Title: Some Thoughts on Dark Energy, Inflation and Extra Dimensions

Date: Jun 02, 2008 03:05 PM

URL: http://pirsa.org/08050058

Abstract:

Dark Energy Theorems and Curious Corollaries

Daniel Wesley (Cambridge)
& PJS, to appear

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complement but NOT directly related to:

no-go theorems based on SuSY, SuGRA
or based on ε or η problems
or constructions leading to string landscape

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more closely related to:

constraints on static deS and assuming SEC ($\rho+3p>0$):

G. Gibbons (1985)
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S. Giddings, S. Kachru and J. Polchinski (2002)
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directly related to:

D. Wesley, arXiv:0802.2106
(establishes the methodology)

Theorem I:

Suppose fundamental physics described by::

1) compactifying from higher dimensions with 13>D>5

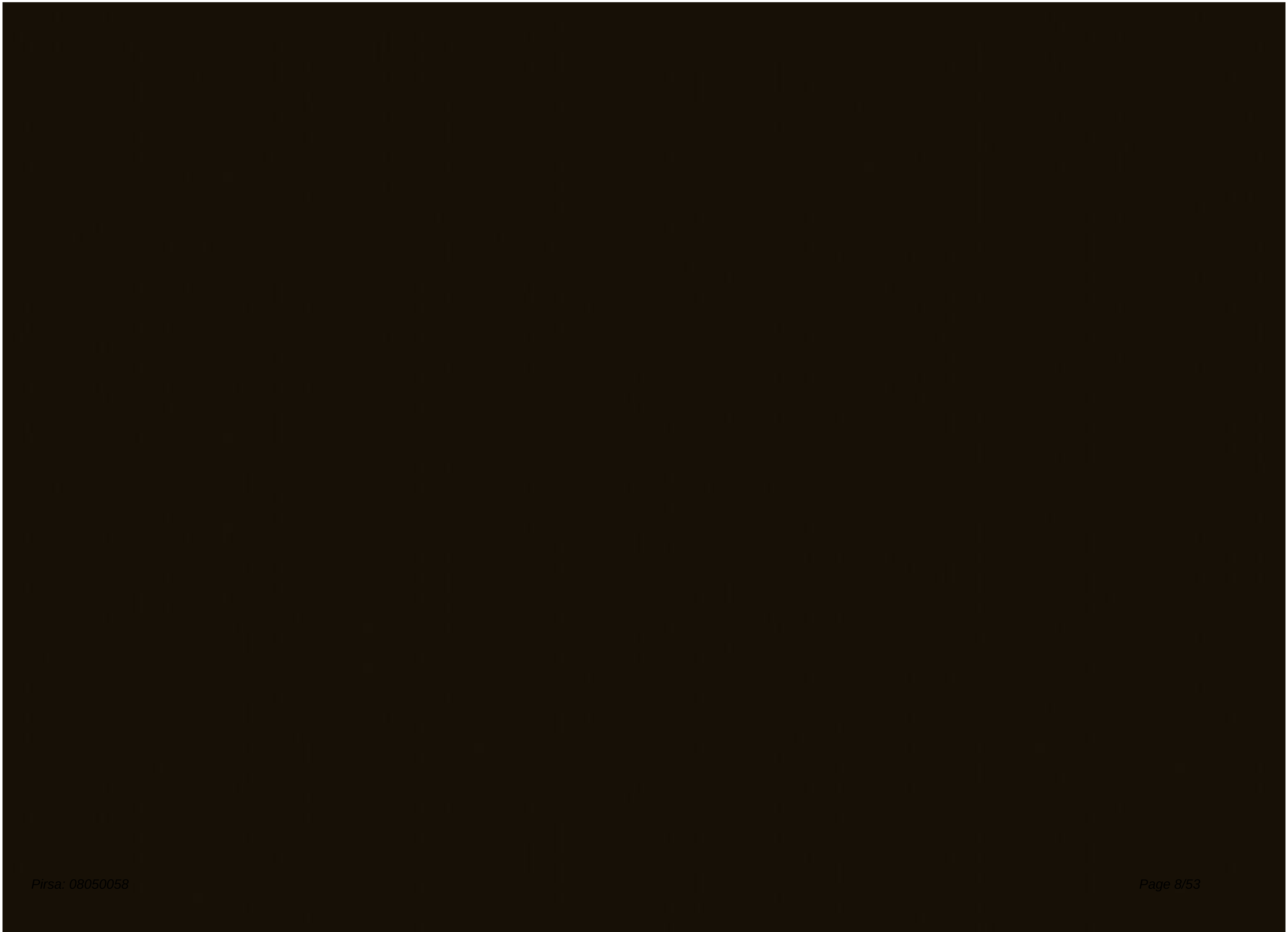
2) where higher D theory described by Einstein GR

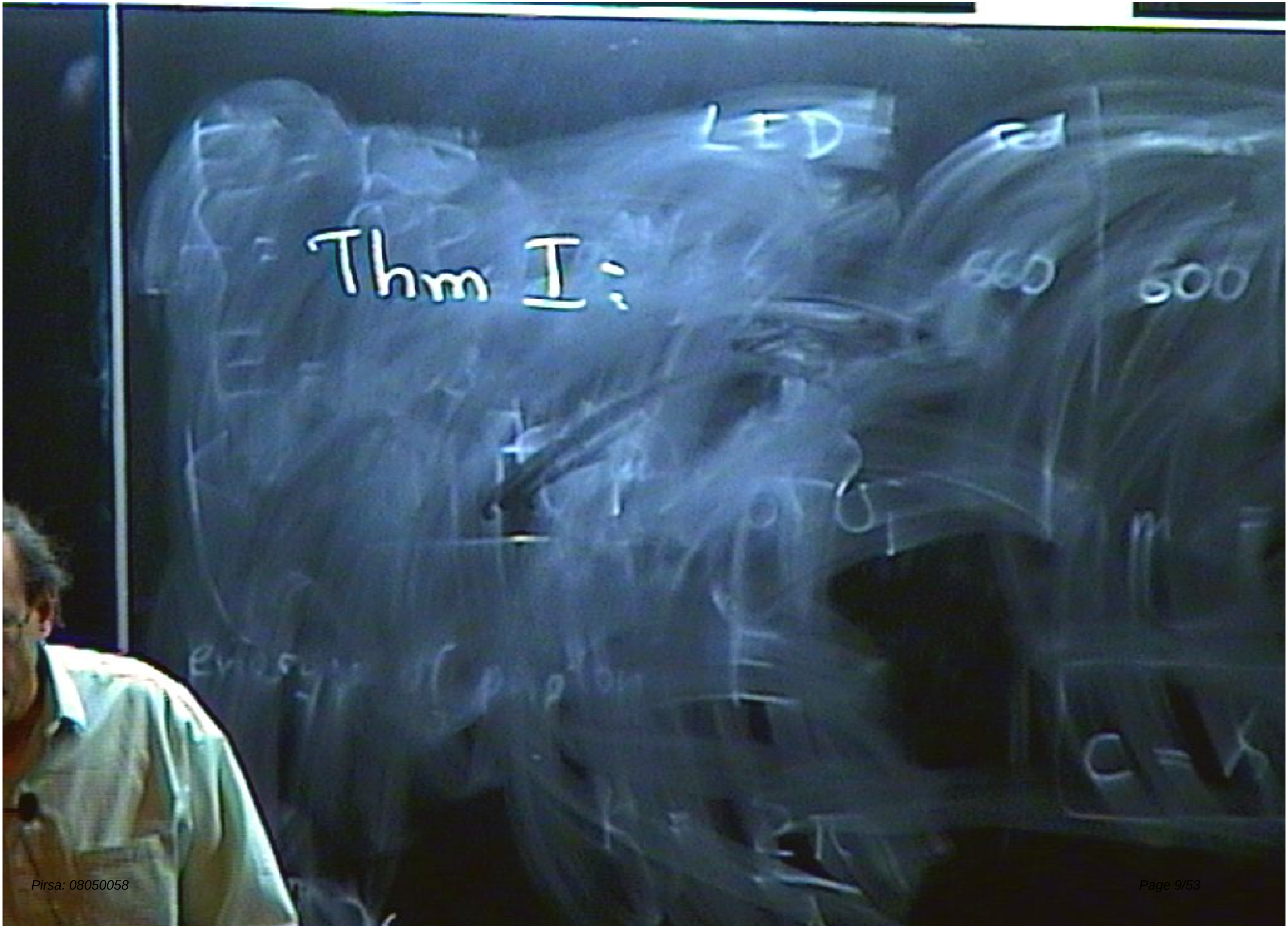
3) where extra D is conformally Ricci-flat* and

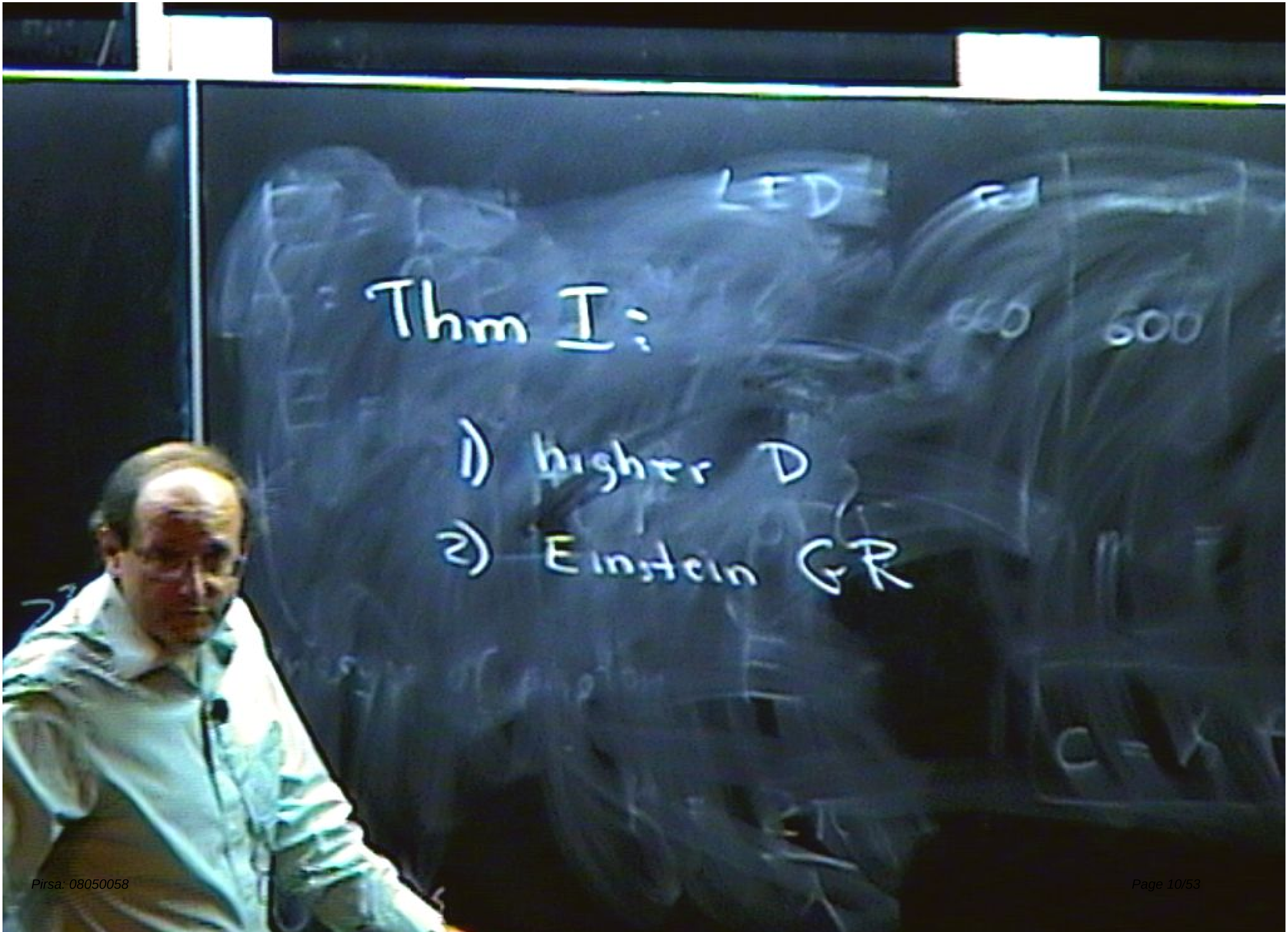
$$ds^2 = e^{-2\Omega}(-dt^2 + a^2(t)\,dx^2) + e^{2\Omega} g_{ab} dy^a dy^b$$

4) and extra D is bounded

5) null energy condition (NEC) is satisfied




LED
Thm I:
660
600


LED
Thm I:
1) higher D
2) Einstein GR
600

LED	red	amber	yellow	green	blue
	660	600	590	565	470

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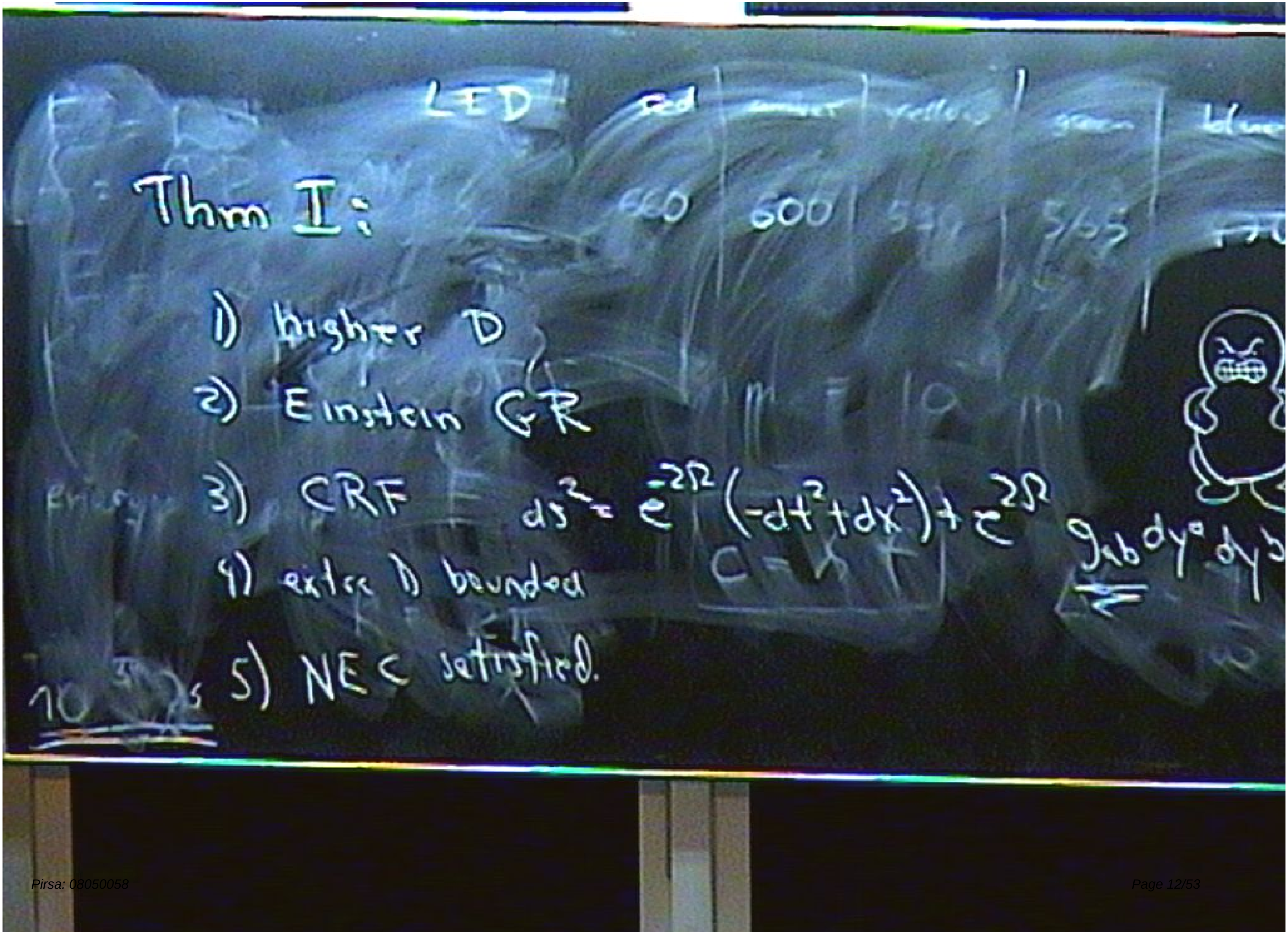
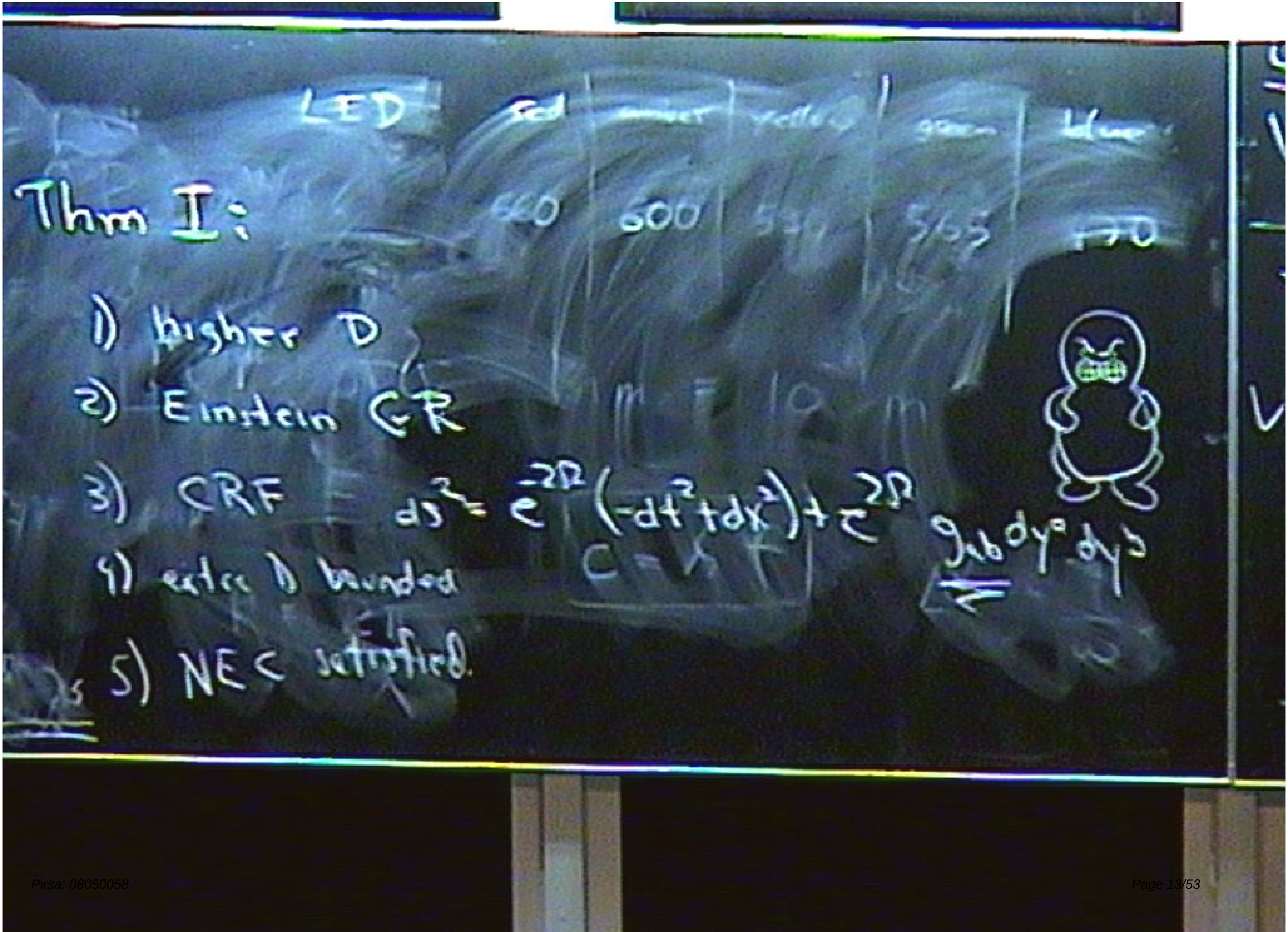
3) CRF $ds^2 = e^{-2\Omega}(-dt^2+dx^2) + e^{2\Omega} g_{ab}\, dy^a dy^b$


LED
red
600
Thm II:
1) higher D
2) Einstein GR
3) CRF
ds² = e^{-2Ω}(-dt²+dx²) + e^{2Ω} g_ab dy^a dy^b
4) extra D bounded
5) NEC satisfied.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Strategy in proving dark energy theorems

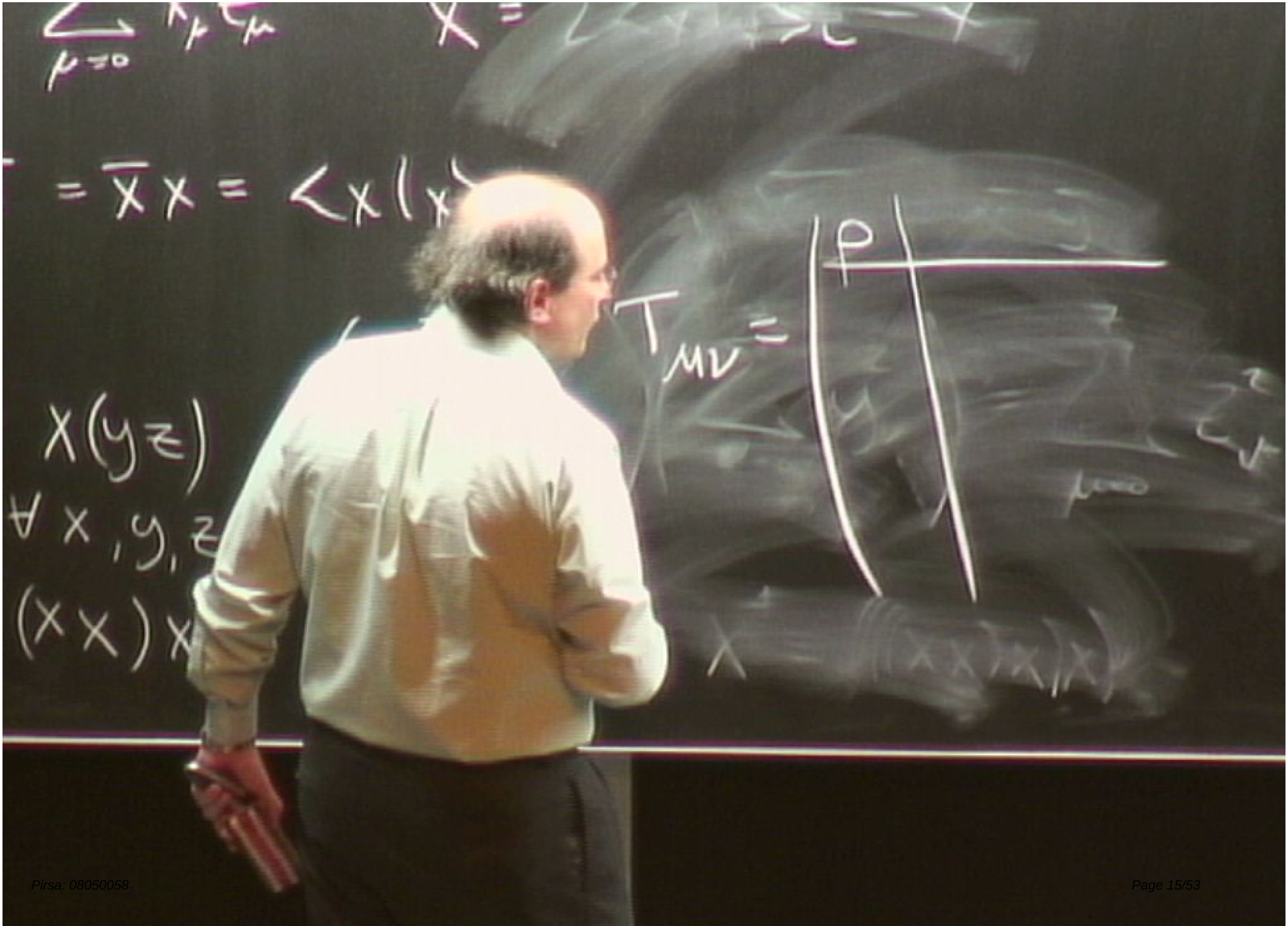
Do all calculations in Einstein frame so interpretation is unambiguous

Treat $T_{\mu\nu}$ space-space components as block diagonal

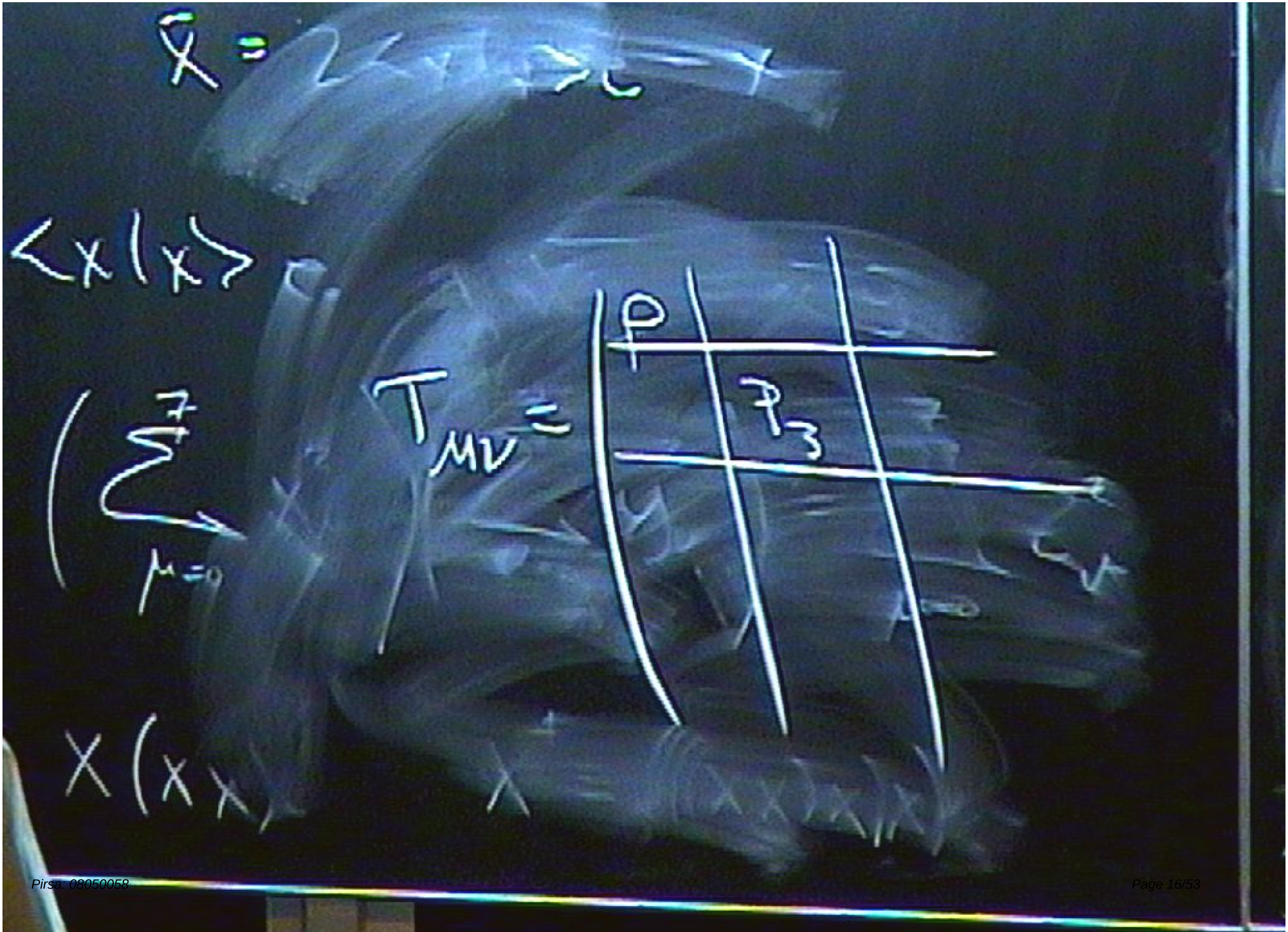
Then try to determine how higher D $\langle \rho + p_k \rangle_A$ & $\langle \rho + p_3 \rangle_A$

relate to the observed ρ^{4d} *and* p^{4d}

(which obey usual Friedmann eqs.)



μ=0
X(yz)
∀ x, y, z
Tμν =
ρ



<x|x>
Tμν =
ρ
p3

$\tilde{X} =$

$\langle X | X \rangle$

$\left(\sum_{\mu=0}^{7} \right.$

$$T_{\mu\nu} = \begin{pmatrix} \rho & & \\ & p_3 & 0 \\ & 0 & P_K \end{pmatrix}$$

$X(XX)$

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A-averaging

NEC $\quad T_{MN}\, n^M n^N \geq 0 \;\; for\;\; every\;\; null\;\; n^M$

A-averaged NEC $\quad \left\langle T_{MN}\, n^M n^N \right\rangle_A \equiv$

$$\left(\int T_{MN}\, n^M n^N \;\; e^{A\Omega} \;\; e^{2\Omega} \sqrt{g}\, d^k y\right) \Big/ \left(\;\; e^{A\Omega} \;\; e^{2\Omega} \sqrt{g}\, d^k y\right) \geq$$

N.B. If A-averaged NEC violated then NEC also violated (but not the converse)

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N.B. If A-averaged NEC violated then NEC also violated
(but not the converse)

Let's get to it . . .

consider general $g_{ab}(t,y)$ *and* $\Omega(t,y)$

$$\frac{d}{dt} g_{ab} = \frac{2}{k}\xi\, g_{ab} + \sigma_{ab}$$

take linear combinations of G_{00} *and* G_{ij} :

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto (\rho^{4d} + p^{4d}) - \frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2 + \text{ neg. semi-def.}$$

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$1 + w_{total} \geq 0$ ≤ 0 ≤ 0

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$1 + 3\,w_{total} < 0$ ≤ 0

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$1 + 3\,w_{total} < 0$

only hope is if this is non-zero

≤ 0

Illustrative example: pure cosmological constant (w_{total} = -1)

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only hope ≤ 0

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Suppose dark energy described by theory obtained by:

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PROVEN

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Curious corollary:

Not only rules out pure Λ universe, but also ΛCDM

That is, if Theorem I assumptions are correct, then $w_{DE} > -1$

Are the Thm I conditions therefore ruled out observationally?

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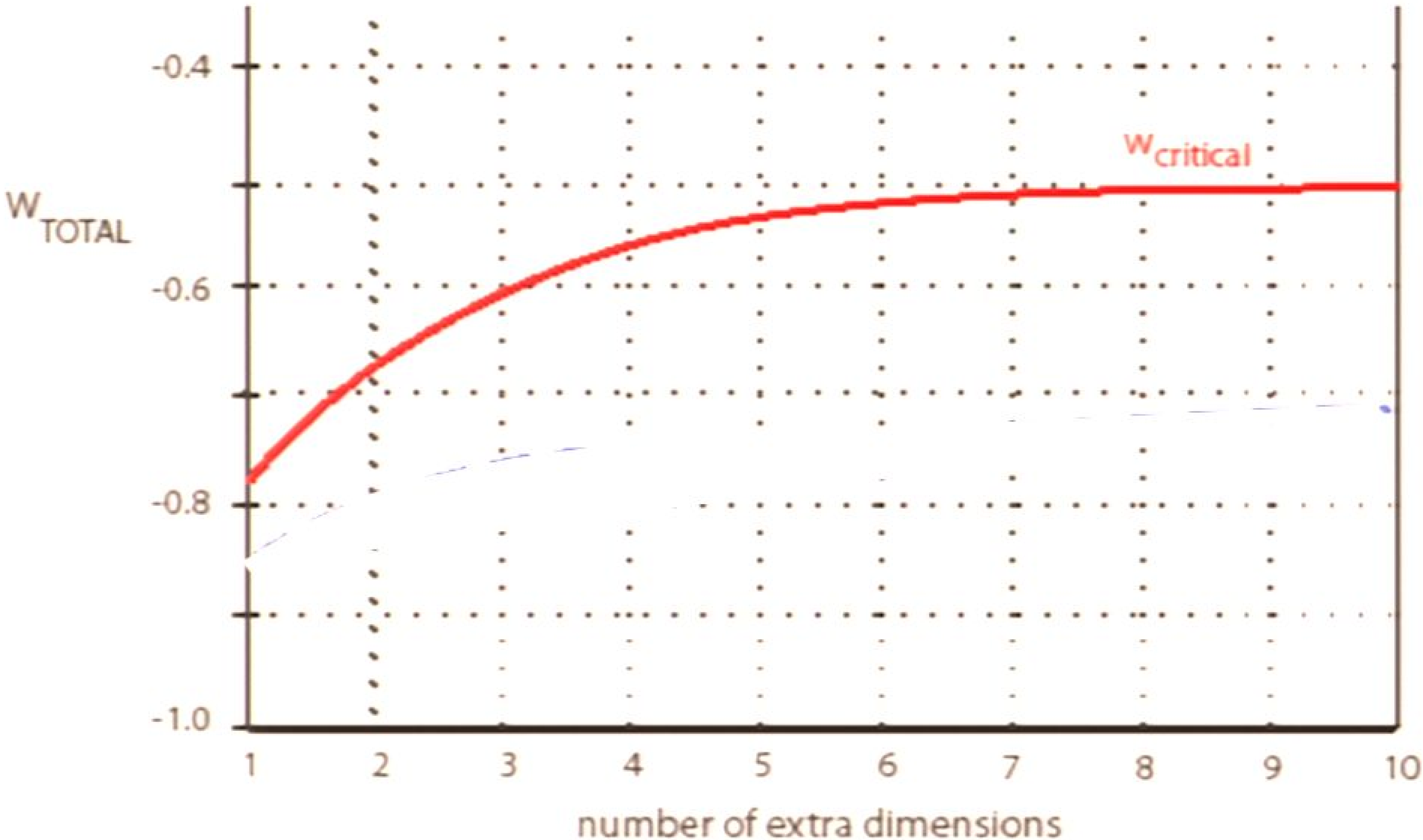
Theorem II: if all conditions of Thm I satisfied and

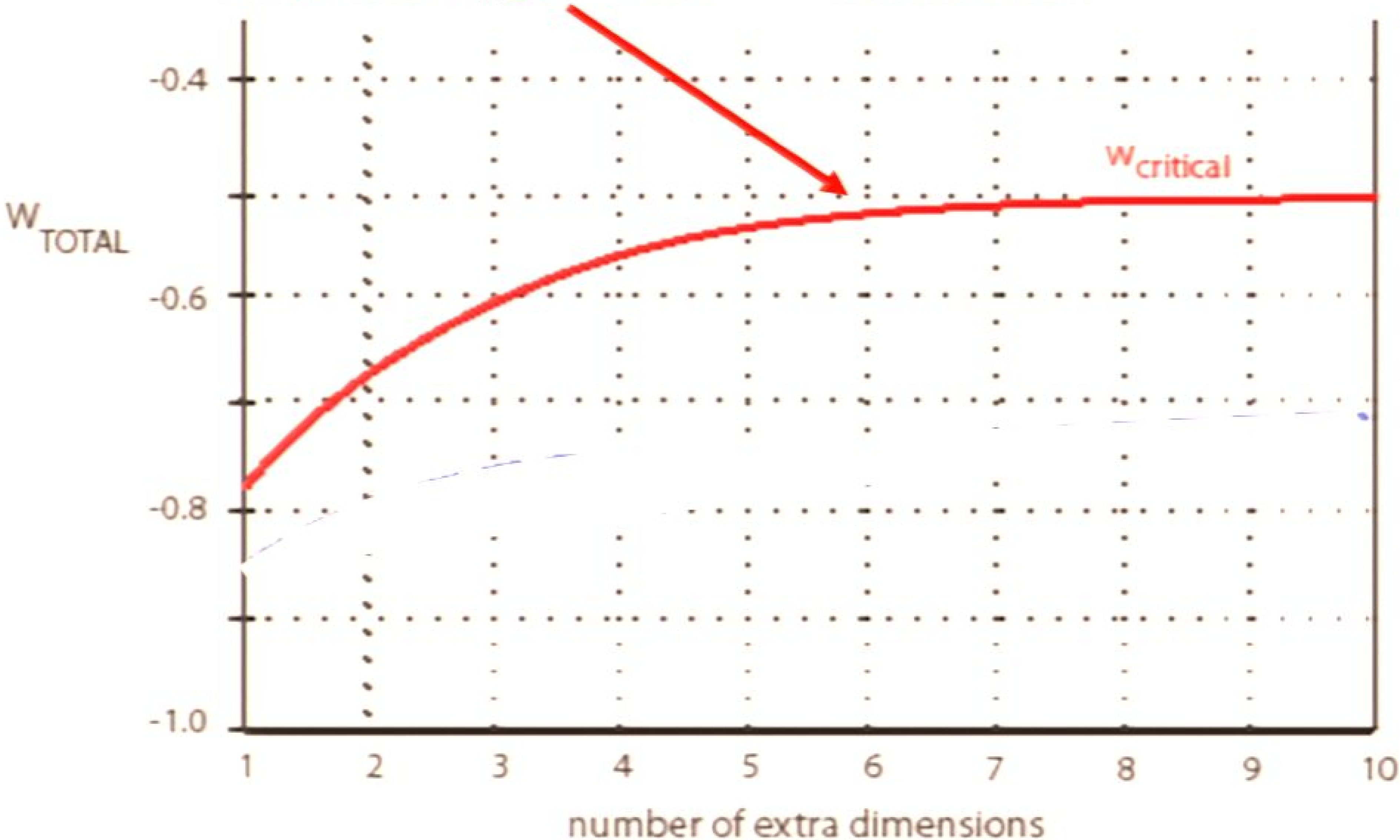
$$-1/3 > w_{total} > w_{critical} > -1$$

. . . can maintain acceleration indefinitely
(but G_N must vary with time to maintain the NEC)



-0.4
-0.6
-0.8
-1.0
W TOTAL
w critical
1
2
3
4
5
6
7
8
9
10
number of extra dimensions

requires $w_{total} > -0.53$ -- ruled out !



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Theorem III: if satisfy all conditions of Thm I and

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can maintain $w < w_{critical}$ for only a brief period;
(w_{DE} and G_N must be <u>rapidly</u> time-varying)

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Practical example: $w_{total} = \Omega_{DE}\, w_{DE} = -0.73$ today

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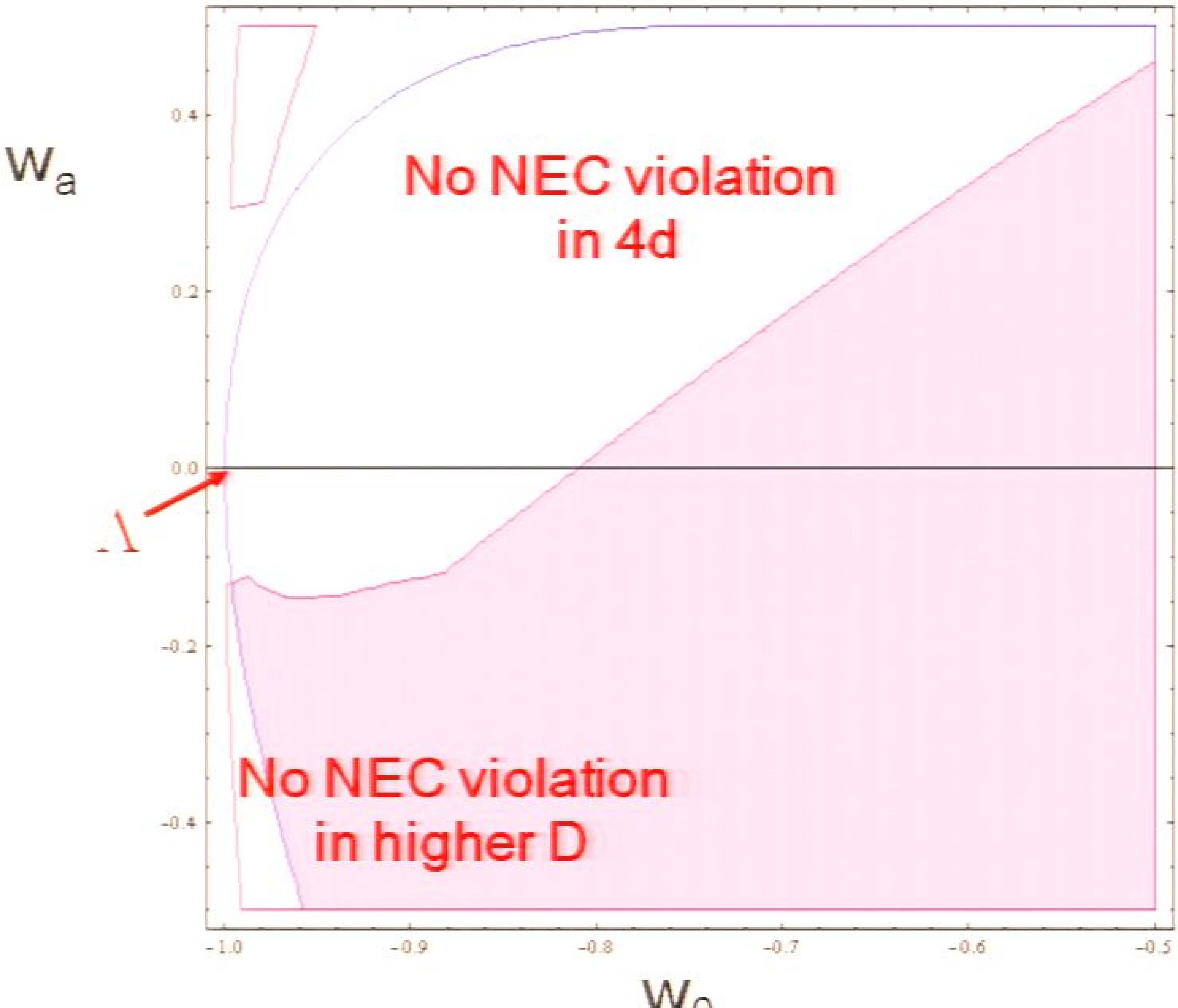
G_N: compare w/ model-independent limits on $\xi = \frac{H_0 \dot{G}}{G}$ & $\frac{|G_{BBN} - G_0|}{G}$

$$|\, d\, w_{total} / d\, z \,| < 1.3$$

Models that satisfy constraints on $G_N(t)$ and NEC

$$w(a) = w_0 + w_a(1-a) + w_b(1-a)^2$$



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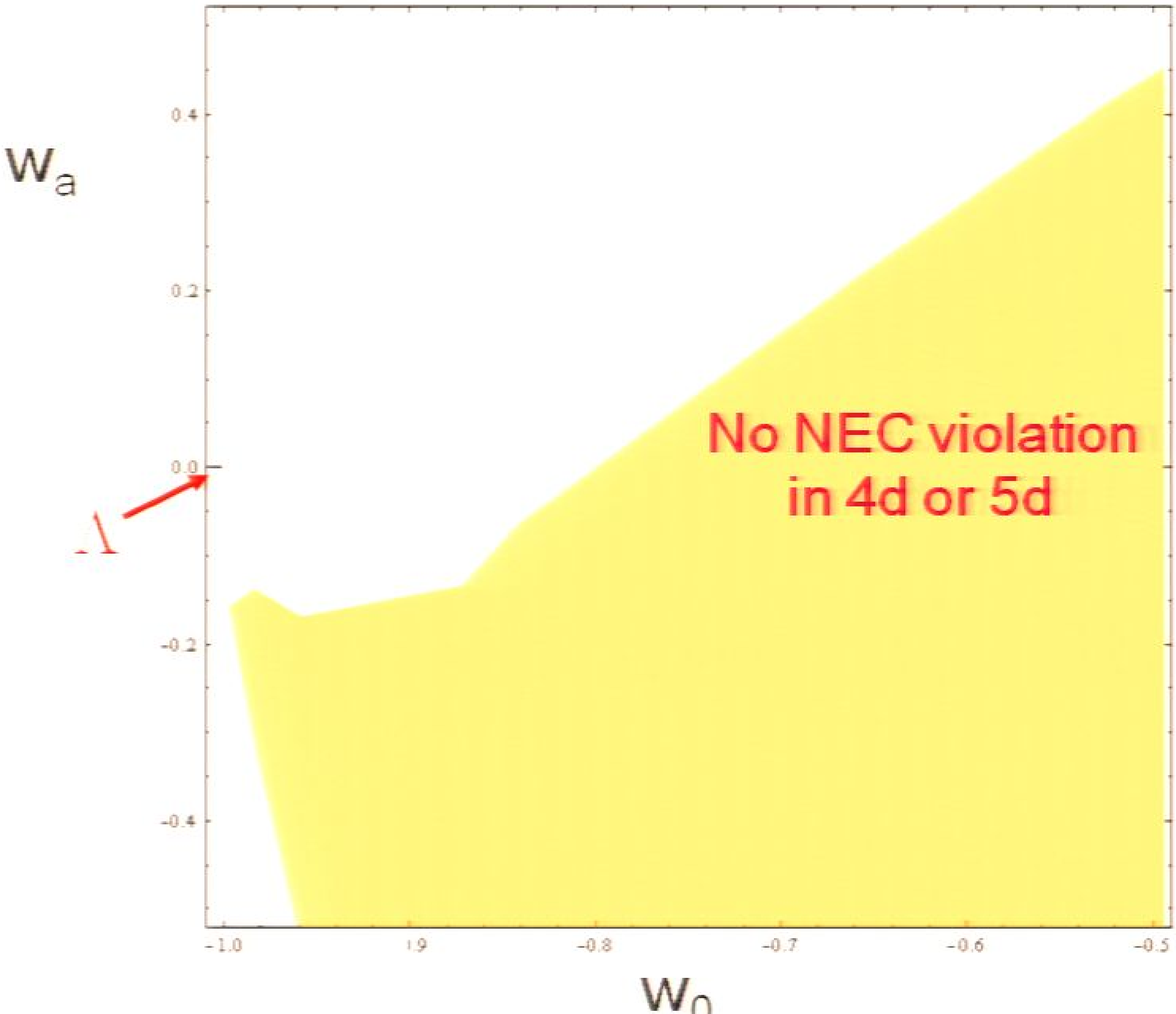

prelimina

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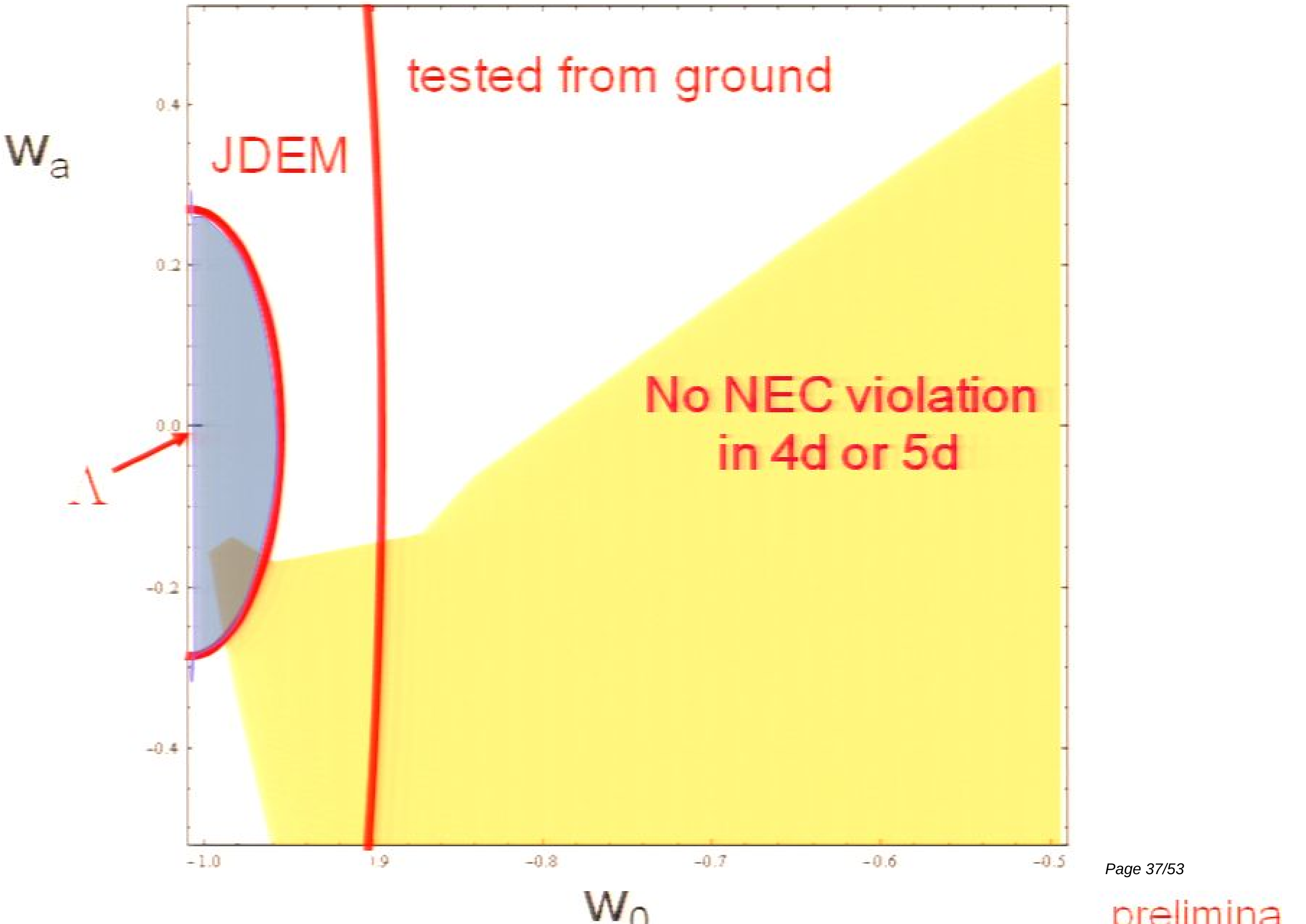

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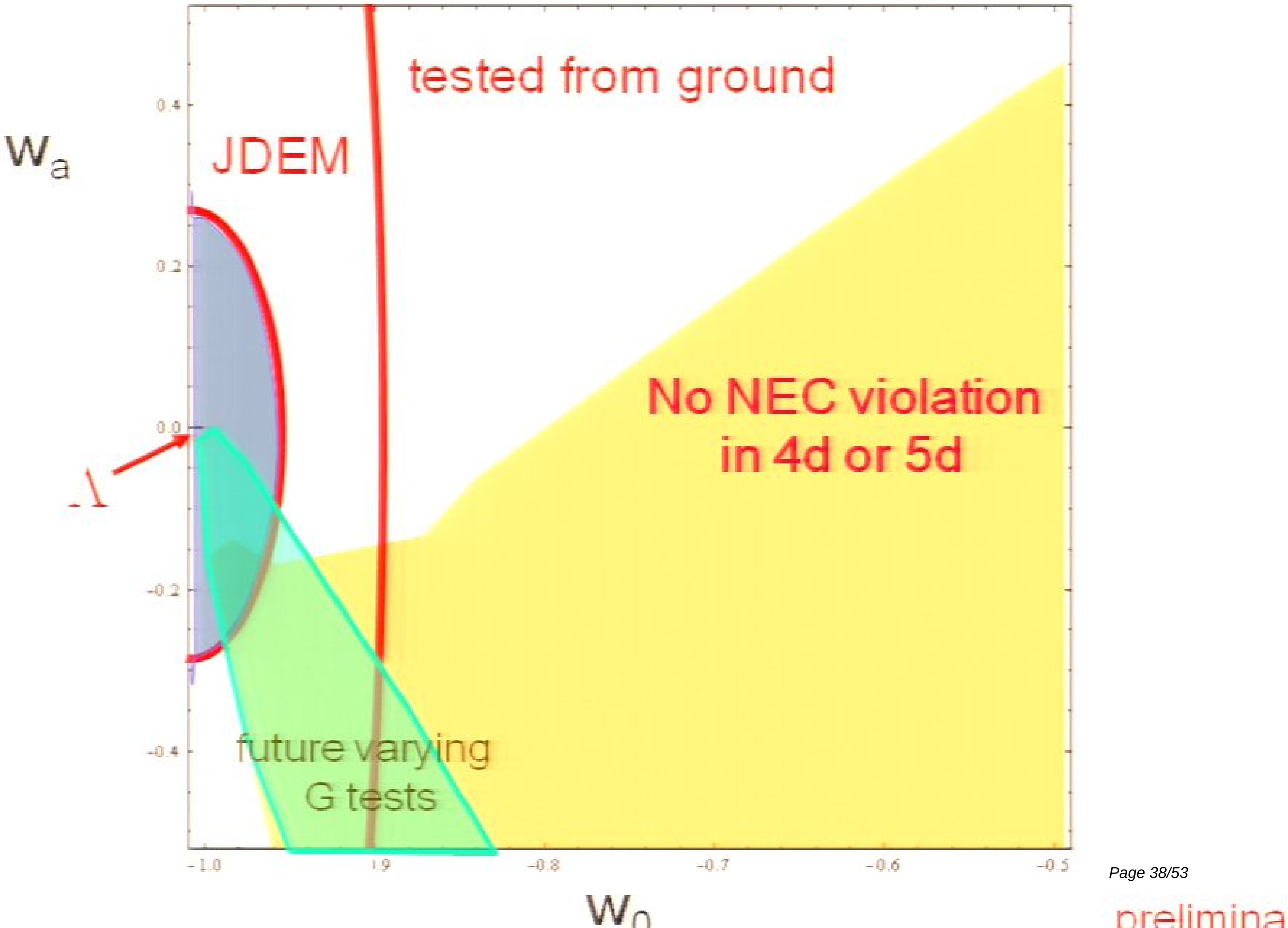

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No, not yet,
although the situation could change
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JDEM (Joint Dark Energy Mission) → JEDM (Joint Extra Dimensions Mission

On the other hand,
Thm I conditions are *completely incompatible* with inflation !

Curious Corollaries:

- inflation requires violating at least one of the Thm I conditions in the early Universe
- Thm I provides interesting counterexample to principle underlying chaotic/eternal inflation

Theorem IV: If fundamental physics is obtained by::

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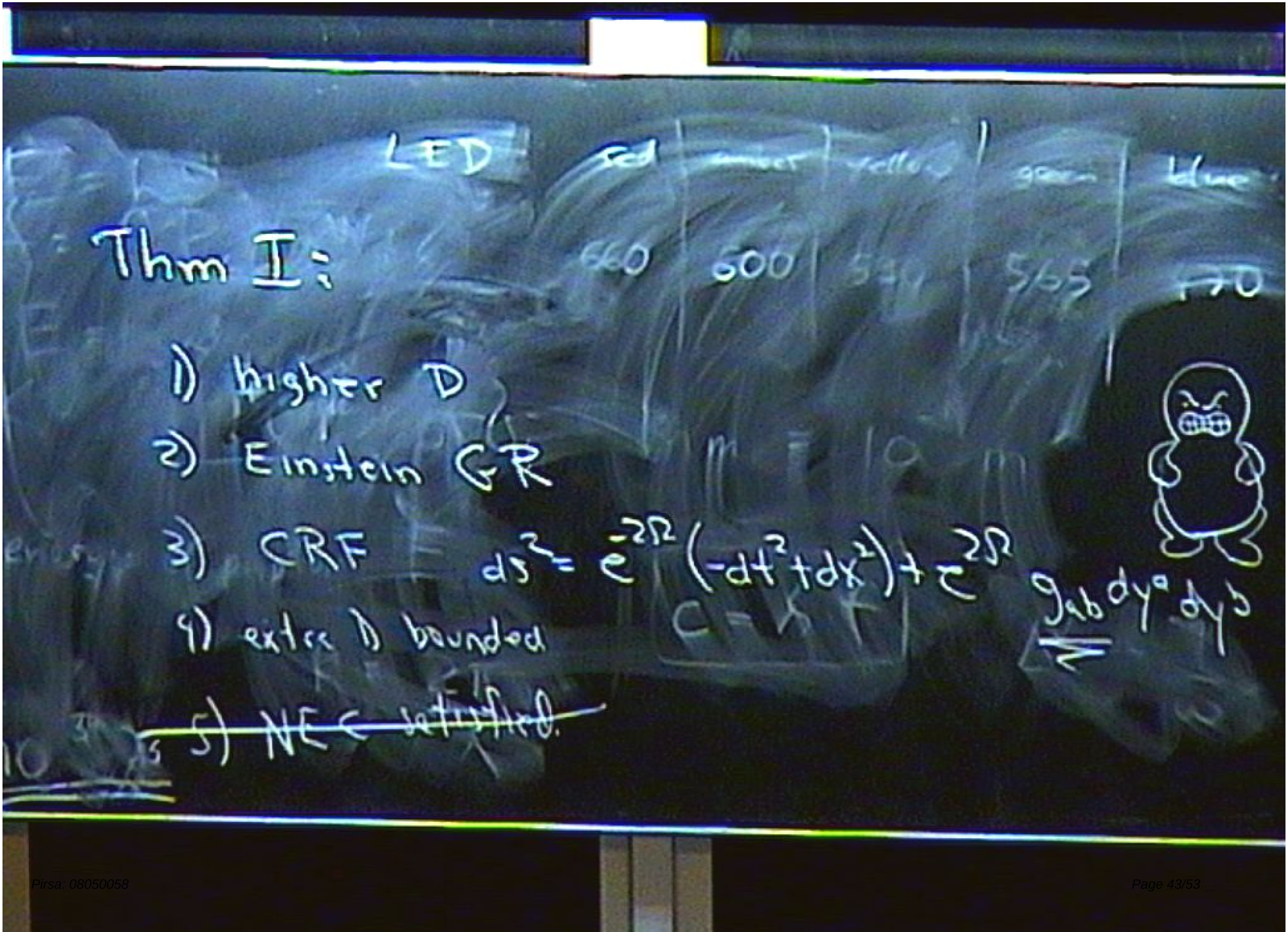
5) null energy condition (NEC) is NOT satisfied in higher D

6) G_N fixed (or very slowly varying)

. . . then, there is an A* such that NEC is violated

for any w_{total} < -1/3 & violation must be in $\langle \rho + p_k \rangle$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. . . then, there is an A* such that NEC is violated

for any w_{total} < -1/3 & violation must be in $\langle \rho + p_k \rangle$

Theorem V: If fundamental physics is obtained by:

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4) and extra D is bounded

5) null energy condition (NEC) is NOT satisfied in higher D

6) G_N fixed (or very slowly varying)

. . . but, also, NEC must be satisfied for same A*

for all $w_{total} > -1/3$

Curious Corollaries:

Constraints on avoiding large G_N variation and
on the nature of NEC violation:

$$p_k{}^{j} = ½\,(-\rho_{4d}{}^{j} + 3\,p_{4d}{}^{j}) = -½\;\rho_{4d}{}^{j}\,(1\text{-}3\,w_{4d})$$

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radiation $p_k{}^r = 0$

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matter $\quad p_k^{m} = -\tfrac{1}{2}\,\rho_{4d}^{m}$

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Constraints on avoiding large G_N variation and on the nature of NEC violation:

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radiation	$p_k^{r} = 0$
matter	$p_k^{m} = -½\, \rho_{4d}^{m}$
Λ	$p_k^{\Lambda} = -2\, \rho_{4d}^{\Lambda}$

But maybe not what you might expect !

e.g. can't be usual Λ or orientifold or Casimir energy alone !

Curious Corollary:

Constraints on avoiding G_N violation and
the kind of NEC violation:

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radiation	$p_k^{r} = 0$
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Λ	$p_k^{\Lambda} = -2\,\rho_{4d}^{\Lambda}$

… also not what you get just by having
scalar fields (inflatons) with flat potentials in 4d

Inflation more problematic than because you must sustain w close to -1 for 60 e-folds: hard to avoid violating NEC by a huge amount

$$\left\langle e^{2\Omega}(\rho+p_3)\right\rangle_A \propto (\rho^{4d}+p^{4d}) - \frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2 + \text{ neg. semi-def.}$$

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neg. semi-def. for range of A

violation of NEC 10^{100} x DE → source of NEC different from DE & must be able to annihilate it more to come . . .

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