Title: Three-body calculations of CBBN reactions for 6Li and 7Li+7Be

Date: May 31, 2008 10:40 AM

URL: http://pirsa.org/08050053

Abstract:

Outline

0) Introduction

1) X^--catalyzed α-transfer reactions

$$(\alpha X) + d \rightarrow {}^6\mathrm{Li} + X$$

$$(\alpha X) + {}^3\mathrm{H} \rightarrow {}^7\mathrm{Li} + X$$

$$(\alpha X) + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be} + X$$

2) X^--catalyzed radiative capture reactions

non-resonant: $({}^7\mathrm{Be}\, X) + p \rightarrow ({}^8\mathrm{B}\, X) + \gamma$

resonant: $({}^7\mathrm{Be}\, X) + p \rightarrow ({}^8\mathrm{B}\, X)_{2p} \rightarrow ({}^8\mathrm{B}\, X)_{1s} + \gamma$

3) X^--catalyzed 3-body breakup reactions

$$({}^6\mathrm{Li}\, X) + p \rightarrow \alpha + {}^3\mathrm{He} + X$$

$$({}^7\mathrm{Li}\, X) + p \rightarrow \alpha + \alpha + X$$

4) Late-time CBBN reactions with (pX)

$$(p\, X) + {}^6\mathrm{Li} \rightarrow \alpha + {}^3\mathrm{He} + X$$

$$(p\, X) + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + X$$

5) Summary

I understand that

my role in this workshop is

to report fully-quantum three-body calculation of

various Catalyzed BBN reactions related to

production and destruction of ^{6}Li and ^{7}Li +^{7}Be

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BBN reactions catalyzed by
a long-lived negatively-charged massive particle,
denoted as X^- .

A candidate of X^- particle is a SUSY particle stau.
(Hamaguchi et al. PL 650 (2007) 268)

But, in this talk, I always refere to X^- .

Discussions about stau will be given by
Hamaguchi in this afternoon.

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5) Summary

An important point in the study of CBBN reactions is that there is, and there will be, no experiment on the reactions.

Any calculation of the CBBN reactions is not a kind of analysis of existing experimental data, but is just a 'prediction' with no experiment in future.

Theoretical prediction on CBBN reactions is then heavily responsible for the network calculation of BBN.

Therefore, you might have questions to me such as

"Does your 3-body calculation have a predictive power?"

"If so, are there any successful examples of your predictions that were verified by experiments performed after your predictions?"

“How accurate is your calculation?”

Yes, our 3-body calculation has a predictive power.

There have been more than 10 examples of successful predictions. But, I have no time to explain them.

The best example of the accuracy of our method for 3-body systems

The best example of the accuracy of our method for 3-body systems

Particle Listings 2007 (Particle Data Group)

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) and 2007 partial update for edition 2008 (URL: http://pdg.lbl.gov)

proton **p** $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗∗

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
938.272029±0.000080	MOHR 05	RVUE	2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •			
938.271998±0.000038	MOHR 99	RVUE	1998 CODATA value
938.27231 ±0.00028	COHEN 87	RVUE	1986 CODATA value
938.2796 ±0.0027	COHEN 73	RVUE	1973 CODATA value

antiproton **p̄** $|m_p - m_{\overline{p}}|/m_p$

A test of *CPT* invariance. Note that the comparison of the $\overline{p}$ and p charge-to-mass ratio, given in the next data block, is much better determined.

VALUE	*CL%*	*DOCUMENT ID*	*TECN*	*COMMENT*
$<1.0 \times 10^{-8}$	90	[1] HORI 03	SPEC	$\overline{p}e^- \, {}^4$He and $\overline{p}e^- \, {}^3$He

↑

This value was determined by our 3-body calculation (10 significant figures) to analyze CERN's lazer spectroscopy data on antiprotonic helium atom

Our calculational method for 3- and 4-body systems

Progress in Particle and Nuclear Physics, 51(2006) 223.(invited review paper

E. Hiyama, Y. Kino and M. Kamimura

[Applications in CBBN]

1] K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino and T.T Yanagida

Particle physics Hadron … Nuclear … Atomic … Particle …

" Stau-catalyzed ^{6}Li production in big-bang nucleosynthesis ",

Phys. Lett. B 650 (2007) 268. ← hep-ph/0702274 (Feb, 2007).

2] A paper on various CBBN reactions for ^{6}Li and ^{7}Li +^{7}Be,

To appear in arXiv in June,

M. Kamimura, Y. Kino and E. Hiyama

Antiproton mass

The winner of 2006 Yukawa Memorial Award

due to the above paper and

due to several successful predictions in

hypernuclear physics

The next talk by Kusakabe

[3] Application of our CBBN reaction rates to a BBN network calculation

M. Kusakabe and T. Kajino (NAO)

The next talk by Kusakabe

[3] Application of our CBBN reaction rates to a BBN network calculation

M. Kusakabe and T. Kajino (NAO) - - - - - - - - →

Kamimura
↓
Matsuse
↓
Kajino
↓
Kusakabe

Section 1.

X^--catalyzed α-transfer reactions

—— Reactions to produce ^{6}Li, ^{7}Li, ^{7}Be ——

$$(\alpha X) + d \rightarrow {}^6\mathrm{Li} + X$$

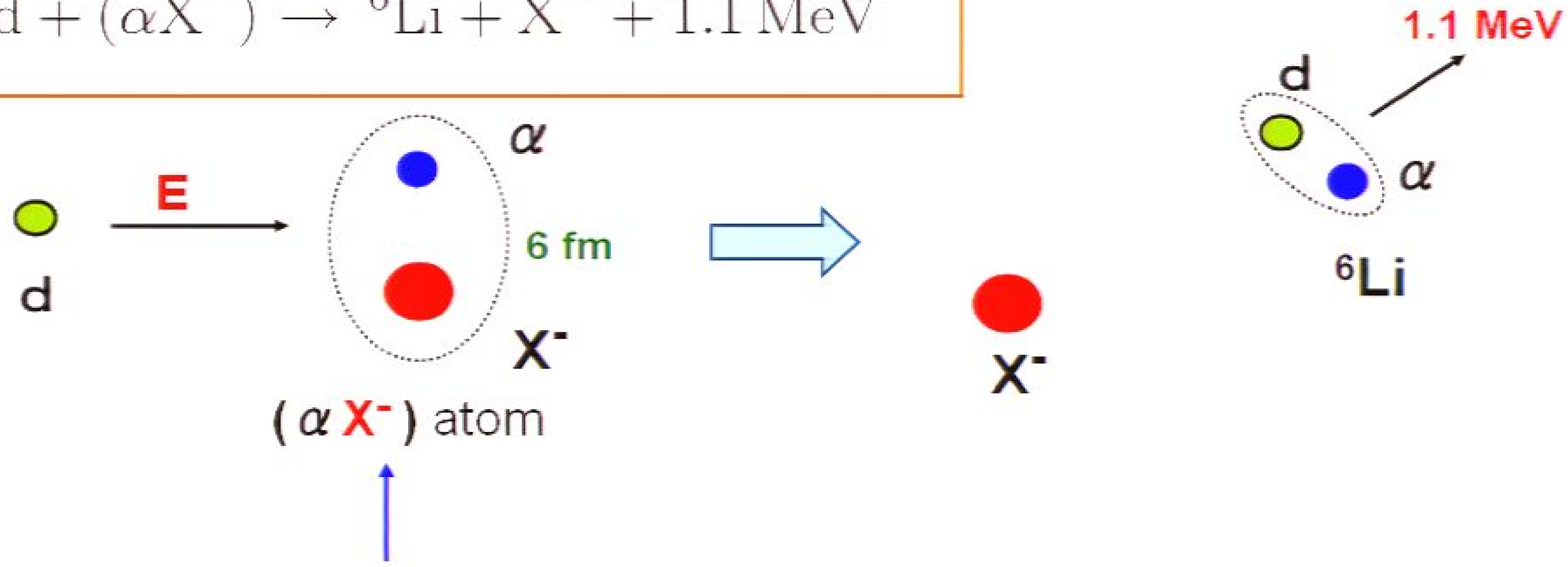
$$(\alpha X) + {}^3\mathrm{H} \rightarrow {}^7\mathrm{Li} + X$$

$$(\alpha X) + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be} + X$$

1) ^{6}Li production

Pospelov, PRL 98 (2007) 231301 [hep-ph/0605215] $\frac{\text{CBBN}}{\text{SBBN}} \sim 10^8$

$$\boxed{\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}}$$



Formation of (α X^-) atom at $T_9 \sim 0.1$

Problem is to calculate the cross section of this reaction at low energies E.

Definition of astrophysical S-factor S(E)

Cross section

$$\sigma(E) = S(E) \underbrace{\exp\left(-bE^{-\frac{1}{2}}\right)} / E$$

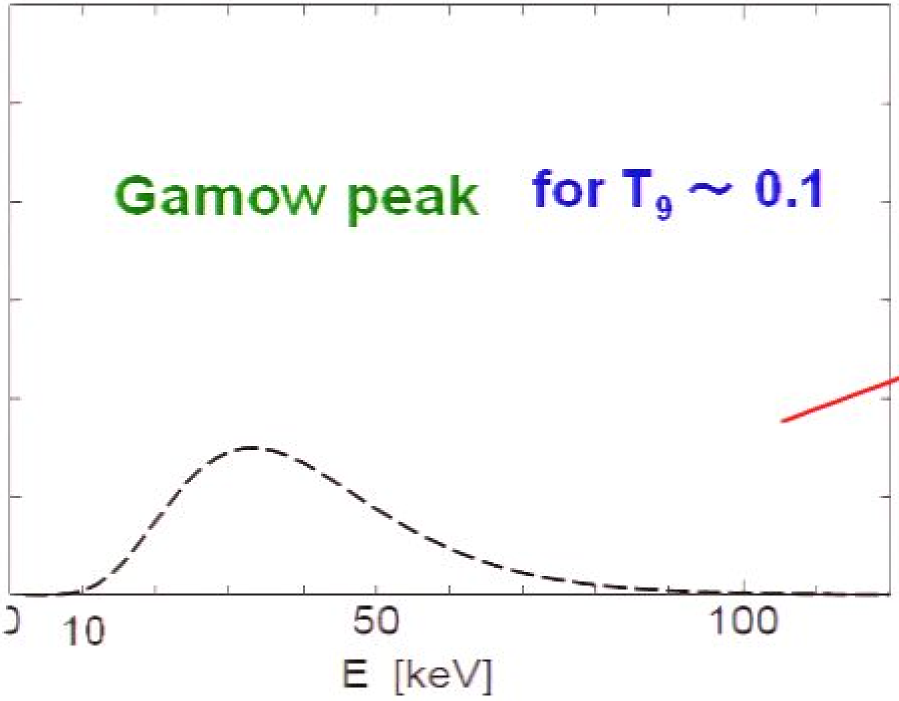
d + (α X) → 6Li + X

Coulomb
penetration

Maxwell-Boltzmann

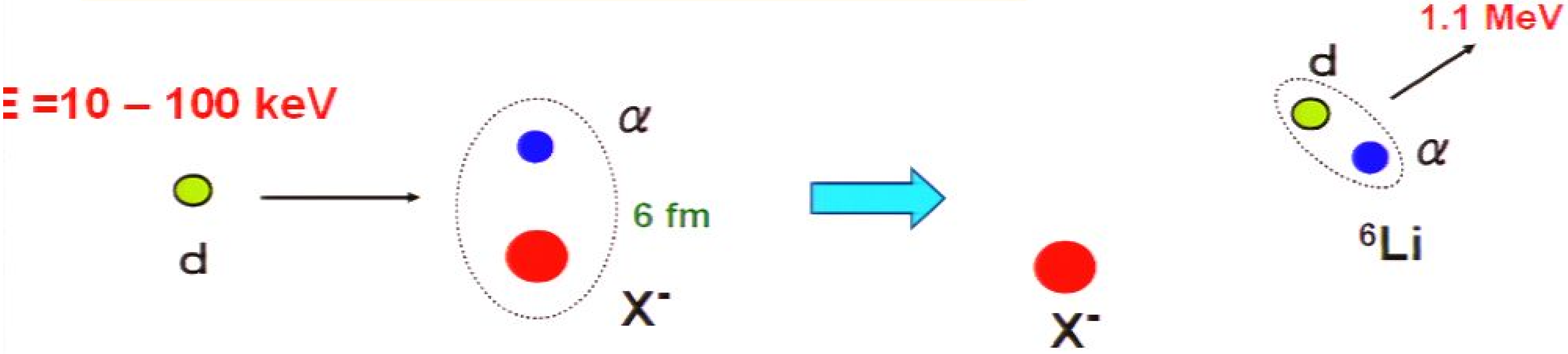
Reaction rate

$$\langle \sigma v \rangle = (\text{const.}) \int_0^\infty S(E) \underbrace{\exp\left(-\frac{E}{kT} - bE^{-\frac{1}{2}}\right)}_{\text{Gamow peak}} dE$$



S(E) is to be calculated at
E=10 – 100 keV

$$\boxed{d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}}$$



Once the potentials between any pairs of

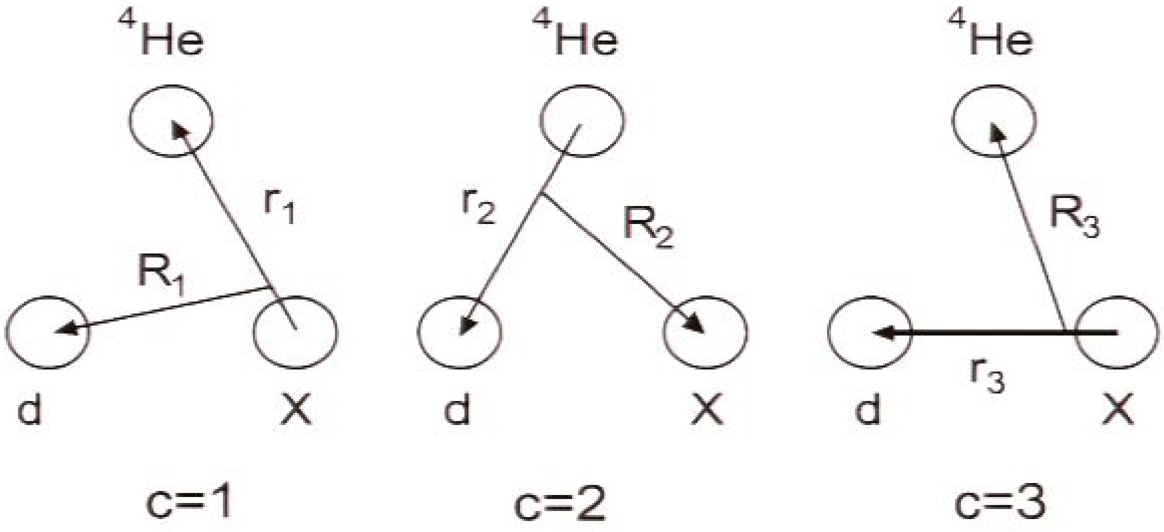
the 3 particles are given,

this is a well-defined quantum 3-body problem.

Let us solve exactly the 3-body Schroedinger equation

and derive the cross sections at E =10 – 100 keV.

3 sets of Jacobi coordinates

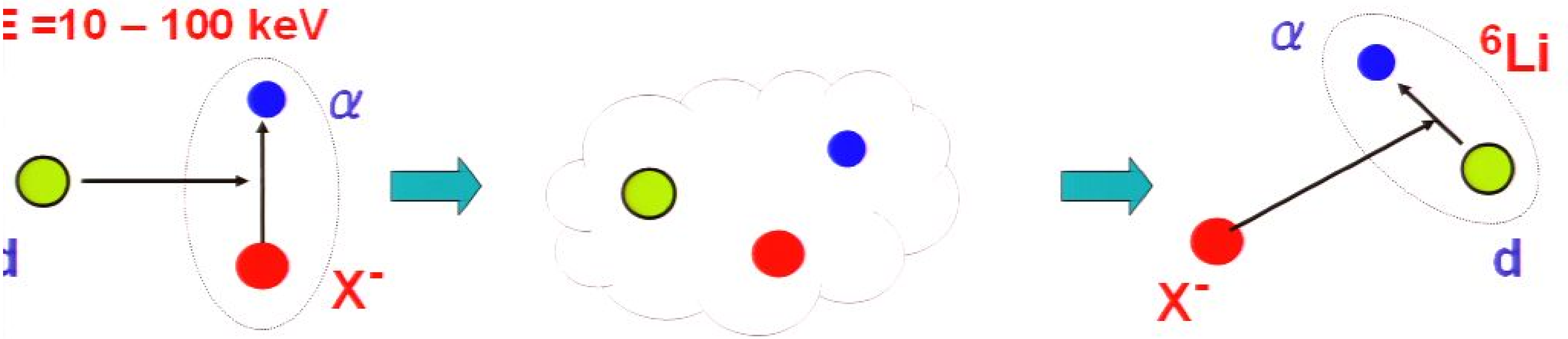


3-body Hamiltonian

$$H = -\frac{\hbar^2}{2m_c}\nabla^2_{\mathbf{r}_c} - \frac{\hbar^2}{2M_c}\nabla^2_{\mathbf{R}_c} + V_{^4\mathrm{He}\text{-}X}(r_1) + V_{^4\mathrm{He}\text{-}d}(r_2) + V_{d\text{-}X}(r_3$$

3-body Schroedinger equation

$$(H - E_{\mathrm{tot}})\Psi_{JM} = 0$$

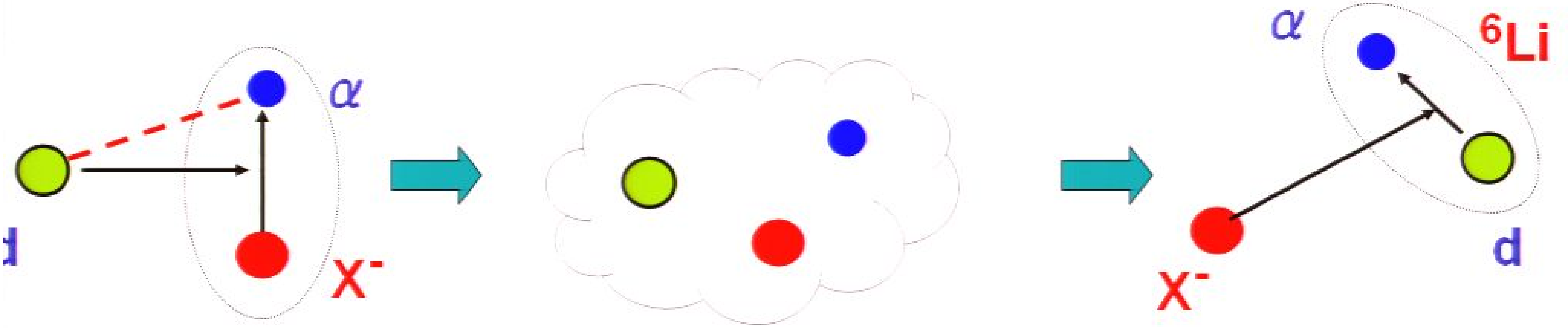


Important points:

1) This is a rearrangement reaction in which, by strong d-α interaction, α particle is tranferred to deuteron to form ^{6}Li.

2) Incident energy is much below the Coulomb barrier (~500 keV). Therefore, the reaction proceeds very slowly. There is enough time to have multi-step transitions between the entrance- and exit- channels.

3) Furthermore, in the interaction region, the 3-particles easily take complicated configurations that are much different from the entrance- and exit-channel wave functions.



E =10 – 100 keV

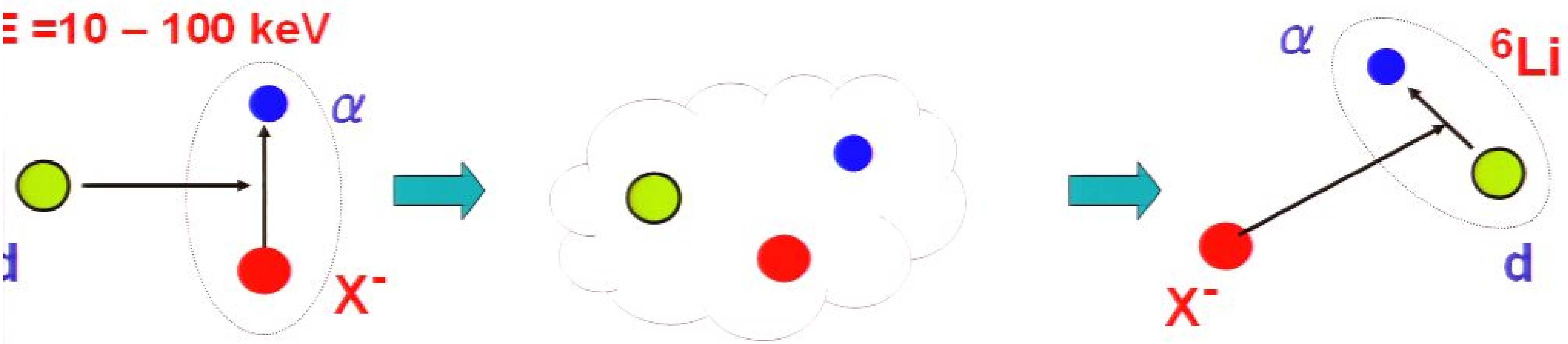
validity of Born approximation

| α - d interaction | << E

But, order of 10 MeV >> 10 – 100 keV

Similarly to other CBBN reactions

Born approximation does not work at all.



Precise 3-body calculation to solve this type of problem is usually a very difficult task of nuclear reaction theory.

But, in the study of muon catalyzed fusion,
Kino and myself have experiences of peforming
such a 3-body calculation of low-energy
muon transfer reactions that are very similar to
the present α-particle transfer reaction.

Section 1.1

Analogical reactions in the muon catalyzed fusion (μ CF) cycle

A review article :

K. Nagamine and M. Kamimura,

Adv. Nucl. Phys. 24 (1998) 151.

low-energy
α - transfer reaction

$$d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$

E ~10 -100 keV

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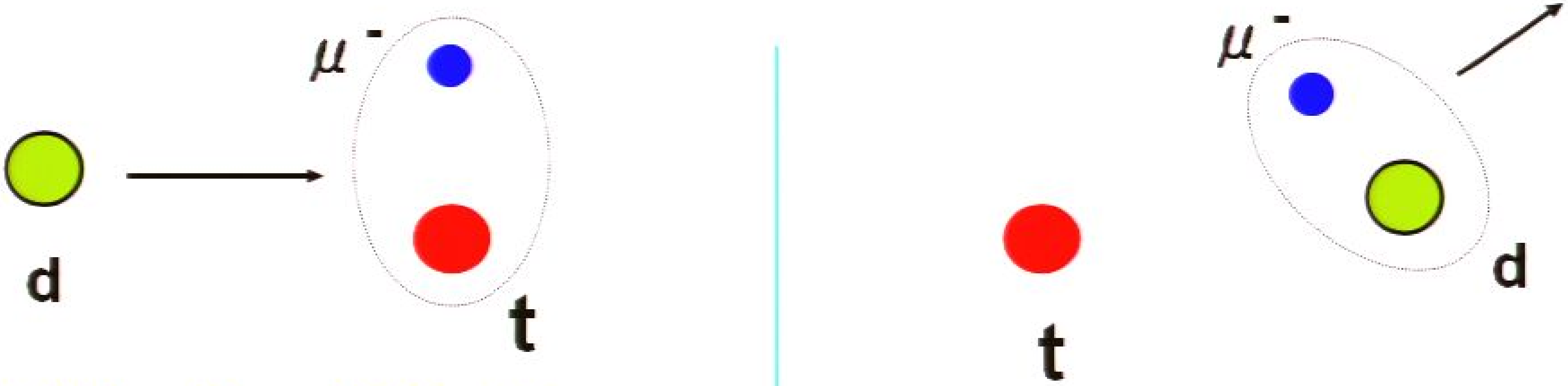
low-energy
μ - transfer reaction

$$t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$$

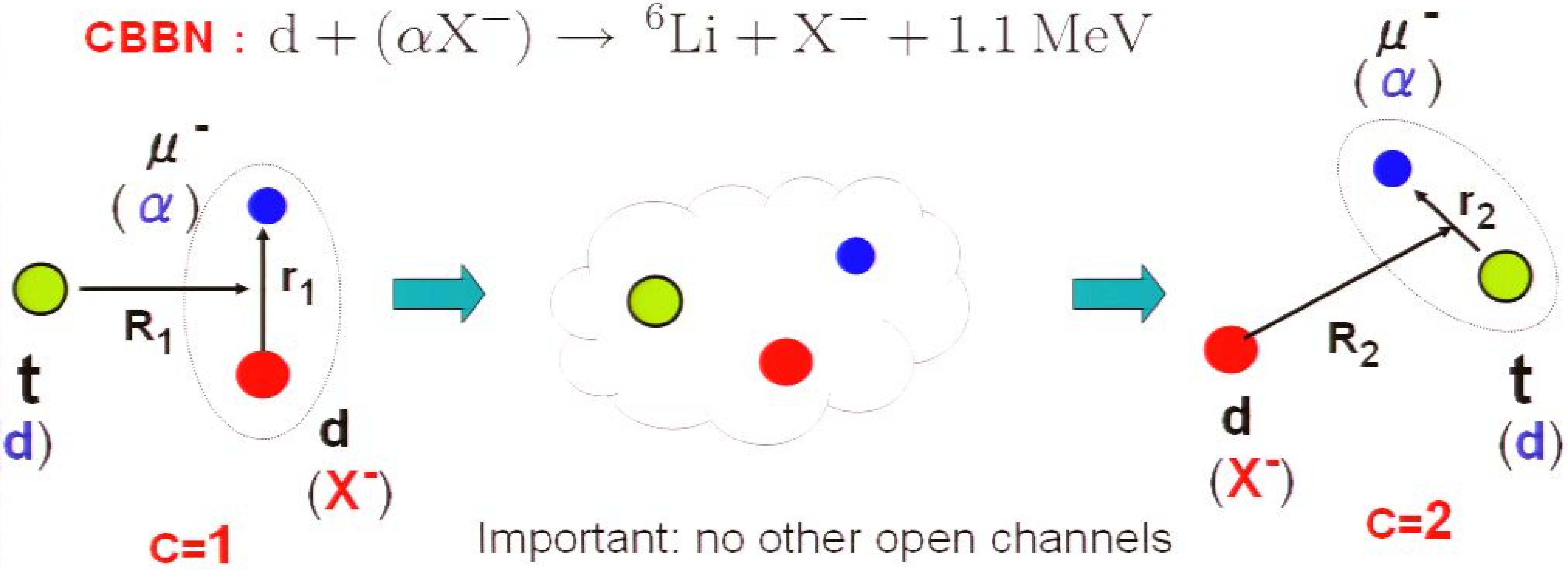


E= 0.001 eV --- 100 eV

Full 3-body calculation of low-energy transfer reaction

μ CF : $t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$

CBBN : $\mathrm{d} + (\alpha\mathrm{X}^-) \rightarrow {}^6\mathrm{Li} + \mathrm{X}^- + 1.1\,\mathrm{MeV}$

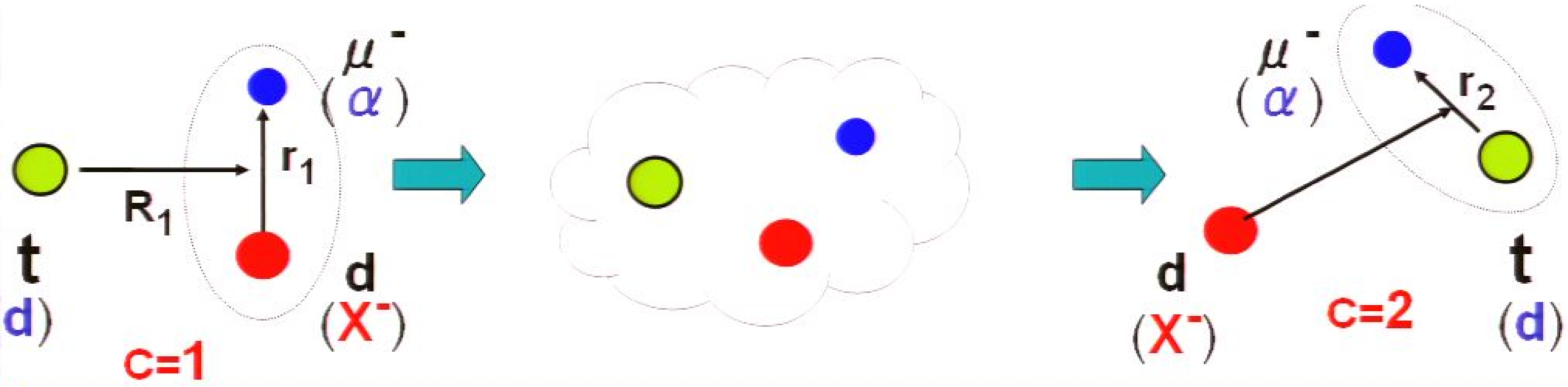


Important: no other open channels

$$\Psi_{JM} = \phi^{(1)}_{\mathrm{g.s.}}(\mathbf{r}_1)\,\underline{\chi^{(1)}_{JM}(\mathbf{R}_1)} + \phi^{(2)}_{\mathrm{g.s.}}(\mathbf{r}_2)\,\underline{\chi^{(2)}_{JM}(\mathbf{R}_2)} + \Psi^{(\mathrm{closed})}_{JM} \ .$$

Scattering wave functions to be solved

c=1 : incoming and outgoing waves c=2 : outgoing wave



$$\Psi_{JM} = \phi^{(1)}_{\rm g.s.}(\mathbf{r}_1)\,\underline{\chi^{(1)}_{JM}}(\mathbf{R}_1) + \phi^{(2)}_{\rm g.s.}(\mathbf{r}_2)\,\underline{\chi^{(2)}_{JM}}(\mathbf{R}_2) + \boxed{\Psi^{(\rm closed)}_{JM}}\,.$$

$$\boxed{\Psi^{(\rm closed)}_{JM}} = \sum_{\nu=1}^{\nu_{\rm max}} \underline{b_{J\nu}}\,\Phi_{JM,\nu}$$

3-body complete set

The 3-rd term, $\boxed{\Psi^{(\rm closed)}_{JM}}$, stands for all the closed channels (virtually-excited channels).

This term is responsible for all the asymptotically-vanishing 3-body amplitudes that are not included in the first two scattering terms.

The term is expanded in terms of the complete set of 3-body basis functions in the finite region

$$\boxed{\Psi_{JM} = \phi^{(1)}_{\rm g.s.}(\mathbf{r}_1)\,\underline{\chi^{(1)}_{JM}(\mathbf{R}_1)} + \phi^{(2)}_{\rm g.s.}(\mathbf{r}_2)\,\underline{\chi^{(2)}_{JM}(\mathbf{R}_2)} + \boxed{\Psi^{\rm (closed)}_{JM}}\,.}$$

$$\boxed{\boxed{\Psi^{\rm (closed)}_{JM}} = \sum_{\nu=1}^{\nu_{\rm max}} \underline{b_{J\nu}}\,\Phi_{JM,\nu}}$$

Scattering boundary condition:

$$\lim_{R_c\to\infty}\Psi_{JM} = \phi^{(c)}_{\rm g.s.}(\mathbf{r}_c)\Big[u_J^{(-)}(k_cR_c)\delta_{c1} - \sqrt{\frac{v_1}{v_c}}\,\underline{\underline{S^J_{1\to c}}}\,u_J^{(+)}(k_cR_c)\Big]Y_{JM}(\widehat{\mathbf{R}}_c). \qquad (c = 1, 2$$

The above is the most general expression of the transfer reaction.

How to solve it starting from $(H - E_{\rm tot})\Psi_{JM} = 0$

is written in Hamaguchi et al., Phys. Lett. B650 (2007) 268.

$$\sigma_{1\to2}(E) = \frac{\pi}{k^2}\sum_{J=0}^{\infty}(2J+1)|\underline{\underline{S^J_{1\to2}}}|^2. \quad \longrightarrow \quad S(E) = \sigma_{1\to2}(E)\,E\exp(2\pi\eta(E$$

We calculated the wave function using two completely different methods:

1) direct numerical method (finite-difference method) --- by Kino

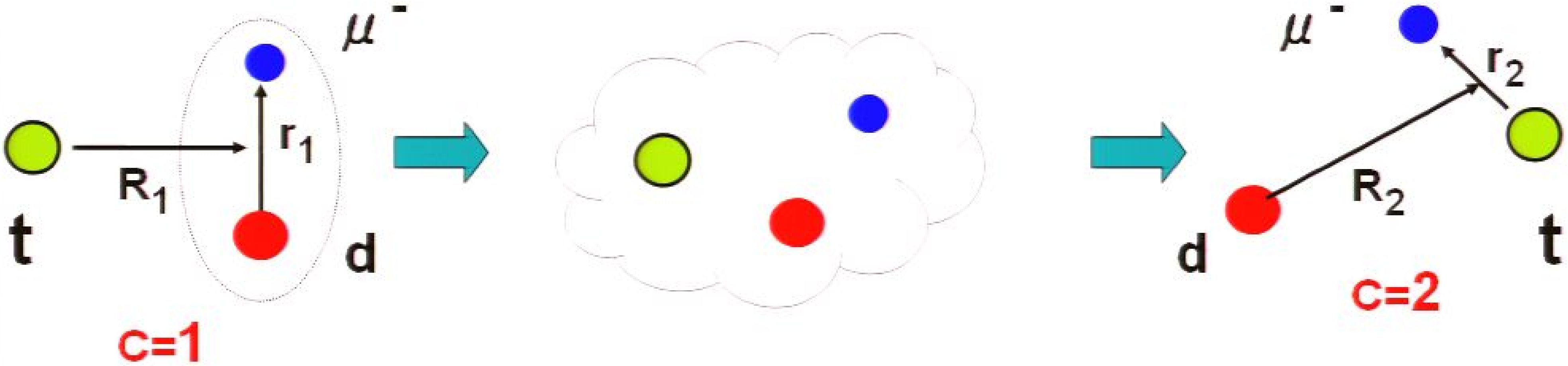
2) Kohn-type variational method --- by Kamimura.

The same result was obtained (we were so careful)

Full 3-body calculation of muon transfer reaction

$$t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$$

$$\Psi_{JM} = \phi^{(1)}_{\mathrm{g.s.}}(\mathbf{r}_1)\,\underline{\chi^{(1)}_{JM}(\mathbf{R}_1)} + \phi^{(2)}_{\mathrm{g.s.}}(\mathbf{r}_2)\,\underline{\chi^{(2)}_{JM}(\mathbf{R}_2)} + \boxed{\Psi^{(\mathrm{closed})}_{JM}}.$$



Large error of approximations

Ratio of cross sections ←

Full 3-body		2-ch CC		Born App. (the best version)
1	:	30		
		1	:	4

I have no time to discuss more about the results and accuracy of the calculation.

But I say, among several literature calculations of the muon transfer reaction, ours is considered to be best and has been used as the benchmark for comparison.

There are examples of successful predictions, before masurement on the following muon transfer reactions via resonant molecular states

$$(d\mu)_{1s} + \ ^3\mathrm{He} \rightarrow (d\,^3\mathrm{He}\mu)_{\mathrm{molecule}} \rightarrow \ (^3\mathrm{He}\mu)_{1s} + d + \gamma$$

$$(p\mu)_{1s} + \ ^4\mathrm{He} \rightarrow (p\,^4\mathrm{He}\mu)_{\mathrm{molecule}} \rightarrow \ (^4\mathrm{He}\mu)_{1s} + p + \gamma$$

Afterwards, experiments at KEK (1993) verified our prediction (1992).

Section 1.2

Result on

$$(\alpha X) + d \rightarrow {}^6\mathrm{Li} + X$$

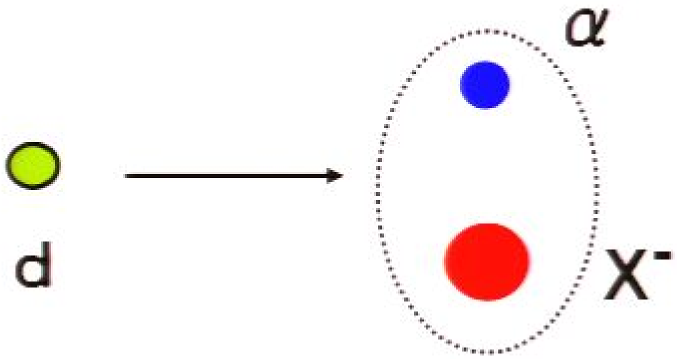
oulomb barrier $\longrightarrow$ $d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$

o Coulomb barrier $\longrightarrow$ $t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$

The former is much more difficult to treat:

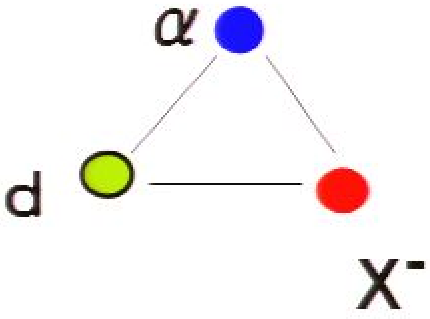
1) X^- catalyzed nuclear fusion takes place much below the Coulomb barrier.

2) We have to treat simultaneously both long-range Coulomb potential and short-range nuclear potential which is the driving force of the transfer reaction.



3) We have to choose a good **α-d** nuclear interaction that explain **α-d** system well.

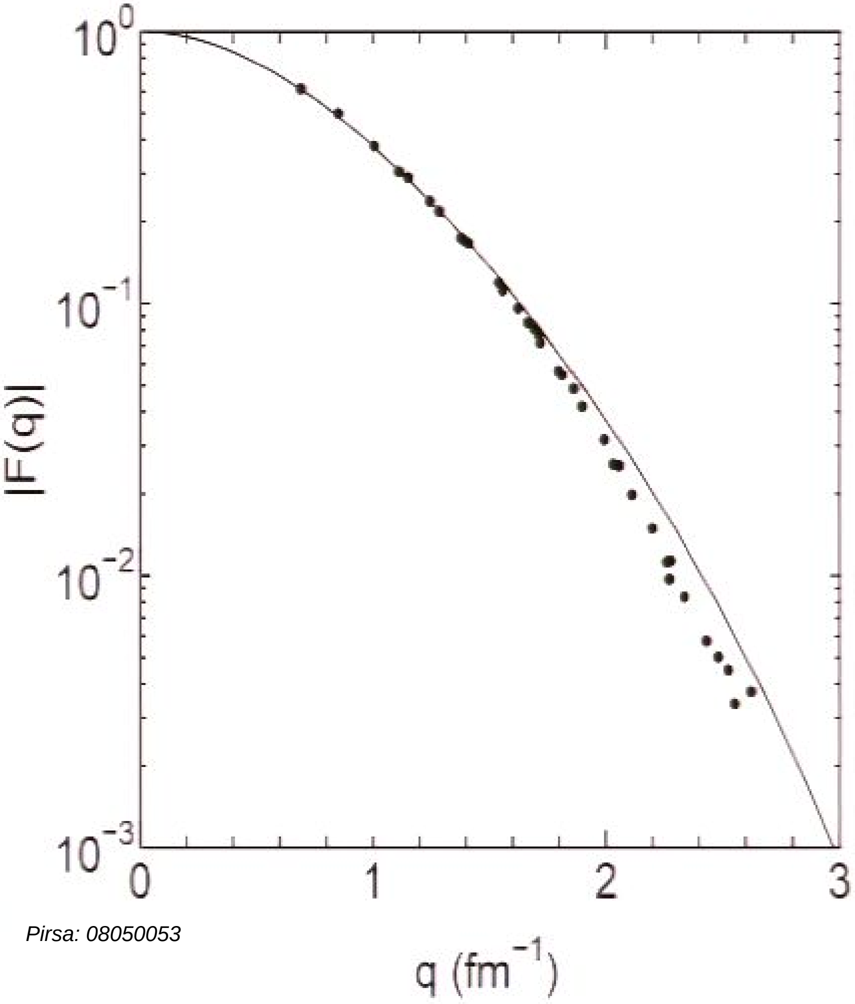
$\alpha - \mathrm{d}$ system



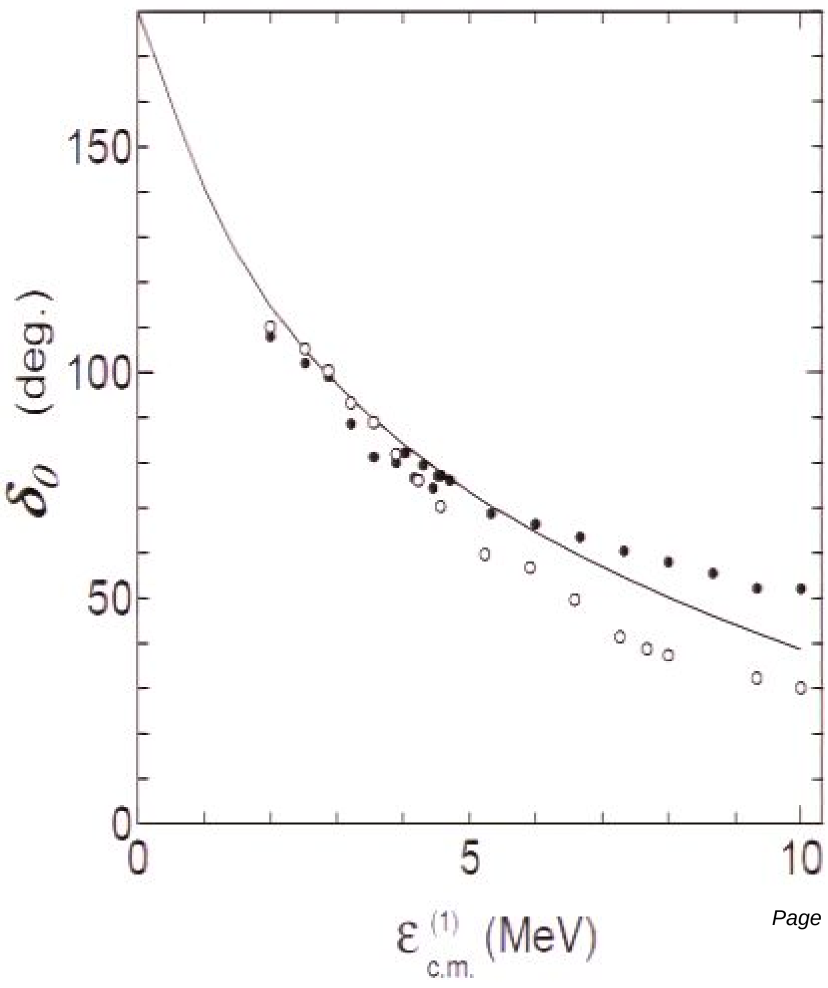
1) **B.E. of ^{6}Li**

2) **r.m.s. charge radius of d, α, ^{6}Li**

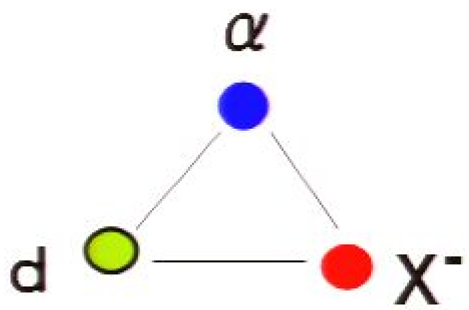
3) **^{6}Li electron-scattering form factor**



4) **$\alpha - \mathrm{d}$ phase shift**



Now, we have



1) good interactions among the 3 particles.

2) an accurate method to solve 3-body Schroedinger equation.

↓

3) predictability for the property of the 3-body system.

(many successful examples)

Spin of **d** is ignored.

No problem in non-resonant scattering

with only one particle having spin.

In Section 2, spin is very important in the resonant reactio

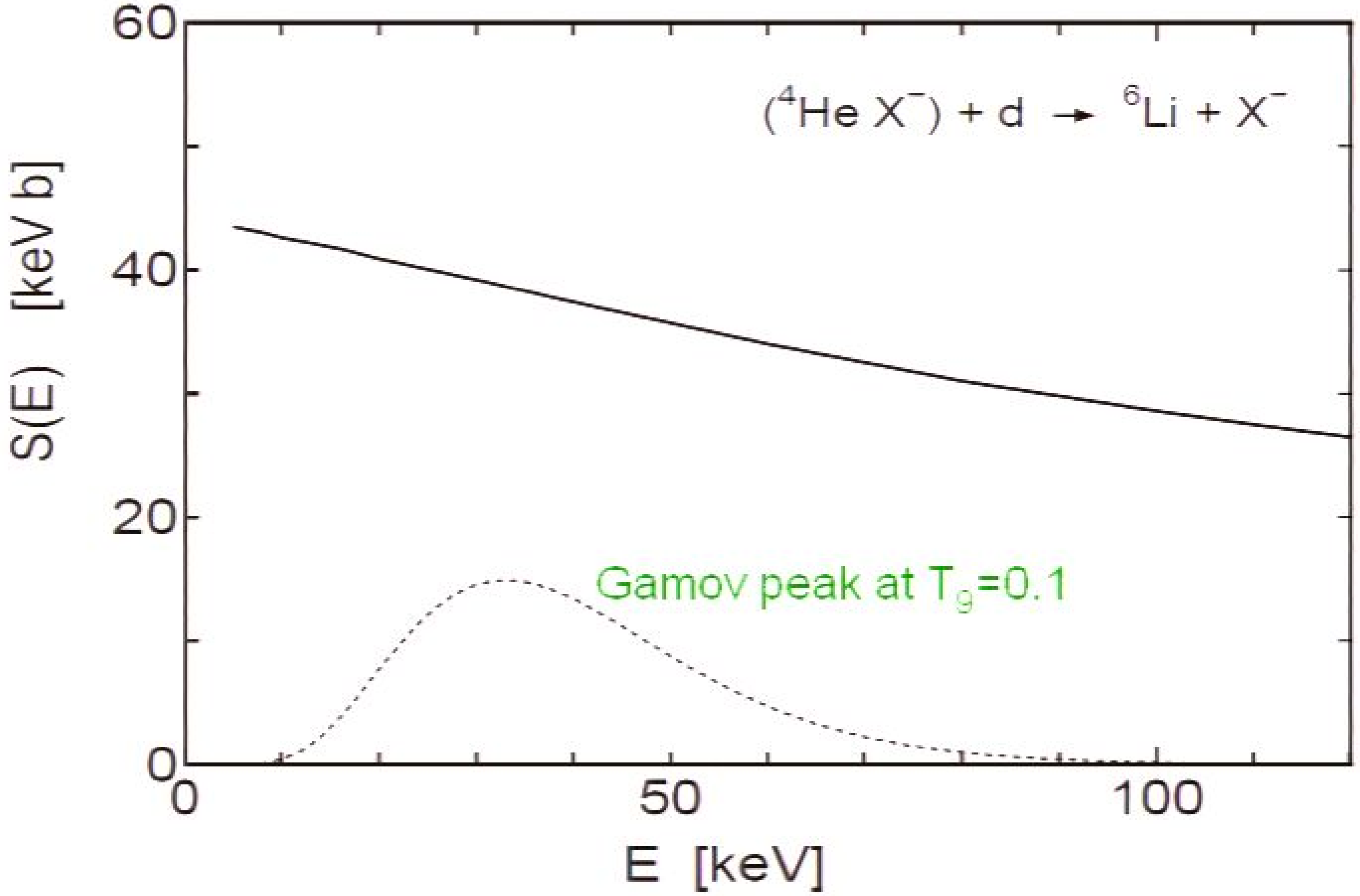
$$({}^7\mathrm{Be}\ X) + p \ \rightarrow\ ({}^8\mathrm{B}\ X)_{2p} \rightarrow\ ({}^8\mathrm{B}\ X)_{1s} + \gamma\ .$$

Result

$$d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$

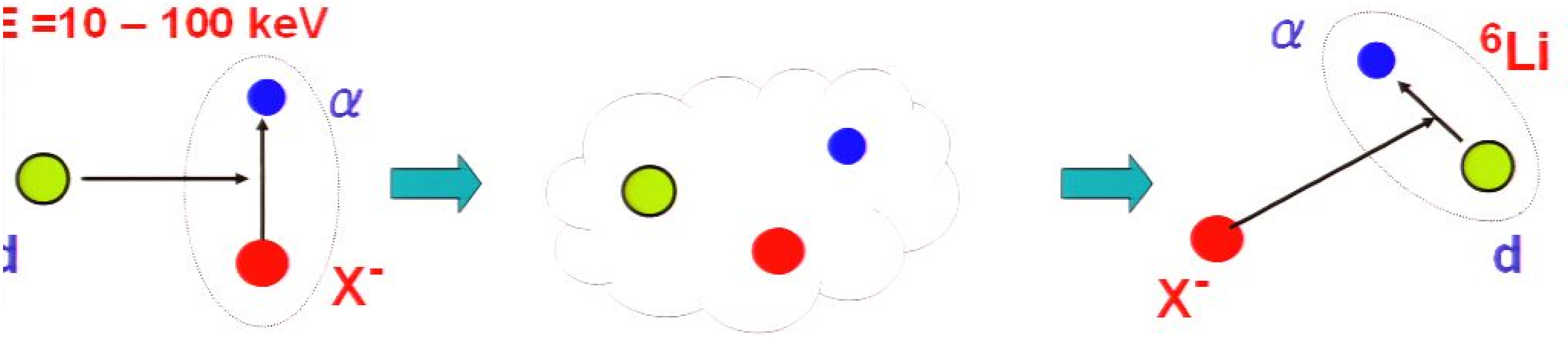
Calculated astrophysical S-factor

Hamaguchi et al.
PL B650
(2007) 268.



Reaction rate

$$N_A\langle\sigma v\rangle = 2.37\times10^8\,(1-0.34\,T_9)\,T_9^{-2/3}\exp\left(-5.33\,T_9^{-1/3}\right)\quad \mathrm{cm}^3\,\mathrm{s}^{-1}\,\mathrm{mol}^{-1}.$$



$$\Psi_{JM} = \phi^{(1)}_{\rm g.s.}(\mathbf{r}_1)\,\chi^{(1)}_{JM}(\mathbf{R}_1) + \phi^{(2)}_{\rm g.s.}(\mathbf{r}_2)\,\chi^{(2)}_{JM}(\mathbf{R}_2) + \boxed{\Psi^{\rm (closed)}_{JM}}.$$

$$\boxed{\Psi^{\rm (closed)}_{JM}} = \sum_{\nu=1}^{\nu_{\rm max}} b_{J\,\nu}\,\Phi_{JM,\,\nu}$$

	Full CC	**2-ch CC**		**Born app.** (the best version)
cross section (ratio)	1	: $\frac{1}{3}$		
		1	:	12

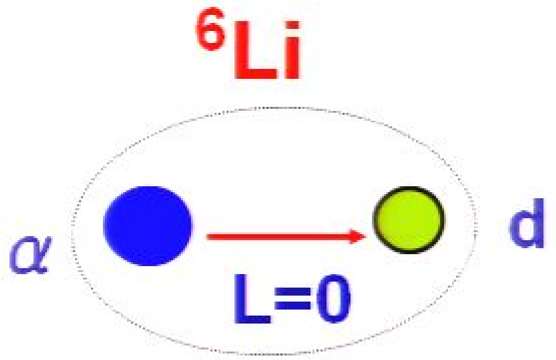
Section 1.3

Result on

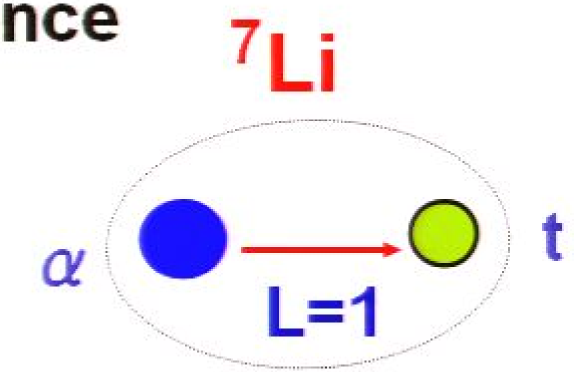
$$(\alpha X) + t \rightarrow {}^{7}\mathrm{Li} + X$$

$$(\alpha X) + {}^{3}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + X$$

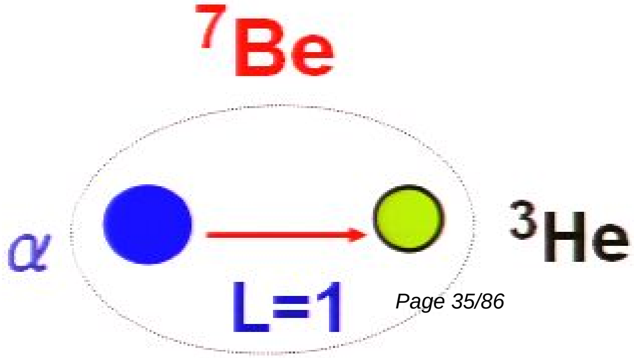
$(\alpha X) + d \rightarrow {}^6Li + X$



big difference



$(\alpha X) + t \rightarrow {}^7Li + X$



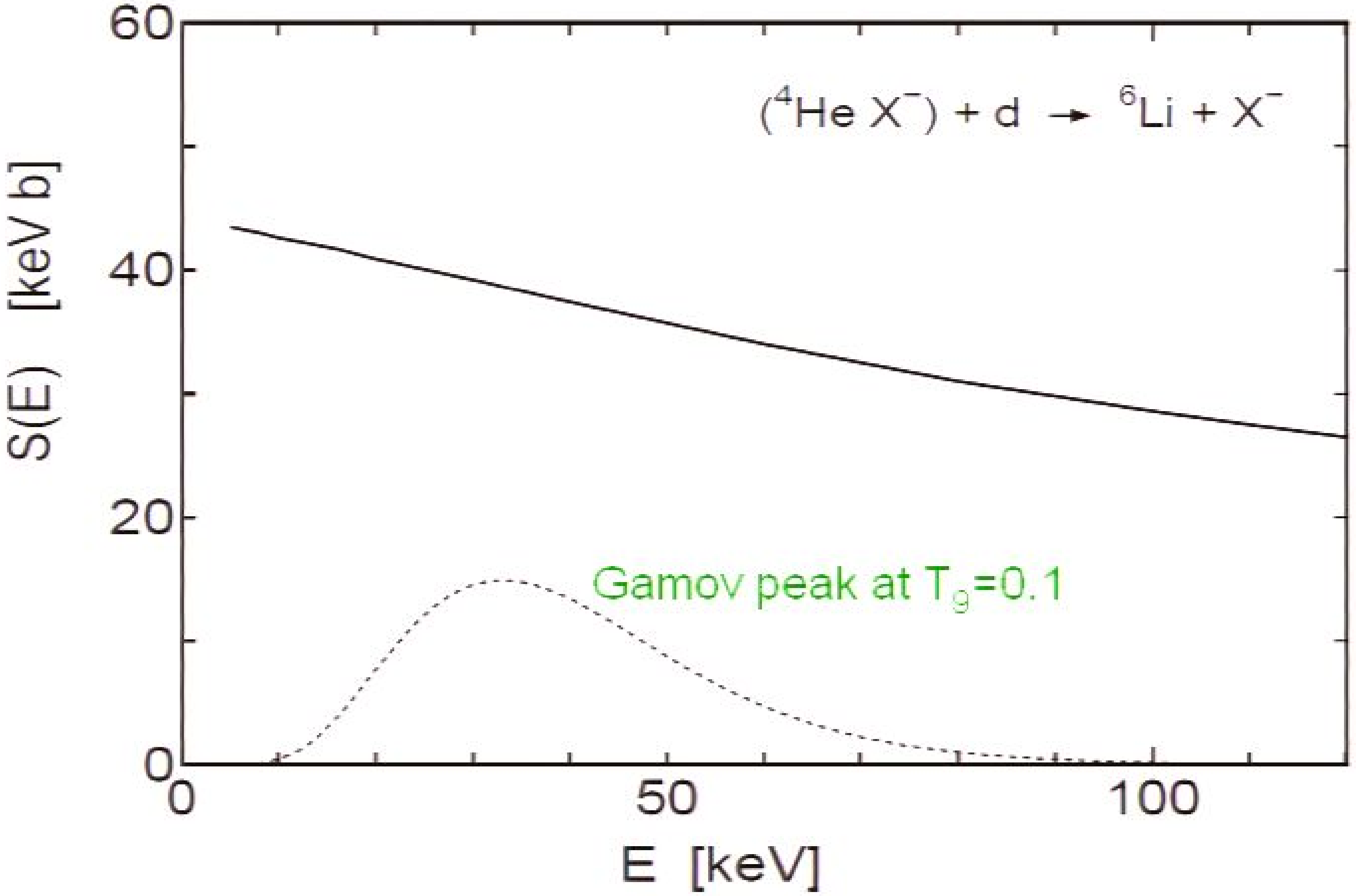
$(\alpha X) + {}^3He \rightarrow {}^7Be + X$

Result

$$d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$

Calculated astrophysical S-factor

Hamaguchi et al.
PL B650
(2007) 268.



Reaction rate

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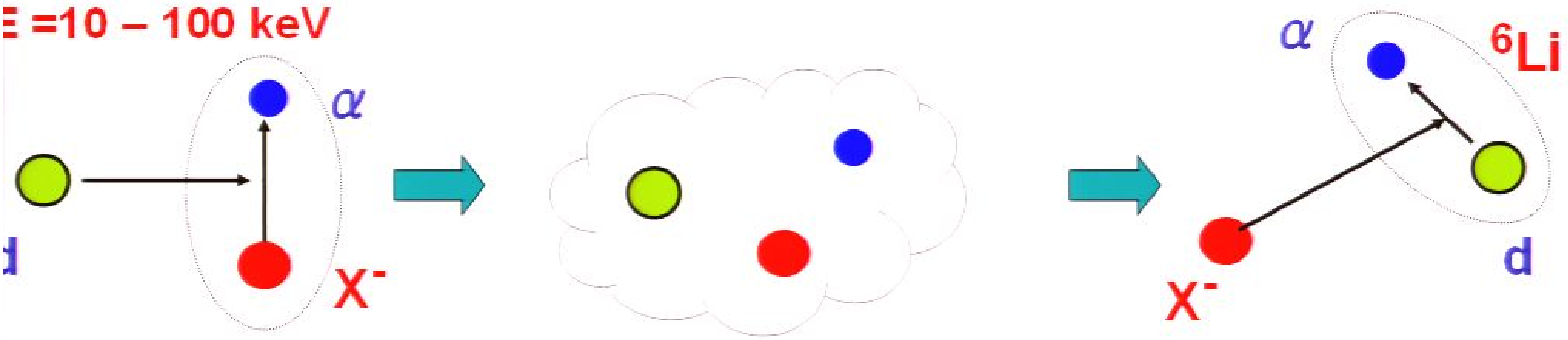

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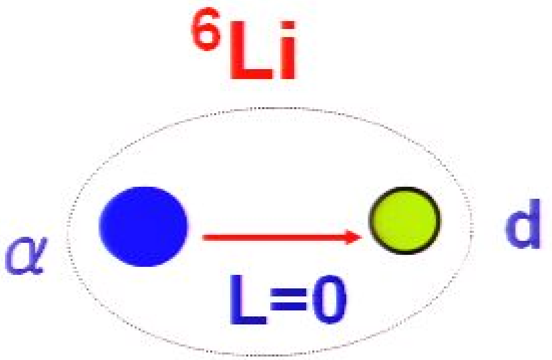
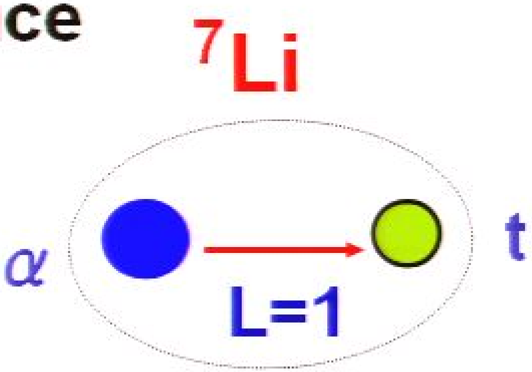
	Full CC		**2-ch CC**	**Born app.** (the best version)
cross section (ratio)	1	:	$\frac{1}{3}$	
			1 :	12

$(\alpha X) + d \rightarrow {}^6\mathrm{Li} + X$

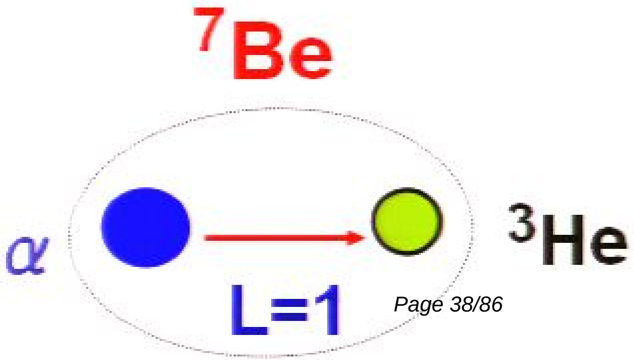


big difference

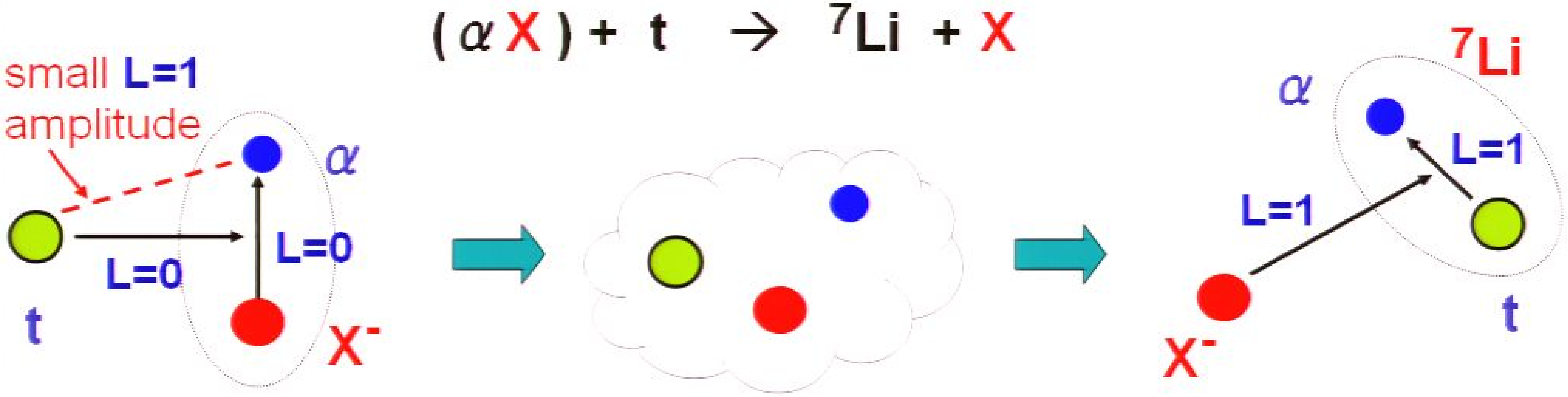
$(\alpha X) + t \rightarrow {}^7\mathrm{Li} + X$



$(\alpha X) + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be} + X$

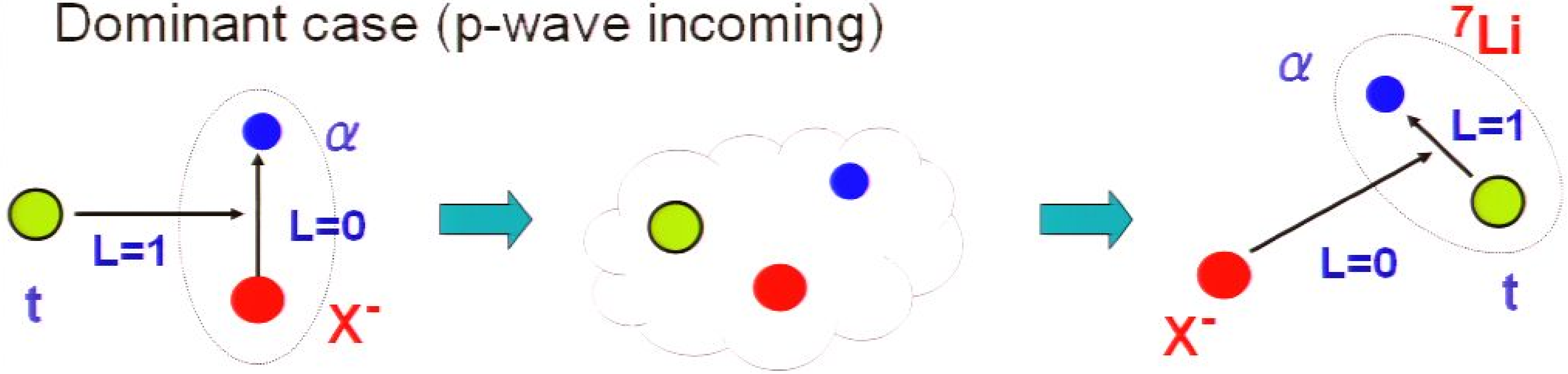


$(\alpha X) + t \rightarrow {}^7\mathrm{Li} + X$



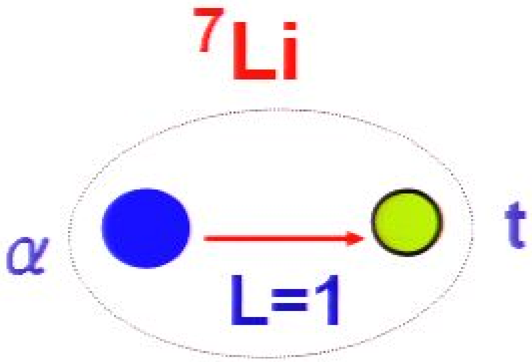
(s-wave incoming)

Dominant case (p-wave incoming)



Cross section (S-factor) will be smaller than that in the 6Li production reaction.

$$(\alpha X) + t \rightarrow {}^{7}\text{Li} + X$$



We determined

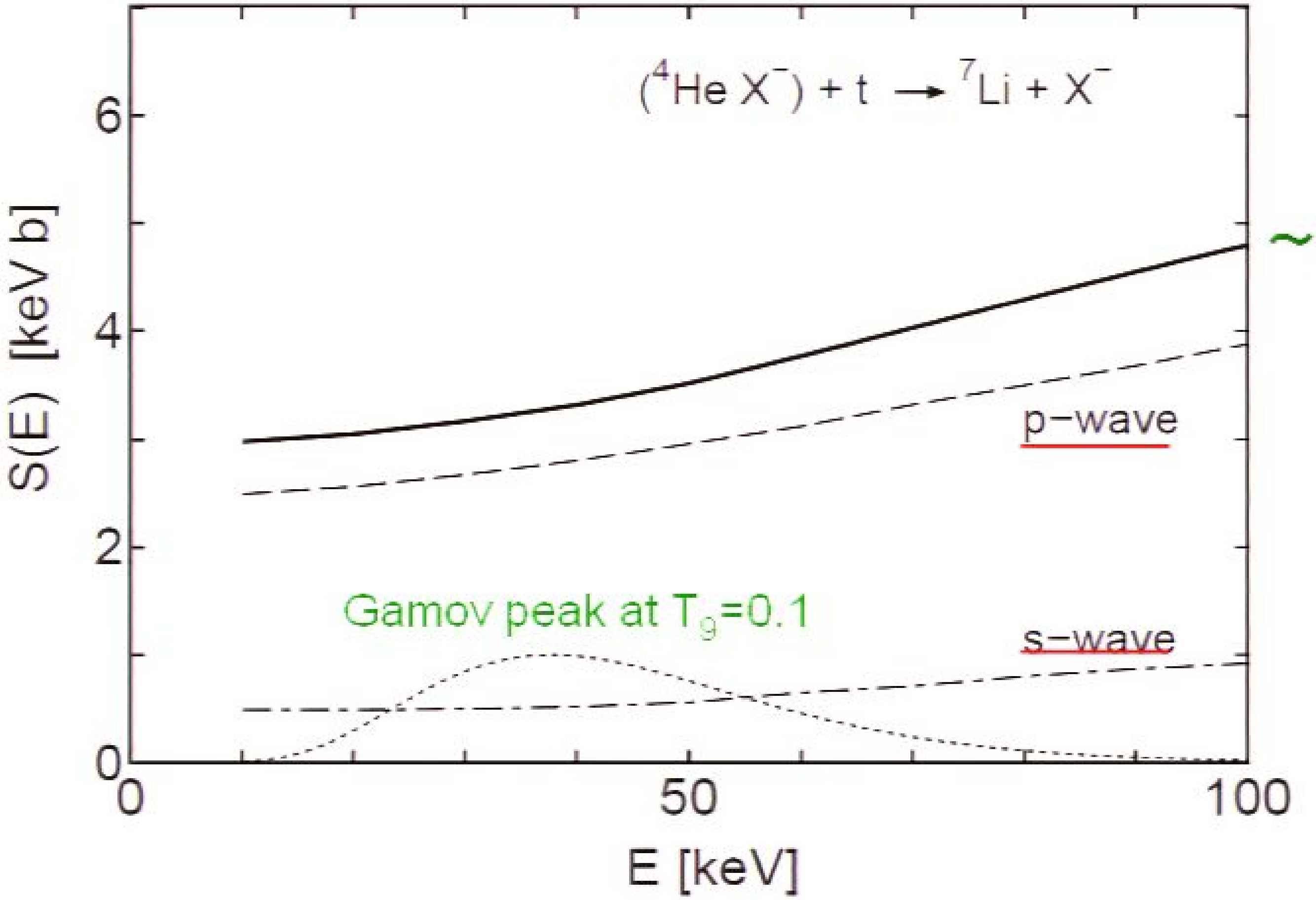
the interaction between α and **t**

to reproduce

binding energy , r.m.s. radius,

low-energy α-**t** scttering phase shifts.

$(\alpha X) + t \to {}^{7}\mathrm{Li} + X$

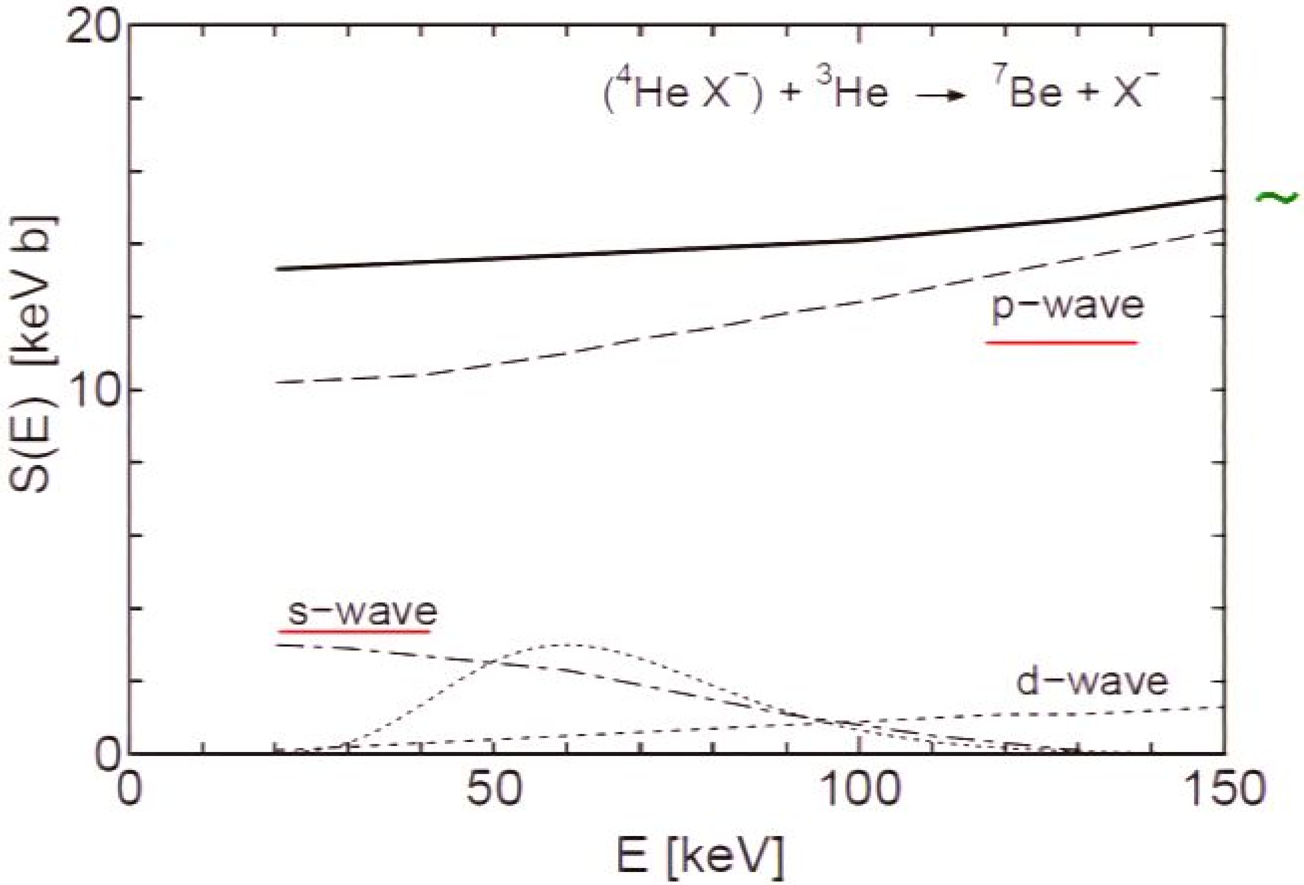


~10 % of 6Li productio

Reaction rate

$$N_A \langle \sigma v \rangle = 1.4 \times 10^{7}\, T_9^{-\frac{2}{3}} \exp\left(-6.08\, T_9^{-\frac{1}{3}}\right)\left(1 + 1.3\, T_9^{\frac{2}{3}} + 0.55\, T_9\right)$$

$(\alpha X) + {}^{3}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + X$



~30 % of 6Li productior

Reaction rate

$$N_A \langle \sigma v \rangle = 9.4 \times 10^{7}\, T_9^{-\frac{2}{3}} \exp\left(-9.66\, T_9^{-\frac{1}{3}}\right) \left(1 + 0.20\, T_9^{\frac{2}{3}} + 0.05\, T_9\right)$$

reaction	S(E_0)	
CBBN: $(\alpha X) + d \rightarrow {}^6Li + X$	38 keV b	
SBBN: $\alpha + d \rightarrow {}^6Li + \gamma$ (E2)	2×10^{-6} keV b	← NACRE

enhancement CBBN / SBBN = **2×10^7**

CBBN: $(\alpha X) + t \rightarrow {}^7Li + X$	3 keV b	
SBBN: $\alpha + t \rightarrow {}^7Li + \gamma$ (E1)	0.1 keV b	← NACRE

enhancement CBBN / SBBN = **30**

CBBN: $(\alpha X) + {}^3He \rightarrow {}^7Be + X$	14 keV b	
SBBN: $\alpha + {}^3He \rightarrow {}^7Be + \gamma$ (E1)	0.5 keV b	← NACRE

enhancement CBBN / SBBN = **30**

ratio CBBN / SBBN

reaction	3-body cal.	Scaling rule	
$(\alpha X) + d \rightarrow {}^{6}\mathrm{Li} + X$	2×10^{7}	7×10^{7}	9×10^{7}
$(\alpha X) + t \rightarrow {}^{7}\mathrm{Li} + X$	30	1×10^{5}	3×10^{4}
$(\alpha X) + {}^{3}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + X$	30	3×10^{5}	8×10^{4}

Estimated by
Cyburt et al.
Astro-h/068562

Estimated by Kamimu
using Eq.(4) of
Pospelov PRL 89
(2007) 111

I have no time to discuss about the scaling rule itself.

Section 2.

X^--catalyzed radiative capture reactions

—— Reactions to destruct ^{7}Be ——

non-resonant: $(^7\mathrm{Be}\,X) + p \rightarrow (^8\mathrm{B}\,X) + \gamma$

resonant: $(^7\mathrm{Be}\,X) + p \rightarrow (^8\mathrm{B}\,X)_{2p} \rightarrow (^8\mathrm{B}\,X)_{1s} + \gamma$

Bird et al. (hep-ph/0703096)

very similar

Prediction for KEK experimen

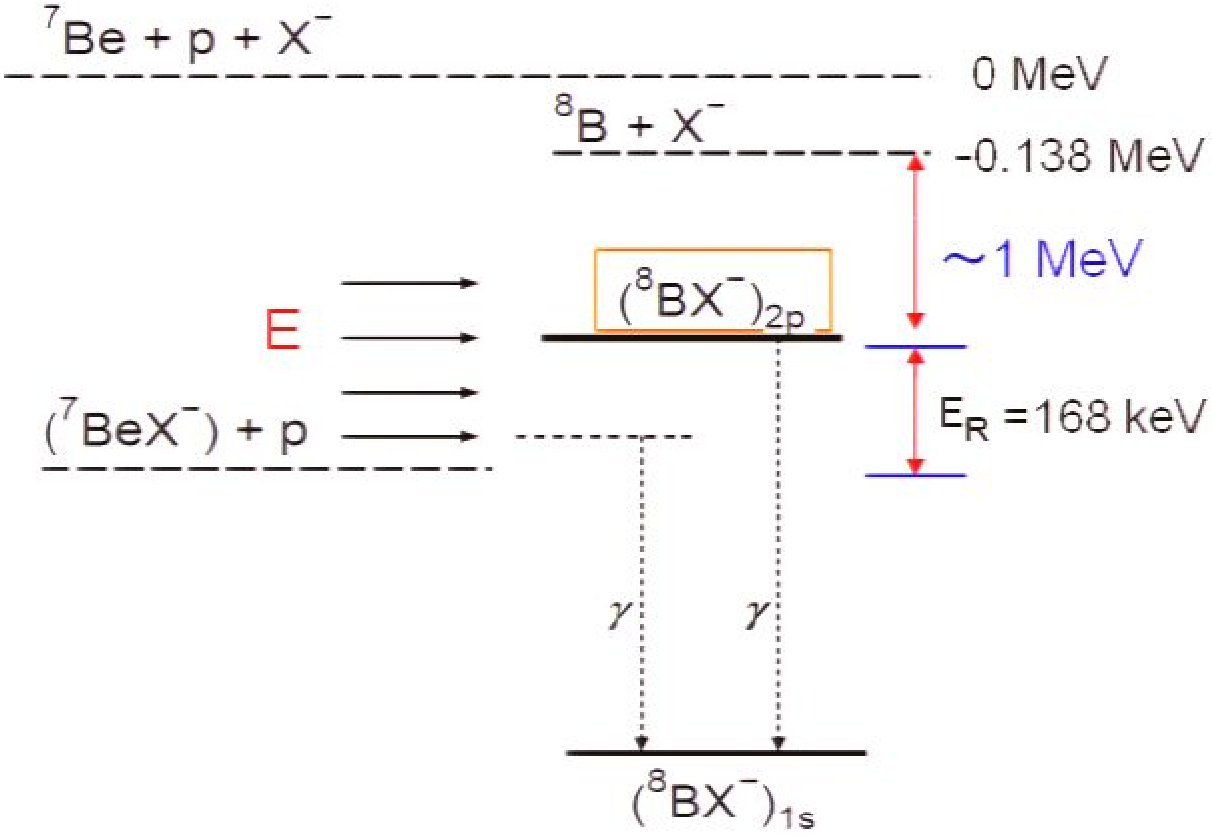
$$(p\mu)_{1s} + \ ^4\mathrm{He} \longrightarrow (p\,^4\mathrm{He}\mu)_{\mathrm{molecule}} \longrightarrow \ (^4\mathrm{He}\mu)_{1s} + p + \gamma$$

$$(d\mu)_{1s} + \ ^3\mathrm{He} \longrightarrow (d\,^3\mathrm{He}\mu)_{\mathrm{molecule}} \longrightarrow \ (^3\mathrm{He}\mu)_{1s} + d + \gamma$$

non-resonant: $(^7\text{Be}\ X) + p \rightarrow (^8\text{B}\ X) + \gamma$

resonant: $(^7\text{Be}\ X) + p \rightarrow (^8\text{B}\ X)_{2p} \rightarrow (^8\text{B}\ X)_{1s} + \gamma$ ← **very intersting mechanism**

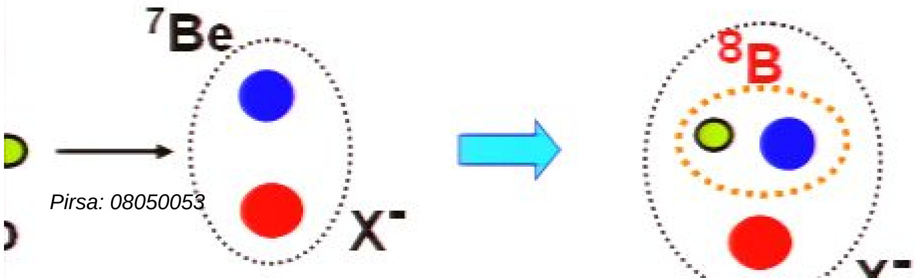
Bird et al. (hep-ph/070309



Reaction rate depends strongly on E_R as $\exp(-E_R/kT)$

At T_9=0.3 (kT=25 keV), 50 keV-change of E_R changes the rate by factor of e^2=7.4

50 keV is only 5% of B.E. (~1 MeV) of $(^8\text{BX})_{2p}$



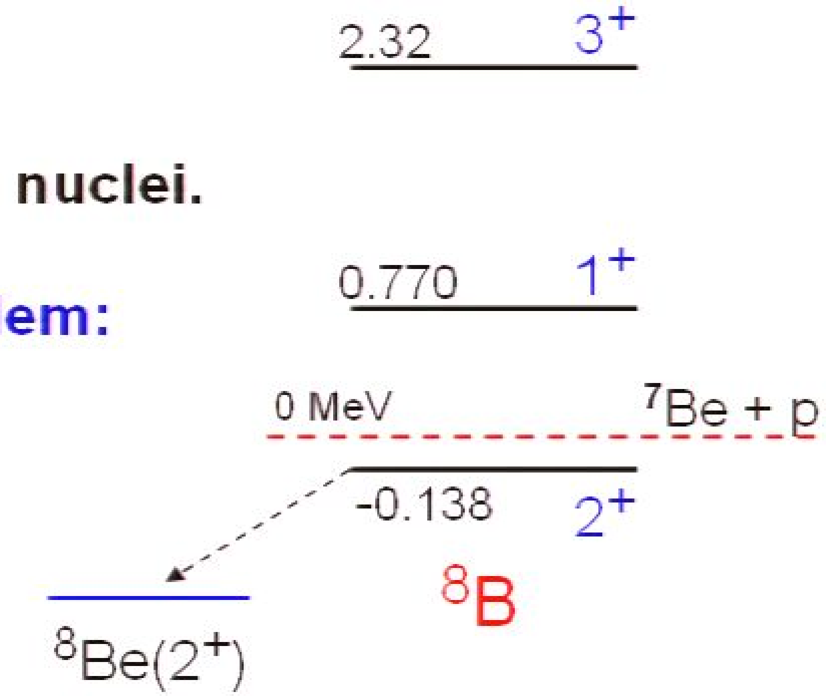
One should calculate E_R carefully

^{8}B nucleus

^{8}B is one of the most extensively studied nuclei.

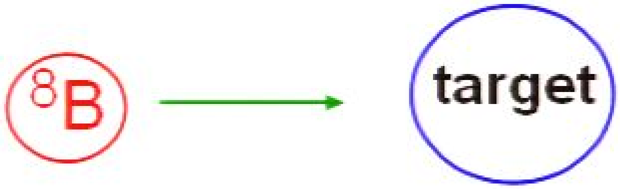
i) A key nucleus in the solar neutrino problem:

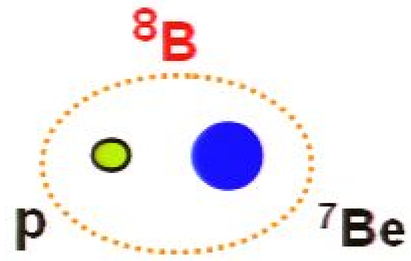
$$\begin{tikzcd} {}^7\mathrm{Be} + p \arrow[r] & {}^8\mathrm{B} + \gamma \arrow[d] & \\ & {}^8\mathrm{Be}(2+) + e^+ + \nu \arrow[dr] & \\ & & \text{Kamiokande} \end{tikzcd}$$



ii) A typical unstable nucleus

used for radioactive ion beam





Structure of ^{8}B nucleus

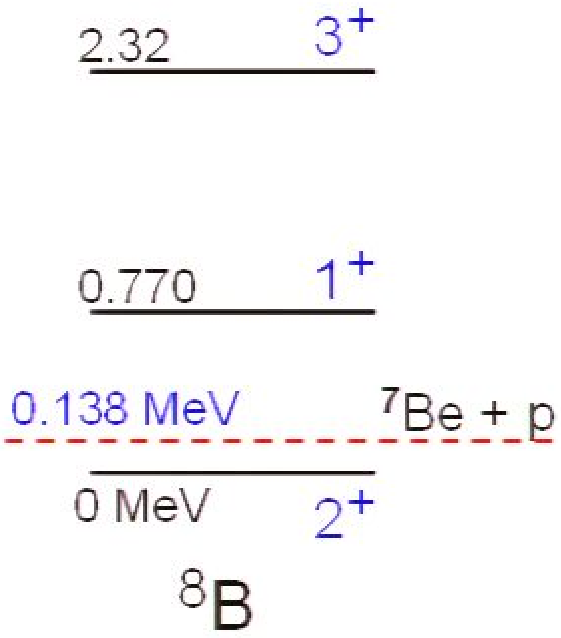
1) ^{7}Be + p structure

Very weakly bound state (-0.138 MeV)

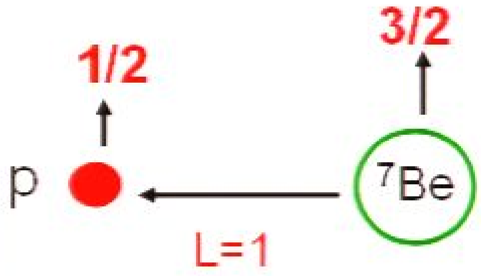
2) Proton halo (long-range tail)

3) p-wave halo

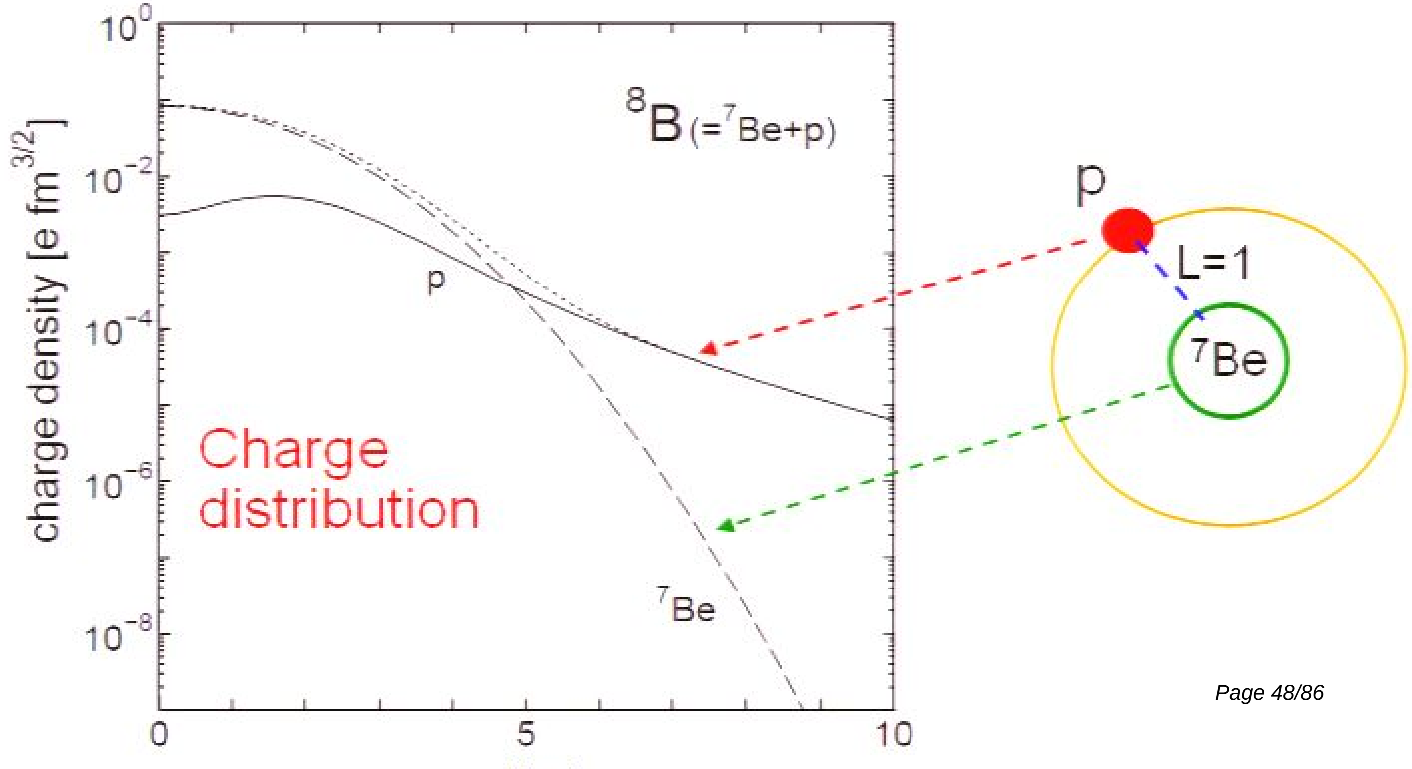
Large Q-moment



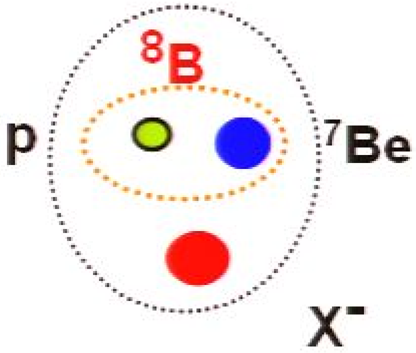
4) Spin = 2^+



Strongly spin-dependent ^{7}Be – p potential



Models for ^{8}B + X$^-$



One can consider the following 4-types of models of the ^{8}B +X system

Model (i) The simplest case:

Assumes Gaussian-shape charge distribution of ^{8}B

Bird et al. (hep-ph/0703096).

---

^{7}Be + p model

Model (ii) Assumes the proton-halo-type charge distribution of ^{8}B but with the monopole part only.

Model (iii) Solves 3-body problem without spins (the quadrupole Coulomb potential is automatically included).

Model (iv) Solves 3-body problem with spins

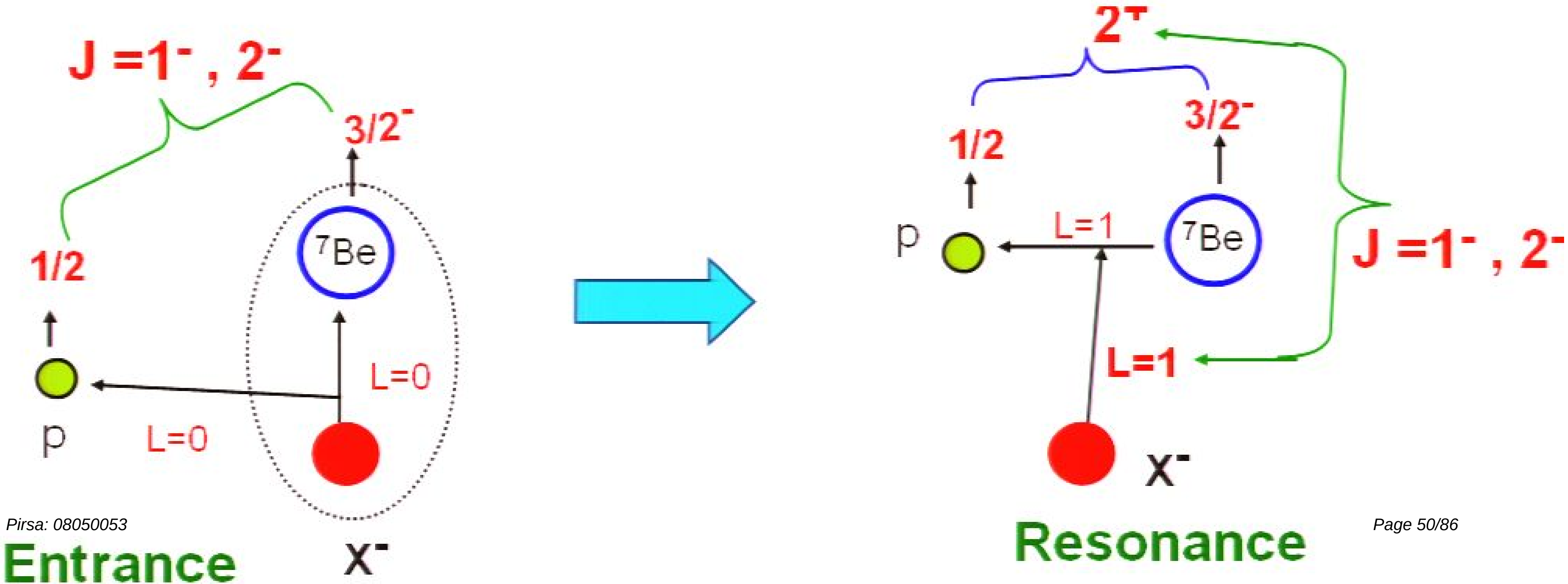
Nuclear potential between **^{7}Be and p**

The most popularly used potential proposed by H. **Esbenson PRC 70 (2004) 047603**

to explain the energy, charge radius and ^{7}Be(p, γ)^{8}B reaction.

Model (iv)

$$\Psi_{JM} = \phi^{(1)}_{00}(\mathbf{r}_1)\,\chi^{(1)}_{00}(\mathbf{R}_1)\left[\xi_{\frac{3}{2}}(^7\mathrm{Be}) \otimes \xi_{\frac{1}{2}}(p)\right]_{JM} + \Psi^{(\mathrm{closed})}_{JM}\ .$$



Model (i) Gaussian-shape charge

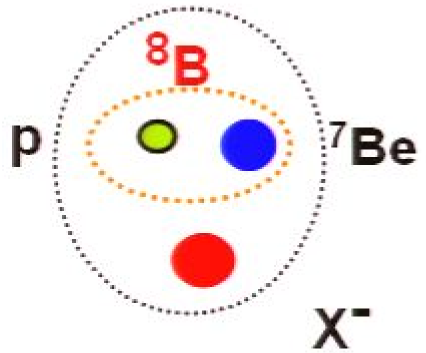
Model (ii) proton-halo-type charge
(monopole only)

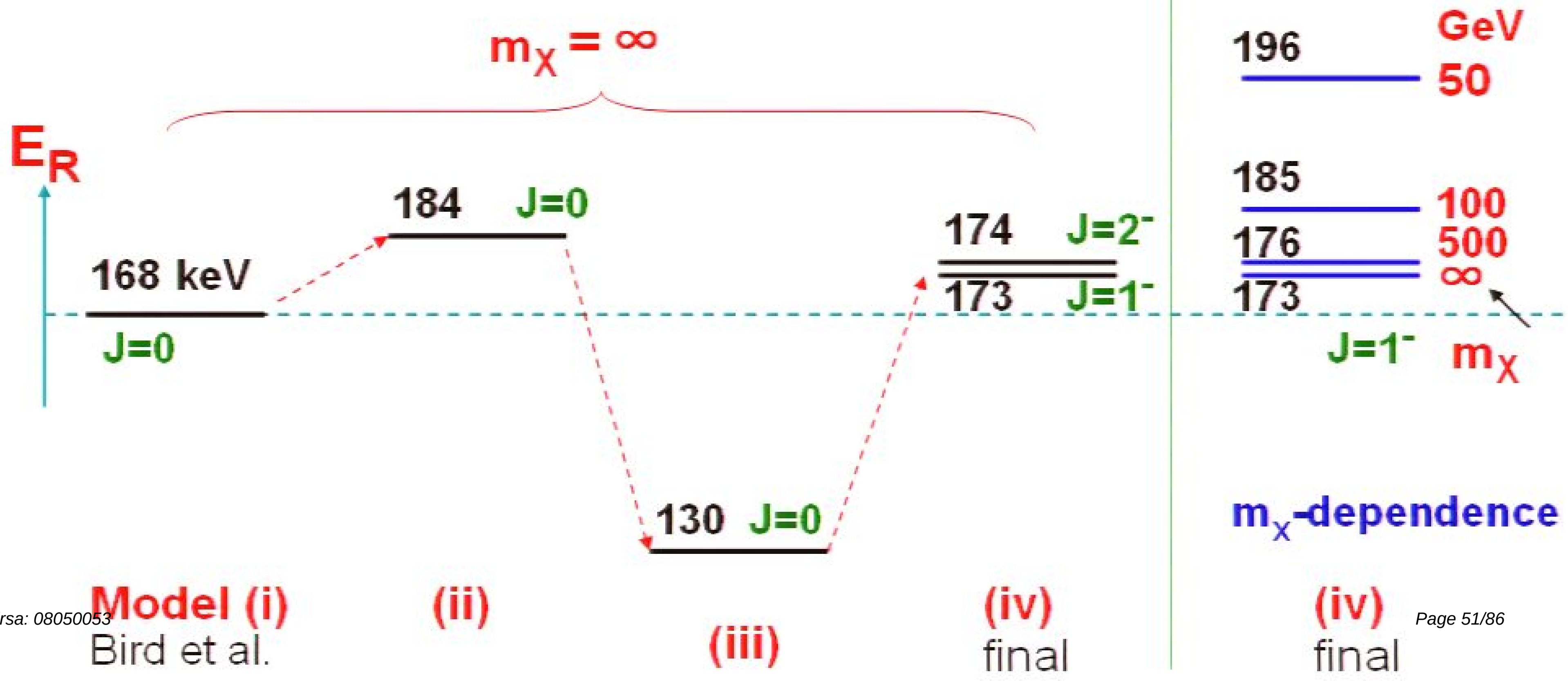
Model (iii) 3-body cal. without spins

Model (iv) 3-body cal. with spins.

Energy of resonance

E_R





Model (i) Gaussian-shape charge

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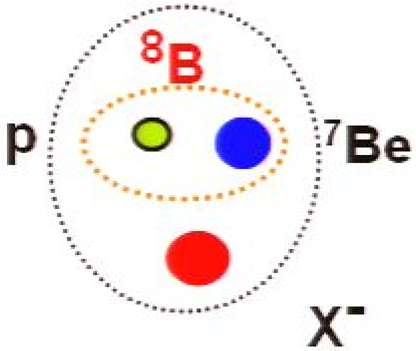
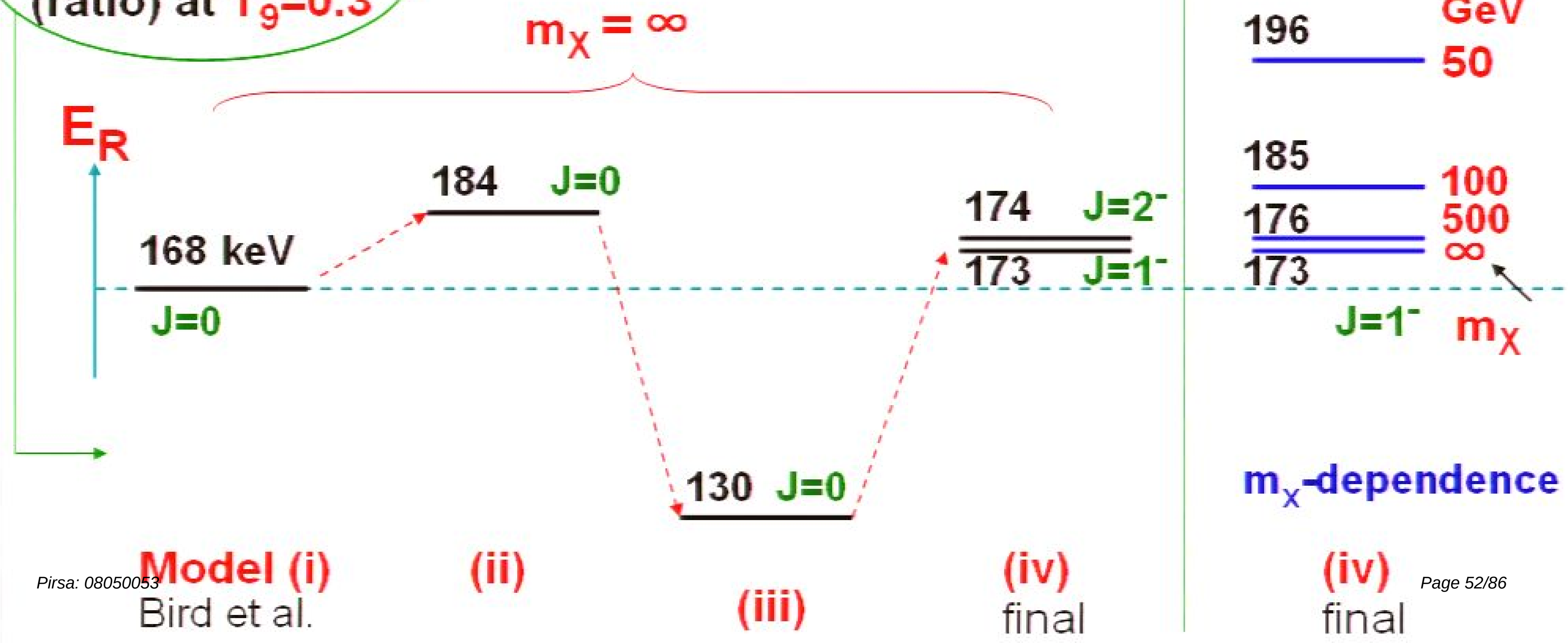
Model (iv) 3-body cal. with spins.

Energy of resonance E_R



Reaction rate (ratio) at $T_9=0.3$



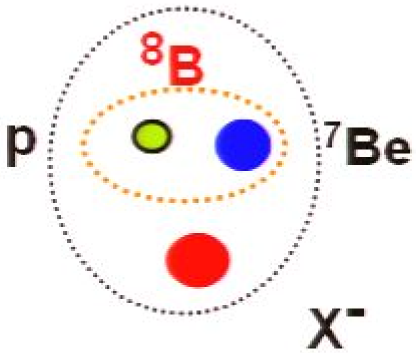
Model (i) Gaussian-shape charge

Model (ii) proton-halo-type charge (monopole only)

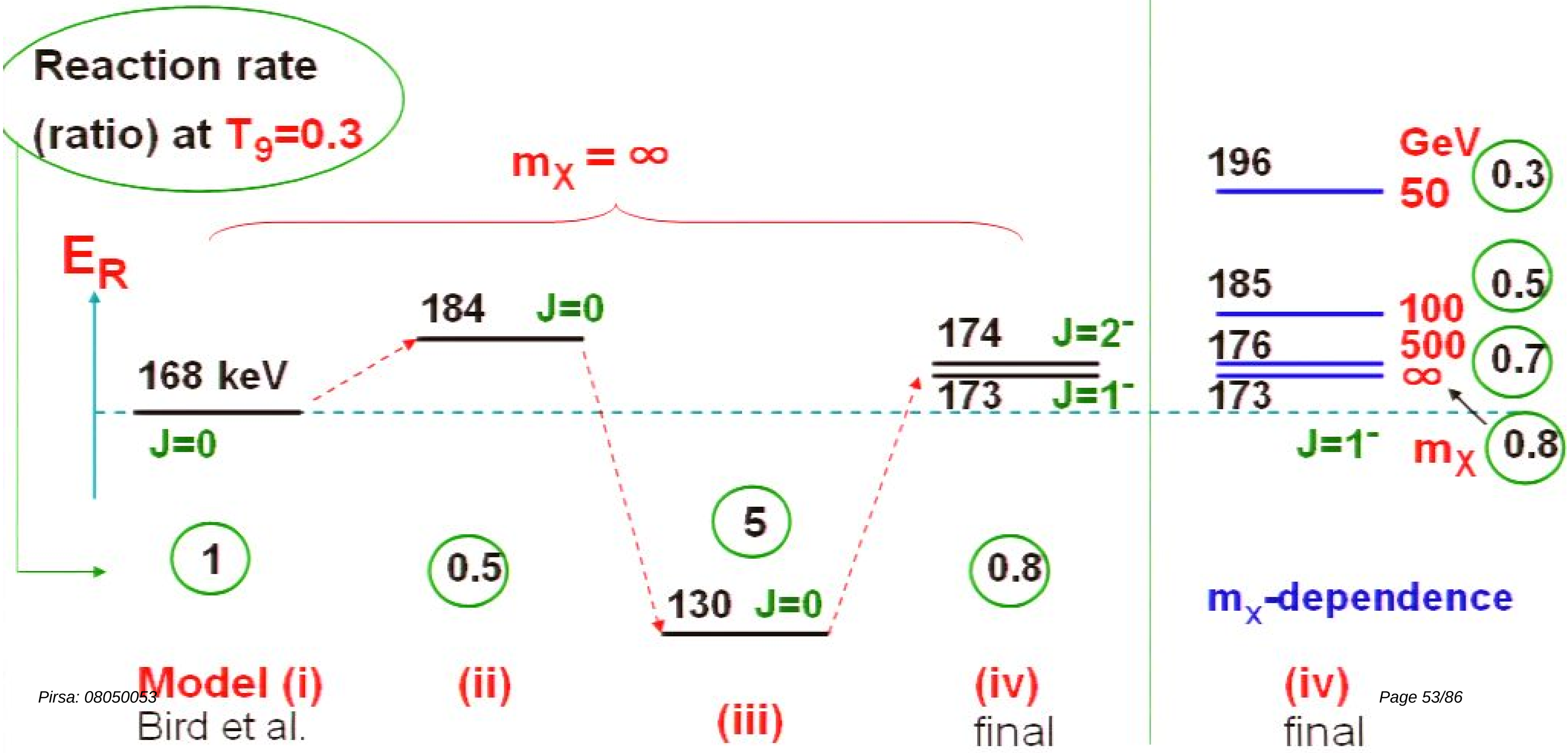
Model (iii) 3-body cal. without spins

Model (iv) 3-body cal. with spins.

Energy of resonance E_R

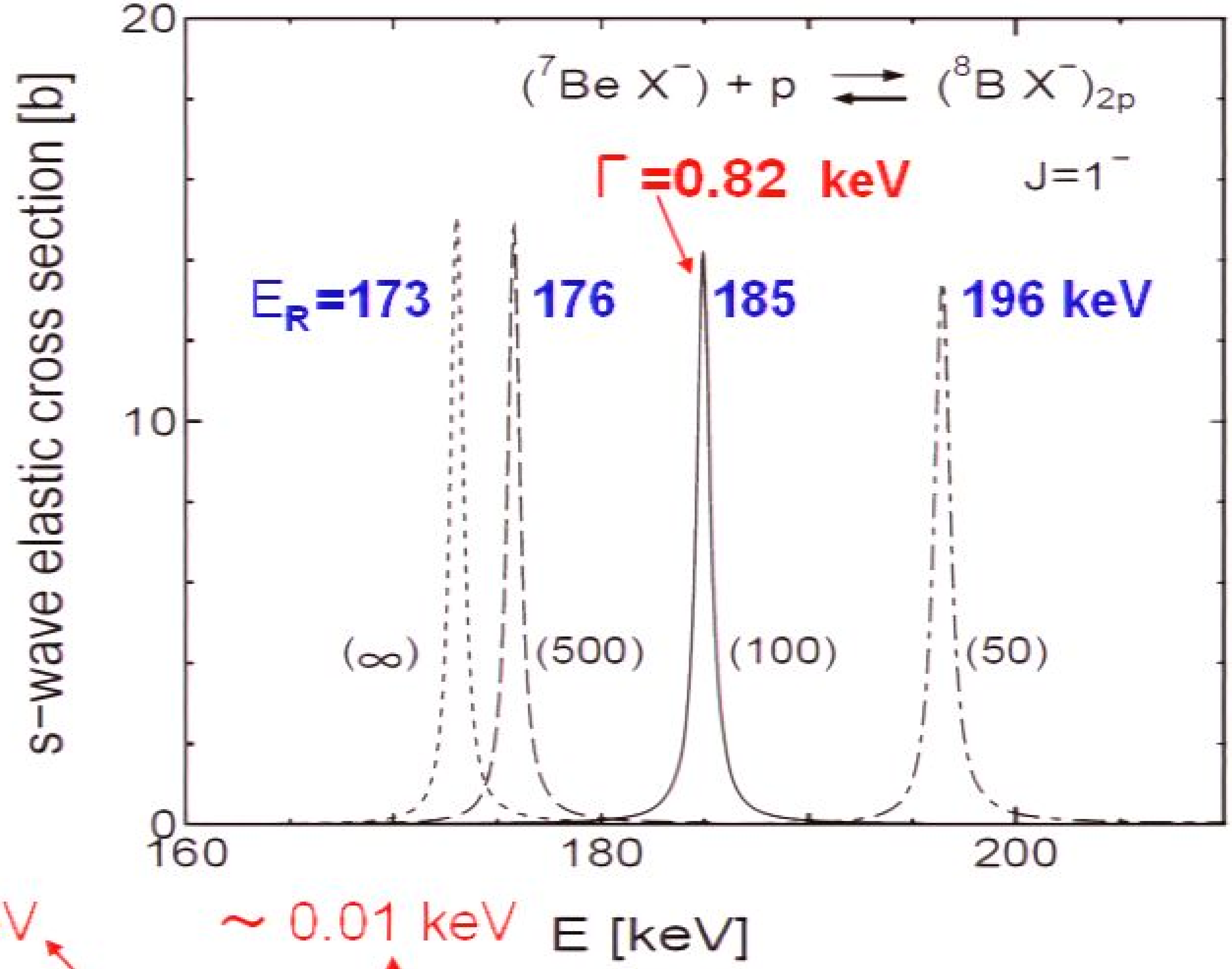


Reaction rate (ratio) at T_9=0.3



3-body scattering calculation of the resonance : Model (iv)

(energy E_R and width Γ depending on m_X)



J=1⁻

(almost the sam

as for J=2⁻)

~ 1 keV

~ 0.01 keV

Γ_p >> Γ_γ is confirmed (Bird et al. assumed *a priori*).

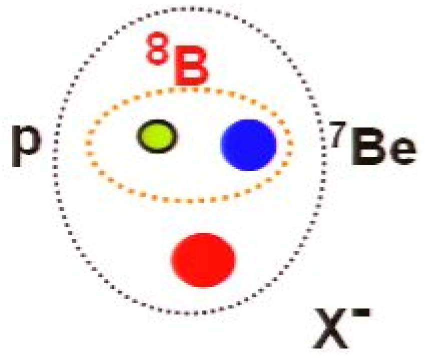
Model (i) Gaussian-shape charge

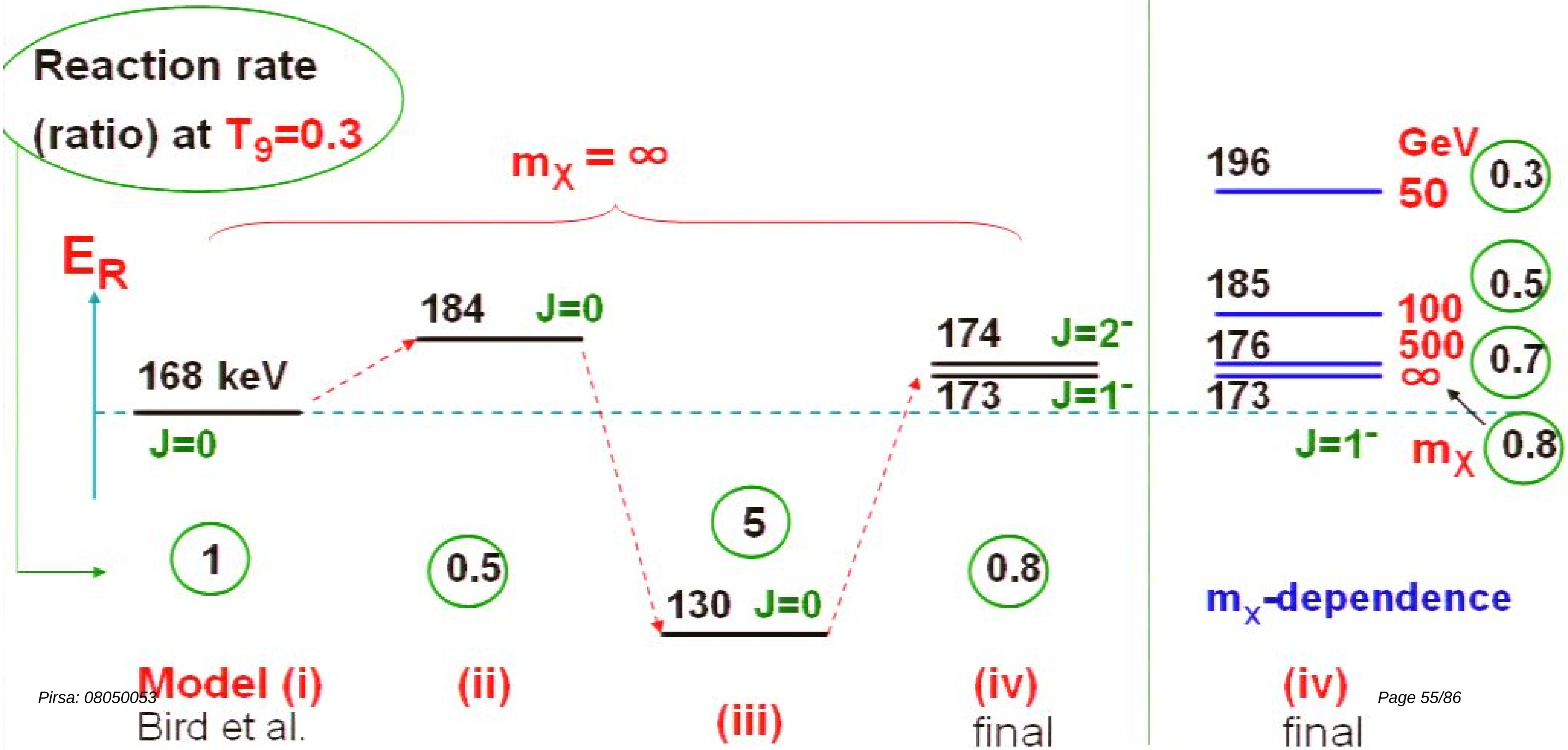
Model (ii) proton-halo-type charge
(monopole only)

Model (iii) 3-body cal. without spins

Model (iv) 3-body cal. with spins.

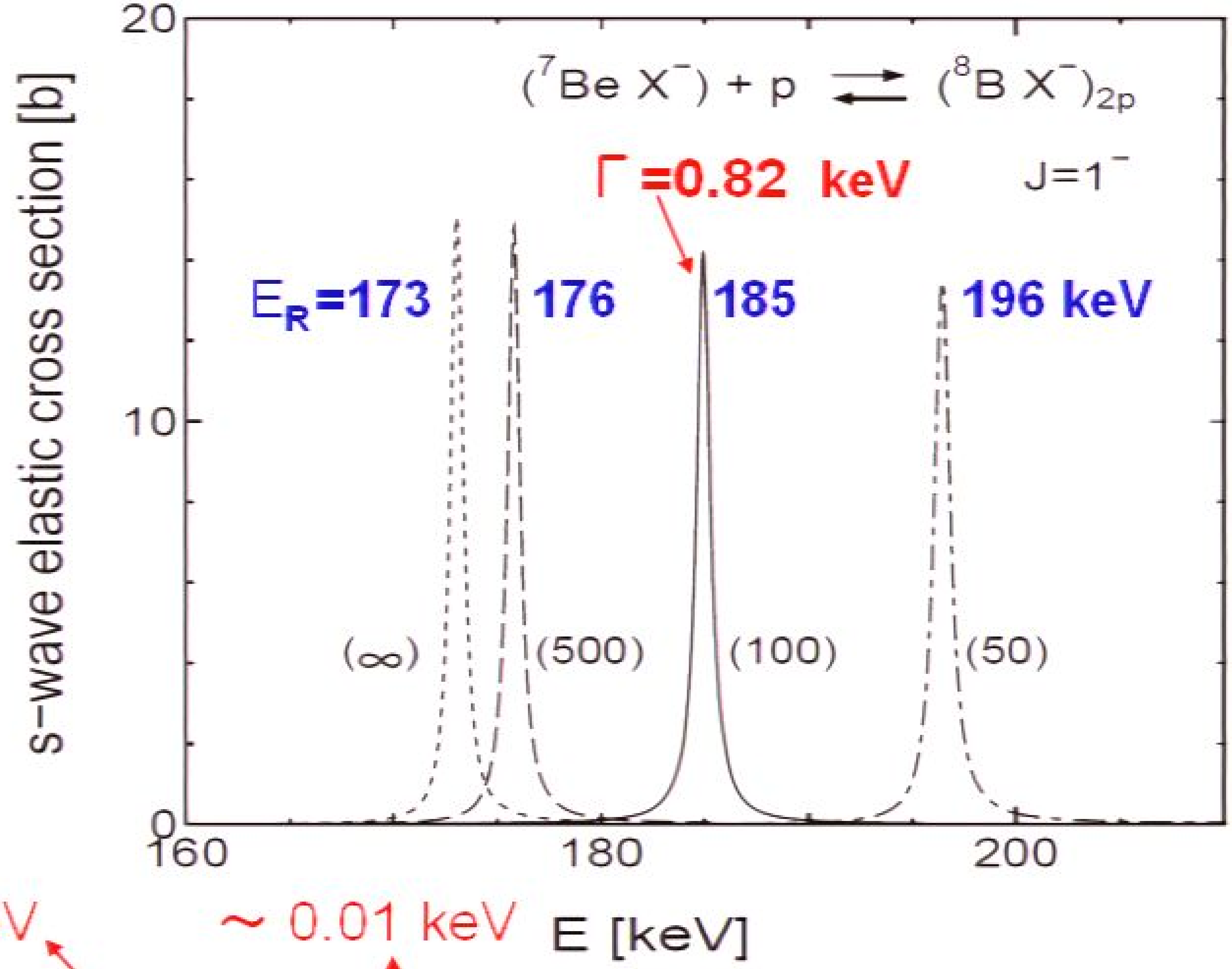
Energy of resonance
E_R




Reaction rate (ratio) at $T_9=0.3$

3-body scattering calculation of the resonance : Model (iv)

(energy E_R and width Γ depending on $\mathbf{m_X}$)



J=1$^-$

(almost the sam

as for J=2$^-$)

~ 1 keV

~ 0.01 keV

Γ_p >> Γ_γ is confirmed (Bird et al. assumed *a priori*).

Summary of resonances (to appear in arXiv in June, 2008)

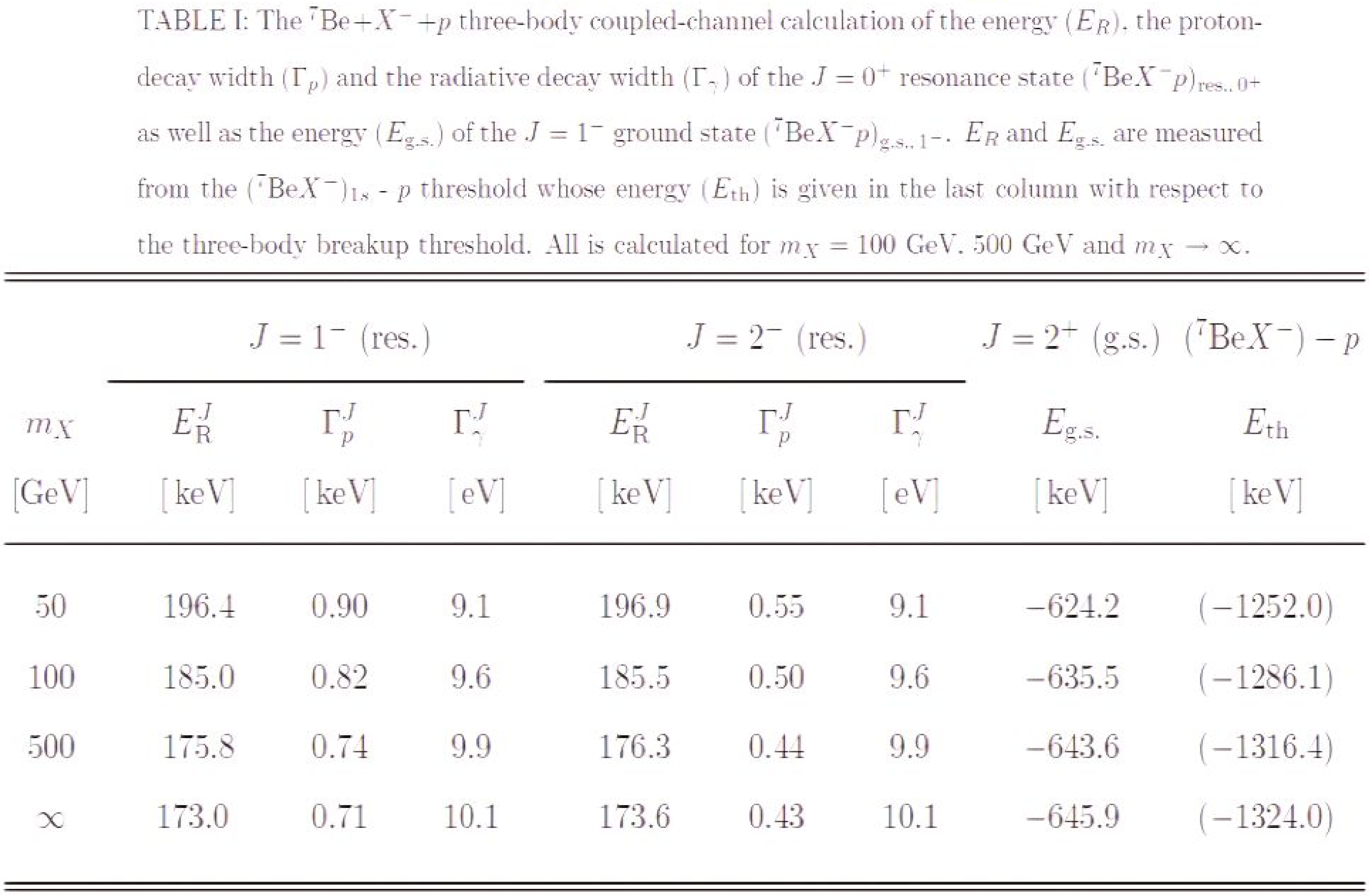


TABLE I: The ^{7}Be+X^-+p three-body coupled-channel calculation of the energy (E_R), the proton-decay width (Γ_p) and the radiative decay width (Γ_γ) of the $J=0^+$ resonance state $(^7\mathrm{Be}X^-p)_{\mathrm{res.},0^+}$ as well as the energy ($E_{\mathrm{g.s.}}$) of the $J=1^-$ ground state $(^7\mathrm{Be}X^-p)_{\mathrm{g.s.},1^-}$. E_R and $E_{\mathrm{g.s.}}$ are measured from the $(^7\mathrm{Be}X^-)_{1s}$ - p threshold whose energy (E_{th}) is given in the last column with respect to the three-body breakup threshold. All is calculated for $m_X = 100$ GeV, 500 GeV and $m_X \to \infty$.

	$J=1^-$ (res.)			$J=2^-$ (res.)			$J=2^+$ (g.s.)	$(^7\mathrm{Be}X^-)-p$
m_X [GeV]	E^J_{R} [keV]	Γ^J_p [keV]	Γ^J_γ [eV]	E^J_{R} [keV]	Γ^J_p [keV]	Γ^J_γ [eV]	$E_{\mathrm{g.s.}}$ [keV]	E_{th} [keV]
50	196.4	0.90	9.1	196.9	0.55	9.1	−624.2	(−1252.0)
100	185.0	0.82	9.6	185.5	0.50	9.6	−635.5	(−1286.1)
500	175.8	0.74	9.9	176.3	0.44	9.9	−643.6	(−1316.4)
∞	173.0	0.71	10.1	173.6	0.43	10.1	−645.9	(−1324.0)

Summary of reaction rate (to appear in arXiv in June, 200

resonant: $(^7\mathrm{Be}\ X) + p \rightarrow (^8\mathrm{B}\ X)_{2p} \rightarrow (^8\mathrm{B}\ X)_{1s} + \gamma$

$$N_A \langle \sigma v \rangle = N_A \hbar^2 \left(\frac{2\pi\hbar^2}{\mu_1 kT} \right)^{\frac{3}{2}} \sum_{J=1}^{2} \frac{2J+1}{(2I_1+1)(2I_2+1)} \frac{\Gamma_p^J \Gamma_\gamma^J}{\Gamma_p^J + \Gamma_\gamma^J} \exp\left(-\frac{E_R^J}{kT} \right)$$

ratio at T_9=0.

$$N_A \langle \sigma v \rangle = 1.37 \times 10^6\, T_9^{-3/2} \exp\left(-2.28/T_9\right), \quad (m_X = 50\ \mathrm{GeV})$$

0.3

$$N_A \langle \sigma v \rangle = 1.44 \times 10^6\, T_9^{-3/2} \exp\left(-2.15/T_9\right), \quad (m_X = 100\ \mathrm{GeV})$$

0.5

$$N_A \langle \sigma v \rangle = 1.48 \times 10^6\, T_9^{-3/2} \exp\left(-2.04/T_9\right), \quad (m_X = 500\ \mathrm{GeV})$$

0.7

$$N_A \langle \sigma v \rangle = 1.51 \times 10^6\, T_9^{-3/2} \exp\left(-2.01/T_9\right), \quad (m_X \rightarrow \infty)$$

0.8

Bird et al.

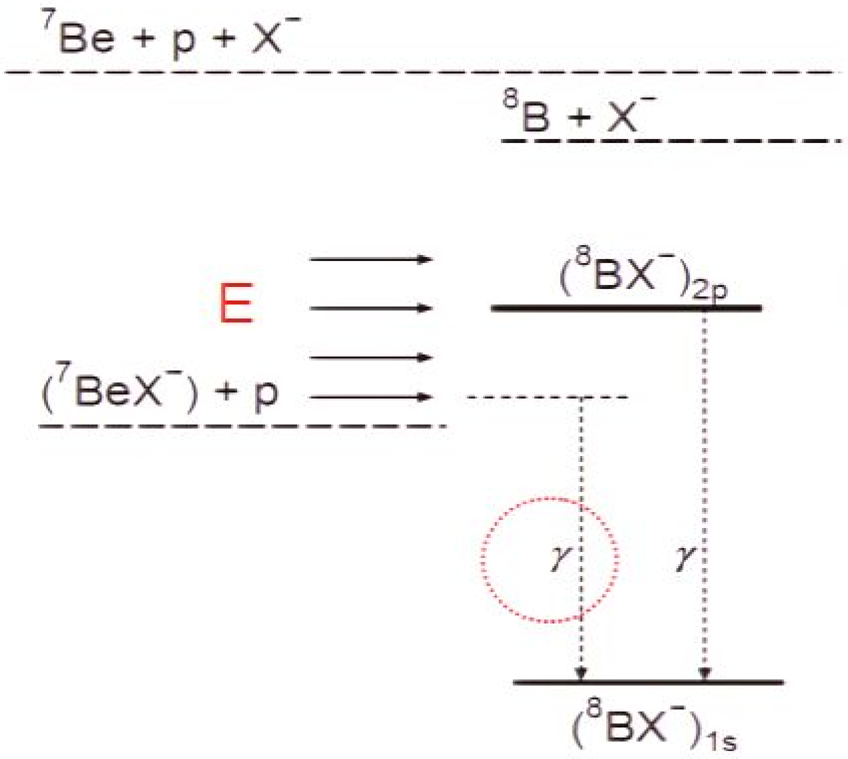
$$N_A \langle \sigma v \rangle = 1.6 \times 10^6\, T_9^{-3/2} \exp(-1.94/T_9) \quad (m_X \rightarrow \infty)$$

1

(in units of $\mathrm{cm}^3\ \mathrm{s}^{-1}\ \mathrm{mol}^{-1}$)

Non-resonant radiative capture

$(^7\mathrm{Be}\,X) + p \rightarrow (^8\mathrm{B}\,X) + \gamma$

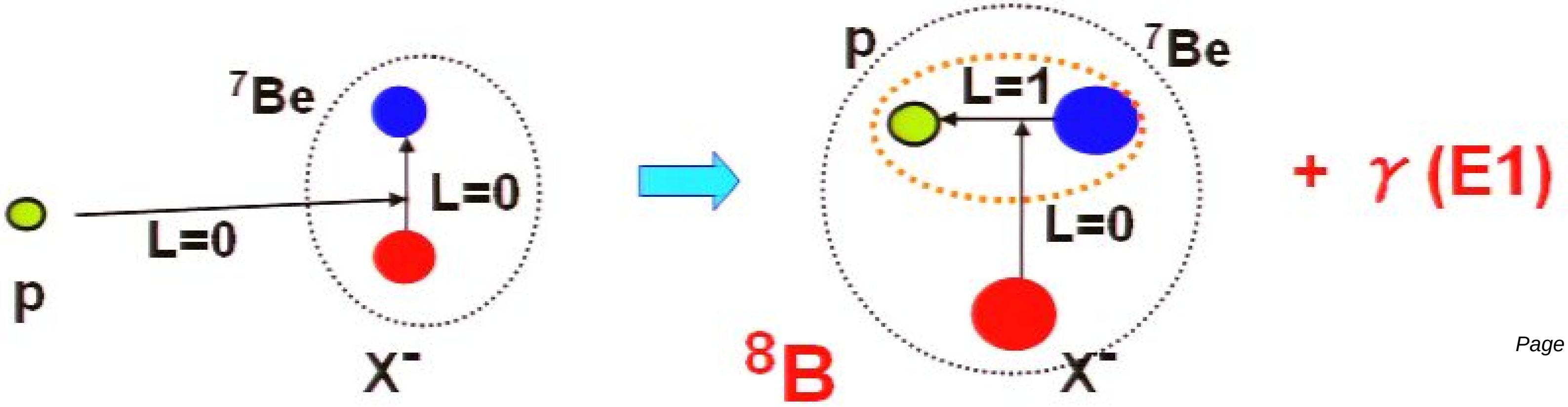


Spins are ignored
(no problem for non-resonant reaction

$$\sigma^{(\mathrm{E1})}_{\mathrm{cap.}}(E) = \frac{16\pi}{9}\frac{k_\gamma^3}{\hbar v}\sum_M |\langle \Phi^{(\mathrm{g.s.})}_{1M} | Q^{(E1)}_{1M} | \Psi_{00}(E)\rangle |$$

$$\sigma^{(\mathrm{E1})}_{cap.}(E) \;= \underline{S(E)}\exp(-2\pi\eta(E))/E\ .$$

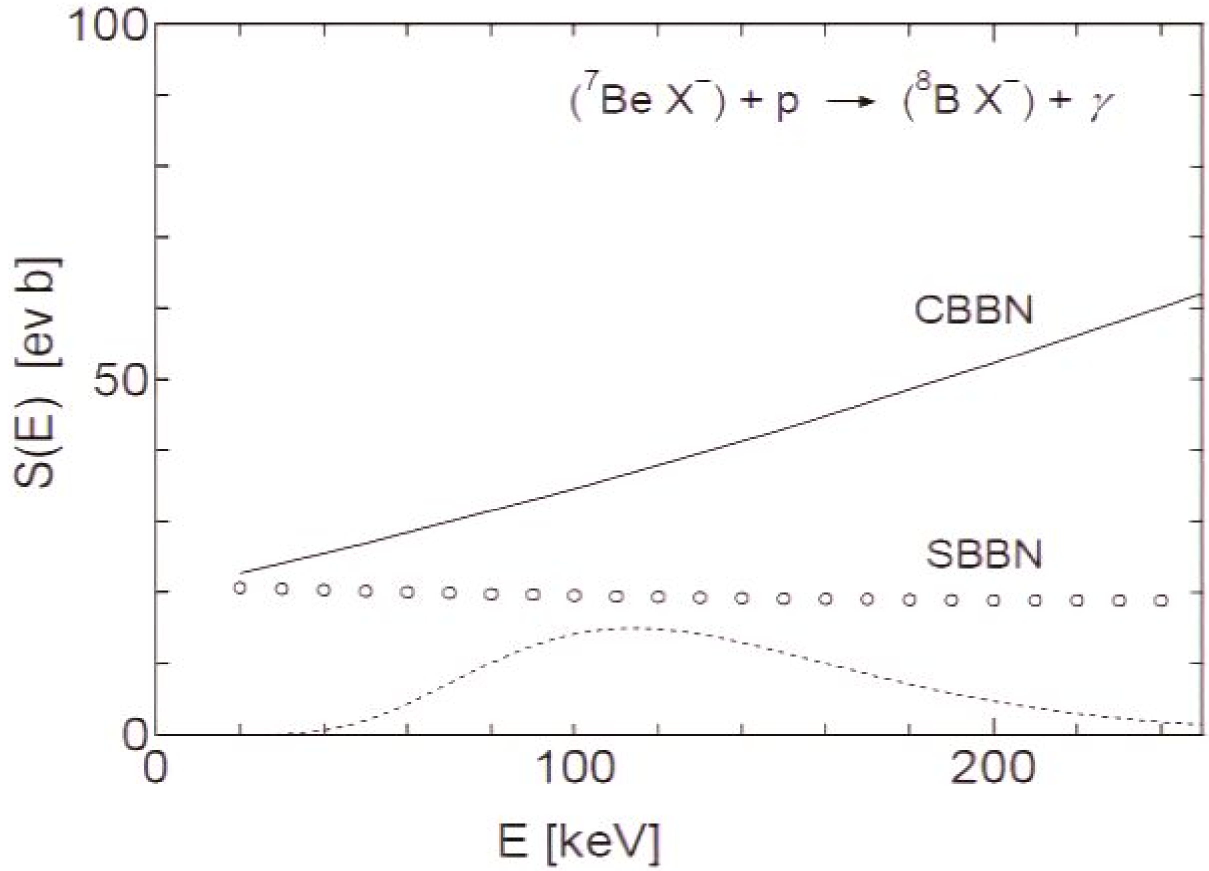
next figure



Non-resonant radiative capture

(to appear in arXiv in June, 200

S-factor S(E) $(^7\mathrm{Be}\ X) + p \rightarrow (^8\mathrm{B}\ X) + \gamma$



Scaling rule

Bird et al. (hep-ph/0703096) $S_{\rm CBBN}(0) \sim 700 \times S_{\rm SBBN}(0) \sim 15$ keV b

3-body calculation $S_{\rm CBBN}(0) \sim S_{\rm SBBN}(0) \sim 0.02$ keV b

Bird et al. (hep-ph/0703096) $S_{\mathrm{CBBN}}(0) \sim 700 \times S_{\mathrm{SBBN}}(0) \sim 15\ \mathrm{keV\ b}$

3-body calculation $S_{\mathrm{CBBN}}(0) \sim S_{\mathrm{SBBN}}(0) \sim 0.02\ \mathrm{keV\ b}$

Main origin of this overestimation by the scaling rule

$$|\langle \Phi_{1M}^{(\mathrm{g.s.})} | Q_{1M}^{(E1)} | \Psi_{00}(E) \rangle|^2$$

Assumption: 3-body matrix element ~ 2-body matrix element

↓ ↓

$({}^7\mathrm{Be}\,X) + p \rightarrow ({}^8\mathrm{B}\,X) + \gamma$ **${}^7\mathrm{Be} + p \rightarrow {}^8\mathrm{B} + \gamma$**

is not appropriate due to the presence of **X^-**.

Reaction rate (non-resonant) **$({}^7\mathrm{Be}\,X) + p \rightarrow ({}^8\mathrm{B}\,X) + \gamma$**

$$N_A \langle \sigma v \rangle = 2.3 \times 10^5\, T_9^{-\frac{2}{3}} \exp(-8.83\, T_9^{-\frac{1}{3}})\,(1 + 1.9\, T_9^{\frac{2}{3}} + 0.54\, T_9) \quad \mathrm{cm^3\, s^{-1}\, mol^{-1}}.$$

for $T_9 \lesssim 0.5$.

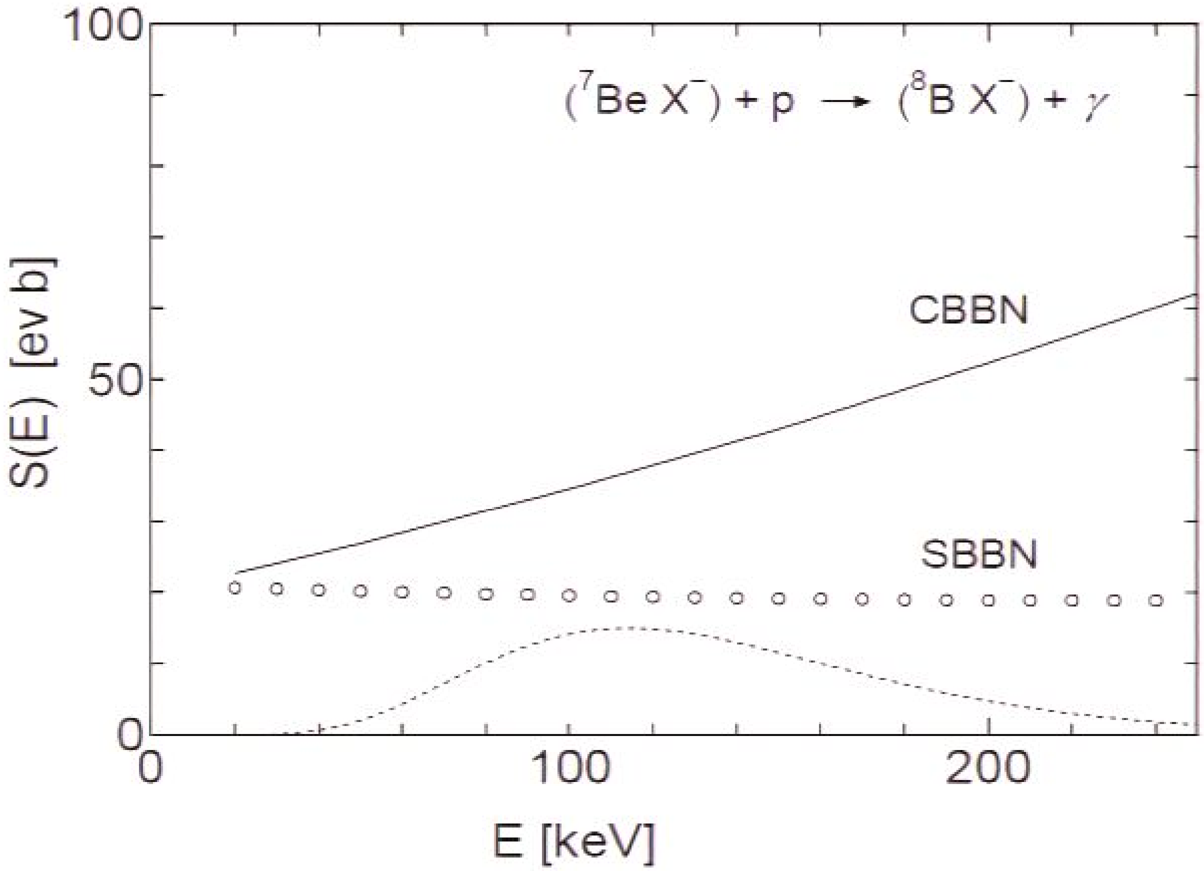
↑

negligible in the CBBN network calculations

Non-resonant radiative capture

(to appear in arXiv in June, 200

S-factor S(E) $(^{7}\mathrm{Be}\ \mathrm{X}) + p \rightarrow (^{8}\mathrm{B}\ \mathrm{X}) + \gamma$



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$$|\langle \Phi^{(\rm g.s.)}_{1M} \,|\, Q^{(E1)}_{1M} \,|\, \Psi_{00}(E) \rangle|^2$$

Assumption: **3-body matrix element** **~ 2-body matrix element**

↓ ↓

$(^7\text{Be}\;X) + p \rightarrow (^8\text{B}\;X) + \gamma$ **$^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$**

is not appropriate due to the presence of **X^-**.

Reaction rate (non-resonant) **$(^7\text{Be}\;X) + p \rightarrow (^8\text{B}\;X) + \gamma$**

$$N_A \langle \sigma v \rangle = 2.3 \times 10^5\, T_9^{-\frac{2}{3}} \exp(-8.83\, T_9^{-\frac{1}{3}})\,(1 + 1.9\, T_9^{\frac{2}{3}} + 0.54\, T_9) \quad \text{cm}^3\,\text{s}^{-1}\,\text{mol}^{-1}.$$

for $T_9 \lesssim 0.5$.

↑

negligible in the CBBN network calculations

Section 4.

X^--catalyzed 3-body breakup reactions

—— Reactions to destruct ^{6}Li and ^{7}Li ——

$$(^6\mathrm{Li}\,X) + p \rightarrow \alpha + {}^3\mathrm{He} + X$$

$$(^7\mathrm{Li}\,X) + p \rightarrow \alpha + \alpha + X$$

In **SBBN**,

$$ {}^{6}\mathrm{Li} + p \rightarrow \alpha + {}^{3}\mathrm{He} $$

$$ {}^{7}\mathrm{Li} + p \rightarrow \alpha + \alpha $$

are the main mechanism to destruct **^{6}Li** and **^{7}Li.**

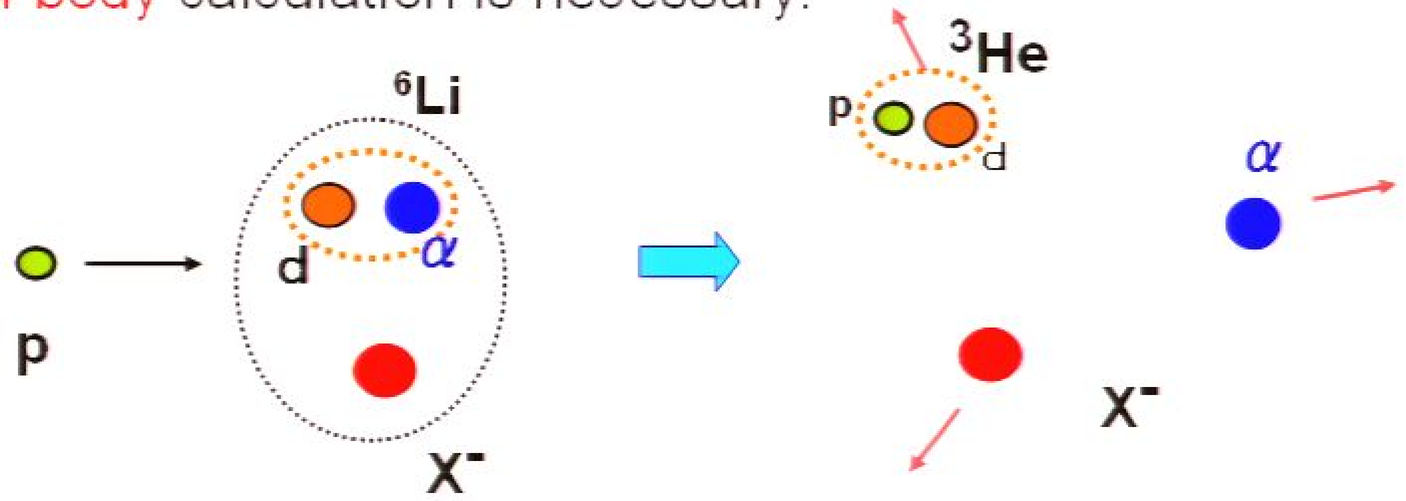
Corresponding **CBBN** reactions :

$$ ({}^{6}\mathrm{Li}\,X) + p \rightarrow \alpha + {}^{3}\mathrm{He} + X $$

$$ ({}^{7}\mathrm{Li}\,X) + p \rightarrow \alpha + \alpha + X $$

How to estimate the cross sections of these complicated reactions?

At least, 4-body calculation is necessary.

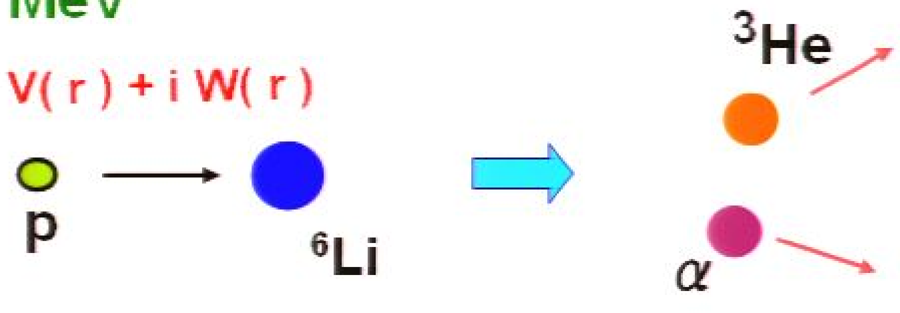


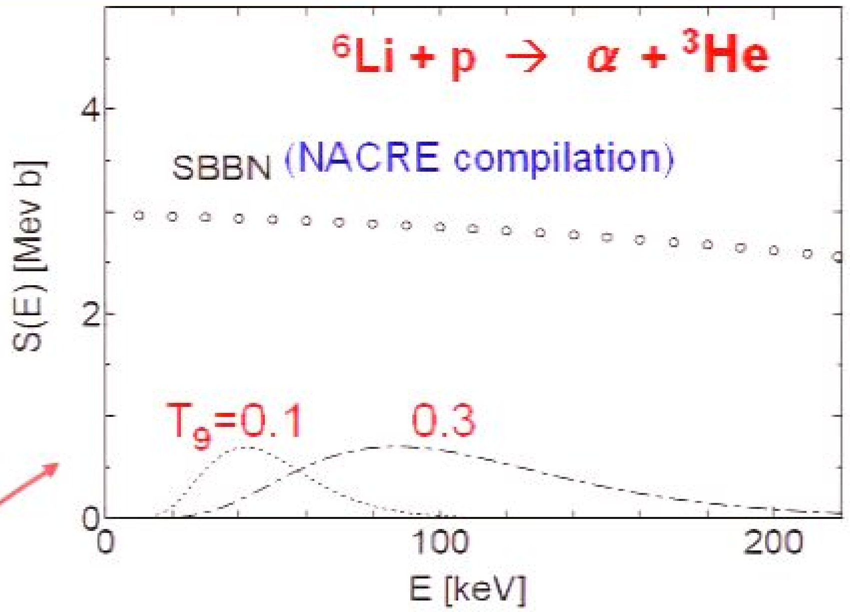
The 4-body calculation itself is not difficult for our group,

but **determination** of appropriate (strongly spin-dependent), low-energy nuclear interactions among α , d and p are very tedious and difficult

An alternative method = **Absorptive-potential model**

SBBN $^{6}Li + p \rightarrow \alpha + {}^{3}He + 4.0$ **MeV**





How to describe this S-factor (EXP)?

idea : 1-st step

1) There are no other open channels at BBN energies.

$V(r) + i\,W(r)$
↑

2) We introduce an absorptive (imaginary) potential between **^{6}Li** and **p**

3) and solve the **^{6}Li + p** Schroedinger equation (1-channel calculation).

4) Elastic scattering S-matrix **$|S_{11}| < 1$** due to the imaginary potential.

5) Probability loss $(1 - |S_{11}|^2)$ just stands for the transition to the exit channel

6) The absorption cross section is nothing but
the reaction cross section for **^{6}Li + p → α + ^{3}He .**

$$\sigma_{\rm reac} = \frac{\pi}{k_1^2}\sum_{J=0}^{\infty}(2J+1)(1-|S^J_{1-1}|^2).$$

$$\sigma_{\rm reac} = \underline{S(E)}\,\exp(2\pi\eta(E))/E$$

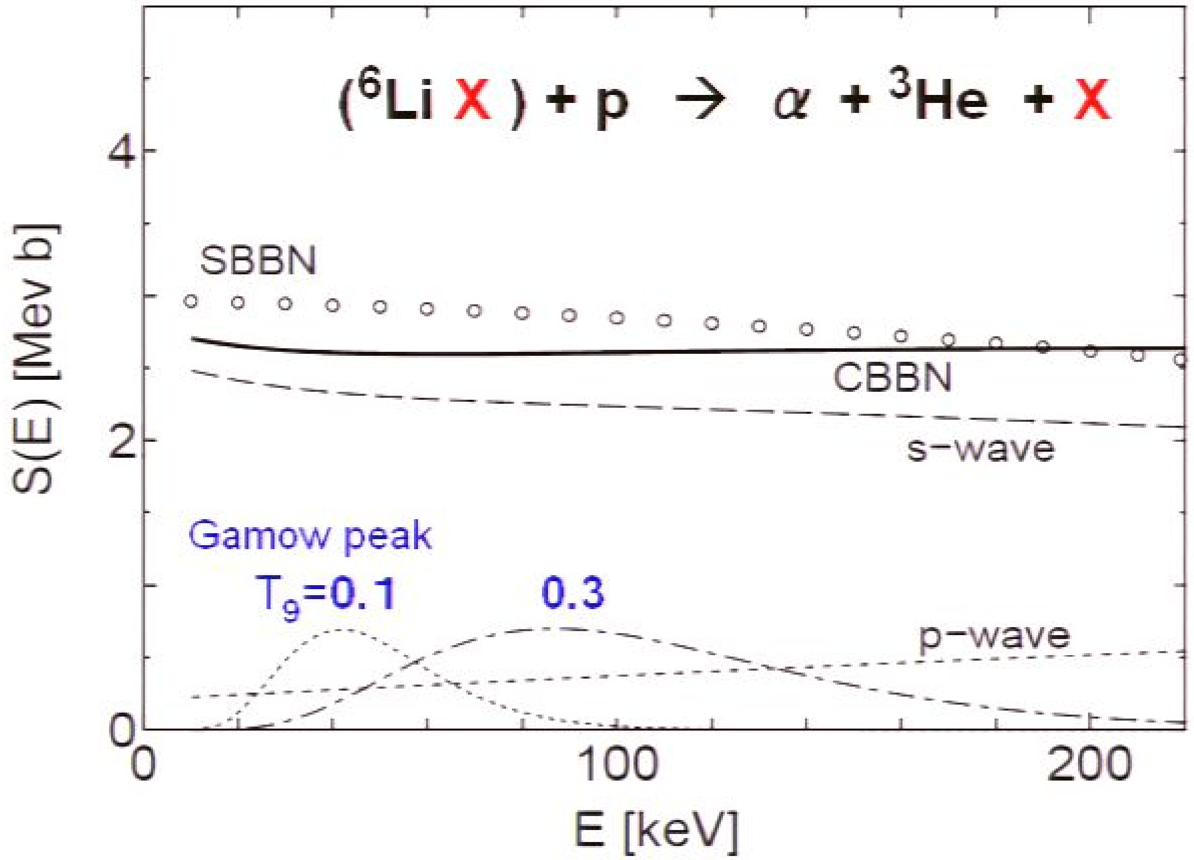
compared to the EXP.

7) We search an optimum potential.

8) Thus, we can explain the reaction cross section (S-factor)
without treating the exit channel explicitly.

Of course , this is not the goal.

3-body calculation



(to appear in arXiv in June, 200

CBBN: $\sigma_{CBBN}(E) = S_{CBBN}(E)\ \ G_{CBBN}(E)\ /E$

$\approx \qquad \gg$

SBBN: $\sigma_{SBBN}(E) = S_{SBBN}(E)\ \ G_{SBBN}(E)\ /E$

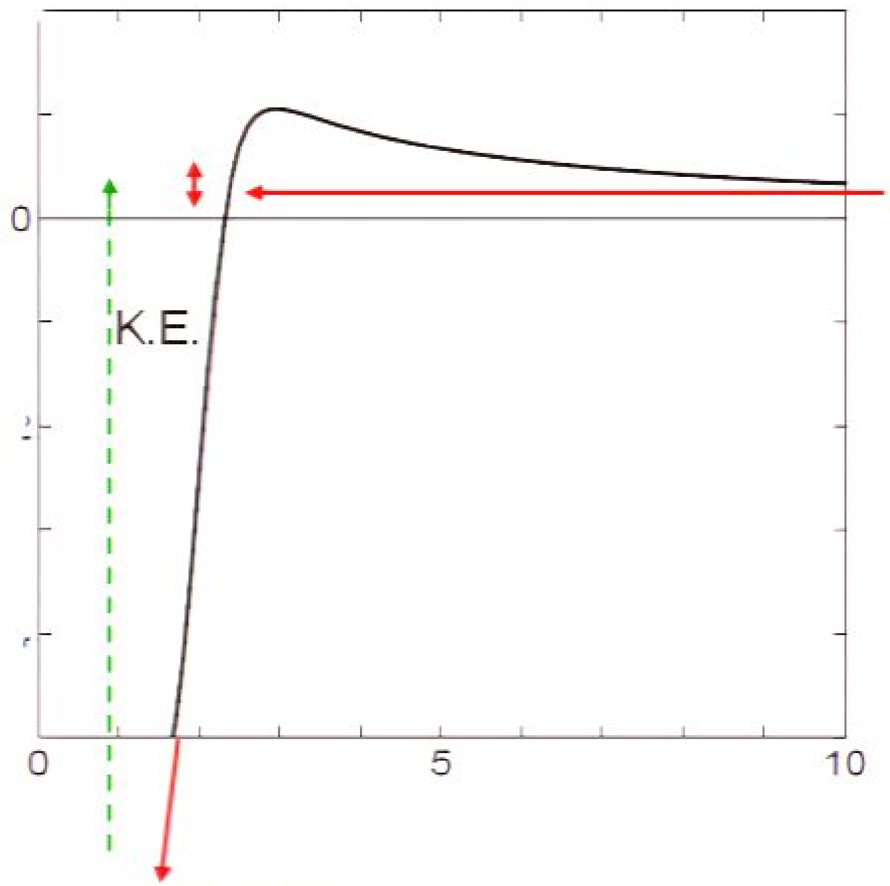
CBBN: $N_A \langle \sigma v \rangle = 2.6 \times 10^{10}\, T_9^{-\frac{2}{3}} \exp(-\underline{6.74}\, T_9^{-\frac{1}{3}})$

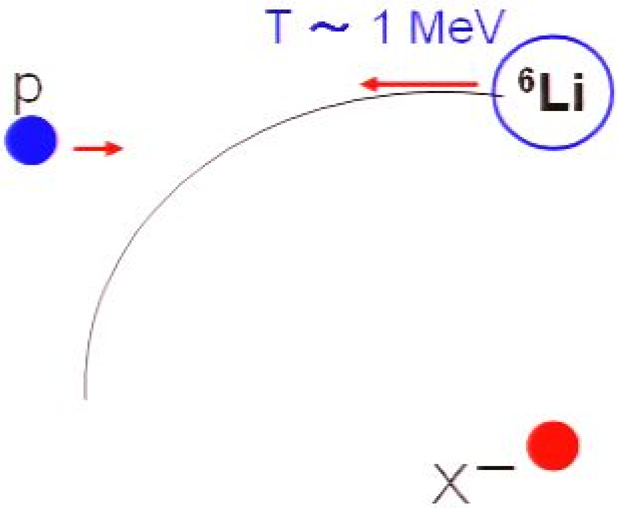
SBBN: $N_A\langle\sigma v\rangle_{gs} = 3.54 \times 10^{10} T_9^{-2/3} \exp(-\underline{8.415} T_9^{-1/3})$

(NACRE)

ratio at	T_9=0.1	0.3
CBBN	30	10
SBBN	1	1

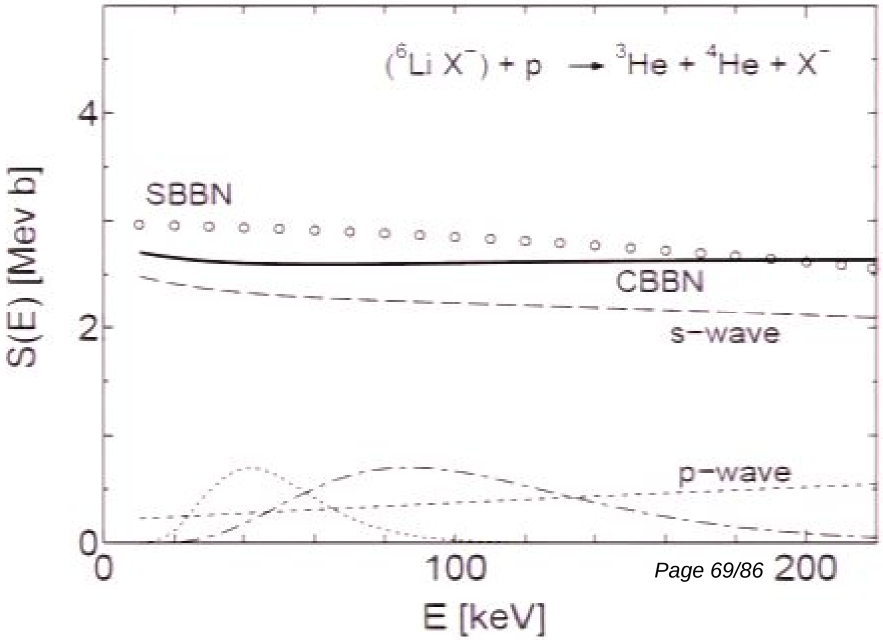
S-factor of non-resonant reaction





Even for the head-on collision.

$E_{c.m.}$ =1/7 x T ~100 keV



CBBN: $\sigma_{\text{CBBN}}(E) = S_{\text{CBBN}}(E)\ \ G_{\text{CBBN}}(E)\ /E$

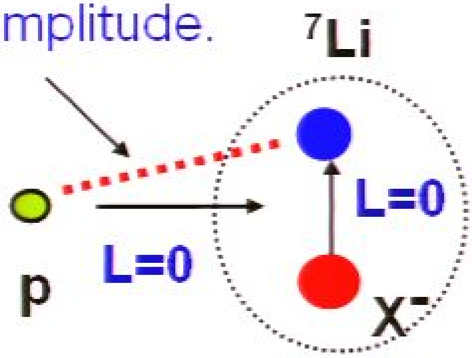
$\wr \qquad \vee$

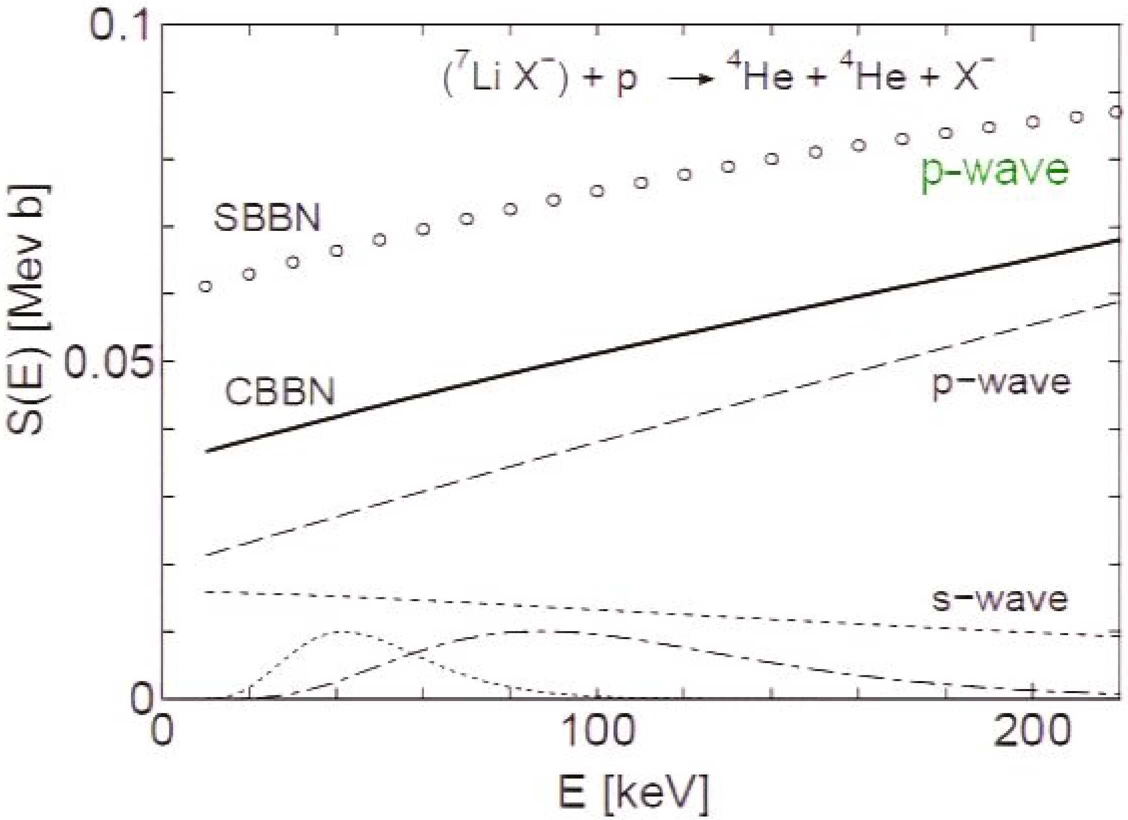
SBBN: σ (E) = S (E) G (E) /E

CBBN: $(^7\mathrm{Li}\,X) + p \rightarrow \alpha + \alpha + X$

SBBN: $\underbrace{^7\mathrm{Li} + p}_{\text{odd L}} \rightarrow \underbrace{\alpha + \alpha}_{\text{even L}}$

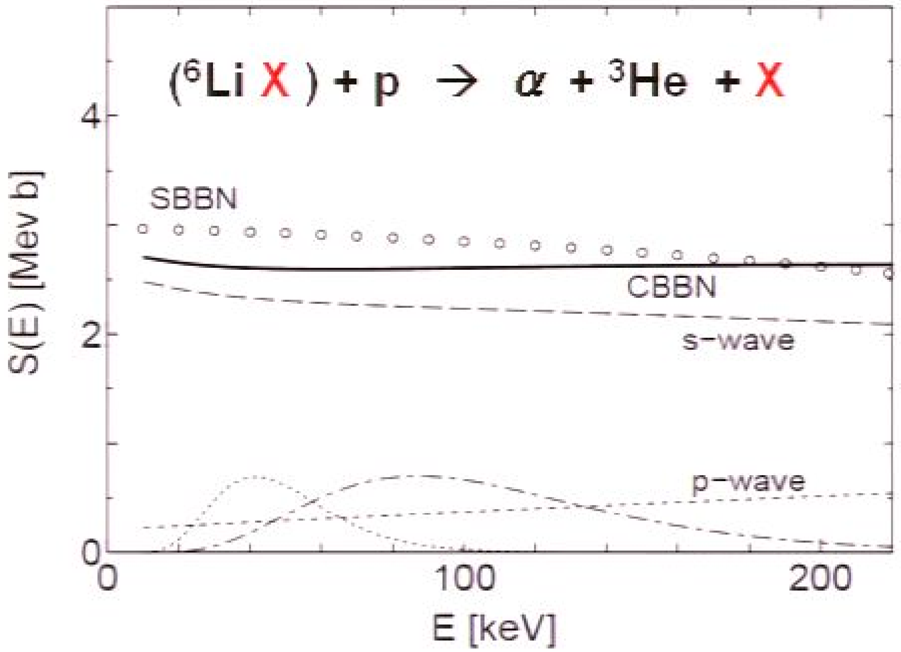
Small **L=1** amplitude.

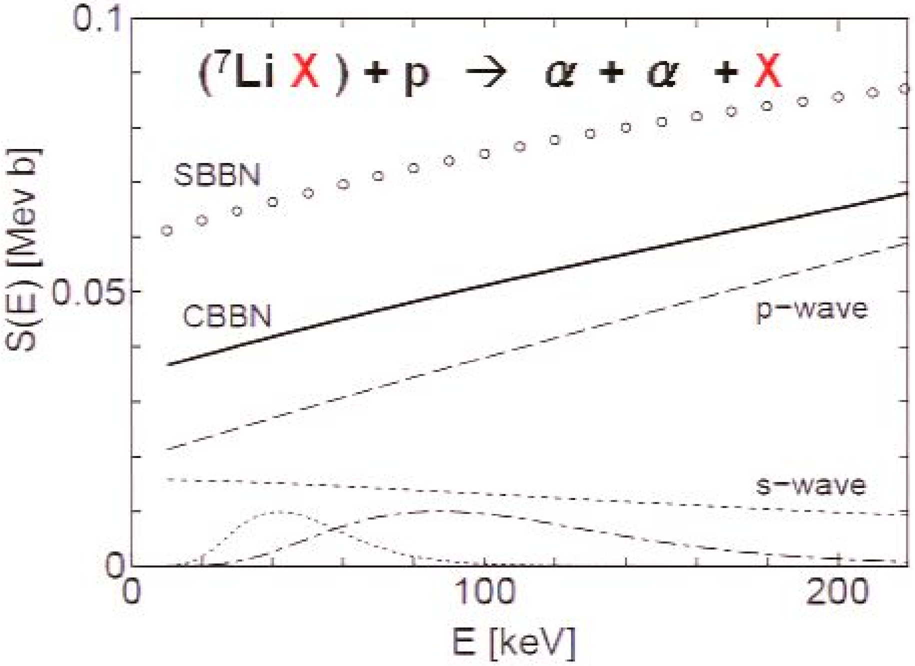




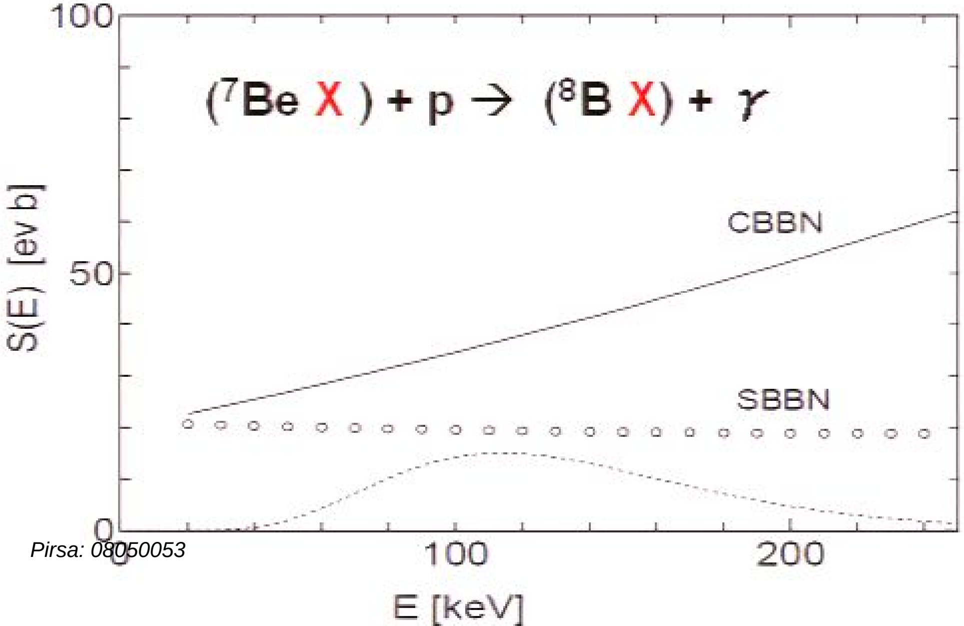
$$\frac{S_{\mathrm{CBBN}}(E)}{S_{\mathrm{SBBN}}(E)} \approx 0.6 \sim 0.8$$

	ratio at T_9=0.1	0.3
CBBN: $N_A\langle\sigma v\rangle = 3.5\times10^8\, T_9^{-\frac{2}{3}} \exp\left(\underline{-6.74}\, T_9^{-\frac{1}{3}}\right)\left(1+0.81\, T_9^{\frac{2}{3}}+0.30\, T_9\right)$	㉒	⑦
SBBN: $N_{\mathrm{A}}\langle\sigma v\rangle_{\mathrm{gs}} = 7.20\times10^8 T_9^{-2/3}\exp(\underline{-8.473}T_9^{-1/3}) + \cdots\cdots\cdots$	①	①





Non-resonant radiative capture



A simple model

$$\frac{S_{CBBN}(E)}{S_{SBBN}(E)} \approx \frac{1}{2} \sim 2$$

for $T_9 \approx 0.1 \sim 0.5$

$$\sigma_{CBBN}(E) \approx S_{SBBN}(E) \quad G_{CBBN}(E)\ /E$$

$^6\mathrm{Li} + p \rightarrow \alpha + {}^3\mathrm{He}$

$^7\mathrm{Li} + p \rightarrow \alpha + \alpha$

$^7\mathrm{Be} + p \rightarrow {}^8\mathrm{B} + \gamma$

SBBN

Corresponding ⟷

$(^6\mathrm{Li}\,X) + p \rightarrow \alpha + {}^3\mathrm{He} + X$

$(^7\mathrm{Li}\,X) + p \rightarrow \alpha + \alpha + X$

$(^7\mathrm{Be}\,X) + p \rightarrow (^8\mathrm{B}\,X) + \gamma$

CBBN

A simple model

$$\frac{S_{\mathrm{CBBN}}(E)}{S_{\mathrm{SBBN}}(E)} \approx \frac{1}{2} \sim 2$$

for $T_9 \approx 0.1 \sim 0.5$

$$\sigma_{\mathrm{CBBN}}(E) \approx S_{\mathrm{SBBN}}(E)\ G_{\mathrm{CBBN}}(E)\ /E$$

SBBN

~~$\alpha + d \rightarrow {}^6\mathrm{Li}$~~

~~$\alpha + {}^3\mathrm{H} \rightarrow {}^7\mathrm{Li}$~~

~~$\alpha + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be}$~~

No correspondence

CBBN

$(\alpha\,X) + d \rightarrow {}^6\mathrm{Li} + X$

$(\alpha\,X) + {}^3\mathrm{H} \rightarrow {}^7\mathrm{Li} + X$

$(\alpha\,X) + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be} + X$

Summary reaction rates by 3-body calculations

Reaction	Reaction rate ($\mathrm{cm}^3\,\mathrm{s}^{-1}\,\mathrm{mol}^{-1}$)	
non-resonant reaction		
a) $({}^4\mathrm{He}X^-) + d \to {}^6\mathrm{Li} + X^-$	$2.78 \times 10^8\, T_9^{-\frac{2}{3}} \exp(-5.33\, T_9^{-\frac{1}{3}})(1 - 0.62\, T_9^{\frac{2}{3}} - 0.29\, T_9)$	
b) $({}^4\mathrm{He}X^-) + t \to {}^7\mathrm{Li} + X^-$	$1.4 \times 10^7\, T_9^{-\frac{2}{3}} \exp(-6.08\, T_9^{-\frac{1}{3}})(1 + 1.3\, T_9^{\frac{2}{3}} + 0.55\, T_9)$	
c) $({}^4\mathrm{He}X^-) + {}^3\mathrm{He} \to {}^7\mathrm{Be} + X^-$	$9.4 \times 10^7\, T_9^{-\frac{2}{3}} \exp(-9.66\, T_9^{-\frac{1}{3}})\,(1 + 0.20\, T_9^{\frac{2}{3}} + 0.05\, T_9)$	
d) $({}^6\mathrm{Li}X^-) + p \to {}^4\mathrm{He} + t + X^-$	$2.6 \times 10^{10}\, T_9^{-\frac{2}{3}} \exp(-6.74\, T_9^{-\frac{1}{3}})$	
e) $({}^7\mathrm{Li}X^-) + p \to {}^4\mathrm{He} + {}^3\mathrm{He} + X^-$	$3.5 \times 10^8\, T_9^{-\frac{2}{3}} \exp(-6.74\, T_9^{-\frac{1}{3}})\,(1 + 0.81\, T_9^{\frac{2}{3}} + 0.30\, T_9)$	
f) $({}^7\mathrm{Be}X^-) + p \to ({}^8\mathrm{B}X^-) + \gamma$	$2.3 \times 10^5\, T_9^{-\frac{2}{3}} \exp(-8.83\, T_9^{-\frac{1}{3}})\,(1 + 1.9\, T_9^{\frac{2}{3}} + 0.54\, T_9)$	
resonant reaction		
g) $({}^7\mathrm{Be}X^-) + p \to ({}^8\mathrm{B}X^-)_{2p}$	$1.37 \times 10^6\, T_9^{-\frac{2}{3}} \exp(-2.28 T_9^{-1})$	$m_X = 50\mathrm{GeV}$
$\to ({}^8\mathrm{B}X^-) + \gamma$	$1.44 \times 10^6\, T_9^{-\frac{2}{3}} \exp(-2.15 T_9^{-1})$	$m_X = 100\mathrm{GeV}$
	$1.48 \times 10^6\, T_9^{-\frac{2}{3}} \exp(-2.04 T_9^{-1})$	$m_X = 500\mathrm{GeV}$
	$1.51 \times 10^6\, T_9^{-\frac{2}{3}} \exp(-2.01 T_9^{-1})$	$m_X \to \infty$

Section 4.

Late-time CBBN reactions by (p X) atom

$$(p\ X) + {}^{6}\mathrm{Li} \rightarrow \alpha + {}^{3}\mathrm{He} + X$$

$$(p\ X) + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + X$$

Late-time catalyzed reactions with (pX).

Yesterday, reported by Jedamzik (astro-ph/07072070).

1) $(p\,X) + {}^6\mathrm{Li} \rightarrow \alpha + {}^3\mathrm{He} + X \longleftarrow ({}^6\mathrm{Li}\,X) + p \rightarrow \alpha + {}^3\mathrm{He} + X$

2) $(p\,X) + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + X \longleftarrow ({}^7\mathrm{Be}\,X) + p \rightarrow ({}^8\mathrm{B}\,X) + \gamma$

Everything has been prepared in these calculations

$$\sigma(E) = S(E)\,\underbrace{\exp\left(-bE^{-\frac{1}{2}}\right)}_{\substack{\parallel \\ 1}}/E$$

1) $(p\,X) + {}^{6}\mathrm{Li} \rightarrow \alpha + {}^{3}\mathrm{He} + X$

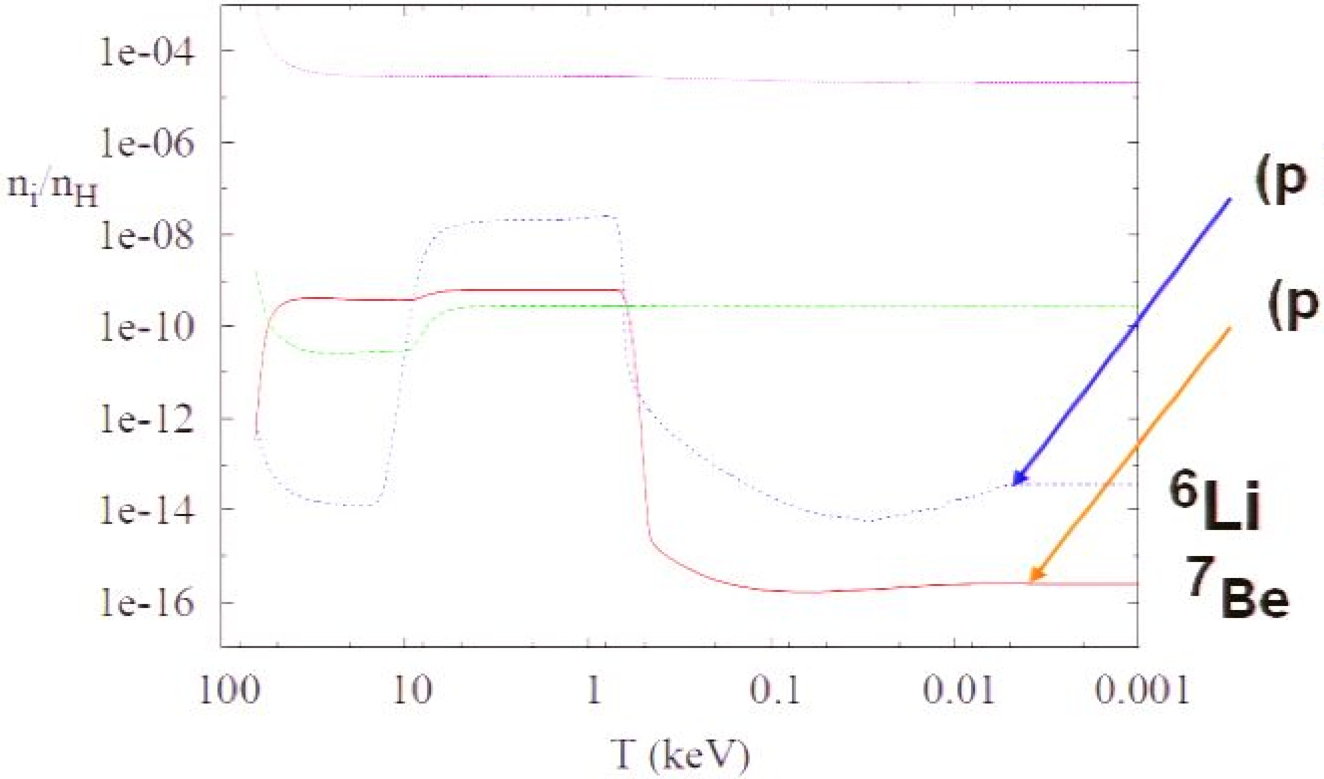
		3-body cal.		BA (Jademzik, Fig.4)
E= 1 keV	S(E)=	0.00012 MeV b	~	1 MeV b
E= 10 keV	S(E)=	0.00012 MeV b	~	1 MeV b

$$\sigma(E) = S(E)\,\underbrace{\exp\left(-bE^{-\frac{1}{2}}\right)}_{\substack{\parallel \\ 1}}\,/E$$

2) (p X) + ^{7}Be → ^{8}B + X

		3-body cal.		BA (Jademzik, Fig.4)
E= 1 keV	S(E)=	0.00016 MeV b	~	1 MeV b
E= 10 keV	S(E)=	0.00018 MeV b	~	1 MeV b

Jademzik (astro-ph/07072070) Fig.11



$(p\,X) + {}^6\mathrm{Li} \rightarrow \alpha + {}^3\mathrm{He} + X$

$(p\,X) + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + X$

The destruction will become much moderate.

Charge-exchange reaction

3) $(p\, X) + {}^4\mathrm{He} \rightarrow ({}^4\mathrm{He}\, X) + p$

Much discussions by Pospelov and Jedamzik yesterday.

I have not calculated it yet.

But, the cross section will be much larger than the previous two reactions.

Section 5.

Summary

No Signal

VGA-1

No Signal

VGA-1

No Signal
VGA-1

No Signal

VGA-1

No Signal

VGA-1

No Signal
VGA-1