

Title: String theory from supergravity and supergravity from string theory

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Abstract: The rich network of string dualities provides powerful constraints in the structure of the theory. The connection of ten-dimensional type II theories to eleven-dimensional supergravity compactified on a circle and on a torus allows one to compute many perturbative high genus terms as well as the complete sum of non-perturbative contributions to a given higher derivative coupling of the string effective action. The same duality connection leads to a series of surprising non-renormalization theorems. These control the UV behavior of maximal supergravity in lower dimensions, and indicate that N=8 supergravity in four dimensions might be a finite quantum gravity theory.

IIA/IIIB 4-j theory
4-particle rule

$$(R^4, DR^4, \dots, D^4 R^4, R^4)$$

Two-level 4-particle amplitude

$$A = \frac{1}{S\Lambda} \frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} = \frac{1}{S\Lambda} \exp \left[2 \sum_{n=1}^{\infty} \frac{S^{(2n+1)}}{2n+1} (S^{(n)} + e^{(n)} + iU^{(n)}) \right]$$

$$S^{(n)}, S^{(n)}U^{(n)} = 0 \quad S = \sum_{n=1}^{\infty}$$

$$K = S^{(n)} \sum_{i=1}^{N_b} S_i^{(n)} \sum_{j=1}^{N_b} K_{ij} \rightarrow K_{tot}, \quad K_{tot} = \frac{1}{4} \sin \eta_{11} \eta_{22} + \dots -$$

$$S(n) = \sum_{k=1}^{\infty} \frac{1}{n!}$$

I. String from Supergravity

IIA/IIB string theory -

q-graviton soln

$$R^4, D^2 R^4, \dots, D^{2k} R^4, R^n \text{ term}$$

$$\zeta_k =$$

Tree-level q graviton amplitude

$$A_q = K \frac{1}{S^{\mu\nu}} \frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} \frac{\zeta_{2k+1}}{2k+1} (S^{(1)} + t^{(1)} + u^{(1)})$$

$$s=0, s+t+u=0 \quad S=\frac{1}{2}(t+u)$$

$$K = \sum_i S_i^{(1)} S_i^{(2)} S_i^{(3)} S_i^{(4)} k_{\mu_1 \nu_1} k_{\mu_2 \nu_2} \dots$$

$$S(P) = \sum_{n=1}^{\infty} \frac{1}{n!} \eta_{\mu_1 \nu_1} \eta_{\mu_2 \nu_2} \dots$$

1. Strong form Supergravity

IIA/IIB string theory -

4-graviton vertex

$$R^4, D^* R^4, \dots, D^{*k} R^4, R^4_{\text{1-loop}}$$

$$\sigma_k = k \sum_{m,n} \frac{\langle P_m \eta_n \rangle}{P_m!} \left(\frac{\alpha'}{2}\right)^m \left(\frac{\alpha'}{2}\right)^n$$

$$\sigma_2 = S^2 + U^2 + M^2$$

$$\sigma_3 = S^3 + U^3 + M^3 = 3 STM$$

Tree-level 4 graviton amplitude

$$A_4 = K \frac{1}{STM} \frac{\Gamma(1-S)\Gamma(1-U)\Gamma(1-M)}{\Gamma(1+S)\Gamma(1+U)\Gamma(1+M)} = \frac{1}{STM} \exp \left[2 \sum_{k=1}^{\infty} \frac{S(2k+1)}{2k+1} (S^{2k+1} + U^{2k+1} + M^{2k+1}) \right]$$

$$S^1 = 1, S^2 + U^2 + M^2 = 0 \quad S = \frac{S^1}{2}$$

$$K = S^1 S^2 S^3 S^4 \dots k_{1234} k_{1235} \dots k_{12345}, \quad K_{ijkl} = \frac{1}{4} STM_{ijkl} \eta_{1234} + \dots$$

$$S(P) = \sum_{n=1}^{\infty} \frac{1}{n!} P^n$$

1. String from Supergravity

IIA/IB string theory -

q - graviton and

$$R^i, D^i R^i, \dots, D^i D^i, R^{i+jm}$$

$$\sigma_k = k \sum_{\text{gravitons}} \frac{\langle p_1 p_2 \dots \rangle}{\Gamma(k)} \left(\frac{s}{2}\right)^k \left(\frac{t}{2}\right)^k$$

$$\sigma_2 = S^2 + t^2 + M^2$$

$$\sigma_3 = S^3 + t^3 + M^3 - 3STM$$

Tree-level q graviton amplitude

$$A_q = K \frac{1}{STM} \frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} = \frac{1}{STM} \exp \left[2 \sum_{k=1}^{\infty} \frac{S(2k+1)}{2k+1} \left(S^{2k+1} + t^{2k+1} + u^{2k+1} \right) \right]$$

$$s+t+u=0 \quad s=t=u$$

$$K = \sum_{\nu} \sum_{\mu} \sum_{\rho} \sum_{\sigma} k_{\nu\mu\rho\sigma}, \quad K_{\nu\mu\rho\sigma} = \frac{1}{4} STM \eta_{\nu\rho} \eta_{\mu\sigma} + \dots$$

$$S(P) = \sum_{n=1}^{\infty} \frac{1}{n! P^n}$$

$$A_4 = K \left(\frac{3}{\delta_3} + A(\delta_1, \delta_3) \right), \quad A(\delta, \mu_0) = \sum_{P, I=0}^{\infty} T_{\mu_0} \delta^P \mu_0^I$$

$$A_1 = K \left(\frac{3}{\sigma_3} + A(\sigma_1, \sigma_3) \right), \quad A(\sigma_1, \sigma_3) = \sum_{P_1, P_3=0}^{\infty} T_{P_1 P_3} \sigma_1^{P_1} \sigma_3^{P_3}$$

$$A(\sigma_1, \sigma_3) = 2\zeta(3) + \zeta(5)\sigma_1 + \frac{2}{3}\zeta(3)^2\sigma_3 + \frac{1}{2}\zeta(7)\sigma_1^2 + \zeta(3)\zeta(5)\sigma_1\sigma_3 + \zeta(7)\sigma_3^3 + \dots$$

$$A_4 = K \left(\frac{3}{\sigma_3} + A(\sigma_1, \sigma_3) \right), \quad A(\sigma_1, \sigma_3) = \sum_{p_1, p_2=0}^{\infty} T_{p_1 p_2} \sigma_1^{p_1} \sigma_3^{p_2}$$

$$A(\sigma_1, \sigma_3) = 2\zeta(3) + \zeta(5)\sigma_1 + \frac{2}{3}\zeta(3)^2\sigma_3 + \frac{1}{2}\zeta(7)\sigma_1^2 + \zeta(5)\zeta(5)\sigma_1\sigma_3 + \zeta(9)\sigma_3^3 + \dots$$

Trunc. level (III A/0.41 cm)

$$S = \int d^3x \sqrt{-g} \tilde{e}^{14} \left(R + 2\zeta(3)R'' + \zeta(5)D^4R'' + \frac{2}{3}\zeta(5)^2 D^6R'' + \frac{1}{2}\zeta(9)\zeta(9)D^{10}R'' + \zeta(11)D^2R'' \right)$$

$$A_4 = \kappa \left(\frac{3}{\delta_3} + A(\delta_1, \delta_3) \right), \quad A(\delta_1, \delta_3) = \sum_{P, I, J=0}^{\infty} T_{P, I, J} \quad \delta_1^P \delta_3^J$$

$$A(\delta_1, \delta_3) = 2\zeta(3) + \zeta(5)\delta_1 + \frac{2}{3}\zeta(7)\delta_1^2 + \frac{1}{2}\zeta(9)\delta_1^3 + \zeta(11)\zeta(5)\delta_1\delta_3 + \zeta(13)\delta_3^2 + \dots$$

Trans. by M. D. Gould

$$S = \int d^3x \sqrt{-g} \tilde{e}^{10} (R + 2\zeta(3)R^4 + \zeta(5)D^4R^4 + \frac{2}{3}\zeta(7)^2 DR^6 + \frac{1}{2}\zeta(11)D^2R^8 + \zeta(13)D^{10}R^4 + \zeta(11)D^2R^4 + \dots)$$

$$\delta_1^3, \delta_3^2$$

\tilde{R}

$$A_4 = K \left(\frac{2}{\sigma_3} + A(\sigma_1, \sigma_3) \right), \quad A(\sigma_1, \sigma_3) = \sum_{p, q=0}^{\infty} T_{pq} \sigma_1^p \sigma_3^q$$

$$A(\sigma_1, \sigma_3) = 2\zeta(3) + \zeta(5)\sigma_1 + \frac{2}{3}\zeta(3)^2\sigma_3 + \frac{1}{2}\zeta(7)\sigma_1^2 + \zeta(3)\zeta(5)\sigma_1\sigma_3 + \zeta(9)\sigma_3^3 + \dots$$

Trans. Icarus III No 41, 1961

$$S = \int d^3x \sqrt{-g} \tilde{e}^{14} \left(R + \frac{1}{2}\zeta(3)R'' + \zeta(5)D^4R'' + \frac{2}{3}\zeta(5)D^6R'' + \frac{1}{2}\zeta(11)D^8R'' + \zeta(13)D^{10}R'' + \zeta(11)D^2R'' + \zeta(13)D^4R'' + \zeta(11)D^6R'' + \zeta(13)D^8R'' + \zeta(11)D^{10}R'' \right)$$



$$A_i = \kappa \left(\frac{3}{\sigma_3} + A(\sigma_1, \sigma_3) \right), \quad A(\sigma_1, \sigma_3) = \sum_{p, q=0}^{\infty} T_{pq} \sigma_1^p \sigma_3^q$$

$$A(\sigma_1, \sigma_3) = 2\zeta(3) + \zeta(5)\sigma_1 + \frac{2}{3}\zeta(3)^2\sigma_3 + \frac{1}{2}\zeta(7)\sigma_1^2 + \zeta(5)\zeta(5)\sigma_1\sigma_3 + \zeta(7)\sigma_3^3 + \dots$$

Trunc. level: $\Pi N b^4/16J^2$

$$S = \int d^3x \sqrt{-g} \tilde{e}^{ab} \left(R + \frac{1}{2}\zeta(3)R^a_a + \zeta(5)D^a D_a + \frac{2}{3}\zeta(5)^2 D^a R_a + \frac{1}{2}\zeta(7)D^a D^b R_{ab} + \zeta(5)\zeta(5)D^a D^b R_{ab} + \zeta(7)\zeta(7)D^a D^b R_{ab} \right. \\ \left. - \Pi b \cdot D^a D^b \right) + \zeta(1) D^a D^b R_{ab} + \dots$$

σ_1^3, σ_3^2



$$A_4 = K \left(\frac{3}{\sigma_3} + A(\sigma_1, \sigma_3) \right), \quad A(\sigma_1, \sigma_3) = \sum_{p, p' \geq 0} T_{p, p'} \sigma_1^p \sigma_3^{p'}$$

$$A(\sigma_1, \sigma_3) = 2\zeta(3) + \zeta(5)\sigma_1 + \frac{2}{3}\zeta(3)^2\sigma_3 + \frac{1}{2}\zeta(7)\sigma_1^2 + \zeta(3)\zeta(5)\sigma_1\sigma_3 + \zeta(7)\sigma_3^3 + \dots$$

Trunc. level (IIA/B, 4-loop)

$$S = \int d^4x \sqrt{-g} \underbrace{\dot{e}^{ab}}_{\text{-- IIb. Dimensionless -- partial + computation}} \left(R + \frac{2}{3}\zeta(3)R'' + \zeta(5)D''R'' + \frac{2}{3}\zeta(5)'DR'' + \frac{1}{2}\zeta(7)D'R' + \zeta(3)\zeta(5)D''R'' \right. \\ \left. + \zeta(7)D''R' \right) \dots$$

-- IIA -- partial -

$$A_4 = \underbrace{K \left(\frac{3}{\sigma_3} + A(\sigma_1, \sigma_3) \right)}, \quad A(\sigma, \rho_\sigma) = \sum_{p, q=0}^{\infty} T_{pq} \sigma^p \rho_\sigma^q$$

$$A(\sigma, \rho_\sigma) = 2 \zeta(3) + \zeta(5) \sigma_1 + \frac{2}{3} \zeta(3)^2 \sigma_3 + \frac{1}{2} \zeta(7) \sigma_1^2 + \zeta(3) \zeta(5) \sigma_1 \sigma_3 + \zeta(9) \sigma_1^3 + \dots$$

Trace-level (IIA/B/C/D)

$$S = \int d^6 \sqrt{-g} \underbrace{R}_{-IIA} + \underbrace{\frac{1}{2} \zeta(3) R^2}_{-IIB} + \zeta(5) D^a R^b + \frac{2}{3} \zeta(3)^2 D^a R^b + \frac{1}{2} \zeta(7) D^a D^b R^c + \zeta(9) S^{ab} D^c R^d + \zeta(11) D^a D^b D^c R^d + \zeta(13) D^a D^b D^c D^d R^e + \zeta(15) D^a D^b D^c D^d D^e R^f + \zeta(17) D^a D^b D^c D^d D^e D^f R^g + \zeta(19) D^a D^b D^c D^d D^e D^f D^g R^h + \zeta(21) D^a D^b D^c D^d D^e D^f D^g D^h R^i + \zeta(23) D^a D^b D^c D^d D^e D^f D^g D^h D^i R^j + \zeta(25) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j R^k + \zeta(27) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k R^l + \zeta(29) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l R^m + \zeta(31) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m R^n + \zeta(33) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n R^o + \zeta(35) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o R^p + \zeta(37) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p R^q + \zeta(39) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q R^r + \zeta(41) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r R^s + \zeta(43) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s R^t + \zeta(45) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t R^u + \zeta(47) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u R^v + \zeta(49) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v R^w + \zeta(51) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w R^x + \zeta(53) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x R^y + \zeta(55) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y R^z + \zeta(57) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z R^w + \zeta(59) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w R^v + \zeta(61) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v R^u + \zeta(63) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u R^t + \zeta(65) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t R^s + \zeta(67) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s R^r + \zeta(69) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r R^q + \zeta(71) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q R^p + \zeta(73) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p R^o + \zeta(75) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o R^n + \zeta(77) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n R^m + \zeta(79) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m R^l + \zeta(81) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l R^k + \zeta(83) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k R^j + \zeta(85) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j R^i + \zeta(87) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i R^h + \zeta(89) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h R^g + \zeta(91) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g R^f + \zeta(93) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f R^e + \zeta(95) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e R^d + \zeta(97) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e D^d R^c + \zeta(99) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e D^d D^c R^b + \zeta(101) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e D^d D^c D^b R^a + \zeta(103) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e D^d D^c D^b D^a R^0 + \zeta(105) D^a D^b D^c D^d D^e D^f D^g D^h D^i D^j D^k D^l D^m D^n D^o D^p D^q D^r D^s D^t D^u D^v D^w D^x D^y D^z D^w D^v D^u D^t D^s D^r D^q D^p D^o D^n D^m D^l D^k D^j D^i D^h D^g D^f D^e D^d D^c D^b D^a D^0 R^0$$

- **II B**: Dimensionless \rightarrow Partial + $\frac{\text{Compton}}{\text{Dimension}}$.

- **II A**: Partial -

III B $T_{mn} T_{(mn)}(\Omega, \Omega)$

$\Omega = X + i e^\phi, \Omega_1 = g_1 \cdot e^\phi$

$$A_i = \underbrace{K \left(\frac{\alpha}{\sigma_i} + A(\sigma_i, \sigma_s) \right)}_{A(\sigma_i, \sigma_s)}, \quad A(\sigma_i, \sigma_s) = \sum_{r, f=0}^{\infty} T_{rf} \sigma_i^r \sigma_s^f$$

$$A(\sigma_i, \sigma_s) = 2\zeta(3) + \zeta(5)\sigma_i + \frac{2}{3}\zeta(5)^2\sigma_s + \frac{1}{2}\zeta(7)\sigma_i^2 + \zeta(5)\zeta(5)\sigma_i\sigma_s + \zeta(7)\sigma_s^3 + \dots$$

Tree level IIA/OSM action

$$S = \int d^6x \sqrt{-g} \tilde{e}^{1/2} \left(R + \frac{1}{2} \zeta(3) R^4 + \zeta(5) D^2 R^2 + \frac{2}{3} \zeta(5)^2 D^4 R^4 + \frac{1}{2} \zeta(7) D^6 R^4 + \zeta(5) \zeta(5) D^2 R^6 + \zeta(7) D^8 R^6 \right)$$

- **II B** Dimension \rightarrow perturb + $\frac{\text{comp}}{D-1}$... Ω^2, Ω^2
- **II A** perturb - **II B** $T_{(0,1)} \rightarrow T_{(p,q)}(\Omega, \bar{\Omega})$

$$\Omega = X + i \tilde{e}^{\frac{1}{2}}, \quad \Omega_2 = g_2 \cdot \tilde{e}^{\frac{1}{2}}$$

$$\Omega \rightarrow e^{\frac{i}{2}} \Omega,$$

$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$
$$a, b, c, d \in \mathbb{Z}$$

$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$

$a, b, c, d \in \mathbb{Z}$

$$g_{\mu\nu}^{\epsilon} \rightarrow g_{\mu\nu}^{\epsilon}$$

CAUTION

DO NOT USE THIS EQUIPMENT
IF YOU ARE DRIVING.
IT IS ILLEGAL TO DRIVE
WHILE DRUNK OR HIGH.

D=11 supergravity.

$$S = \frac{1}{2\kappa^2} \int d^Dx \left(R_{11} + |G_1|^2 + G_3 \wedge G_4 \wedge G_5 + \beta_{\text{matter}} \right).$$

D=11 superspace

$$S = \frac{1}{2k_1^2} \int d^d x \left[R_{(1)} + |G_1|^2 + G_3 \wedge G_4 \wedge G_5 + \beta_{\text{matter}} \right].$$

UV divergences $\Lambda = \frac{1}{l_p}$

D=11 supergravity

$$S = \frac{1}{2\kappa_1^2} \int d^Dx \left(R_{(11)} + |G_4|^2 + G_3 \wedge G_4 \wedge G_4 + \text{higher terms} \right).$$

UV dimension $\Lambda = \frac{1}{\lambda_P}$

$$S(R_{11}) \quad A_4^{(11)}(S, t, R_{11}) \quad (R_{11}^2 S)$$

$\boxed{S^2 = R_{11}^2}$

$\mathcal{D} = \{1, \dots, k\}$.

$$S = \frac{1}{2k^2} \int d^2x \left(R_{(1)} + |G_1|^2 + G_3 \wedge G_1 \wedge G_1 + \beta_{mn} \right).$$

UV divergenz.

$$\Lambda = \frac{1}{\lambda_p}$$

$$S(R_i)$$

$$A_{ii}^{(0)}(s, \lambda, R_i) \quad (R_i^2 S)$$

$$g_s^2 = R_{ii}^2$$

$$A_S = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3}$$

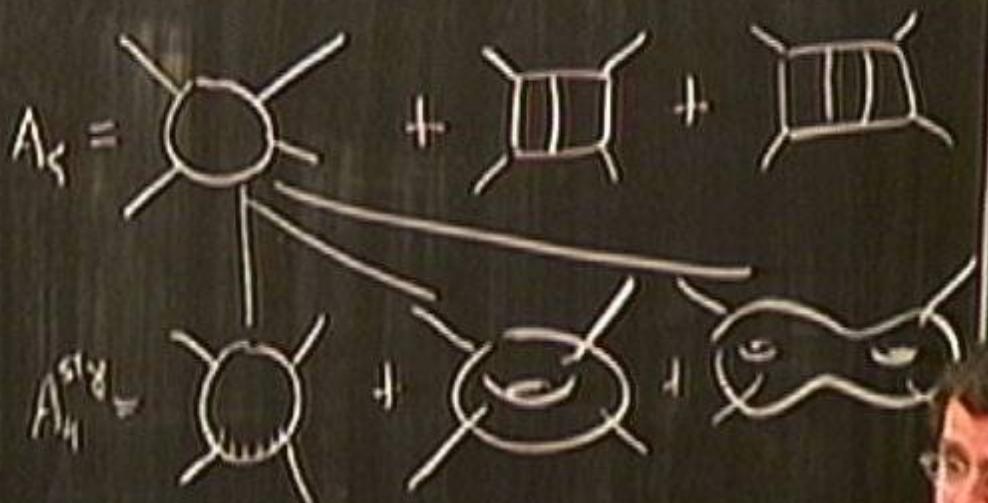
$$A_{ii}^{S^2 S} = \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6}$$

$\mathcal{D} = \{1, \dots, n_{\text{parameters}}, k\}$.

$$\mathcal{S} = \frac{1}{2k^2} \int d^2x \left(R_{(1)} + |G_1|^2 + G_3 \wedge G_4 \wedge G_5 + \beta_{mn} \right).$$

UV diverg. $\Lambda = \frac{1}{\lambda_P}$

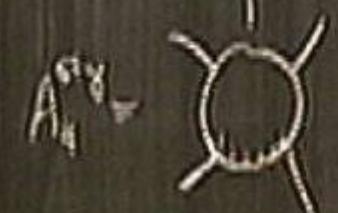
$$S(R_i) \quad A_{\mu}^{(1)}(s, \lambda, R_i) \quad (R_{11}^2 S)$$
$$g_s^2 = R_{11}^2$$
$$L=1$$



$$P = \left(1 - n_F \int_{-\infty}^{\infty} dk \right).$$

$$S = \frac{1}{2\sqrt{t}} \int d^2x \left[R_{(1)} + |G_1|^2 + G_2 G_3 + G_4 + \mu_{mn} \right].$$

UV decomposition. $\Lambda = \frac{1}{\lambda}$

$S(R_i)$ $A^{(1)}_{i_1}(s, t, R_i) \quad (R_i s)$ $L=1$ $\delta s = R_i L$	$A_1 =$  +  +  $A^{(2)}_1 =$  +  + 
-------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

$D = \{I\}$ Hyperplane .

$$S = \frac{1}{2k^2} \int d^2x \left(R_{\parallel\parallel} + |G_2|^2 + G_3 \wedge G_3 \wedge G_4 + \beta_{\text{matter}} \right).$$

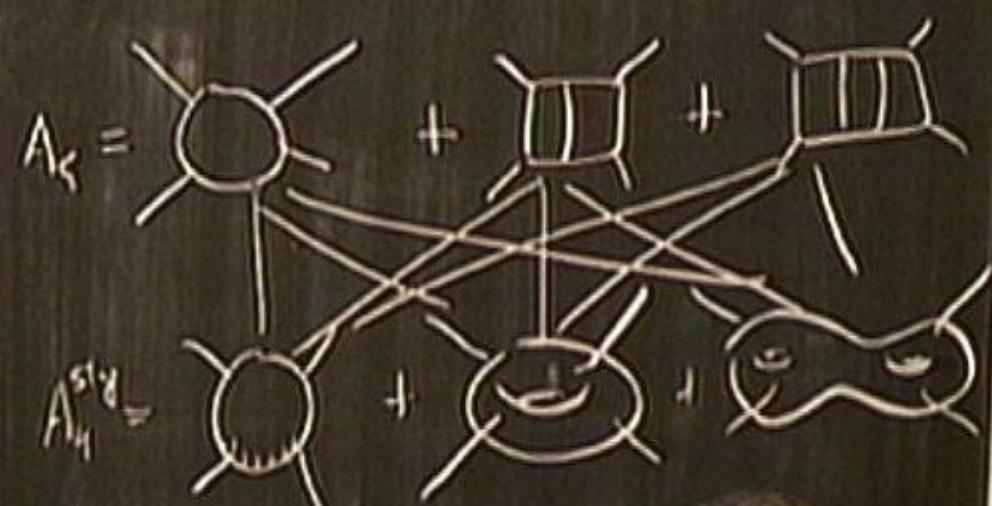
UV divergenz. $\Lambda = \frac{1}{L}$

$$S(R_i) \quad A_{\mu}^{(0)}(s, \lambda, R_i)$$

$$\delta s = R_{\parallel\parallel}^2$$

$$L=1$$

$$(R_{\parallel\parallel}^2 S)$$



$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$

$$g_{\mu\nu}^{\epsilon} \rightarrow g_{\mu\nu}^{\epsilon}$$



$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$

$a, b, c, d \in \mathbb{Z}$

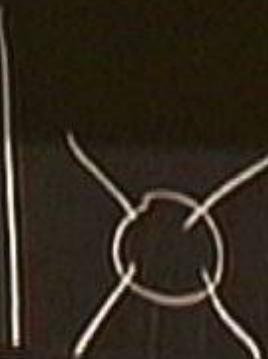
$$g_{\mu\nu}^{\epsilon} \rightarrow g_{\mu}^{\epsilon}$$

$$= \left(\frac{1}{g^2} S^{(3)} + S^4 \right) R^4 + g^2 S^{(4)} D^4 R^4 \\ + g^4 S^{(6)} D^6 R^4, \dots$$

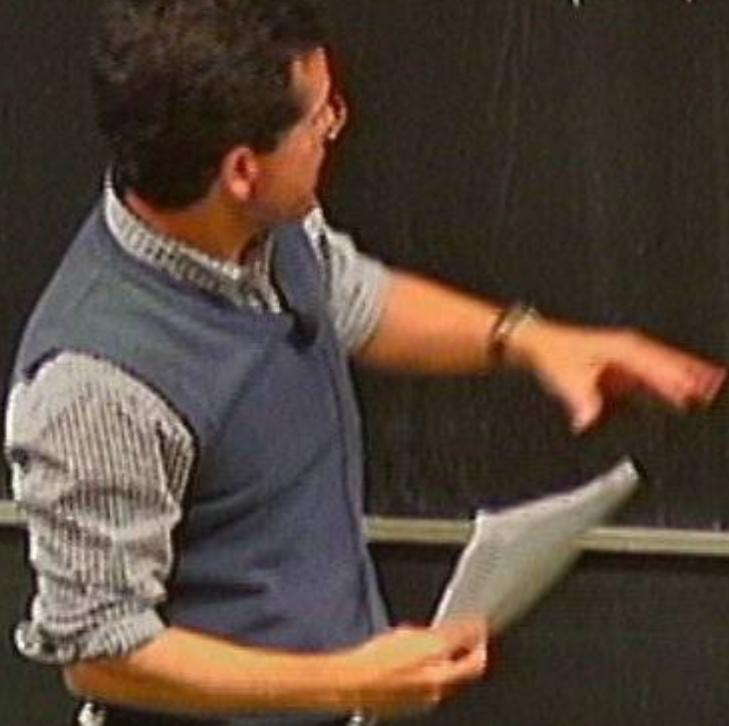
$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$

$a, b, c, d \in \mathbb{Z}$

$$g^{\epsilon}_{\mu\nu} \rightarrow g^{\epsilon}_{\mu\nu}$$



$$= \left(\frac{1}{g^2} S^{(3)} + \Lambda^3 \right) R^4 + g^2 S^{(4)} D^4 R^4 + g^4 S^{(6)} D^6 R^4$$



$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc = 1$$

$a, b, c, d \in \mathbb{Z}$

$$g_{\mu\nu}^{\epsilon} \rightarrow g_{\mu\nu}^{\epsilon}$$



$$= \left(\frac{1}{g^2} S^{(3)} + \frac{1}{S} \right) R^4 + \frac{g^2 S^{(4)}}{2} D^4 R^4 + g^4 S^{(6)} D^6 R^4, \dots$$

$$\frac{1}{pq} \left(\frac{\alpha}{2}\right) \left(\frac{\beta}{3}\right)$$

$$\mu^2$$

$$\mu^2 + \mu^3 = 3 \sin \mu$$

$$\text{ii) } \underbrace{(s^{2k+1} + t^{2k+1})}_{\equiv \zeta_k} - u^{(k+1)}$$

$$S(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

$$\Omega \rightarrow \frac{a\Omega + b}{c\Omega + d}, \quad ad - bc \sim 1 \\ a, b, c, d \in \mathbb{Z}$$

$$R^q \xrightarrow{g^q_{\nu\nu}} g^q_{\mu\nu}$$

CAUTION

I²

$$\text{Diagram} = \int d\mathbf{R} \, r_i \, F_g \left(\frac{\Delta}{F_b} R^i + Z_{Y_i} R^i + \sum_{k=1}^{\infty} \frac{1}{F_b^k} P_{k,i} D^k R^i \right)$$

I²

$$\text{Diagram} = \int d\Omega \Gamma_0 \Gamma_0 \left(\frac{\Delta}{\Gamma_0} R^i + \sum_{g=0, g=1} Z_g R^i + \sum_{k \in L} \frac{1}{\Gamma_0} R_{k,i} D^i R^i \right)$$

$$A_i(s, \Omega, v) \stackrel{R_i, R_o}{\sim}$$

Nernst.
Emulsion

$$\sum_{k \in L} = \delta_{iws} \quad k + \text{min. max.}$$

e⁻¹

$\underline{\underline{I^2}}$

$$\text{Diagram} = \int dR \, r_i \, F_g \left(\frac{A}{F_0} R' + Z_{\mu_1} R' + \sum_{n=1}^{\infty} \frac{1}{F_0} R_{k_n} D^n R' \right)$$

$$A_i(s, \Omega, v) \stackrel{P_i P_0}{\longrightarrow}$$

Nord. hell.
Emiss.

$$Z_{\mu_1} = g_{\mu_1 \mu_2} + g_{\mu_1 k} + m_{\mu_1 \mu_2} e^{-\frac{R'}{L}}$$



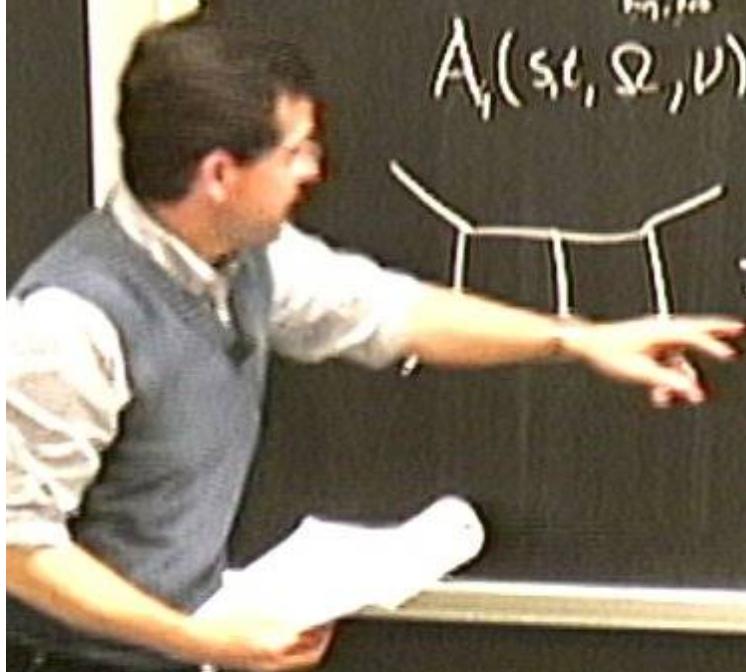
$\underline{\underline{T^2}}$

$$\text{Diagram} = \int d\Omega \Gamma_1 \Gamma_2 \left(\frac{\Delta}{R^2} R^4 + \boxed{2g_2 R^4} \right) \sum_{k=2} \frac{1}{R^k} Z_{k1} D^k R.$$

$$A_i(s_i, \Omega, V)$$

$$\text{Non-hol. Env. } Z_{k1} = 1 + g_{k1} k + \text{non-perturb. } e^{-k}$$

$$\begin{aligned}
 \underline{\underline{T}}^2 &= \sum_{\text{internal}} \frac{\partial r}{\text{internal}} \\
 \text{Diagram} &= \int dR \Gamma_1 \Gamma_2 \left(\frac{\Delta^2}{R^2} R^4 + \boxed{\frac{2\delta g_2 R^4}{g_{m0} g_{e1}}} \right) \sum_{k=2} \frac{1}{R^k} \Gamma_k D' R \\
 A_i(s_i, \Omega, V) &\quad \text{Non-hol. Env.} \quad \Gamma_{\frac{1}{2}} = g_{m0} k + g_{e1} L + \text{non-pot.} \\
 &\quad e^{-k}
 \end{aligned}$$



$$I^2 = \sum_{k=0}^{\infty} \frac{r_k}{|k+1|^2} R^k$$


$$= \int d\omega r_\omega \Gamma \cdot g \left(\frac{\Delta}{R_0} R^k + \boxed{\frac{Z(\omega) R^k}{g_{m0}, g_{e1}}} \right) \sum_{k=0}^{\infty} \frac{1}{R_0^k} Z_{k+1} D^k R^k.$$

$$A_i(s_i, \Omega, V)$$

$$\begin{matrix} P_1, P_2 \\ \text{Nor. hol.} \\ E_{m1, e1} \end{matrix}$$

$$Z_{k+1} = g_{m0} k + g_{e1} k + m_{m, e1} \dots$$

$$\boxed{(\Delta_0 - \mu(r, \omega)) Z_r = 0} e^{-\frac{1}{2} k}$$

$$\text{Diagram of a square loop with four internal lines.} = D^k R^k Z_{k+1} + \varepsilon_1 D^k R^k + \varepsilon_2 D^k R^k + F D^{k+1} R^k + G D^k R^k$$

$$(\Delta_0 - \lambda) Z_r = Z_r Z_r$$

$$\text{Diagram} = \int d\Omega \int \Gamma_R \left(\frac{\Delta}{R^4} + \left| \begin{matrix} Z^{(g)} R^4 \\ g_{\mu\nu}, g_{\alpha\beta} \end{matrix} \right| \right) \sum_{k=2} \frac{1}{l_k^4} \epsilon_{k+1}$$

$$A_i(s_i, \Omega, V)$$

Non-hol.
Em!ns

$$Z_{\mu_2} = \delta_{\mu_2} \omega_2 + g_{\mu_2 k} \epsilon_{k+1} e^{-\frac{i}{\hbar} \phi}$$

$$\text{Diagram} = D^i R^i Z_{\mu_1} + \epsilon_{\mu_1} D^i R^i + \tilde{\epsilon}_{\mu_1} D^i R^i + F^i D^i R^i + G^i D^i R^i$$

$$\Delta_\Omega = \Omega^2 \left(\frac{\partial^2}{\partial x^1} + \frac{\partial^2}{\partial x^2} \right) (\Delta_\Omega - \lambda) \epsilon_{\mu_1} = Z_{\mu_1} Z_1$$

$\underline{I^2}$

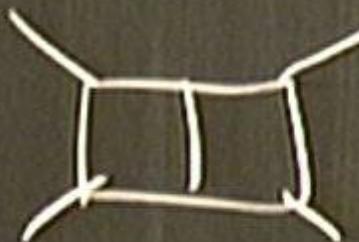
$$Z_r = \sum_{i=1}^{N_{\text{sites}}} \frac{\sigma_i}{|r_i + r_0|^{\alpha}}$$



$$A_r(s, \Omega, V) \stackrel{R_s, R_o}{=} \text{Non-hol. Env.}$$

$$= \int d\Omega \, r_0 \, \vec{g} \cdot \left(\frac{\partial}{\partial \vec{r}_0} \vec{R}^* + \boxed{\vec{Z}_{r=0}^* \vec{R}^*} \right) \sum_{k=L} \frac{1}{r_0^k} \sum_{k'} D^k R^{k'}$$

$$\vec{Z}_{r=0}^* = g_{ws} \, \vec{1} + g_{ws} \, \vec{k} + \text{non-perturb.} \\ \boxed{(A_r - n(r=0)) \vec{Z}_{r=0}} e^{-\frac{r}{r_0}}$$



$$= D^* R^* \vec{Z}_{r=0}^* + \epsilon D^* R^* + \tilde{\epsilon} D^* R^* + F D^* R^* \\ + G D^* R^*$$

$$\Delta_\Omega = \Omega \left(\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial t^2} \right) \quad (\Delta_\Omega - \lambda) \mathcal{E}_{rt} = Z_r Z_t$$

I^2

$$I^2 = \sum_{k=1}^{\infty} \frac{\omega_k}{4\pi k^2 R^4}$$

$$= \int dR \Gamma_0 \Gamma_R \left(\frac{8}{R^3} R^4 + \boxed{\sum_{j=0, j=1}^{j=0} R^4} \right) \sum_{k=1}^{\infty} \frac{1}{R^4} Z_{k+1} D' R^4.$$

$$A_r(s_r, \Omega, V)$$

P_m, P_n

Non-hol.
Env. ω

$$Z_{k+1} = g_{m+k} + g_{m-k} + \text{non-polar.}$$

$\boxed{(\Delta_r - \eta(r)) Z_r = 0}$

 $e^{-\frac{1}{2}}$

$$\text{Diagram} = D' R^4 Z_{k+1} + \varepsilon_1 D' R^4 + \sum_{m,k} D' R^4 + F D^{10} R^4 + G D^2 R^4$$

$$\Delta_r = \Omega_r \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \right) (\Delta_r - \lambda) \varepsilon_r = Z_r Z_1$$

$$\text{Diagram} = \int d\mathbf{x} \Gamma_0 \Gamma_0 \left(\frac{\Delta}{\Gamma_0^2} R'' + \boxed{Z_{\frac{1}{2}}^{(g)} R''}_{g=0, g=1} \right) \sum_{k=0} \frac{1}{\Gamma_0^k} Z_k D k$$

$$A_r(s_t, \Omega, v) \stackrel{R_1, R_0}{\sim}$$

Non-hol.
Exclusion

$$Z_{\frac{1}{2}} = g_{\text{mass}} \frac{1}{2} + g_m k + \text{non-perturb.}$$

$$\boxed{(\Delta_r - \eta(r-1)) Z_r = 0} \quad e^{-\frac{k}{m^2}}$$

$$\text{Diagram} = D' R'' Z_{\frac{1}{2}}^{(g)} + \mathcal{E}_r D' R'' + \sum_{\text{pert}} D' R'' + F D^{10} R'' + G D^2 R''$$

$$\Delta_r = \Omega_r^2 \left(\frac{\partial^2}{\partial r_1^2} + \frac{\partial^2}{\partial r_2^2} \right) \quad (\Delta_r - \lambda) \mathcal{E}_r = Z_r Z_{\frac{1}{2}}$$

$$S = S^{(1)} + S^{(2)} + S^{(3)} + S^{(4)} + \dots$$

$$Z_r = \sum_{\substack{(n, k) \\ (l, m)}} \frac{S_l}{|n + m \Omega|^2},$$

$$= \int d\omega r_\theta \Gamma_\theta \left(\frac{\Delta}{r_0} R'' + \boxed{Z_{\theta\theta}^{(0)} R''} \right) \sum_{k=0}^{\infty} \frac{1}{r_0^k} Z_{kk} D'' R''.$$

$$A_r(s, t, \Omega, V) \quad \begin{matrix} R_s, R_d \\ \text{Non-hol.} \\ \text{Emiss.} \end{matrix}$$

$$Z_{\theta\theta}^{(0)} = g_{\theta\theta} s_1 + g_{\theta\theta} k + \text{min. rel. or.} \\ \boxed{(\Delta_r - \eta(r)) Z_r = 0} \quad e^{-\frac{1}{r}}$$

$$= D'' R'' Z_{\theta\theta}^{(0)} + \sum_{k=0}^{\infty} D'' R'' Z_{kk} + \sum_{k=0}^{\infty} D'' R'' Z_{kk} + \sum_{k=0}^{\infty} D'' R'' Z_{kk} + G D'' R''$$

$$\Delta_\Omega \left(\frac{\partial^2}{\partial \Omega^2} + \frac{\partial^2}{\partial \bar{\Omega}^2} \right) (\Delta_\Omega - \lambda) \mathcal{E}_r = Z_r Z_1$$

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \dots \quad \delta S = 0$$

I

$$\text{Diagram} = \int d\Omega R_i F_g \left(\frac{\Delta}{R_0} R'' + \boxed{Z_{g1} R''} \right) \sum_{k=2}^{\infty} \frac{1}{R_0^k} Z_{k+1} D' R.$$

$$A_i(s_i, \Omega, v) \stackrel{R_i, P_i}{=} \text{Non-hol. Extens.}$$

$$Z_{g1} = g_{\mu\nu} \partial_\mu k + \text{non-pot.} \\ \boxed{(\Delta - \lambda(r)) Z_r = 0} \quad e^{-\frac{r}{R_0}}$$

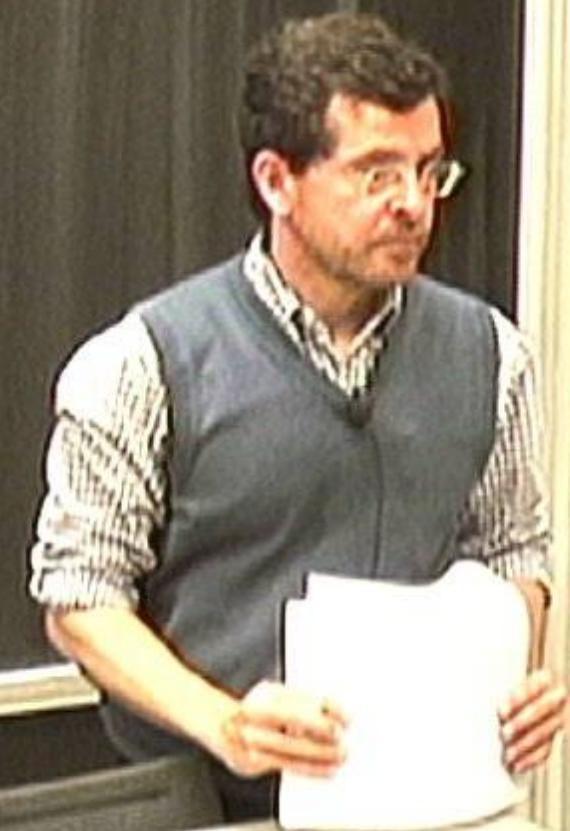
$$\text{Diagram} = D'R'' \underline{Z_{g1}} + \sum_{k=2}^{\infty} \frac{D' R''}{R_0^k} Z_k + \sum_{k=2}^{m^2} \frac{D' R''}{R_0^k} Z_k + F D'' R'' + G D' R''$$

$$\Delta_0 = \Omega_i \left(\frac{\partial}{\partial x^i} + \sum_j \right) (\Delta - \lambda) E_i = Z_r Z_i$$

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \dots \quad \partial S = 0$$

II.

$$\Delta = \frac{2}{\lambda_r}$$

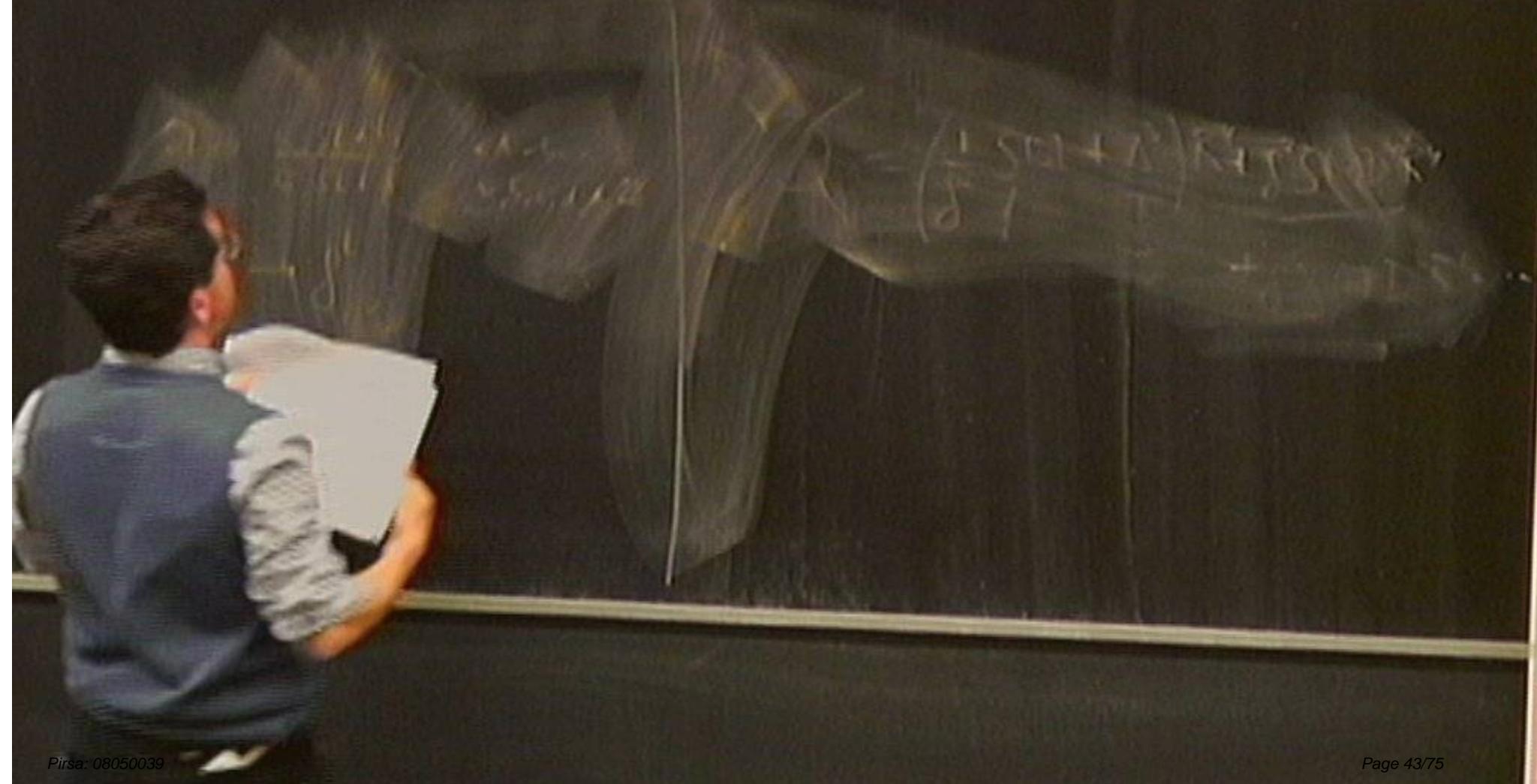


II. Supergravity from String Theory



$$\Delta L = \lambda + iC^*, \quad \Delta L_i = g_i \cdot C^*$$

$$A_q^L = S^h (1+\alpha s) R$$



$$A_q = S^k (1+\alpha s) R^k$$

$$\alpha_1=0$$

$$\alpha_2=2$$

$$\alpha_3=3$$

$$A_q = \left(\frac{1}{\delta} S^k + R \right) R^k + \beta_3 S^k R^k$$



$$A_q = S^q (1 + \alpha(s)) R$$

$$\alpha = 0$$

$$\beta_2 = 2$$

$$\beta_3 = 3$$

$$S_L = K_{\parallel}^{2(L+1)} \Delta R_n^m \int d\kappa'' \sqrt{\delta} D^{2\mu+2r} R^4 \quad r = 0, 1, \dots$$

Type IIA

$$S = \Delta \oint_A \int^{2(h - \frac{L}{2} + \frac{m}{3} + 2)} \sqrt{-g} D^3 R \Big)_A$$

$$A_q = S^h (1 + \alpha s) R$$

$$\alpha = 0$$

$$\beta_2 = 2$$

$$\beta_3 = 3$$

$$S_L = U_{11}^{2(L-1)} \Delta^m R^{-m} \int d\zeta'' \sqrt{-g} D^{2\mu_1 \gamma r} R^4 \quad r = 0, 1, \dots$$

Type IIA

$$S = \Delta^m \int_A d\zeta \quad 2(L - \frac{m}{2} + \frac{1}{3} + 2) = 2k - 2, \quad h = g^{mn} -$$

$$0 \leq n \leq 9L - 6 - 2k$$

$$\int d\zeta (\sqrt{-g} D^a R)_A$$

$$\Rightarrow \boxed{h \leq k}$$

$$A_q^L = S^h (1 + \alpha g) R$$

$$\alpha_1 = 0$$

$$\alpha_2 = 2$$

$$\alpha_3 = 3$$

$$S_L = \mu_{\parallel}^{2(L+1)} \Delta R_{\parallel}^{-m} \int d\mathbf{x}^n \sqrt{g} D^{2h+2r} R^n \quad r = 0, 1, \dots$$

Type II A

$$S = \Delta \oint_A$$

$$0 \leq n \leq 2L - h - 2r$$

$$2(L-h) + \frac{n}{3} + 2 = 2h - 2, \quad h = j \text{ mod } 3$$

$$\int d\mathbf{x}^n (\sqrt{g} D^h R)$$

$$\Rightarrow \boxed{h \leq k} \quad \text{because } L=1$$

II. Supergravity from String Theory

$$\frac{g_{\text{grav}}}{\text{String}} = A_4^k = K_{\text{dil}}$$

II. Supergravity from String Theory

$$\frac{g_{\text{string}}}{\text{String}} \quad A_4^h = \frac{\alpha'^{2(h-1)}}{k_{(h)}} \sqrt{\alpha'^{3-4h+1}} \quad S^{F_h} \quad (1 + O(s)) \quad R^h$$



II. Supersymmetry from String Theory

$$\frac{\text{gauge}}{\text{String}} \quad A_4^h = \frac{2(h-1)}{K_{10}} \alpha'^{3-4h+12} S^{B_4} (1 + O(s)) R^4$$

$$\alpha' \rightarrow 0$$

$$(K_{10}) = \text{fixed}$$

$$\Delta$$

II. Supersymmetry from String Theory

$$\frac{\text{gauge}}{\text{String}} \quad A_4^h = K_{00}^{2(h-1)} e^{3-4h+1/2} S^{\beta_h} (1 + O(s)) R^4$$

$$\alpha' \rightarrow 0 \\ (K_{10}) = \text{fixed}$$

$$\Delta = \frac{1}{\sqrt{\alpha'}}$$

II. Supergravity from String Theory

$$\frac{g_{\text{string}} h}{\text{String}} \quad A_4 = K_{10}^{2(h-1)} \wedge^{\alpha^{3-4h+P_2}} S^{P_1} (1 + O(s)) R^4$$

$$\alpha' \rightarrow 0 \quad (K_{10}) = \text{fixed} \quad \Delta = \frac{\Lambda}{\sqrt{\kappa}}$$

$$= K_6^{2(h-1)} \wedge^{8h-6-2P} S^P (1 + O(\alpha')) R^4$$

$$\frac{s(j)\omega}{\omega} A_4 = K_{(10)} \wedge \omega^{\lceil \frac{d}{2} \rceil - 1} S^p \left(1 + O(s) \right) \kappa$$

$$K_{(10)} = \text{fixed} \quad \Delta = \frac{\Lambda}{\sqrt{\kappa'}}$$

$$= K_6^{2^{(4n+1)}} \wedge^{8h-6-2p} S^p \left(1 + O(\omega^4 s) \right) R^h$$

$$A_{4,0}^h = K_1 \wedge^{\lceil \frac{d}{2} \rceil - h} \Delta^{(d-2)h}$$

$\underline{\text{S}(\gamma^{\text{new}})}$

$$A_1 =$$

$K_{(10)}$

\wedge

$S^P \left(1 + O(s) \right) K$

$$(K_{10}) = \frac{1}{2} \omega d \quad \stackrel{\alpha \rightarrow 0}{\sim}$$

$$\Delta = \frac{1}{\sqrt{s}}$$

$$= K_6^{2(h+1)} \wedge^{8h-6-2p} S^P \left(1 + O(s) \right) R^4$$

$$A_{4,q}^h = K_d^{2(h+1)} \wedge^{(d-2)h-6-2\beta_h} S^P \left(1 + O(s) \right) R^4$$

say

$$A_q = K_{v_0}$$

$$S^P \left(1 + O(s) \right) K$$

$$(K_{10}) = \text{fixed} \quad \overset{\alpha \rightarrow 0}{\Delta} =$$

$$\Delta = \frac{1}{\sqrt{s'}}$$

$$= K_b^{2(h-1)} \Delta^{8h-6-2P} S^P \left(1 + O(s) \right) R^4$$

$$A_{q,0}^h = K_d^{2(h-1)} \Delta^{(d-2)h-6-2\beta_h} S^P \left(1 + O(s) \right) R^h$$

$$\text{Sylow } A_q = K_{(n)} \wedge S^{r^n} ((1 + O(s))) \kappa$$

$$(\mu_{10}) = 1 \times d \quad \Delta = \frac{d}{\sqrt{n}}$$

$$= K_0^{\gamma(n+1)} \wedge^{8h - 6 - 2\beta_0} S^p ((1 + O(s))) R^h.$$

$$A_{q,q}^h = K_d^{\gamma(n)} \wedge^{(d,2)h - 6 - 2\beta_0} S^p ((1 + O(s))) R^h$$

gas! $\beta_1 = 0$
gas? $\beta_1 = 2$

$$A_q^L = S^h (1 + \alpha_s) R$$

$$\alpha_1 = 0$$

$$\beta_2 = 2$$

$$\rho_3 = 3$$

$$S_L = \int d\kappa'' \sqrt{-g} D^{2h+2r} R^4 \quad r = 0, 1, \dots$$

Type IIA

$$S = \int d^{\infty} x \left(\sqrt{-g} D^{\alpha} R' \right)_A$$

$$0 \leq n \leq 7L - 6 - 2h$$

$$2(L-n+\frac{m}{3}+2) = 2h-2 \quad h = j^{mn}$$

$$\Rightarrow \boxed{h \leq k} \quad h = k \text{ const. } L = 1$$

$$A_q^L = S^h (1 + \alpha(s)) R$$

$$\beta_1 = 0$$

$$\beta_2 = 2$$

$$\beta_3 = 3$$

$$S_L = \langle \epsilon_{\mu_1} \rangle^{2(L+1)} \Delta^m R_{\mu_1}^{-m} \int d\epsilon'' \sqrt{-g} D^{2h+2r} R^4 \quad r=0, 1, \dots$$

Type IIA

$$S = \Delta^m \int^3 \delta^h \quad 2(h - \frac{m}{2} + \frac{m}{3} + 2) = 2h - 2, \quad h = jmn - \\ 0 \leq n \leq jL - 6 - 2jL$$

$$\int d^10 \sqrt{-g} D^h R^4$$

$$\Rightarrow h \leq k$$

$h = k$ (cons)

$$\text{say } A_1 = K_{10} \wedge S^p (1 + O(s)) \wedge$$

$$\alpha^{1 \rightarrow 0} \\ (K_{10}) = \int d\omega d\theta \quad \Delta = \frac{\Lambda}{\sqrt{s}}$$

$$= K_6^{2(h+1)} \wedge^{8h-6+2p} S^p (1 + O(s)) R^h.$$

$$A_{4,q} = K_d^{2(h+1)} \wedge^{(d-2)h-6-2p_d} S^p (1 + O(s)) R^h$$

$\begin{cases} \text{case 1} & p_1 = 0 \\ \text{case 2} & p_1 = 2 \\ \text{case 3} & p_1 = h \end{cases}$

$$\text{sky}^{(h)} \quad A_1 = K_{\alpha_1} \wedge S^{\alpha_1} (1 + O(s)) R$$

$$(K_{10}) = \int d\Omega d\theta \quad \Delta = \frac{1}{\sqrt{3}}$$

$$= K_6^{p(h,1)} \wedge^{8h-6-2p} S^p (1 + O(s)) R^h$$

$$A_{n,h}^h = K_d^{p(h,1)} \wedge^{(d,2)h - 6 - 2p_h} S^{p_h} (1 + O(s)) R^h$$

gus1 $\beta_1 = 0$

gus2 $\beta_1 = 2$

$\beta_h = h$

$$\text{Sugiyama} \quad A_1 = K_{\infty} \wedge S^r ((1+O(s)) R)$$

$$(K_{10}) = \int d\omega d\theta \quad \Delta = \frac{\Delta}{\sqrt{s}}$$

$$= K_0^{p(4-h)} \wedge^{8h-6-2p} S^p ((1+O(s)) R^h)$$

$$A_{n,q}^h = K_d^{q(h)} \wedge^{(d,2)h-6-2\beta_n} S^{p_n} ((1+O(s)) R^h)$$

$$d < 2 + \frac{2\beta_n + 6}{h} \rightarrow \boxed{d < h + \frac{6}{h}}$$

case 1 $\beta_1 = 0$
 case 2 $\beta_1 = 2$
 $\beta_n = h$

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4			
5			
6			
7			
8			
9			
10			

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4			
5			
6			
7			
8	✓	✓	✓
9	✓	✓	✓
10	✓	✓	✓

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4			
5			
6			
7			
8	A U U	U	U
9		U	U
10		U	U

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4			
5			
6			
7			
8	1	1	1
9	1	1	1
10	1	1	1

$P_{RN} \propto R^{1.5}$
(Unknown)

$$\begin{cases} h \geq 2 \\ h \geq 3 \end{cases}$$

$$2h+2r \quad R^2$$

$$(2h+2r) = 2h-2, \quad h =$$

$$1 \times (\sqrt{g} D)^2 \Rightarrow Th \leq$$

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

PINN MR. 05
(backwards)

$$\begin{cases} h \geq 2 \\ h \geq 3 \end{cases}$$

$$2h+2r \leq R^2$$

$$2(h-2) + 2r = 2h - 4 + 2r = 2h + 2r - 4$$

$$1 \times (\sqrt{-g} D^2) \Rightarrow \boxed{\sqrt{h} \leq \dots}$$

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4	6		
5			
6			
7			
8	A	U	
9		I	
10		F	(A)

PROOF METHOD (backward)

$$\begin{cases} h \geq 2 \\ h \geq 3 \end{cases}$$

$$2h+2r \quad R^2$$

$$2(h-1) + 2(r-1) = 2h-2, \quad h=$$

$$S^{10} \times (\sqrt{-g} D^2)$$

$$\Rightarrow \underline{Th} \leq$$

D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4	6		
5	4		
6	3		
7	2		
8	1		
9	1		
10	1		

PML MR 15
(Bakewell)

$$\begin{cases} h \geq 2 \\ h \geq 3 \end{cases}$$

$$2h+2r \quad R^4$$

$$(h-2) = 2h-2, \quad h=$$

$$1 \times (\sqrt{g} D)^2$$

$$\Rightarrow Th \leq$$

$$\begin{aligned}
 &= K_b \Lambda^{8h - 6 - 2\beta} S^{\beta} ((1 + o(1))) R^h \\
 A_h^h &= K_d \Lambda^{(d+2)h - 6 - 2\beta_d} S^{\beta_d} ((1 + o(1))) R^h \\
 &\quad \boxed{d < 2 + \frac{2\beta_d + 6}{h}} \rightarrow \boxed{d < h + \frac{6}{h}}
 \end{aligned}$$

guess 1 $\beta_1 = 0$
 guess 2 $\beta_2 = 2$
 $\beta_d = h$

D	Known	$\alpha = h$ up to $h=5$	$\beta = h$ for all h
4	6		
5	4		
6	3	3	3
7	2	2	2
8	1	1	1
9	1	1	1
10	1	1	1

→

PMI (Proof by
(Induction))

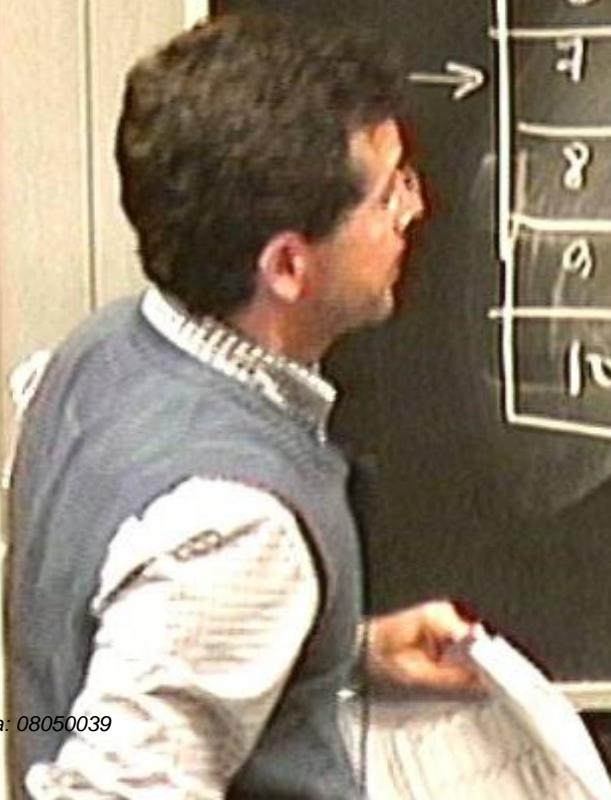
$$\begin{aligned} A_h &\geq 2 \\ A_h &\geq 3 \end{aligned}$$

2A12r

$$(h+1) + (h+2) = 2h+2 ,$$

$$1 \times (\sqrt{8})$$

$$\Rightarrow \frac{1}{\sqrt{h}}$$



D	Known	$\beta = h$ up to $h=5$	$\beta = h$ for all h
4	6	9	
5	4	6	6
6	3	3	3
7	2	2	2
8	1	1	1
9	1	1	1
10	1	1	1

$P_{\text{all}}(X \geq h)$
(succinct.)

$$\begin{aligned} A_h &\geq 2 \\ A_h &\geq 3 \end{aligned}$$

2A+2r

$$(2h-2) \rightarrow 2h-2$$

$$1 \times (\sqrt{-\delta})$$

$$\Rightarrow \sqrt{h}$$

$$A_{q,q}^h = K_d \Lambda^{(d-2)\beta_h - 6 - 2\beta_h} S^{p_h(1+O(\epsilon))} R^h$$

$d < 2 + \frac{2\beta_h + 6}{h}$

$\beta_h \geq 6$

guess 1	$p_1 = 0$
guess 2	$p_2 = 2$
$\beta_h = h$	

$$R_i = \sqrt{a^i} \\ a^i \rightarrow c \frac{K_d^2}{\rho_i \nu_d} d.$$

$$R^4 \quad r = 0, 1, 2$$

$$h = \text{gauss}$$

$$(\sqrt{-g} D^\alpha R)_{\alpha}$$

$$\boxed{\pi \leq k} \quad h = k \text{ const for } L = 1$$

$$R_i = \sqrt{a'} \quad K_j^2 = (a')^{\frac{(d-1)}{2}} e^{2j}$$

$$a' \rightarrow 0 \quad \underline{K_d^2} \quad \text{phys.}$$

$$2n+2r \quad R^4 \quad r = 0, 1, 2$$

$$h = j^{and-}$$

$$k \times (\sqrt{g} D^\alpha R')$$

$$\underline{h = k \text{ const. from } L=1}$$

$$\boxed{h \leq k}$$

$$Z_{d+1}^{(h+1)}$$

$$\Delta$$

$$(d-2)h = 6 - 2\beta_h$$

$$S^{\beta_h} \left(1 + O(s) \right) K$$

gen 2

$$d < 2 + 2\beta_h + 6$$

$$\frac{h}{h}$$

$$\beta \geq 6$$

$$d < A + \frac{6}{h}$$

SAR MIM
N=9 SYM