-CHSH or Kochen-Specker

CTM

- KBP affects only some of the variables, while others are completely indeterminate.
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- Relaxed valid operations, from ontic to epistemic determinism
- Two toy bit group isomorphic to $\mathcal{EC}(2)/U(1)$
- Model physically unreasonable (operations not completely validity preserving)
- Validity preserving maps used as test for correlations in STM
- Relaxed states and measurements, adding a parameter that is completely indeterminate
- CTM exhibits the Kochen-Specker property

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Questions

- Can KBP be altered to allow for qudits? What about a continuous set of states?
- Three or more parties?
- Any processing tasks that cannot be performed in STM but can be performed in CTM?

THANK YOU

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