Title: Identifying phases of matter that are universal for quantum computation

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Abstract: A recent breakthrough in quantum computing has been the realization that quantum computation can proceed solely through single-qubit measurements on an appropriate quantum state - for example, the ground state of an interacting many-body system. It would be unfortunate, however, if the usefulness of a ground state for quantum computation was critically dependent on the details of the system\'s Hamiltonian; a much more powerful result would be the existence of a robust ordered phase which is characterized by the ability to perform measurement-based quantum computation (MBQC). To identify such phases, we propose to use nonlocal correlation functions that quantify the fidelity of quantum gates performed between distant qubits. We investigate a simple spin-lattice system based on the cluster-state model for MBQC, and demonstrate that it possesses a zero temperature phase transition between a disordered phase and an ordered \'cluster phase\' in which it is possible to perform a universal set of quantum gates.

Identifying phases of matter that are universal for quantum computing

Stephen Bartlett
The University of Sydney
AUSTRALIA
School of Physics

in collaboration with
Andrew Doherty (UQ)

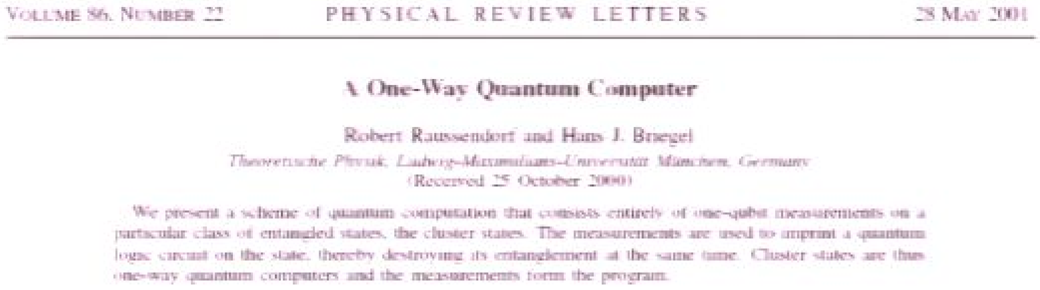
Quantum computing with a *cluster state*

Quantum computing can proceed through *measurements* rather than unitary evolution

Measurements are strong and incoherent: easier

Uses a *cluster state*:

- a universal circuit board
- a 2-d lattice of spins in a specific *entangled* state



Volume 86, Number 22 PHYSICAL REVIEW LETTERS 28 May 2001

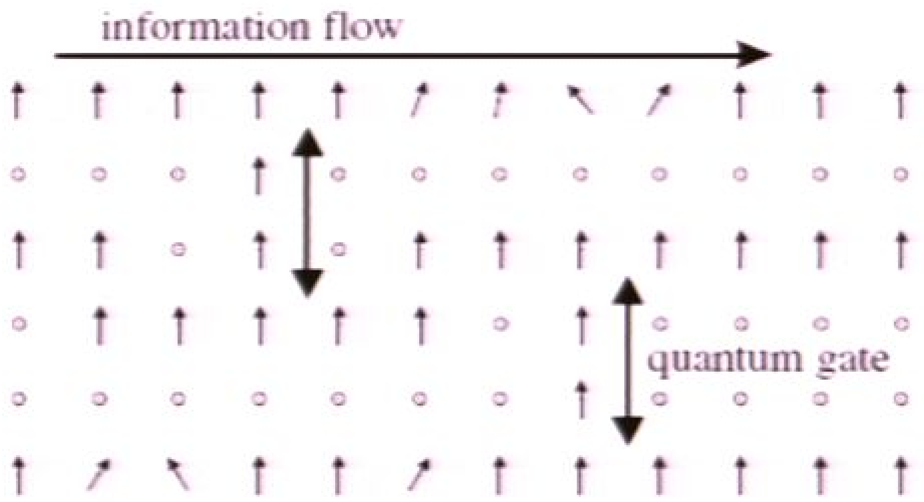
A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel

Theoretische Physik, Ludwig-Maximilians-Universität München, Germany

(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.



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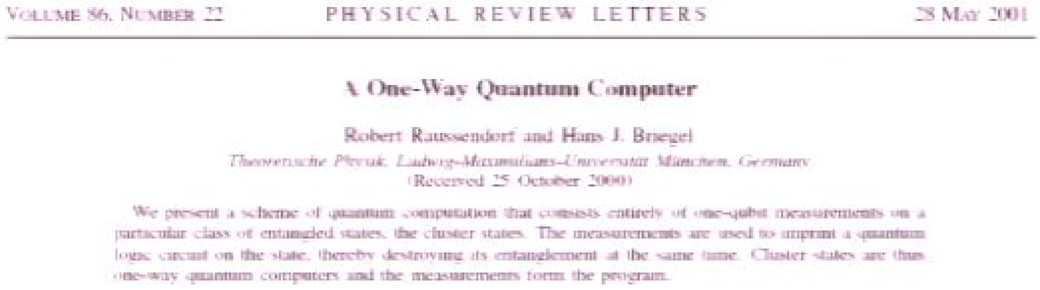

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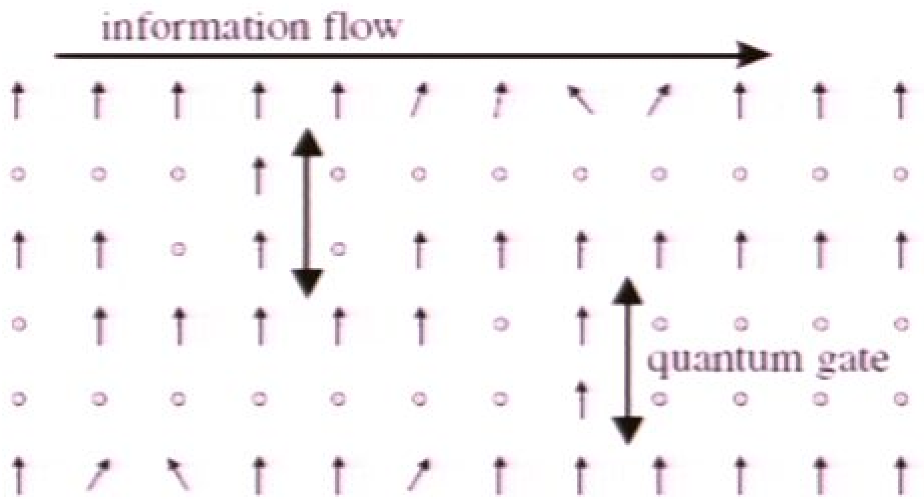


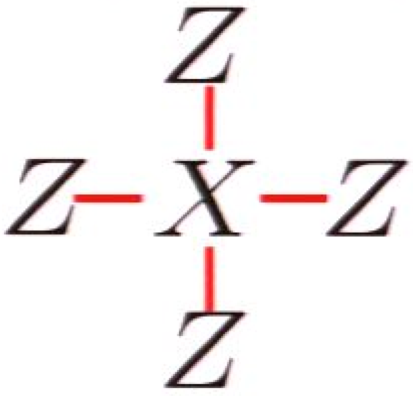
Q: What makes the cluster state work?

Q: What properties of a state are necessary?

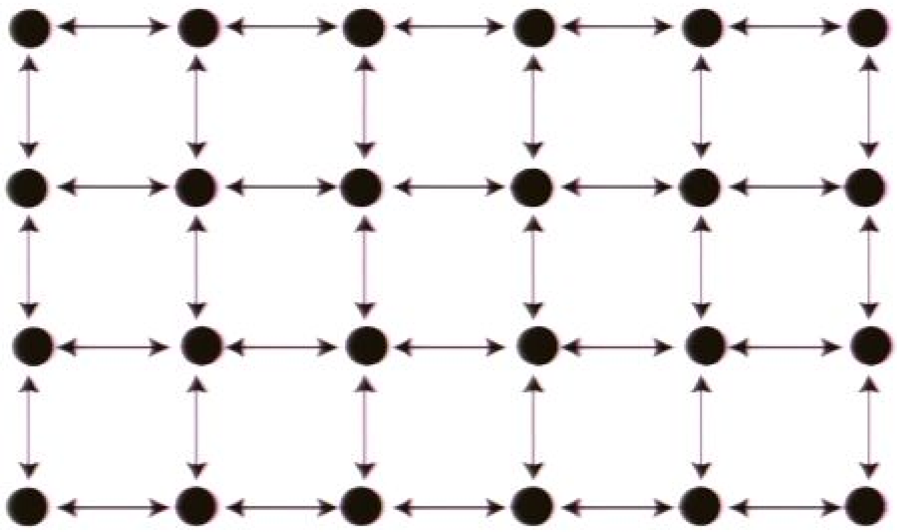
So what is a cluster state?

- Describe via the eigenvalues of a complete set of commuting observables

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Stabilizer



- Cluster state is the +1 eigenstate of all stabilizers

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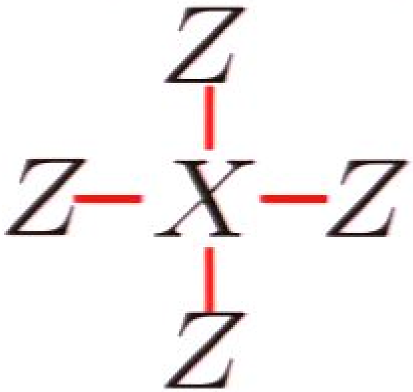

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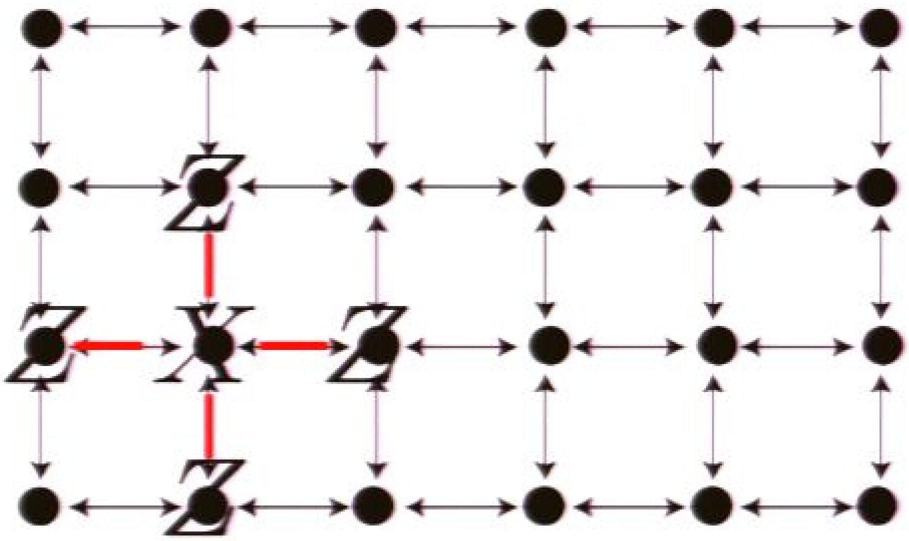

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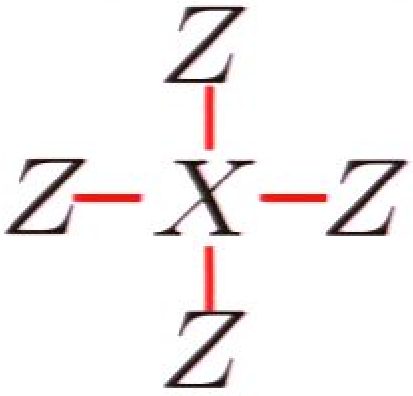

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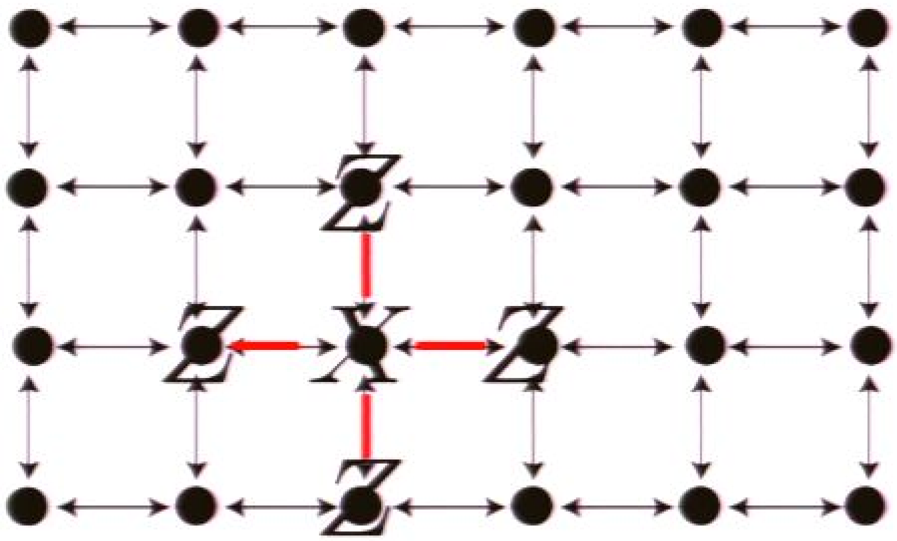

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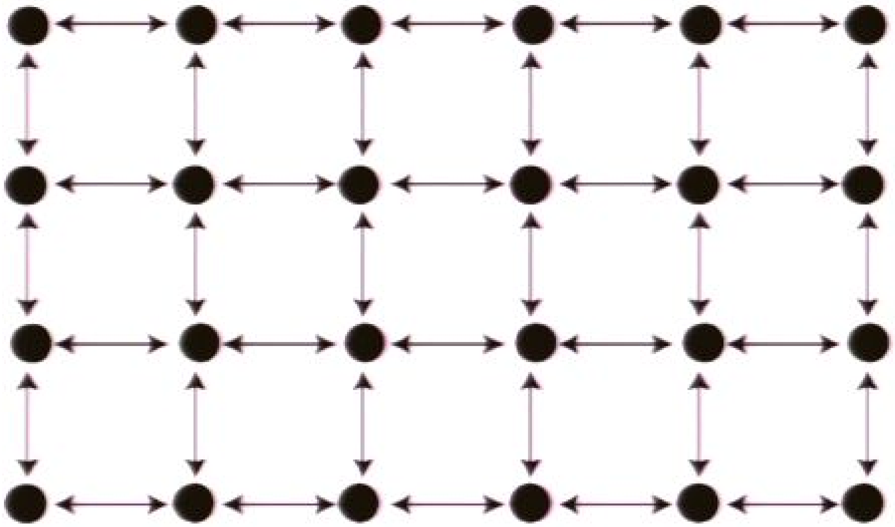
Quantum gates in cluster state QC

- With a cluster state, you can teleport a qubit (identity gate) between any two points on the lattice using local measurements on the qubits in between



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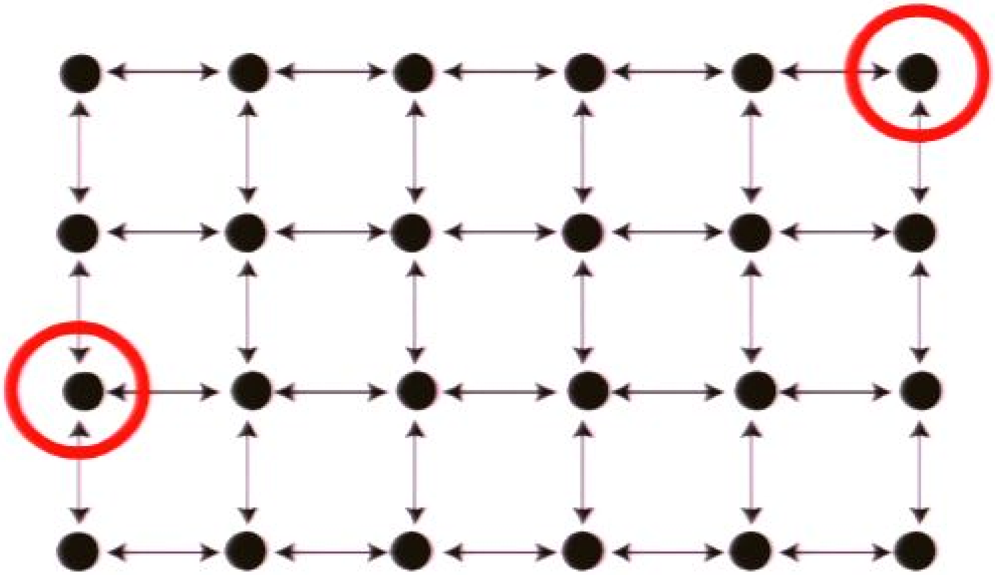

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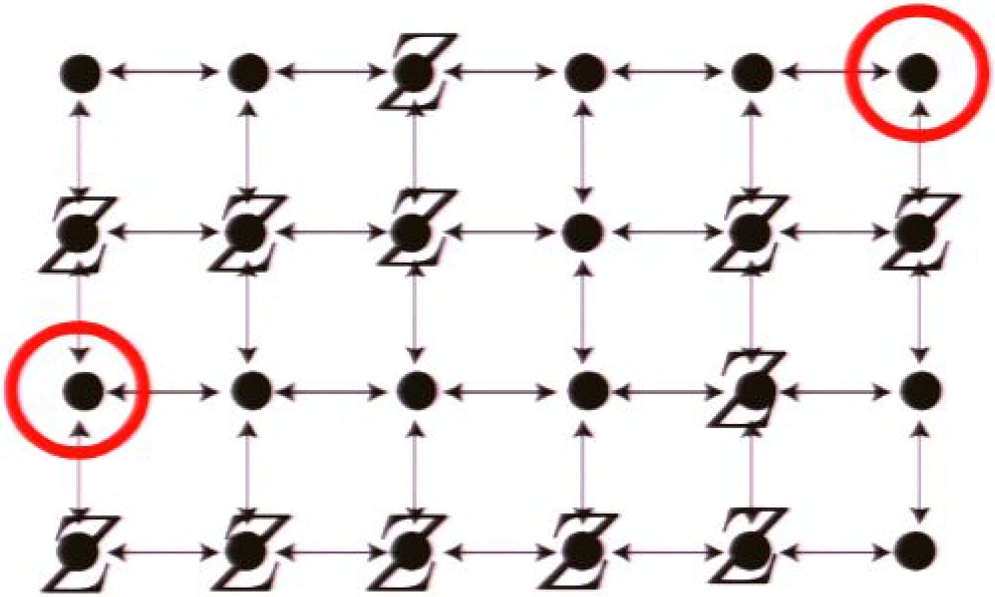

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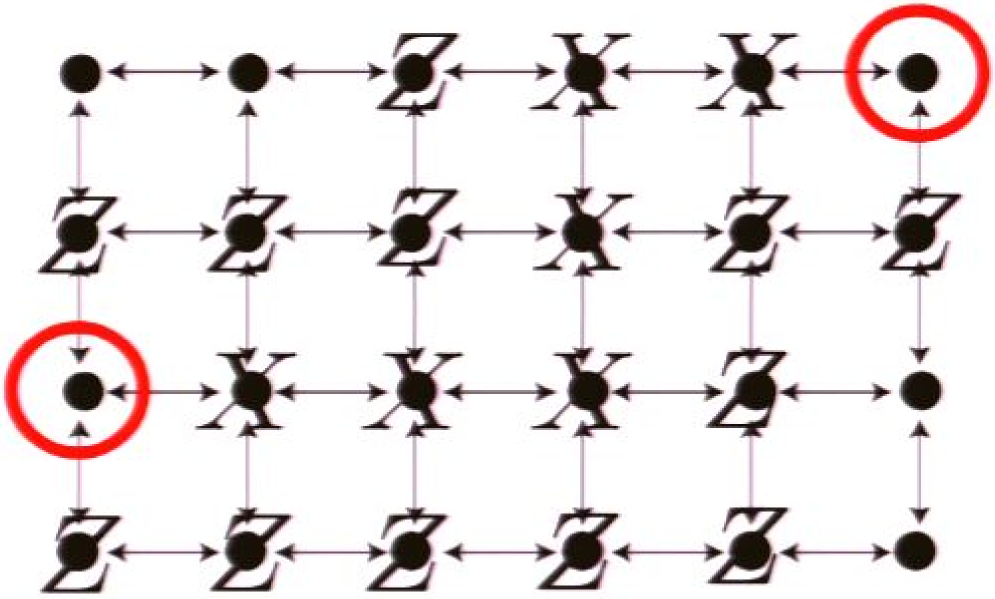

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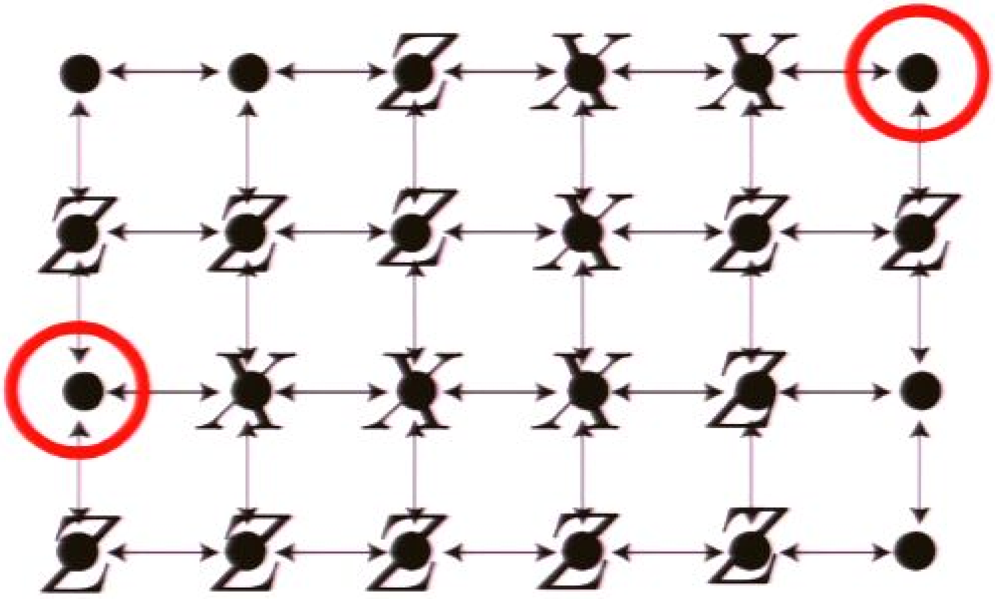

☐ Measurement outcomes determine a Pauli correction to gate

☐ Other gates are given by different measurement patterns

☐ Performance of a gate is quantified by the fidelity of the resulting entangled state

Universal resources for MBQC

Questions:

- What states are universal resources for MBQC?
- How do we characterise the usefulness and universality of a state for a given MBQC scheme?

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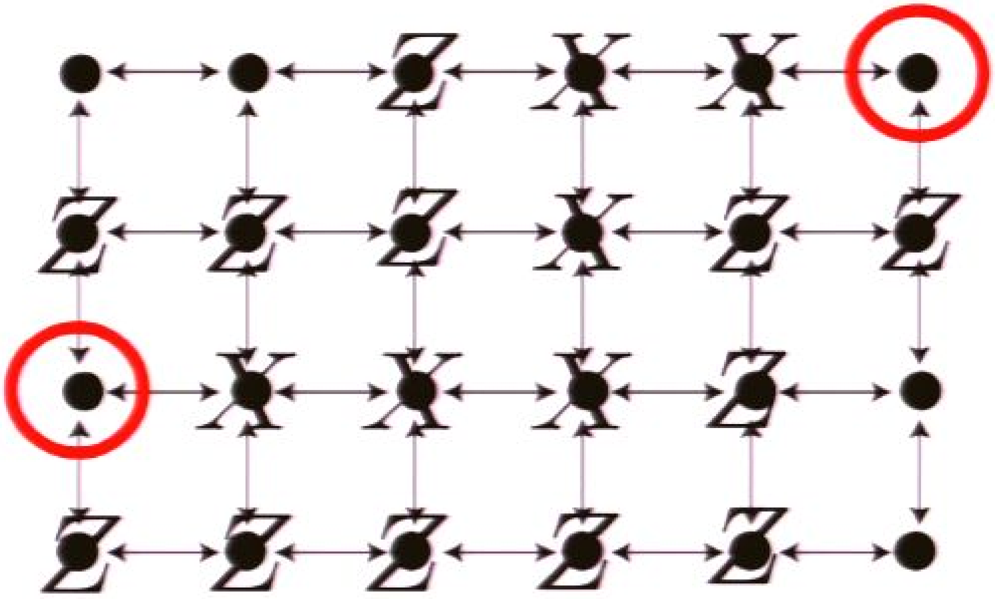

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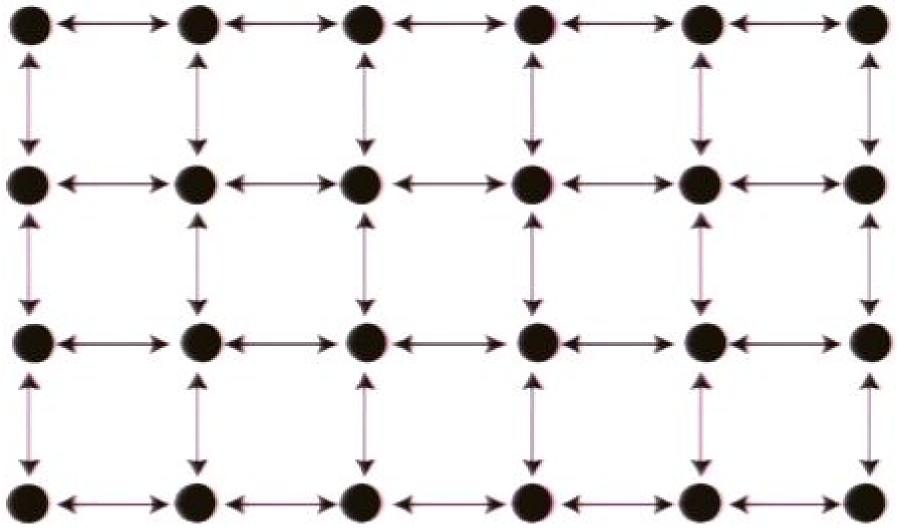
Approach:

- Quantum many-body physics: order parameters
- Correlation functions as quantum gates

  → order parameters to characterise *phases* which are universal for MBQC

Making cluster states:
Can Nature do the work?

- Is the cluster state the ground state of some system?
- If it was (and system is gapped), we could cool the system to the ground state and get the cluster state for free!



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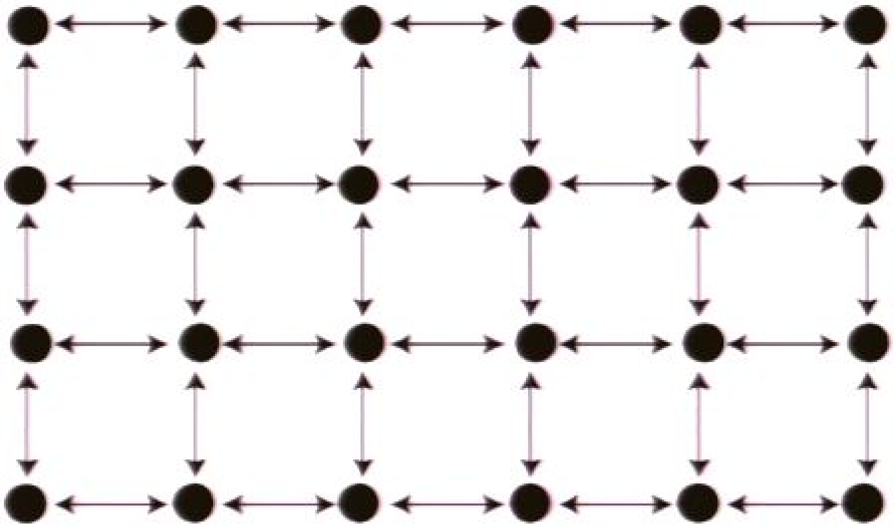

$$H = -\sum_{\text{sites}} Z - \overset{\displaystyle Z \atop |}{\underset{| \atop \displaystyle Z}{X}} - Z$$

- Nielsen 2005 – gives proof: *no* 2-body nearest-neighbour *H* has the cluster state as its exact ground state
- Bartlett & Rudolph 2006 – can obtain an encoded cluster state

Ground states of a cluster phase?

- What if our Hamiltonian was only "close" to the desired cluster Hamiltonian?
- Example: add a local X field to the cluster Hamiltonian

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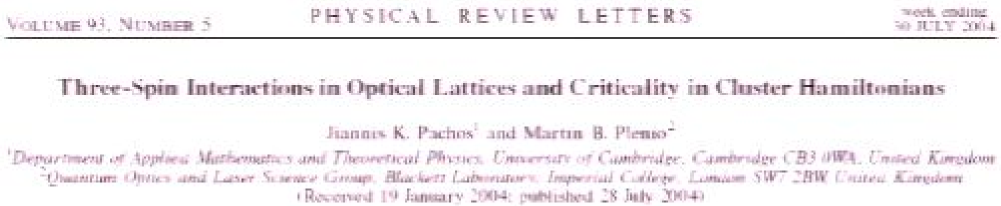
1-D cluster state

- Pachos & Plenio 2004
- Ground state of cluster Hamiltonian with local field

$$H = -\sum ZXZ - B\sum X$$

exhibits a quantum phase transition at B=1

- Localizable entanglement length in the ground state remains infinite for all values of B<1

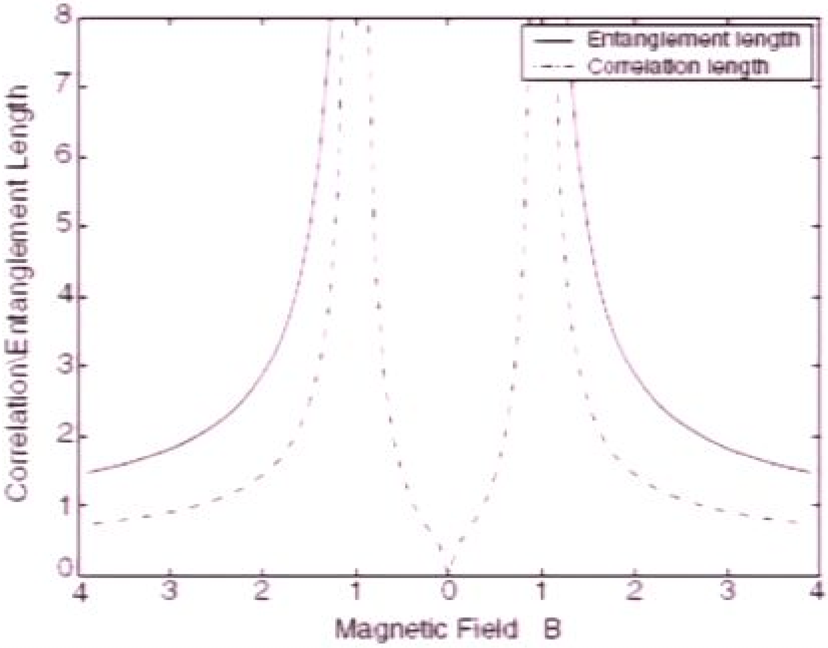


VOLUME 93, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 30 JULY 2004

Three-Spin Interactions in Optical Lattices and Criticality in Cluster Hamiltonians

Jiannis K. Pachos[1] and Martin B. Plenio[2]

[1]Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom
[2]Quantum Optics and Laser Science Group, Blackett Laboratory, Imperial College, London SW7 2BW United Kingdom
(Received 19 January 2004; published 28 July 2004)



1-D cluster state redux

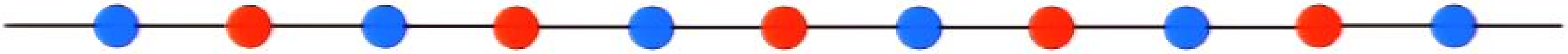
Outline:

- map the cluster Hamiltonian with local field to a (pair of) known models
- relate the correlation functions and critical behaviour of the two models

Colouring the 1-D system

- Cluster Hamiltonian with local field

$$H = -\sum ZXZ - B\sum X$$



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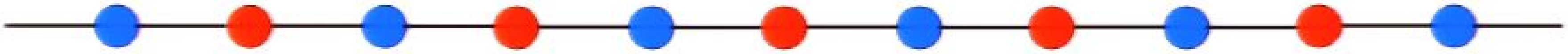

$$H = H_r + H_b \quad \text{where} \quad \begin{aligned} H_r &= -\sum ZXZ - B\sum X \\ H_b &= -\sum ZXZ - B\sum X \end{aligned}$$

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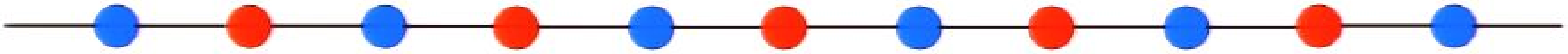

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- These two Hamiltonians commute
- (Note: ignored boundary conditions, but easy to incorporate)

A transformation

- Perform a canonical transformation on the spins:

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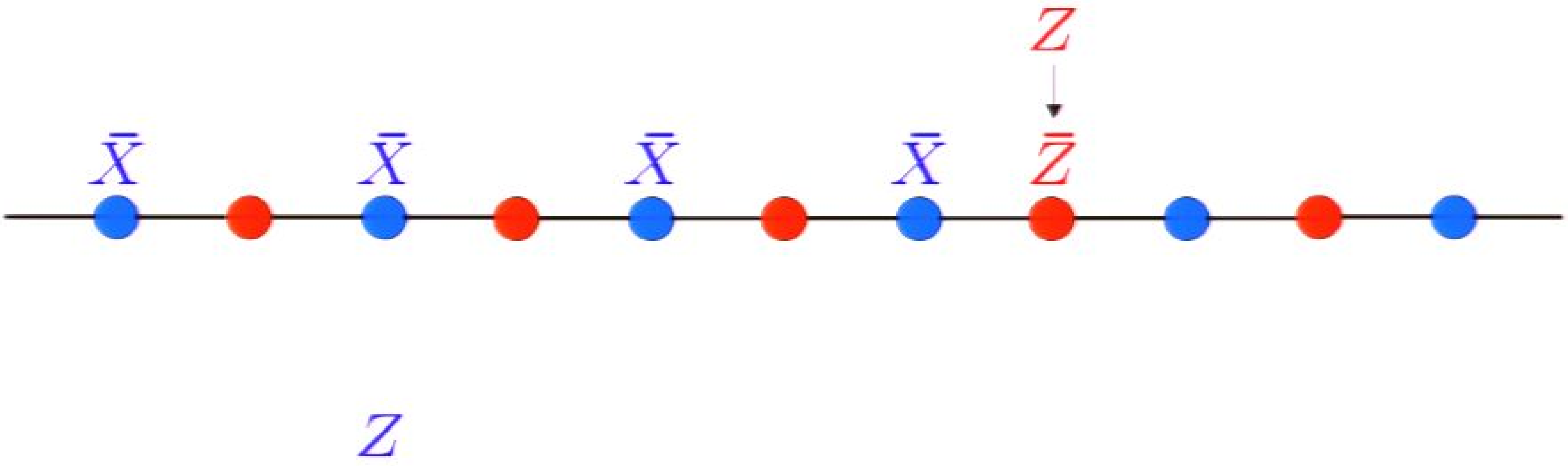

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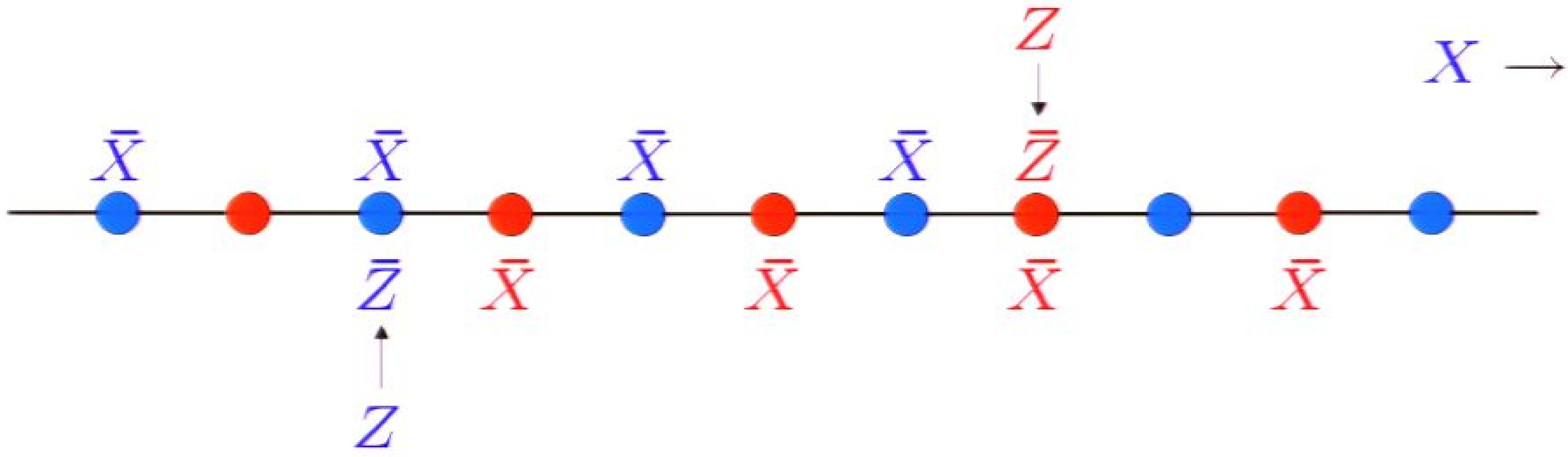

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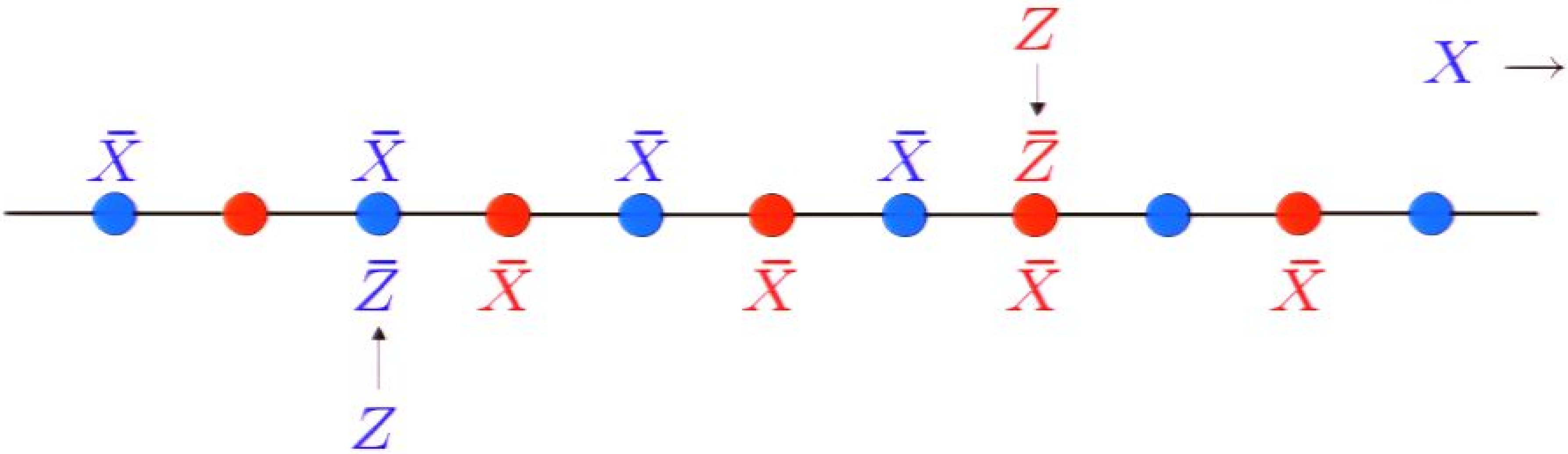

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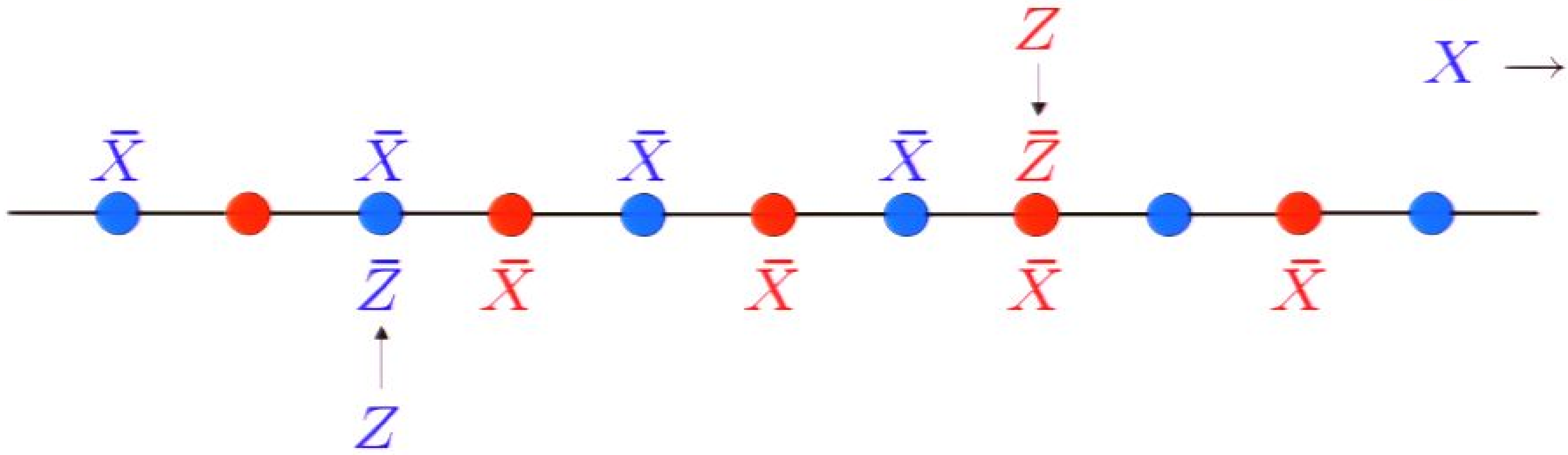


$$H_r = -\sum ZXZ - B\sum X \longrightarrow \bar{H}_r = -\sum \bar{Z}\bar{Z} - B\sum \bar{X}$$

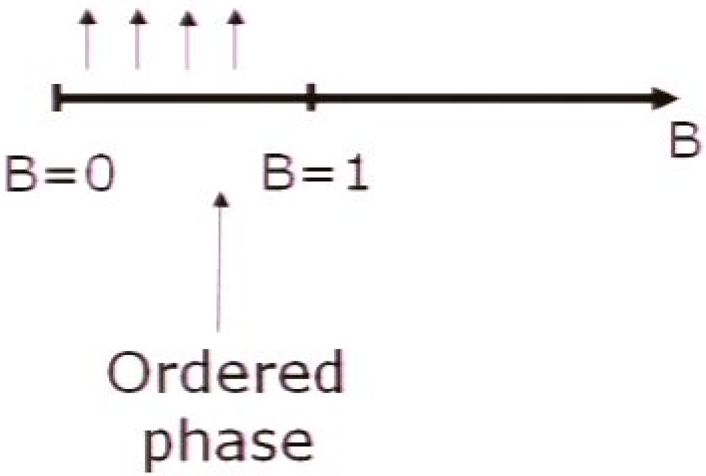
$$H_b = -\sum ZXZ - B\sum X \longrightarrow \bar{H}_b = -\sum \bar{Z}\bar{Z} - B\sum \bar{X}$$

transverse field Ising model!

Transverse field Ising model

- Two copies of the transverse field Ising model

$$H = -\sum \bar{Z}\bar{Z} - B\sum \bar{X}$$



- Completely solved
- Ground state has a single quantum phase transition at B=1
- In the "ordered" phase (B<1), the correlation function

$$\langle \bar{Z}_i \bar{Z}_{i+\Delta} \rangle$$

is long-ranged at zero temperature

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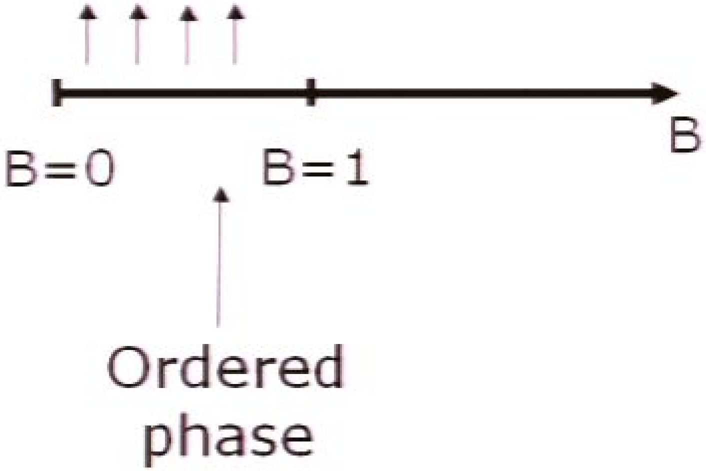


$$\lim_{\Delta\to\infty} \langle \bar{Z}_i \bar{Z}_{i+\Delta} \rangle = (1 - B^2)^{1/4}$$

Correlations on the cluster model

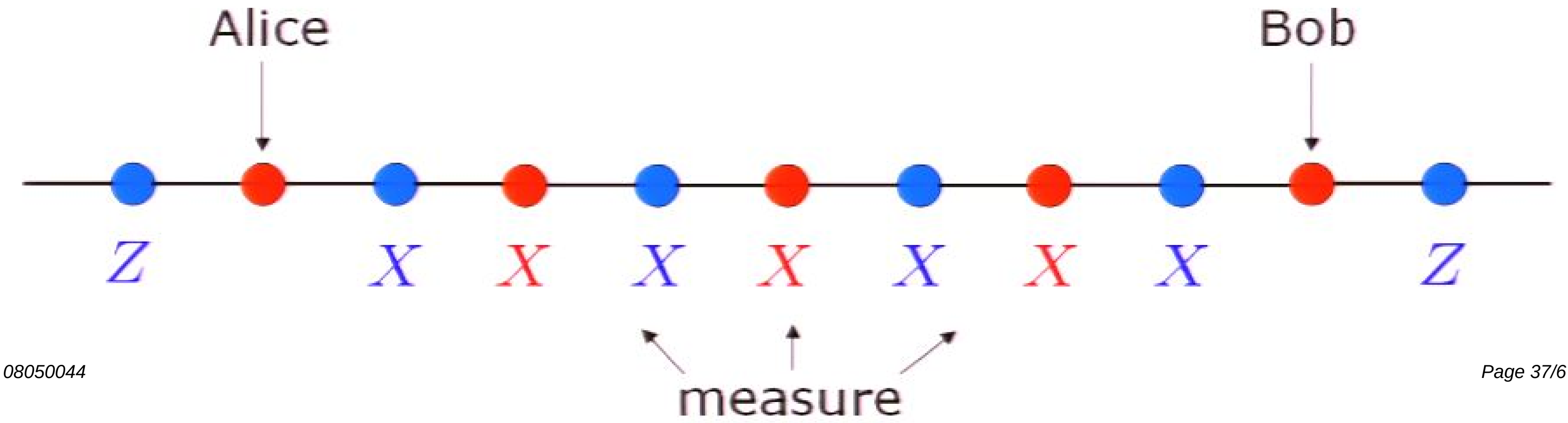
- ☐ Reverse the canonical transformation:

$$\langle \bar{Z}_{2i}\bar{Z}_{2j}\rangle \rightarrow \langle Z_{2i}\left(\prod_{k=i}^{k=j-1} X_{2k+1}\right) Z_{2j}\rangle$$

$$\langle \bar{Z}_{2i-1}\bar{Z}_{2j-1}\rangle \rightarrow \langle Z_{2i-1}\left(\prod_{k=i}^{k=j-1} X_{2k}\right) Z_{2j-1}\rangle$$

Ising model correlation functions tell us if we can do the identity gate!

- ☐ Recall: teleportation using the cluster state



1-D cluster state - summary

Summary:

- cluster Hamiltonian with local field maps to a pair of transverse field Ising models
- all ground state properties, correlation functions, and critical behaviour are known
- immediately shows the existence of:
  - ☐ quantum phase transition at $B=1$
  - ☐ long range "identity gate" for $B<1$

Key point:

- ground state of the cluster Hamiltonian with local field behaves like a cluster state for the entire *cluster phase* $B<1$

2-D cluster Ham on a square lattice

- Square lattice is also bi-colourable

$$H = -\left[\sum_{\text{red } s} Z \overset{Z}{\underset{Z}{X_s}} Z + B\sum_{\text{blue } s} X_s\right] - \left[\sum_{\text{blue } s} Z \overset{Z}{\underset{Z}{X_s}} Z + B\sum_{\text{red } s} X_s\right]$$

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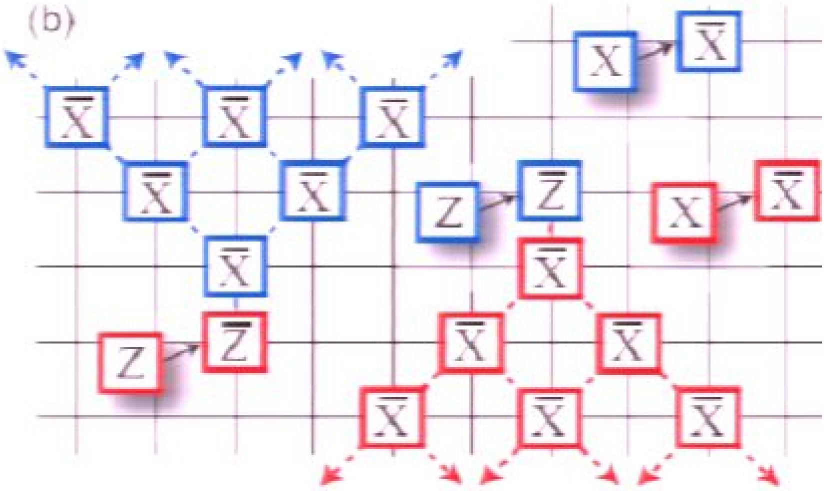

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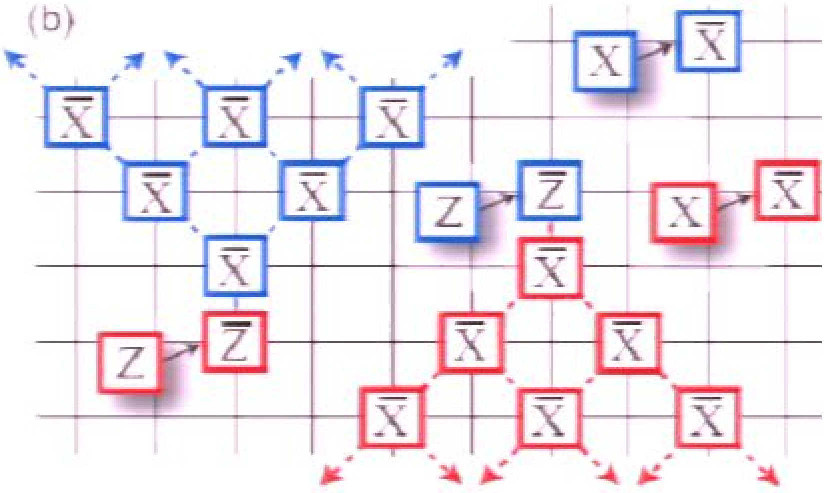

- A new canonical transformation:

- Effect on stabilizers:

$$\begin{matrix} & Z & \\ Z & X & Z \\ & Z & \end{matrix} \to \begin{matrix} & \bar{Z} & \\ \bar{Z} & & \bar{Z} \\ & \bar{Z} & \end{matrix}. \qquad \begin{matrix} & Z & \\ Z & X & Z \\ & Z & \end{matrix} \to \begin{matrix} & \bar{Z} & \\ \bar{Z} & & \bar{Z} \\ & \bar{Z} & \end{matrix}$$

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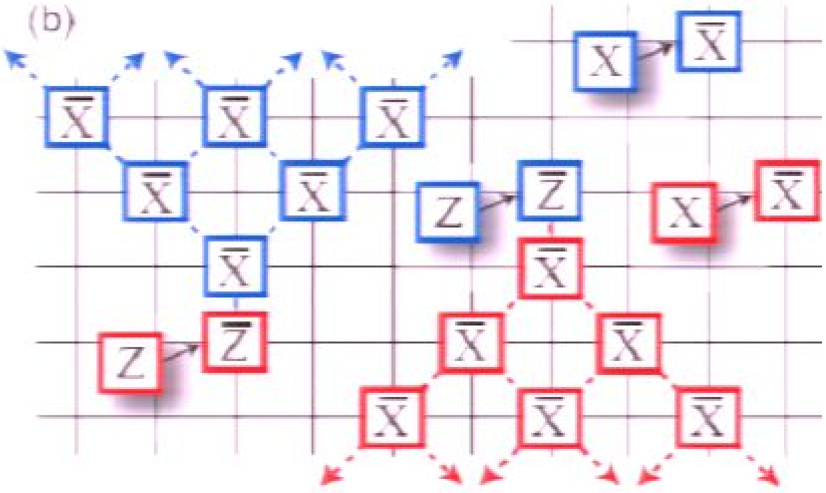

Result: two copies of

$$\bar{H} = -\sum \begin{matrix} \bar{Z} & \bar{Z} \\ \bar{Z} & \bar{Z} \end{matrix} - B\sum \bar{X}$$

2-D quantum Xu-Moore model

$$\bar{H} = -\sum \begin{matrix} \bar{Z} & \bar{Z} \\ \bar{Z} & \bar{Z} \end{matrix} - B \sum \bar{X}$$

- Phase transition
    - self-dual, with a single quantum phase transition at B=1
    - B<1 phase has long-range "bond order"
- Symmetries
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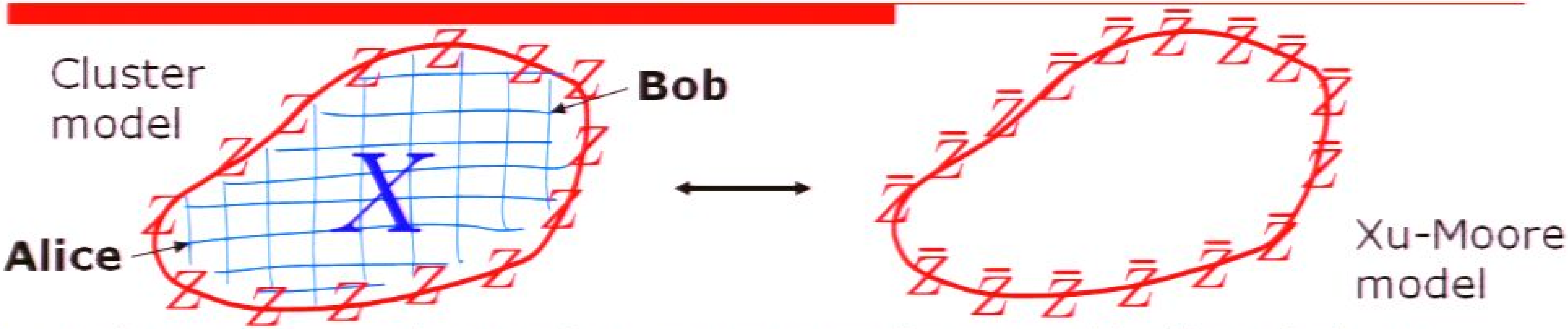
Correlation functions in 2-D



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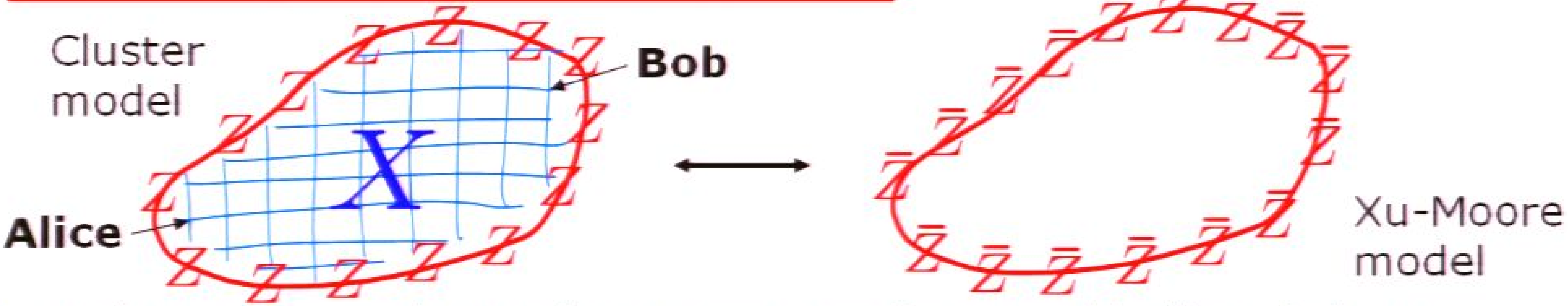

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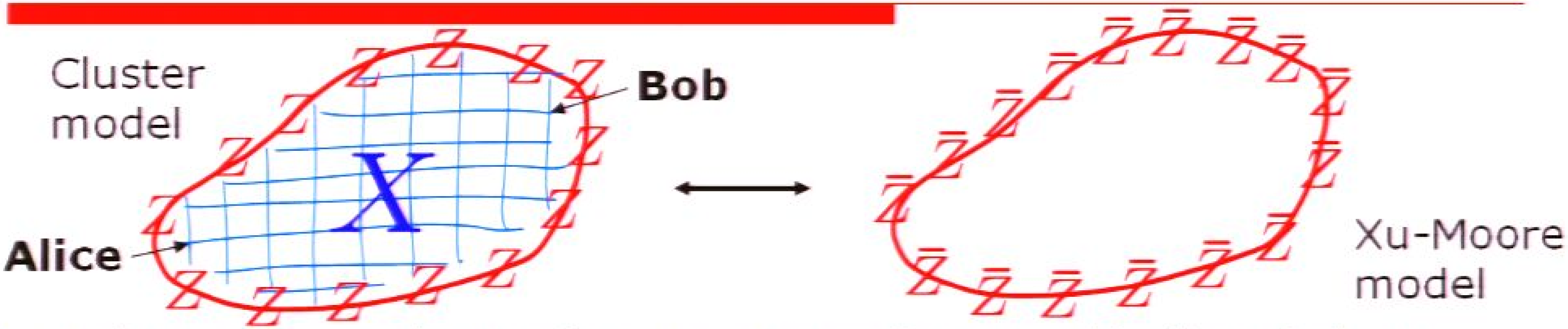

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- ☐ **The anisotropic quantum compass model**

Correlation functions in 2-D

- Anisotropic quantum compass model has long-ranged Ising order parameters in the ordered B<1 phase

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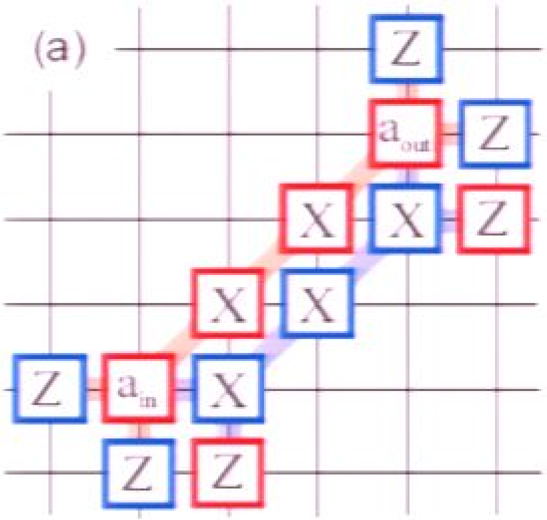

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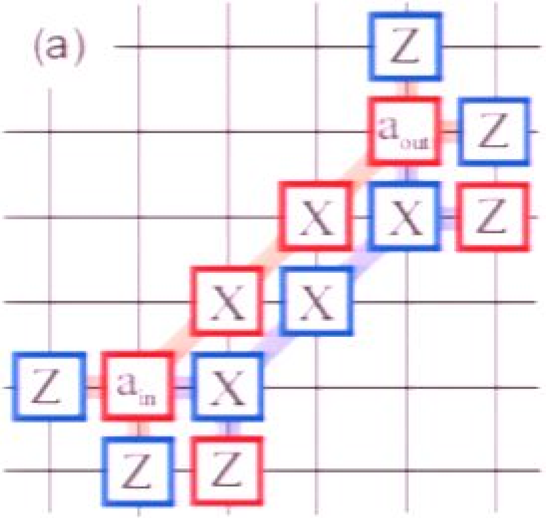

- As with the 1-D model, identity gate correlation function serves as an order parameter, labelling the cluster phase

Correlation functions in 2-D

- On a 2-D cluster state, measurement-based quantum computation can be performed with a universal set of gates

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(See Raussendorf, Browne, Briegel 2003)

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Correlation functions in 2-D

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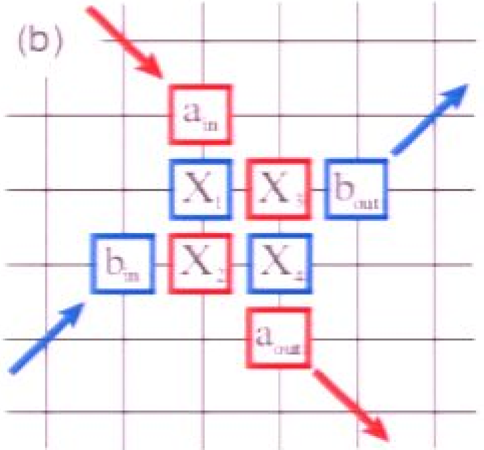
quantum gate ⟷ correlation function

(See Raussendorf, Browne, Briegel 2003)

- ☐ **Key question:** does the cluster phase possess long range order for all correlations functions corresponding to a universal gate set?
- ☐ **Yes!** 1- and 2-qubit Clifford gates (including CSIGN), plus a 1-qubit non-Clifford gate



Conclusions – A new approach

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- In a model with a cluster Hamiltonian and a local X field, there exists a **cluster phase**
  - What happens in higher dimensions (in particular, 3-D)?
  - Does this phase exist at finite temperature?
  - Similar results hold for a local Z field

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IQC seminar
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- ☐ Quantum gates, as correlation functions, can serve as order parameters to identify **universal phases** for MBQC

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2-D quantum Xu-Moore model

$$\bar{H} = -\sum \begin{matrix} \bar{Z} & \bar{Z} \\ \bar{Z} & \bar{Z} \end{matrix} - B\sum \bar{X}$$

- Phase transition
  - self-dual, with a single quantum phase transition at B=1
  - B<1 phase has long-range "bond order"
- Symmetries
  - full symmetry of the lattice (translations, rotations by $\pi/2$)
  - *not* a gauge model – 4-body terms are not plaquettes of the Ising gauge model or Kitaev's toric code model (spins sit on the vertices and not on the links)
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1-D cluster state - summary

Summary:

- cluster Hamiltonian with local field maps to a pair of transverse field Ising models
- all ground state properties, correlation functions, and critical behaviour are known
- immediately shows the existence of:
  - ☐ quantum phase transition at B=1
  - ☐ long range "identity gate" for B<1

Key point:

- ground state of the cluster Hamiltonian with local field behaves like a cluster state for the entire *cluster phase* B<1

Correlations on the cluster model

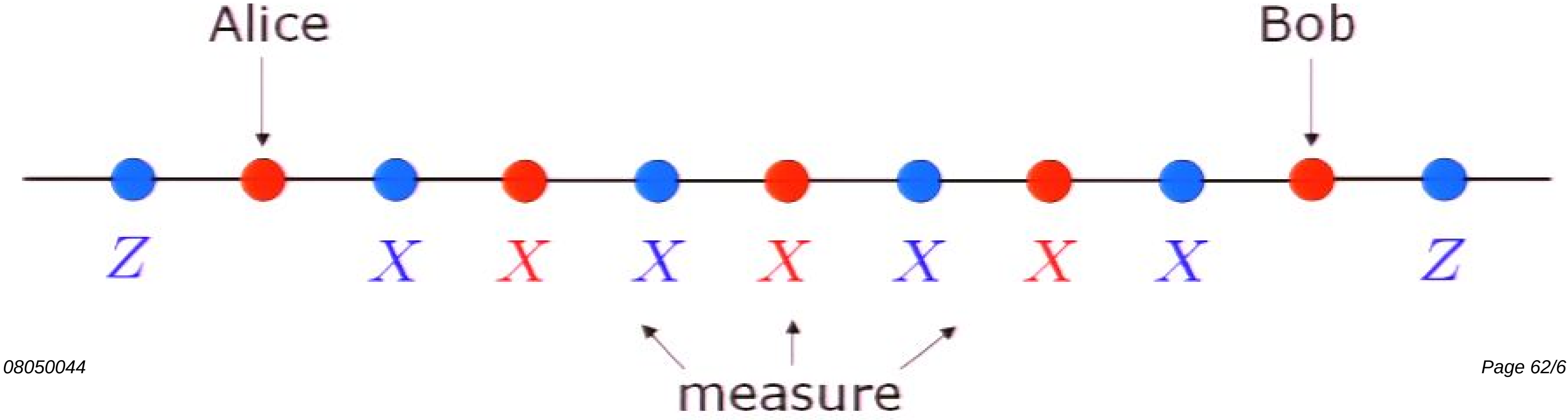
- ☐ Reverse the canonical transformation:

$$\langle \bar{Z}_{2i}\bar{Z}_{2j}\rangle \rightarrow \langle Z_{2i}\left(\prod_{k=i}^{k=j-1} X_{2k+1}\right) Z_{2j}\rangle$$

$$\langle \bar{Z}_{2i-1}\bar{Z}_{2j-1}\rangle \rightarrow \langle Z_{2i-1}\left(\prod_{k=i}^{k=j-1} X_{2k}\right) Z_{2j-1}\rangle$$

Ising model correlation functions tell us if we can do the identity gate!

- ☐ Recall: teleportation using the cluster state



Alice

Bob

Z X X X X X X X Z

measure

Transverse field Ising model

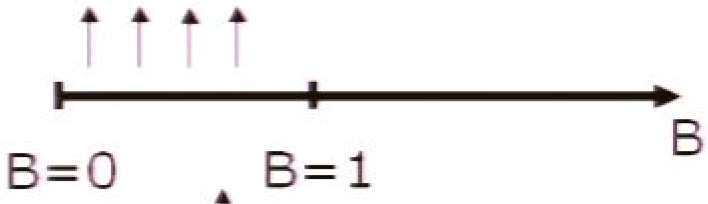
- Two copies of the transverse field Ising model

$$H = -\sum \bar{Z}\bar{Z} - B\sum \bar{X}$$

- Completely solved
- Ground state has a single quantum phase transition at B=1
- In the "ordered" phase (B<1), the correlation function

$$\langle \bar{Z}_i \bar{Z}_{i+\Delta} \rangle$$

is long-ranged at zero temperature



Ordered phase

$$\lim_{\Delta\to\infty} \langle \bar{Z}_i \bar{Z}_{i+\Delta} \rangle = (1 - B^2)^{1/4}$$

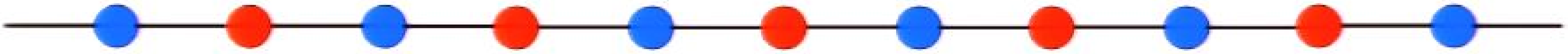
A transformation

- Perform a canonical transformation on the spins:

Colouring the 1-D system

☐ Cluster Hamiltonian with local field

$$H = -\sum ZXZ - B\sum X$$



$$H_r = -\sum ZXZ - B\sum X$$

$H = H_r + H_b$ where

$$H_b = -\sum ZXZ - B\sum X$$

1-D cluster state redux

Outline:

- map the cluster Hamiltonian with local field to a (pair of) known models
- relate the correlation functions and critical behaviour of the two models

Ground states of a cluster phase?

- What if our Hamiltonian was only “close” to the desired cluster Hamiltonian?
- Example: add a local X field to the cluster Hamiltonian

$$H = -\sum_{\text{sites}} Z - \begin{array}{c} Z \\ | \\ X \\ | \\ Z \end{array} - Z \; - B \sum_{\text{sites}} X$$

- Investigate order and correlation functions in the ground state as a function of B
- **Key result:** in 1- and 2-D, there is a cluster phase