Title: Gravitational waves from phase transitions

Date: Jun 10, 2008 02:00 PM

URL: http://pirsa.org/08060000

Abstract: In this talk I will analyse the stochastic background of gravitational waves coming from a first order phase transition in the early universe. The signal is potentially detectable by the space interferometer LISA. I will present a detailed analytical model of the gravitational wave production by the collision of broken phase bubbles, together with analytical results for the gravitational wave power spectrum. Gravitational wave production by turbulence and magnetic fields will also be briefly discussed.

Gravitational Waves from Phase Transitions

(an analytical study)

Chiara Caprini, IPhT
Ruth Durrer, University of Geneva
Geraldine Servant, CERN, IPhT

arXiv: 0711.2593

Outline

- motivation
- sources of GW from phase transition: expected frequency and amplitude
- analytical model of the sources
- results for the spectra

A. Kosowsky and M. Turner, 1993
M. Kamionkowski, A. Kosowsky and M. Turner, 1994

A. Kosowsky, A. Mack and T. Kahniashvili 2002
A. Dolgov, D. Grasso and A. Nicolis 2002
C. Caprini and R. Durrer 2006
G. Gogoberidze, T. Kahniashvili and A. Kosowsky 2007

Why primordial sources?

Small perturbations in FRW metric: $(h_i{}^i = h_i{}^j{}_{|j} = 0)$

$$ds^2 = a^2(\eta)(d\eta^2 - (\delta_{ij} + 2h_{ij})dx^i dx^j) \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{k},\eta) + \frac{2}{\eta}\dot{h}_{ij}(\mathbf{k},\eta) + k^2 h_{ij}(\mathbf{k},\eta) = 8\pi G a^2(\eta)\Pi_{ij}(\mathbf{k},\eta)$$

Source: $\Pi_{ij}(\mathbf{k},\eta)$ anisotropic stress

- Once emitted, propagate without interaction
- Direct probe of physical processes in the early universe (gravitons)
- Primordial source: stochastic background of GWs (example: inflation)

GW energy density: $\Omega_G = \dfrac{\langle \dot{h}_{ij}\dot{h}^{ij}\rangle}{G\rho_c} = \displaystyle\int \frac{dk}{k}\frac{d\Omega_G(k)}{d\log(k)}$

Characteristic frequency

- Characteristic frequency of GWs produced at time η_*

$$\mathcal{H}_*^{-1} = \frac{a}{\dot{a}}\Big|_* = \eta_* \quad \text{causality:} \quad k_* \geq \mathcal{H}_*$$

- Frequency window from a cosmological source: $\begin{cases} k_{\rm eq} \simeq 10^{-15}\,\mathrm{Hz} \\ k_{\rm inf} \simeq 1\mathrm{GHz} \end{cases}$

100 GeV (EW phase transition): $k_{100\mathrm{GeV}} \geq 10^{-5}\mathrm{Hz}$

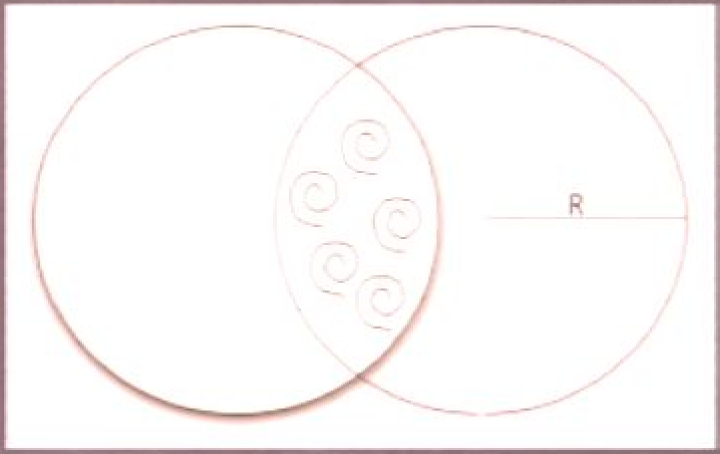
- LISA detection at low frequency:

$10^{-4}\mathrm{Hz} \leq k \leq 1\mathrm{Hz}$ $\qquad \Omega_G \sim 10^{-12}$ at about 1 mHz

GW from phase transitions

FIRST ORDER:

- Collision of bubbles walls
- Turbulent motions
- Magnetic fields



$\beta^{-1} \simeq 0.01\,\mathcal{H}_*^{-1}$ duration of the phase transition

$v_b \leq 1$ speed of the bubbles walls

$R \simeq v_b \beta^{-1}$

$$f \simeq \frac{1}{R} \simeq \frac{10^{-2}}{v_b} \frac{\beta}{\mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{mHz}$$

Scaling of the GW amplitude

- **Bubbles:** $T_{ij} = (\rho + p)\gamma^2 v_i v_j$ (phase transition in a thermal bat

$\frac{4}{3}\rho_*$ enthalpy density $\qquad v_i$ velocity profile in the bubble

Energy density in bubble walls to radiation energy density: $\frac{\Omega^*_{\text{kin}}}{\Omega^*_{\text{rad}}} = \frac{4}{3}\frac{v^2}{1-v^2}$

- **Turbulence:** $T_{ij} = (\rho + p)v_i v_j \qquad \frac{\Omega^*_T}{\Omega^*_{\text{rad}}} = \frac{2}{3}\langle v^2 \rangle \qquad \langle v^2 \rangle \leq \frac{1}{3}$

- **Magnetic fields:** $T_{ij} = \frac{1}{8\pi}B_i B_j \qquad \frac{\Omega^*_B}{\Omega^*_{\text{rad}}} = \frac{\langle B^2 \rangle}{8\pi\rho_{\text{rad}}}$

Scaling of the GW amplitude

$$\delta G_{ij} = 8\pi G T_{ij}$$

$$\beta^2 h \sim 8\pi G T$$

characteristic time of evolution

tensor perturbation

energy density: $\rho_G \sim \frac{\dot{h}^2}{16\pi G}$

$$\dot{h} \sim \frac{8\pi G T}{\beta}$$

$$T \sim \rho_{\mathrm{rad}} \frac{\Omega^*_{\mathrm{kin}}}{\Omega^*_{\mathrm{rad}}}$$

$$\Rightarrow \quad \Omega_G \sim \underbrace{\Omega_{\mathrm{rad}}}_{10^{-5}} \underbrace{\left(\frac{\mathcal{H}_*}{\beta}\right)^2}_{10^{-4}} \underbrace{\left(\frac{\Omega^*_{\mathrm{kin}}}{\Omega^*_{\mathrm{rad}}}\right)^2}_{10^{-2}}$$

Analytic study of the GW signal

GW power spectrum:

$$\frac{d\Omega_G}{d\ln k}=\frac{k^3|\dot{h}|^2}{G\rho_c} \qquad \langle\dot{h}_{ij}(\mathbf{k},\eta)\dot{h}^*_{ij}(\mathbf{q},\eta)\rangle=\delta(\mathbf{k}-\mathbf{q})|\dot{h}|^2(k,\eta)$$

Wave equation: $h_{ij}(\mathbf{k},\eta)=\int_{\eta_{\rm in}}^{\eta}d\tau\mathcal{G}(\tau,\eta)\Pi_{ij}(\mathbf{k},\tau)$

Anisotropic stress power spectrum: $\langle\Pi_{ij}(\mathbf{k},\tau_1)\Pi^*_{ij}(\mathbf{q},\tau_2)\rangle=\delta(\mathbf{k}-\mathbf{q})\Pi(k,\tau_1,\tau_2)$

Energy momentum tensor: $\Pi_{ij}=\mathcal{P}_{ij}^{lm}T_{lm}$

$$T_{ij}(\mathbf{k},\tau)=\frac{w(\tau)}{1-v^2(\tau)}\int d^3p\,v_i(\mathbf{k}-\mathbf{p},\tau)v_j(\mathbf{p},\tau)$$

4 point correlation function → Wick theorem → **Power spectrum of the source**

Scaling of the GW amplitude

$$\delta G_{ij} = 8\pi G T_{ij}$$

$$\beta^2 h \sim 8\pi G T$$

characteristic time of evolution

tensor perturbation

energy density: $\rho_G \sim \dfrac{\dot{h}^2}{16\pi G}$

$$\dot{h} \sim \frac{8\pi G T}{\beta}$$

$$T \sim \rho_{\rm rad} \frac{\Omega^*_{\rm kin}}{\Omega^*_{\rm rad}}$$

$$\Rightarrow \quad \Omega_G \sim \Omega_{\rm rad} \left(\frac{\mathcal{H}_*}{\beta}\right)^2 \left(\frac{\Omega^*_{\rm kin}}{\Omega^*_{\rm rad}}\right)^2$$

10^{-5}

10^{-4}

10^{-2}

Analytic study of the GW signal

GW power spectrum:

$$\frac{d\Omega_G}{d\ln k} = \frac{k^3|\dot{h}|^2}{G\rho_c} \qquad \langle \dot{h}_{ij}(\mathbf{k},\eta)\dot{h}^*_{ij}(\mathbf{q},\eta)\rangle = \delta(\mathbf{k}-\mathbf{q})|\dot{h}|^2(k,\eta)$$

Wave equation: $$h_{ij}(\mathbf{k},\eta) = \int_{\eta_{\rm in}}^{\eta} d\tau \mathcal{G}(\tau,\eta)\Pi_{ij}(\mathbf{k},\tau)$$

Anisotropic stress power spectrum: $$\langle \Pi_{ij}(\mathbf{k},\tau_1)\Pi^*_{ij}(\mathbf{q},\tau_2)\rangle = \delta(\mathbf{k}-\mathbf{q})\Pi(k,\tau_1,\tau_2)$$

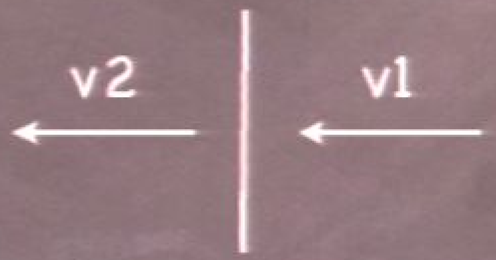
Energy momentum tensor: $\Pi_{ij} = \mathcal{P}_{ij}^{lm}T_{lm}$

$$T_{ij}(\mathbf{k},\tau) = \frac{w(\tau)}{1-v^2(\tau)}\int d^3p\, v_i(\mathbf{k}-\mathbf{p},\tau)v_j(\mathbf{p},\tau)$$

4 point correlation function ⟹ Wick theorem ⟹ Power spectrum of the source

Bubble walls power spectrum

Hydrodynamics of bubble growth at late times:



broken phase

$$\rho_2 = aT_2^4$$
$$p_2 = aT_2^4/3$$

symmetric phase

$$\rho_1 = aT_1^4 + \rho_{\rm vac}$$
$$p_1 = aT_1^4/3 - \rho_{\rm vac}$$

combustion front

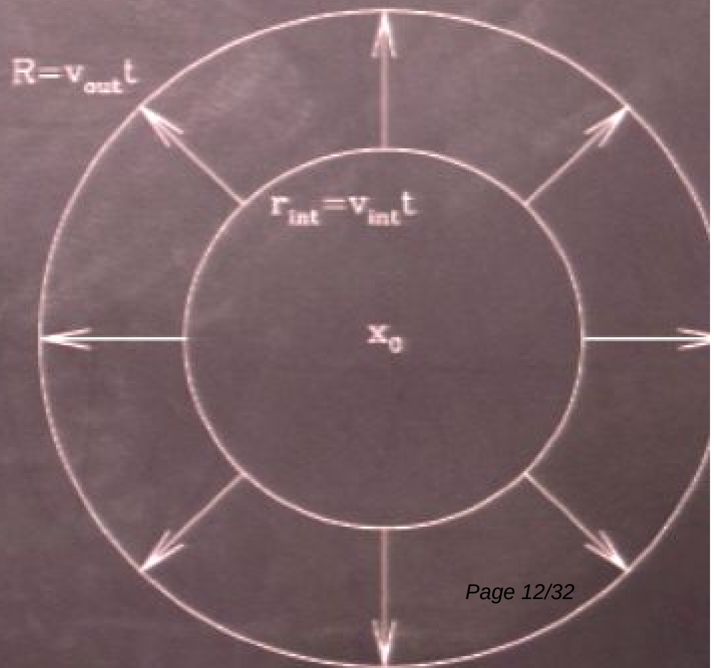
(conservation of energy and momentum)

- DETONATIONS: $v_1 > c_s \, , v_2 = c_s$

  symmetric phase at rest
  (Steinhardt 82)

- DEFLAGRATIONS: $v_1 < c_s \, , v_2 < c_s$

  broken phase at rest
  shock wave in the symmetric phase



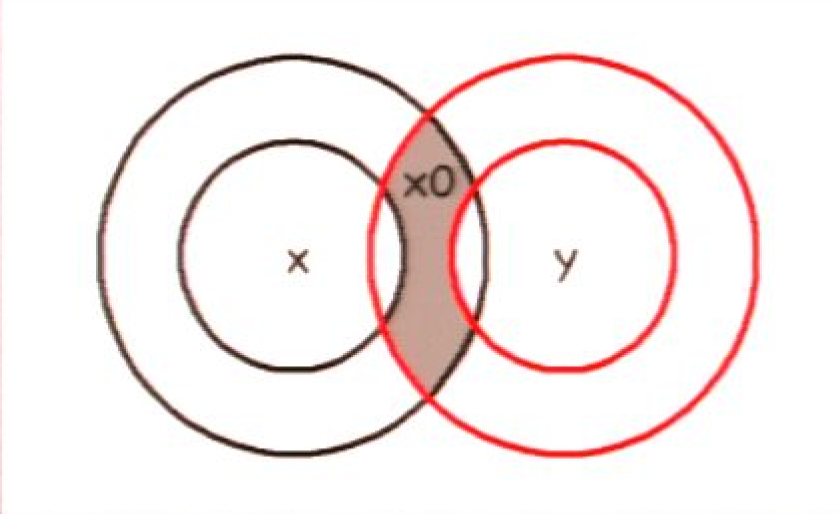
Bubble walls power spectrum

Velocity profile of a spherical bubble:

$$v_i(\mathbf{x},t) = \begin{cases} (v_f/R)\,(\mathbf{x}-\mathbf{x}_0)_i & \text{for} \quad r_{\rm int} < |\mathbf{x}-\mathbf{x}_0| < R \\ 0 & \text{otherwise} \end{cases}$$

Two point correlation function $\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle$ non-zero only if

- $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$
- x and y in the same bubble
- the velocity is not zero



$$\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle = \frac{v_f^2}{R^2}\frac{p}{V}\int_V d^3x_0 (\mathbf{x}-\mathbf{x}_0)_i (\mathbf{y}-\mathbf{x}_0)_j$$

p: probability that there is a centre in the intersection region

Bubble walls power spectrum

Power spectrum: Fourier transform of 2 p. correlation function

$$\langle v_i(\mathbf{k},t)v_j^*(\mathbf{q},t)\rangle = \delta(\mathbf{k}-\mathbf{q})v_f^2R(t)^3P(t)[A(kR)\delta_{ij}+B(kR)\hat{k}_i\hat{k}$$

- $P(t)$ probability that a point is in the broken phase at time t

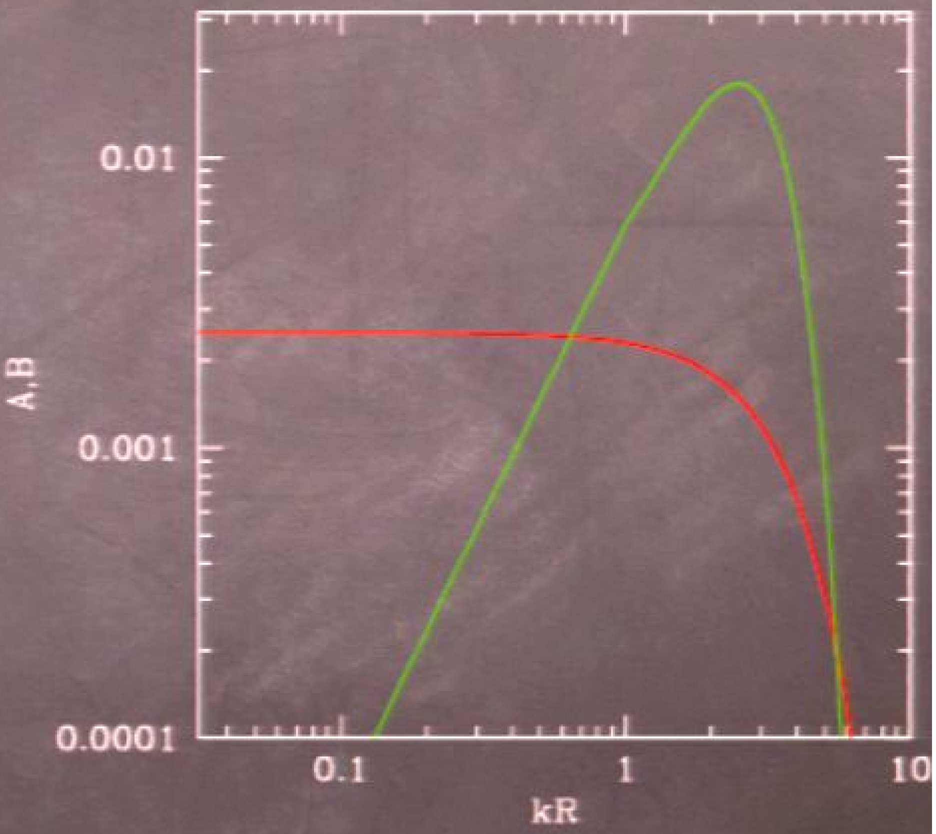
$$P(t)\simeq 1-\exp(-\Gamma(t))$$

- correlation function with compact support $|\mathbf{x}-\mathbf{y}|\leq 2R(t)$

analytic power spectrum

$$A(k\to 0)\propto k^0$$

$$B(k\to 0)\propto k^2$$



- small scale part $A(k\gg\pi/R)\sim B(k\gg\pi/R)\propto k^{-4}$

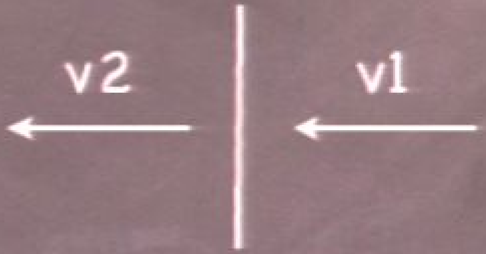
Bubble walls power spectrum

Hydrodynamics of bubble growth at late times:

broken phase

$\rho_2 = aT_2^4$

$p_2 = aT_2^4/3$



symmetric phase

$\rho_1 = aT_1^4 + \rho_{\rm vac}$

$p_1 = aT_1^4/3 - \rho_{\rm vac}$

combustion front

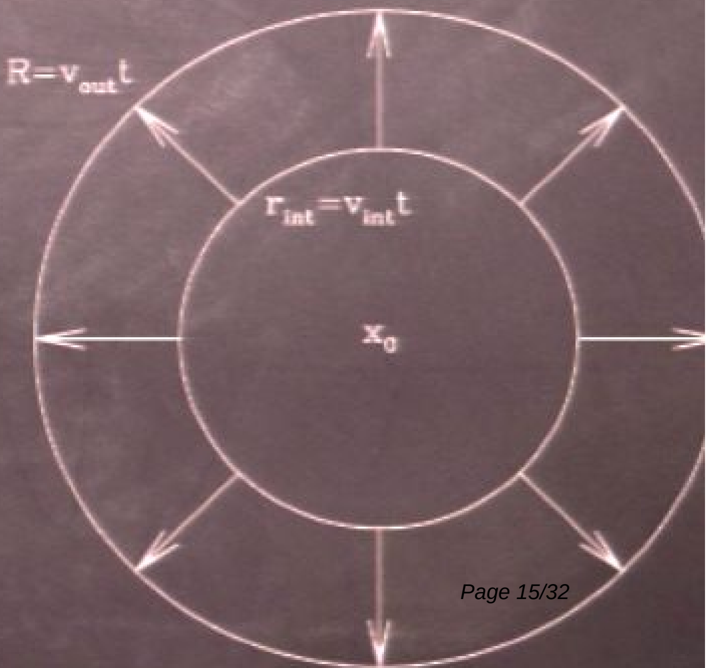
(conservation of energy and momentum)

- DETONATIONS: $v_1 > c_s \, , v_2 = c_s$

  symmetric phase at rest
  (Steinhardt 82)

- DEFLAGRATIONS: $v_1 < c_s \, , v_2 < c_s$

  broken phase at rest
  shock wave in the symmetric phase



Analytic study of the GW signal

GW power spectrum:

$$\frac{d\Omega_G}{d\ln k} = \frac{k^3|\dot{h}|^2}{G\rho_c} \qquad \langle \dot{h}_{ij}(\mathbf{k},\eta)\dot{h}^*_{ij}(\mathbf{q},\eta)\rangle = \delta(\mathbf{k}-\mathbf{q})|\dot{h}|^2(k,\eta)$$

Wave equation: $$h_{ij}(\mathbf{k},\eta) = \int_{\eta_{\rm in}}^{\eta} d\tau \mathcal{G}(\tau,\eta)\Pi_{ij}(\mathbf{k},\tau)$$

Anisotropic stress power spectrum: $$\langle \Pi_{ij}(\mathbf{k},\tau_1)\Pi^*_{ij}(\mathbf{q},\tau_2)\rangle = \delta(\mathbf{k}-\mathbf{q})\Pi(k,\tau_1,\tau_2)$$

Energy momentum tensor: $$\Pi_{ij} = \mathcal{P}_{ij}^{lm}T_{lm}$$

$$T_{ij}(\mathbf{k},\tau) = \frac{w(\tau)}{1-v^2(\tau)}\int d^3p\, v_i(\mathbf{k}-\mathbf{p},\tau)v_j(\mathbf{p},\tau)$$

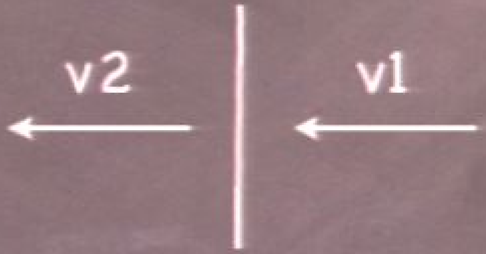
4 point correlation function ⟹ Wick theorem ⟹ Power spectrum of the source

Bubble walls power spectrum

Hydrodynamics of bubble growth at late times:

broken phase

$$\rho_2 = aT_2^4$$
$$p_2 = aT_2^4/3$$



symmetric phase

$$\rho_1 = aT_1^4 + \rho_{\rm vac}$$
$$p_1 = aT_1^4/3 - \rho_{\rm vac}$$

combustion front

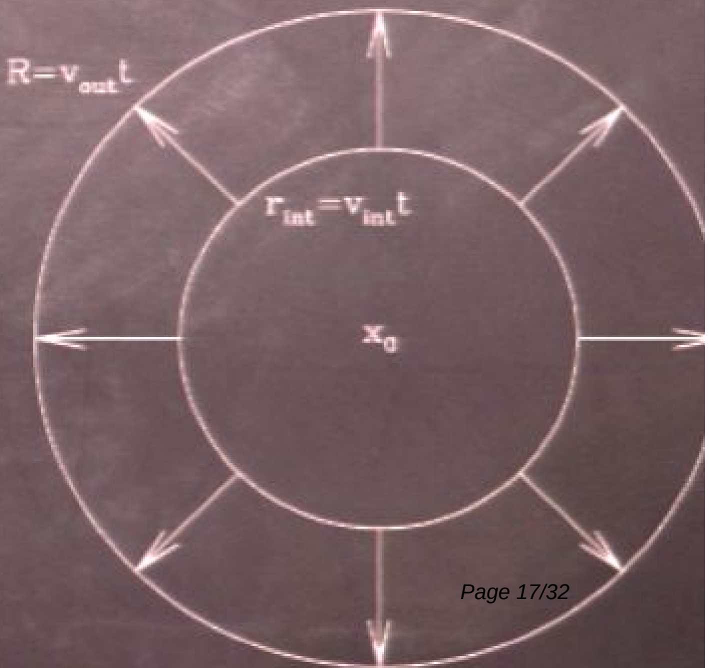
(conservation of energy and momentum)

- DETONATIONS: $v_1 > c_s \, , v_2 = c_s$

  symmetric phase at rest
  (Steinhardt 82)

- DEFLAGRATIONS: $v_1 < c_s \, , v_2 < c_s$

  broken phase at rest
  shock wave in the symmetric phase



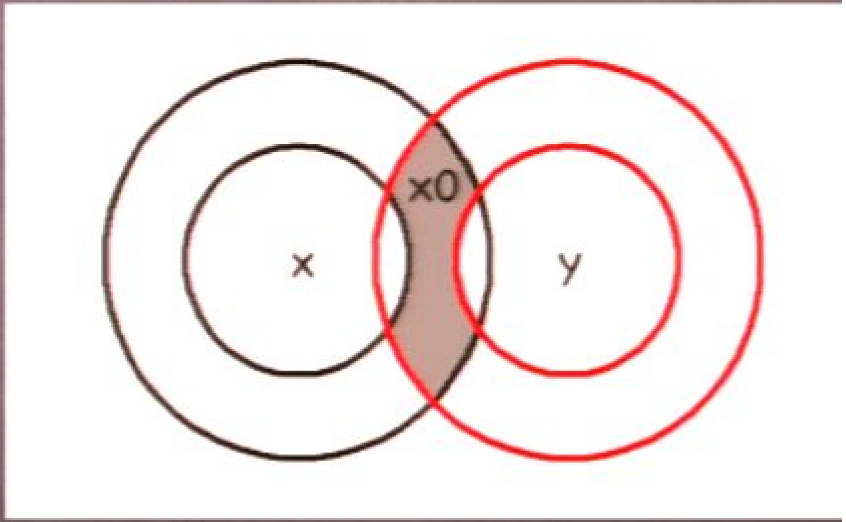
Bubble walls power spectrum

Velocity profile of a spherical bubble:

$$v_i(\mathbf{x},t) = \begin{cases} (v_f/R)\,(\mathbf{x}-\mathbf{x}_0)_i & \text{for} \quad r_{\rm int} < |\mathbf{x}-\mathbf{x}_0| < R \\ 0 & \text{otherwise} \end{cases}$$

Two point correlation function $\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle$ non-zero only if

- $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$
- x and y in the same bubble
- the velocity is not zero



$$\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle = \frac{v_f^2}{R^2}\,\frac{p}{V}\int_V d^3x_0 (\mathbf{x}-\mathbf{x}_0)_i(\mathbf{y}-\mathbf{x}_0)_j$$

p: probability that there is a centre in the intersection region

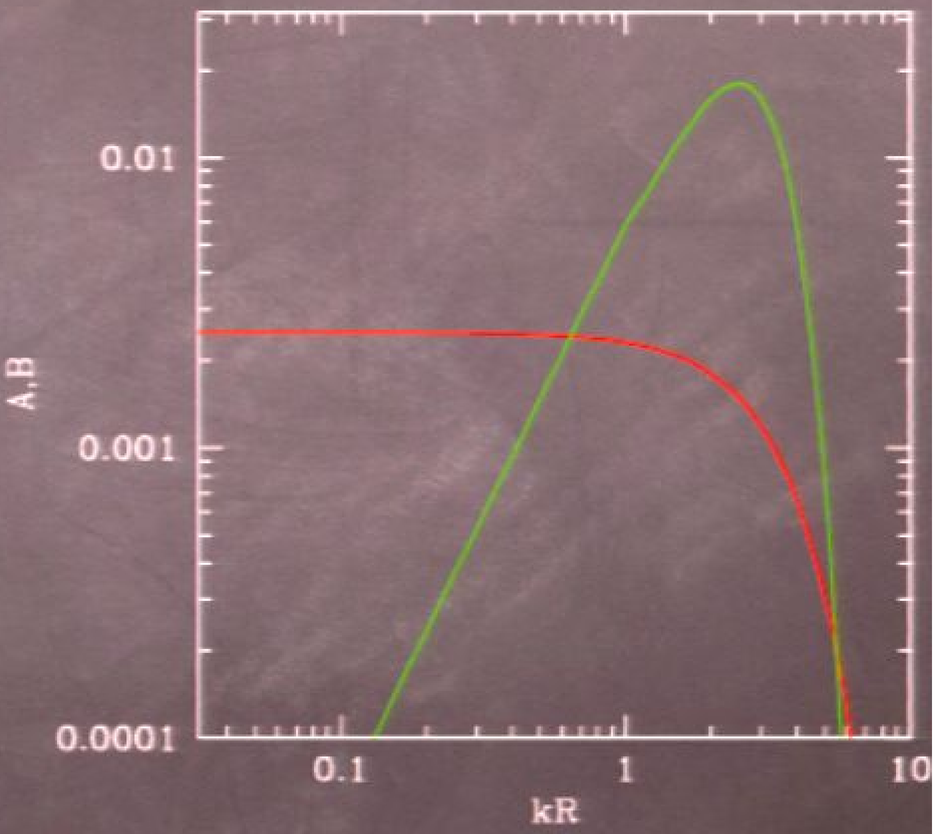
Bubble walls power spectrum

Power spectrum: Fourier transform of 2 p. correlation function

$$\langle v_i(\mathbf{k},t)v_j^*(\mathbf{q},t)\rangle = \delta(\mathbf{k}-\mathbf{q})v_f^2R(t)^3P(t)[A(kR)\delta_{ij} + B(kR)\hat{k}_i\hat{k}$$

- $P(t)$ probability that a point is in the broken phase at time t

$$P(t) \simeq 1 - \exp(-\Gamma(t))$$



- correlation function with compact support $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$

  analytic power spectrum

$$A(k \to 0) \propto k^0$$
$$B(k \to 0) \propto k^2$$

- small scale part $A(k \gg \pi/R) \sim B(k \gg \pi/R) \propto k^{-4}$

Turbulence and magnetic field power spectra

- Correlated over the scale $2R$ ⟹ peak at $k \sim \pi/R$
- Divergence-free vector fields

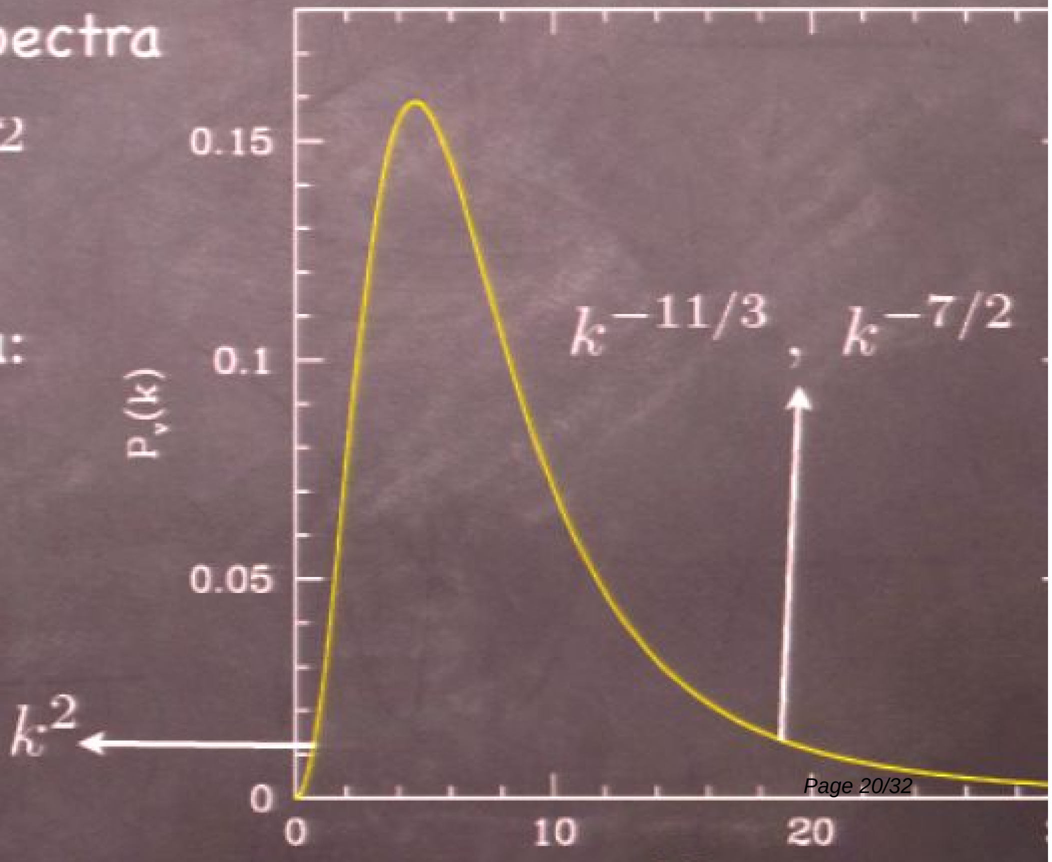
$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q})\,(\delta_{ij} - \hat{k}_i \hat{k}_j) P_v(kR)$$

⟹ large scale part of the spectra

$$P_v(k \ll \pi/R) \propto k^2$$

- Small scale part of the spectra:

Kolmogorov (turbulence)
Iroshnikov Kraichnan
(magnetic field)



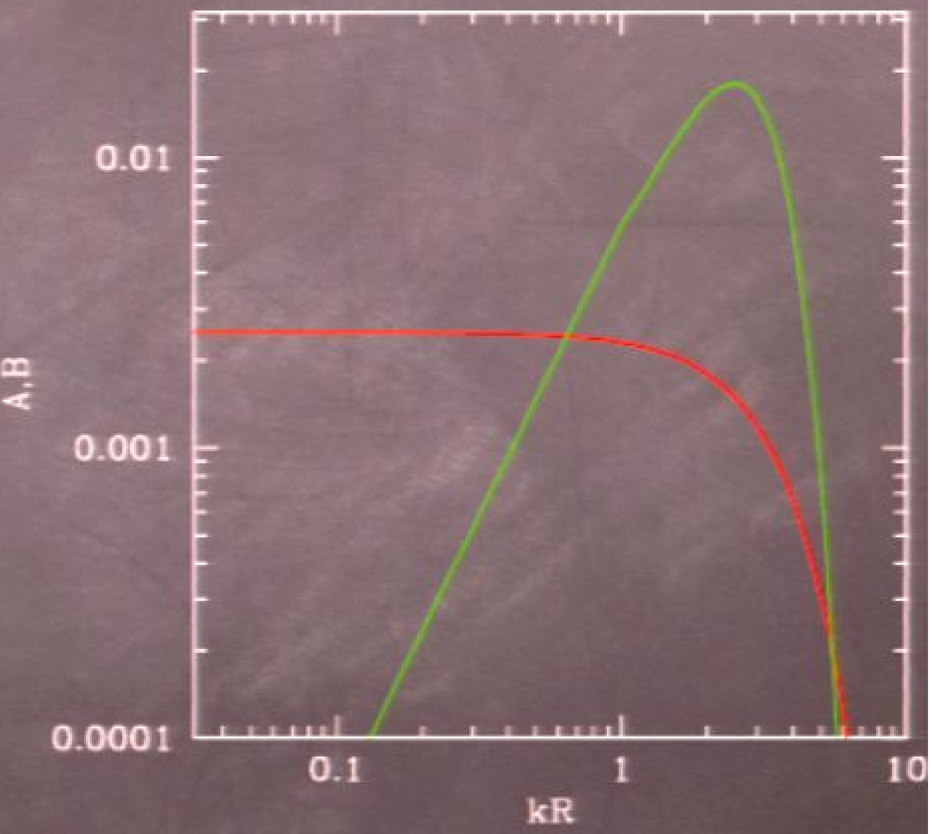
Bubble walls power spectrum

Power spectrum: Fourier transform of 2 p. correlation function

$$v_i(\mathbf{k},t)v_j^*(\mathbf{q},t)\rangle = \delta(\mathbf{k}-\mathbf{q})v_f^2 R(t)^3 P(t)[A(kR)\delta_{ij} + B(kR)\hat{k}_i\hat{k}$$

- $P(t)$ probability that a point is in the broken phase at time t

$$P(t) \simeq 1 - \exp(-\Gamma(t))$$



- correlation function with compact support $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$

analytic power spectrum

$$A(k \to 0) \propto k^0$$

$$B(k \to 0) \propto k^2$$

- small scale part $A(k \gg \pi/R) \sim B(k \gg \pi/R) \propto k^{-4}$

Turbulence and magnetic field power spectra

- Correlated over the scale $2R$ ⟹ peak at $k \sim \pi/R$

- Divergence-free vector fields

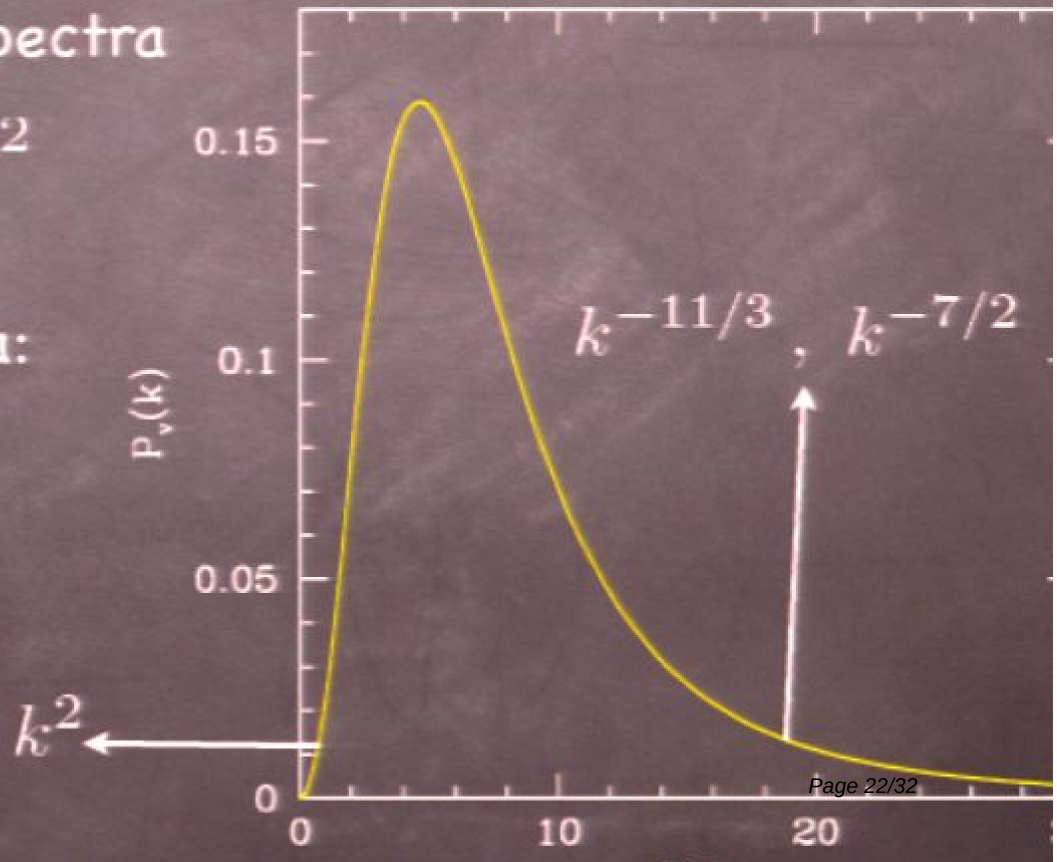
$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{q})\rangle = \delta(\mathbf{k}-\mathbf{q})\,(\delta_{ij} - \hat{k}_i\hat{k}_j) P_v(kR)$$

⟹ large scale part of the spectra

$$P_v(k \ll \pi/R) \propto k^2$$

- Small scale part of the spectra:

Kolmogorov (turbulence)
Iroshnikov Kraichnan
(magnetic field)



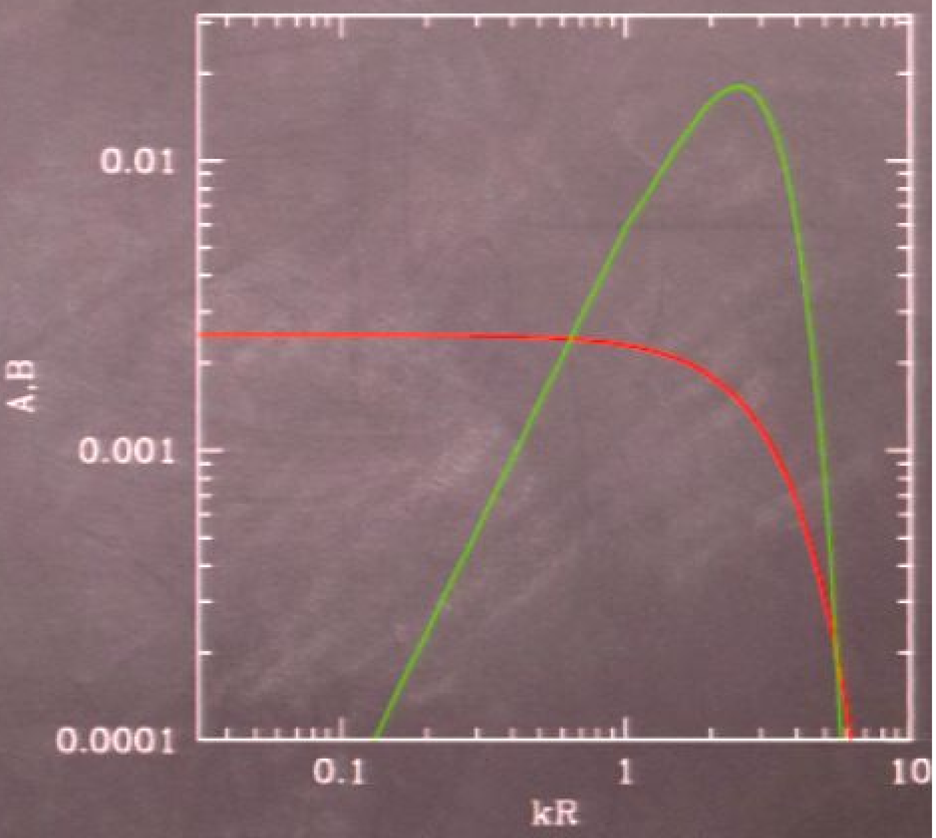
Bubble walls power spectrum

Power spectrum: Fourier transform of 2 p. correlation function

$$v_i(\mathbf{k},t)v_j^*(\mathbf{q},t)\rangle = \delta(\mathbf{k}-\mathbf{q})v_f^2 R(t)^3 P(t)[A(kR)\delta_{ij} + B(kR)\hat{k}_i\hat{k}$$

- $P(t)$ probability that a point is in the broken phase at time t

$$P(t) \simeq 1 - \exp(-\Gamma(t))$$



- correlation function with compact support $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$

  analytic power spectrum

$$A(k\to 0) \propto k^0$$
$$B(k\to 0) \propto k^2$$

- small scale part $A(k \gg \pi/R) \sim B(k \gg \pi/R) \propto k^{-4}$

Turbulence and magnetic field power spectra

- Correlated over the scale $2R$ ⟹ peak at $k \sim \pi/R$

- Divergence-free vector fields

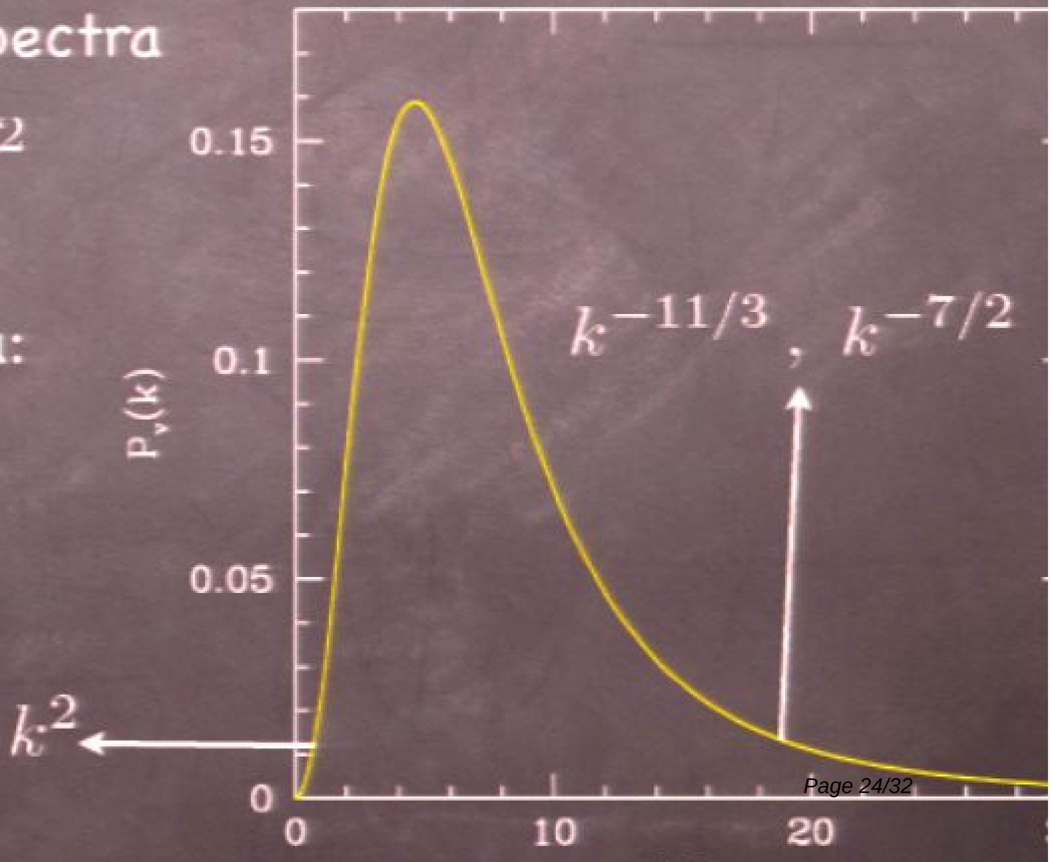
$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{q}) \rangle = \delta(\mathbf{k}-\mathbf{q})\,(\delta_{ij} - \hat{k}_i \hat{k}_j) P_v(kR)$$

⟹ large scale part of the spectra

$$P_v(k \ll \pi/R) \propto k^2$$

- Small scale part of the spectra:

Kolmogorov (turbulence)
Iroshnikov Kraichnan
(magnetic field)



Large scale part of the GW spectrum: k^3

Generic **CAUSAL** power spectrum:

$$P(k) \propto v^2 R^3 \begin{cases} (Rk)^n & \text{for } Rk < \pi \\ (Rk)^m & \text{for } Rk > \pi \end{cases}$$

The **anisotropic stress power spectrum** is the CONVOLUTION:

$$\Pi(k) \propto \int_0^\infty dq\, q^2\, P(|\mathbf{k}-\mathbf{q}|)P(q) \to v^4 R^3 \left(\frac{1}{2n+3} - \frac{1}{2m+3} \right)$$

$$\downarrow$$

$$k \to 0$$

$$n > -3/2$$

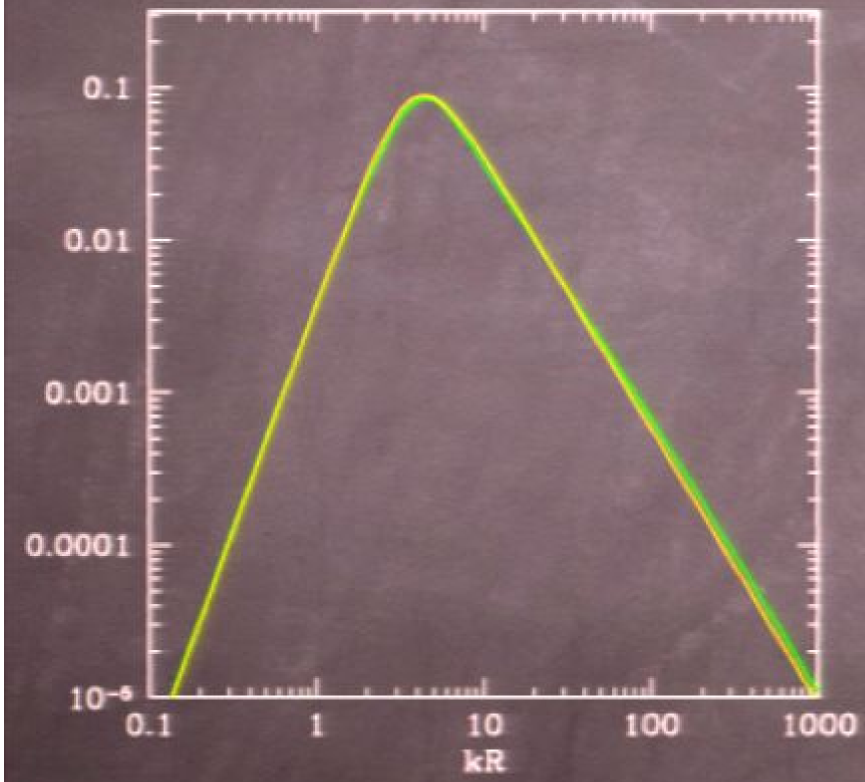
$$m < -3/2$$

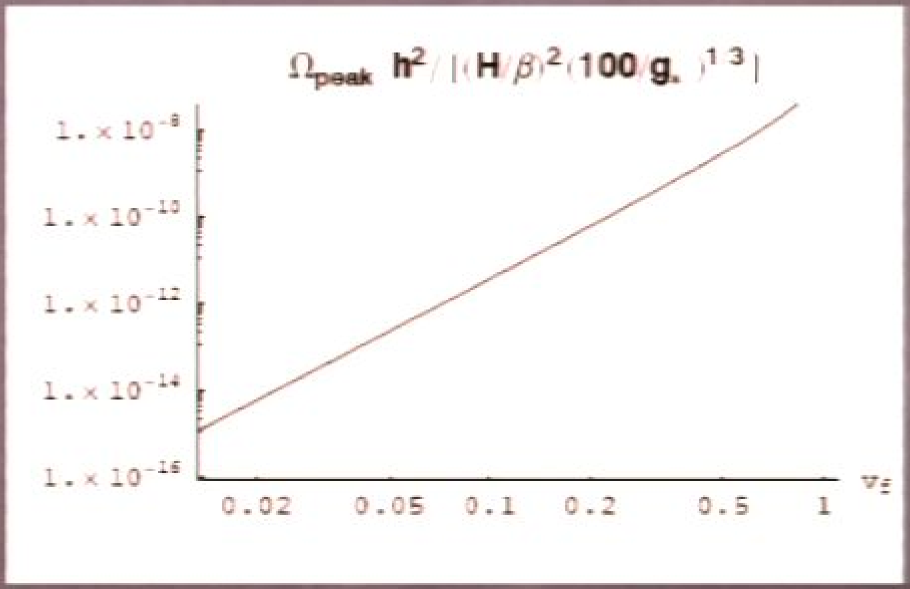
White noise for the anisotropic stress $\rightarrow k^3$ for the GW energy densit

$$\frac{d\Omega_G}{d\ln k} = \frac{k^3 |\dot{h}|^2}{G\rho_c}$$

Result: GW from bubble walls (detonations)

$$\frac{d\Omega_G}{d\ln k} \simeq 10^{-2}\Omega_{\rm rad}\left(\frac{\mathcal{H}_*}{\beta}\right)^2\left(\frac{\Omega^*_{\rm kin}}{\Omega^*_{\rm rad}}\right)^2 \frac{\left(\frac{kR}{3.8}\right)^3}{1+\frac{1}{2}\left(\frac{kR}{3.8}\right)^2+\left(\frac{kR}{3.8}\right)^{4.8}}$$





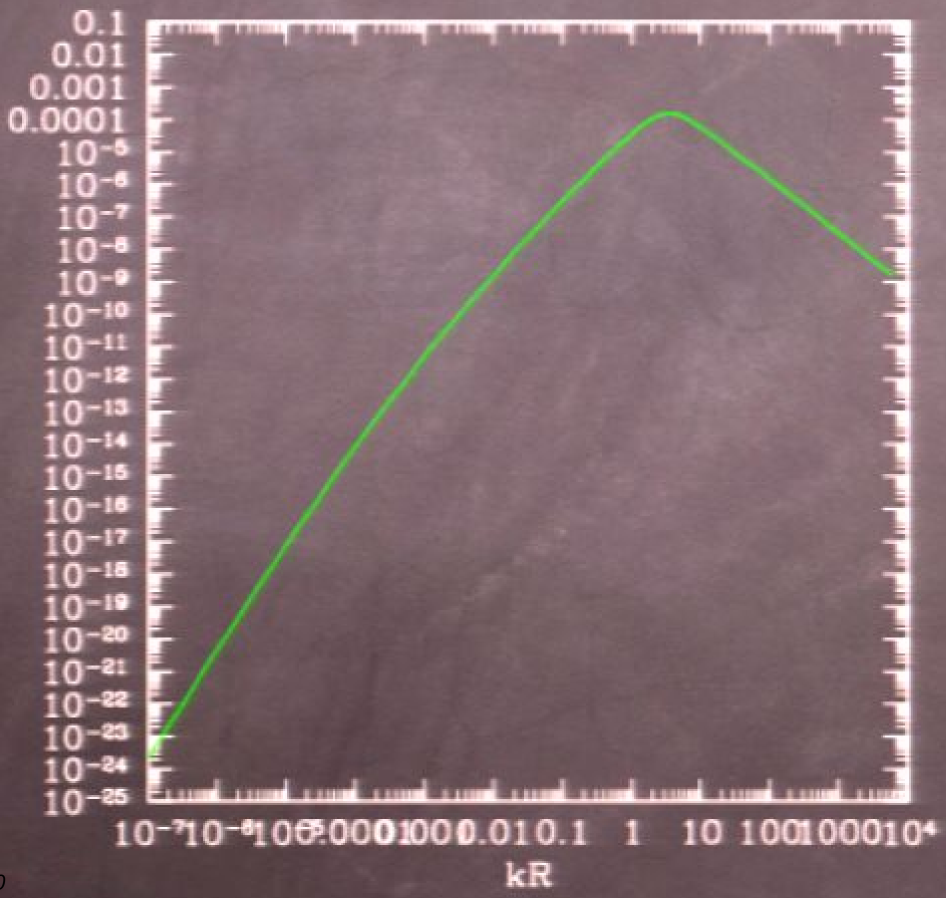
$\Omega_G \sim 10^{-12}$ for

$v_f \sim 0.5 \quad \beta/\mathcal{H}_* \sim 100$

$v_f \sim 0.2 \quad \beta/\mathcal{H}_* \sim 10$

Very preliminary: GW from turbulence

$$\frac{d\Omega_G}{d\ln k} \simeq \Omega_{\rm rad}\left(\frac{\Omega^*_{\rm T}}{\Omega^*_{\rm rad}}\right)^2 \begin{cases} (kR)^3 & \text{for } kR < 4 \\ (kR)^{-5/3} & \text{for } kR > 4 \end{cases}$$



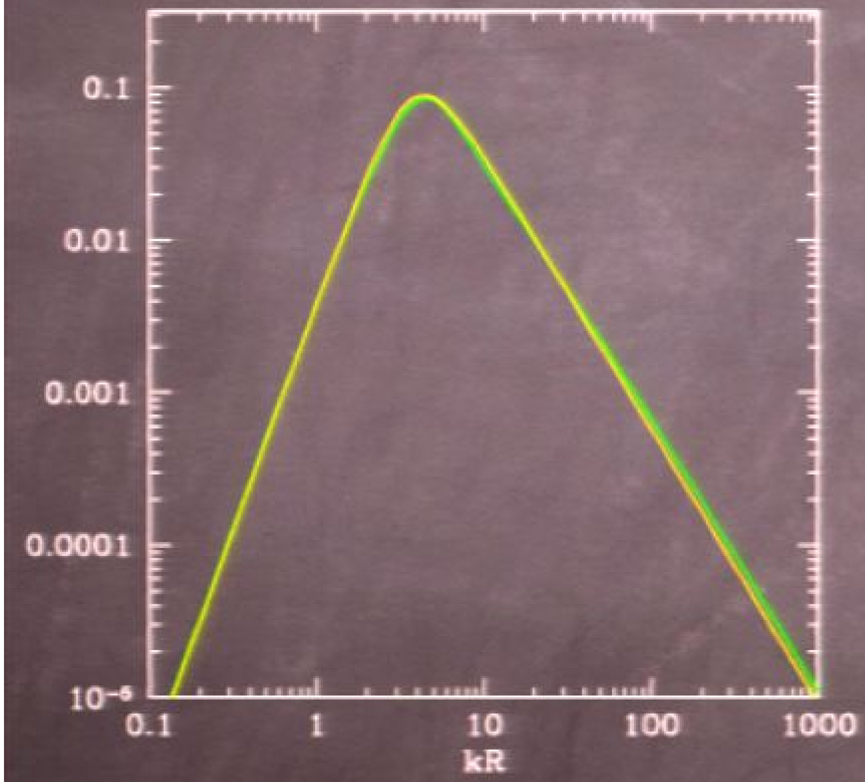
$$\Omega_G \sim 10^{-10}$$

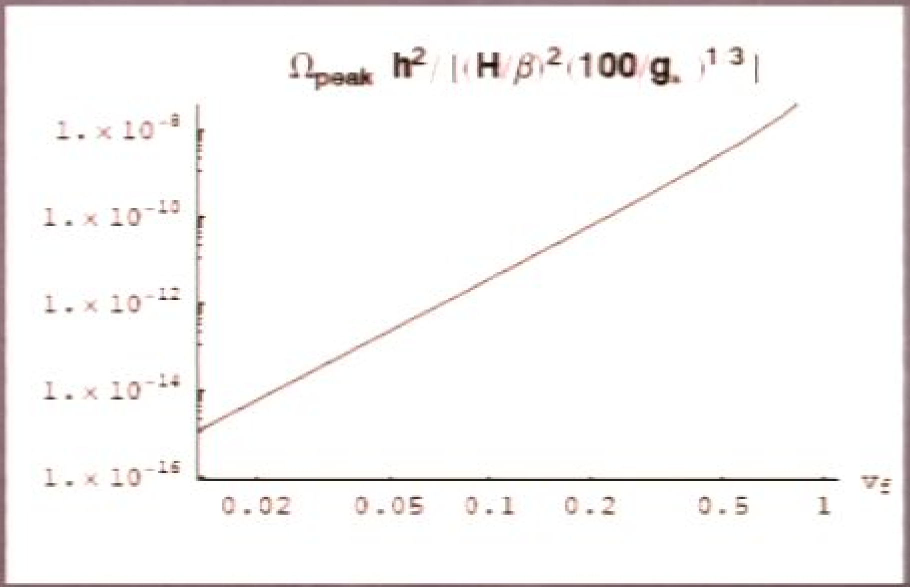
for

$$\frac{\Omega^*_T}{\Omega^*_{\rm rad}} \simeq \frac{2}{9}$$

Result: GW from bubble walls (detonations)

$$\frac{d\Omega_G}{d\ln k} \simeq 10^{-2}\Omega_{\rm rad}\left(\frac{\mathcal{H}_*}{\beta}\right)^2\left(\frac{\Omega^*_{\rm kin}}{\Omega^*_{\rm rad}}\right)^2\frac{\left(\frac{kR}{3.8}\right)^3}{1+\frac{1}{2}\left(\frac{kR}{3.8}\right)^2+\left(\frac{kR}{3.8}\right)^{4.8}}$$





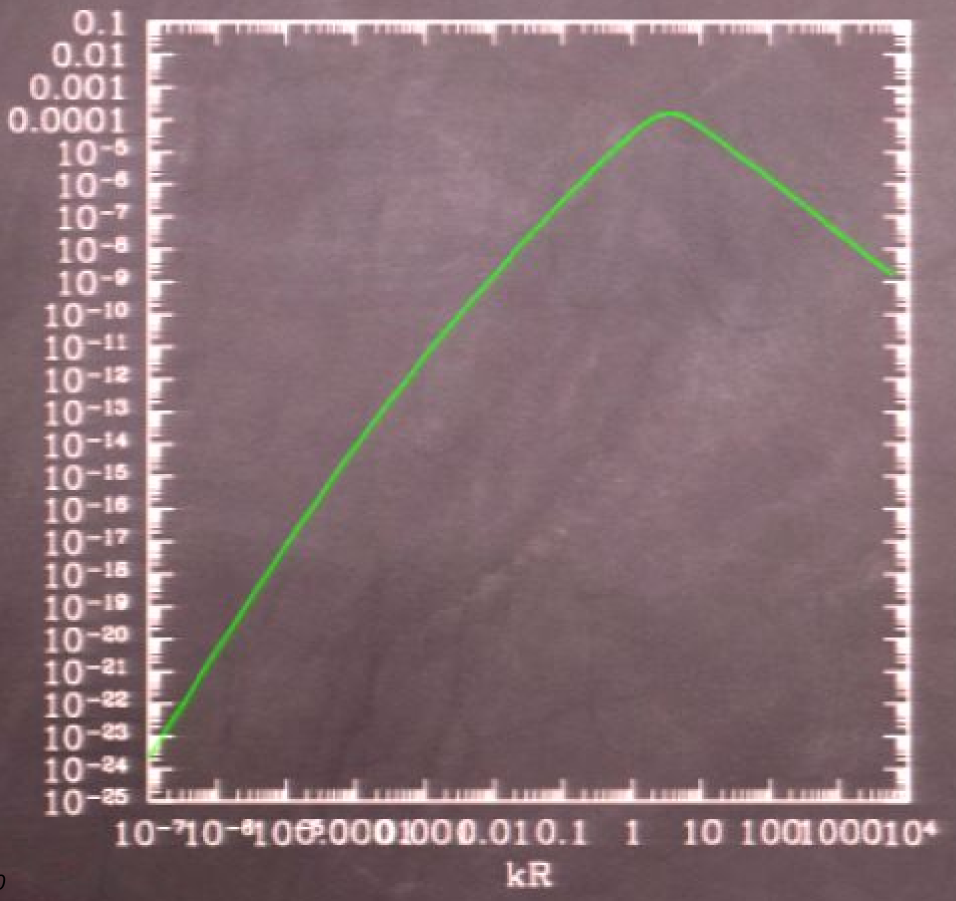
$\Omega_G \sim 10^{-12}$ for

$v_f \sim 0.5 \quad \beta/\mathcal{H}_* \sim 100$

$v_f \sim 0.2 \quad \beta/\mathcal{H}_* \sim 10$

Very preliminary: GW from turbulence

$$\frac{d\Omega_G}{d\ln k} \simeq \Omega_{\rm rad}\left(\frac{\Omega^*_{\rm T}}{\Omega^*_{\rm rad}}\right)^2 \begin{cases} (kR)^3 & \text{for } kR < 4 \\ (kR)^{-5/3} & \text{for } kR > 4 \end{cases}$$



$$\Omega_G \sim 10^{-10}$$

for

$$\frac{\Omega^*_T}{\Omega^*_{\rm rad}} \simeq \frac{2}{9}$$

Conclusions

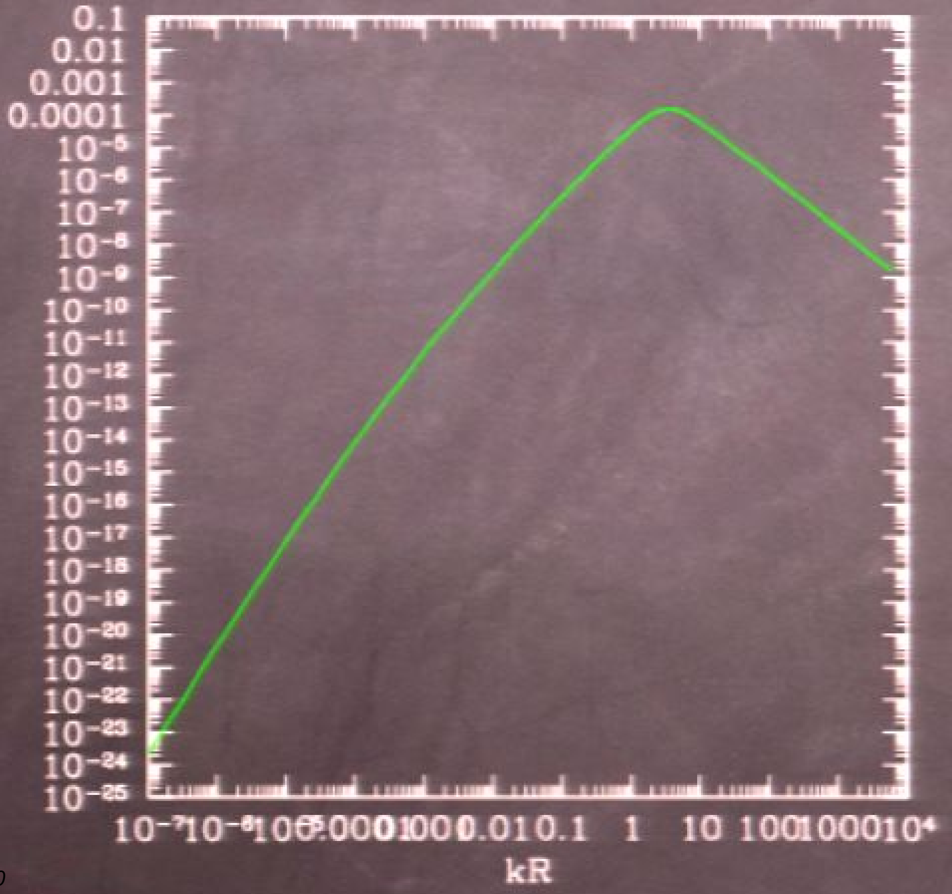
- The spectra rise as k^3 and peak at frequency of about 1/R (order 1 mHz for I order EWPT)
- GW from bubbles observable for high bubble wall velocity or long lasting phase transition
- GW from turbulence observable for speed of sound motions

Comparison with previous analysis

- Extension to the deflagration case (no thin wall approximation, model fo the bubble velocity profile)
- High and low frequency slopes of the GW spectrum
- Peak frequency connected to the size of the bubbles R: depends on the strength of the phase transition

Very preliminary: GW from turbulence

$$\frac{d\Omega_G}{d\ln k} \simeq \Omega_{\rm rad}\left(\frac{\Omega^*_{\rm T}}{\Omega^*_{\rm rad}}\right)^2 \begin{cases} (kR)^3 & \text{for } kR<4 \\ (kR)^{-5/3} & \text{for } kR>4 \end{cases}$$



$$\Omega_G \sim 10^{-10}$$

for

$$\frac{\Omega^*_T}{\Omega^*_{\rm rad}} \simeq \frac{2}{9}$$

Conclusions

- The spectra rise as k^3 and peak at frequency of about 1/R (order 1 mHz for I order EWPT)
- GW from bubbles observable for high bubble wall velocity or long lasting phase transition
- GW from turbulence observable for speed of sound motions

Comparison with previous analysis

- Extension to the deflagration case (no thin wall approximation, model for the bubble velocity profile)
- High and low frequency slopes of the GW spectrum
- Peak frequency connected to the size of the bubbles R: depends on the strength of the phase transition