

Title: Numerical study of black hole spacetimes

Date: May 28, 2008 02:00 PM

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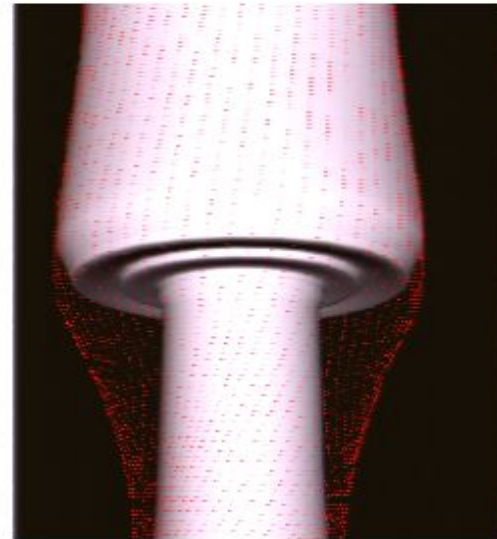
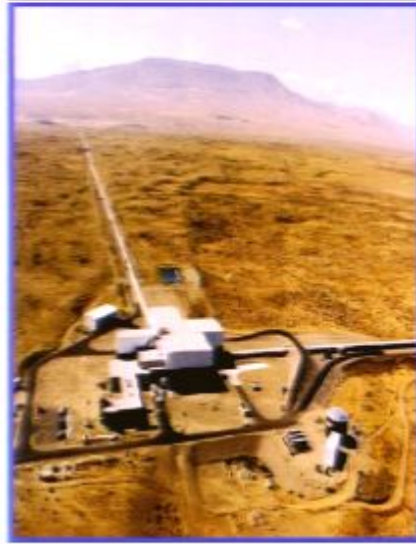
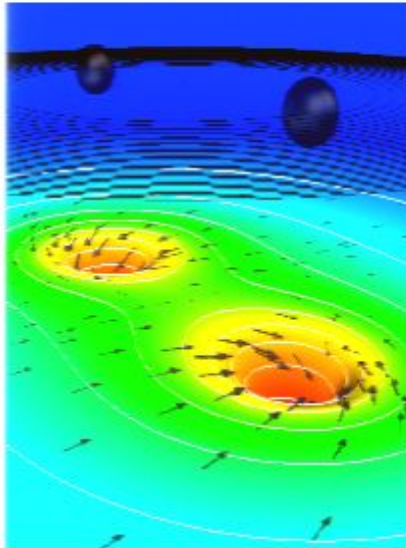
Abstract: The last years have seen tremendous progress in simulations of inspiral and coalescence of binary black holes. I will present recent results of the Caltech/Cornell collaboration simulating inspiral and collision of two black holes. Furthermore, while currently no talk on numerical relativity seems to be complete without a discussion of binary black hole coalescence, there are many more aspects of Einstein's equations that can be probed numerically. I will discuss some of these unexpected and intriguing features, among them black holes with five horizons and super-extremal black holes.

Numerical study of black hole spacetimes

Harald Pfeiffer

California Institute of Technology

Perimeter Institute, May 28, 2008

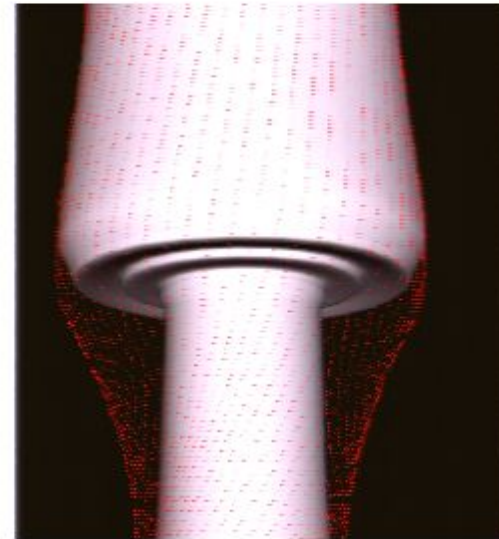
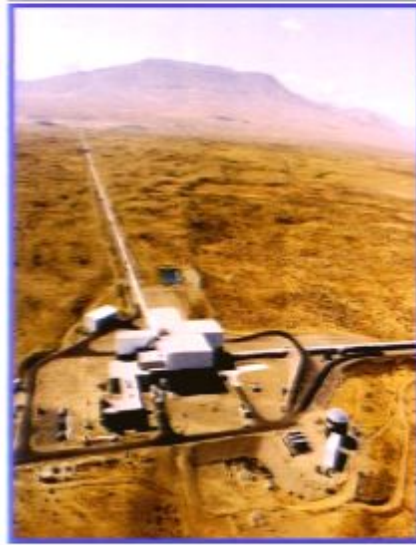
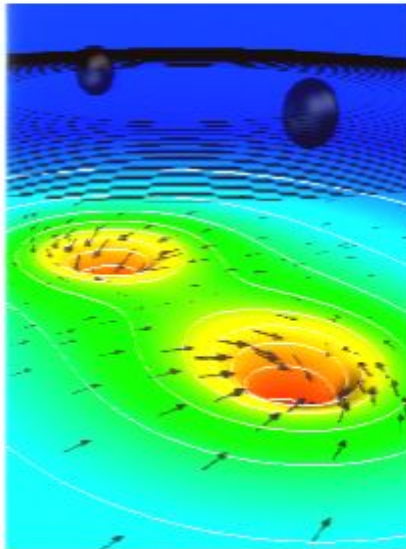


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Gravitational Waves

- Einstein's equations admit wave-solutions

$$g_{ab} = \eta_{ab} + h_{ab} \quad \square \bar{h}_{ab} = 0$$

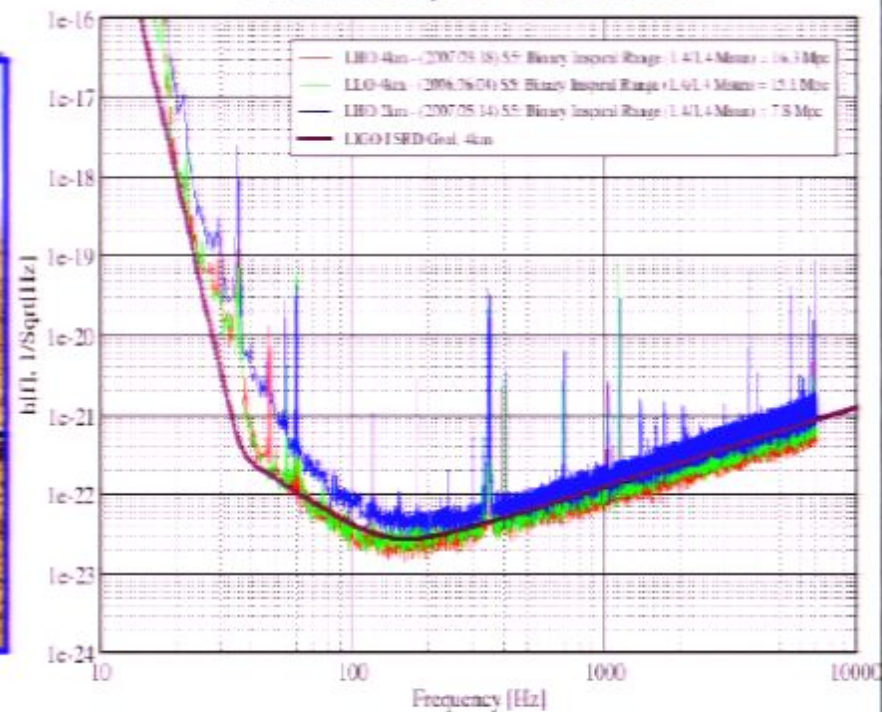
- Efforts are underway to detect these gravitational waves



LIGO Hanford

Strain Sensitivity of the LIGO Interferometers

S5 Performance - May 2007 LIGO-G070366-00-E

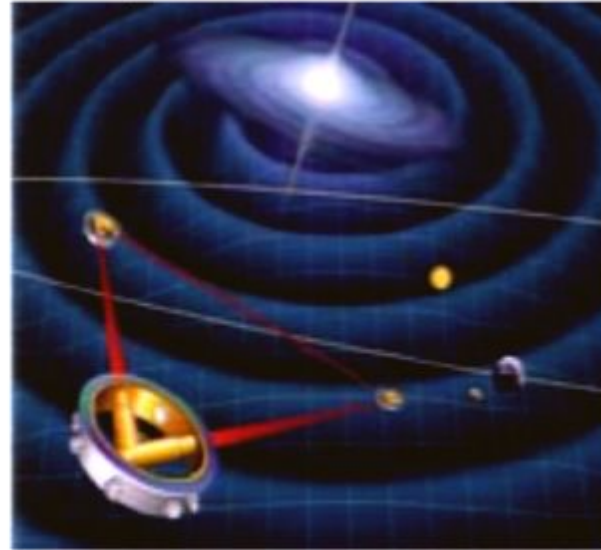


Gravitational wave detectors

LIGO (2 sites)



LISA (planned)



GEO 600



VIRGO



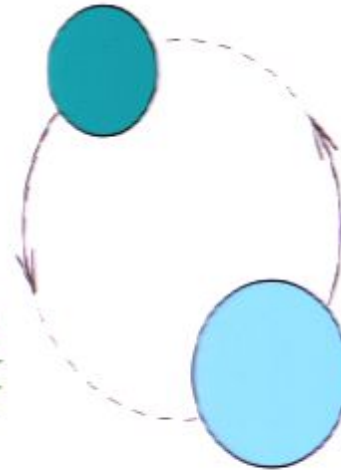
Gravitational Wave Sources

- Generated by changing quadrupole moments

$$h_{ij} = \frac{1}{r} \ddot{Q}_{ij}$$

- **Compact object binaries**

$$\Omega^2 r^3 = GM, \quad f_{\text{GW}} = 2 \frac{\Omega}{2\pi}$$



- Close to merger

$$r \sim 10GM/c^2 \Rightarrow f_{\text{GW}} \sim 2\text{kHz} \left(\frac{M}{M_{\odot}} \right)^{-1}$$

- $M = 1 \dots 100 M_{\odot} \Rightarrow$ LIGO
- $M = 10^4 \dots 10^7 M_{\odot} \Rightarrow$ LISA

Matched Filtering

- Tiny signal, $h = \frac{\Delta L}{L} \sim 10^{-21}$
- Detector output s , waveform template h_T

$$\text{SNR} = \frac{\langle s, h_T \rangle}{(\langle h_T, h_T \rangle)^{1/2}}$$

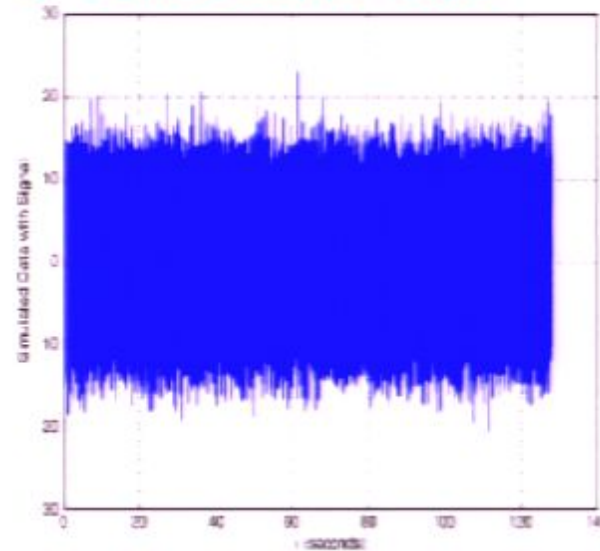
$$\langle a, b \rangle = \int df \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_h(f)}$$

- Phase of h_T crucial

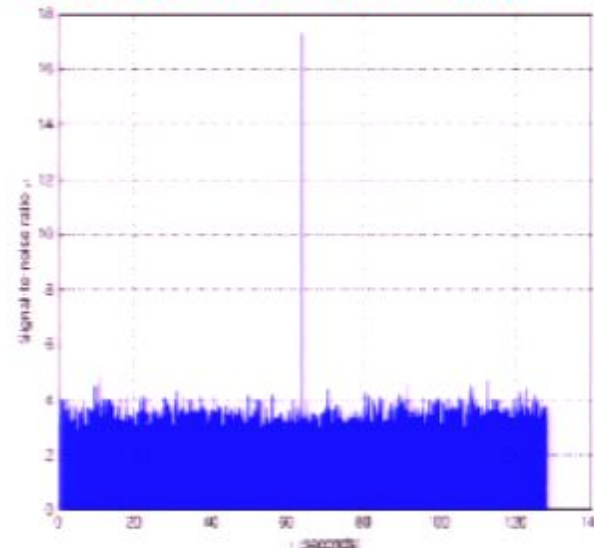
$$\delta\phi \lesssim 1/\text{SNR}$$

SNR = 8 ... 50 for advanced LIGO

Simulated detector output

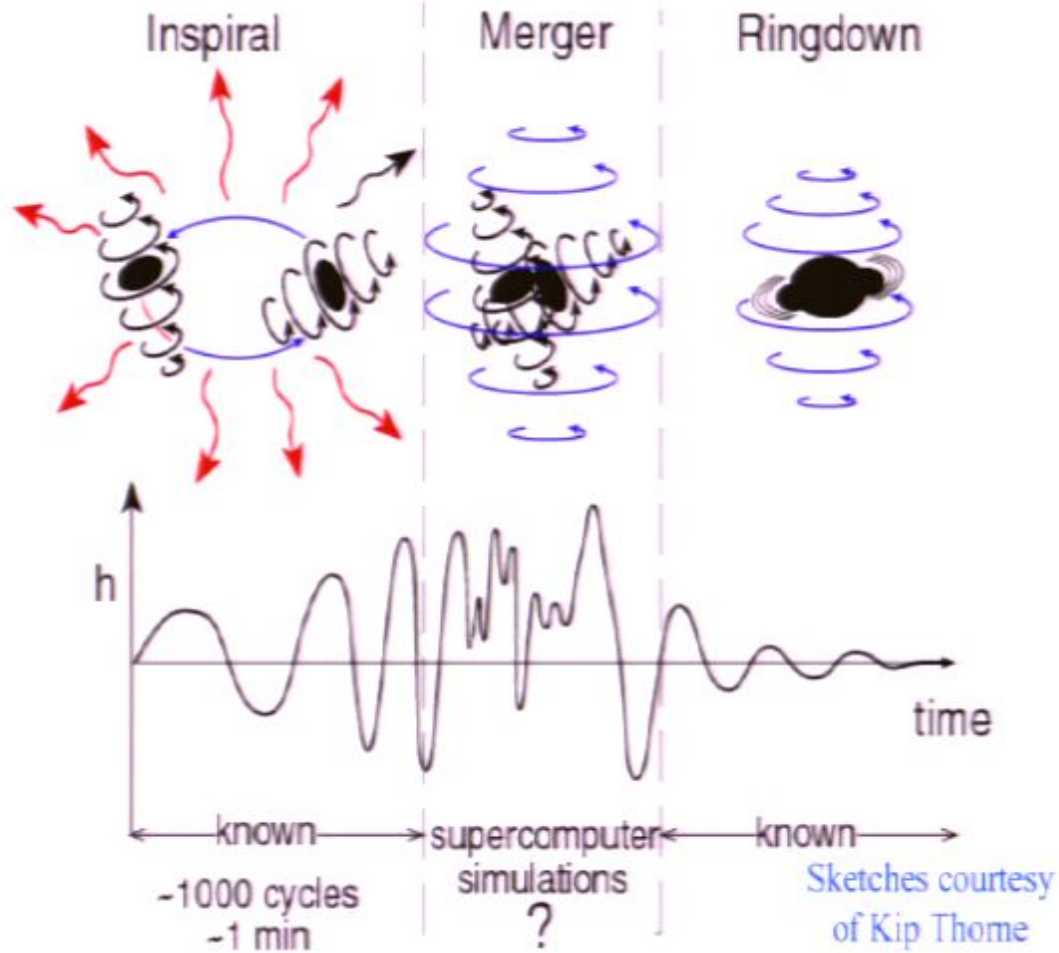


SNR vs. coalescence time



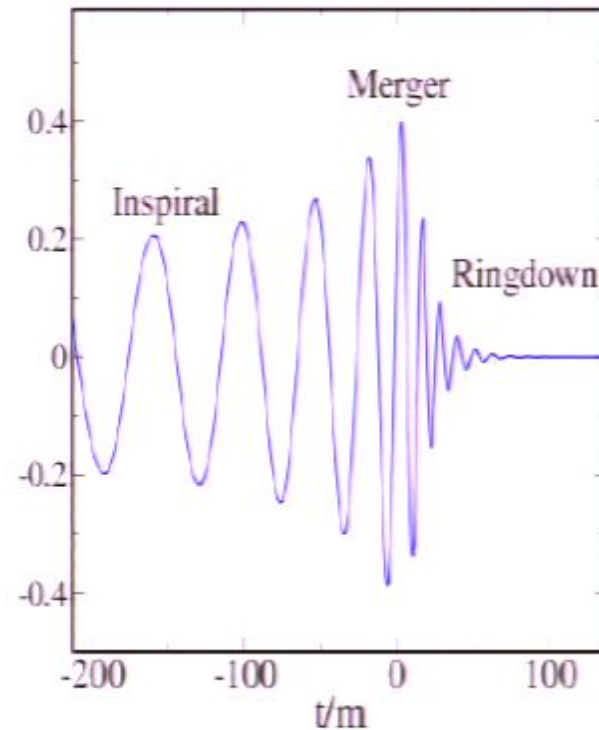
(D. Brown)

Stages of binary black hole evolution



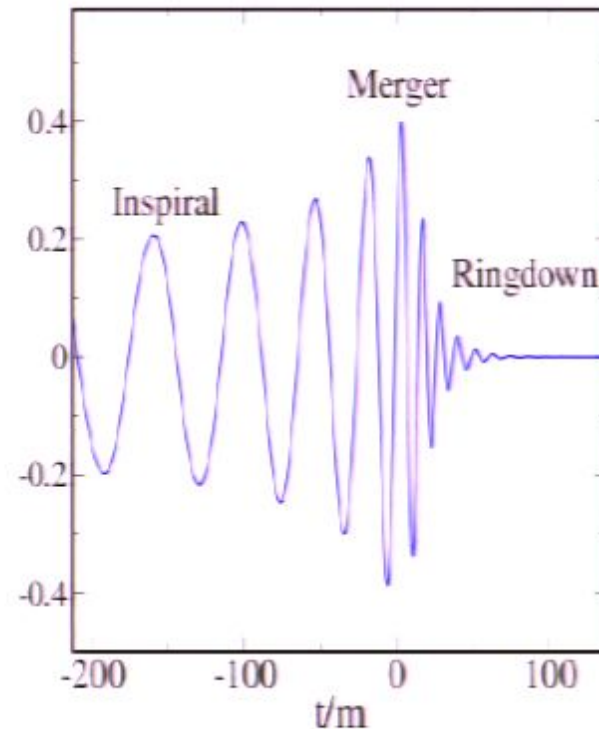
Tools for computing the waveform

- **Inspiral**
 - $v \ll c$: perturbative expansion in v/c (post-Newtonian expansion)
 - v/c large: Numerical relativity
- **Merger**
 - Numerical relativity
- **Ringdown**
 - BH perturbation theory
 - Numerical relativity



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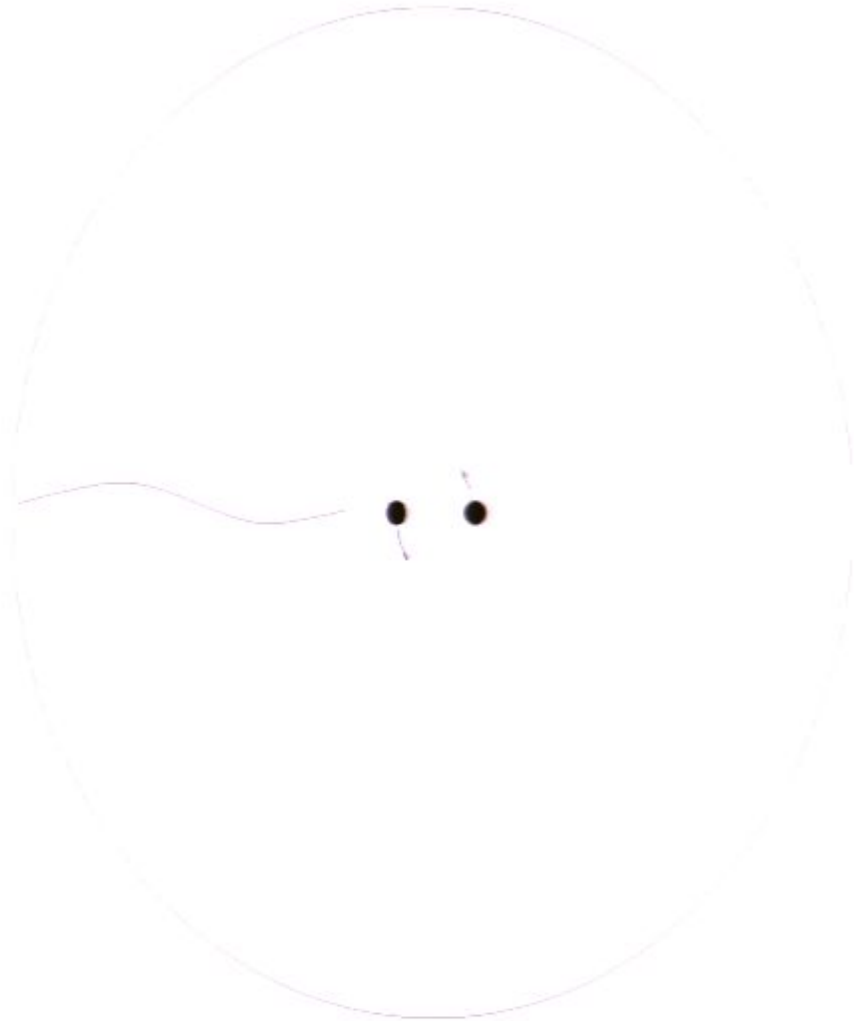
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- **Tasks for Numerical relativity:**
 - simulate “late” inspiral and merger
 - determine what “**late**” means



BBH Simulations – Overview

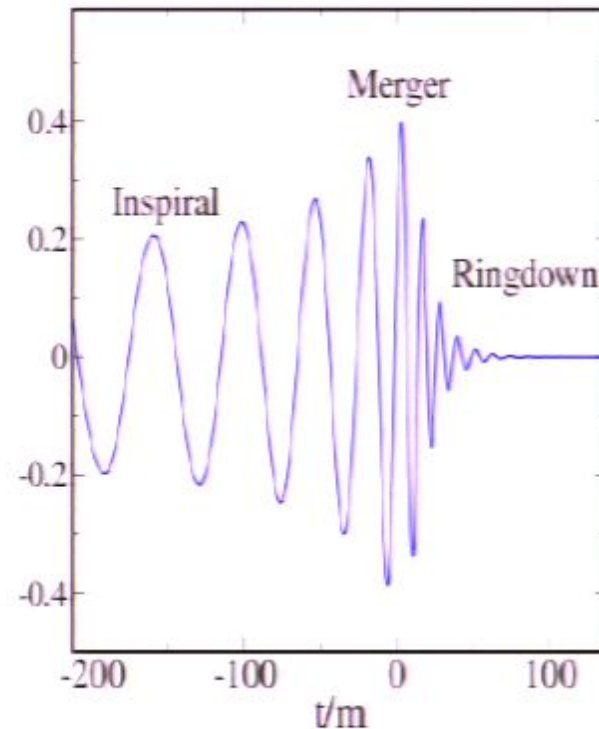
Problem characteristics

- **Multiple length scales**
 - ▶ Size of BH's ~ 1
 - ▶ Separation ~ 10
 - ▶ Wavelength $\lambda \sim 100$
 - ▶ Wave extraction at several λ
- **Gravitational wave flux small**
 - ▶ $\dot{E}/E \sim 10^{-5}$
 - ▶ \dot{E} drives inspiral
- **High accuracy required**
 - ▶ Absolute phase error $\delta\phi \ll 1$
- **Solutions are smooth**



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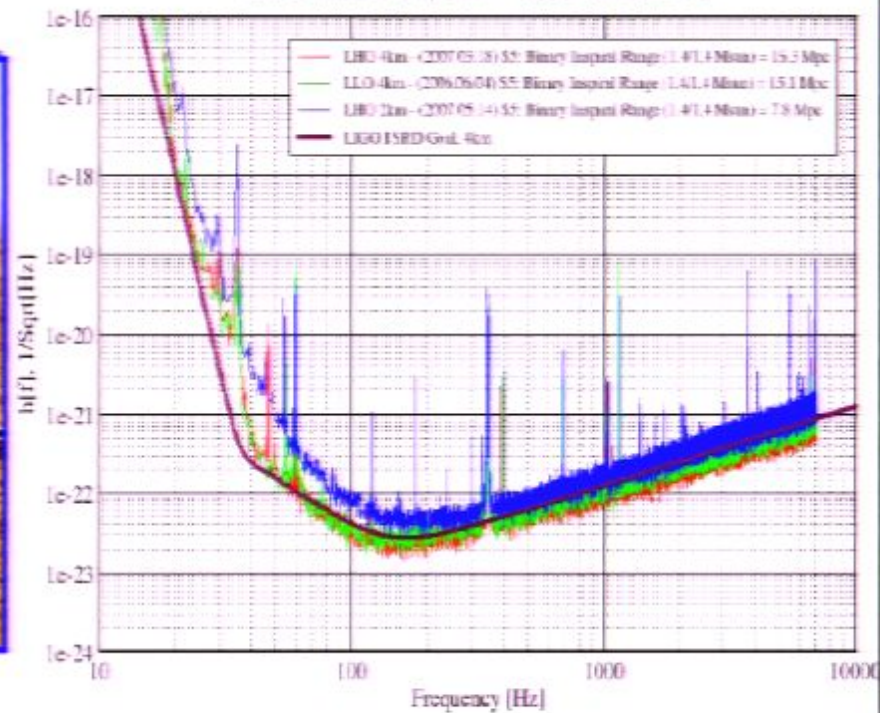
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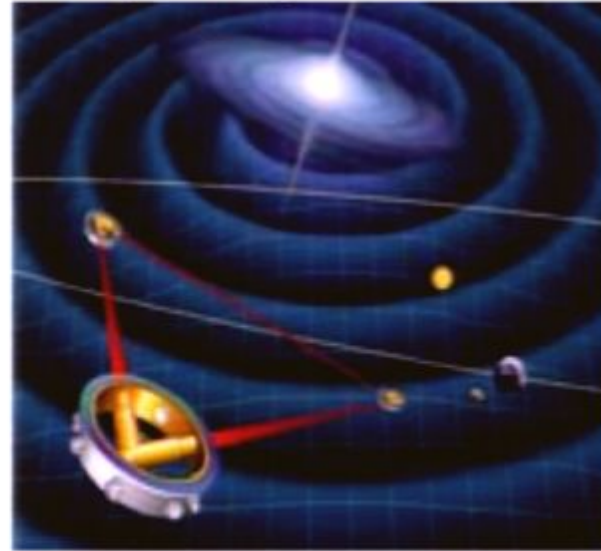


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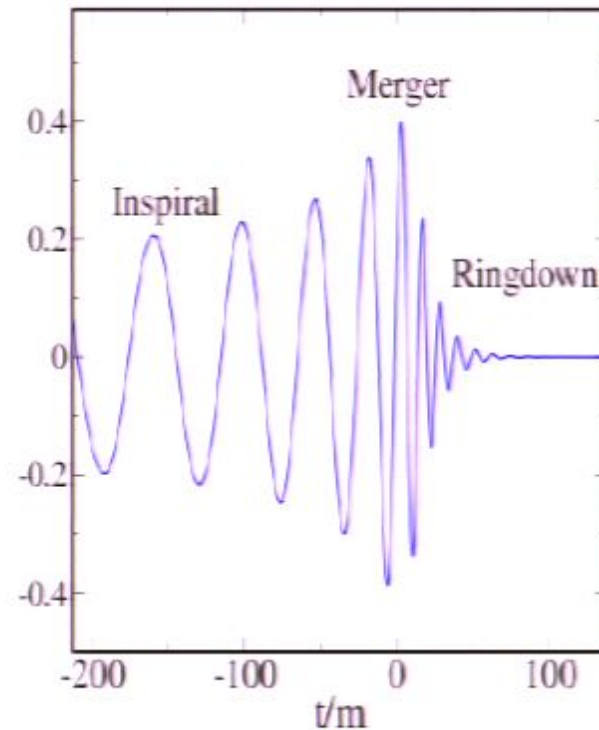


VIRGO



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Computational approaches

- **Finite difference AMR**
 - ▶ Albert-Einstein Institut (Berlin), Goddard, Jena (Germany), LSU, Penn State, Princeton, Rochester
 - ▶ Impressive short inspirals with mergers (**BH-kicks**)
 - ▶ Accurate long inspirals difficult
- **Multi-domain spectral methods**
 - ▶ Cornell/Caltech collaboration
 - ▶ Impressive long inspirals
 - ▶ Merger difficult, but possible

Computational Framework I

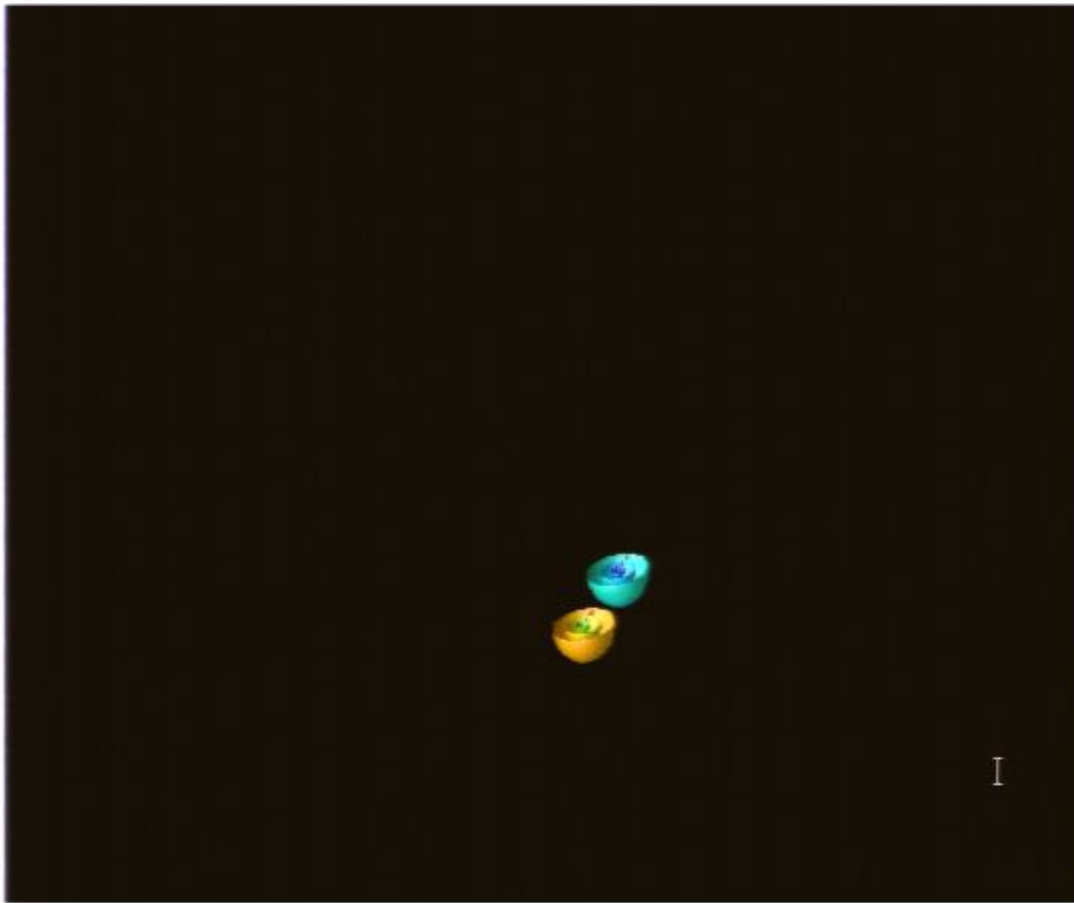
- Pseudo-spectral code

$$u(x, t) = \sum_{k=1}^N \tilde{u}_k(t) \Phi_k(x)$$

- Evaluate derivatives in spectral space, non-linear terms in physical space
- Elliptic problems
Solve large set of algebraic equations for \tilde{u}_k (HP et al. 2003)
- Hyperbolic problems
Evolve $\tilde{u}_k(t)$ with method of lines
- Principal code developers Larry Kidder, Mark Scheel, HP; 250,000 lines

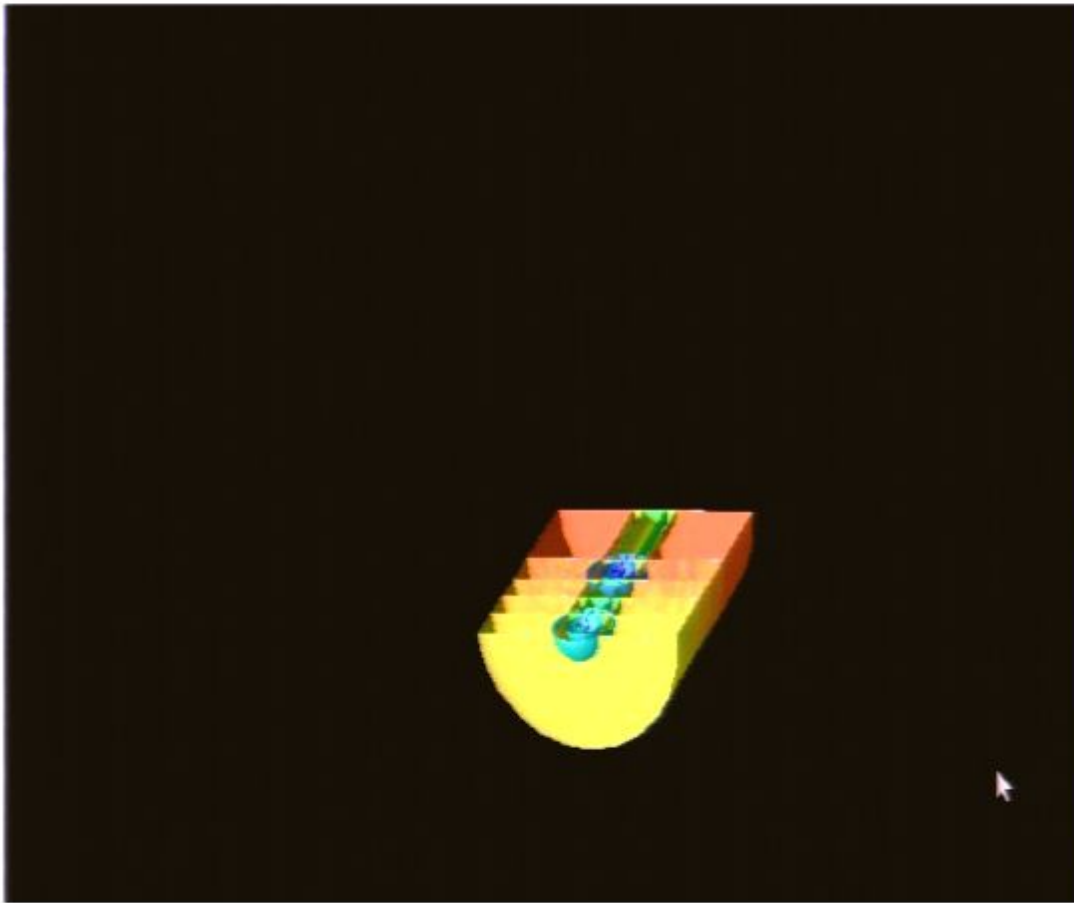
Computational Framework II

- Domain-decomposition



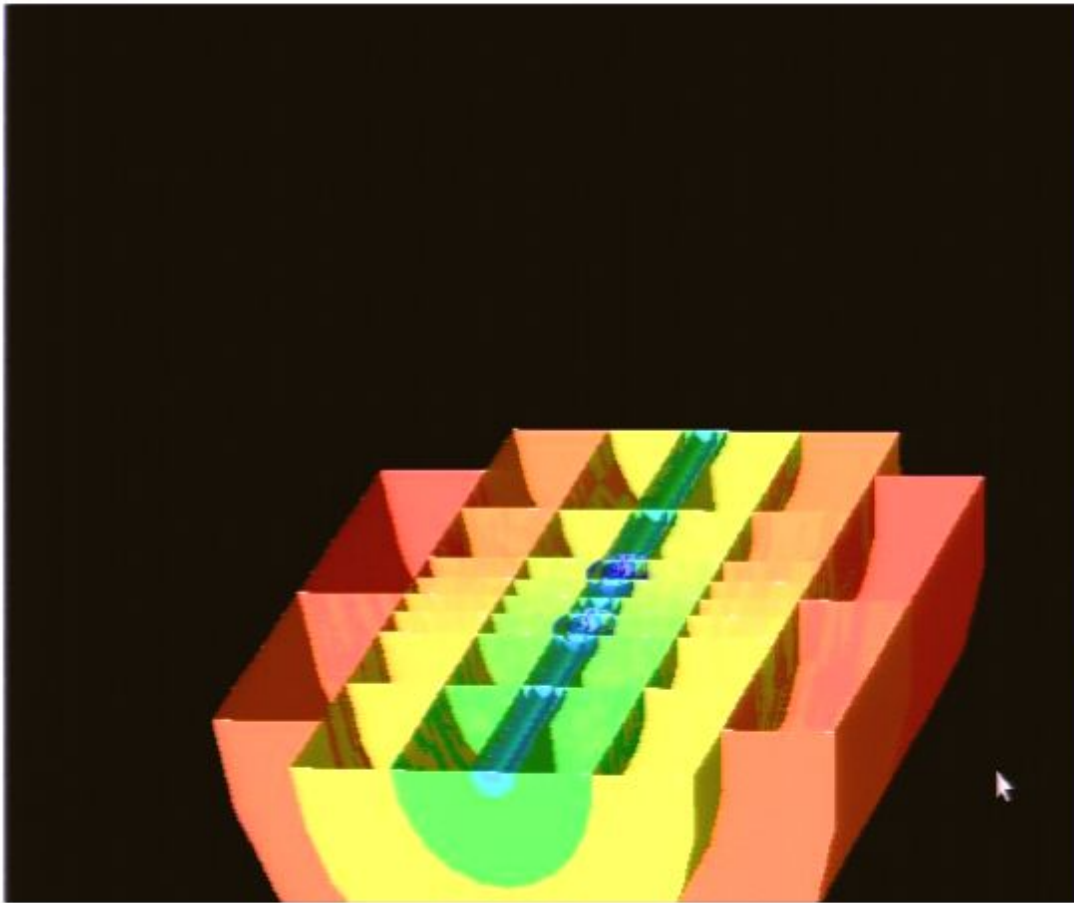
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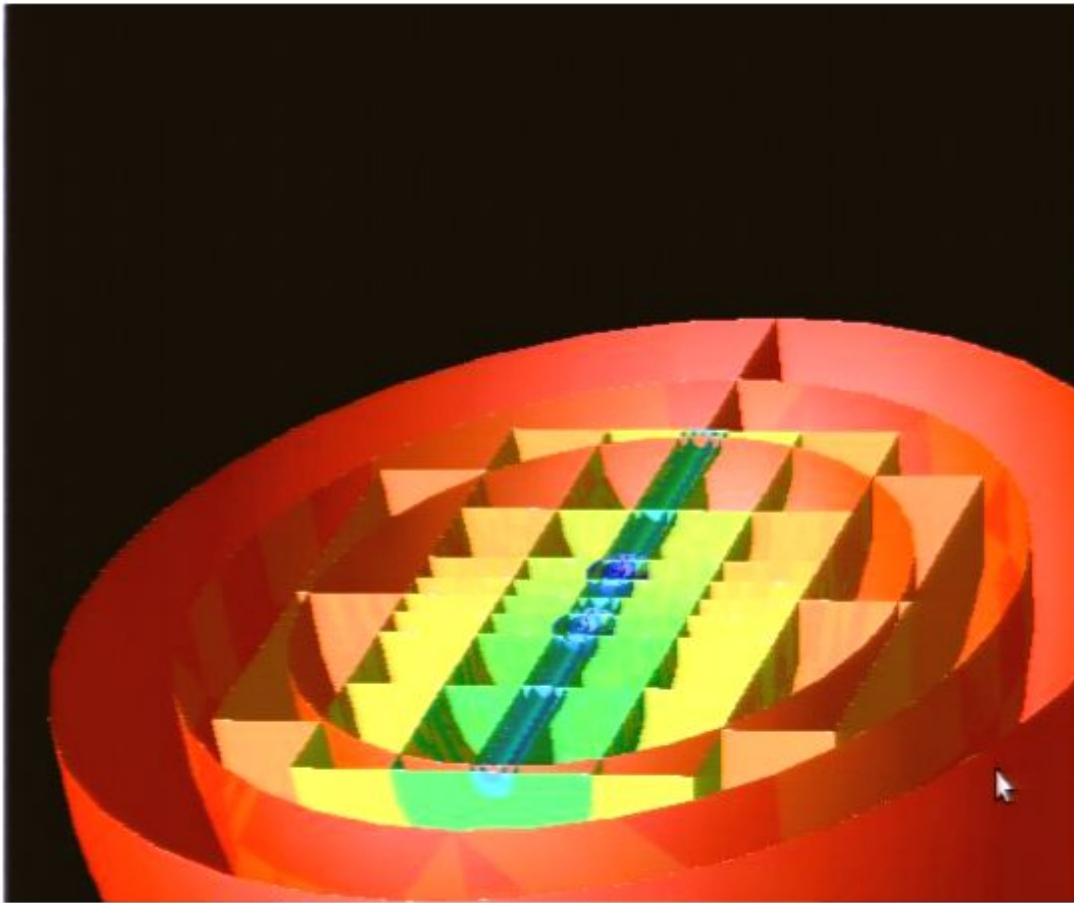
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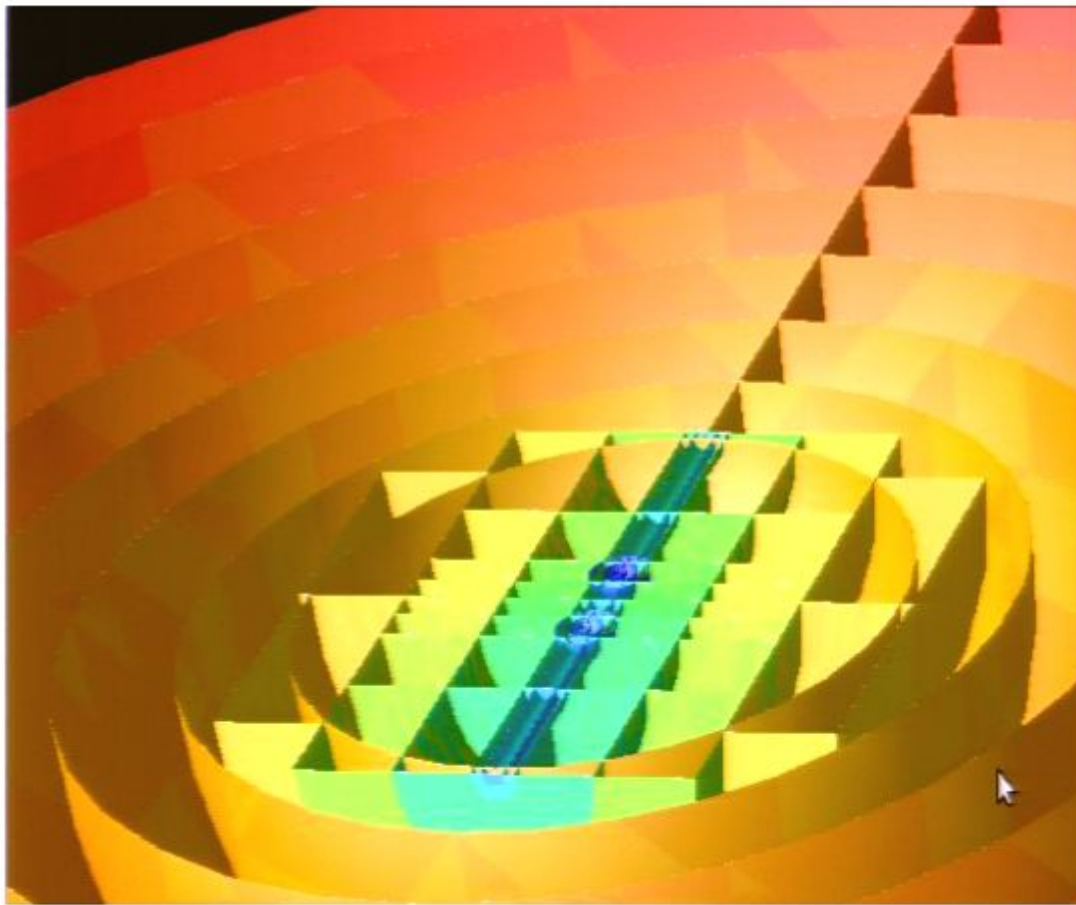
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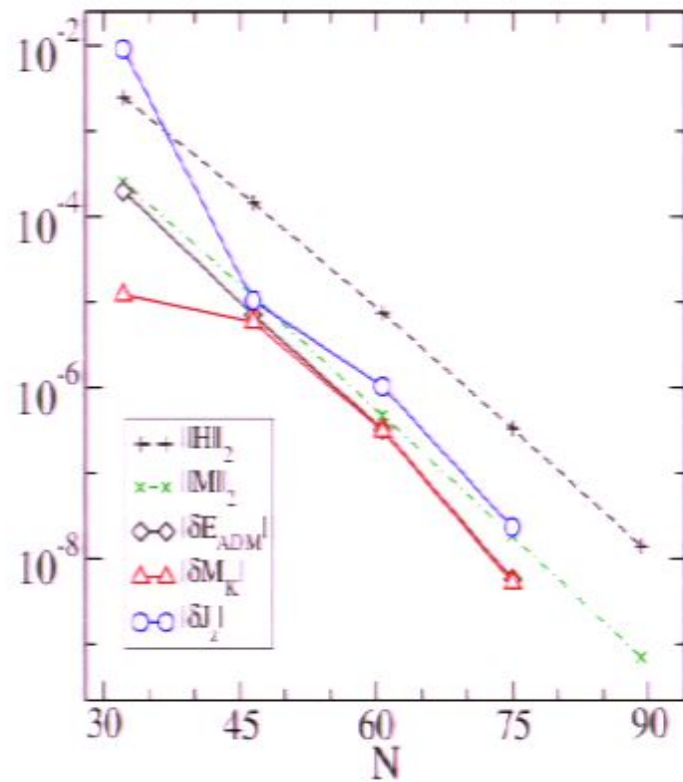
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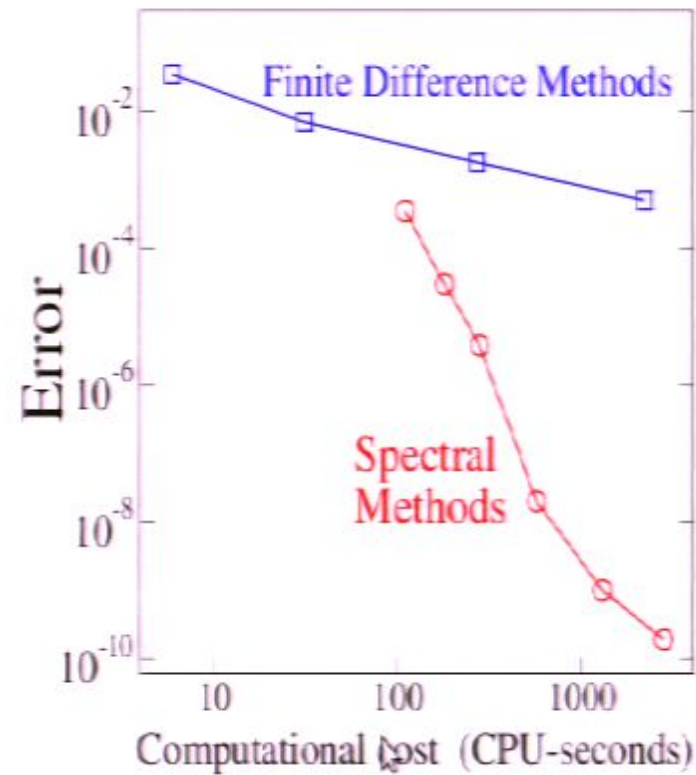


Why spectral methods?

Smooth solutions \Rightarrow exponential convergence



Cook, HP 2004



HP et al. 2003

Computational Framework III

- Domain-decomposition using simple topologies (DUST)

Building blocks

- I_1 interval
- S_1 circle
- S_2 sphere
- B_2 disk
- B_3 ball

Tensor-product



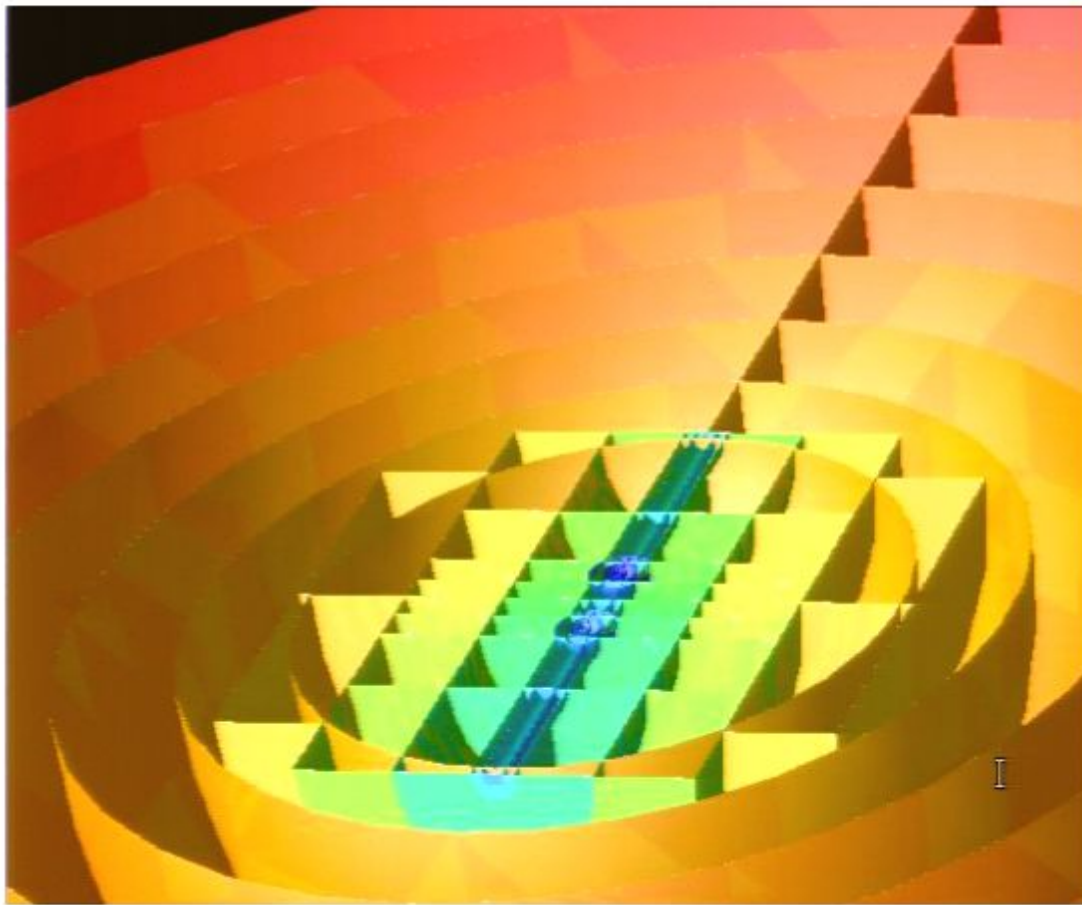
Composite topologies

- n-dim Block $I_1 \otimes I_1 \otimes \dots \otimes I_1$
- Cylinder $B_2 \otimes I_1, I_1 \otimes S_1 \otimes I_1$
- Sphere $B_3, I_1 \otimes S_2$
- 4-D w/ compactified dim
 $S_1 \otimes B_3, S_1 \otimes I_1 \otimes S_2$
- ...

I

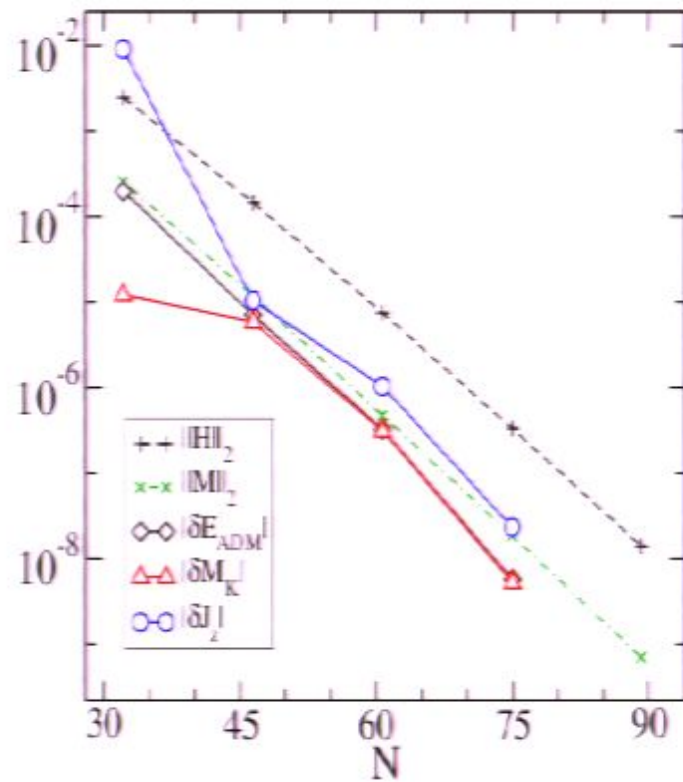
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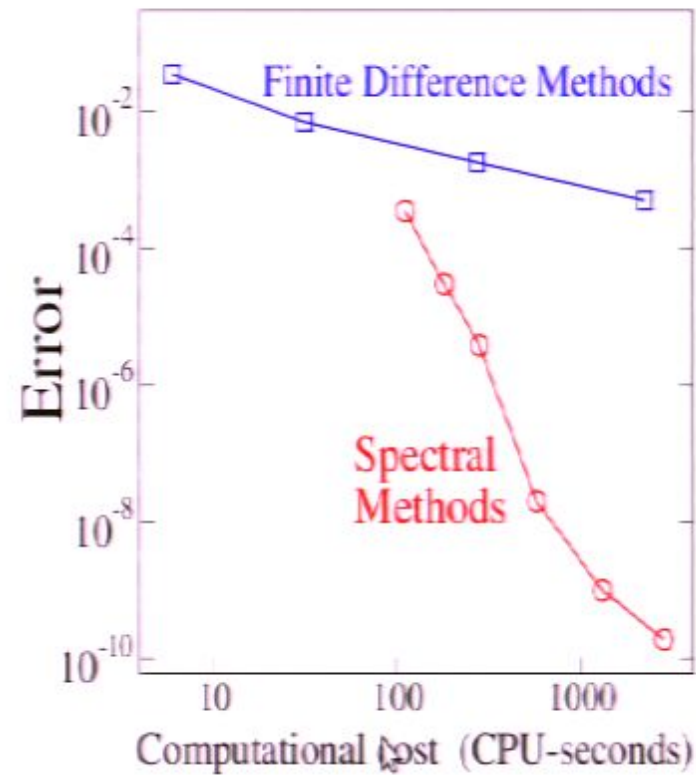


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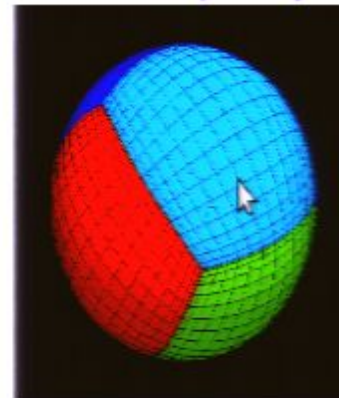


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- ...

Combine several subdomains to get desired shape/topology

Six distorted squares = sphere



Evolution equations

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\square g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms.} \quad \Gamma_a = -g_{ab}\square x^b.$$

- Generalized harmonic coordinates $g_{ab}\square x^b \equiv H_a(x^a, g_{ab})$
(Friedrich 1985, Pretorius 2005; note that $H \equiv 0$ has been used since 1920's)

$$\square g_{ab} = \text{lower order terms.}$$

$$\Rightarrow \text{Constraint } C_a \equiv H_a - g_{ab}\square x^b = 0$$

- **Constraint damping** (Gundlach, et al., Pretorius, 2005)

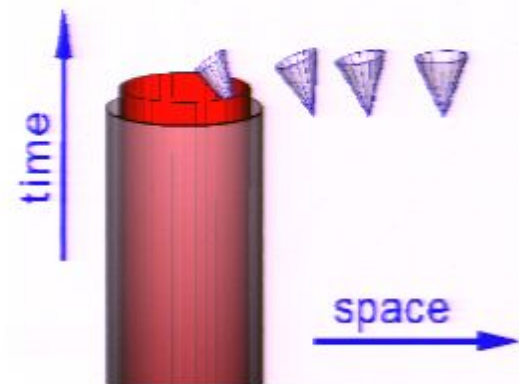
$$\square g_{ab} = \gamma \left[t_{(a} C_{b)} - \frac{1}{2} g_{ab} t^c C_c \right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

Boundary conditions

- **Black hole singularity excision**

- Place artificial inner boundary just inside horizon
- Causality \Rightarrow pure outflow condition, no BC applied (Unruh, 80's)
- Technical details require dual coordinate frames (Scheel et al., 2006)



- **Outer boundary**

- Constraint preserving (Kidder et al. 05; Sarbach, Tiglio 05; Lindblom et al. 06)
- Transparent to outgoing gravitational waves (Lindblom et al., 2006)
- No incoming gravitational waves (Lindblom et al., 2006)
- No reflections of gauge-modes (Rinne et al. 2007)

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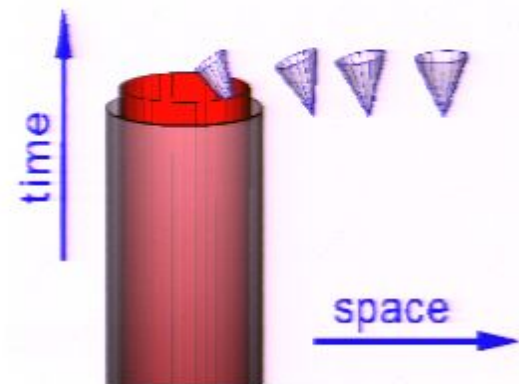
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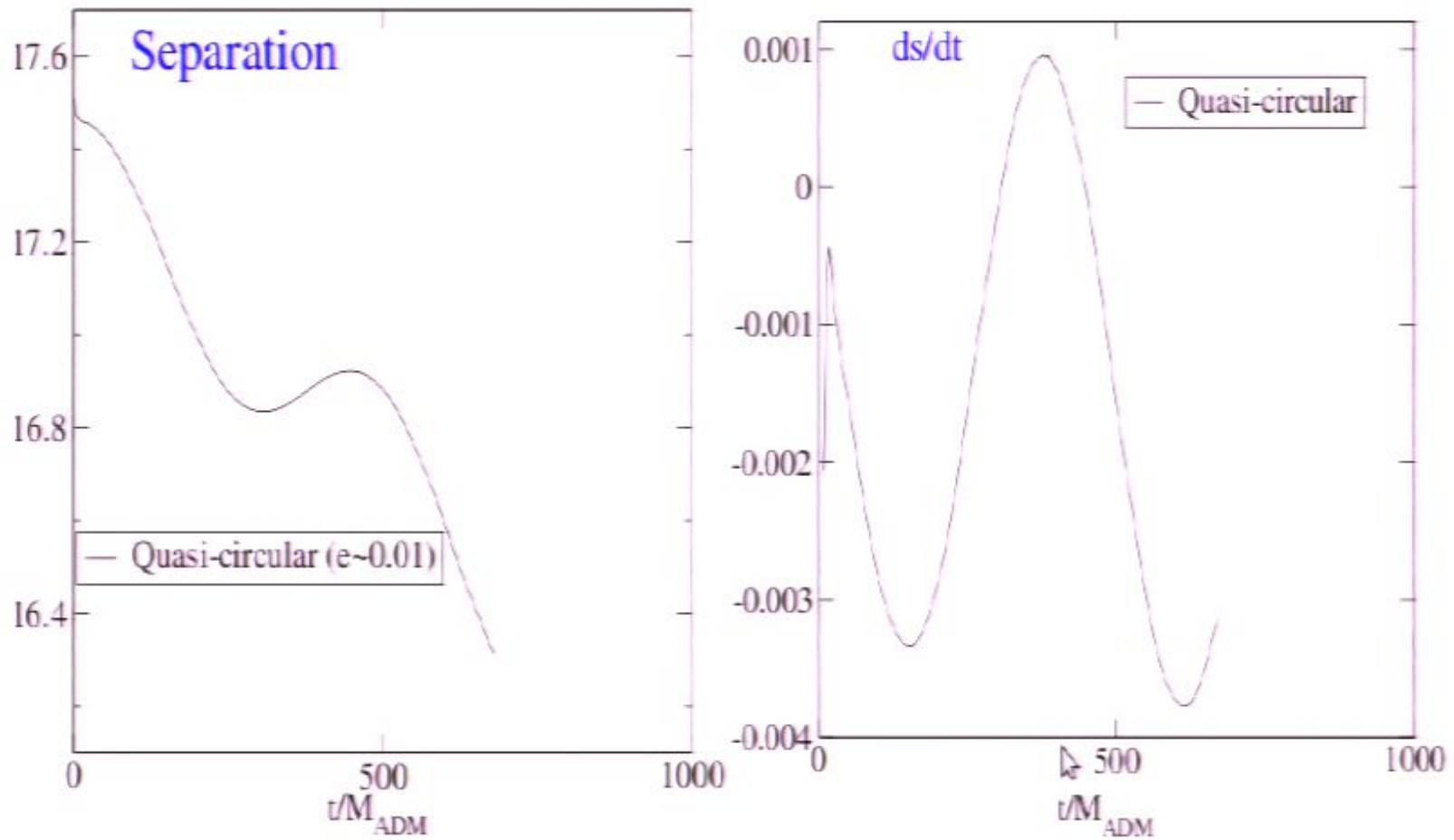
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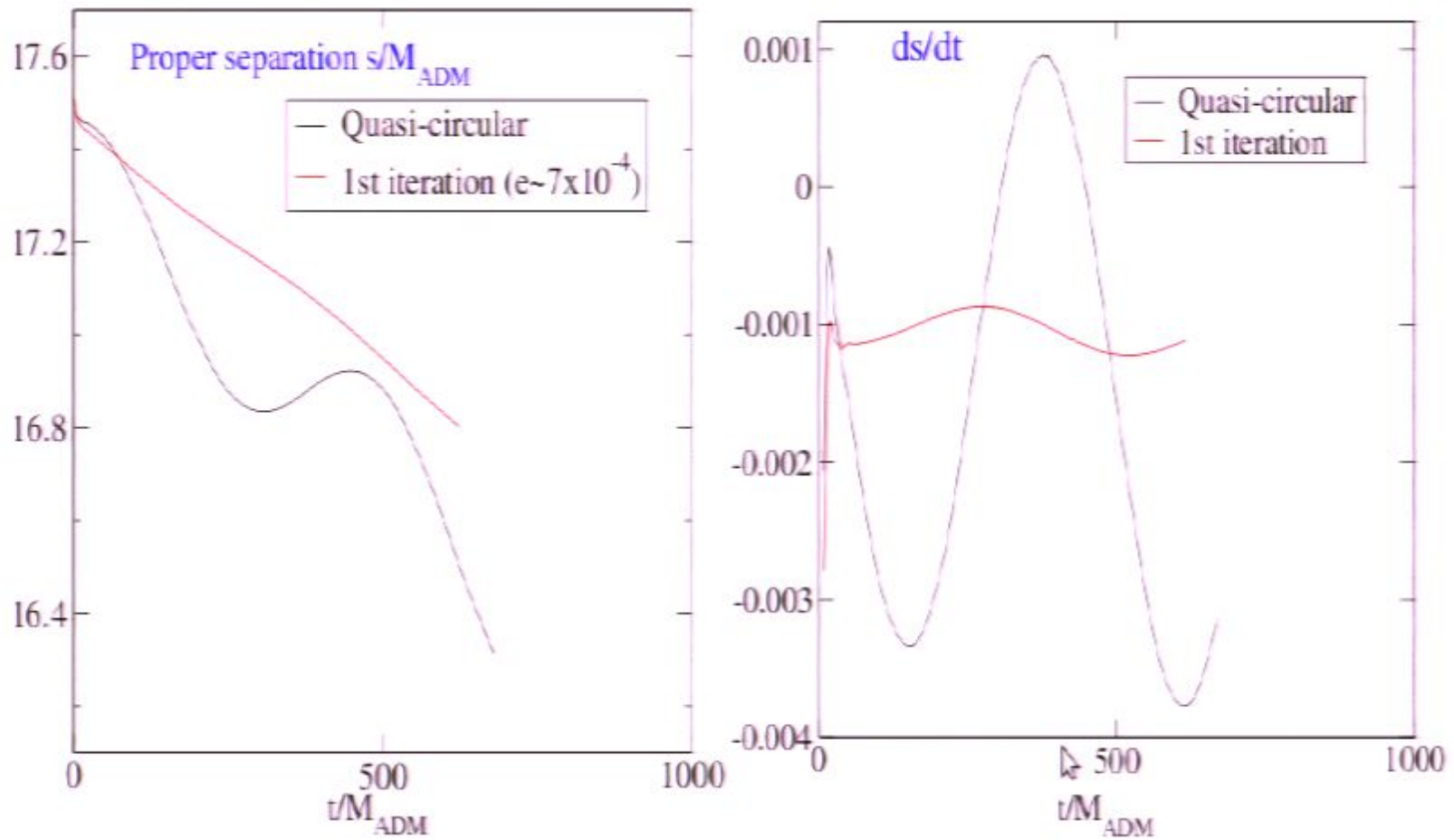
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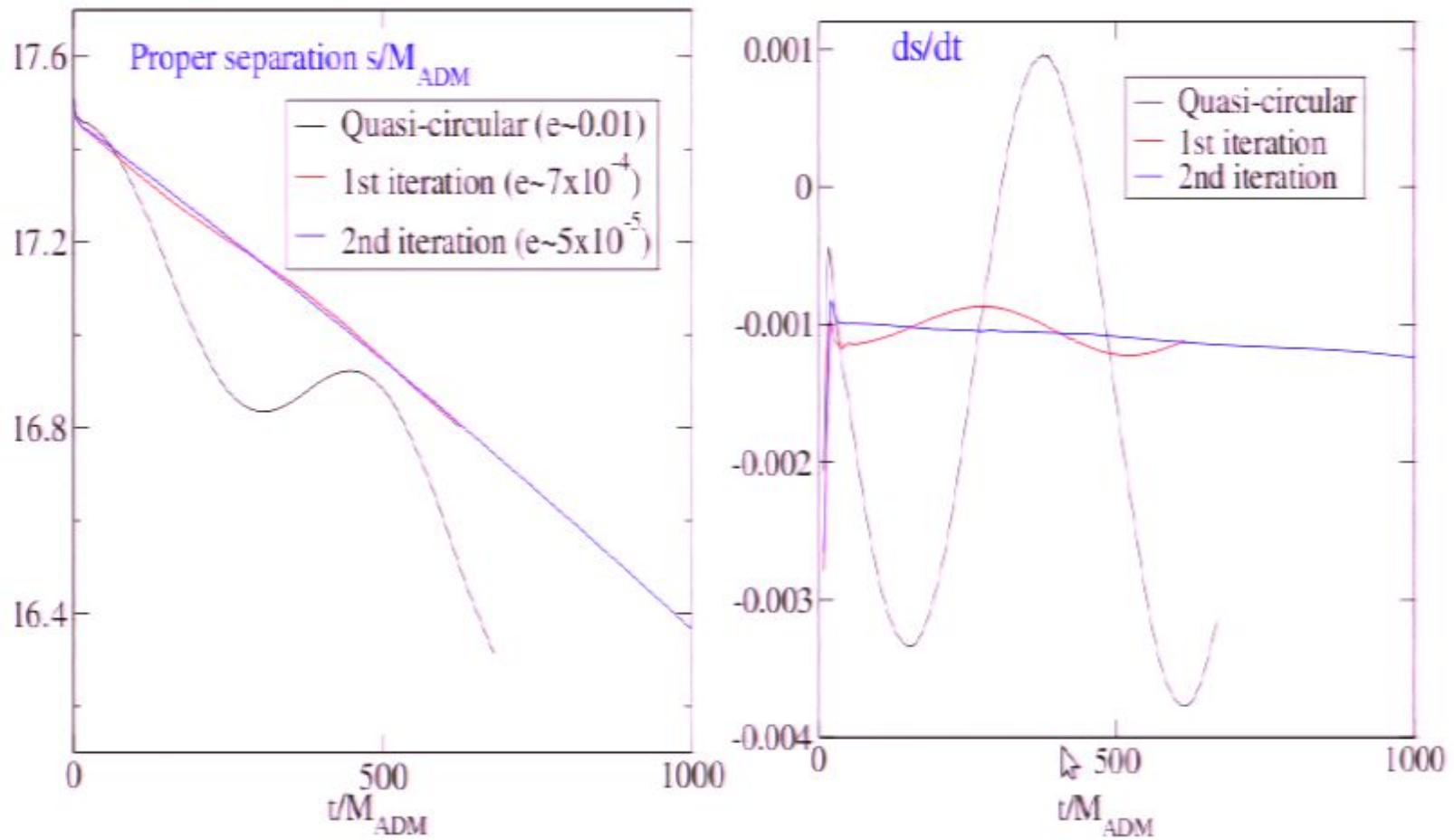
Iteratively control eccentricity (HP et al., 2007)



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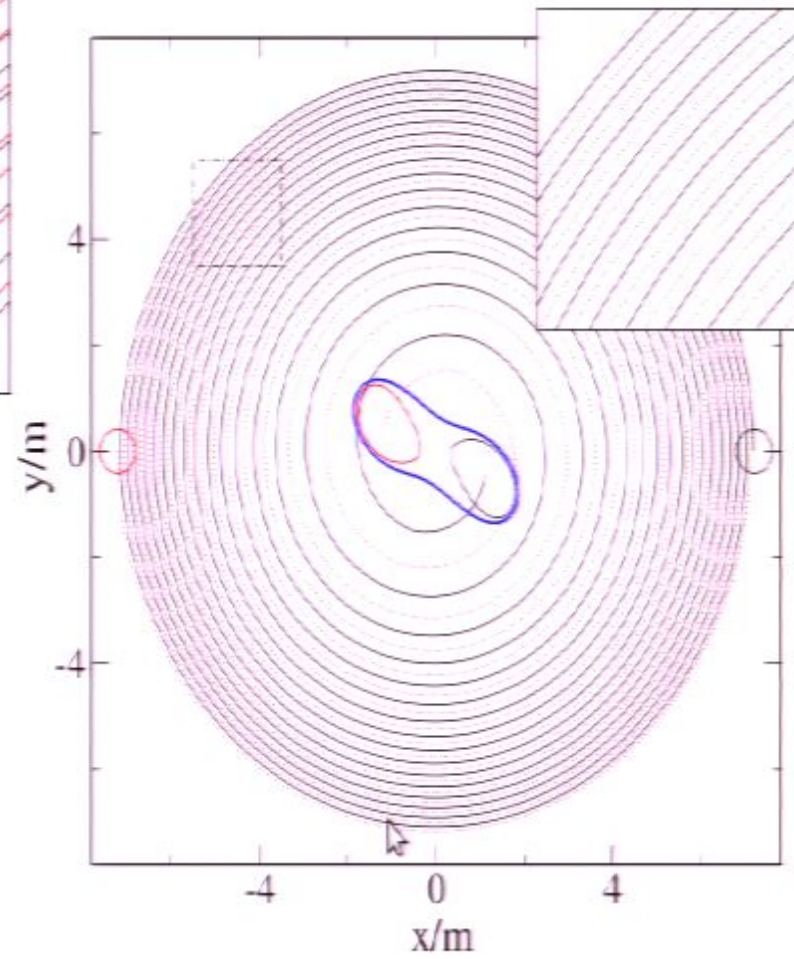
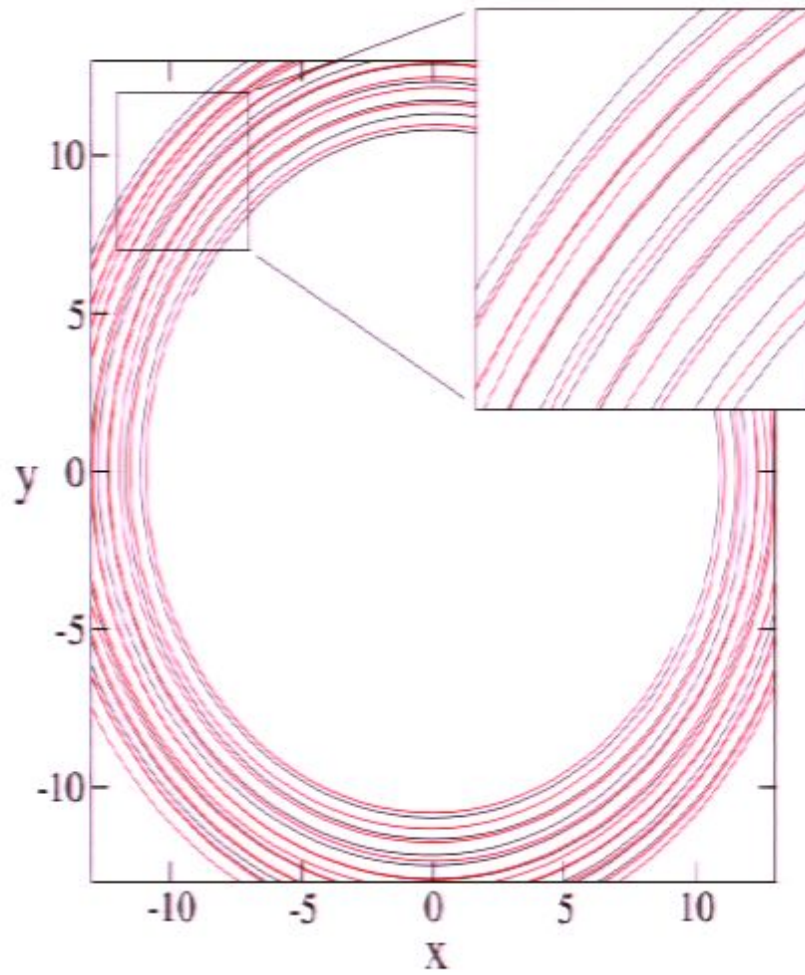
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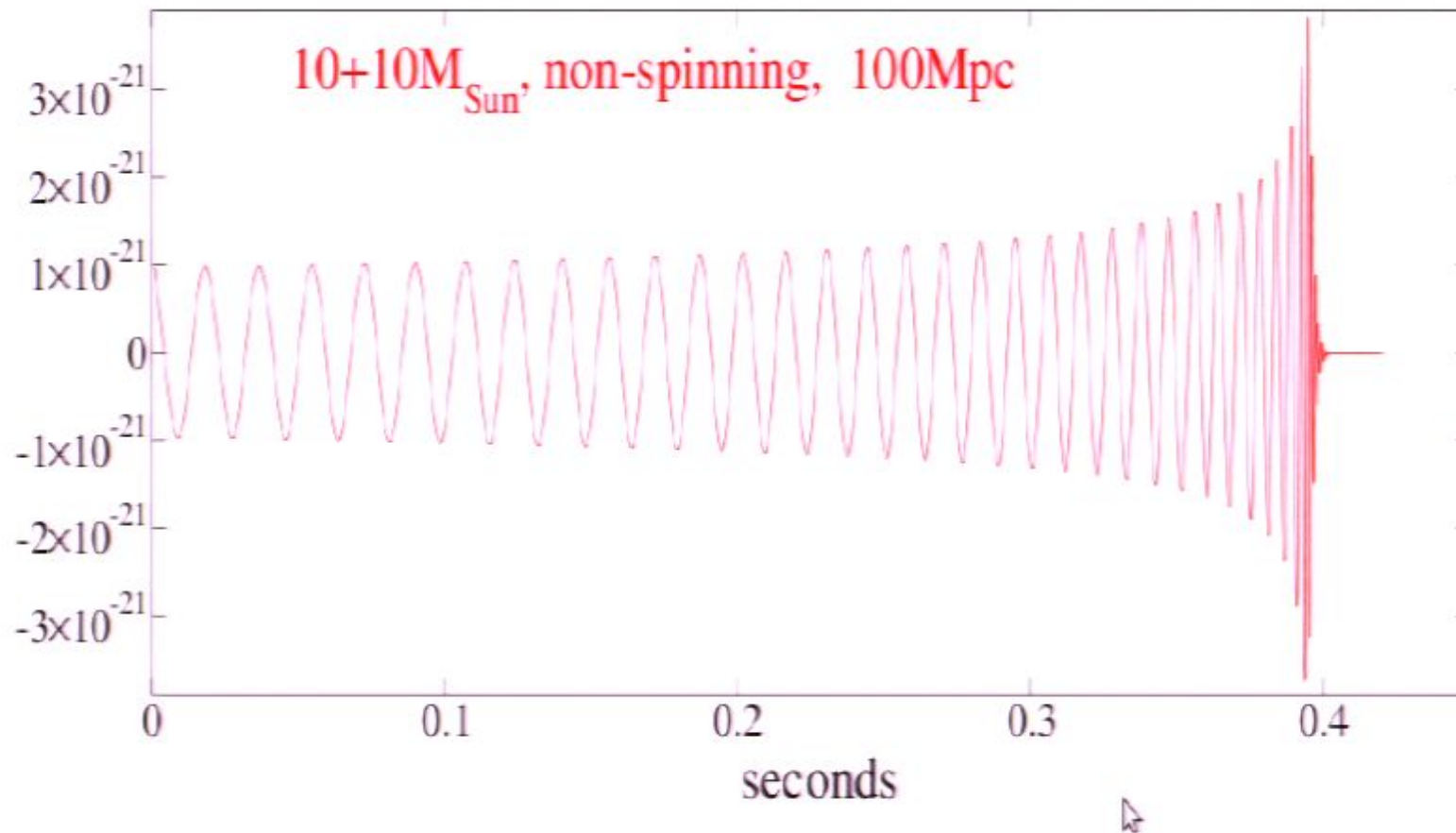
Orbital trajectory

$e = 0.01$

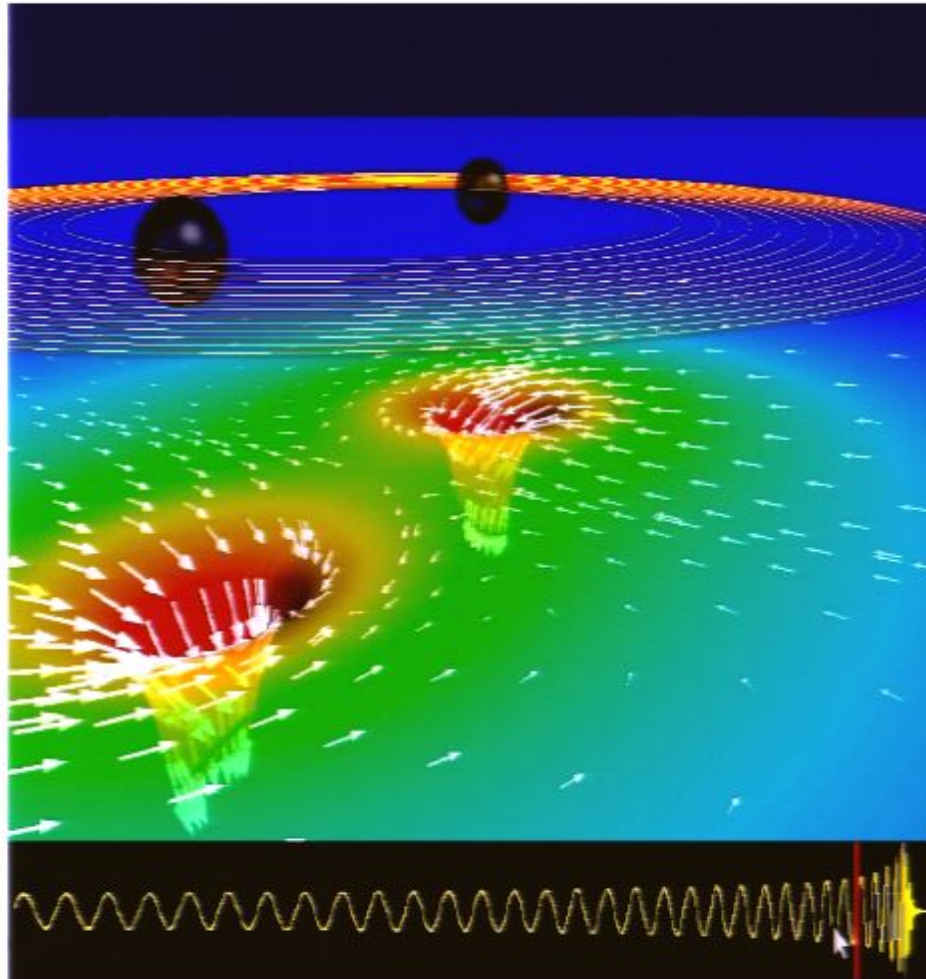
$e = 5 \times 10^{-5}$



At last – a waveform!!

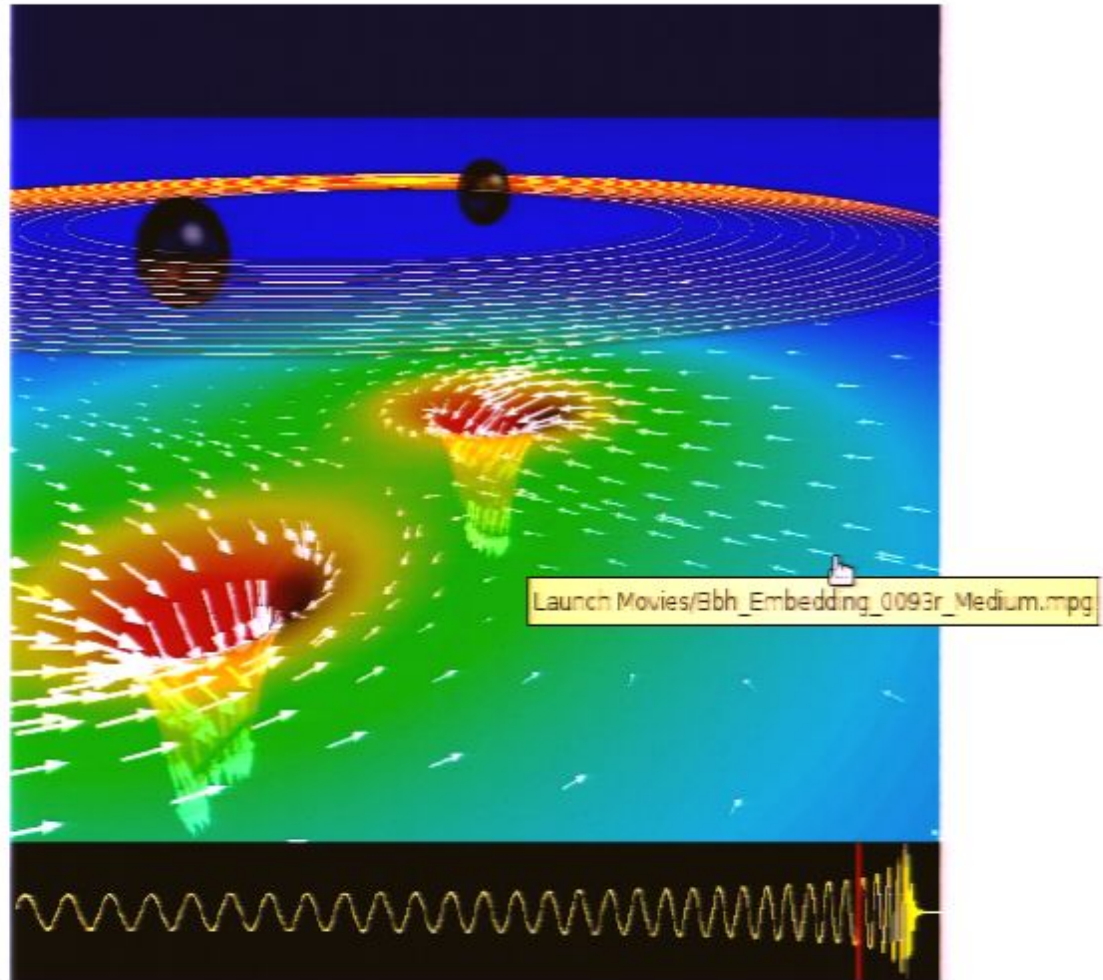


Movies I



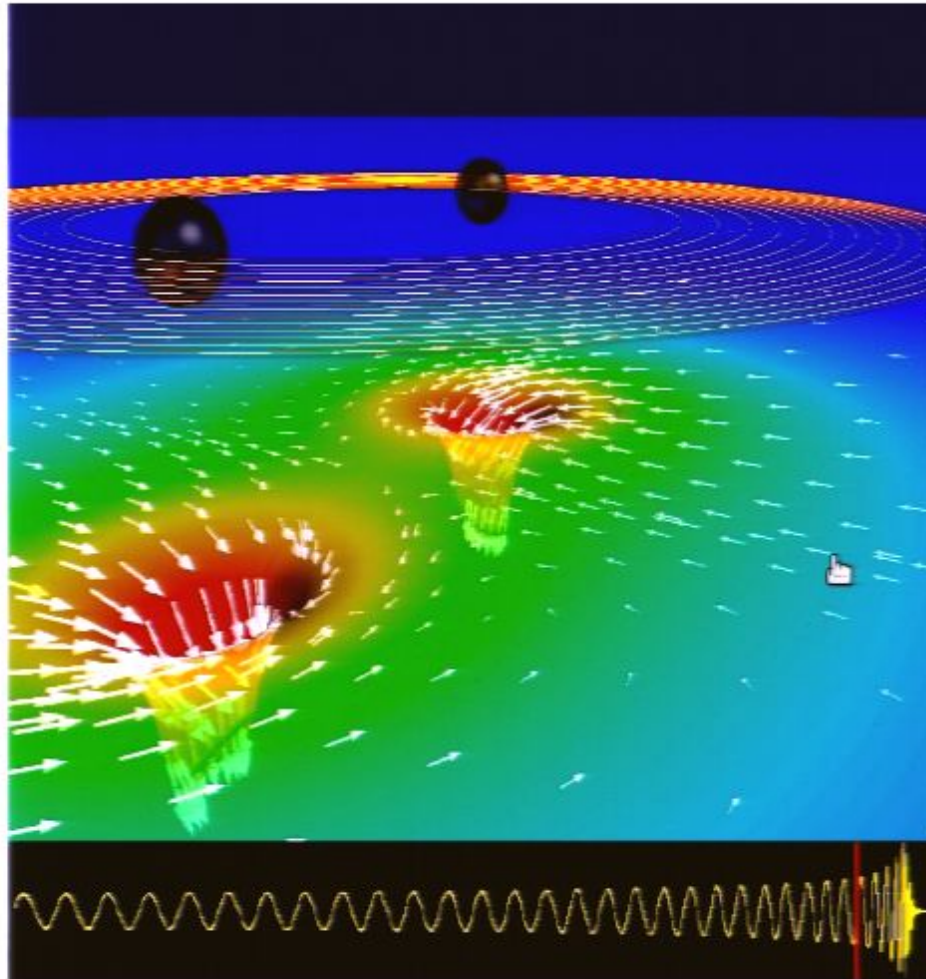
www.black-holes.org/explore2.html

Movies I



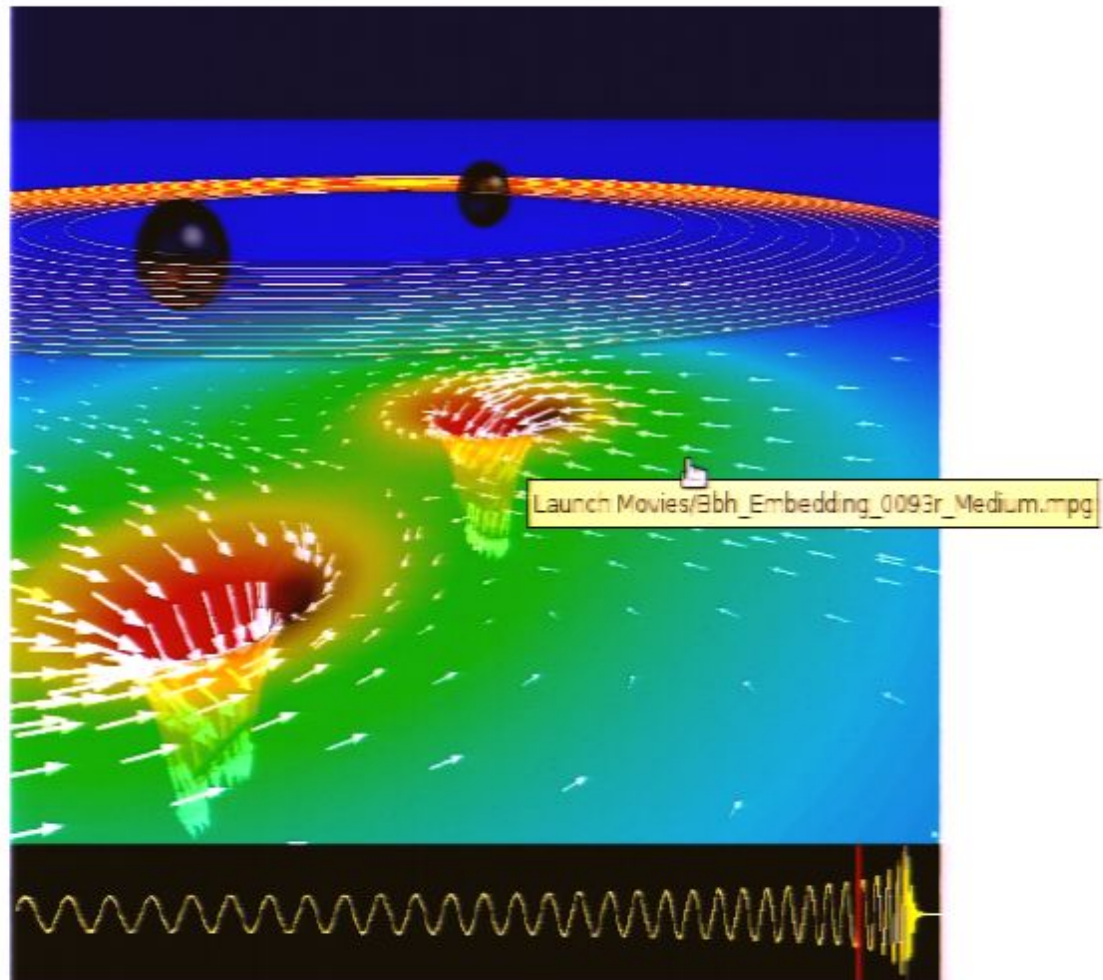
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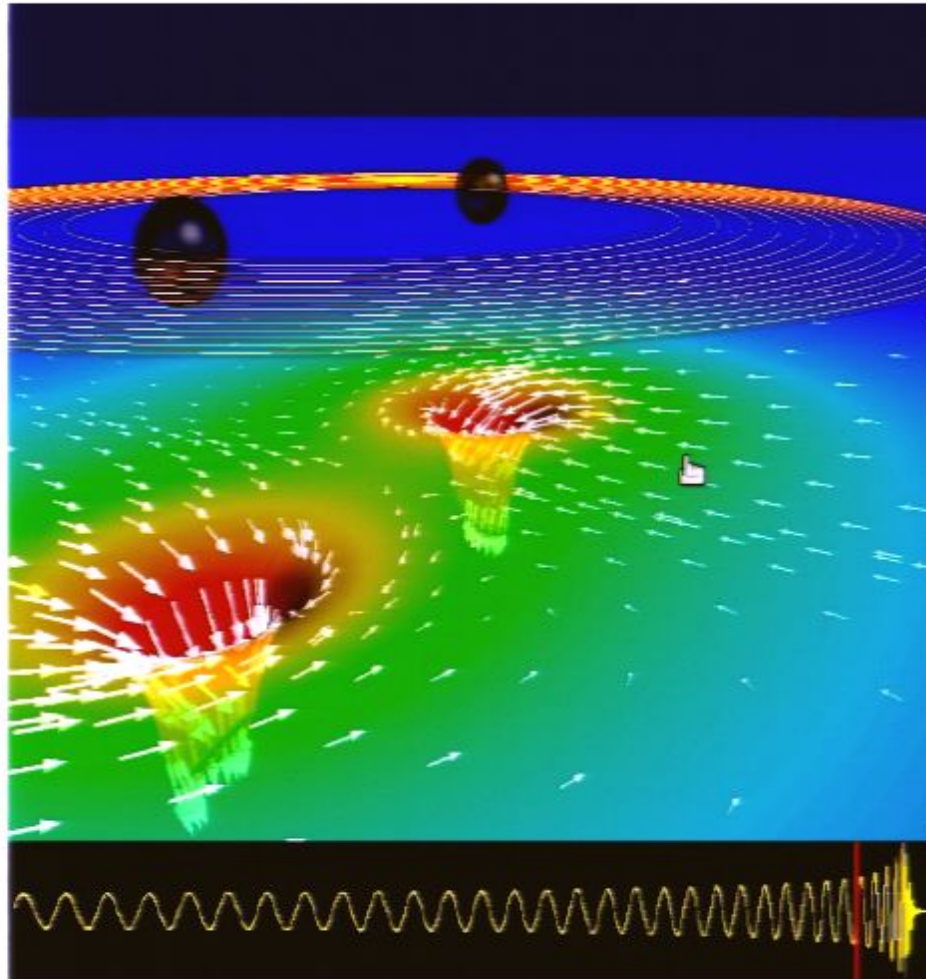
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Movies I



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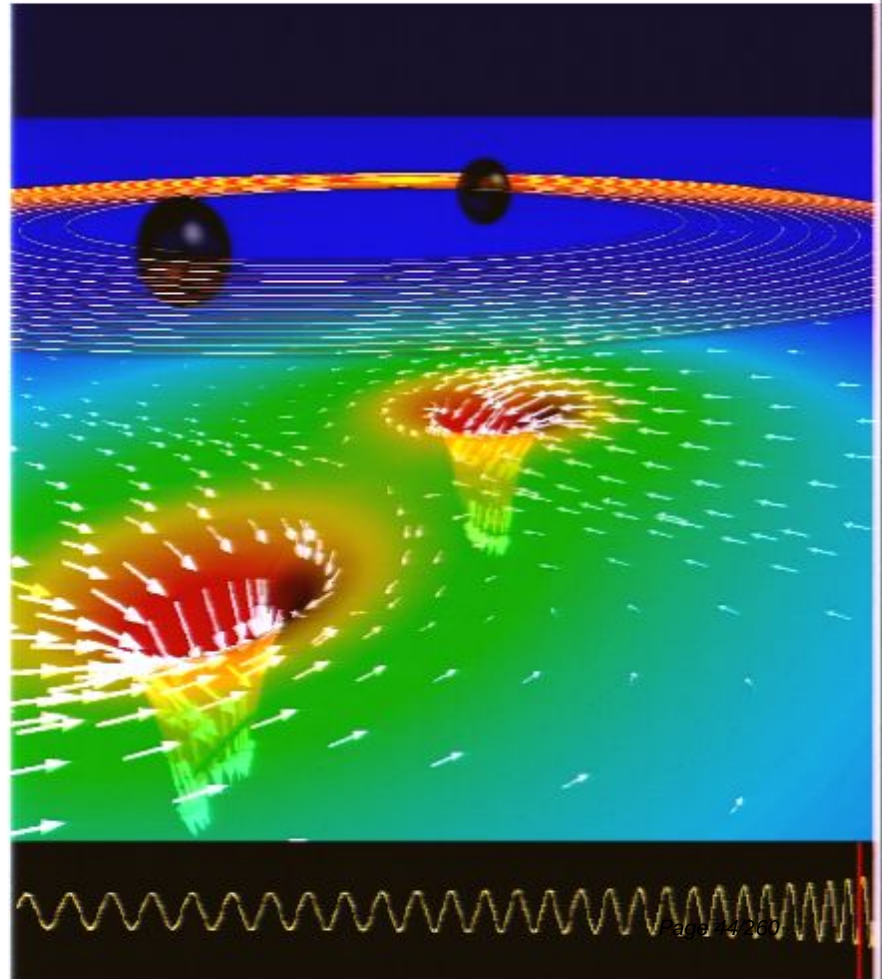
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Previous Next

27 of 67

Best Fit

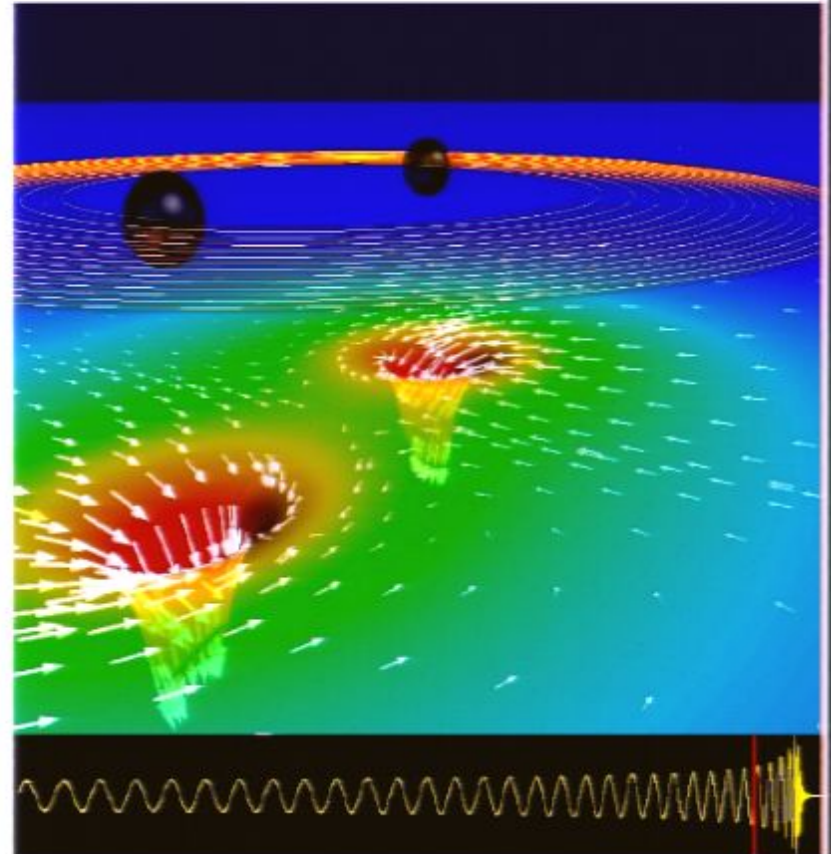
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← Previous 27 of 67 Best Fit ▾ → Next

Movies I

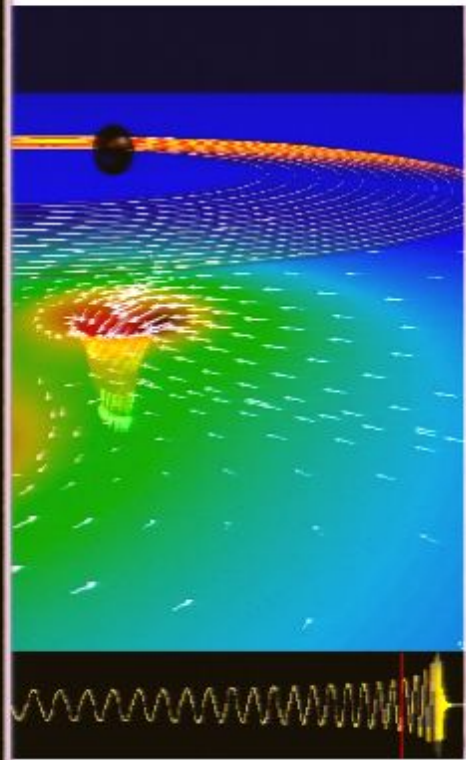
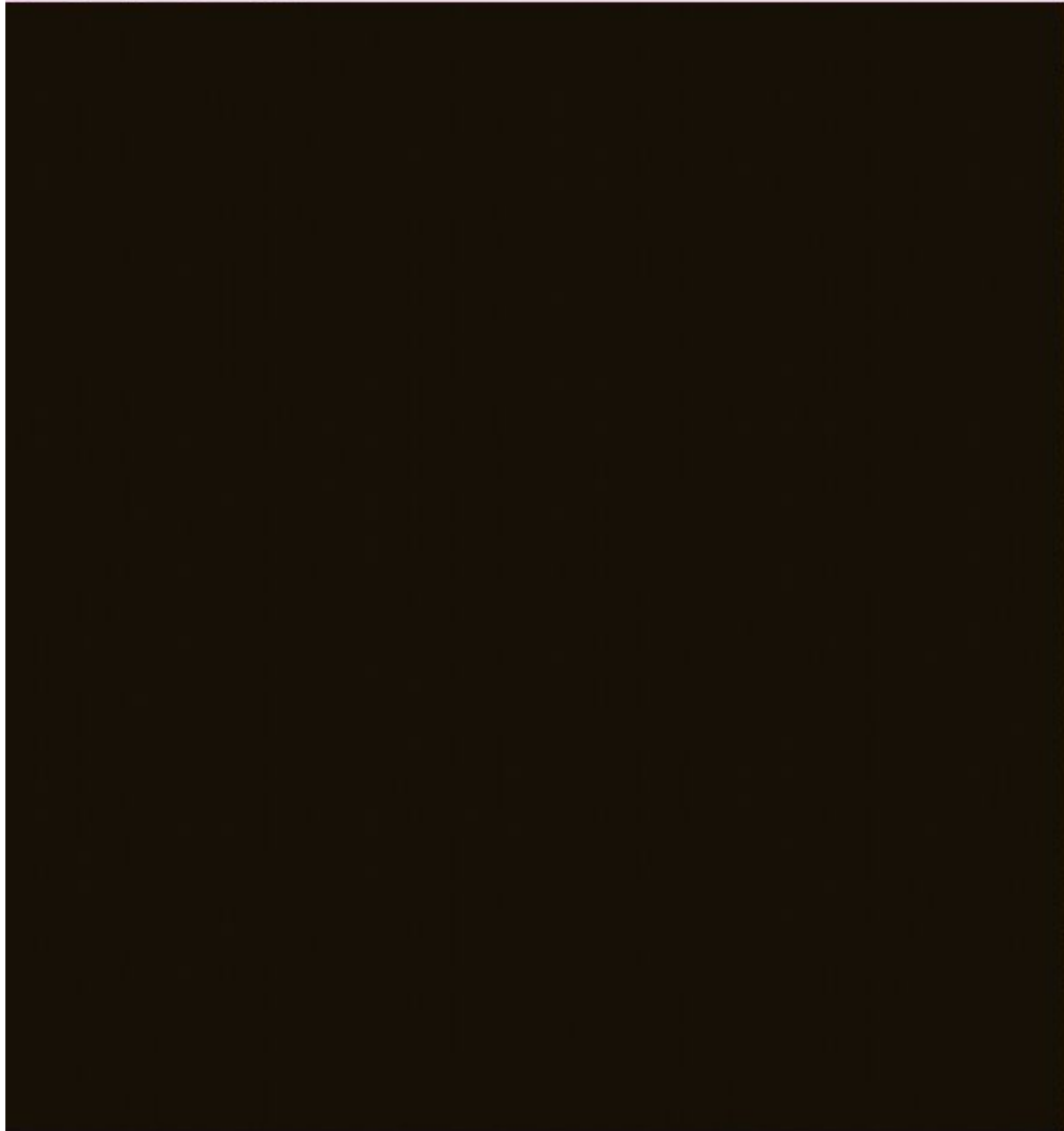


www.black-holes.org/explore2.html

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of black hole spacetimes

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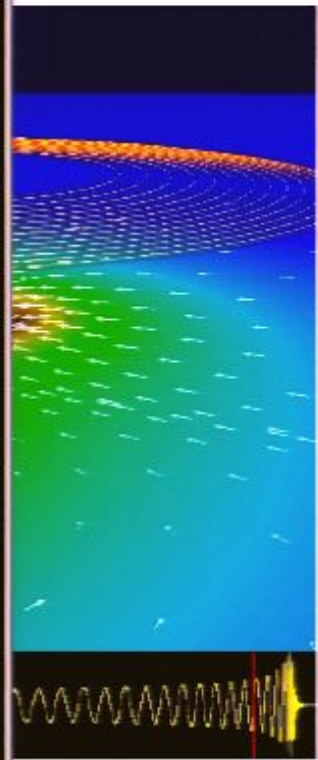
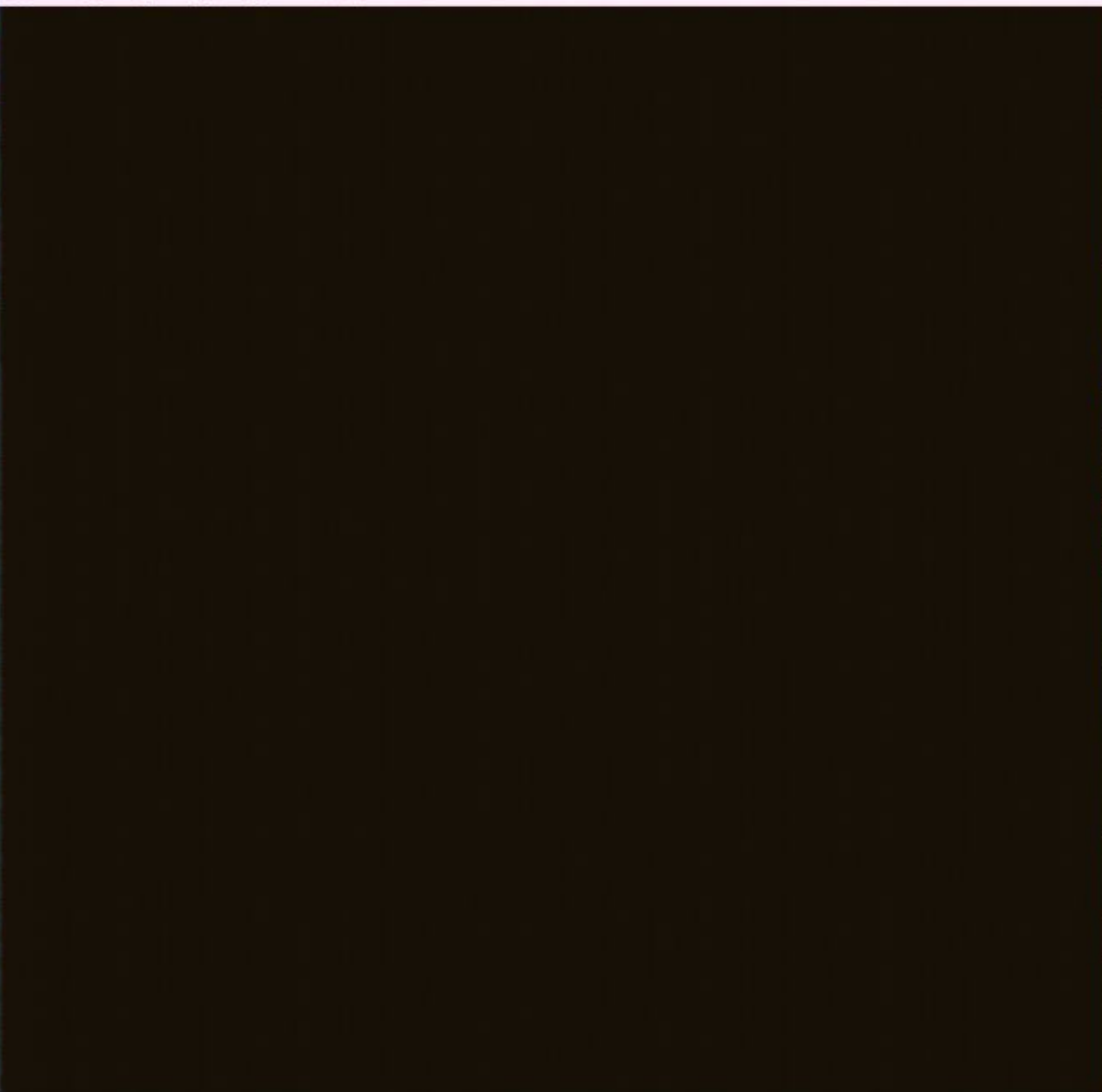


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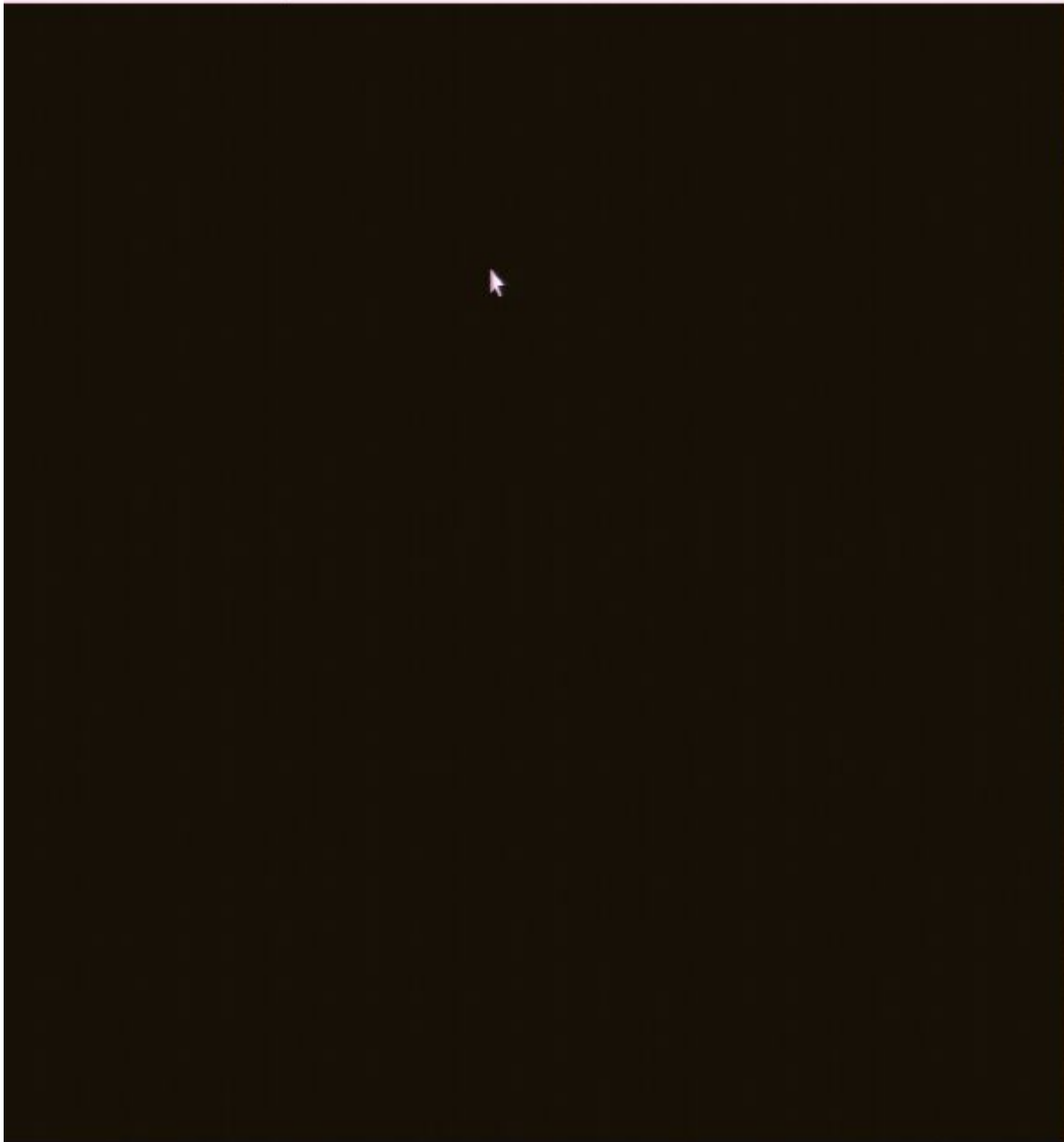
Bbh_Embedding_0093r_Medium.mpg

spacetimes 

Movie Edit View Go Sound Help



explore2.html



A vertical sidebar with a blue and yellow gradient background and a white top section containing window control buttons.

No Signal

VGA-1

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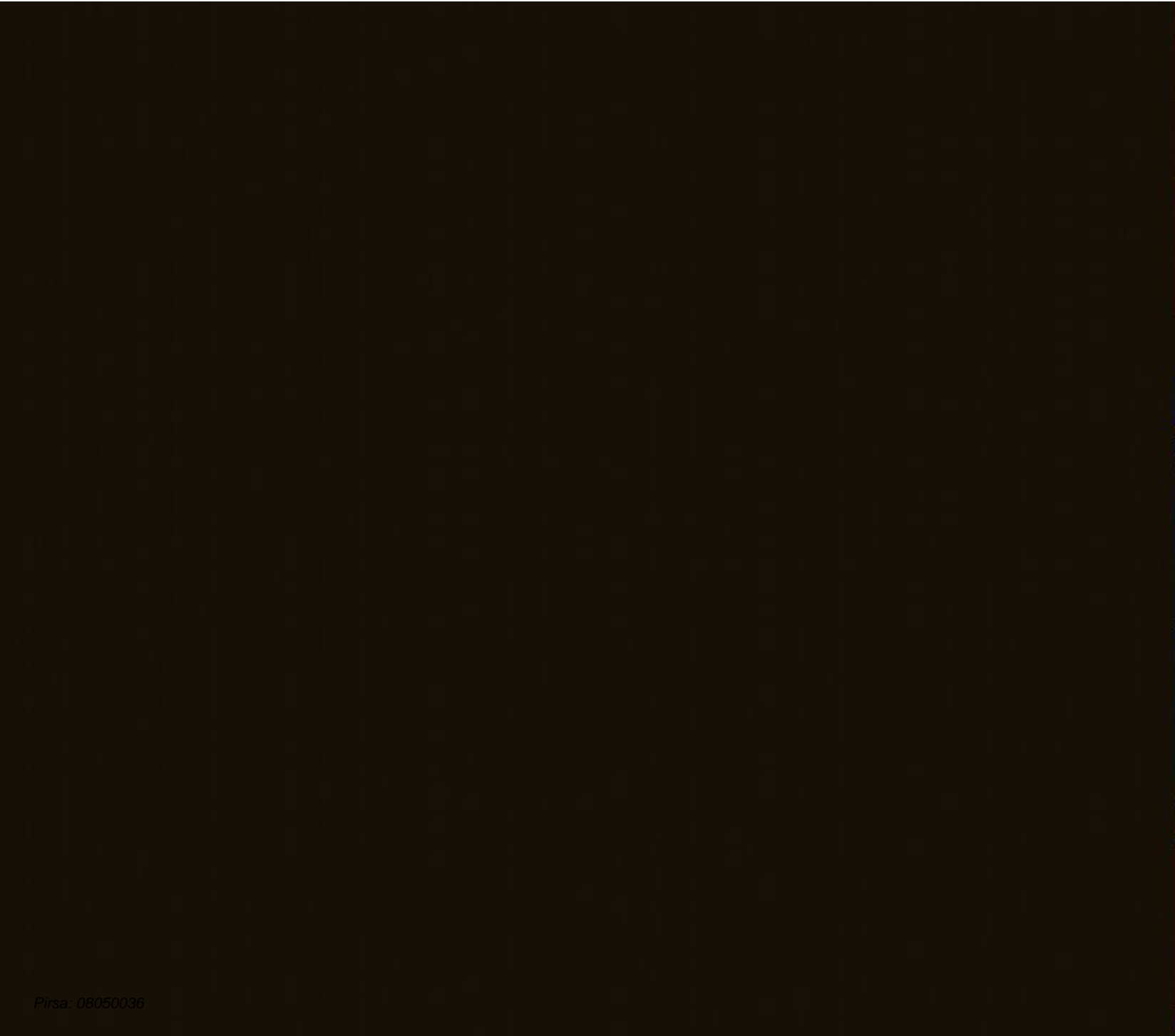
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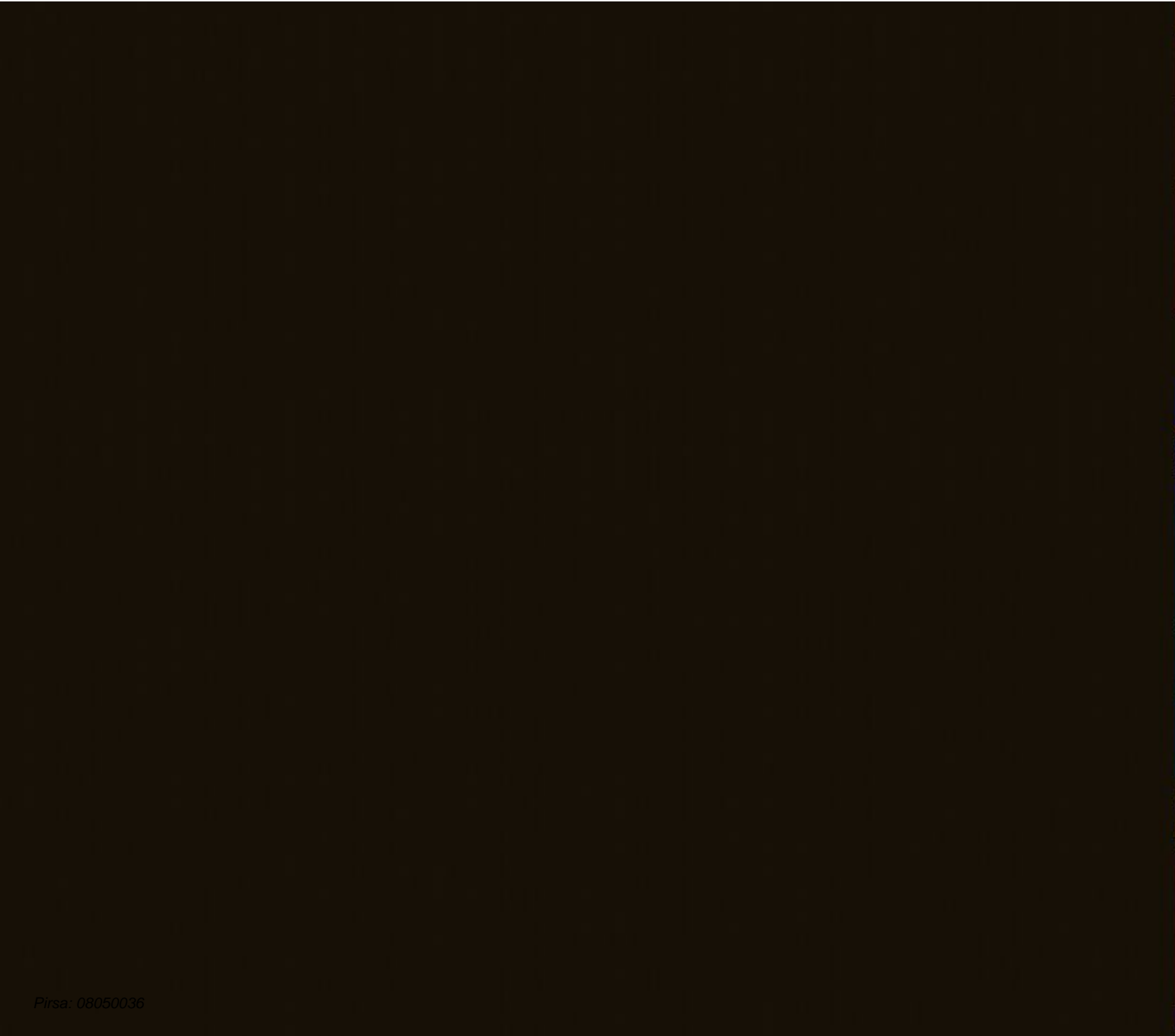
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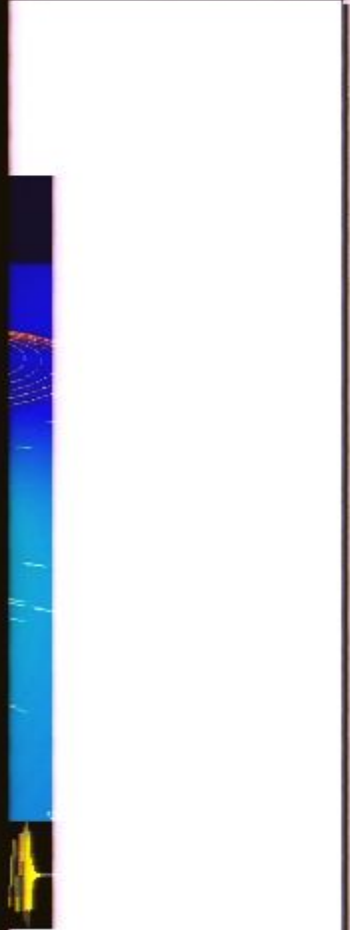
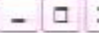
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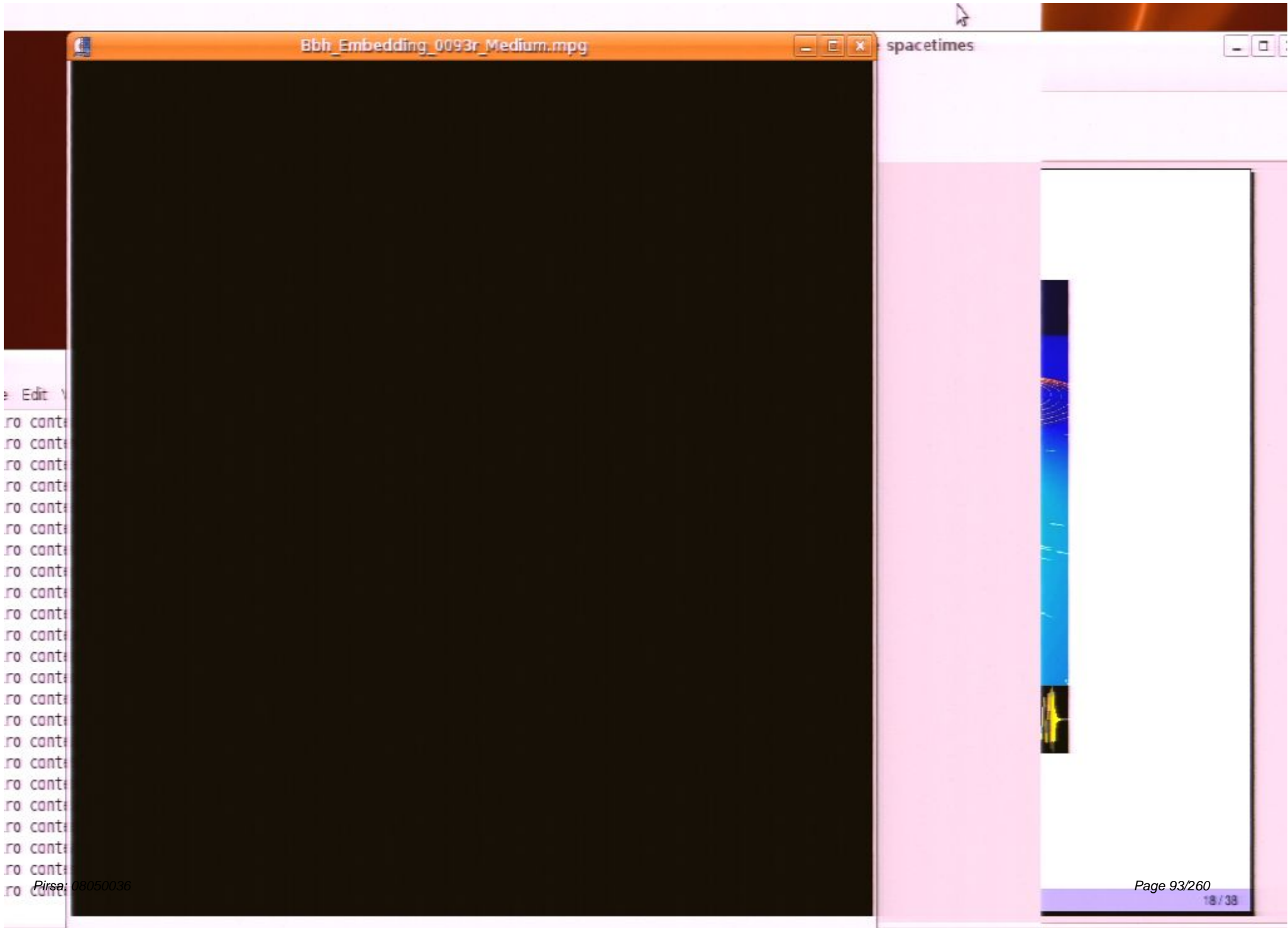
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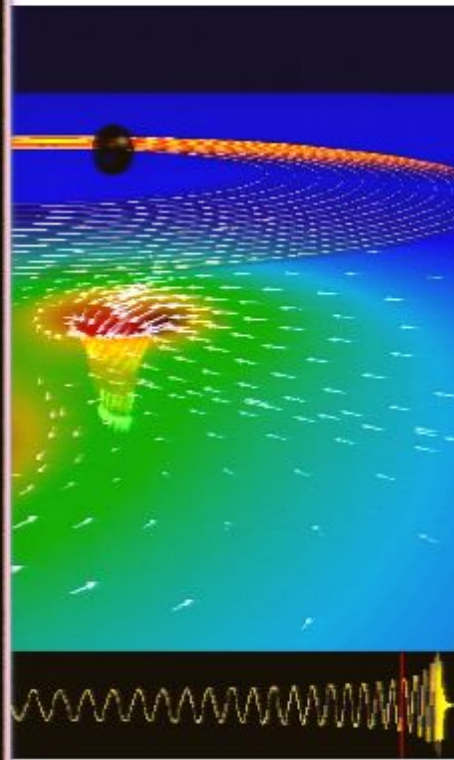
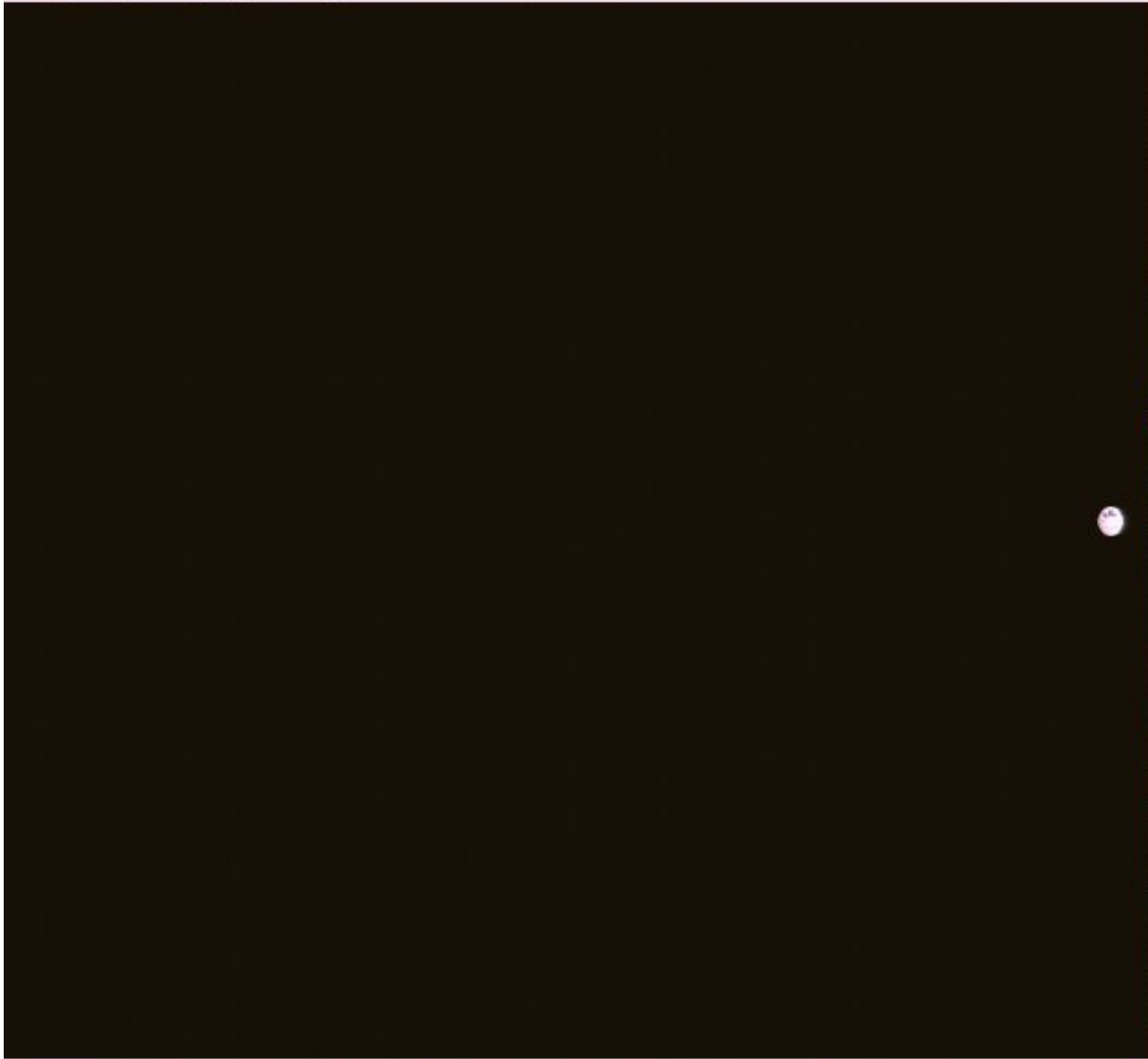
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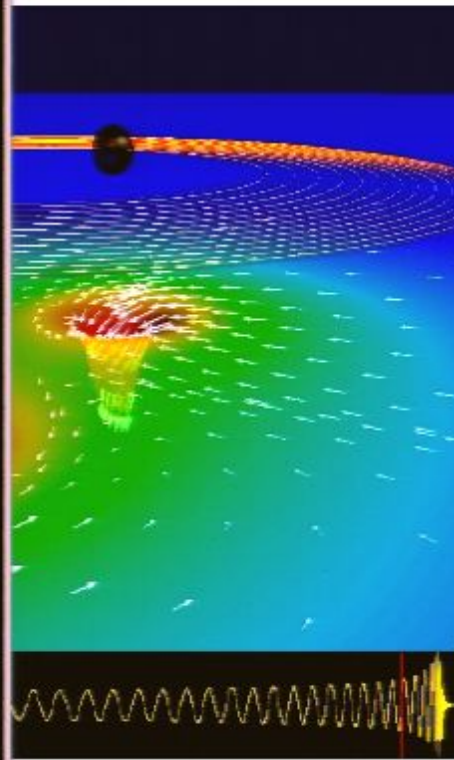
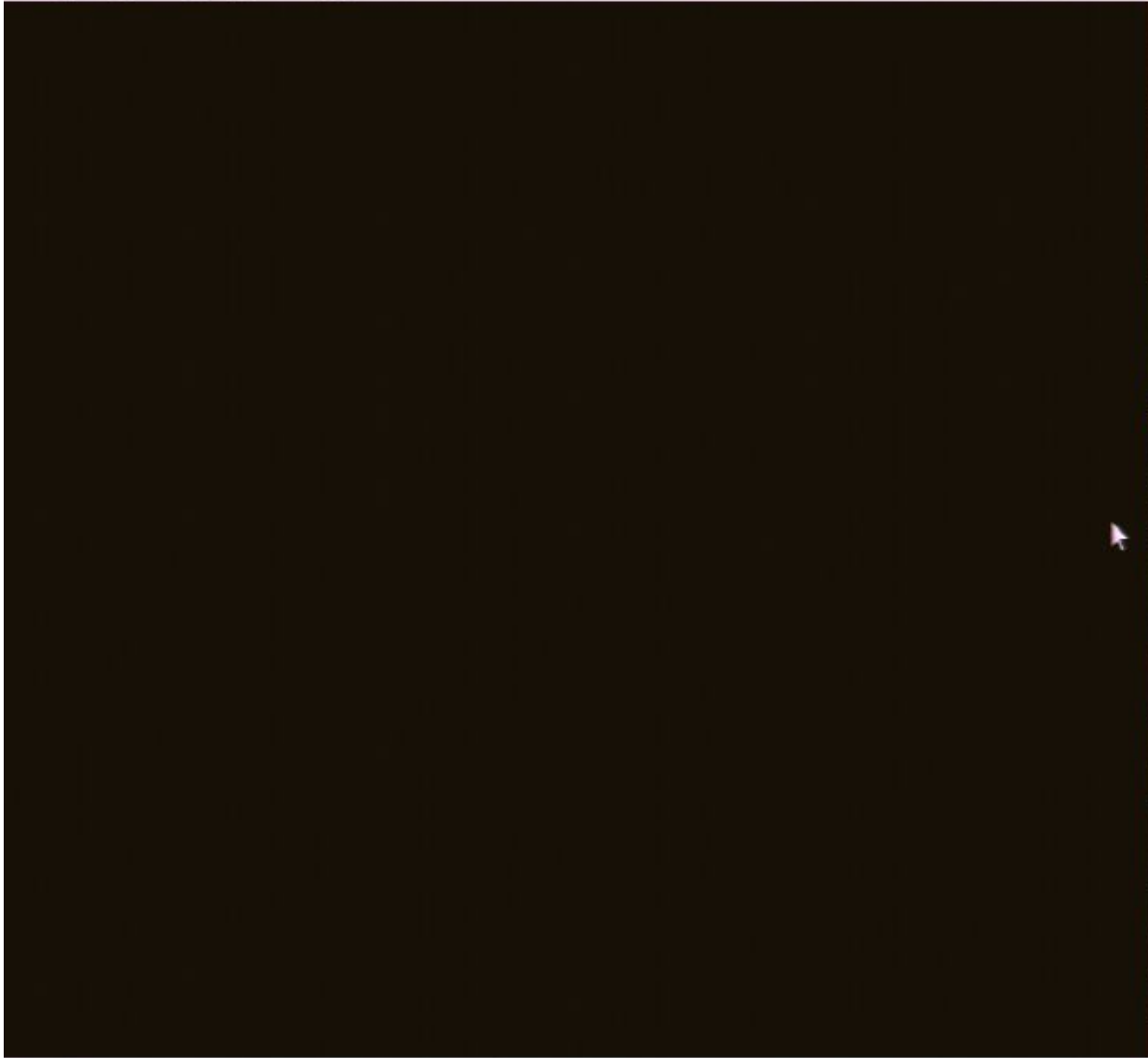
Progress bar with a slider and a play button icon.

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▶ Sidebar

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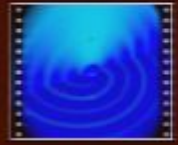
...les.org/explore2.html

Progress bar with a slider and a volume icon.

▶ Sidebar

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harald@Neville:AA~/research/Talks/D8M

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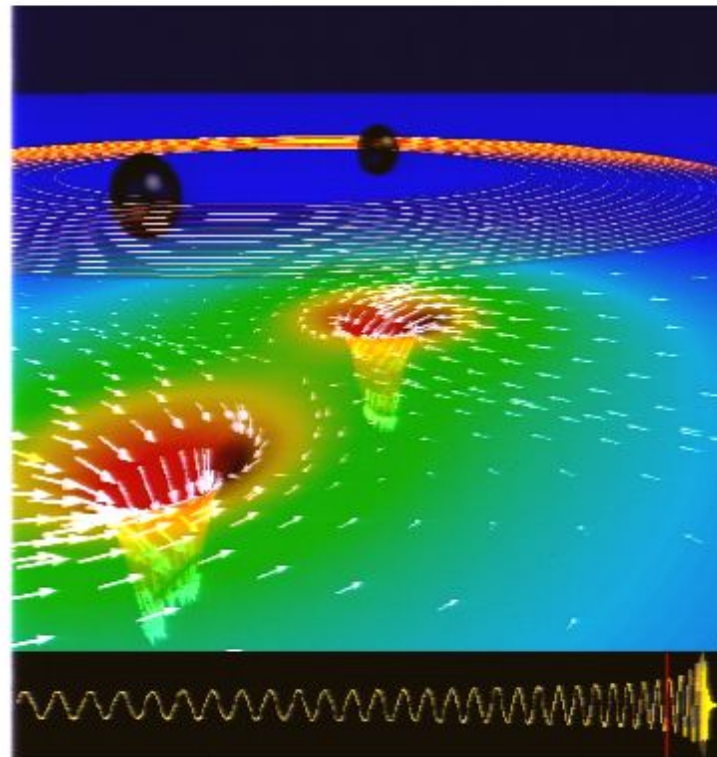
rald@Neville:~/May_Perimeter\$ cd Movies
rald@Neville:~/Movies\$ mpeg

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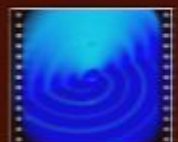
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www.black-holes.org/explore2.html



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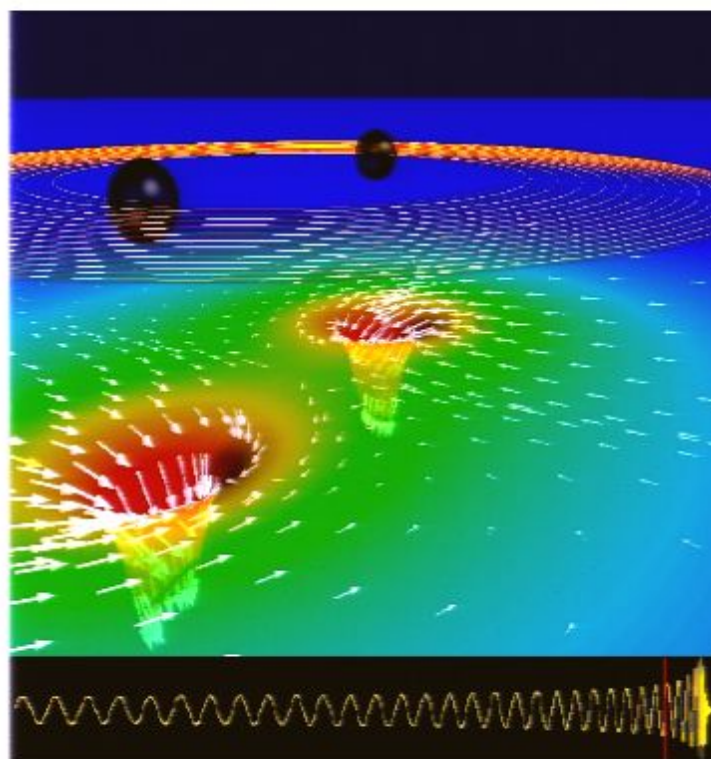
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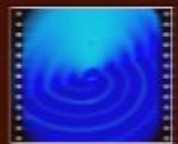
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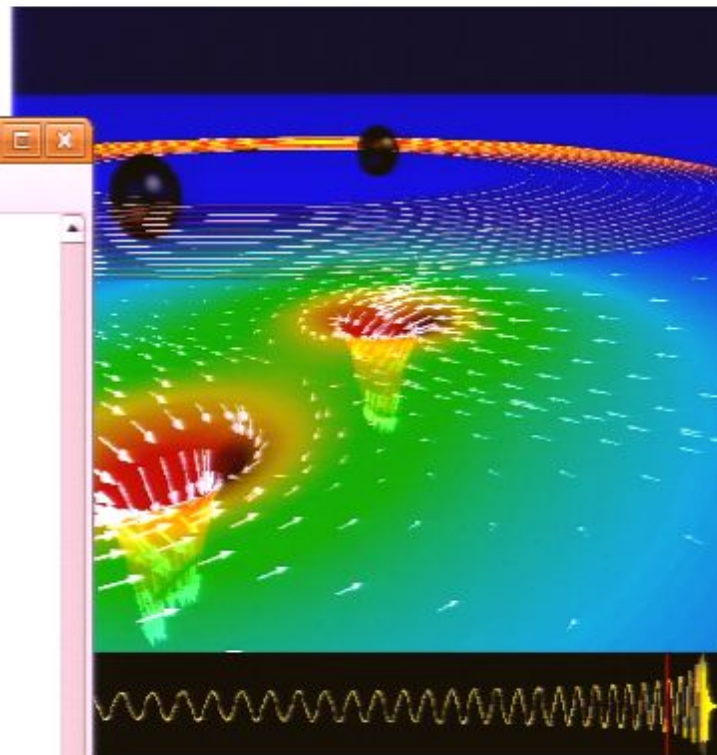
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rald@Neville Movies]$ mpeg_play Em
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rald@Neville Movies]$ mpeg_play Bbh
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rald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_

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Previous Next 27 of 67 Best Fit

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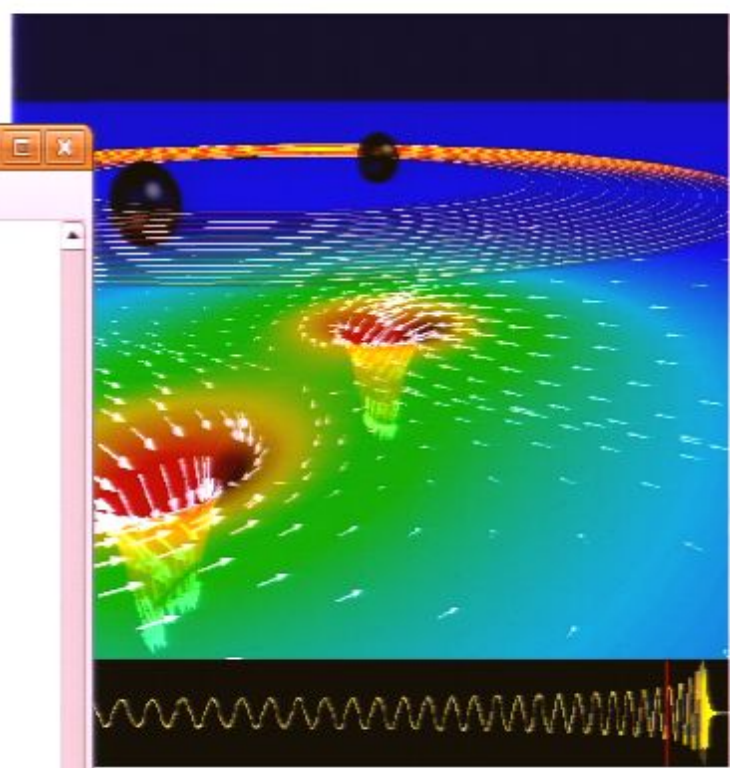
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harald@Neville Movies]$ mpeg_play Bbh
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Me
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Medium.

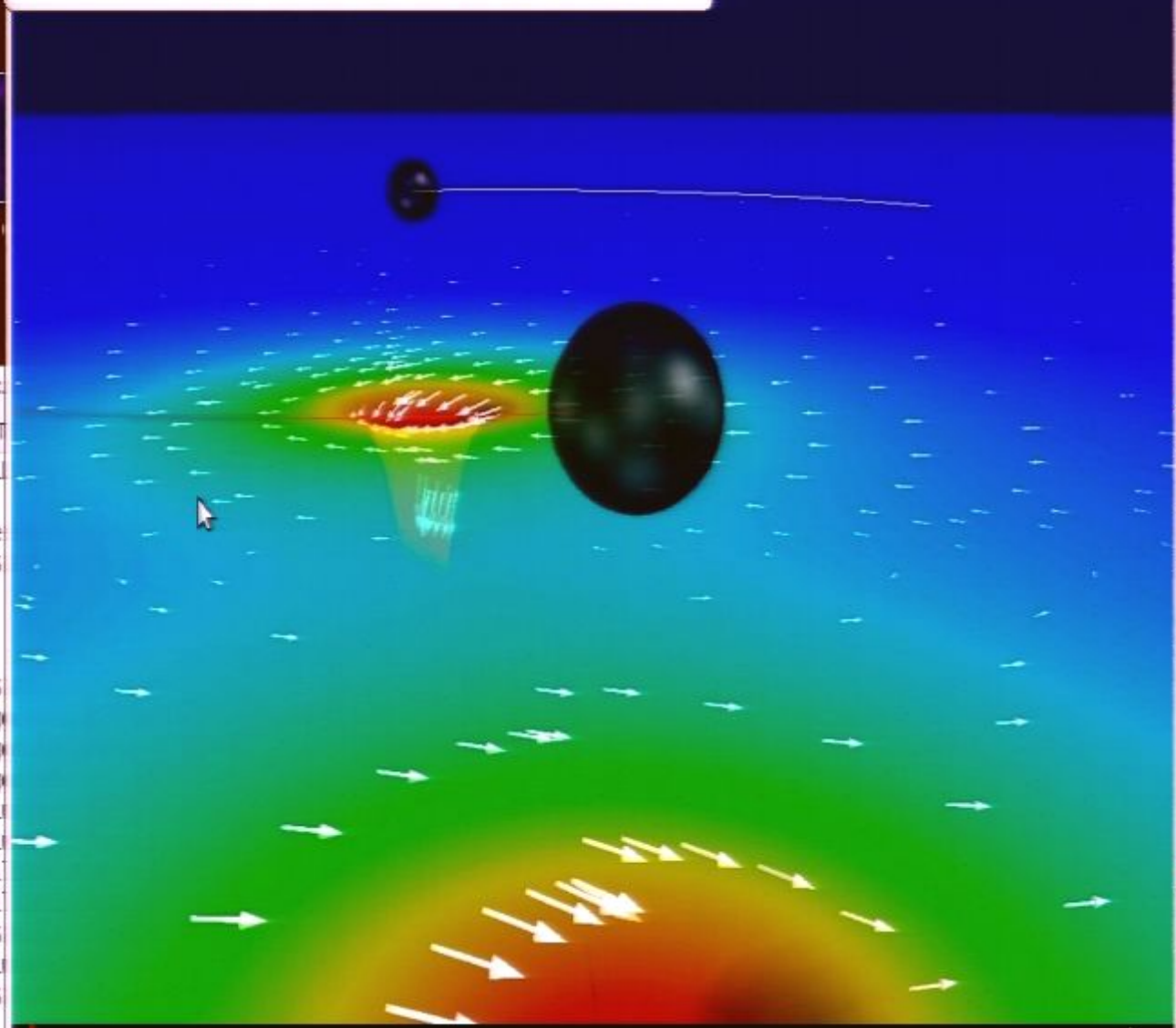
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www.black-holes.org/explore2.html

MPEG Player Controls

Frames/Rate 49/25.3 Rewind Pause Step Play Loop OFF Exit



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rald@Neville Movies]$

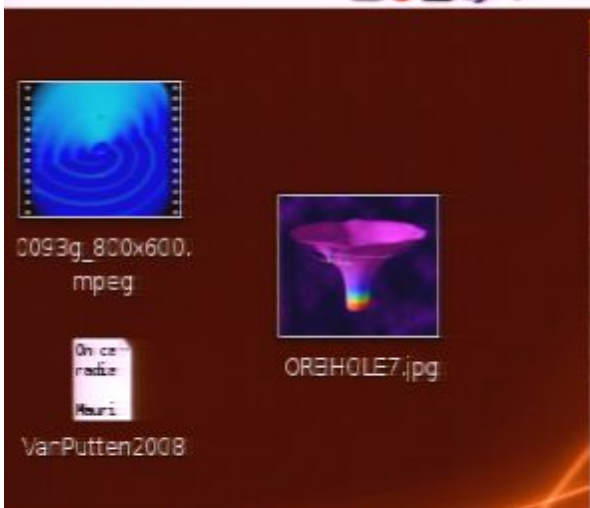
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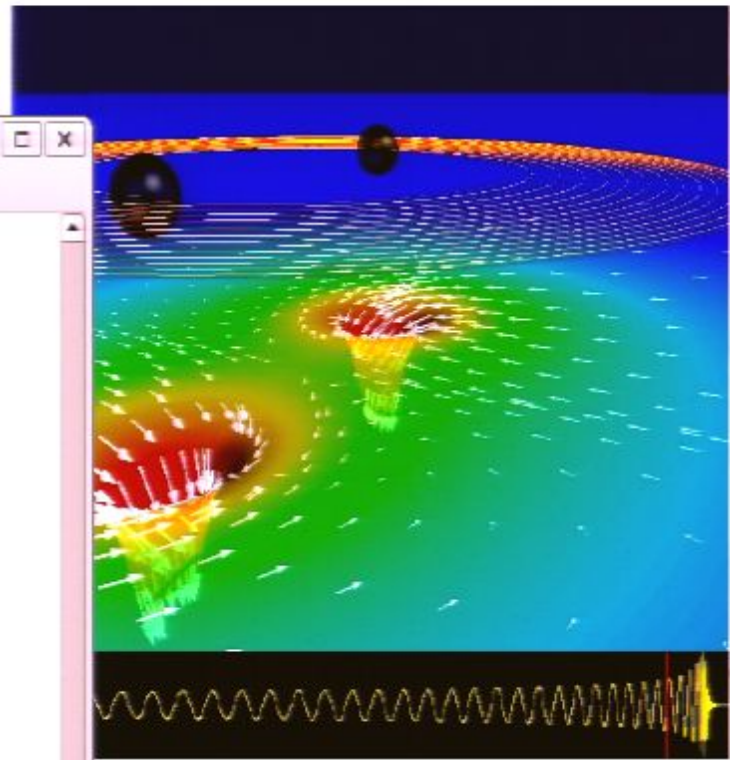


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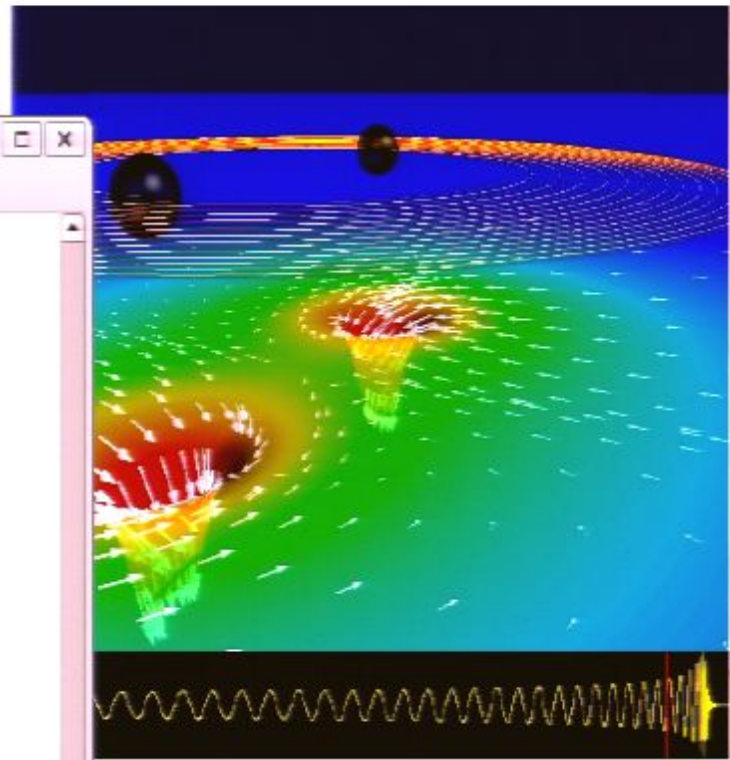
Movies I



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harald@Neville 08May_Perimeter]$ cd Movies
harald@Neville Movies]$ mpeg_play Em
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harald@Neville Movies]$ mpeg_play Bbh
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r Me
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Medium.mpg

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harald@Neville Movies]$
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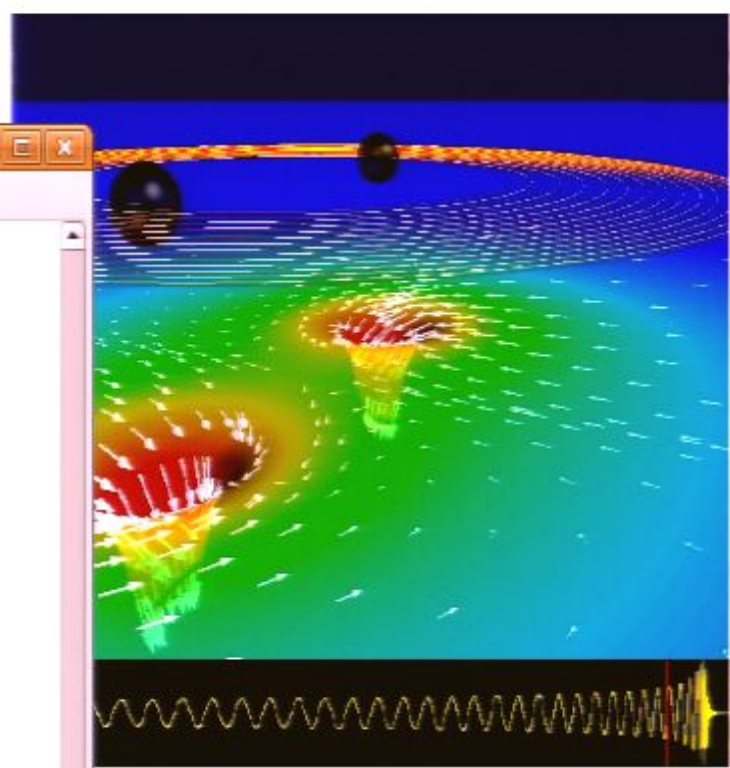
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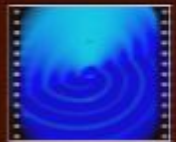
harald@Neville:AA~/research/Talks/08May_Perimeter/Movies
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harald@Neville 08May_Perimeter]$ cd Movies
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Medium.mpg

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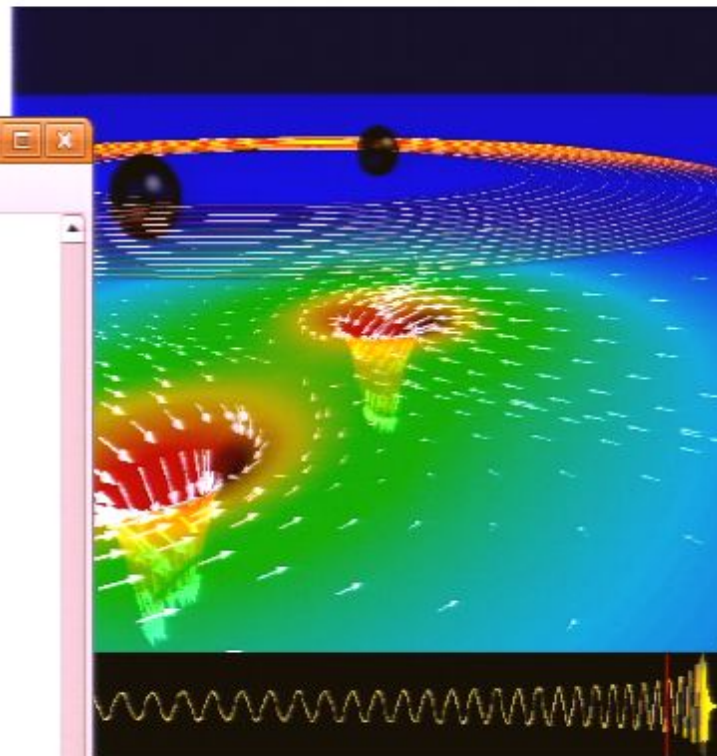
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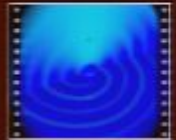
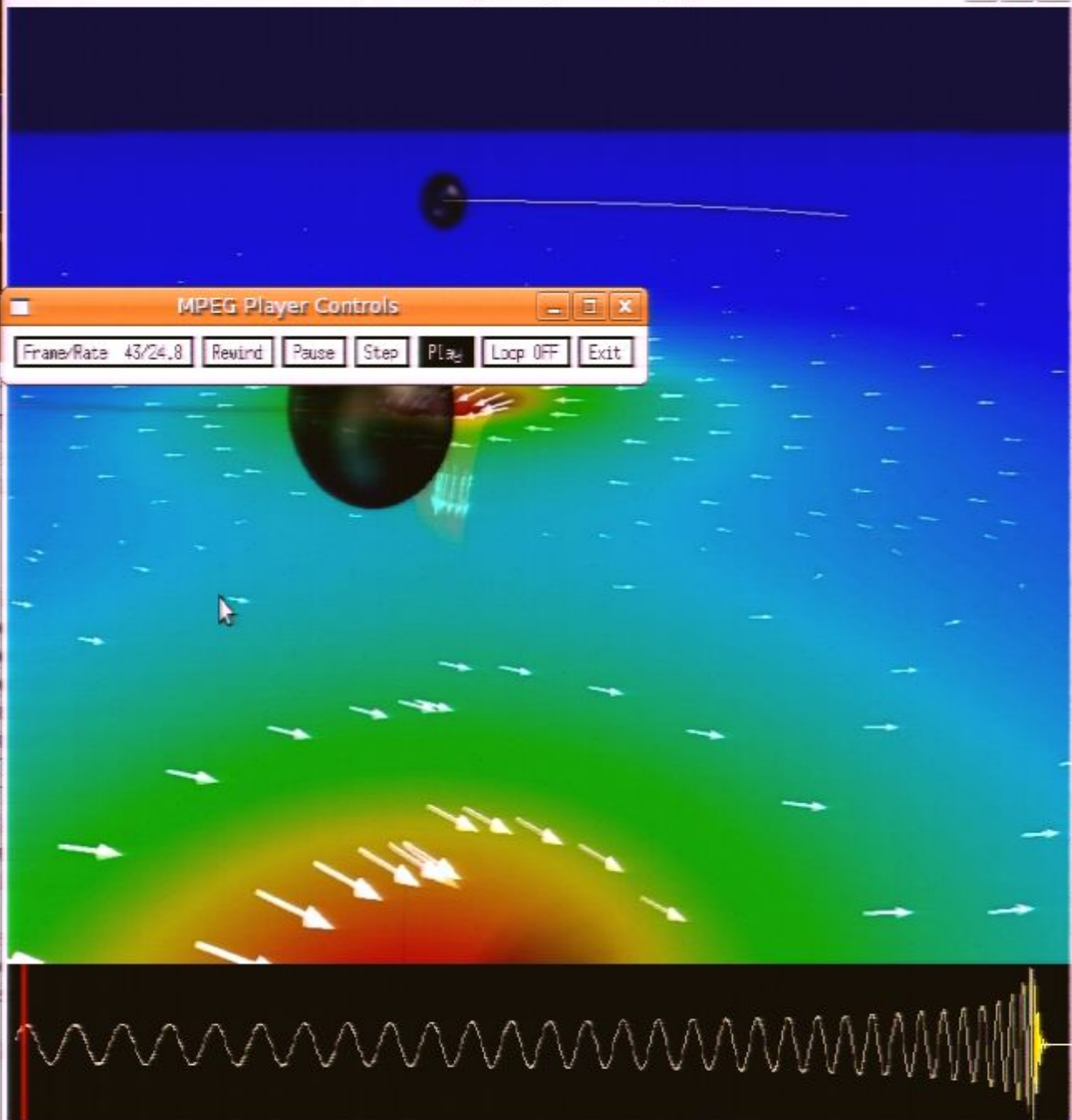
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harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Medium.mpg

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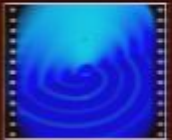
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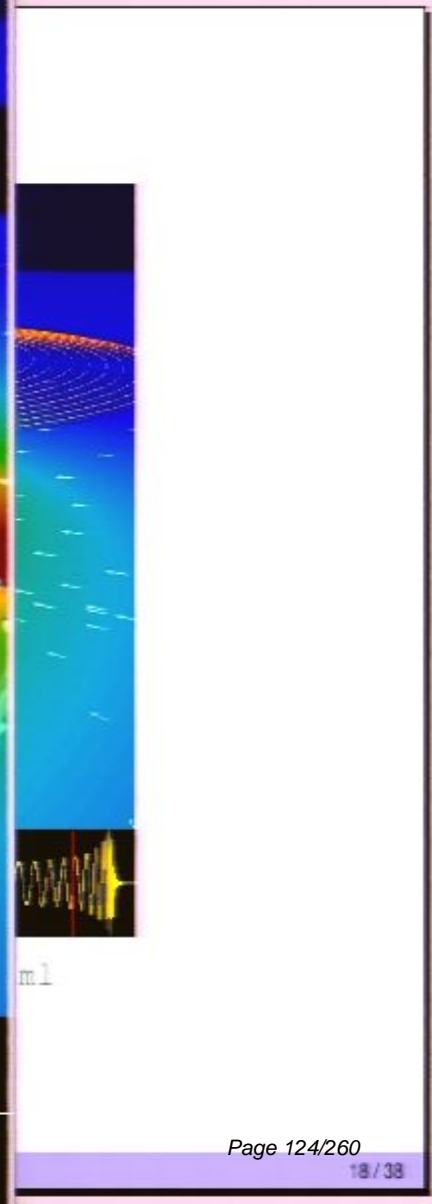
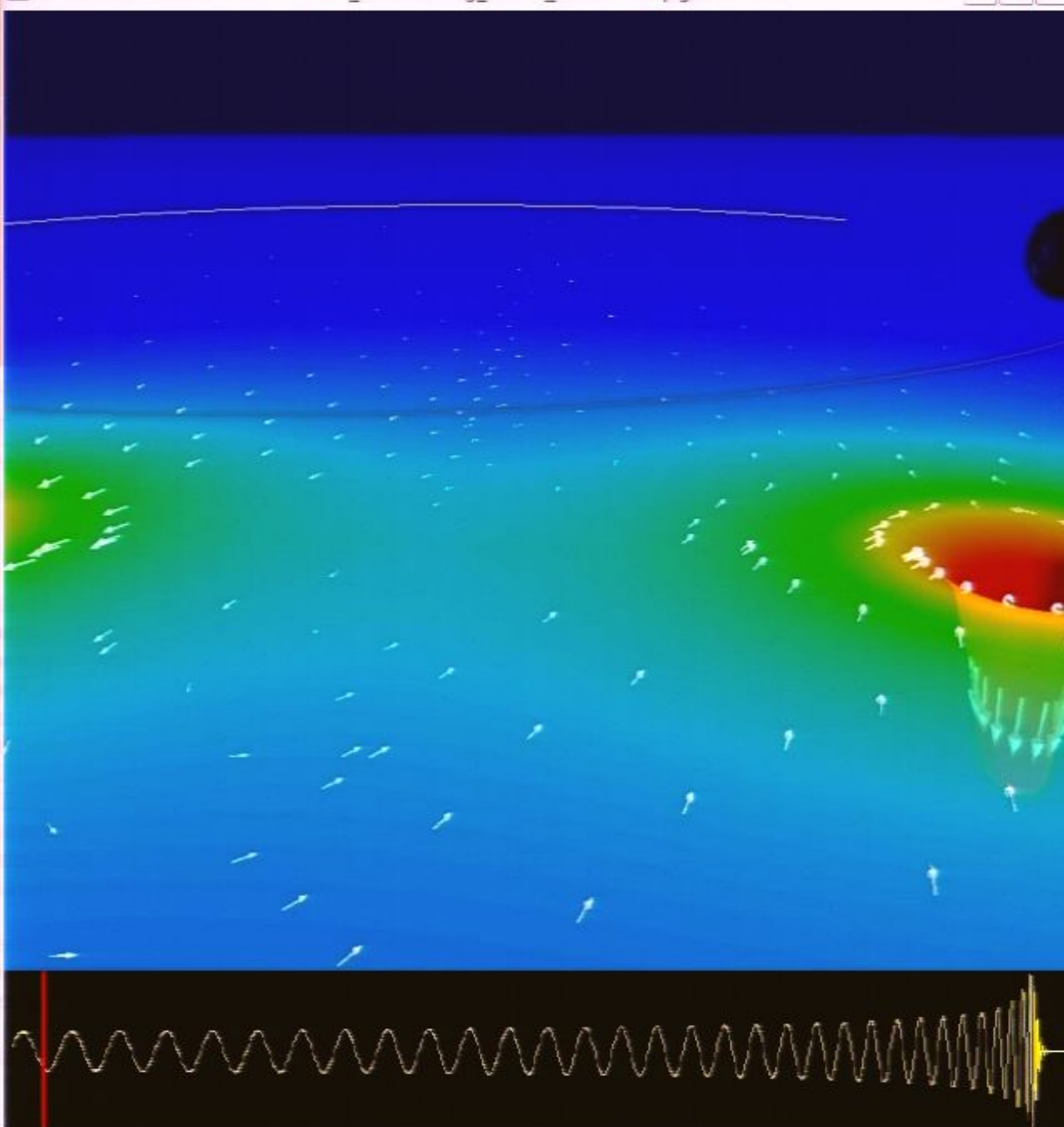
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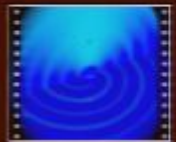
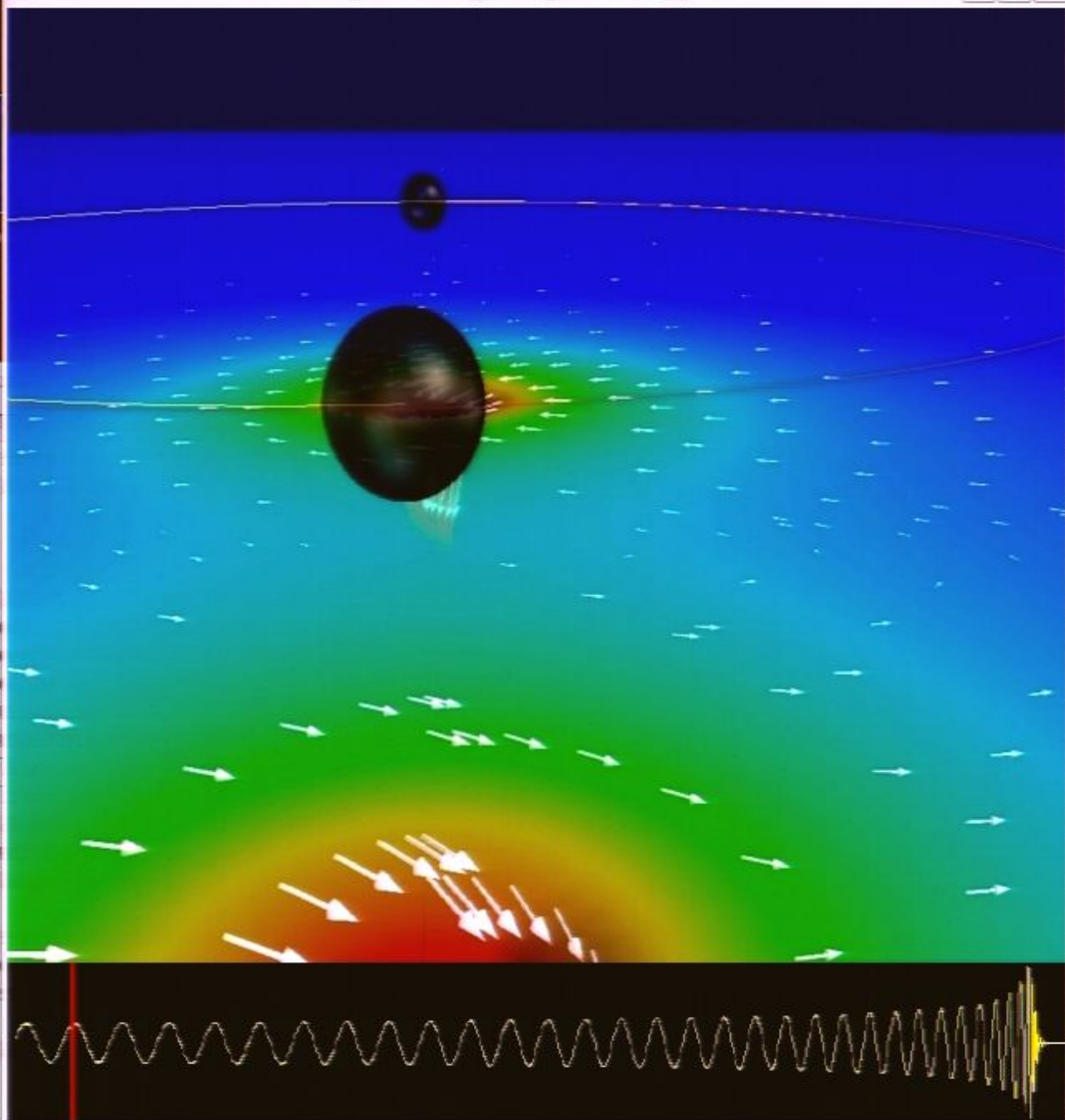
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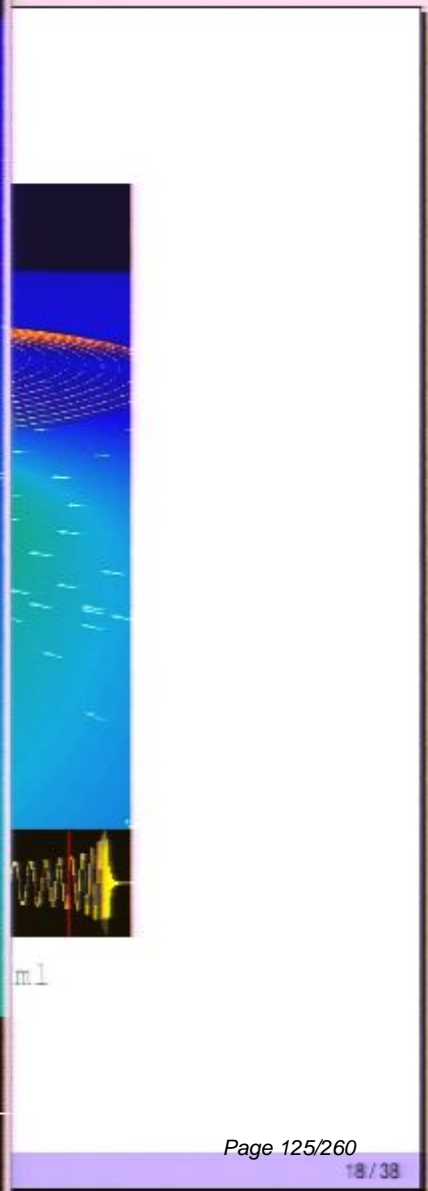
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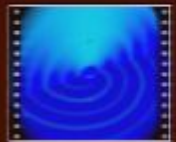
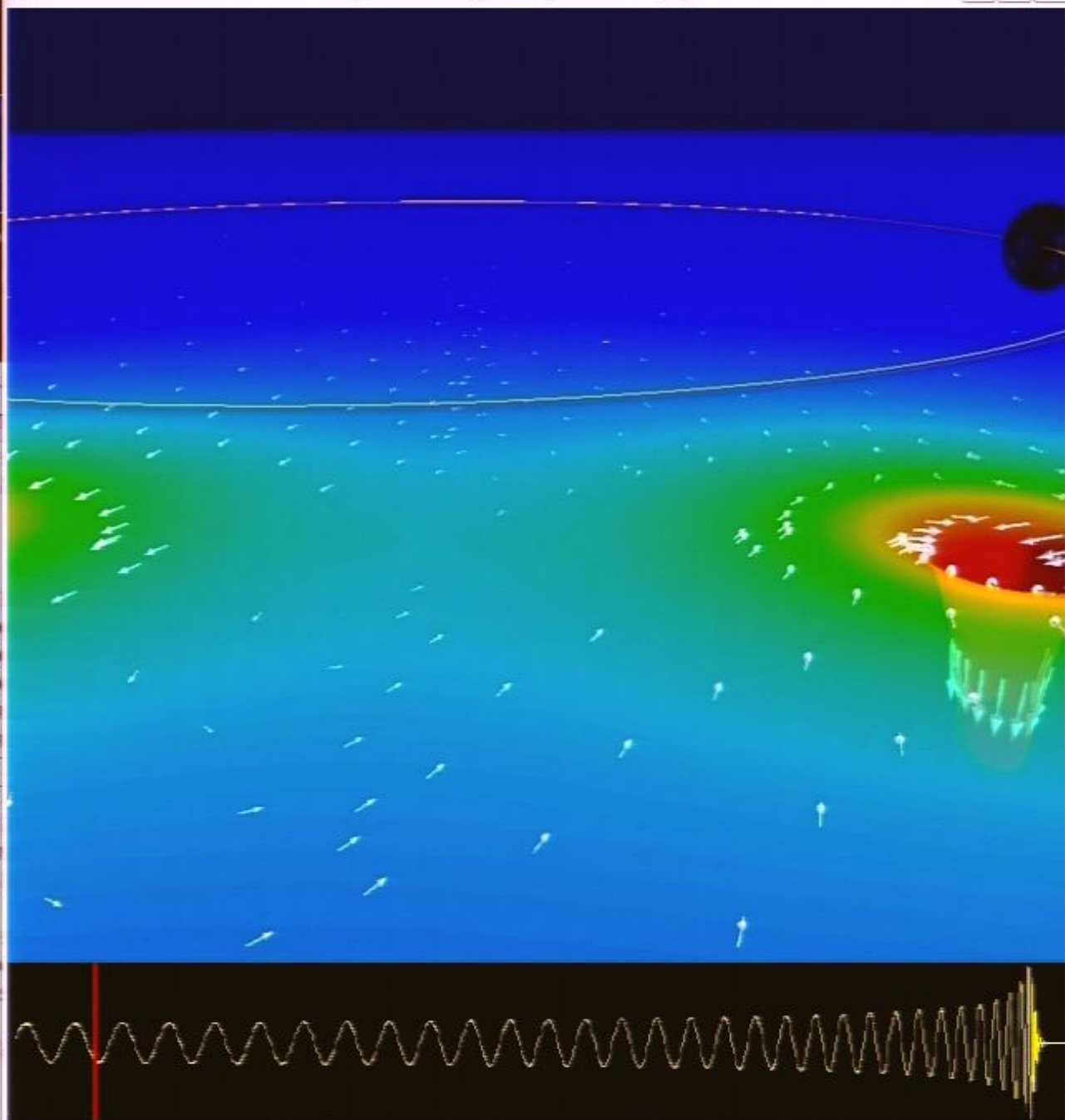
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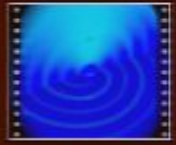
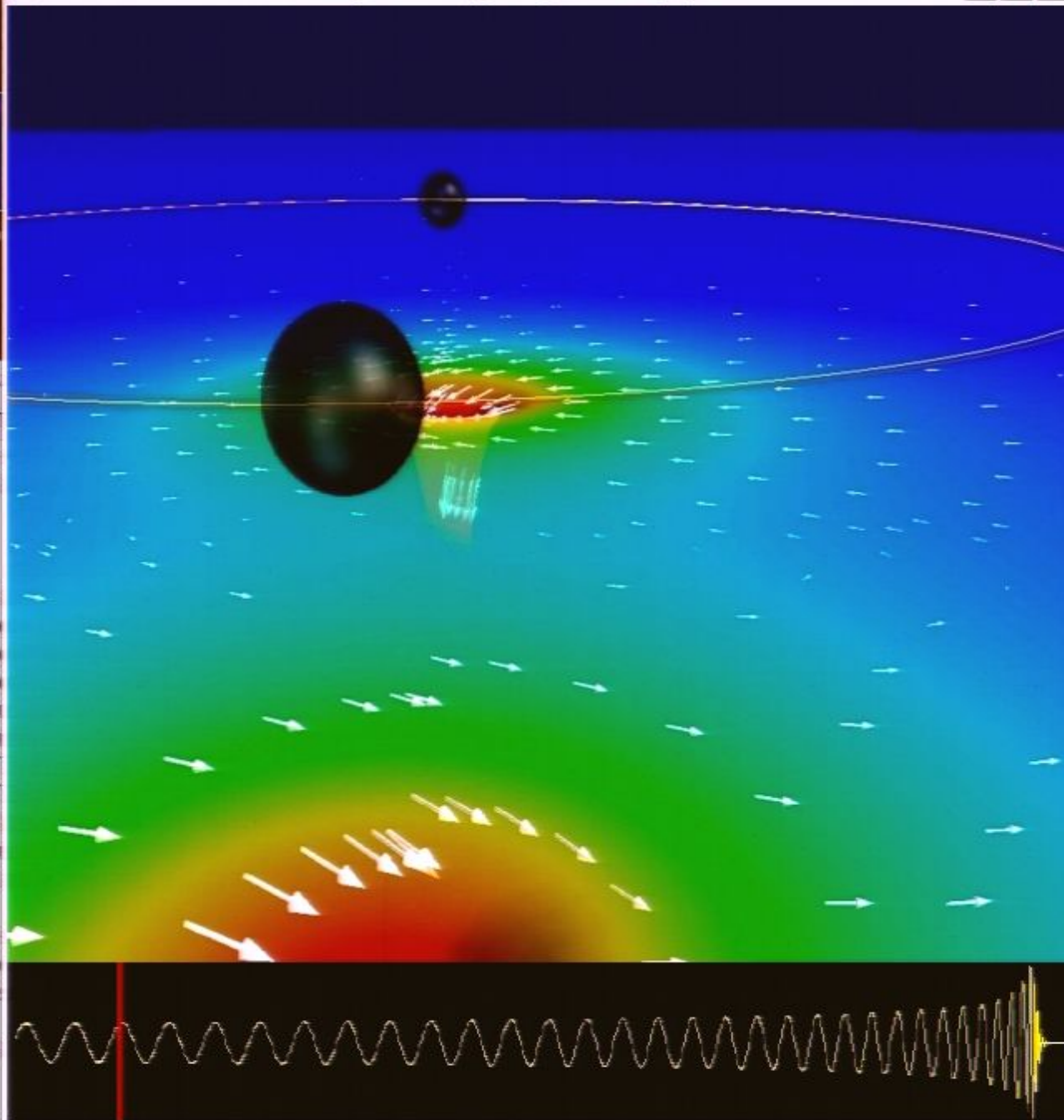
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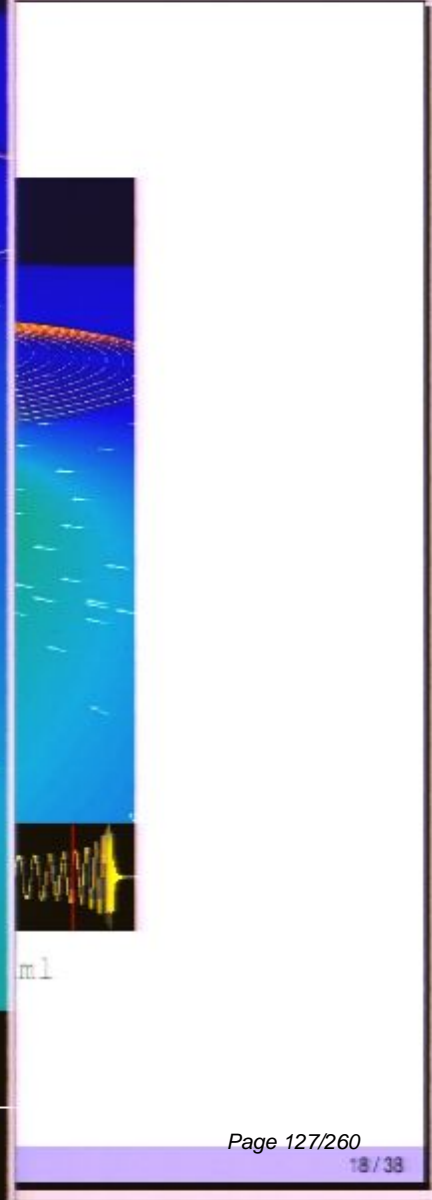
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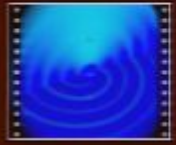
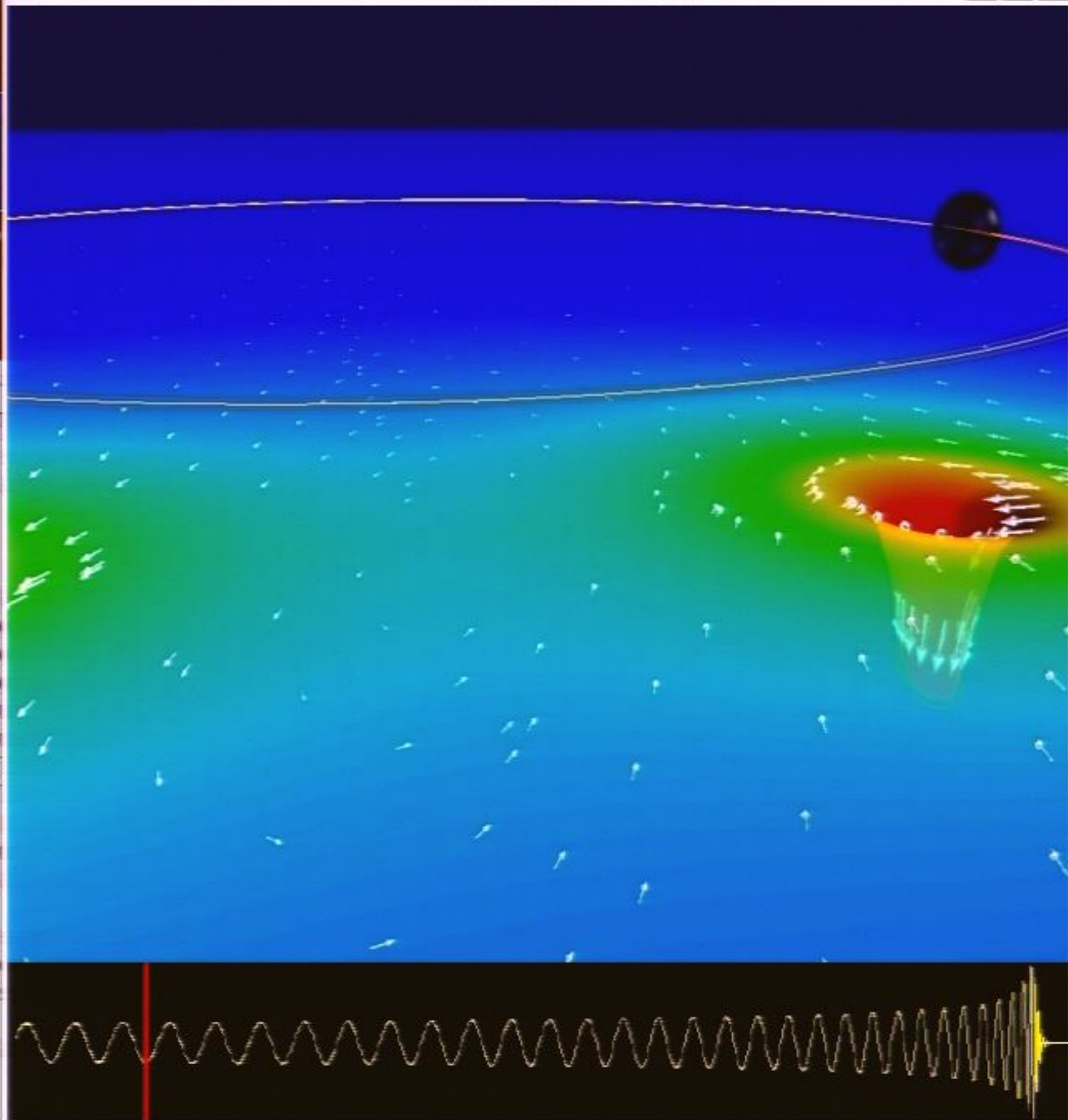
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Numerical study of black hole spacetimes

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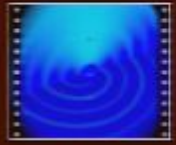
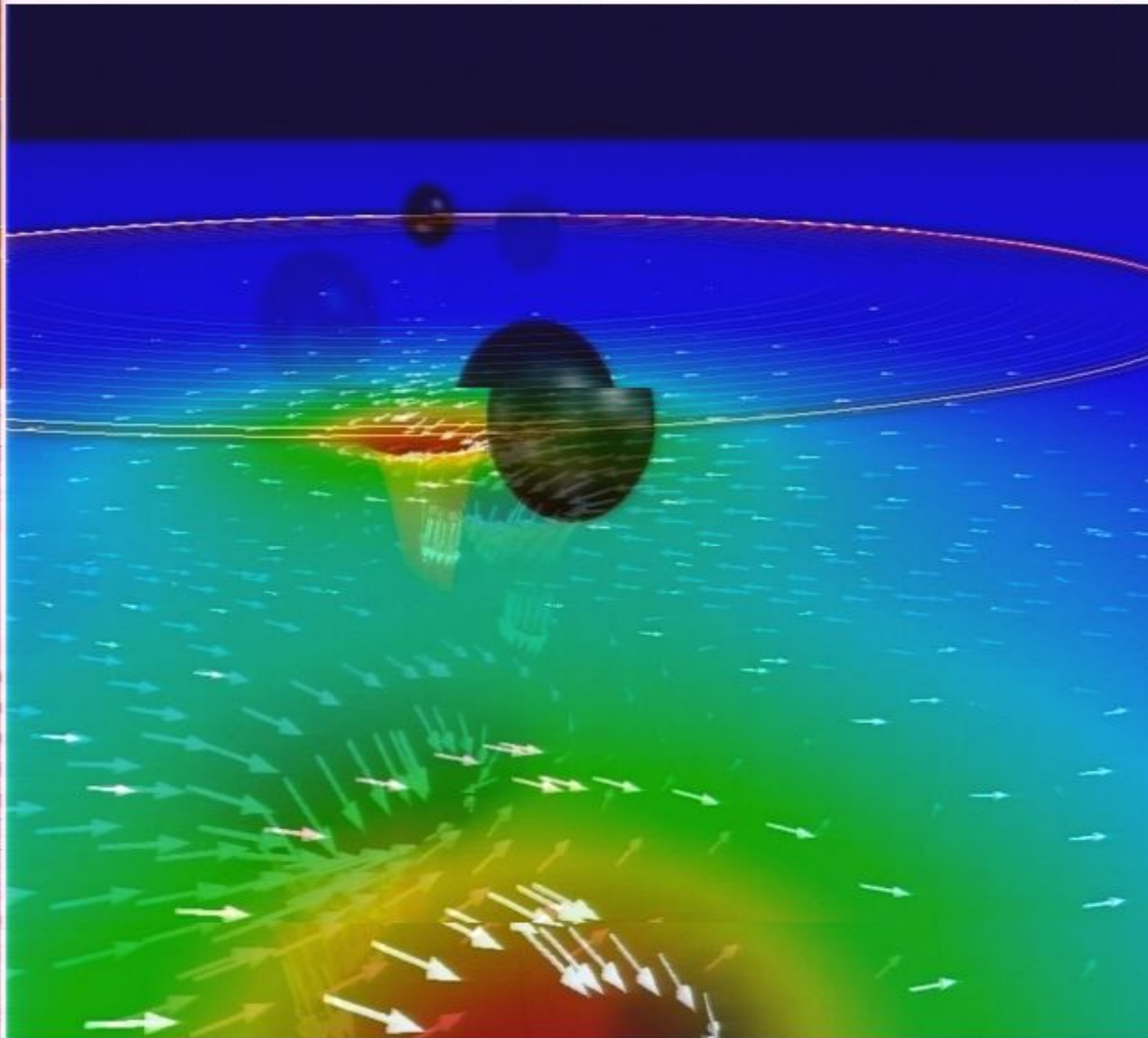
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Numerical study of black hole spacetimes

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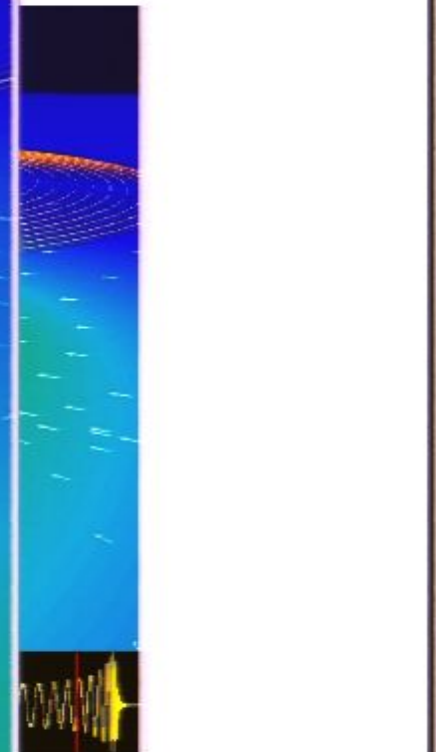
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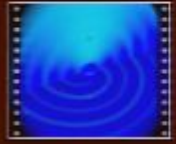
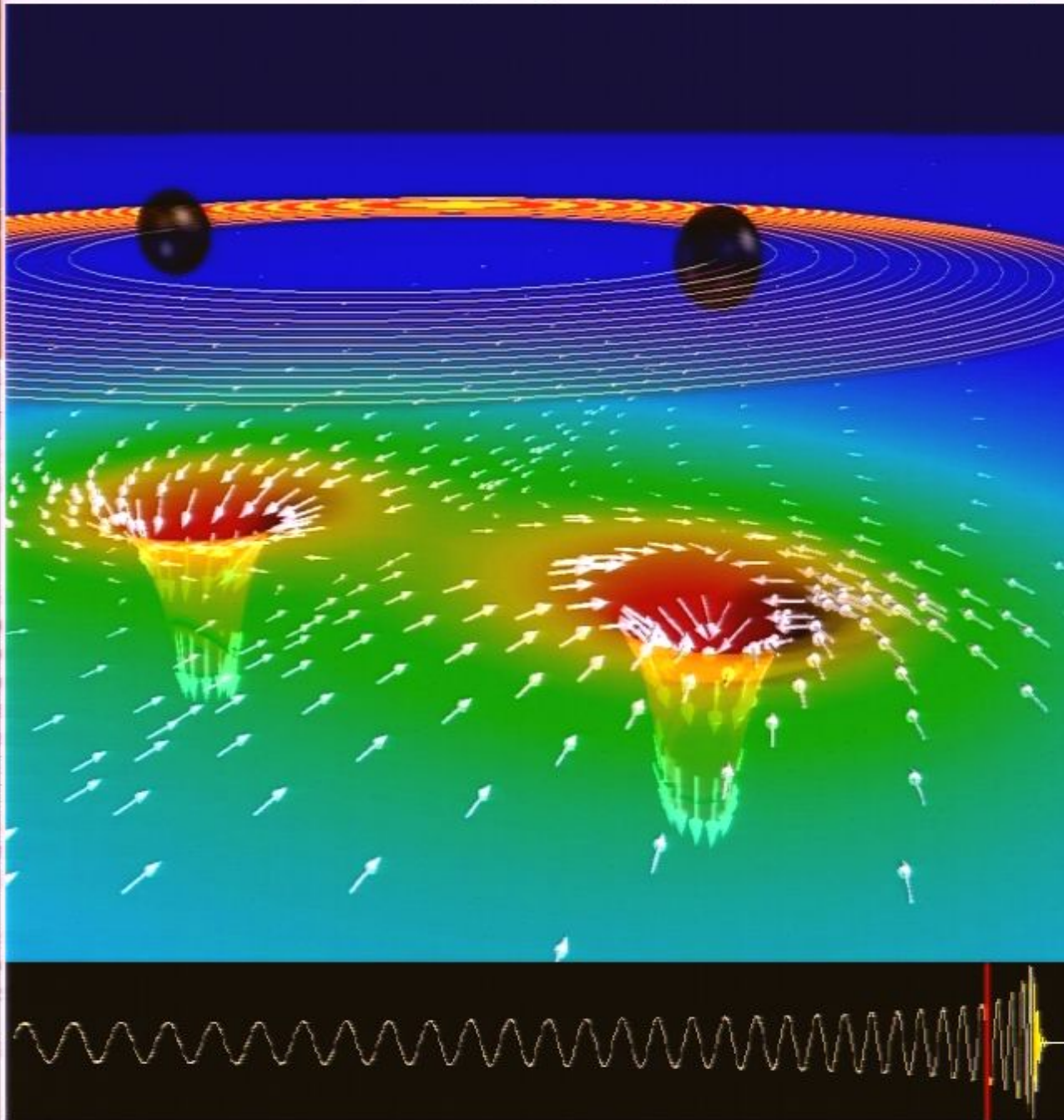
Pirsa: 08050036



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Numerical study of black hole spacetimes

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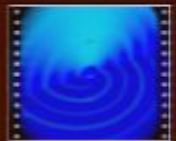
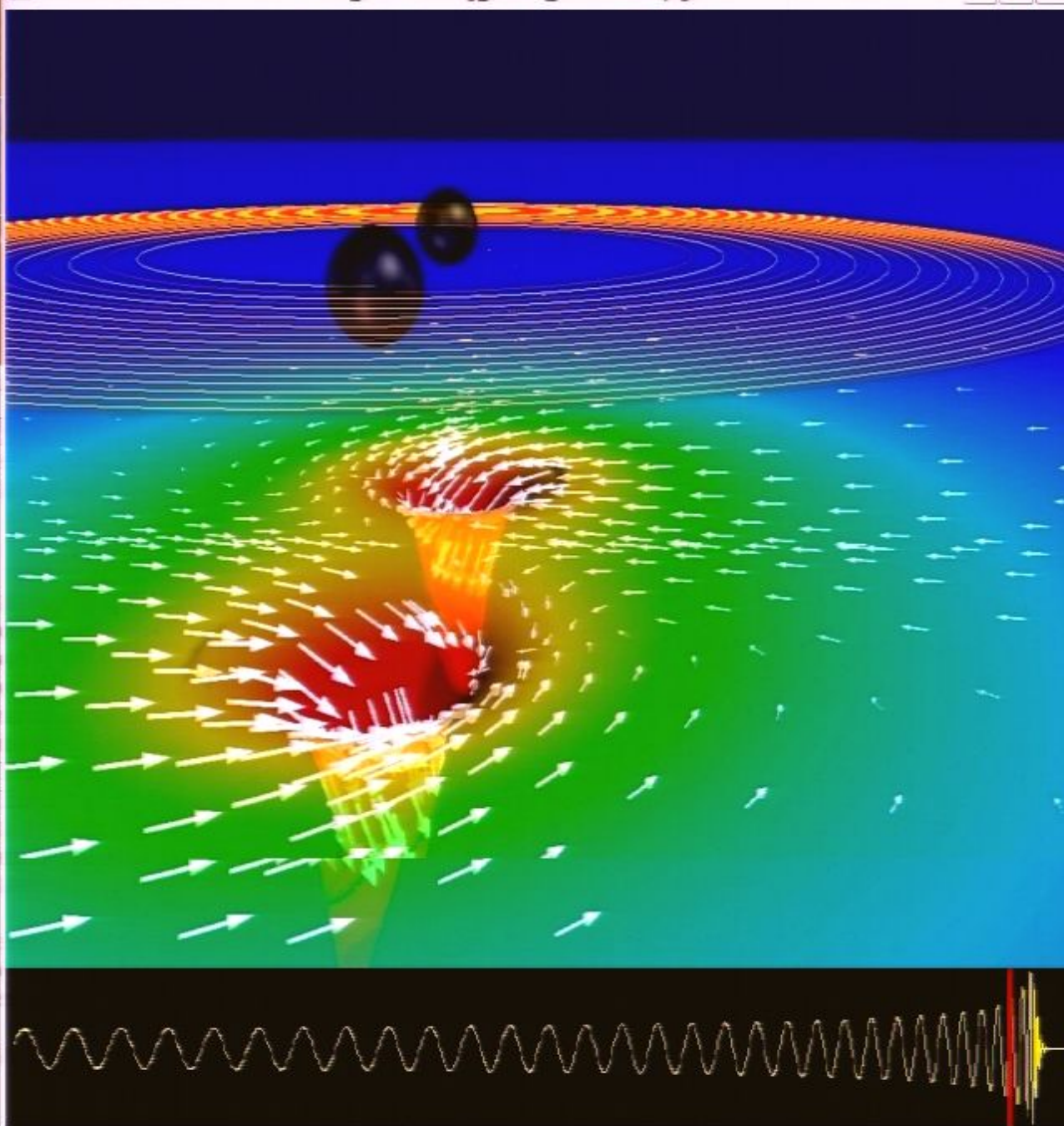
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Numerical study of black hole spacetimes

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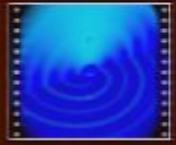
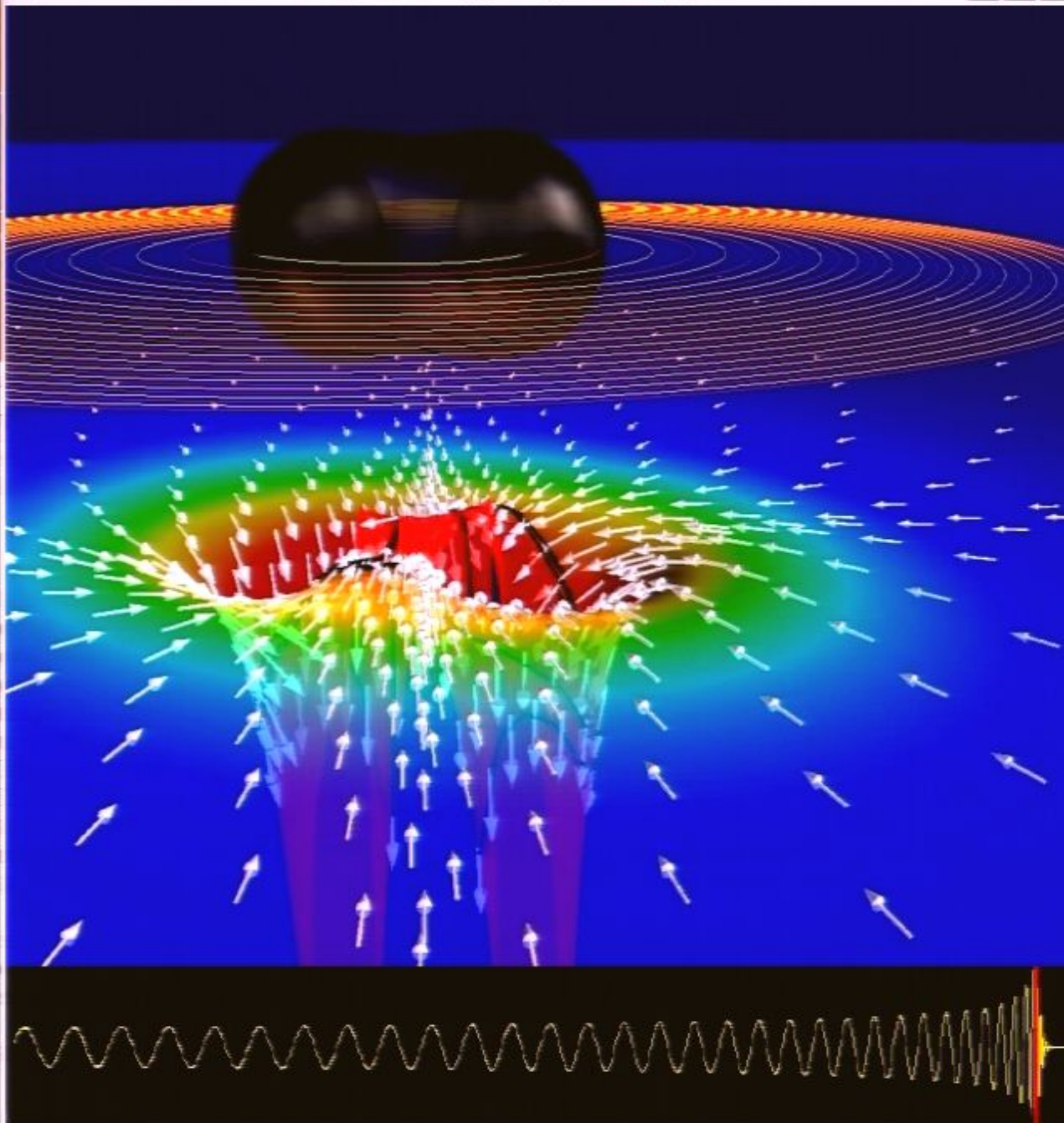
harald@Neville Movies]\$

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Numerical study of black hole spacetimes

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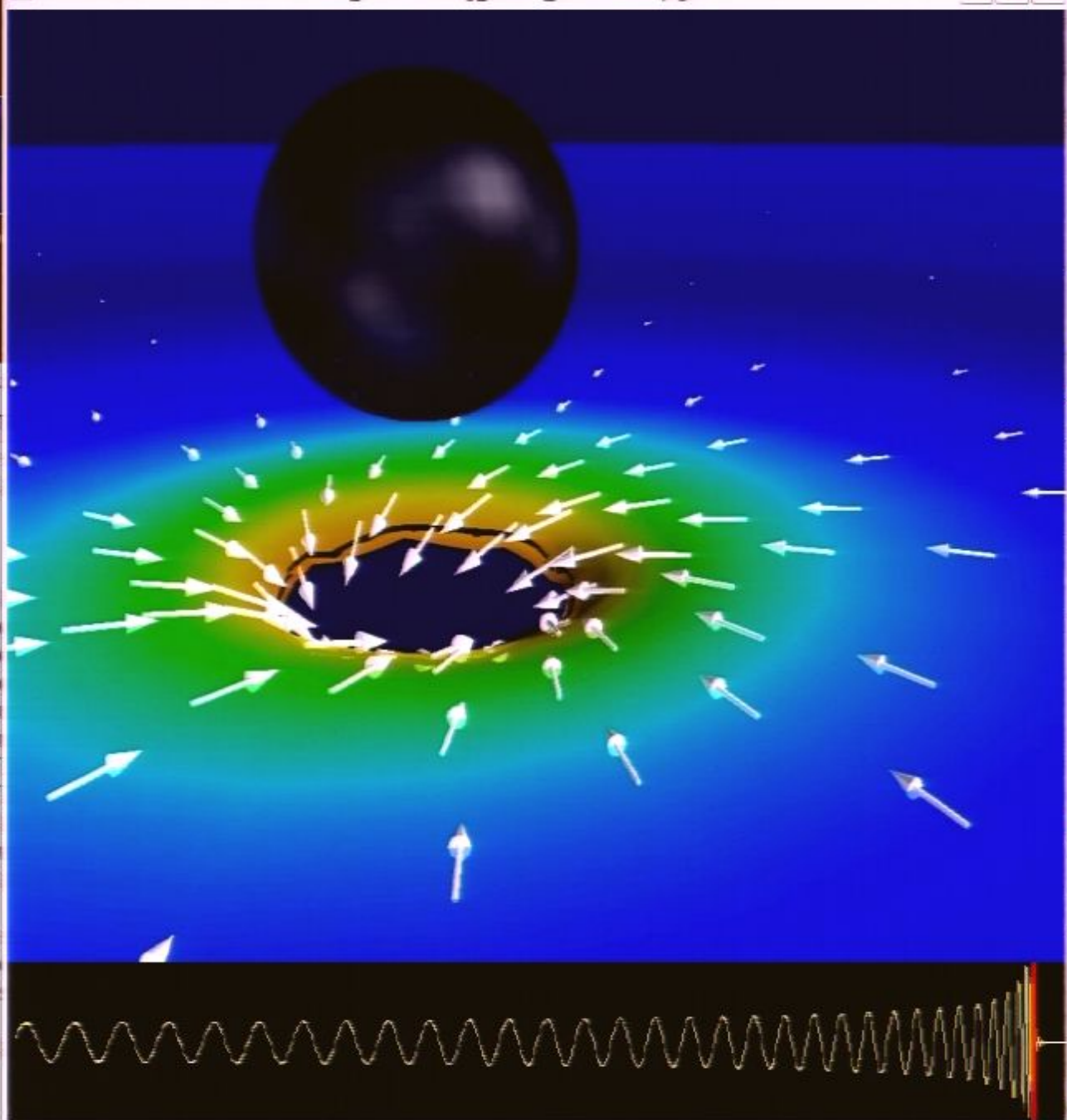
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Numerical study of black hole spacetimes

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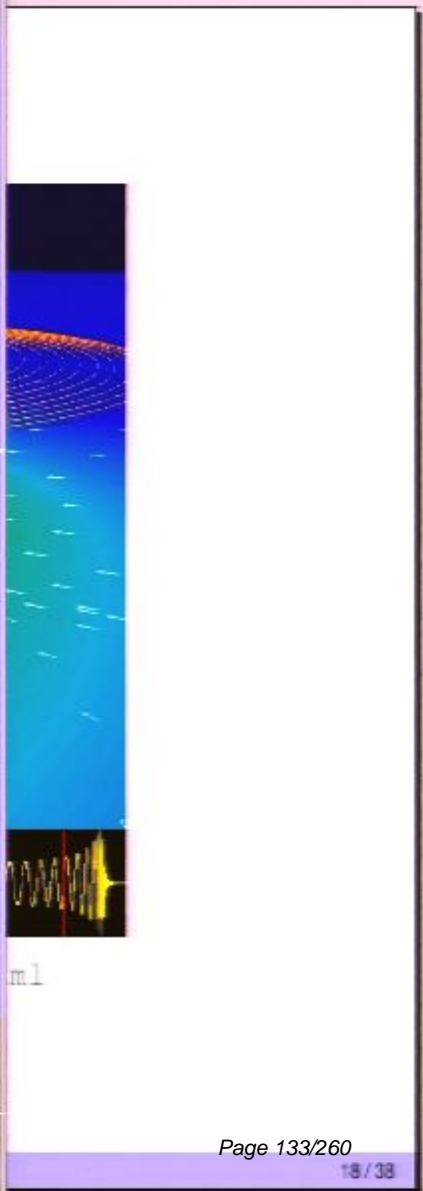
Loop OFF Exit

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24 bit displays: use
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harald@Neville Movies]$

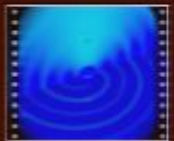
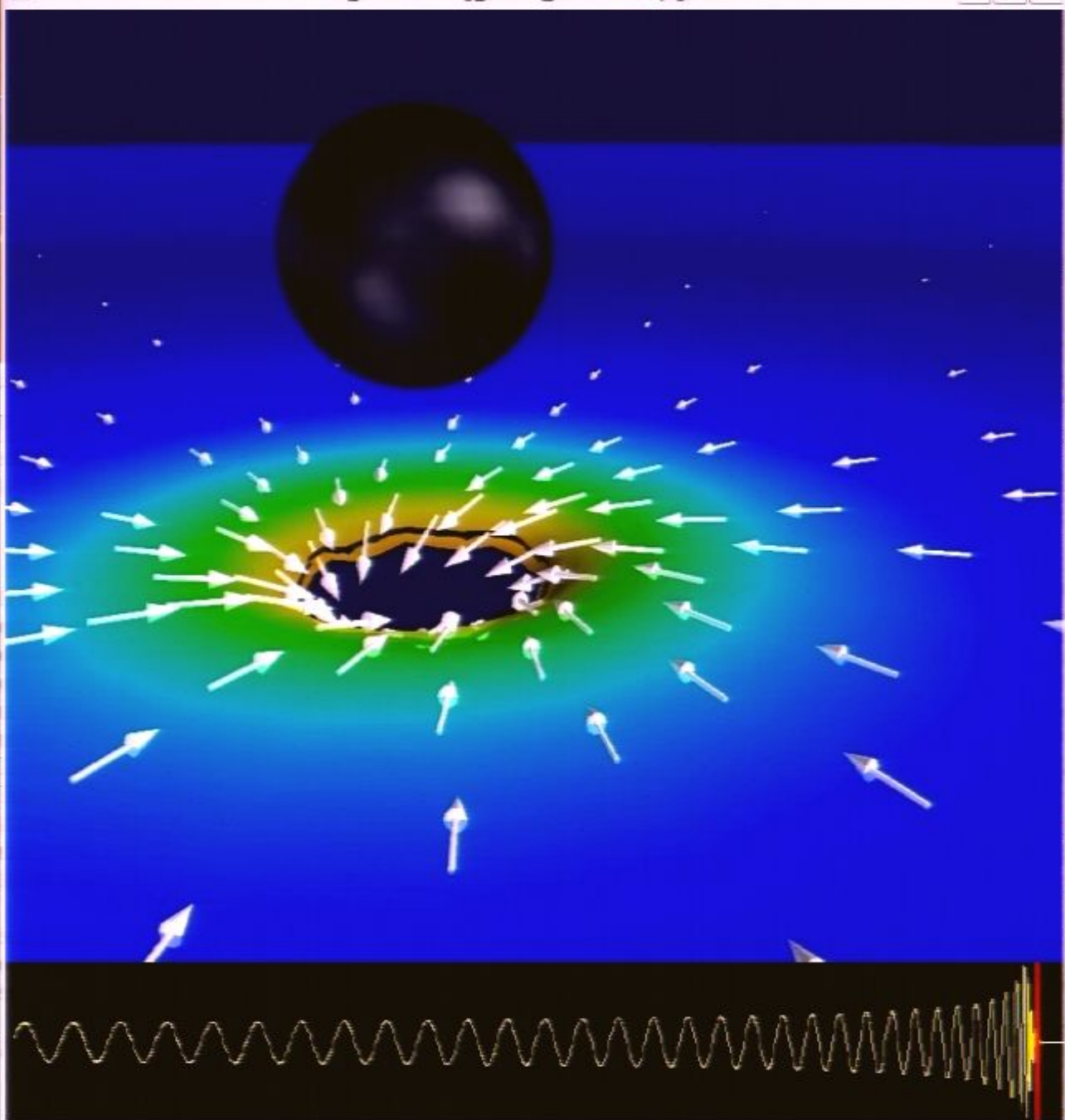
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Pirsa: 08050036



Numerical study of black hole spacetimes

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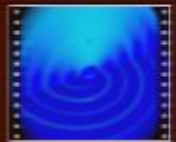
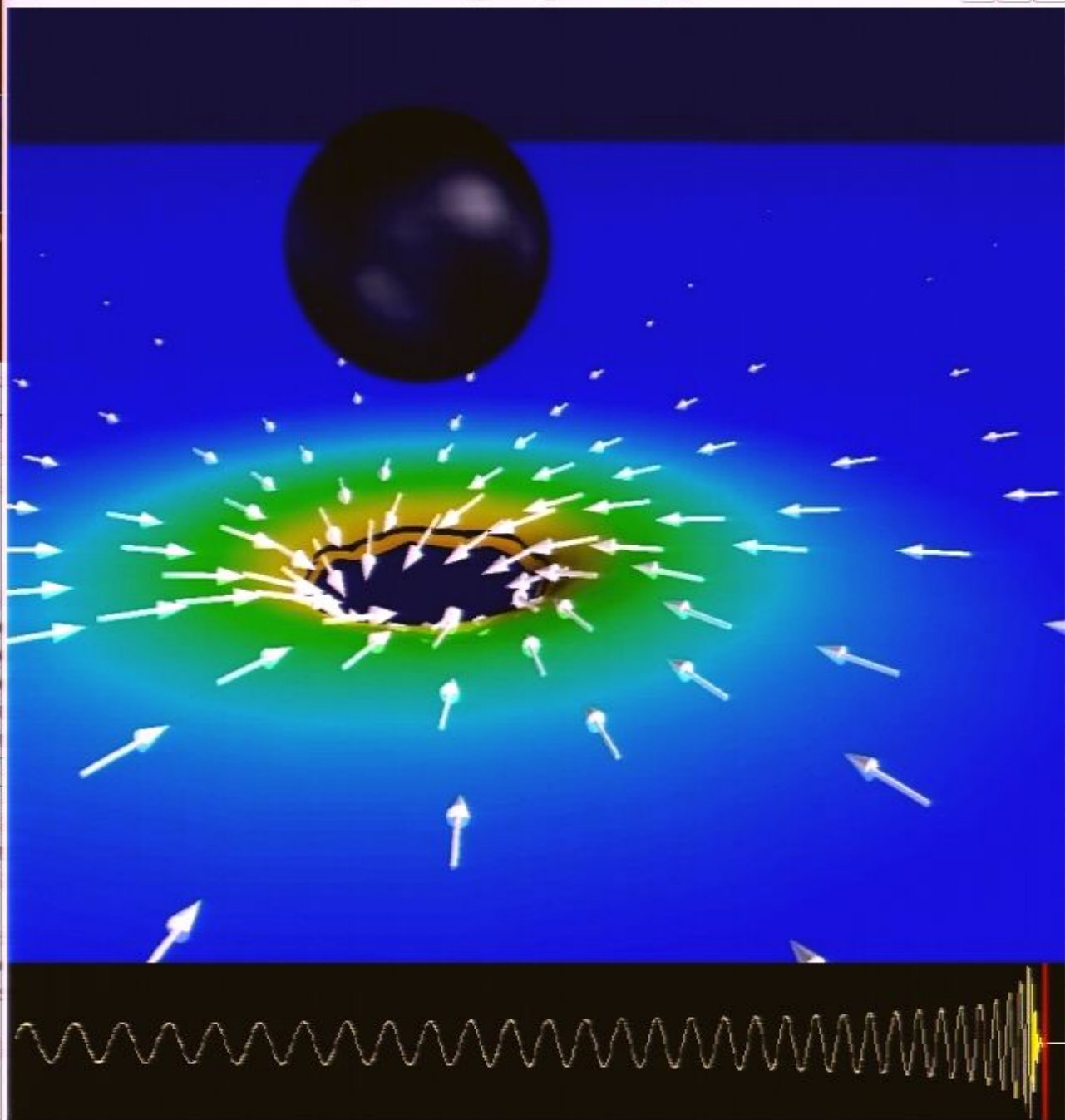
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Numerical study of black hole spacetimes

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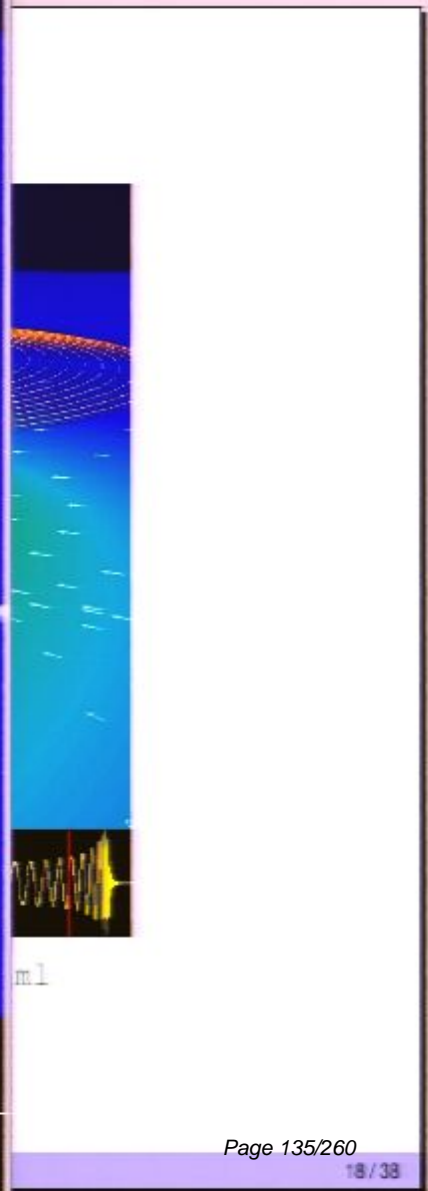
Loop OFF Exit

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harald@Neville Movies]S

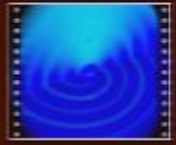
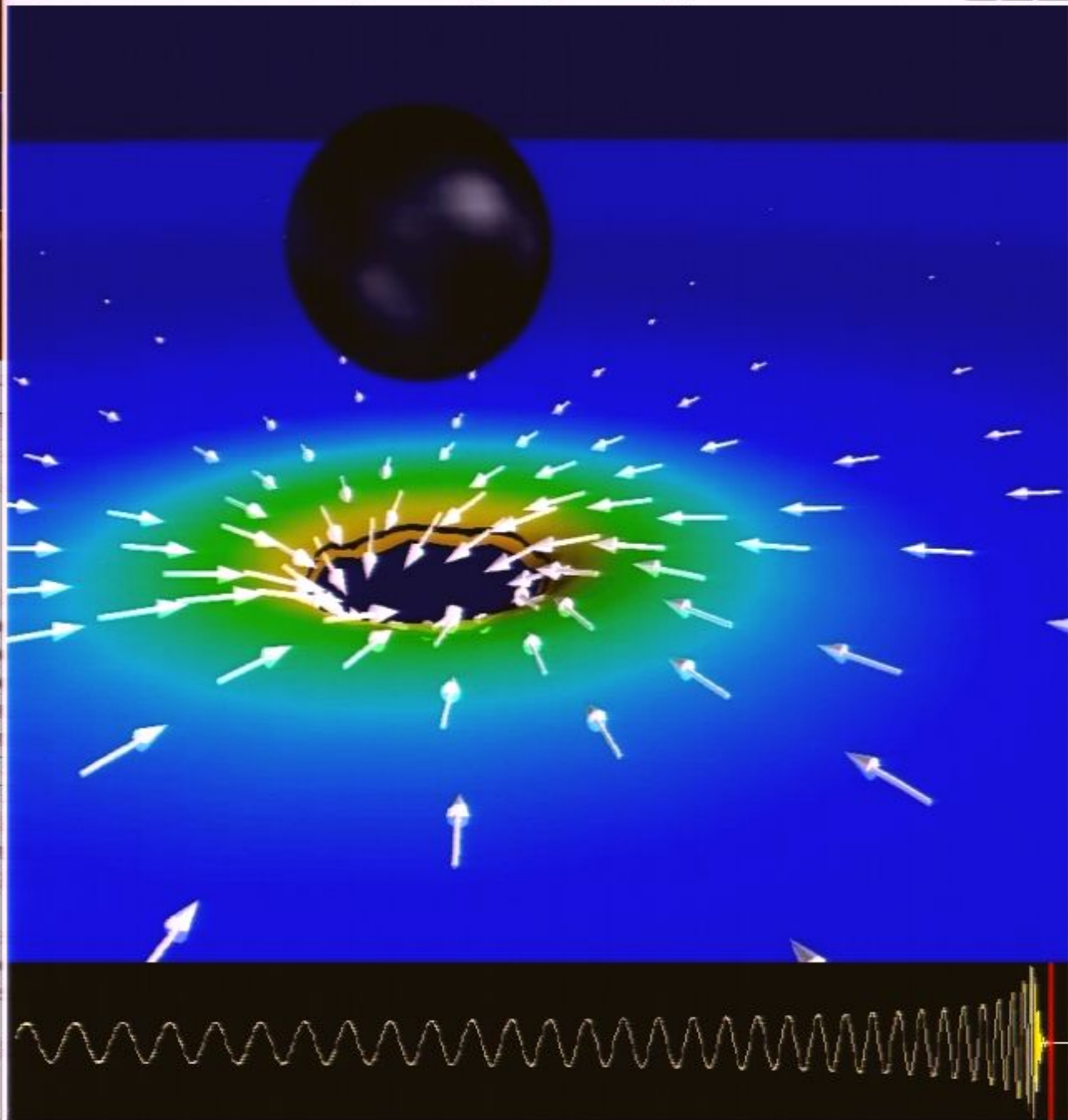
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Numerical study of black hole spacetimes

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harald@Neville Movies]$

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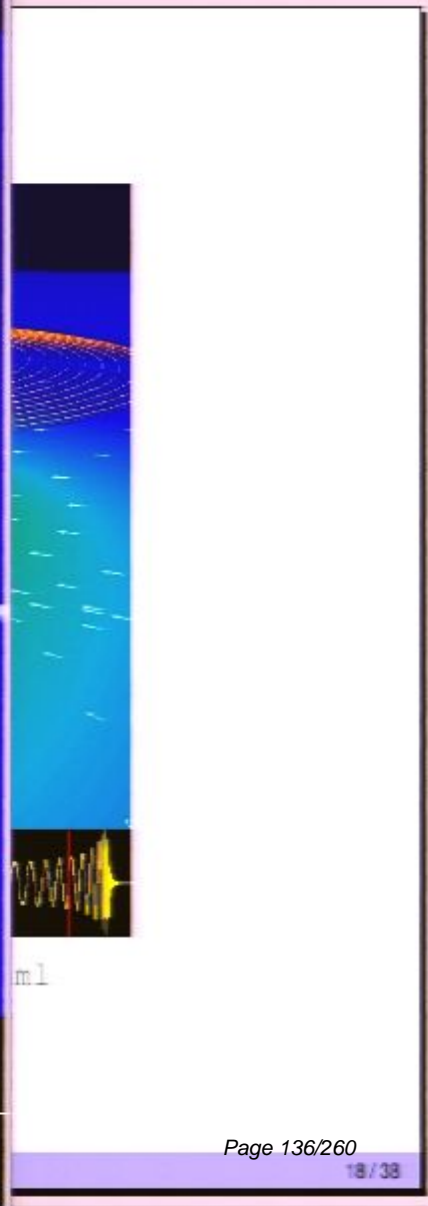
Loop OFF Exit

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24 bit displays: use
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harald@Neville Movies]$

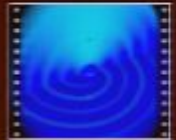
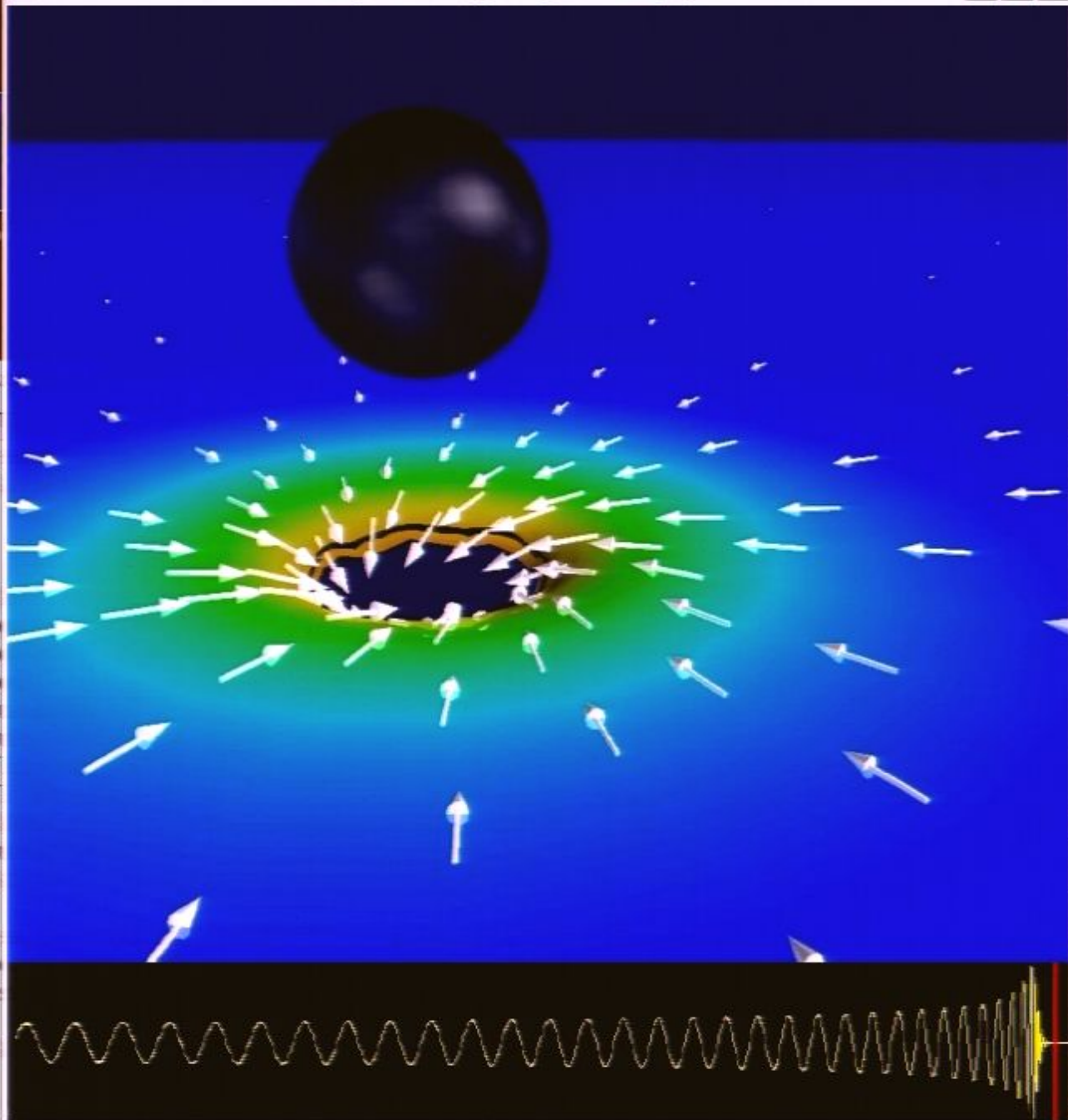
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Numerical study of black hole spacetimes

Bbh_Embedding_0093r_Medium.mpg



0093g_800x600.mpeg

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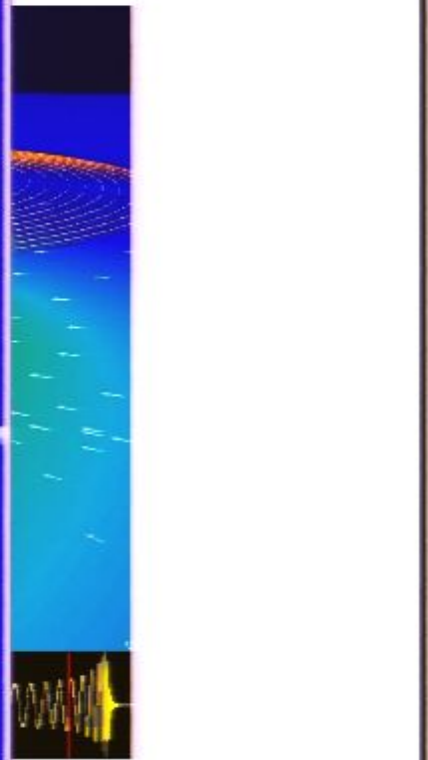
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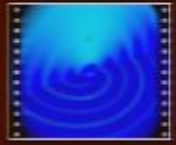
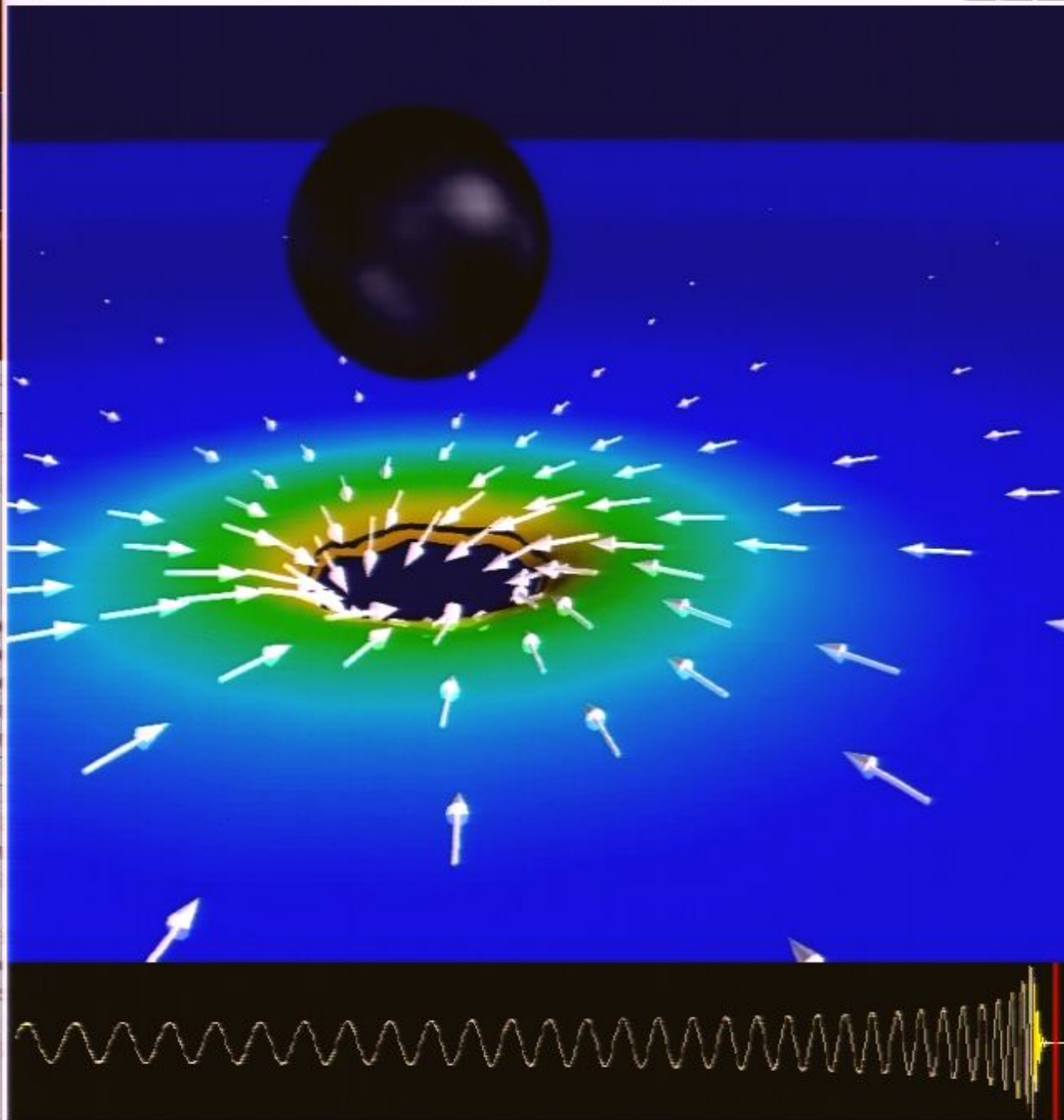
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Numerical study of black hole spacetimes

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Once again

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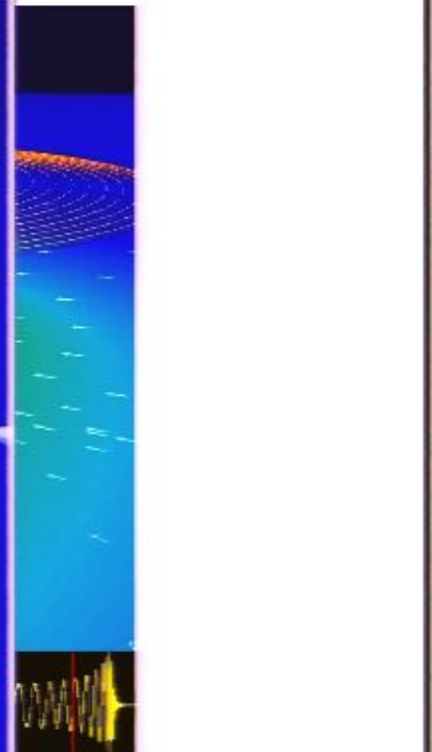
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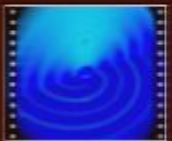
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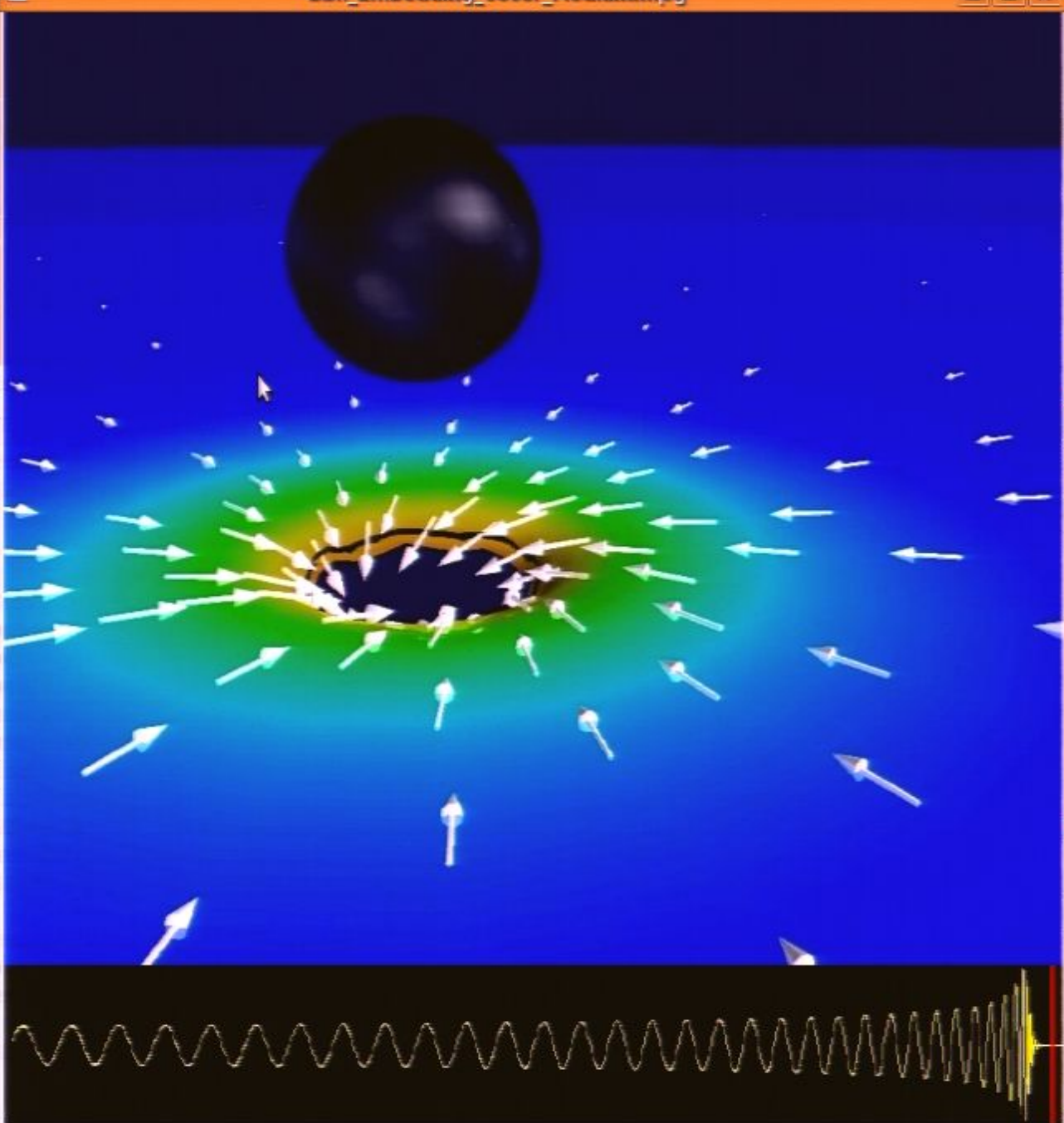
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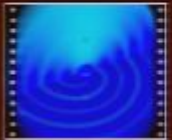
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Numerical study of black hole spacetimes

Bbh_Embedding_0093r_Medium.mpg



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harald@Neville Movies]$
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Loop OFF Exit
harald@Neville Movies]$

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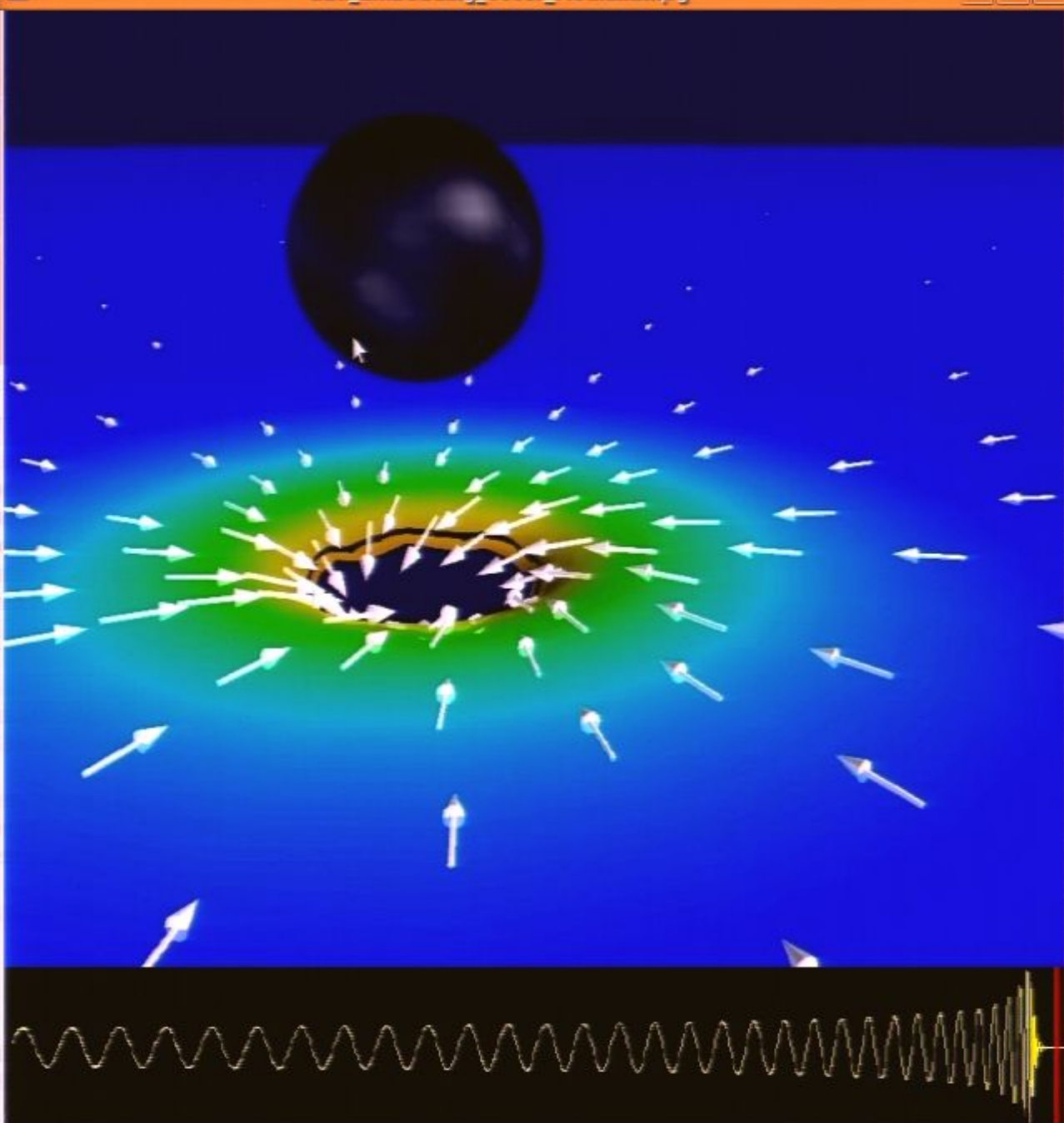
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cified dither require
ing -dither color
harald@Neville Movies]$

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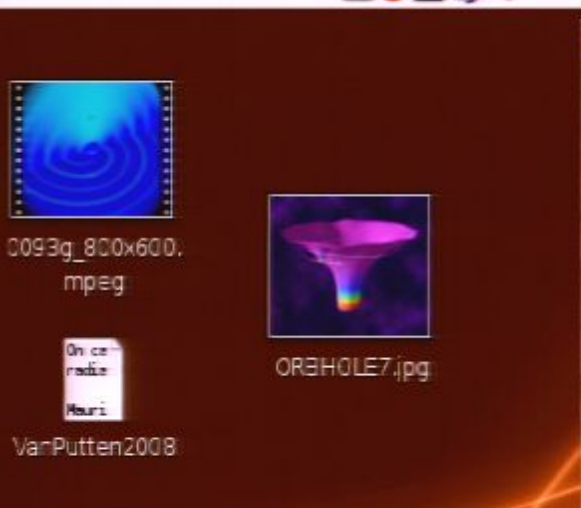
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Numerical study of black hole spacetimes

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← 27 of 67 Best Fit

Next →

Movies I

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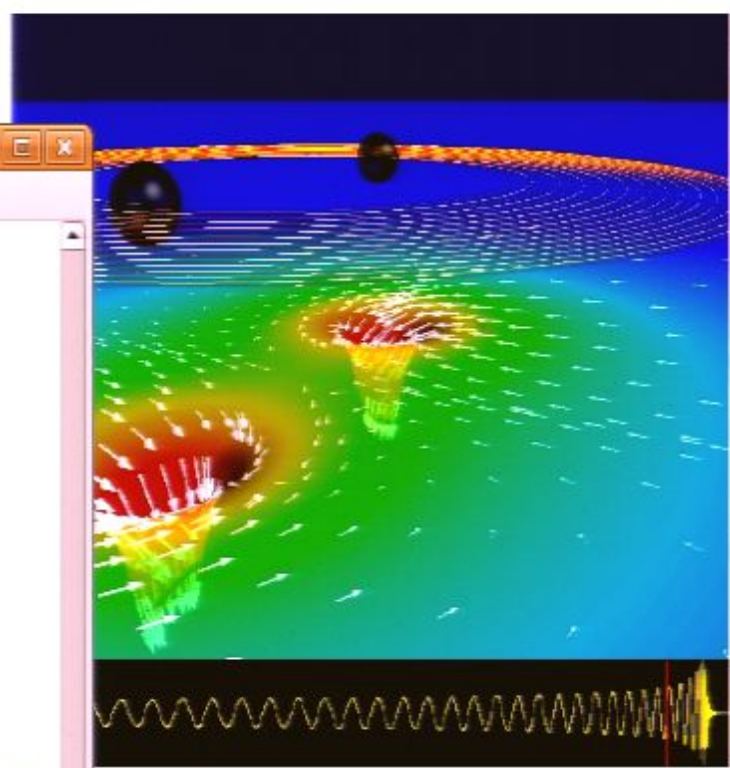
harald@Neville:AA~/research/Talks/08May_Perimeter/Movies
Edit View Terminal Tabs Help
harald@Neville Movies]$ mpeg_play Bbh
Embedding_0093r_Large_BeforeMerger.png
Embedding_0093r_Large.mpg
Embedding_0093r_Large.png
Embedding_0093r_Medium.mpg
Embedding_0093r_Medium.png
Embedding_0093r_Small_BeforeMerger.jpg
Embedding_0093r_Small.mpg
harald@Neville Movies]$ mpeg_play Bbh Embedding_0093r Me
Embedding_0093r_Medium.mpg Bbh Embedding_0093r_Medium.png
harald@Neville Movies]$ mpeg_play Bbh Embedding_0093r_Medium.mpg

24 bit displays: use -dither color to get full color
ordered dither is the default.
simplified dither requires 8 bit display
using -dither color
harald@Neville Movies]$ mpeg_play -dither color Bbh Embedding_0093r_Medium.mpg

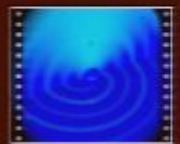
e!

Time Spent (After Initializations): 39.581727 secs.
Frames/sec: 24.961013
harald@Neville Movies]$

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www.black-holes.org/explore2.html



0093g_800x600.mpeg



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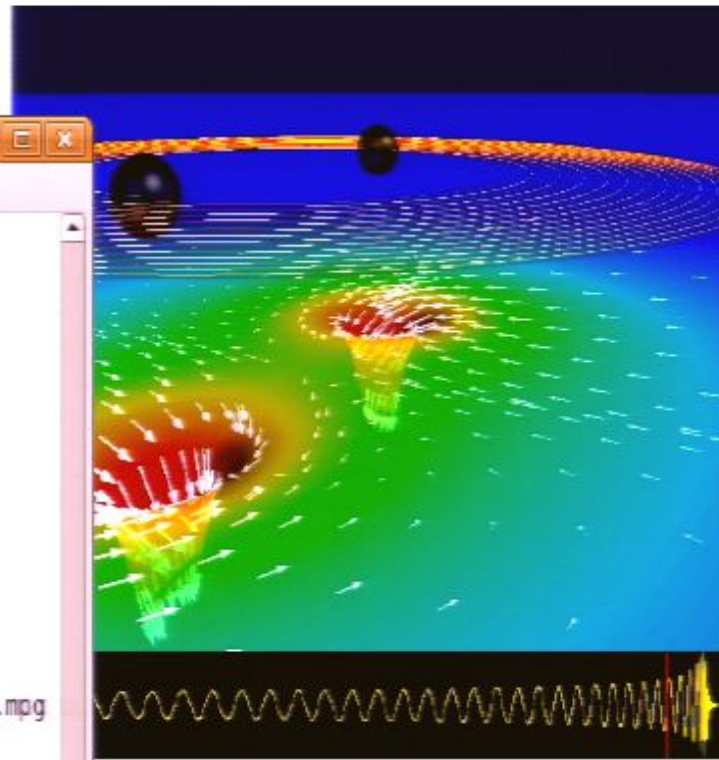
Numerical study of black hole spacetimes

File Edit View Go Help

← 27 of 67 Best Fit

Next →

Movies I



www.black-holes.org/explore2.html

harald@Neville:AA~/research/Talks/08May_Perimeter/Movies

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Edit View Terminal Tabs Help
Embedding_0093r_Large.mpg
Embedding_0093r_Large.png
Embedding_0093r_Medium.mpg
Embedding_0093r_Medium.png
Embedding_0093r_Small_BeforeMerger.jpg
Embedding_0093r_Small.mpg
harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Me
Embedding_0093r_Medium.mpg Bbh_Embedding_0093r_Medium.png
harald@Neville Movies]$ mpeg_play Bbh_Embedding_0093r_Medium.mpg

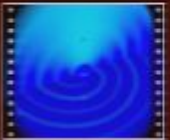
24 bit displays: use -dither color to get full color
ordered dither is the default.
simplified dither requires 8 bit display
adding -dither color
harald@Neville Movies]$ mpeg_play -dither color Bbh_Embedding_0093r_Medium.mpg

e!

Time Spent (After Initializations): 39.581727 secs.
Frames/Sec: 24.961013
harald@Neville Movies]$ mpeg_play 0093g
Bg_000x000_000.png 0093g_800x600.mpeg
harald@Neville Movies]$ mpeg_play 0093g_800x600

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0093g_800x600.mpeg



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harald@Neville Movies]\$
Embedding 0093r Medi
harald@Neville Movies]\$

24 bit displays: use
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3g_800x600_Late.png
harald@Neville Movies]\$

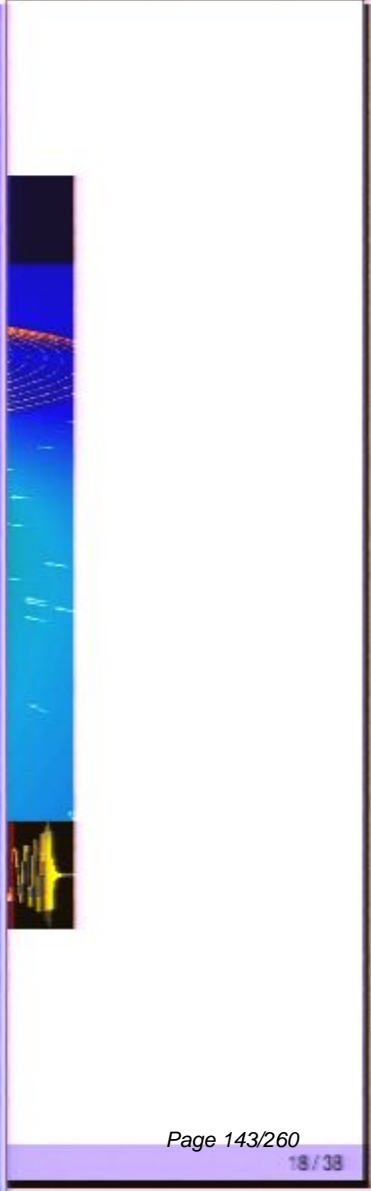
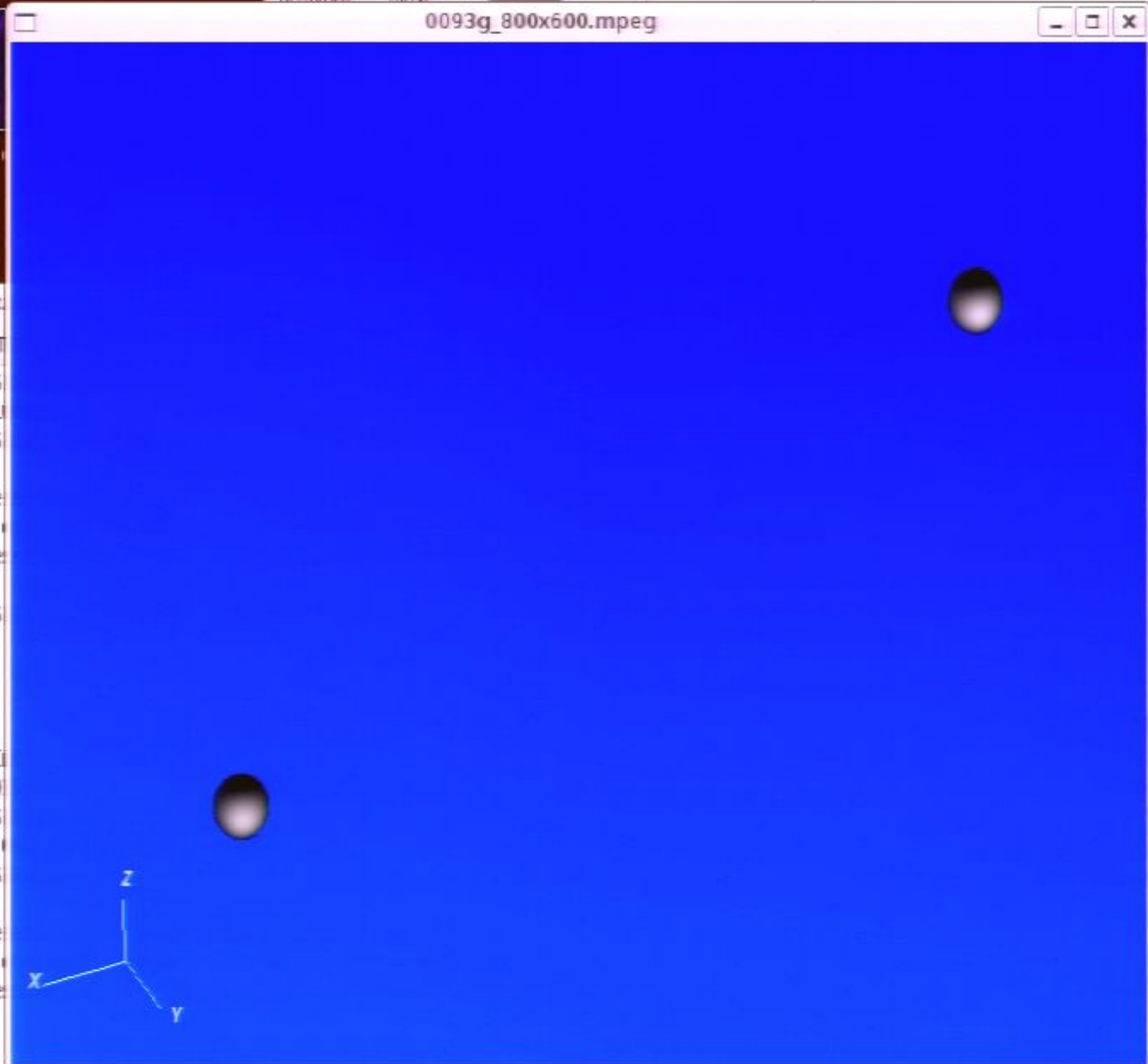
24 bit displays: use
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specified dither require
ing -dither color

Pirsa: 08050036

MPEG Player Controls

FrameRate 38/25.2 Rewind Pause Step Play Loop OFF Exit

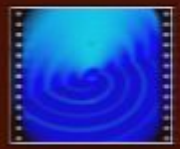
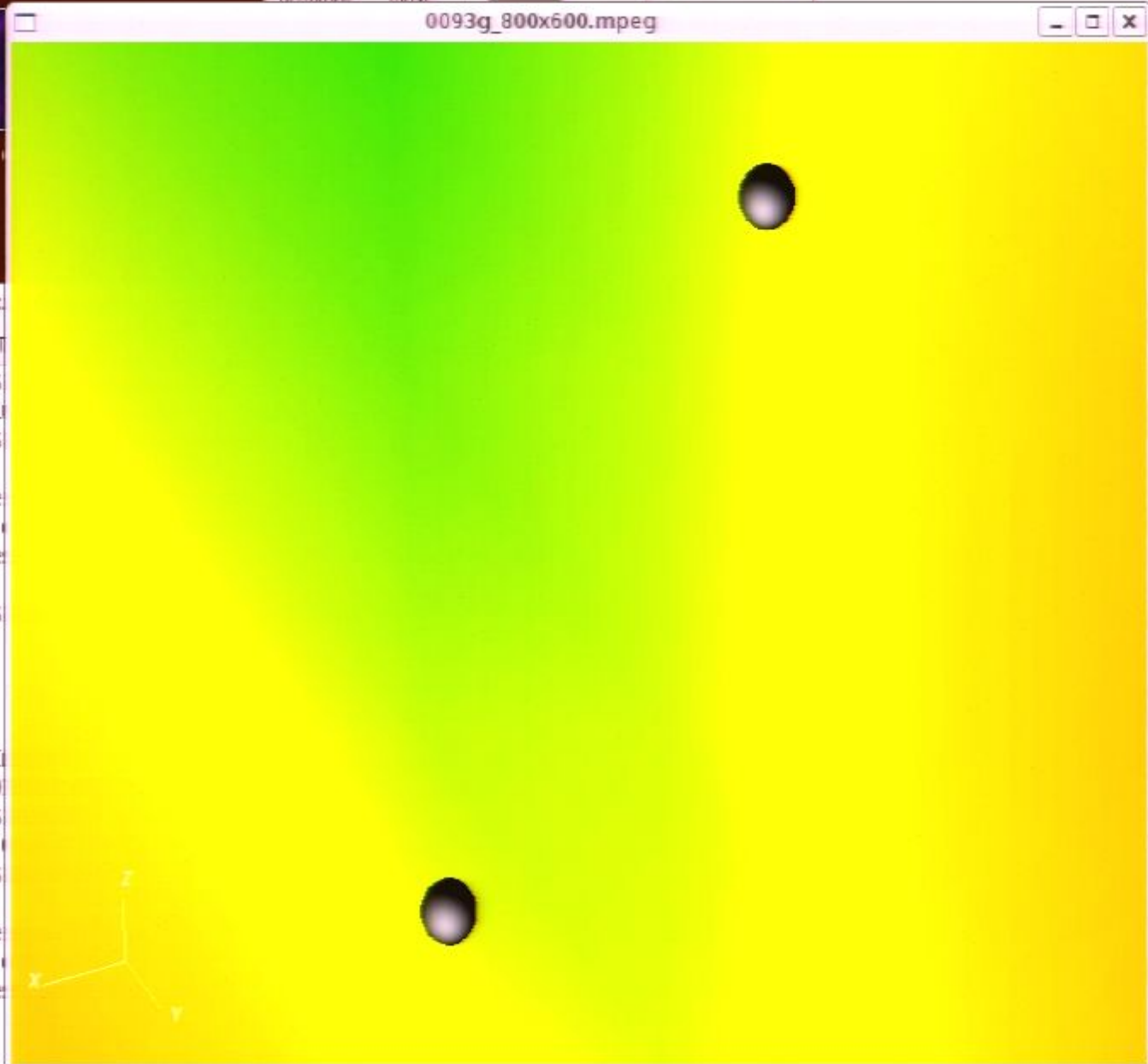
Previous Next Best Fit



MPEG Player Controls

FrameRate 102/25.1 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit



0093g_800x600.mpeg



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rald@Neville Movies]$
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rald@Neville Movies]$

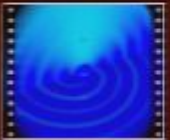
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rald@Neville Movies]$
3g_800x600_Late.png
rald@Neville Movies]$

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harald@Neville Movies]$
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harald@Neville Movies]$
3g_800x600_Late.png
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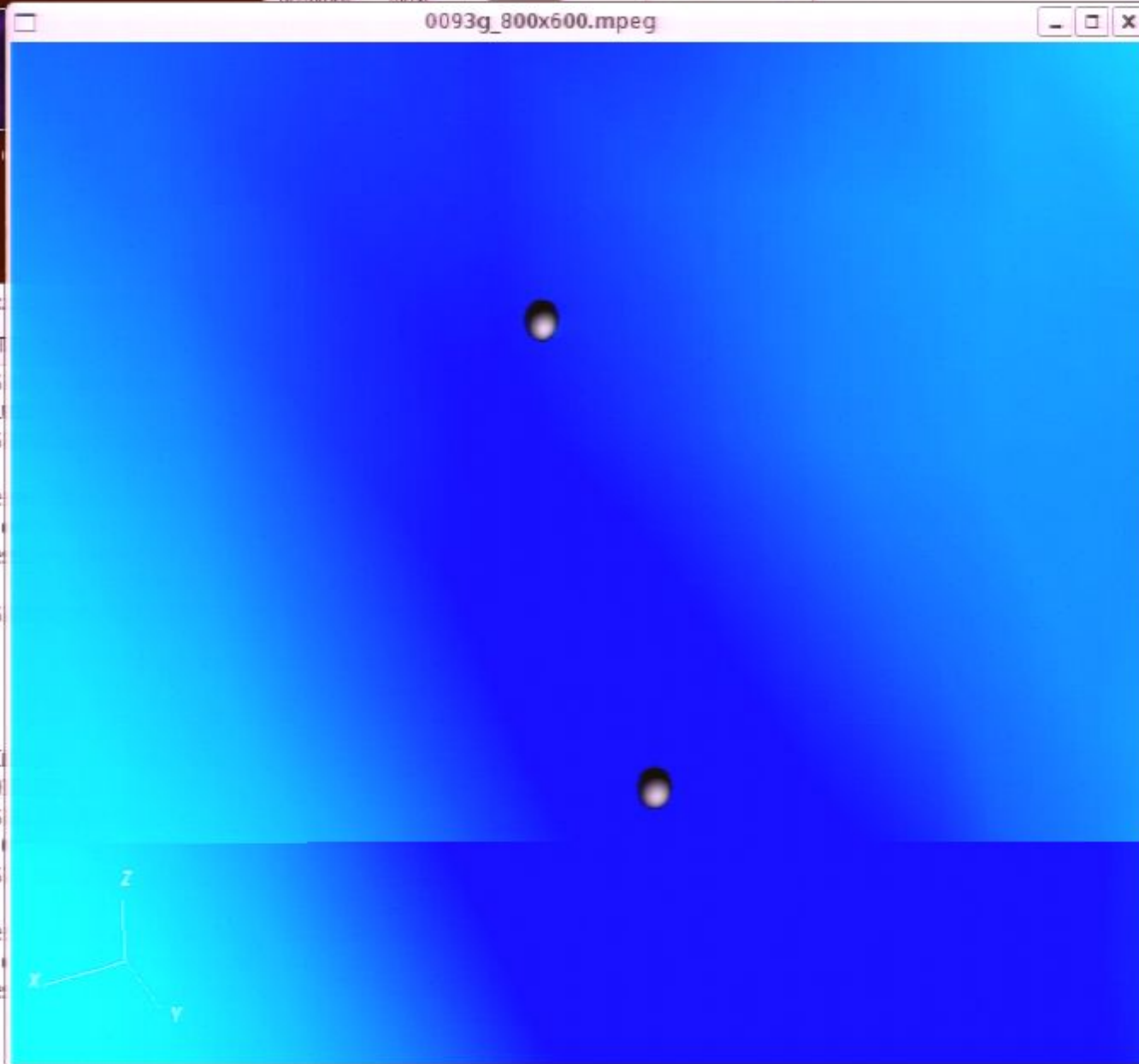
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24 bit displays: use
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Pirsa: 08050036

MPEG Player Controls

Frame/Rate 168/25.0 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit

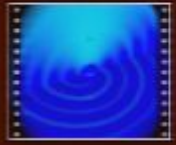
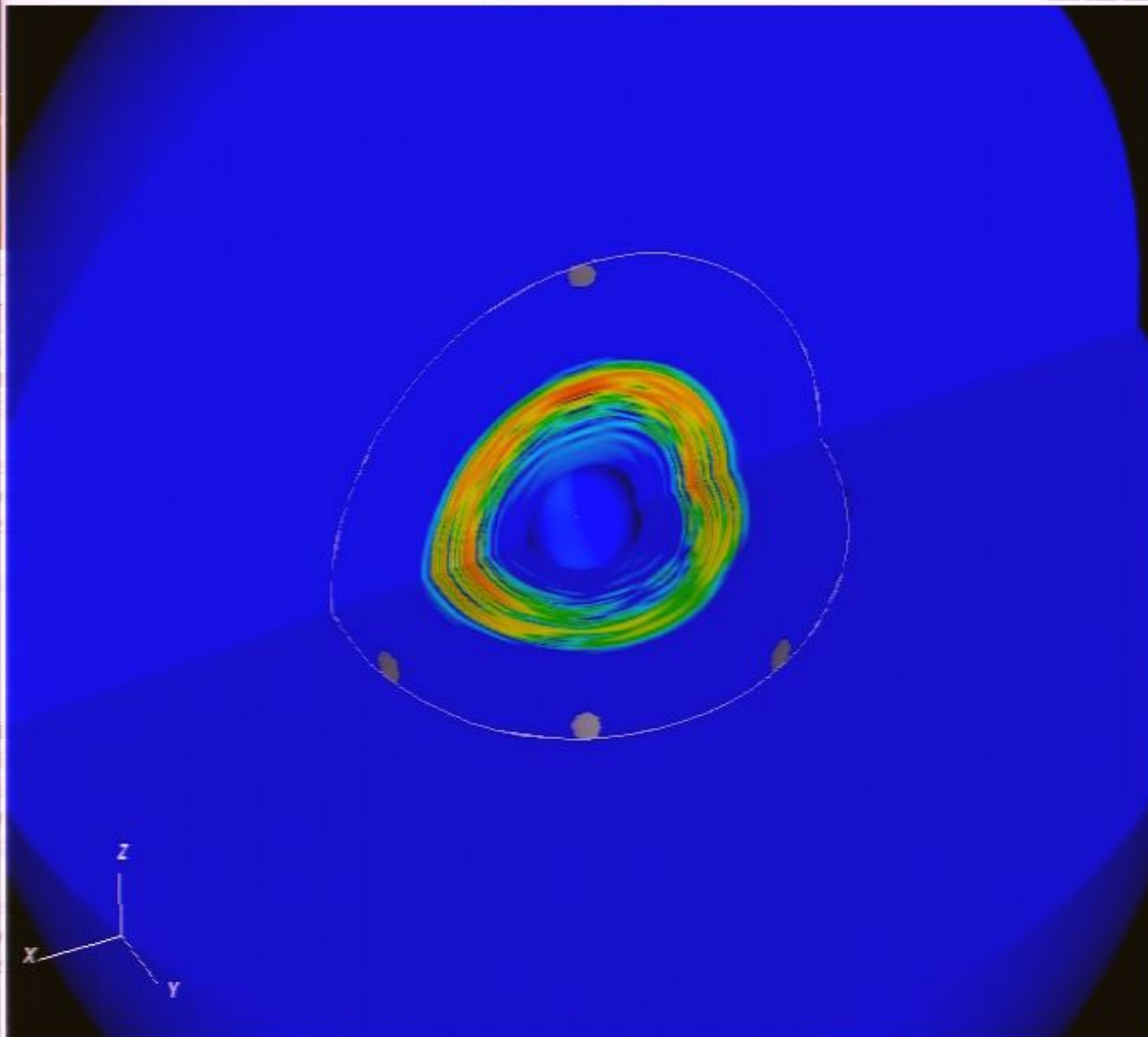


MPEG Player Controls

FrameRate 232/24.9 Rewind Pause Step Play Loop OFF Exit

Best Fit

0093g_800x600.mpeg



0093g_800x600.mpeg



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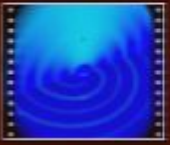
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0093g_800x600.mpeg



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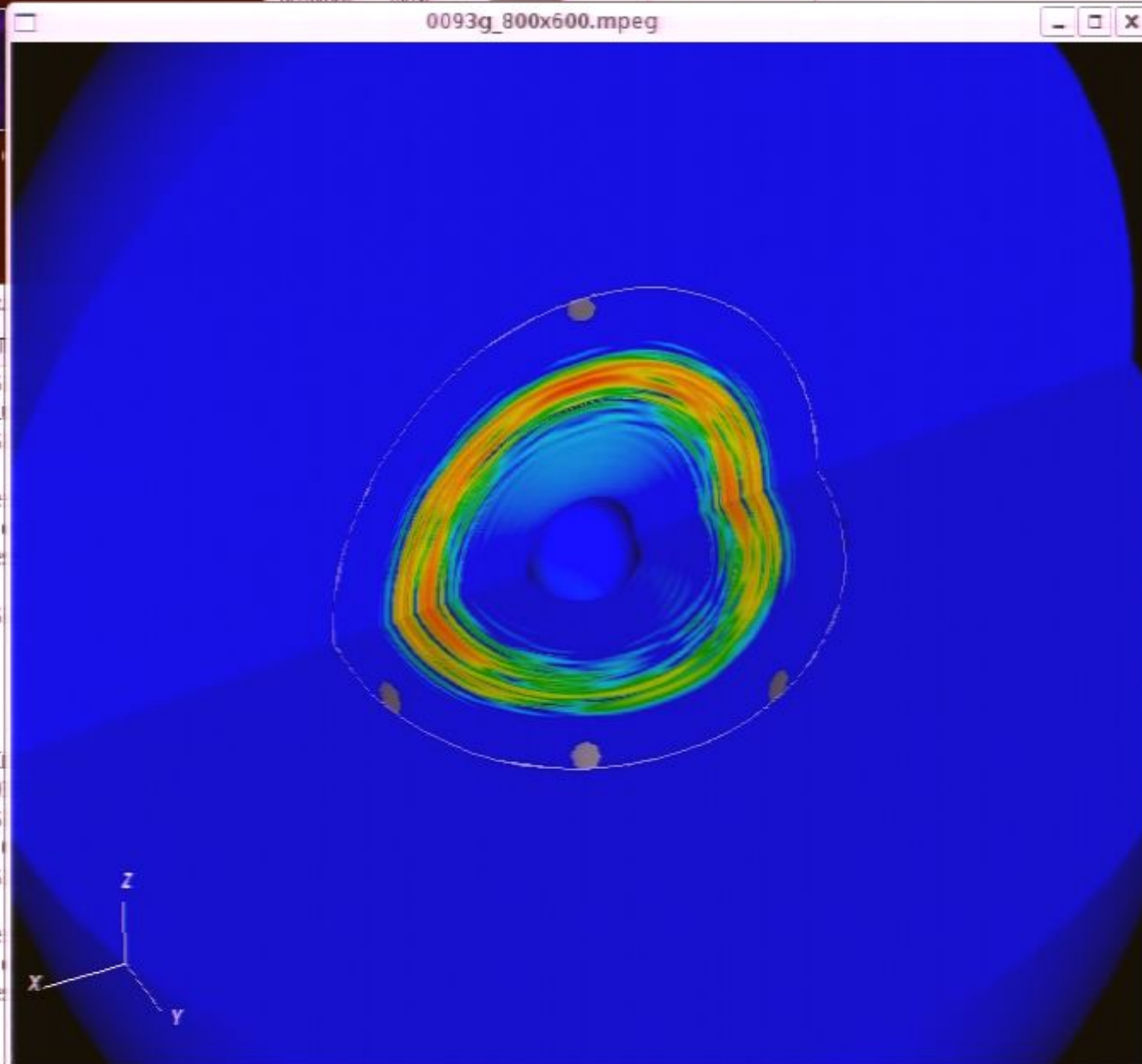
24 bit displays: use
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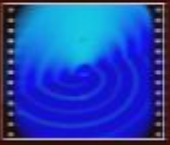
Pirsa: 08050036

MPEG Player Controls

FrameRate 296/24.9 Rewind Pause Step Play Loop OFF Exit

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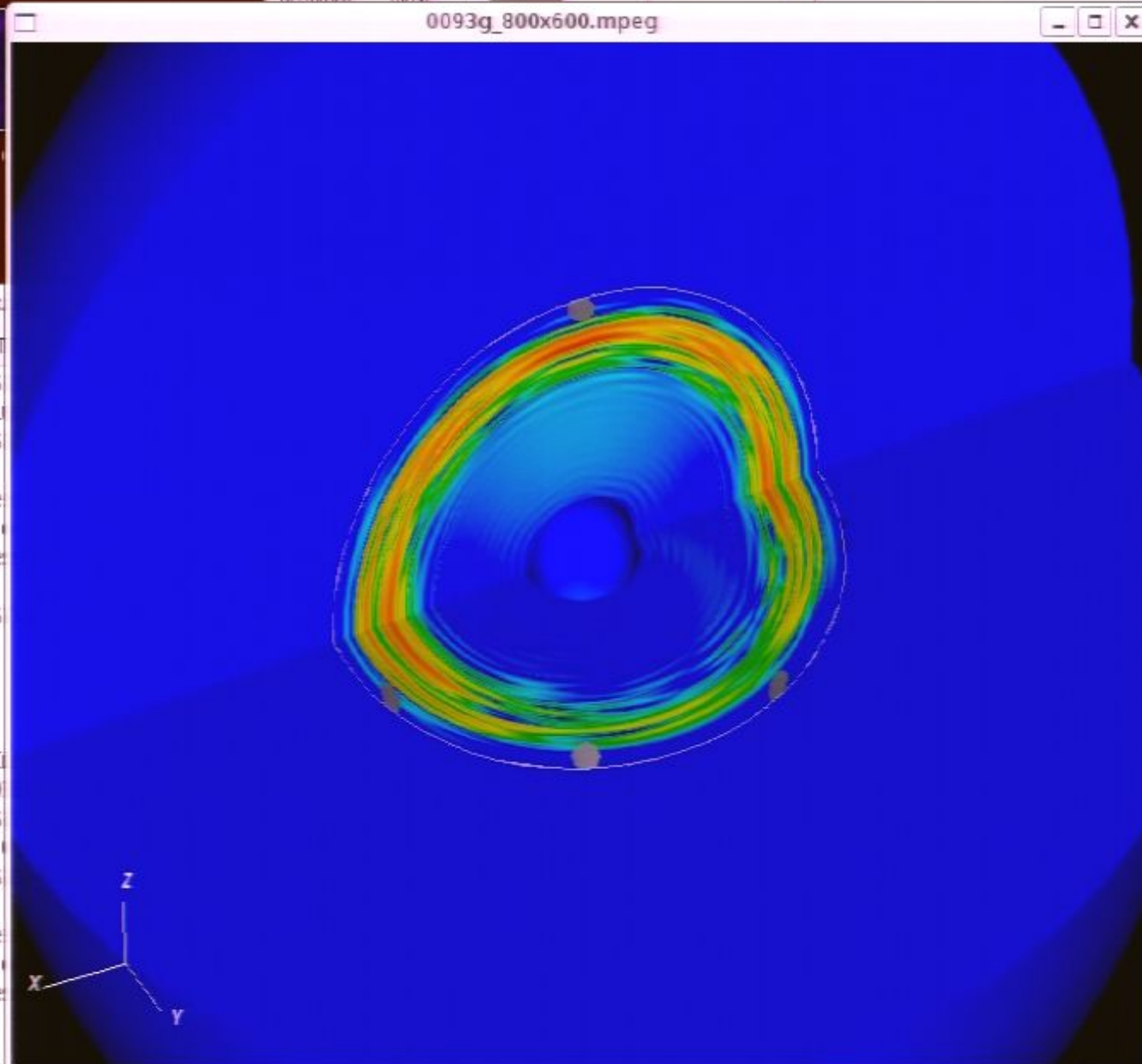
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MPEG Player Controls

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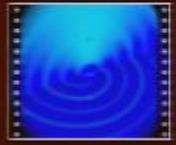
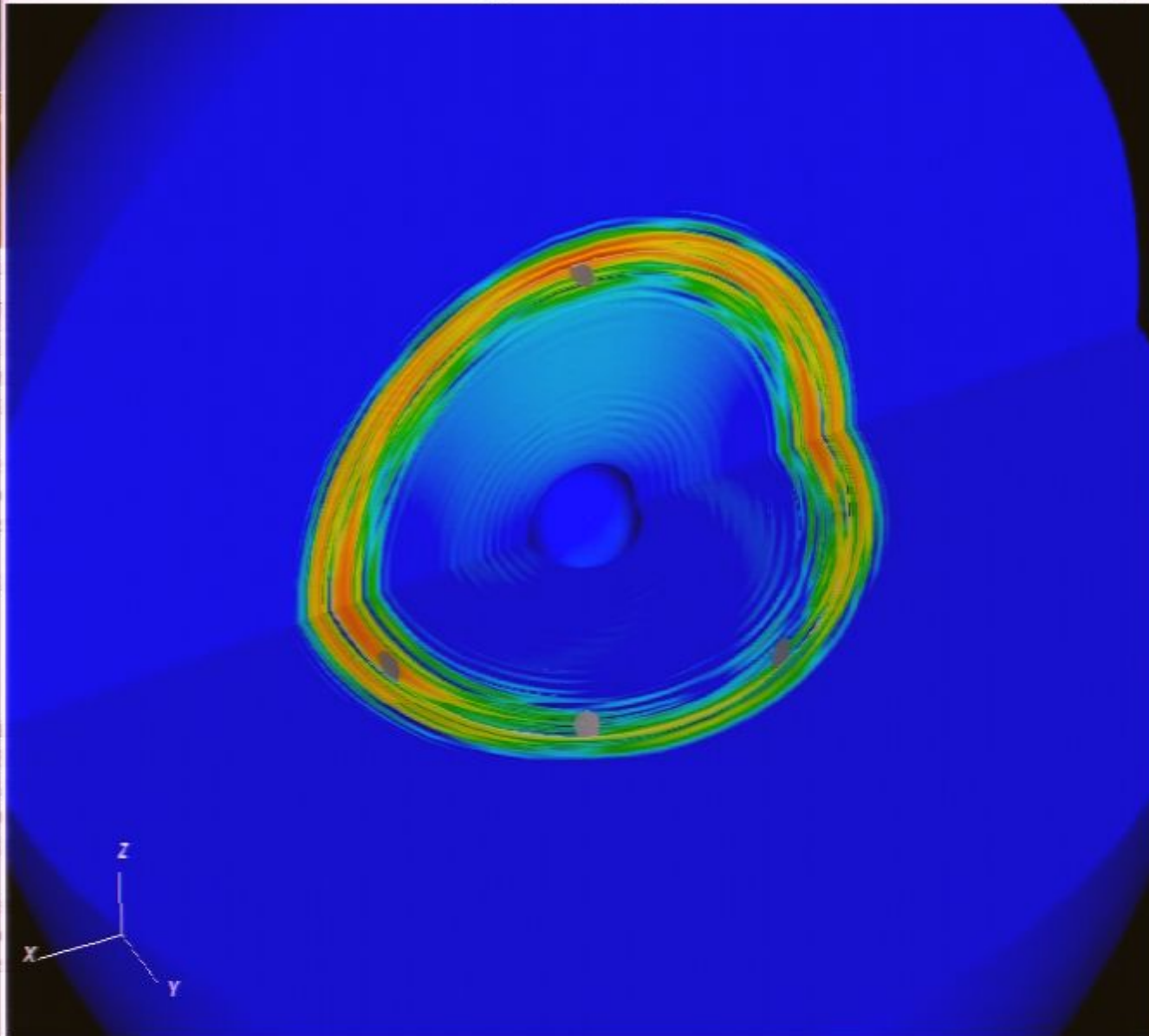
Previous Next Best Fit



MPEG Player Controls

FrameRate 430/24.9 [Rewind] [Pause] [Step] [Play] [Loop OFF] [Exit]

0093g_800x600.mpeg



0093g_800x600.mpeg



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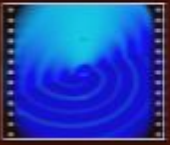
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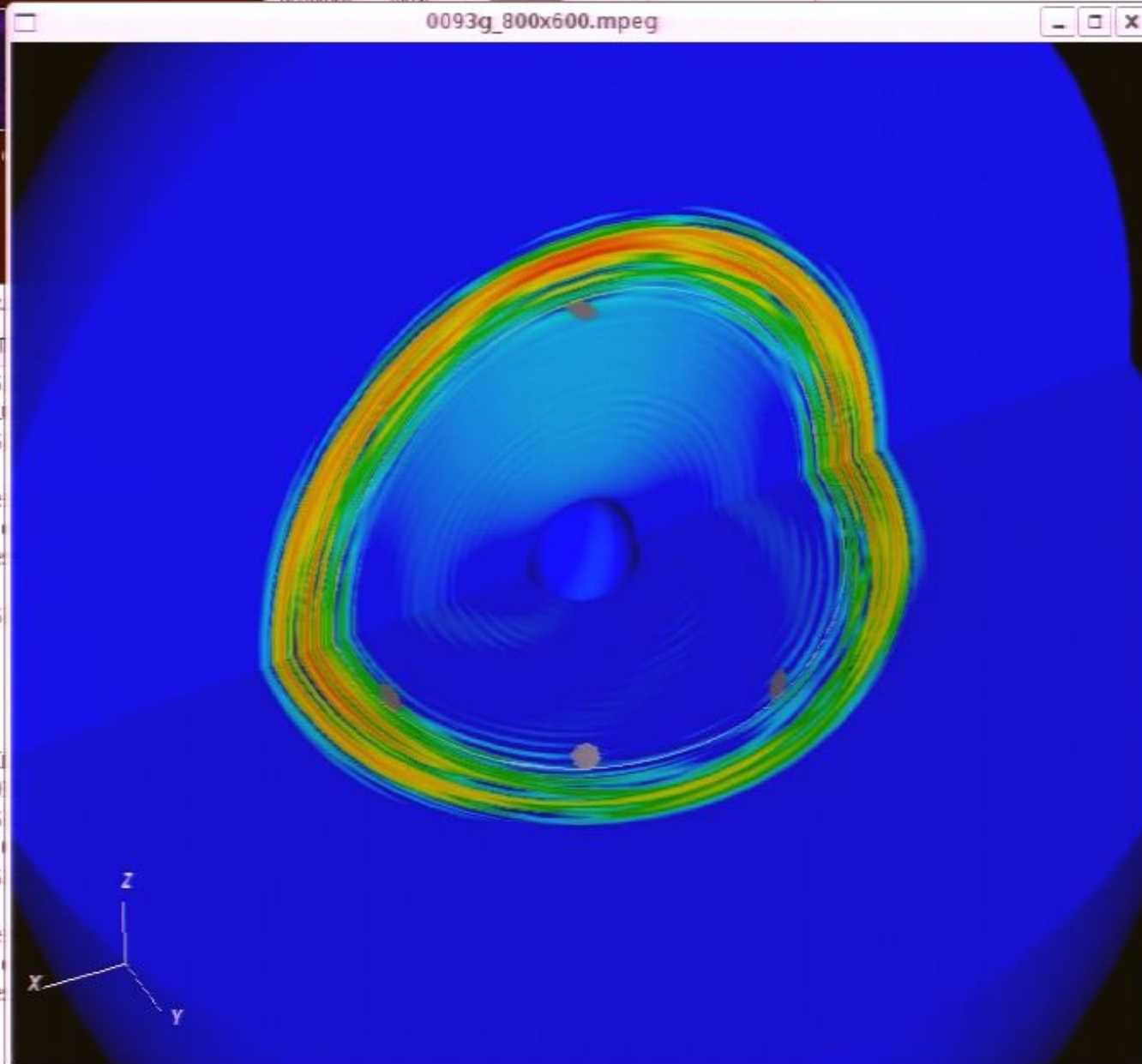
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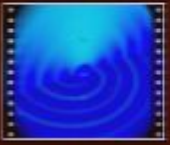
Pirsa: 08050036

MPEG Player Controls

FrameRate 491/24.9 Rewind Pause Step Play Loop OFF Exit

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0093g_800x600.mpeg



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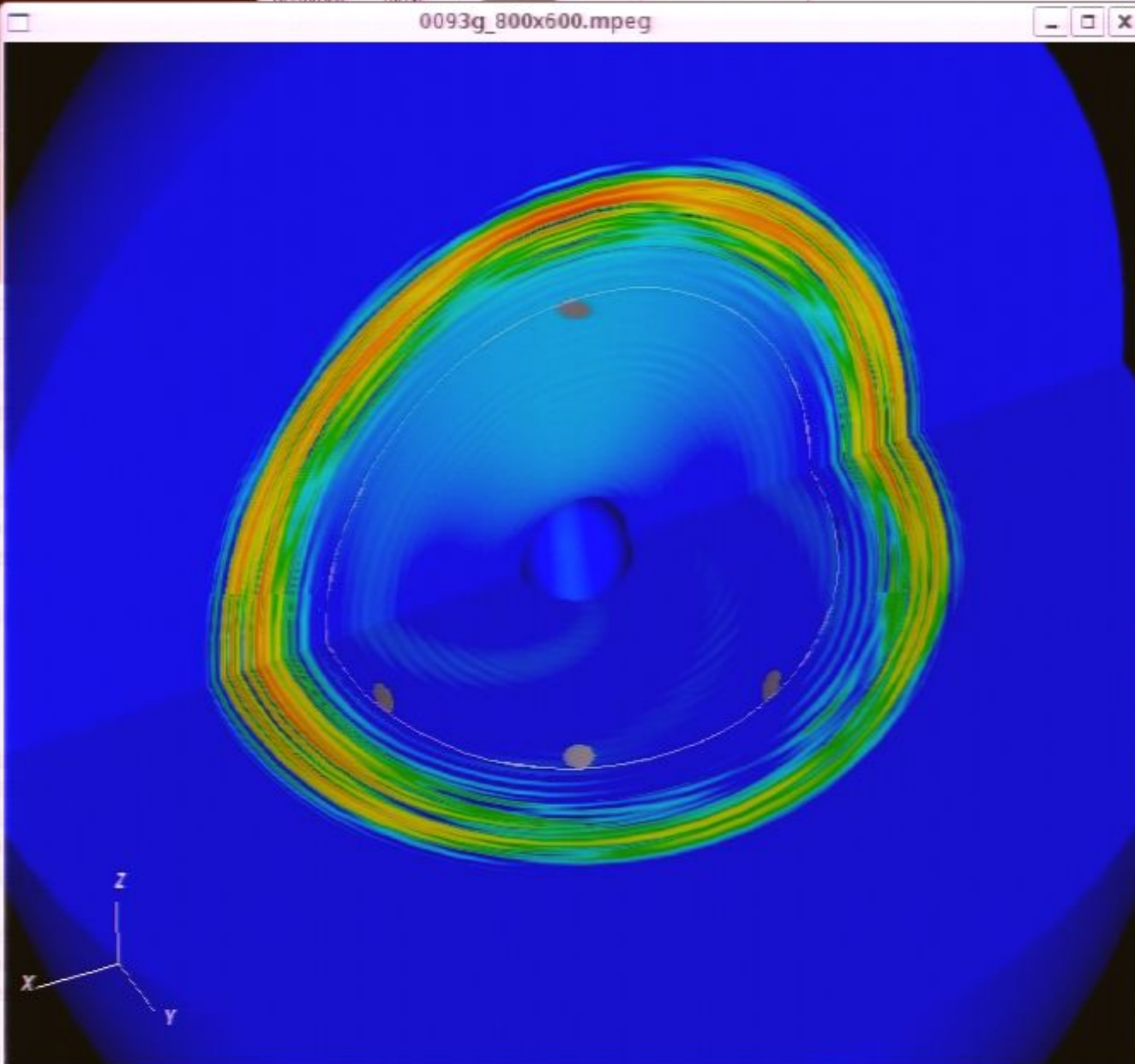
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Pirsa: 08050036

MPEG Player Controls

FrameRate 560/24.9 Rewind Pause Step Play Loop OFF Exit

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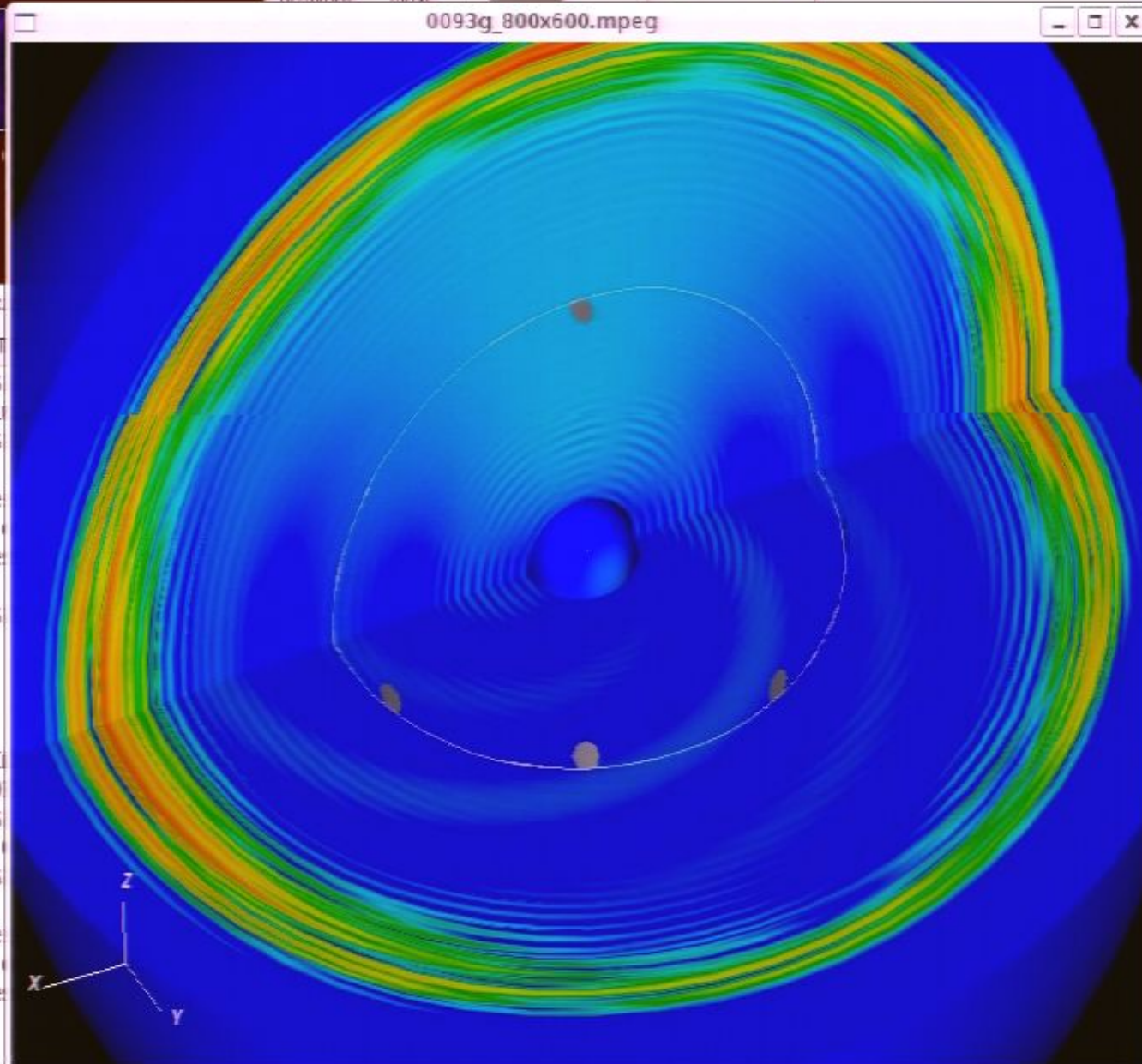
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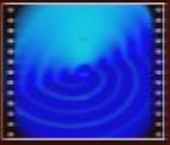
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MPEG Player Controls

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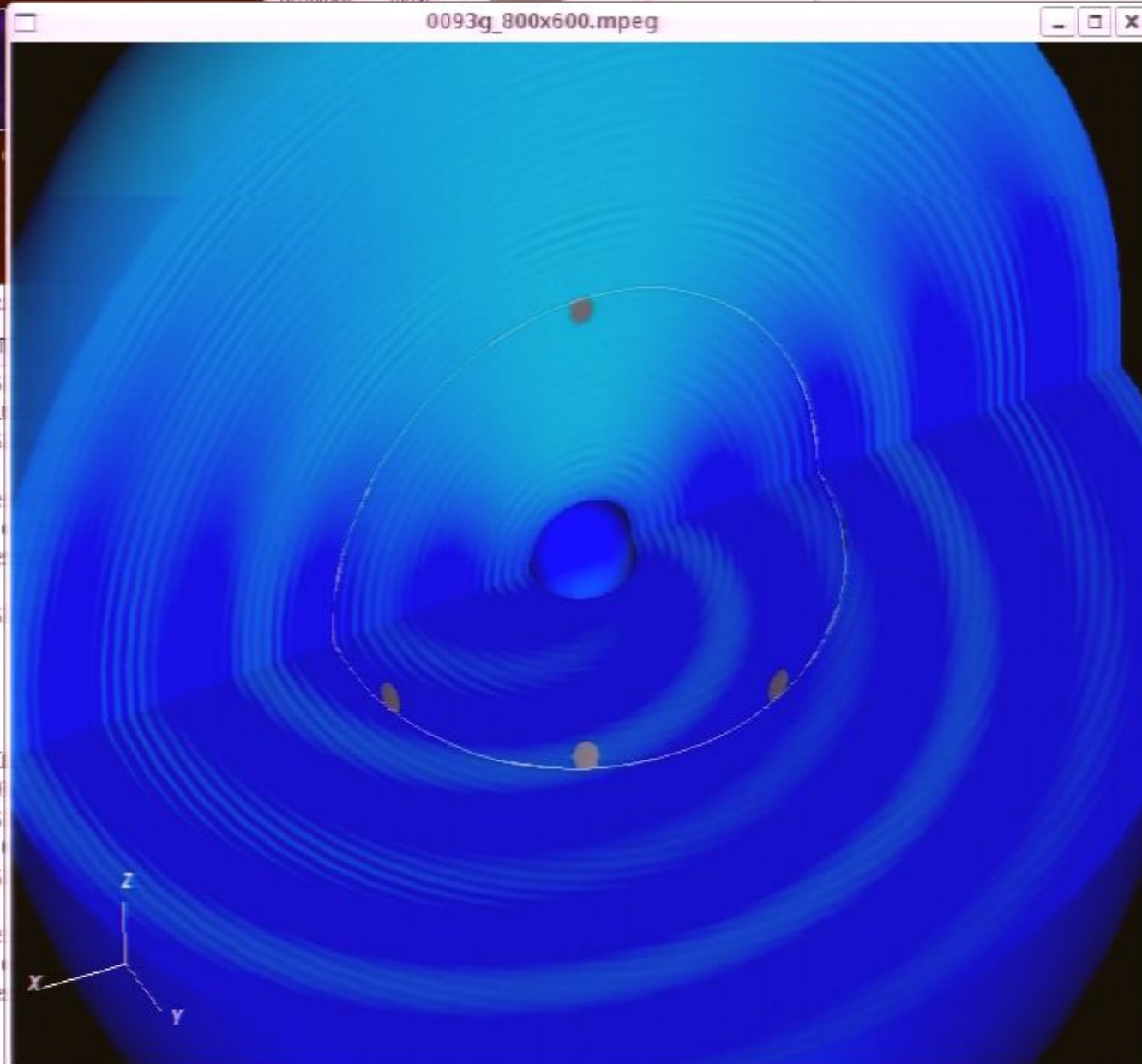
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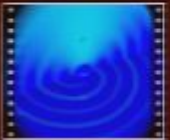
MPEG Player Controls

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Previous Next Best Fit



Numerical study of black hole spacetimes



0093g_800x600.mpeg

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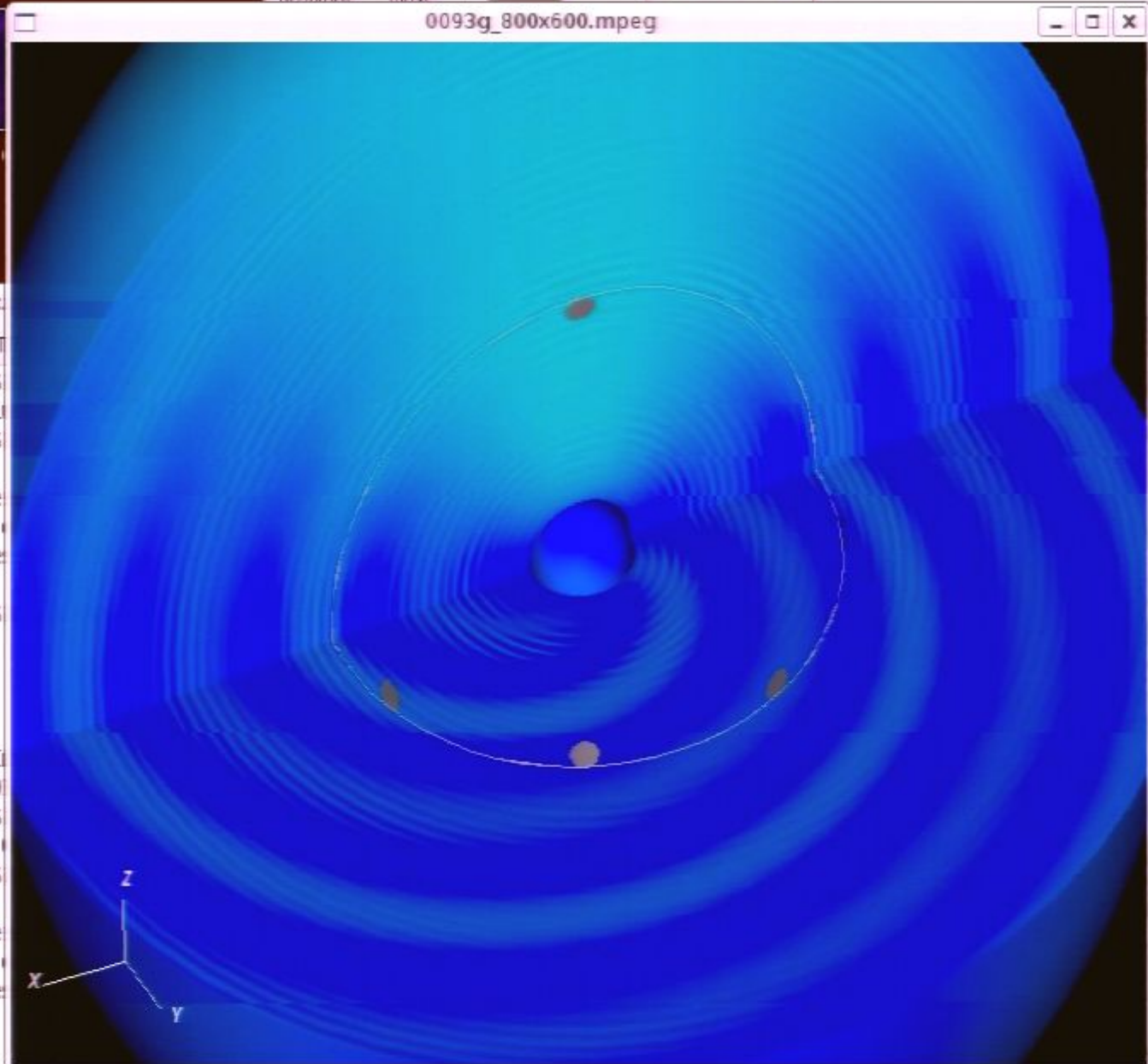
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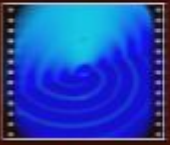
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MPEG Player Controls

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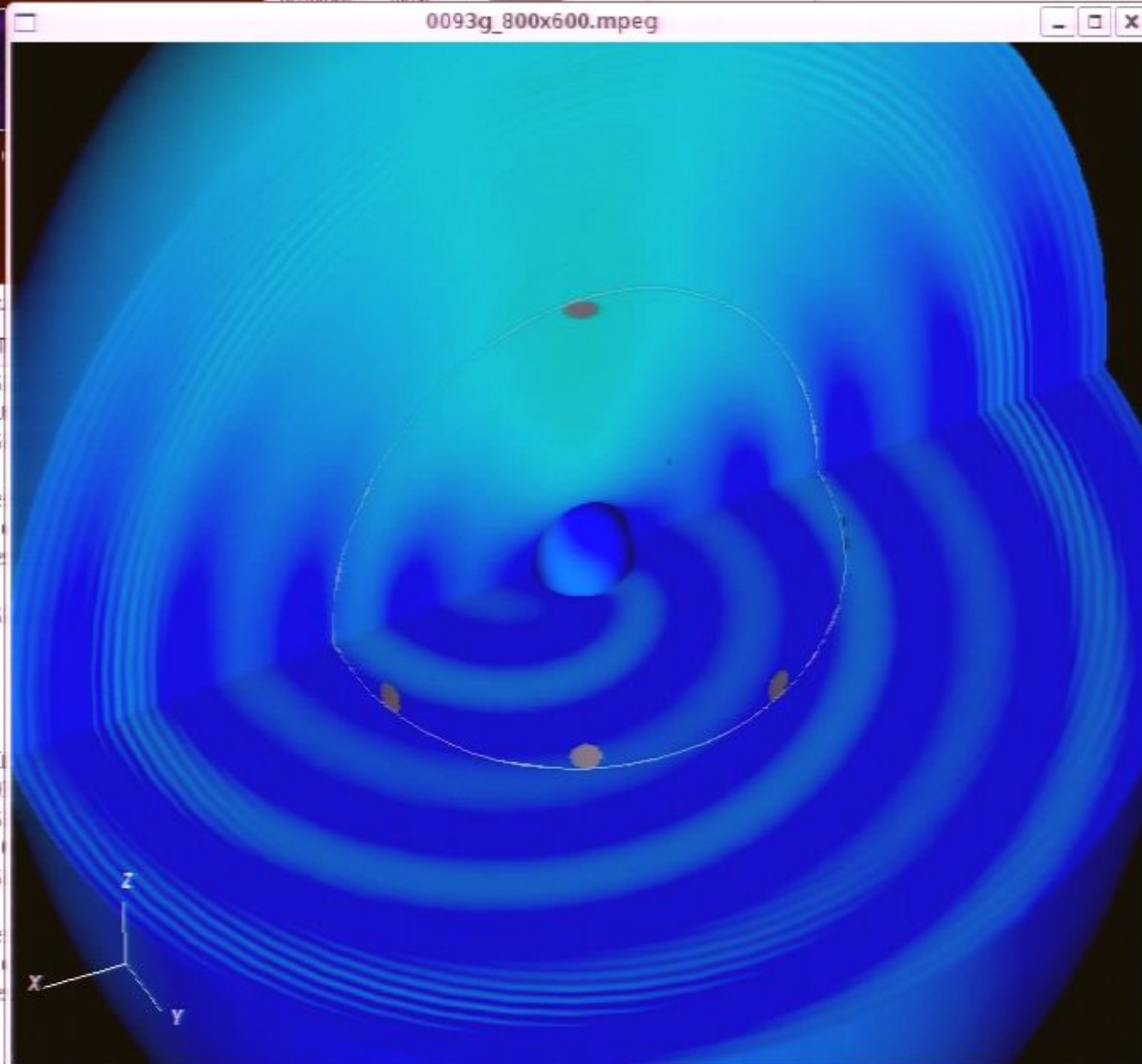
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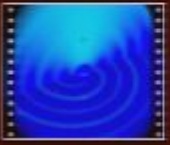
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MPEG Player Controls

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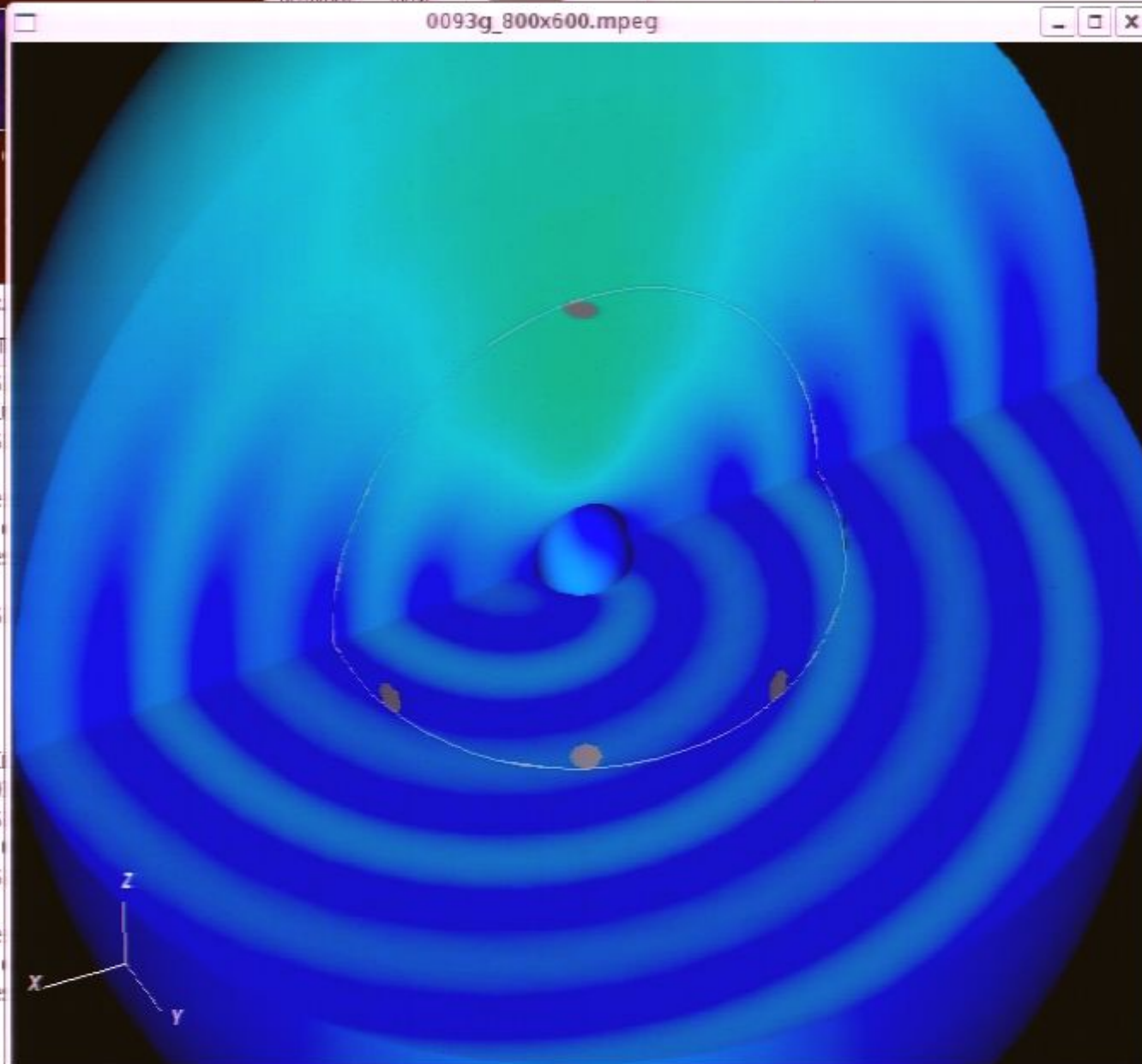
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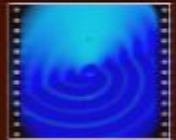
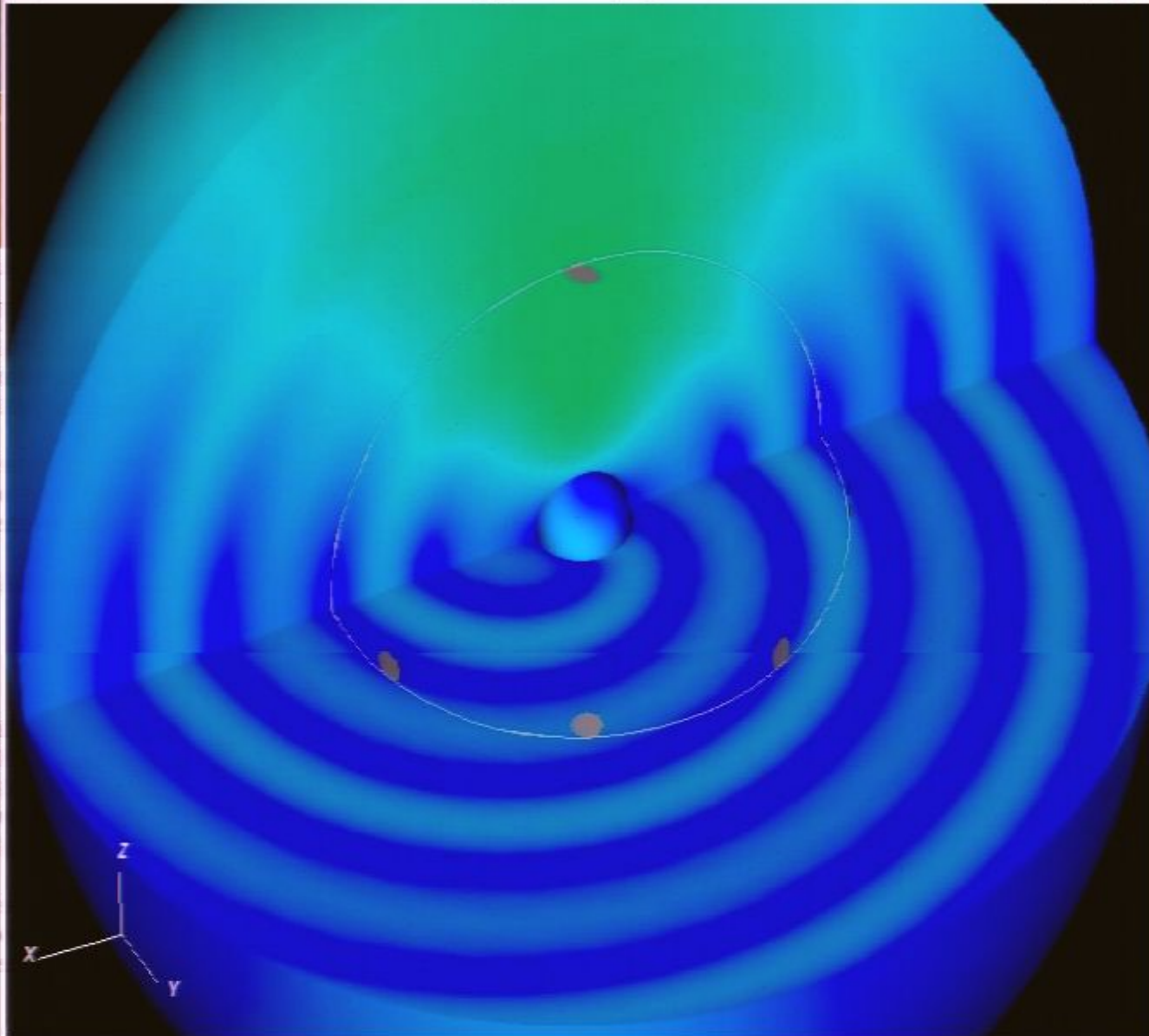


MPEG Player Controls

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Best Fit

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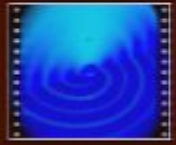
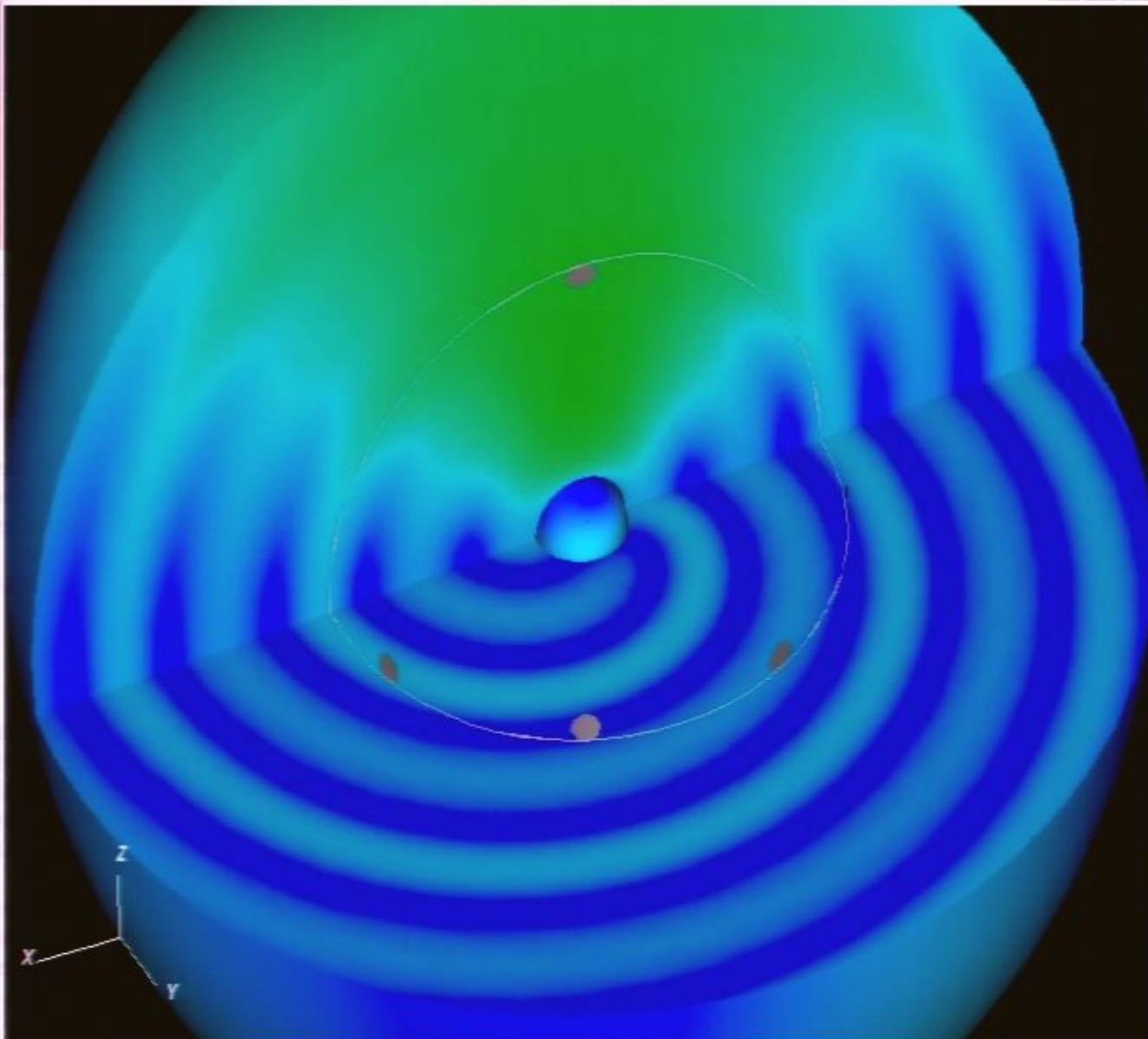
Pirsa: 08050036

Numerical study of black hole spacetimes

MPEG Player Controls

FrameRate: 1020/24.9 [Buttons: Rewind, Pause, Step, Play, Loop OFF, Exit]

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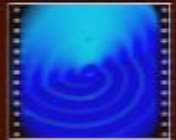
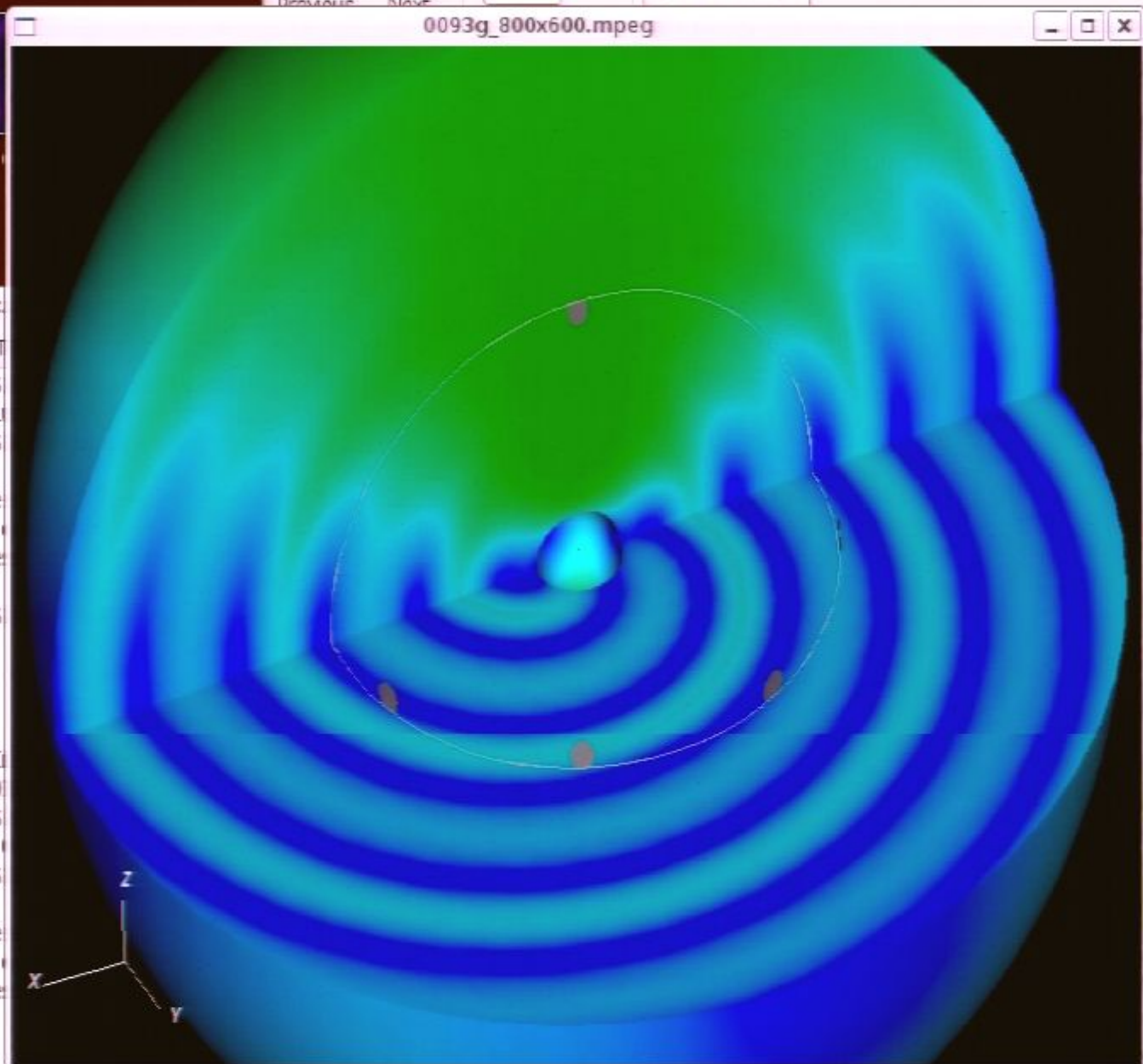
Pirsa: 08050036

Numerical study of black hole spacetimes

MPEG Player Controls

FrameRate: 1085/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit



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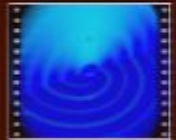
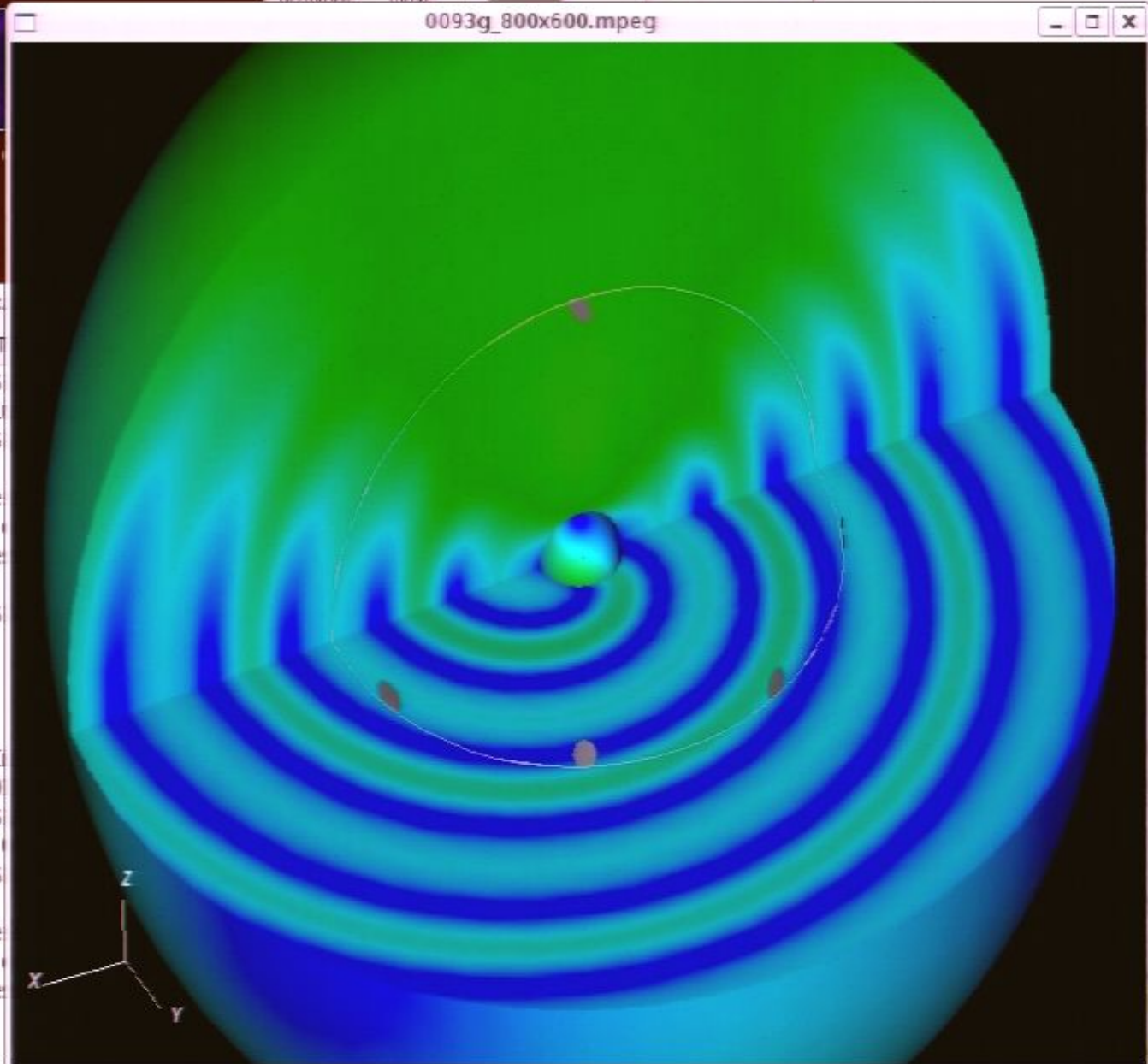
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Numerical study of black hole spacetimes

MPEG Player Controls

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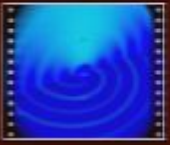
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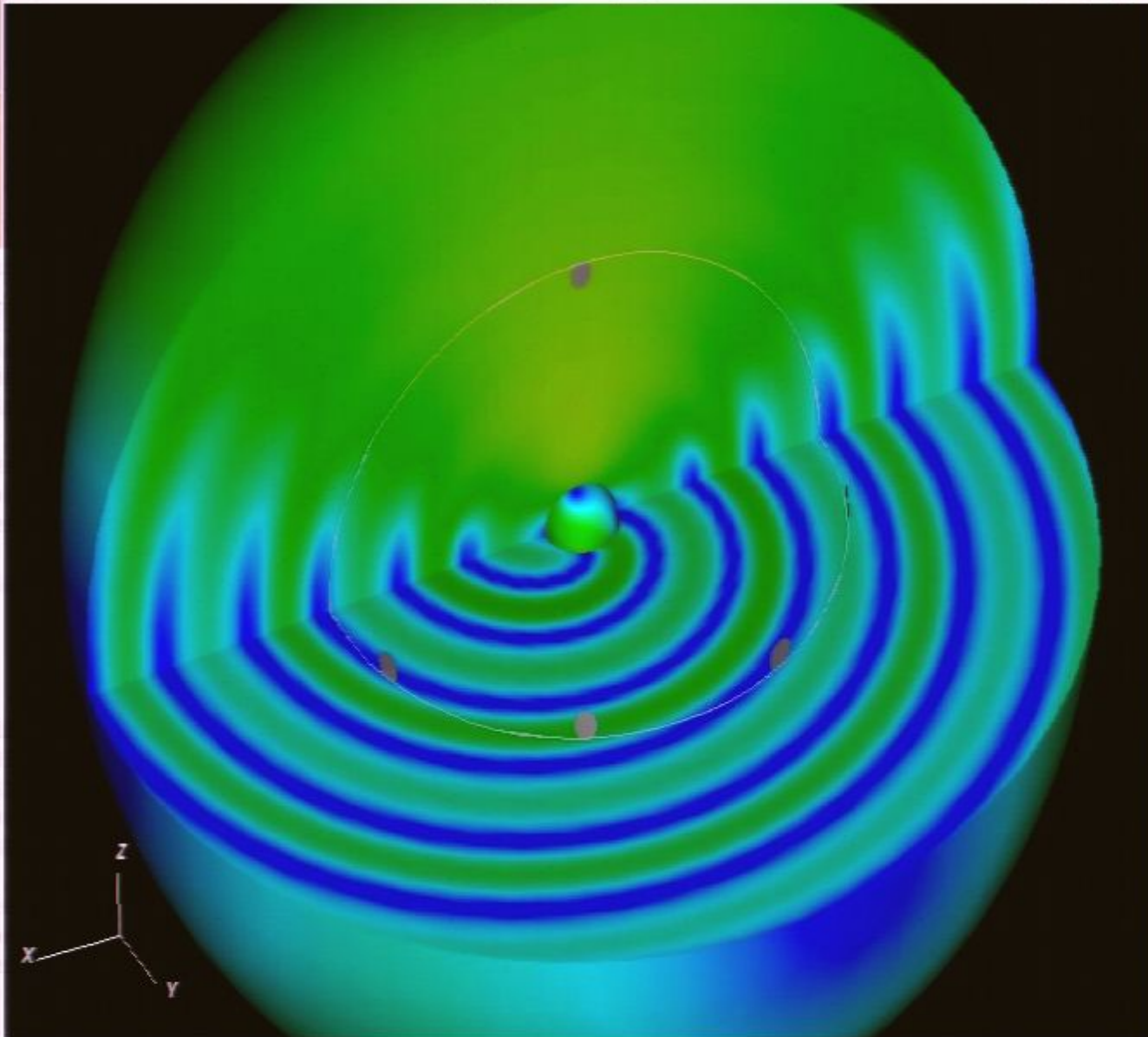
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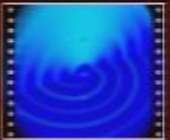
MPEG Player Controls

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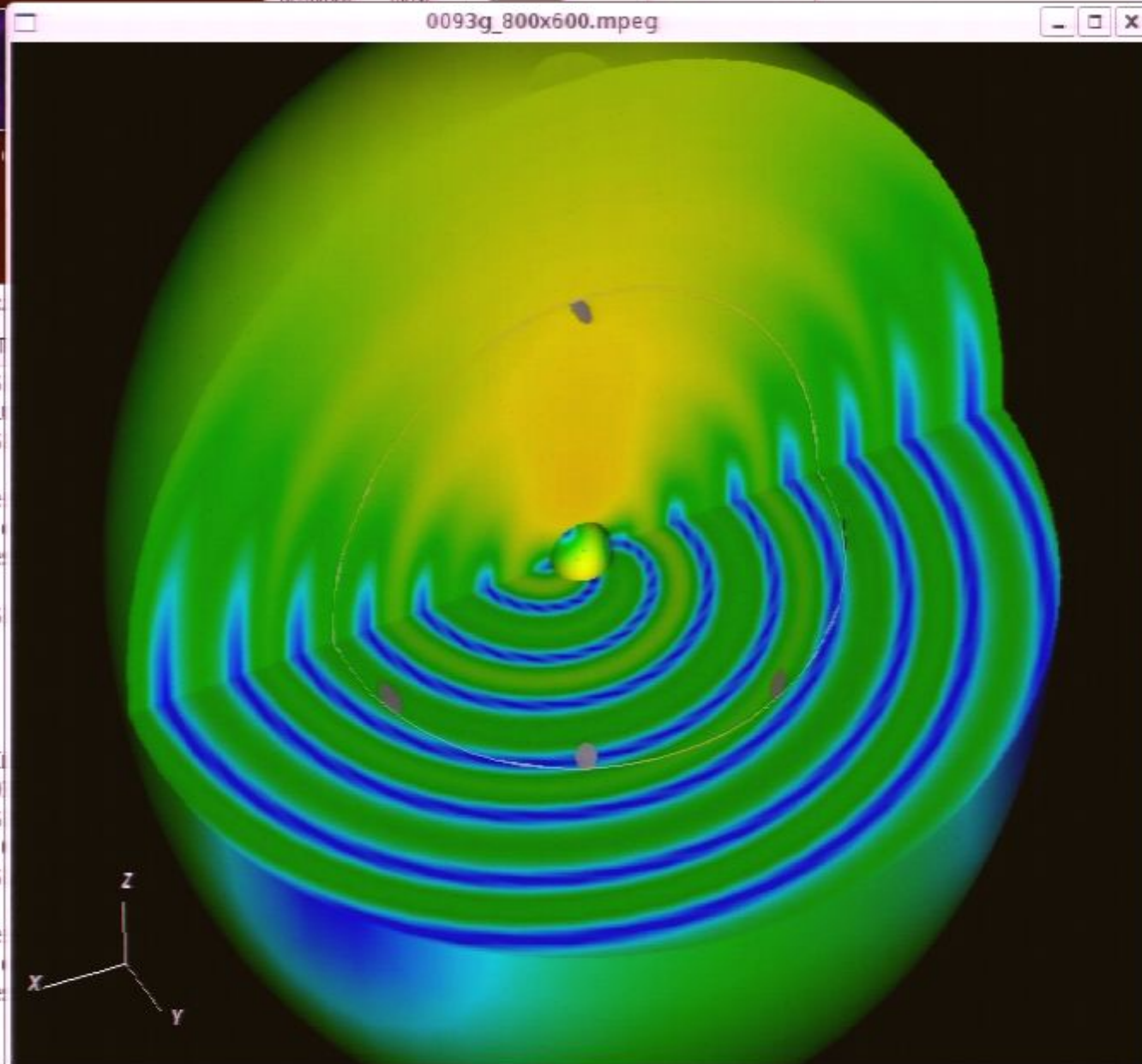
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MPEG Player Controls

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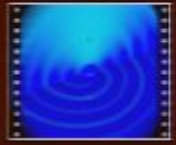
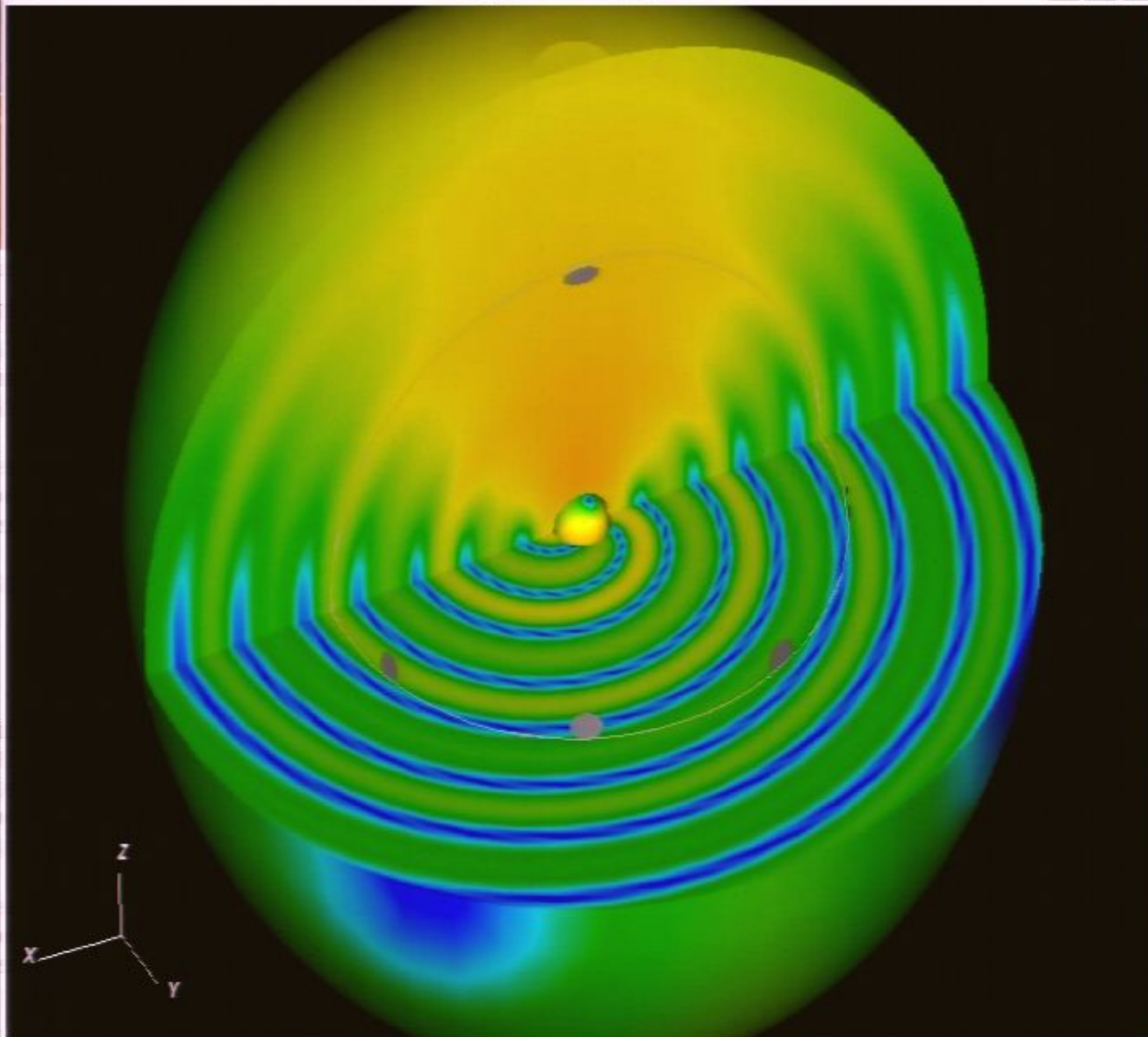
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MPEG Player Controls

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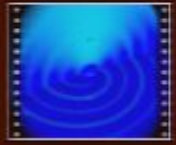
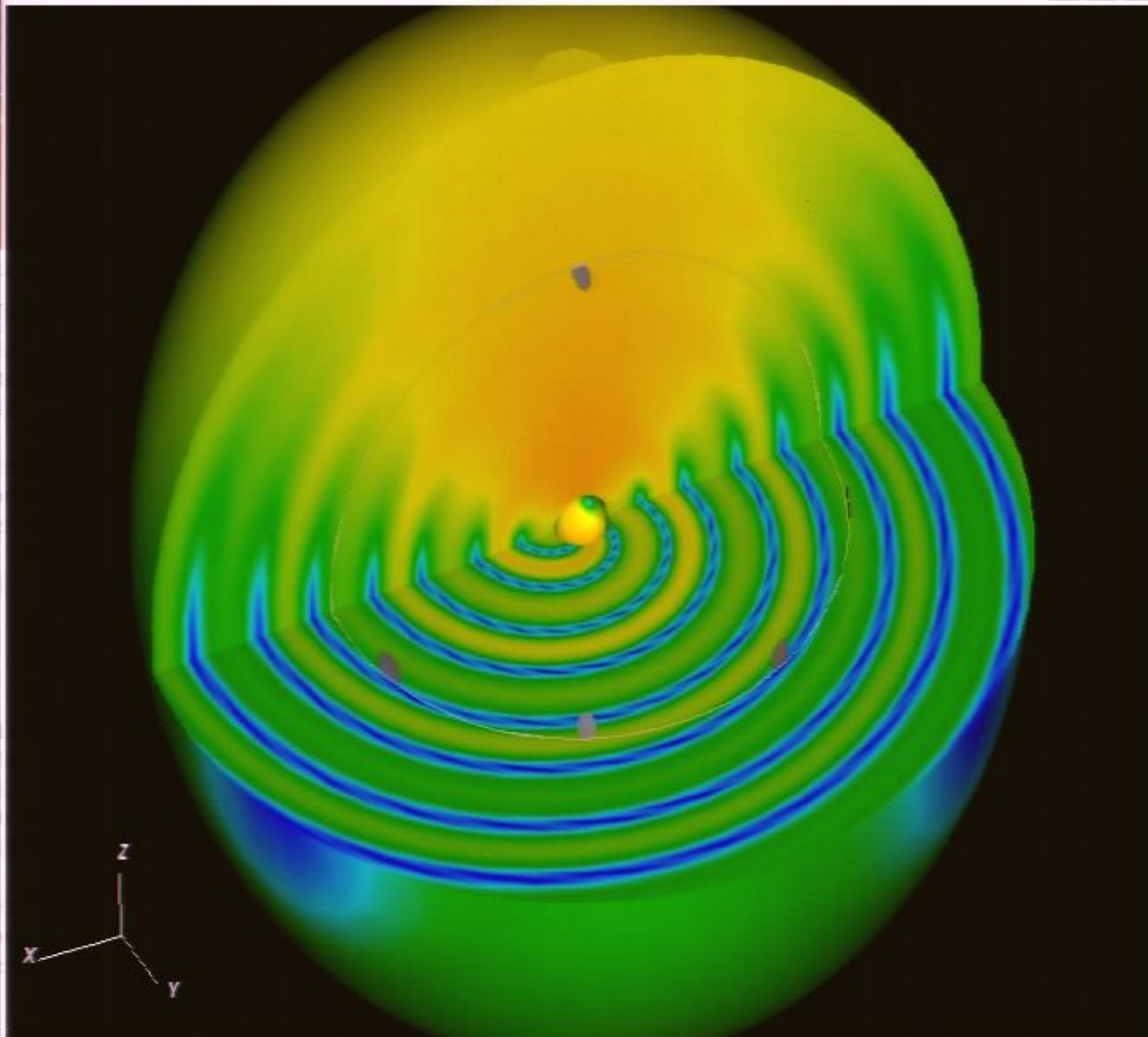
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Pirsa: 08050036

MPEG Player Controls

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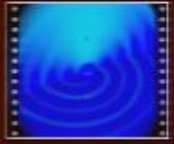
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MPEG Player Controls

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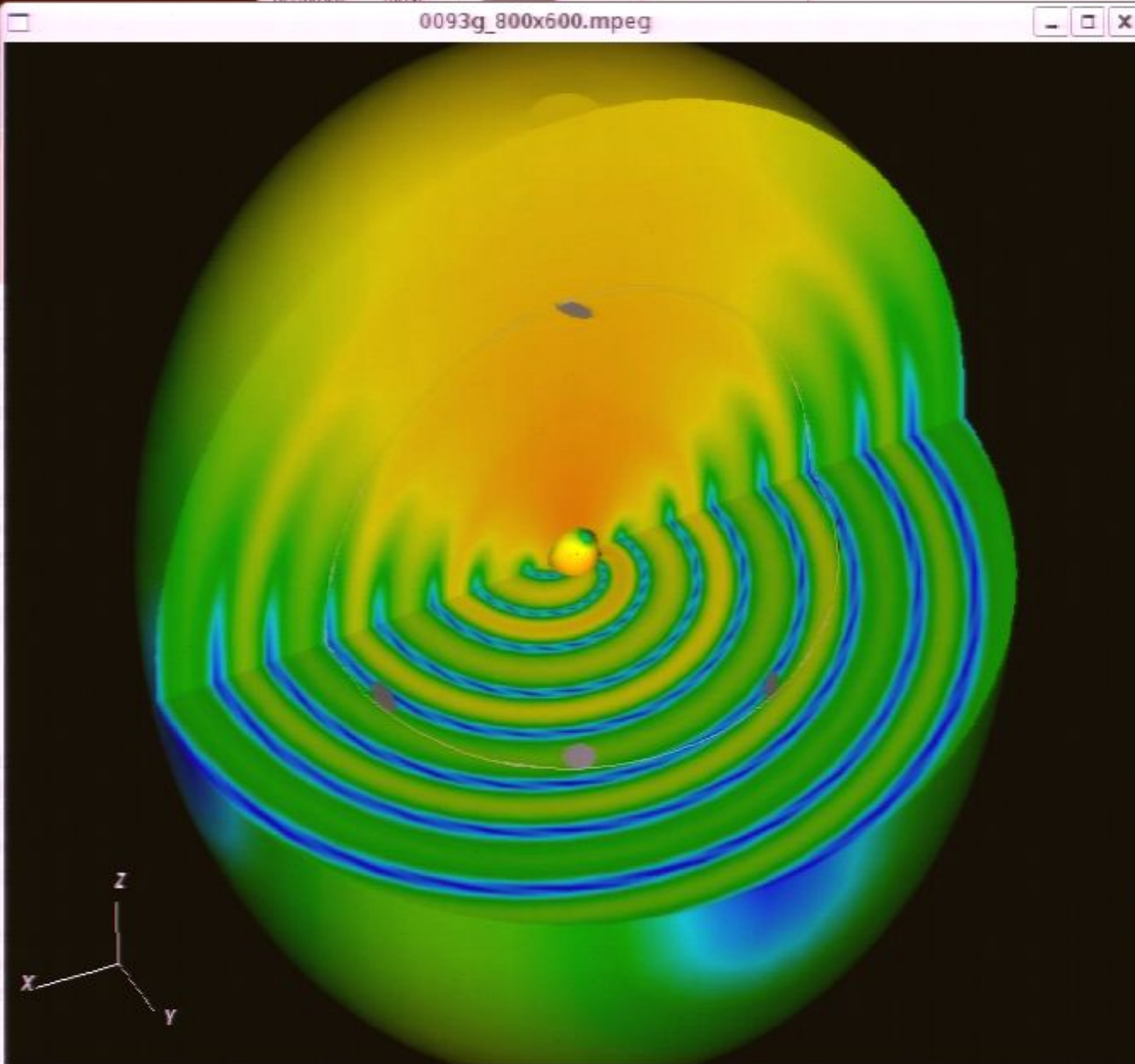
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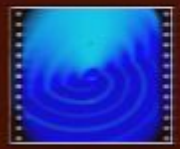
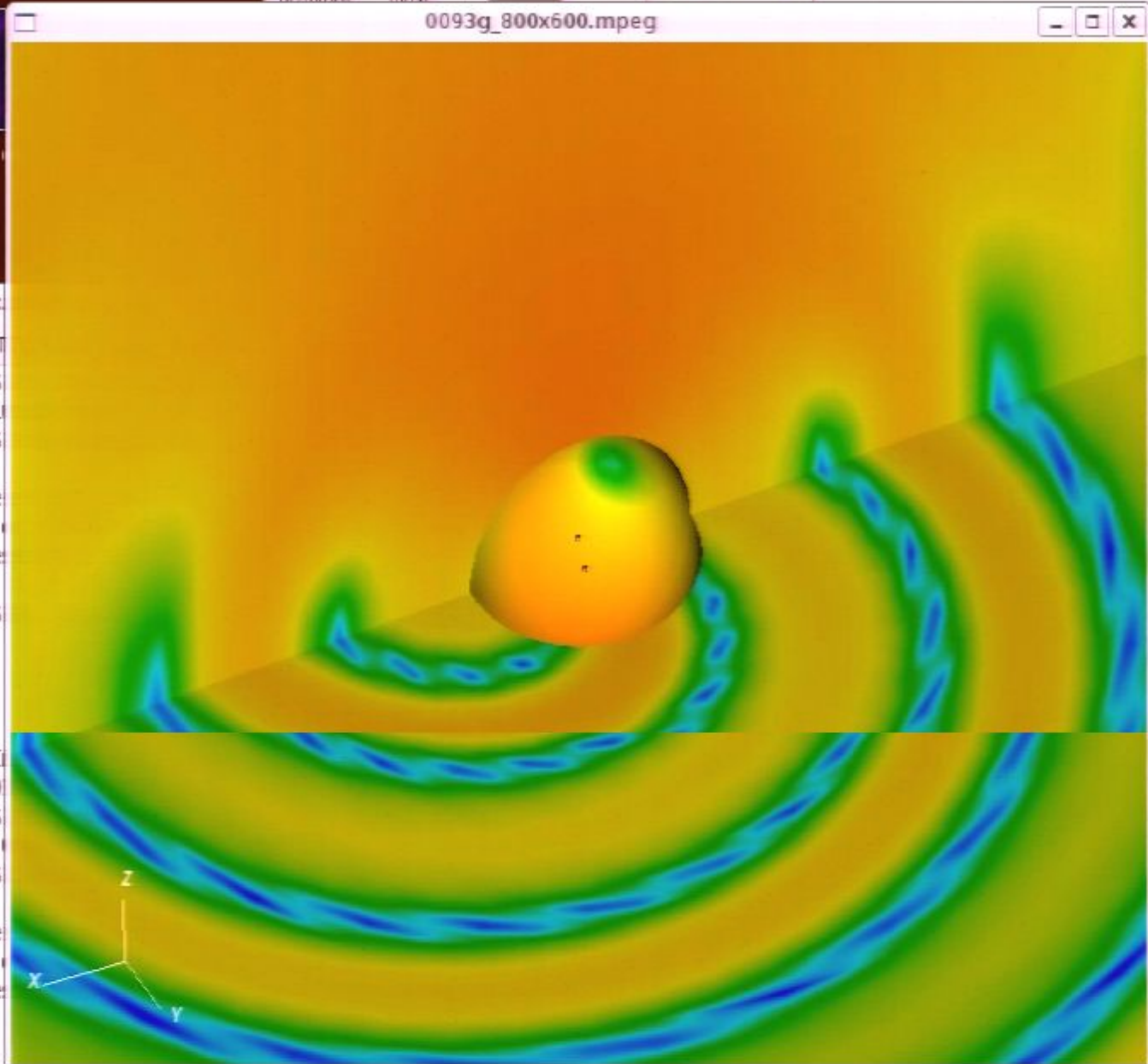
Pirsa: 08050036



MPEG Player Controls

Frame/Rate 1545/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

```
harald@Neville Movies]$  
Embedding 0093r Medi  
harald@Neville Movies]$
```

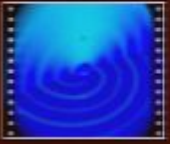
```
24 bit displays: use  
ordered  
cified dither require  
ing -dither color  
harald@Neville Movies]$
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e!

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Time Spent (After I  
Frames/Sec: 24.9610  
harald@Neville Movies]$  
3g_800x600_Late.png  
harald@Neville Movies]$
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24 bit displays: use  
ordered  
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ing -dither color
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Pirsa: 08050036



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

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harald@Neville Movies]$  
Embedding 0093r Medi  
harald@Neville Movies]$
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24 bit displays: use  
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ing -dither color  
harald@Neville Movies]$
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Time Spent (After I  
Frames/Sec: 24.9610  
harald@Neville Movies]$  
3g_800x600_Late.png  
harald@Neville Movies]$
```

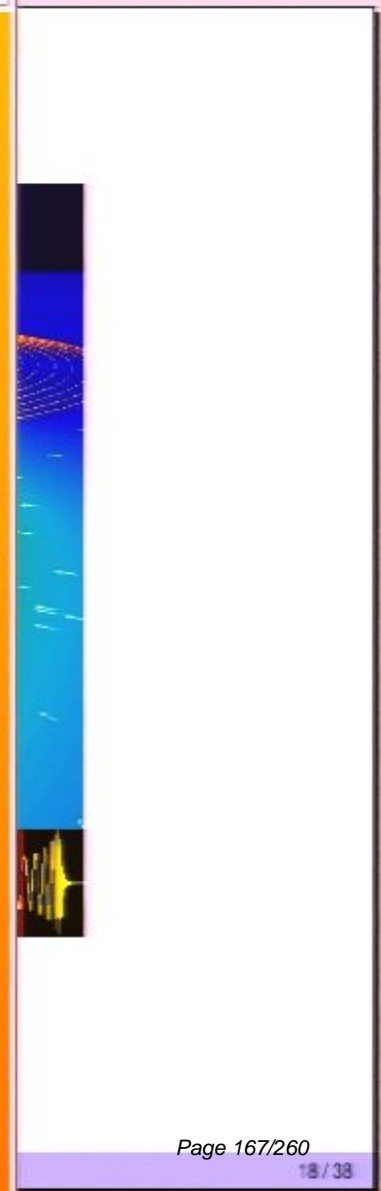
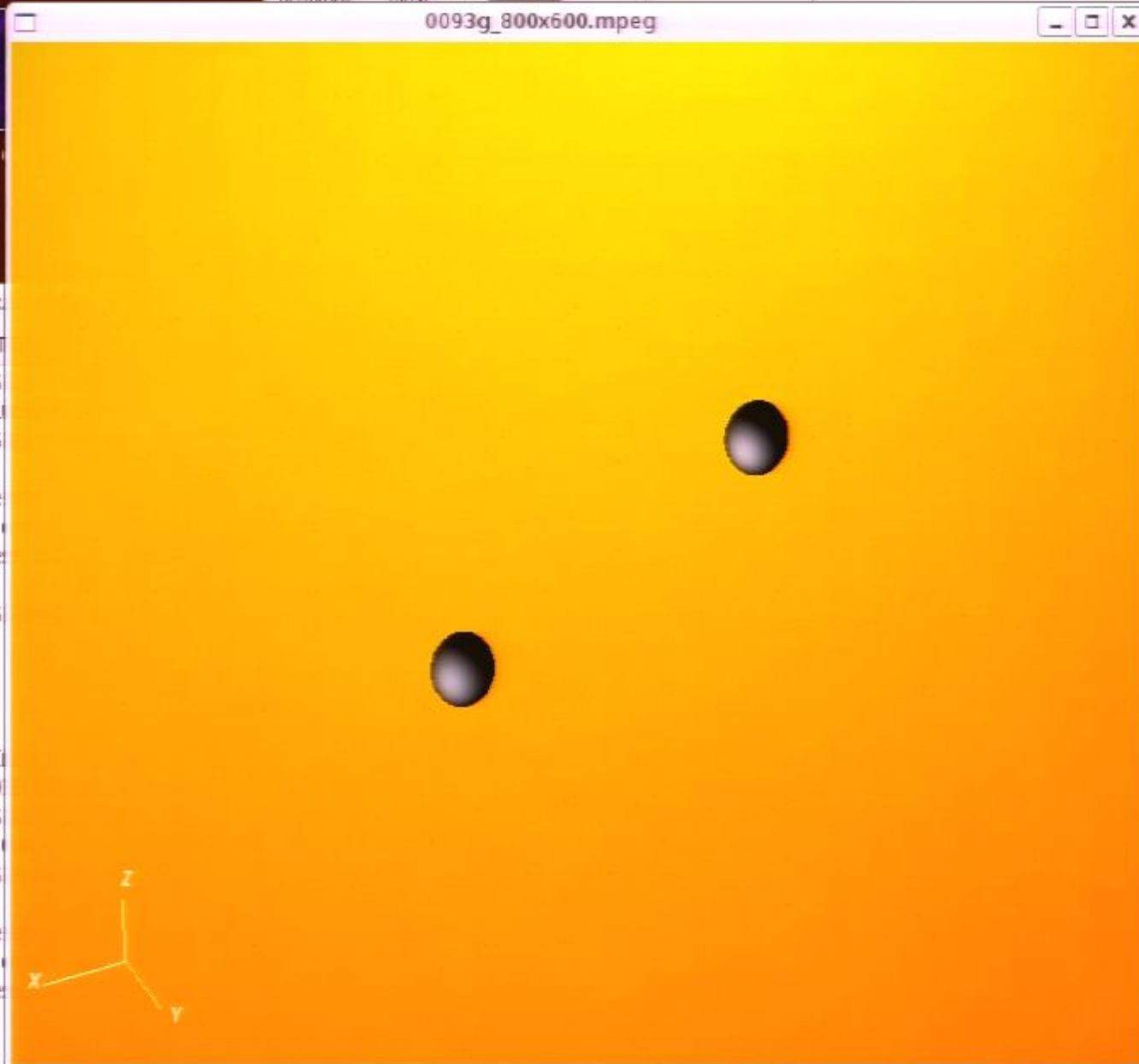
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24 bit displays: use  
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cified dither require  
ing -dither color
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Pirsa:08050036

MPEG Player Controls

FrameRate1610/24.9 Rewind Pause Step Play Loop OFF Exit

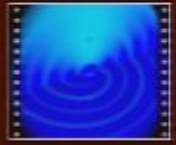
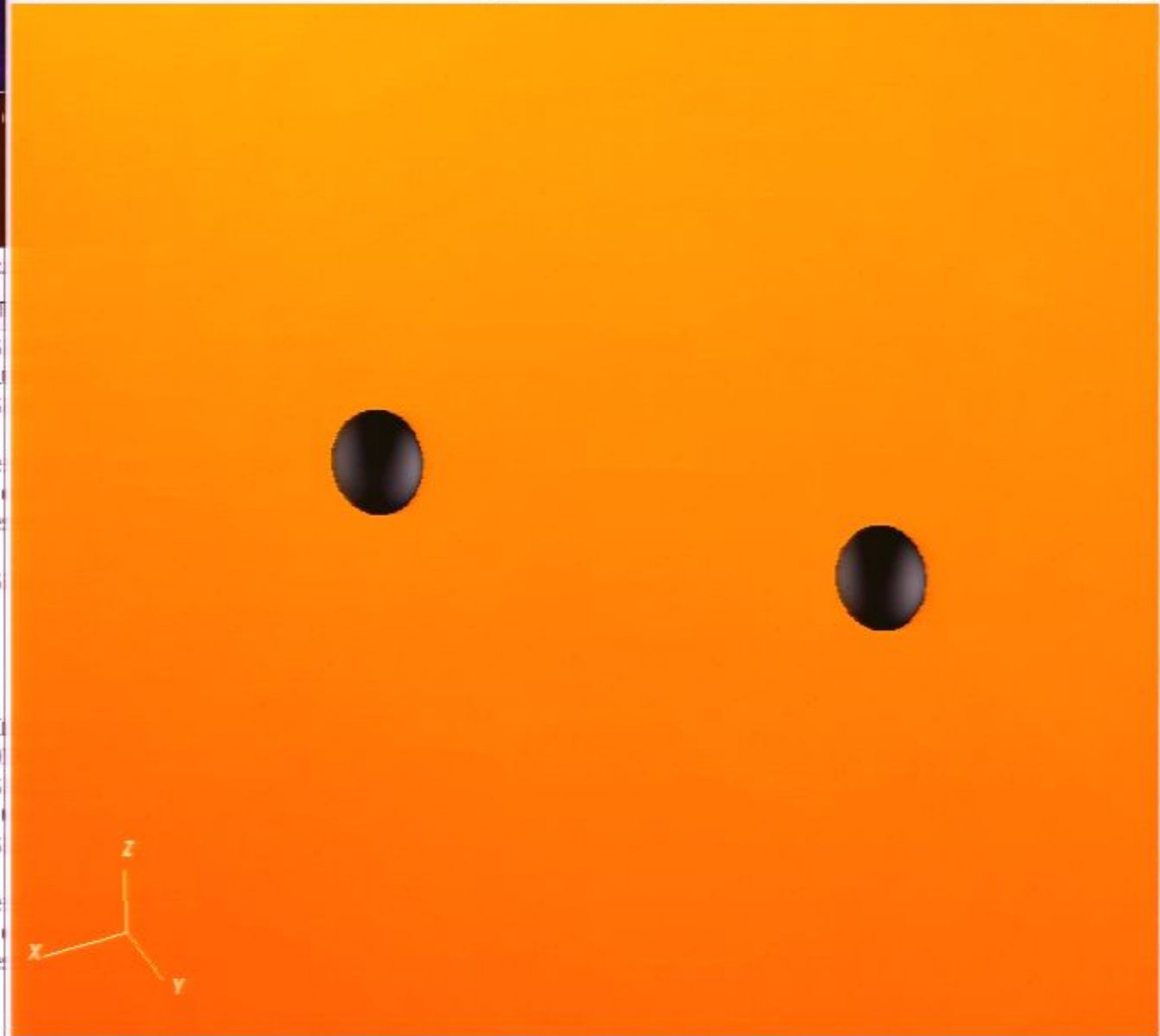
Previous Next Best Fit



MPEG Player Controls

FrameRate1675/24.9

0093g_800x600.mpeg



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

File Edit View Terminal

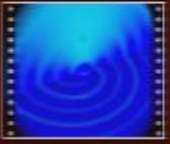
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harald@Neville Movies]$  
Embedding @093r Medi  
harald@Neville Movies]$
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24 bit displays: use  
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cified dither require  
ing -dither color  
harald@Neville Movies]$
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e!

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Time Spent (After I  
Frames/Sec: 24.9610  
harald@Neville Movies]$  
3g_800x600_Late.png  
harald@Neville Movies]$
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```
24 bit displays: use  
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cified dither require  
ing -dither color
```



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

File Edit View Terminal
harald@Neville Movies]\$
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harald@Neville Movies]\$

24 bit displays: use
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cified dither require
ing -dither color
harald@Neville Movies]\$

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Time Spent (After I
Frames/Sec: 24.9610
harald@Neville Movies]\$
3g_800x600_Late.png
harald@Neville Movies]\$

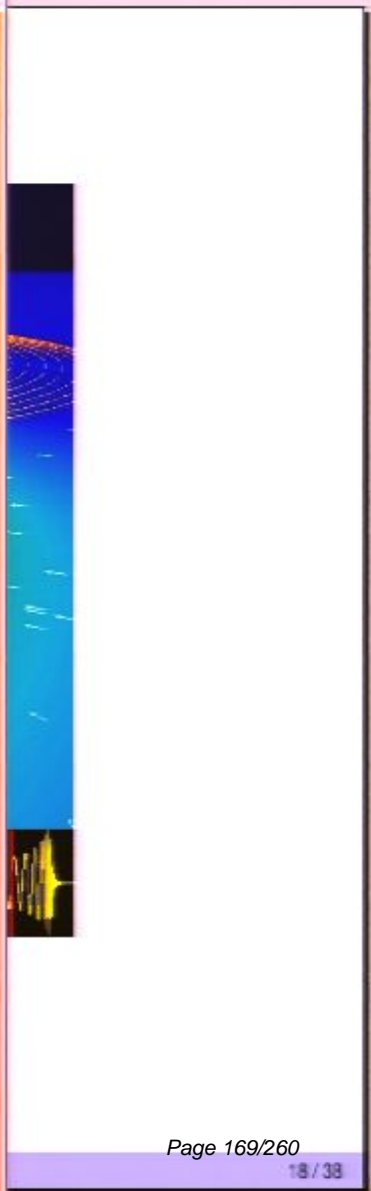
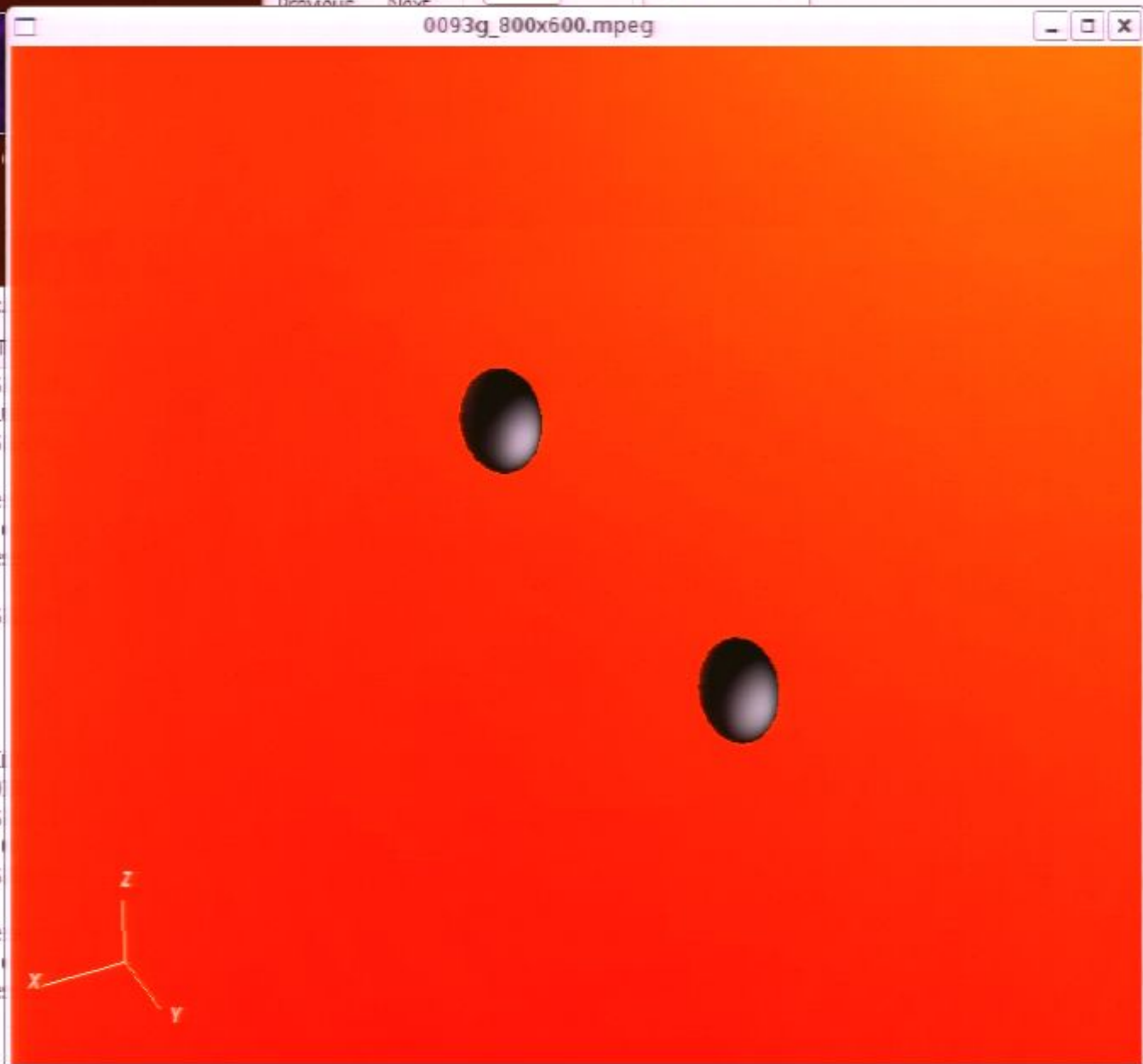
24 bit displays: use
ordered
cified dither require
ing -dither color

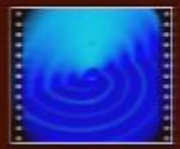
Pisa: 08050036

MPEG Player Controls

FrameRate: 1738/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit





0093g_800x600.mpeg

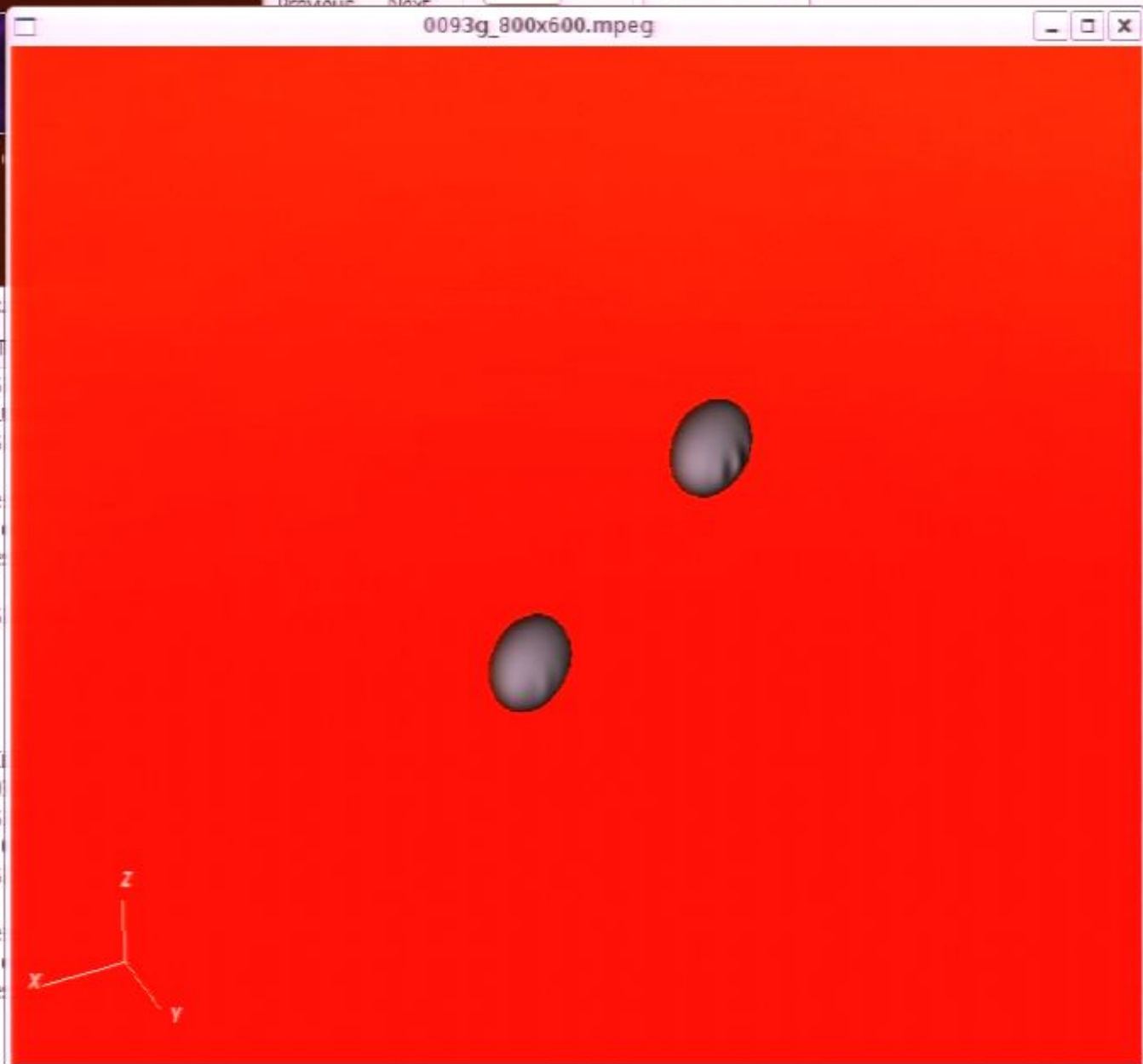


VanPutten2008

MPEG Player Controls

FrameRate1794/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit



harald@Neville:

File Edit View Terminal

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harald@Neville Movies]$  
Embedding 0093r Medi  
harald@Neville Movies]$
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24 bit displays: use  
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cified dither require  
ing -dither color  
harald@Neville Movies]$
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e!

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Time Spent (After I  
Frames/Sec: 24.9610  
harald@Neville Movies]$  
3g_800x600_Late.png  
harald@Neville Movies]$
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24 bit displays: use  
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cified dither require  
ing -dither color
```

Pirsa: 08050036



0093g_800x600.mpeg



VarPutten2008

harald@Neville:

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harald@Neville Movies]$  
Embedding 0093r Medi  
harald@Neville Movies]$
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24 bit displays: use  
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cified dither require  
ing -dither color  
harald@Neville Movies]$
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e!

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Time Spent (After I  
Frames/Sec: 24.9610  
harald@Neville Movies]$  
3g_800x600 Late.png  
harald@Neville Movies]$
```

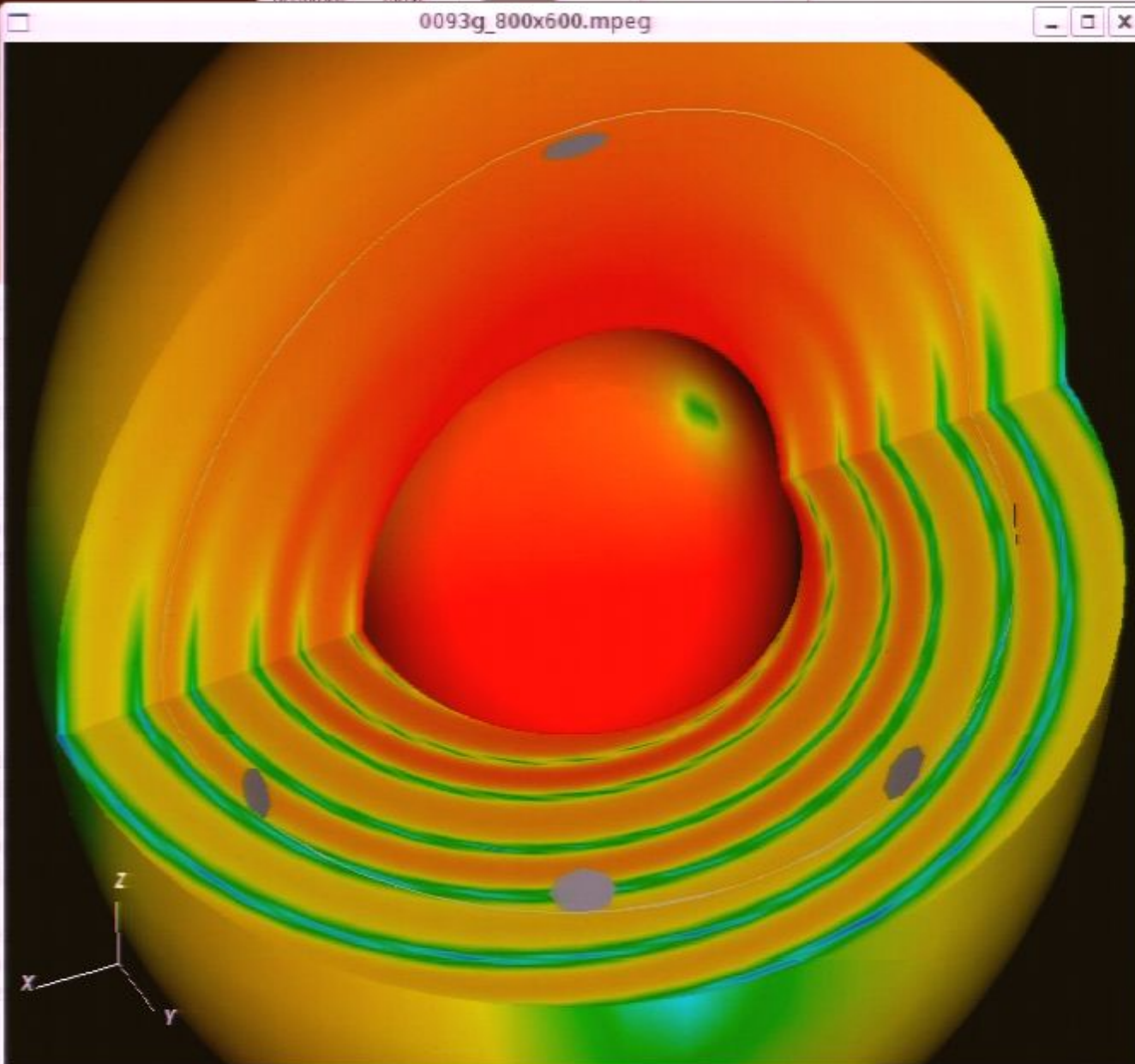
```
24 bit displays: use  
ordered  
cified dither require  
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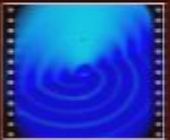
Pirsa: 08050036

MPEG Player Controls

Frame/Rate: 1954/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit





0093g_800x600.mpeg



VanPutten2008

harald@Neville:

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harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$
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24 bit displays: use
ordered
cified dither require
ing -dither color
rald@Neville Movies]$
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e!

```
Time Spent (After I
Frames/Sec: 24.9610
rald@Neville Movies]$
3g_800x600_Late.png
rald@Neville Movies]$
```

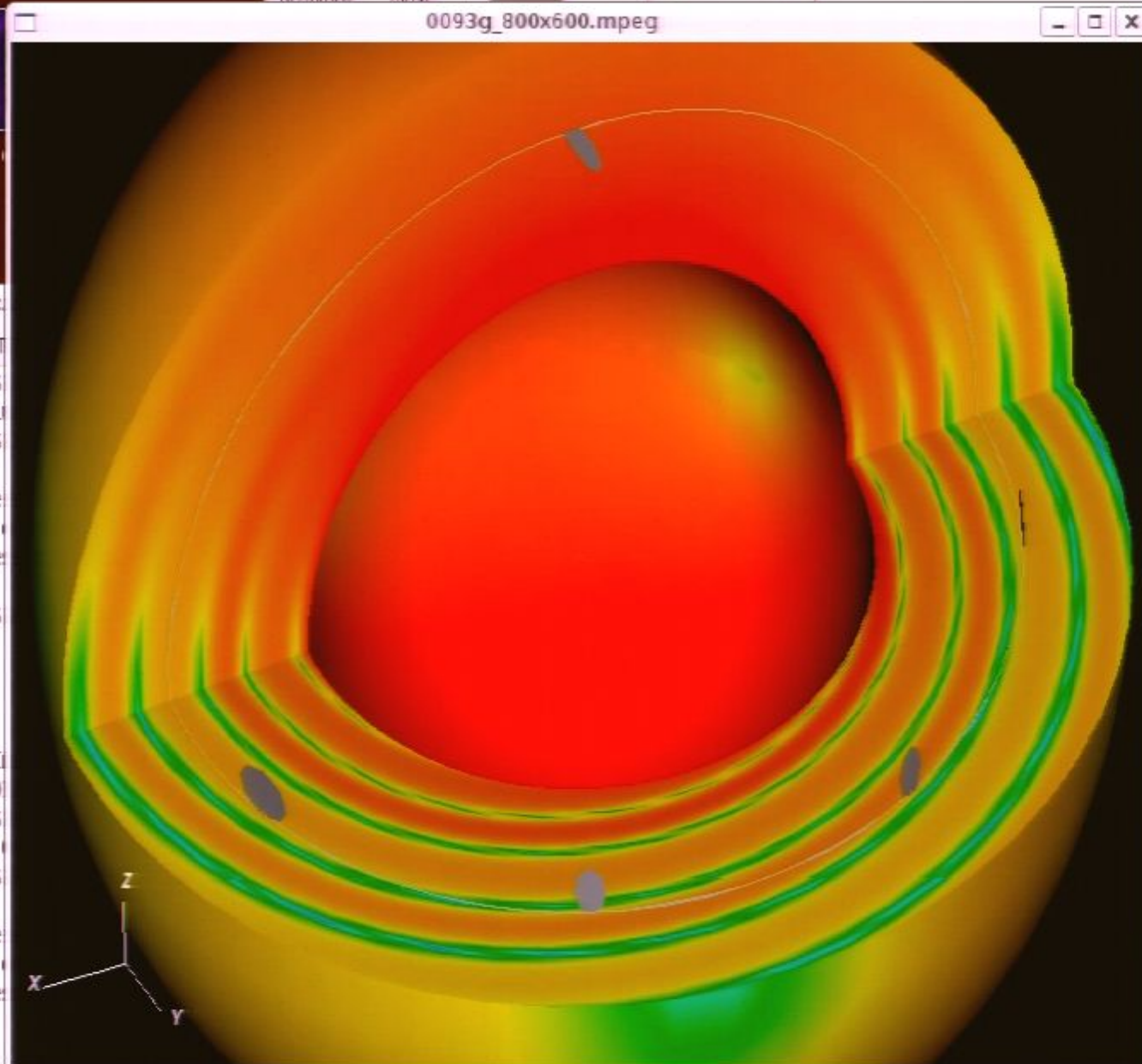
```
24 bit displays: use
ordered
cified dither require
ing -dither color
```

Pirsa: 08050036

MPEG Player Controls

FrameRate: 2014/24.9 Rewind Pause Step **Play** Loop OFF Exit

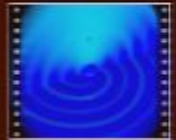
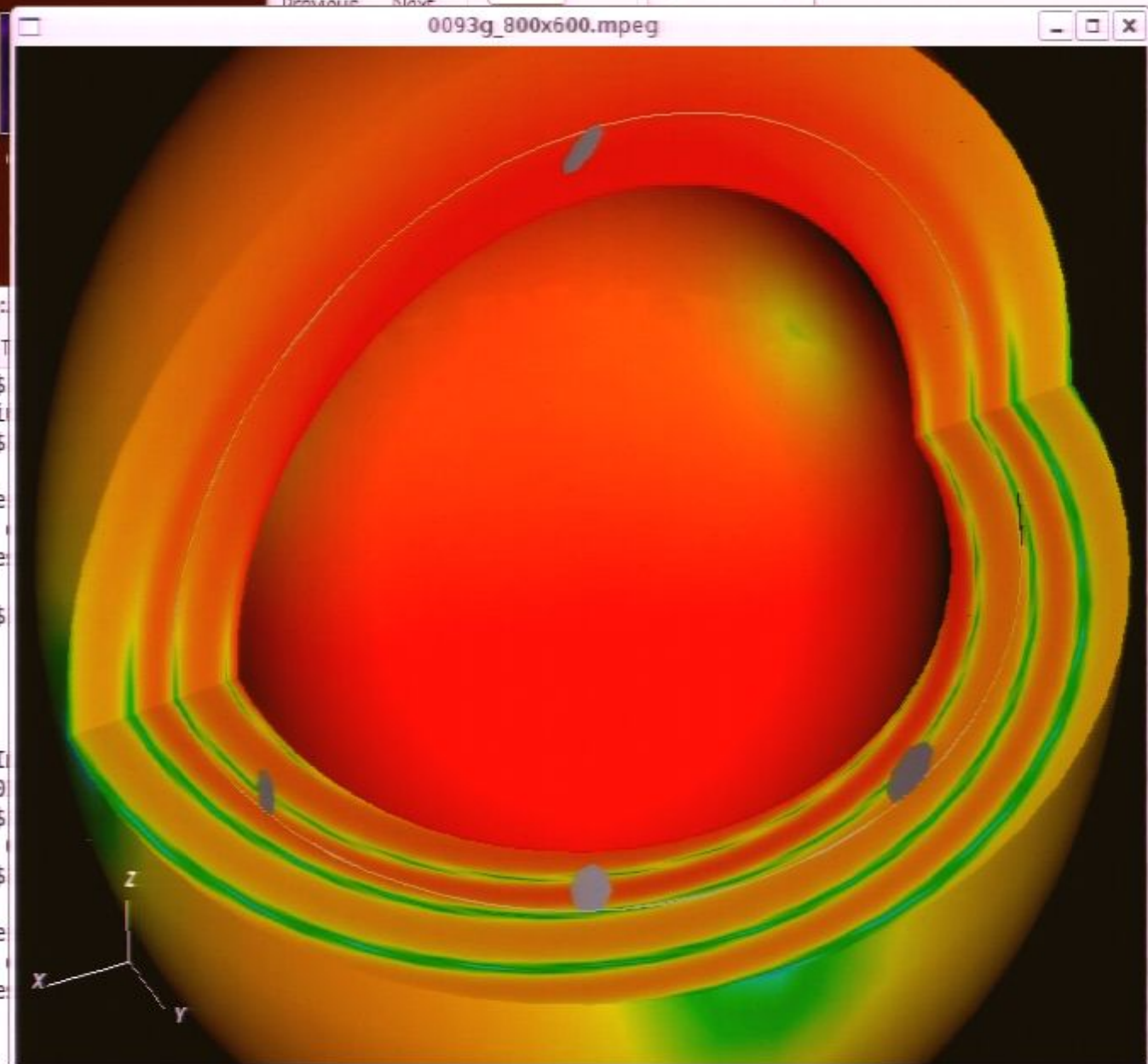
Previous Next Best Fit



MPEG Player Controls

FrameRate: 2085/24.9 Rewind Pause Step Play Loop OFF Exit

Previous Next Best Fit



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

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Edit View Terminal T
harald@Neville Movies]$
Embedding 0093r Medi
harald@Neville Movies]$

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24 bit displays: use
ordered
cified dither require
ing -dither color
harald@Neville Movies]$

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e!

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Time Spent (After I
Frames/Sec: 24.9610
harald@Neville Movies]$
3g_800x600 Late.png
harald@Neville Movies]$

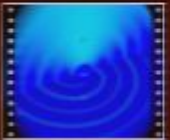
```

```

24 bit displays: use
ordered
cified dither require
ing -dither color

```

Pisa: 00050036



0093g_800x600.mpeg



VarPutten2008

harald@Neville:

File Edit View Terminal T
harald@Neville Movies]\$
Embedding 0093r Medi
harald@Neville Movies]\$

24 bit displays: use
ordered
cified dither require
ing -dither color
harald@Neville Movies]\$

e!

Time Spent (After I
Frames/Sec: 24.9610
harald@Neville Movies]\$
3g_800x600 Late.png
harald@Neville Movies]\$

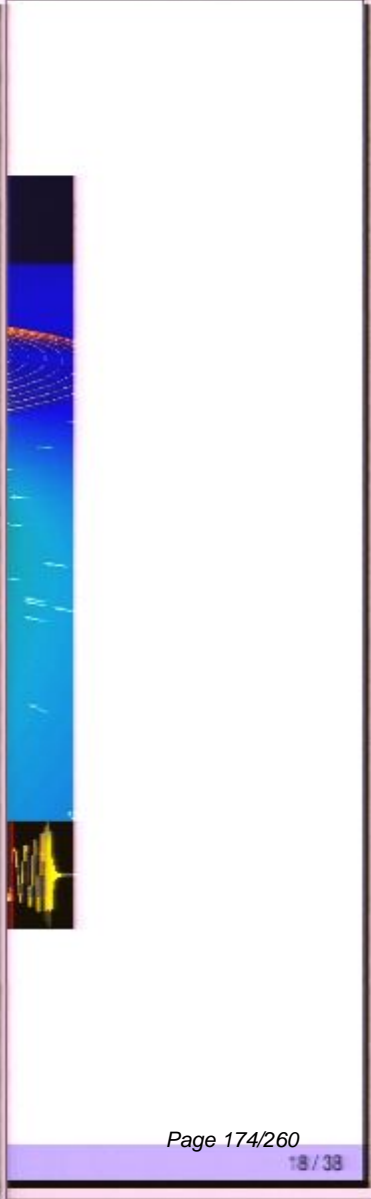
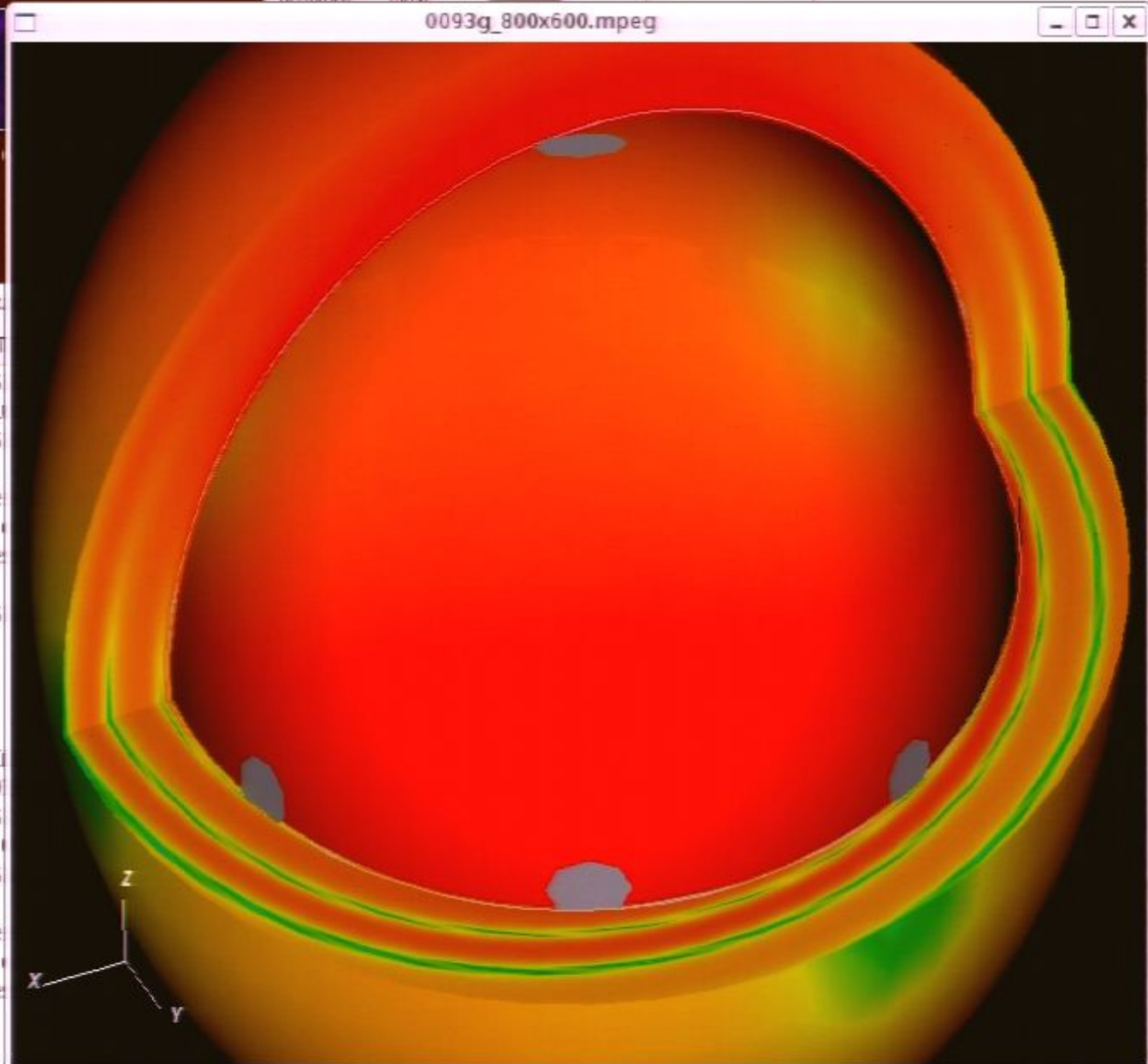
24 bit displays: use
ordered
cified dither require
ing -dither color
Pirsa: 08050036

MPEG Player Controls

Frame/Rate: 2144/24.9

Buttons: Rewind, Pause, Step, Play, Loop OFF, Exit

Buttons: Previous, Next, Best Fit

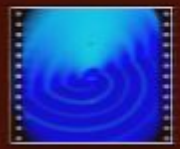
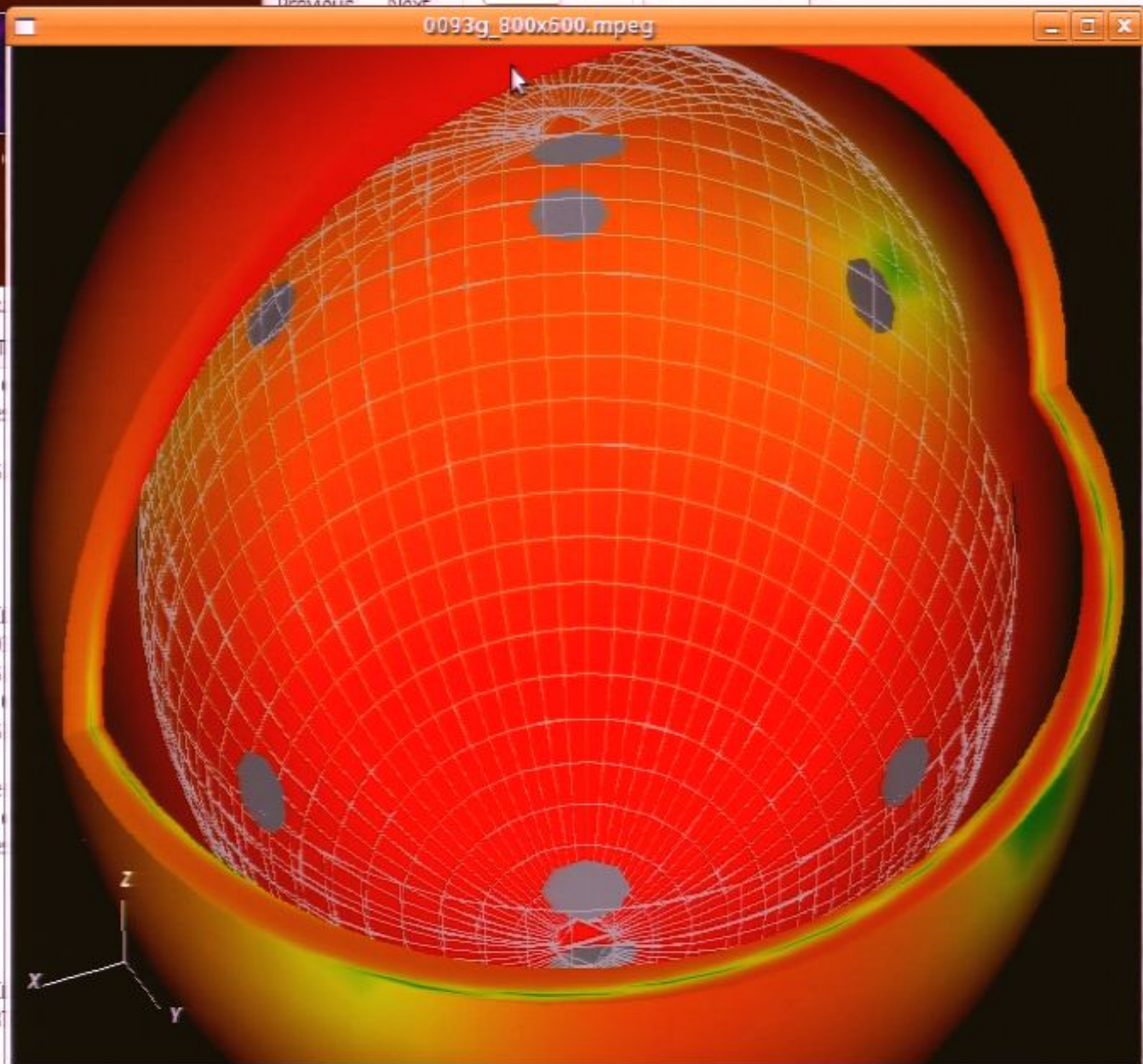


MPEG Player Controls

FrameRate: 2210/24.9

Buttons: Rewind, End, Step, Play, Loop OFF, Exit

Previous, Next, Best Fit



0093g_800x600.mpeg



VanPutten2008

harald@Neville:

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edit View Terminal T
ordered
specified dither require
ing -dither color
rald@Neville Movies]$
```

e!

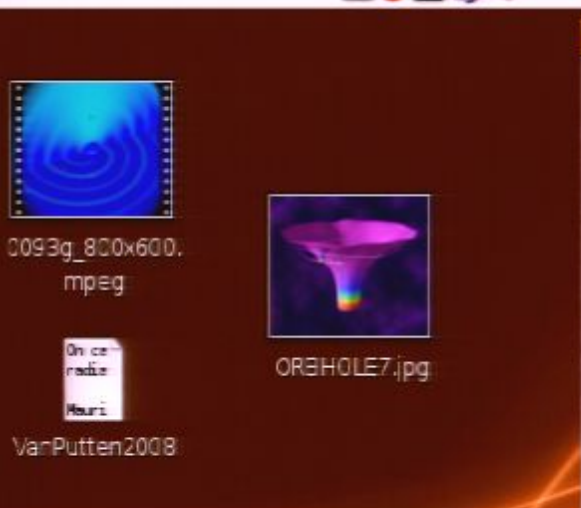
```
Time Spent (After I
Frames/Sec: 24.9610
rald@Neville Movies]$
3g 800x600 Late.png
rald@Neville Movies]$
```

```
24 bit displays: use
ordered
specified dither require
ing -dither color
```

e!

```
Time Spent (After I
Frames/Sec: 24.9373
```

Pisa: 08050036



Numerical study of black hole spacetimes

File Edit View Go Help

Previous Next 27 of 67 Best Fit

Movies I

```

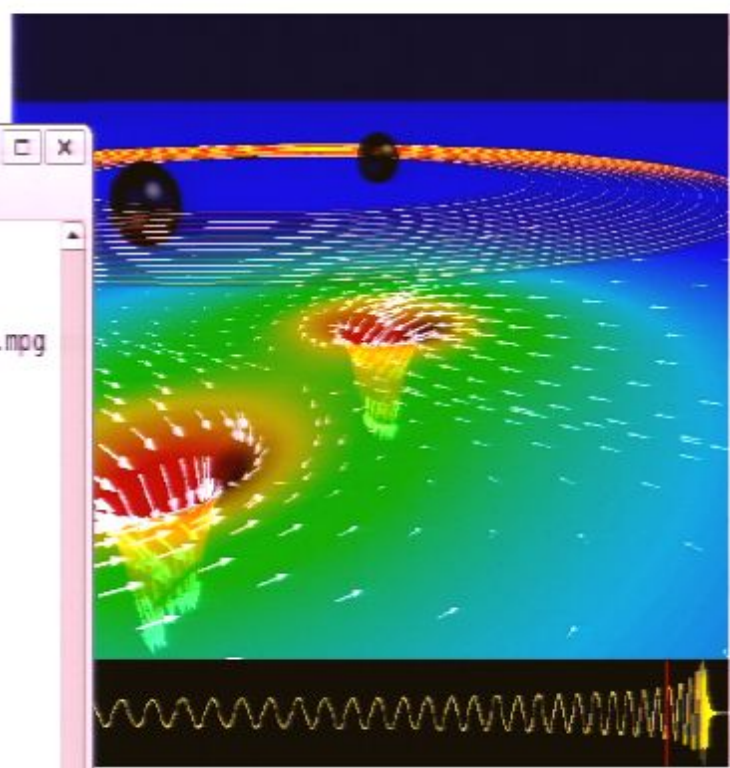
harald@Neville:AA~/research/Talks/08May_Perimeter/Movies
Edit View Terminal Tabs Help
ordered dither is the default.
cified dither requires 8 bit display
ing -dither color
rald@Neville Movies]$ mpeg_play -dither color Bbh_Embedding_0093r_Medium.mpg
e!

Time Spent (After Initializations): 39.581727 secs.
Frames/Sec: 24.961013
rald@Neville Movies]$ mpeg_play 0093g
3g_800x600_Late.png 0093g_800x600.mpeg
rald@Neville Movies]$ mpeg_play 0093g_800x600.mpeg

24 bit displays: use -dither color to get full color
ordered dither is the default.
cified dither requires 8 bit display
ing -dither color
e!

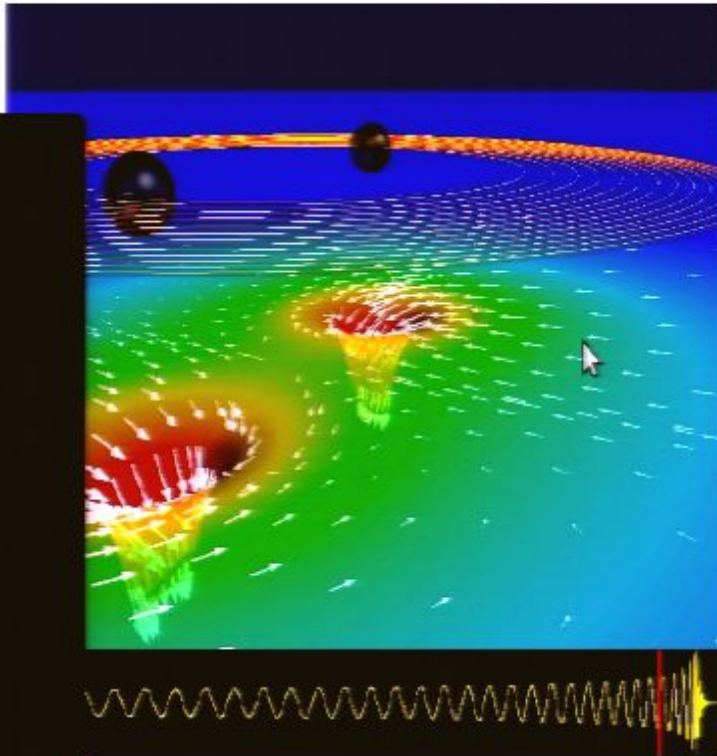
Time Spent (After Initializations): 88.621993 secs.
Frames/Sec: 24.937376
rald@Neville Movies]$

```



www.black-holes.org/explore2.html

Movies I



www.black-holes.org/explore2.html

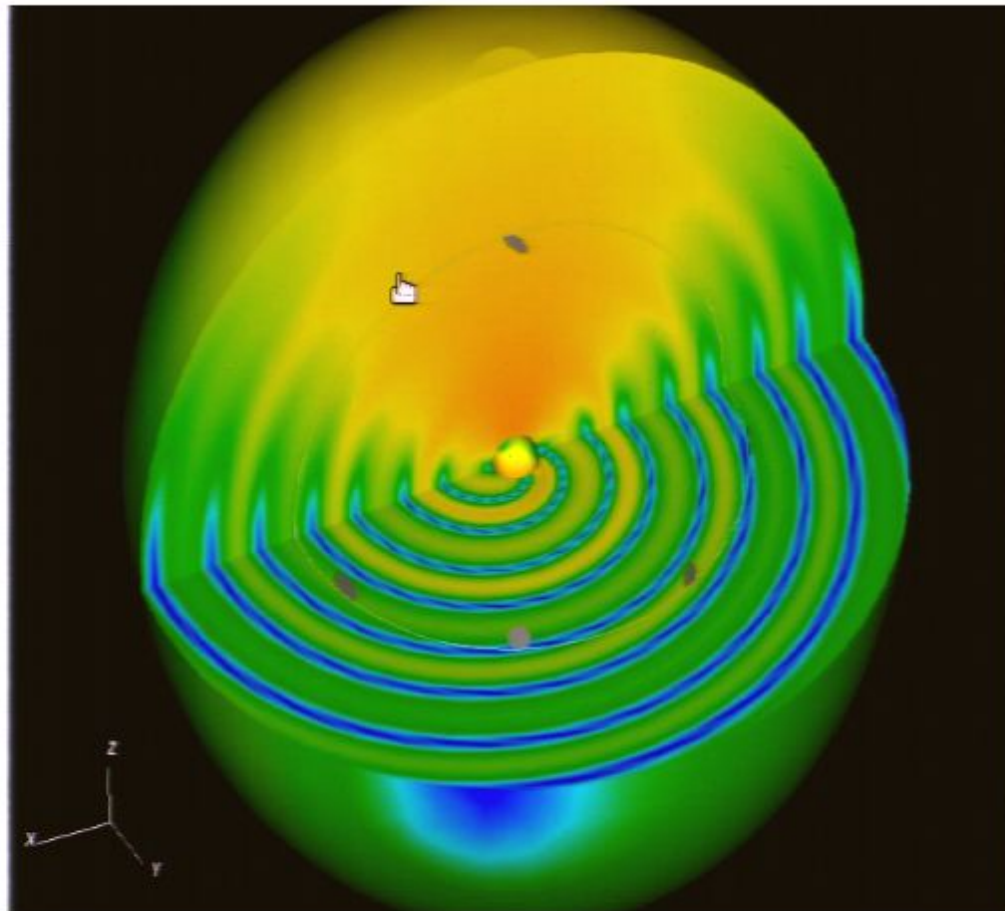
Black hole simulations

18 / 38

Black hole simulations

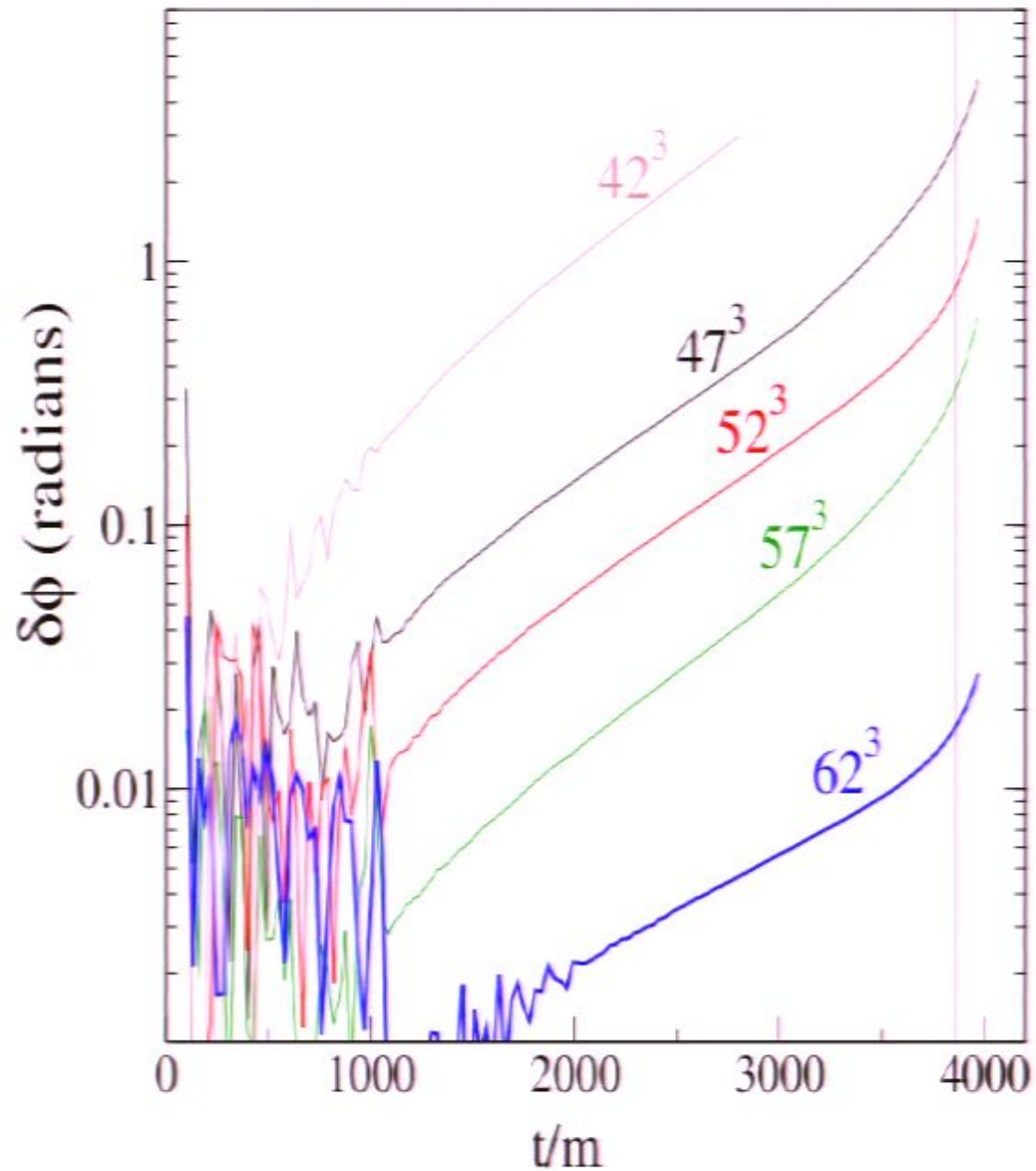
18 / 38

Movies II



www.black-holes.org/explore2.html

Phase-Accuracy (out of 200 radians)



Post-Newtonian theory

Blanchet, Damour, Iyer, Schäfer, Jaranowski, Faye; Will, Wiseman, Kidder. ...

- Expansion in velocity $v = v/c$

- For a binary in a circular orbit

- ▶ Energy
$$E(v) = -\frac{\mu}{2}v^2 \left(1 + \sum_{k=1}^7 a_k v^k \right)$$

- ▶ GW-Flux
$$F(v) = \frac{32\mu^2}{5}v^{10} \left(1 + \sum_{k=1}^7 b_k v^k \right)$$

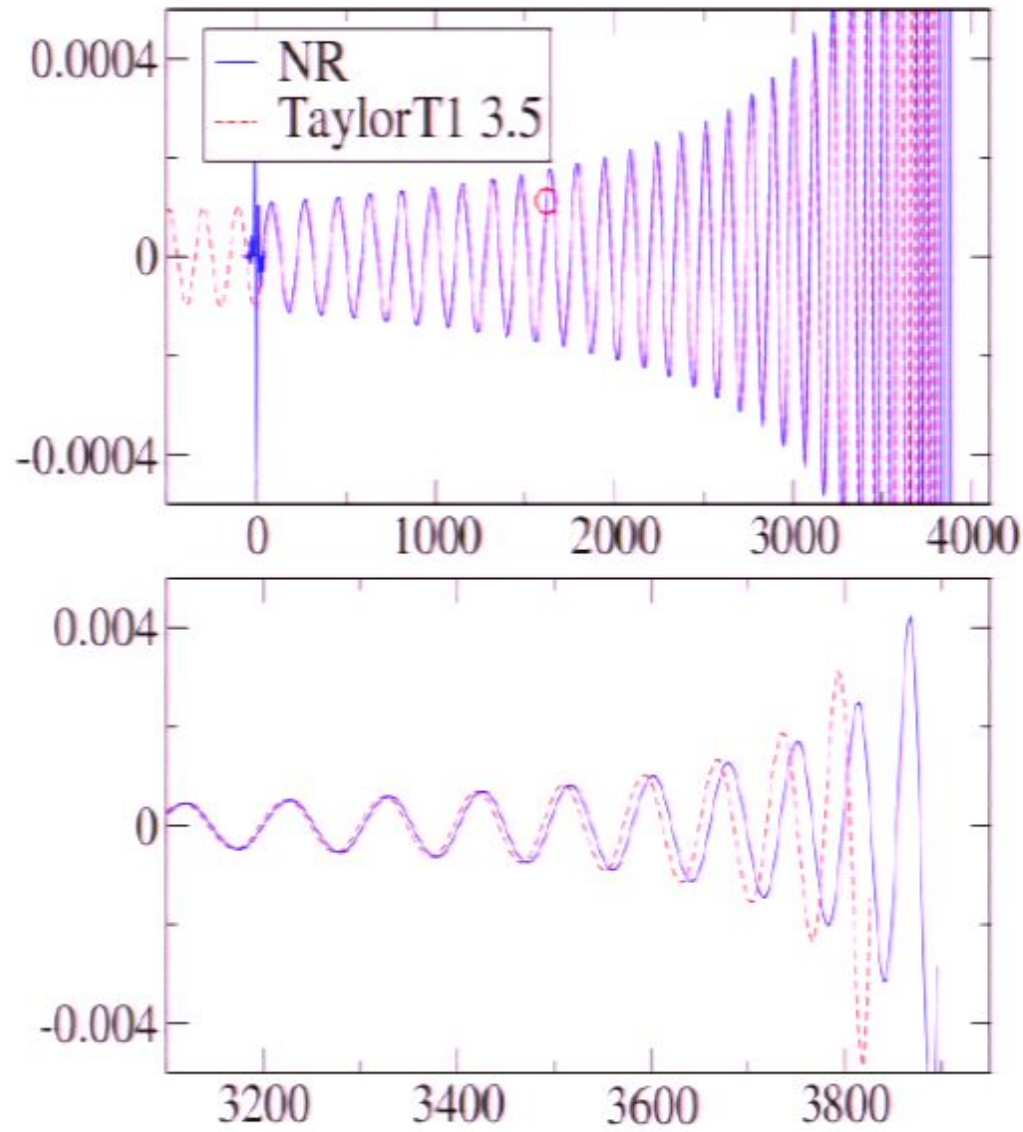
- Energy-balance gives time-evolution:

$$\frac{dE}{dt} = -F \quad \Rightarrow \quad \frac{dv}{dt} = -\frac{F}{dE/dv}$$

- Difficulty: $v/c \sim 0.3$ during late inspiral

- ▶ Slow convergence
- ▶ Uncontrolled higher-order terms seizeable!

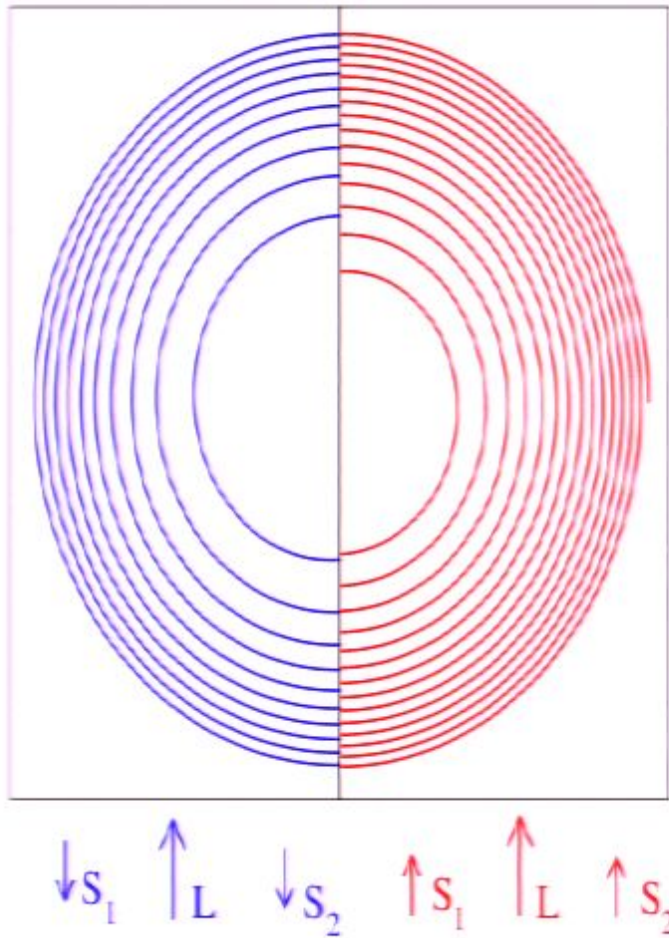
Comparing Waveforms



In the pipeline

Spinning BHs

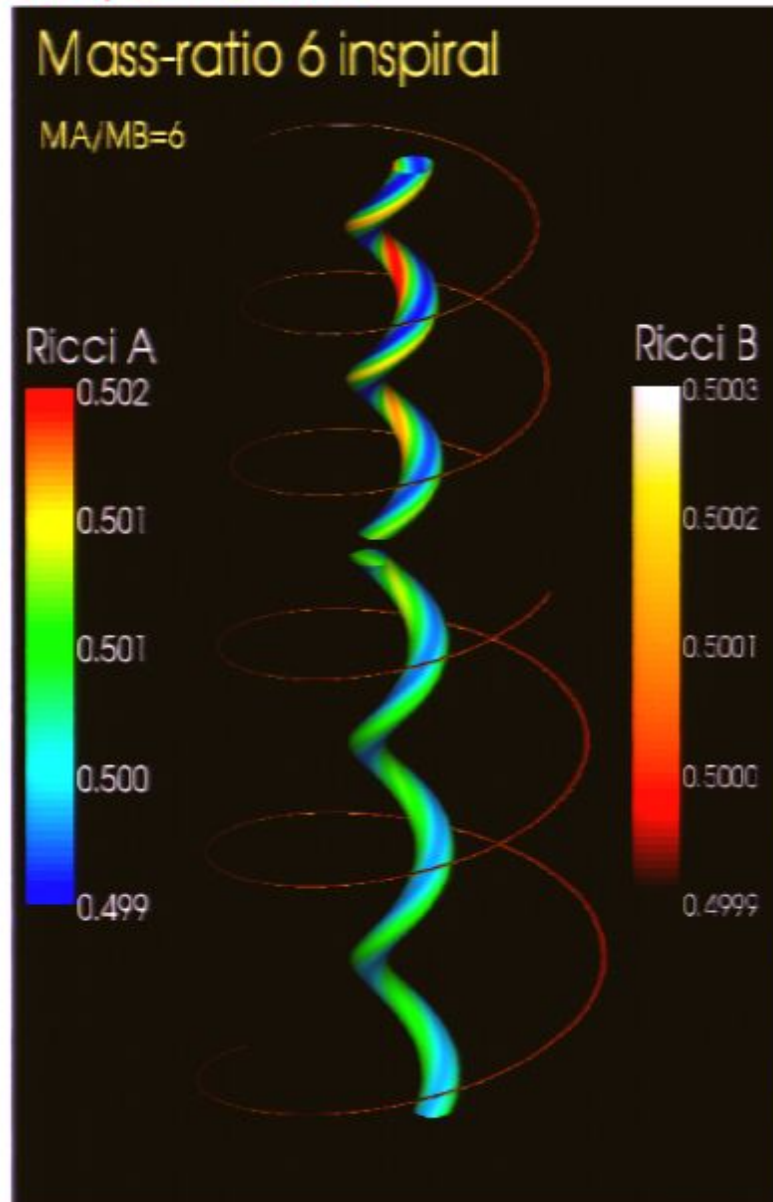
(Trajectory of BH A)



Unequal mass BHs

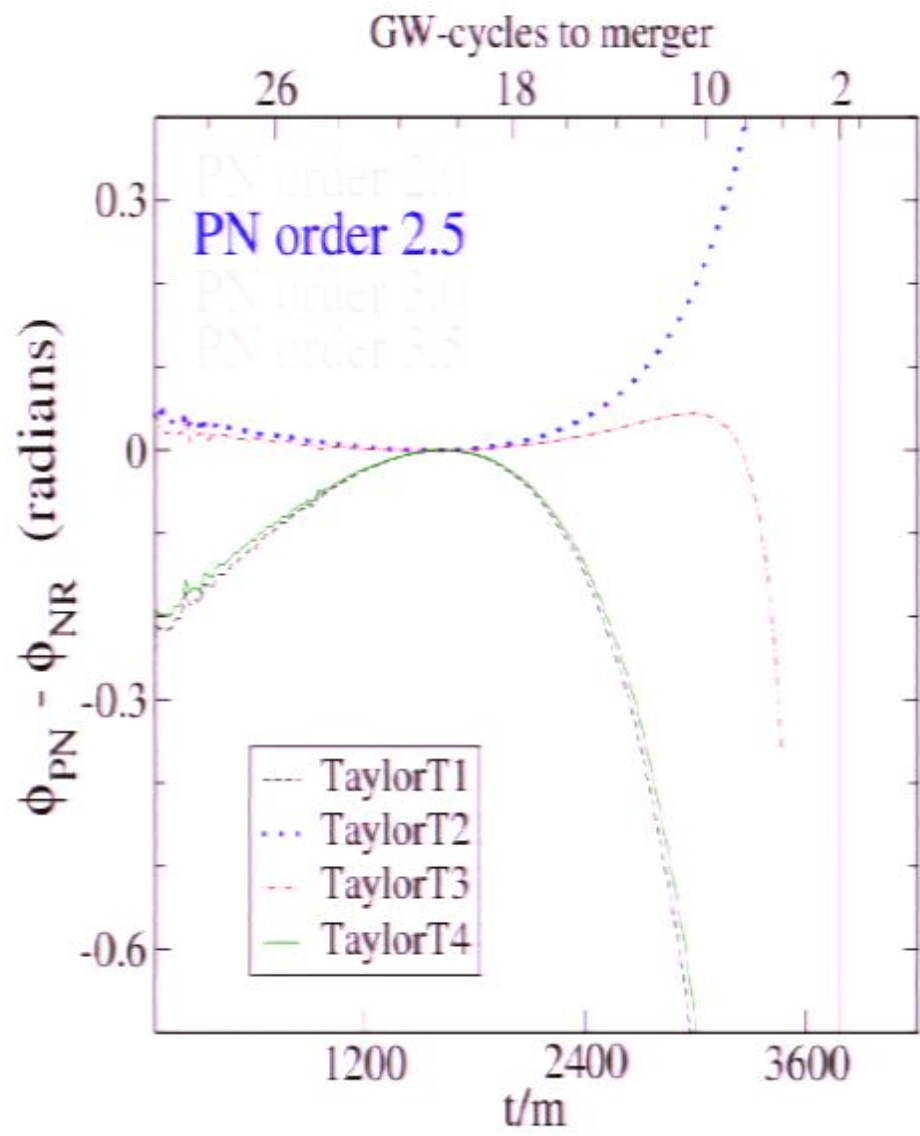
Mass-ratio 6 inspiral

$M_A/M_B=6$



Numerical relativity vs. post-Newtonian

Boyle et al., 2007



PN-approximants

- Different treatment of uncontrolled higher-order terms, e.g.

- ▶ Use of energy-balance equation (Damour, Iyer, Sathyaprakash, 01)

$$\frac{dv}{dt} = -\frac{F}{dE/dv} \quad \text{TaylorT1}$$

$$\frac{dv}{dt} = -\text{Series} \left[\frac{F}{dE/dv}, v \right] \quad \text{TaylorT4}$$

$$\text{Series} \left[\frac{dE/dv}{F}, v \right] \frac{dv}{dt} = -1 \quad \text{TaylorT2 \& T3}$$

- ▶ Padé-resummation of $F(v)$ (Damour et al. 98; Buonanno et al. 98)
- ▶ Effective-One-Body formalism (Damour and collaborators)

Post-Newtonian theory

Blanchet, Damour, Iyer, Schäfer, Jaranowski, Faye; Will, Wiseman, Kidder, ...

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- For a **binary in a circular orbit**

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$$E(v) = -\frac{\mu}{2}v^2 \left(1 + \sum_{k=1}^7 a_k v^k \right)$$

- ▶ GW-Flux
$$F(v) = \frac{32\mu^2}{5}v^{10} \left(1 + \sum_{k=1}^7 b_k v^k \right)$$

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PN-approximants

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$$\frac{dv}{dt} = -\text{Series} \left[\frac{F}{dE/dv}, v \right] \quad \text{TaylorT4}$$

$$\text{Series} \left[\frac{dE/dv}{F}, v \right] \frac{dv}{dt} = -1 \quad \text{TaylorT2 \& T3}$$

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Post-Newtonian theory

Blanchet, Damour, Iyer, Schäfer, Jaranowski, Faye; Will, Wiseman, Kidder. ...

- Expansion in velocity $v = v/c$
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$$E(v) = -\frac{\mu}{2}v^2 \left(1 + \sum_{k=1}^7 a_k v^k \right)$$

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$$F(v) = \frac{32\mu^2}{5}v^{10} \left(1 + \sum_{k=1}^7 b_k v^k \right)$$

- Energy-balance gives time-evolution:

$$\frac{dE}{dt} = -F \quad \Rightarrow \quad \frac{dv}{dt} = -\frac{F}{dE/dv}$$

- Difficulty: $v/c \sim 0.3$ during late inspiral
 - ▶ Slow convergence
 - ▶ Uncontrolled higher-order terms seizeable!

PN-approximants

- Different treatment of uncontrolled higher-order terms, e.g.

- ▶ Use of energy-balance equation (Damour, Iyer, Sathyaprakash, 01)

$$\frac{dv}{dt} = -\frac{F}{dE/dv} \quad \text{TaylorT1}$$

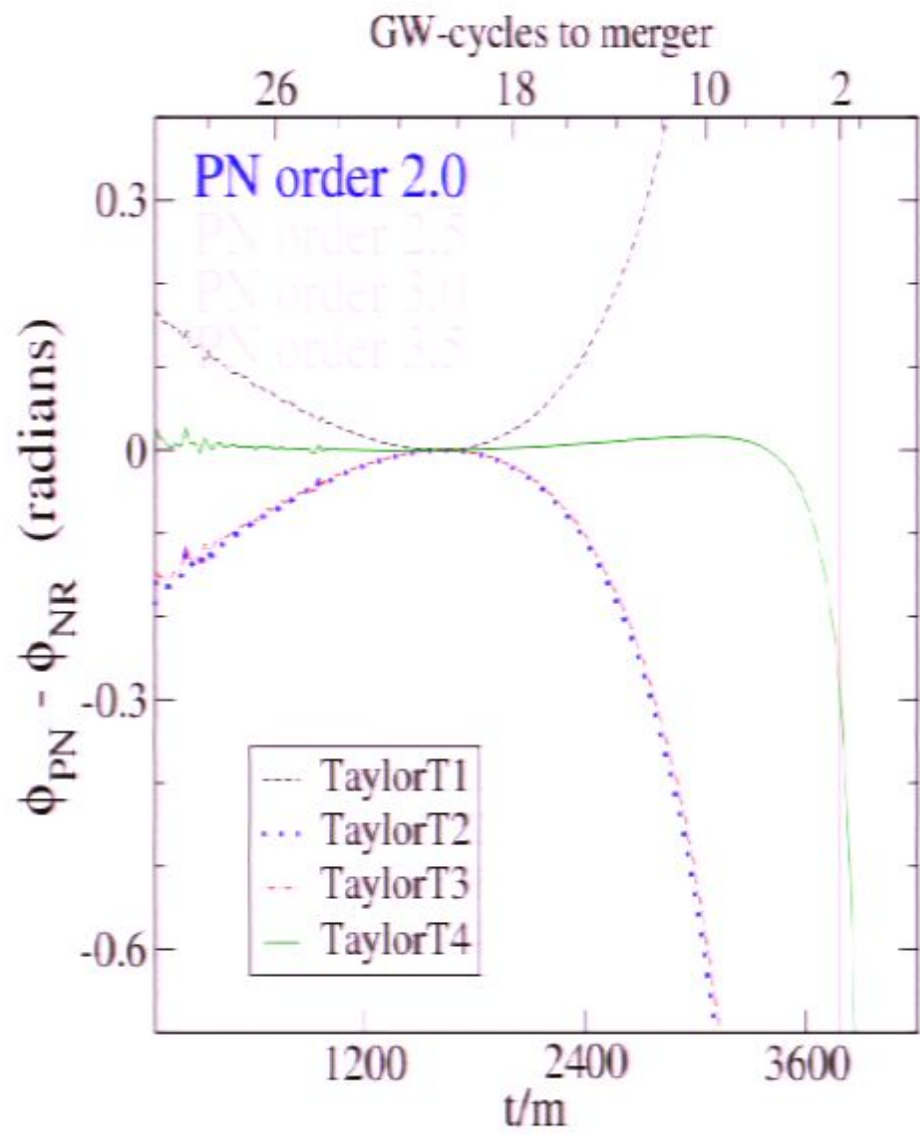
$$\frac{dv}{dt} = -\text{Series} \left[\frac{F}{dE/dv}, v \right] \quad \text{TaylorT4}$$

$$\text{Series} \left[\frac{dE/dv}{F}, v \right] \frac{dv}{dt} = -1 \quad \text{TaylorT2 \& T3}$$

- ▶ Padé-resummation of $F(v)$ (Damour et al. 98; Buonanno et al. 98)
- ▶ Effective-One-Body formalism (Damour and collaborators)

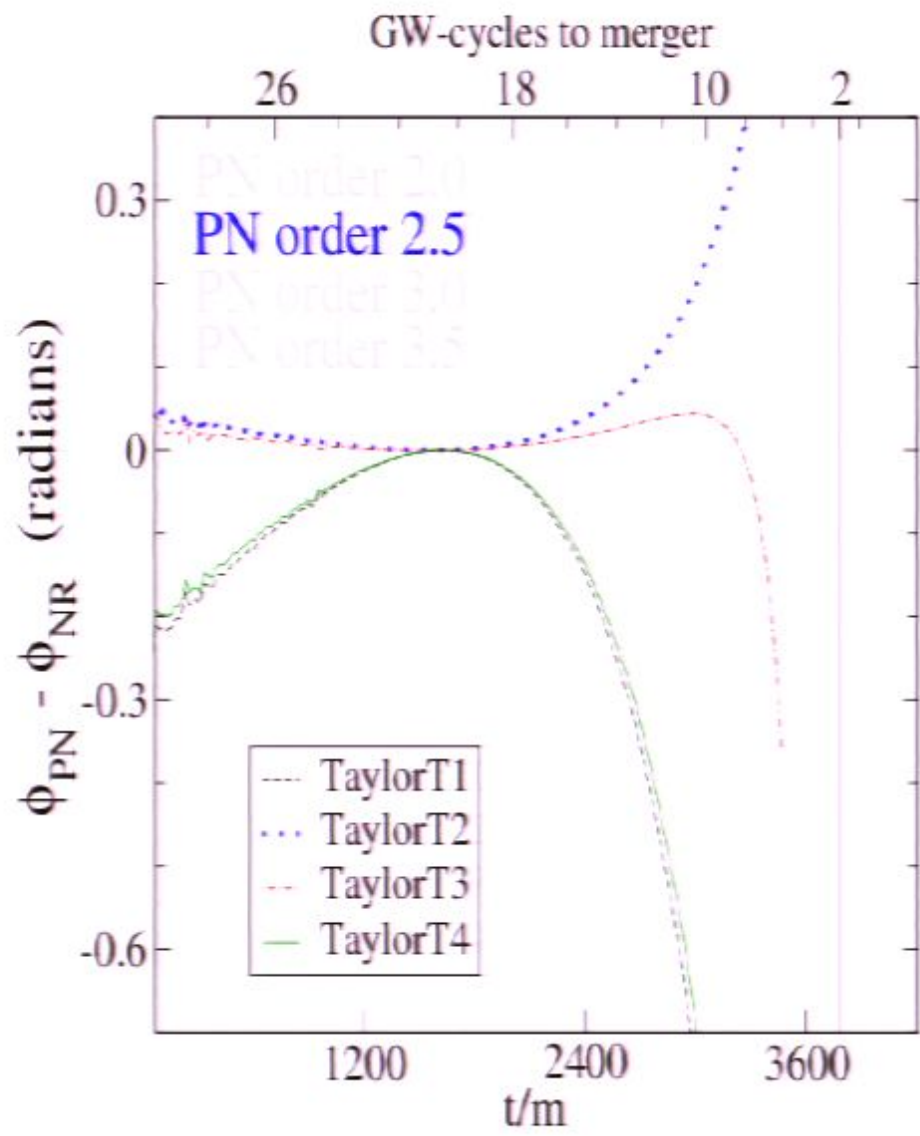
Numerical relativity vs. post-Newtonian

Boyle et al., 2007



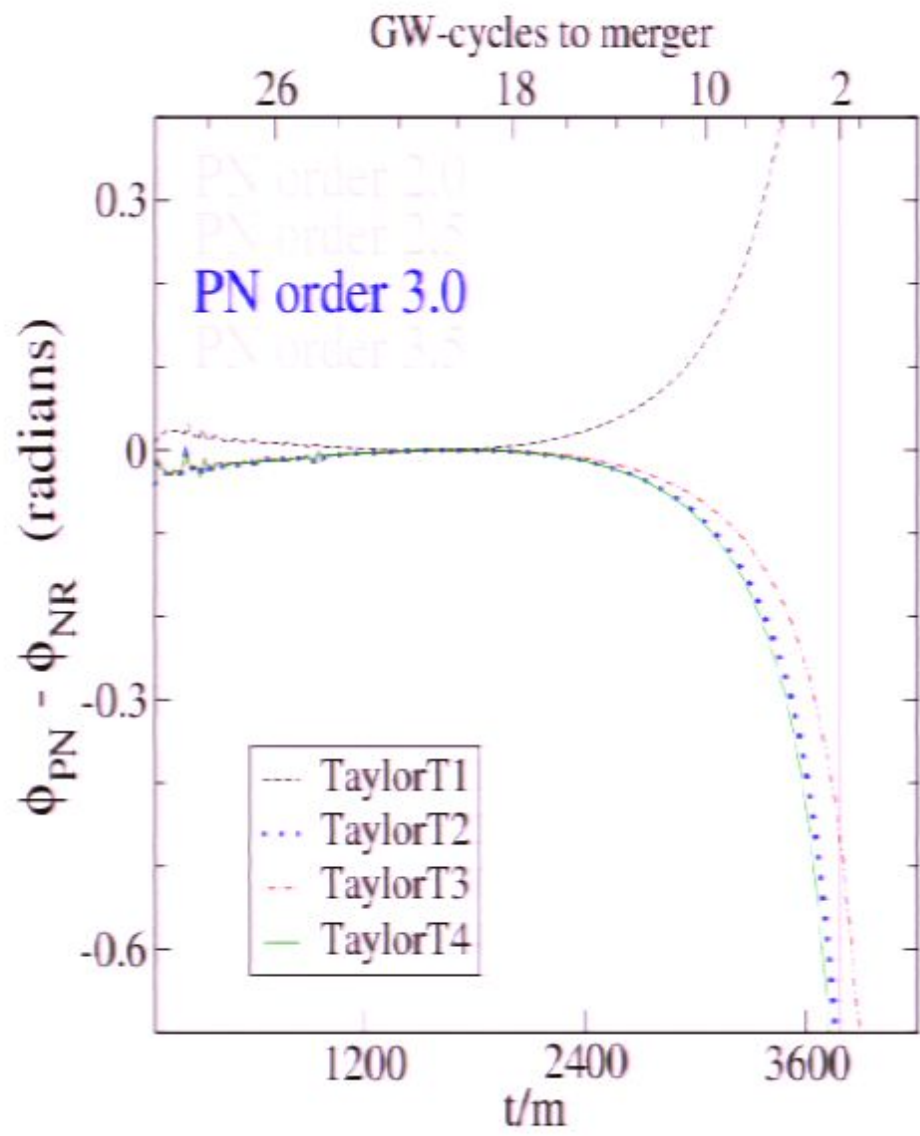
Numerical relativity vs. post-Newtonian

Boyle et al., 2007



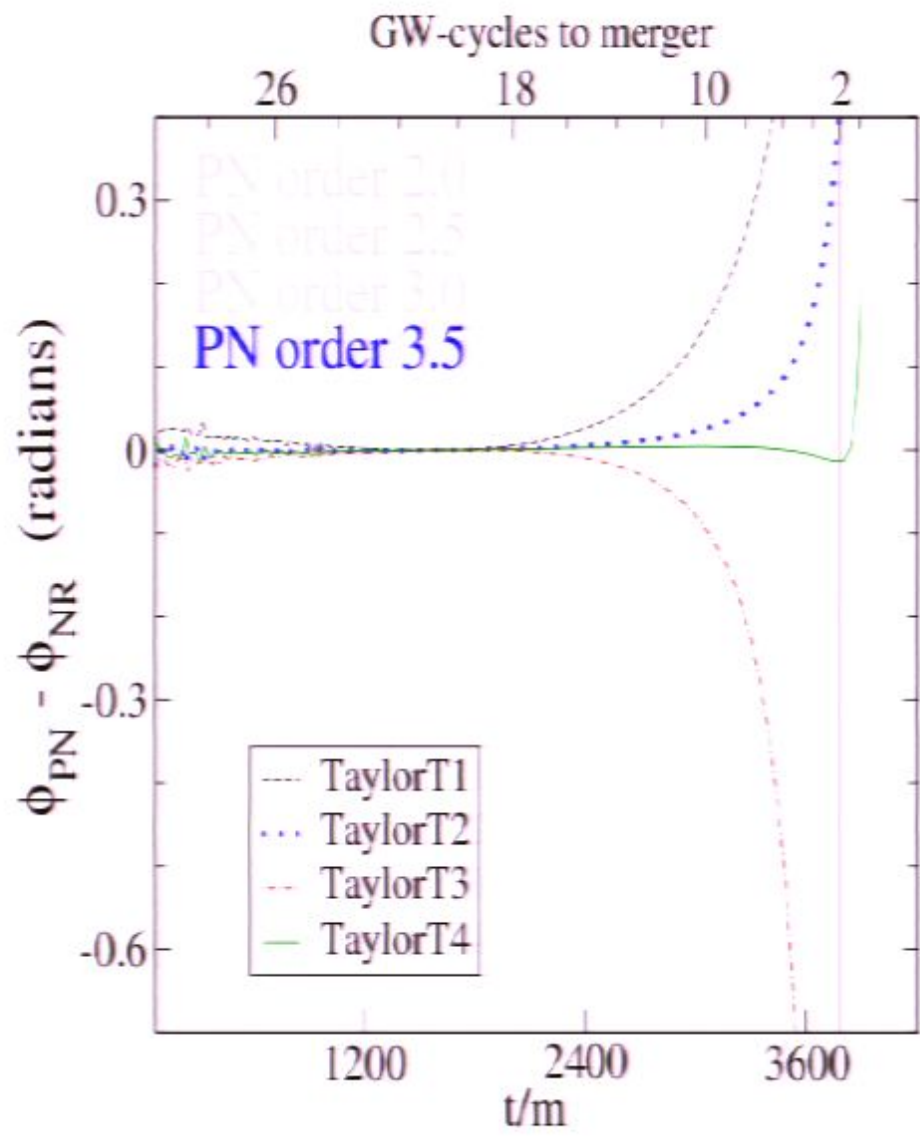
Numerical relativity vs. post-Newtonian

Boyle et al., 2007



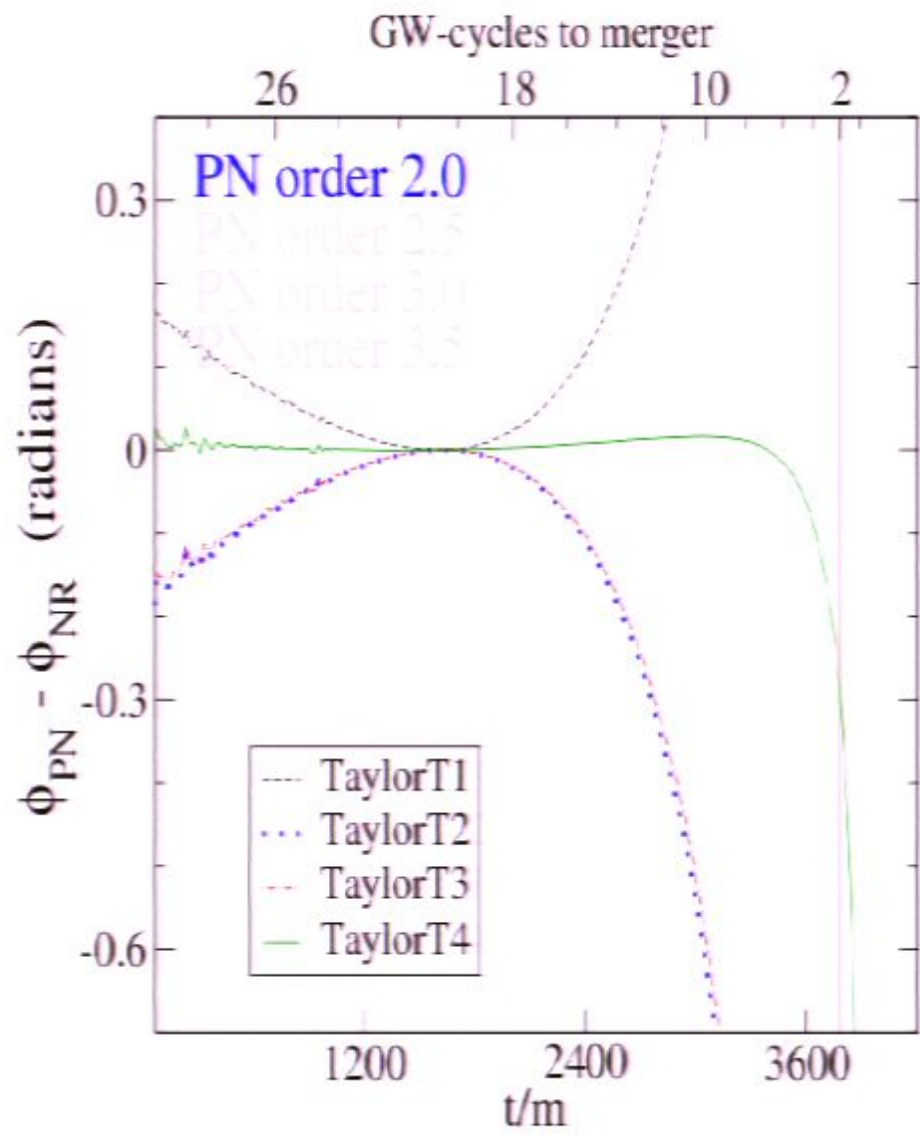
Numerical relativity vs. post-Newtonian

Boyle et al., 2007



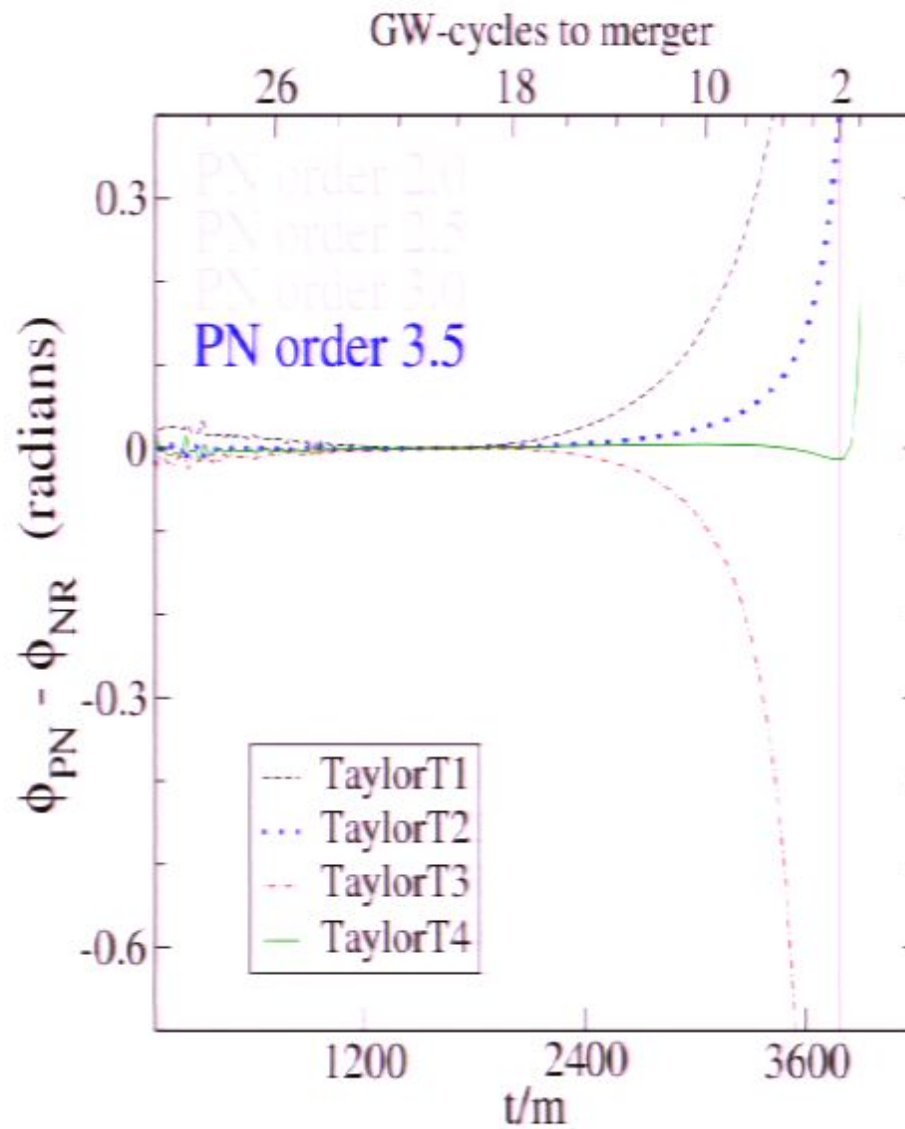
Numerical relativity vs. post-Newtonian

Boyle et al., 2007

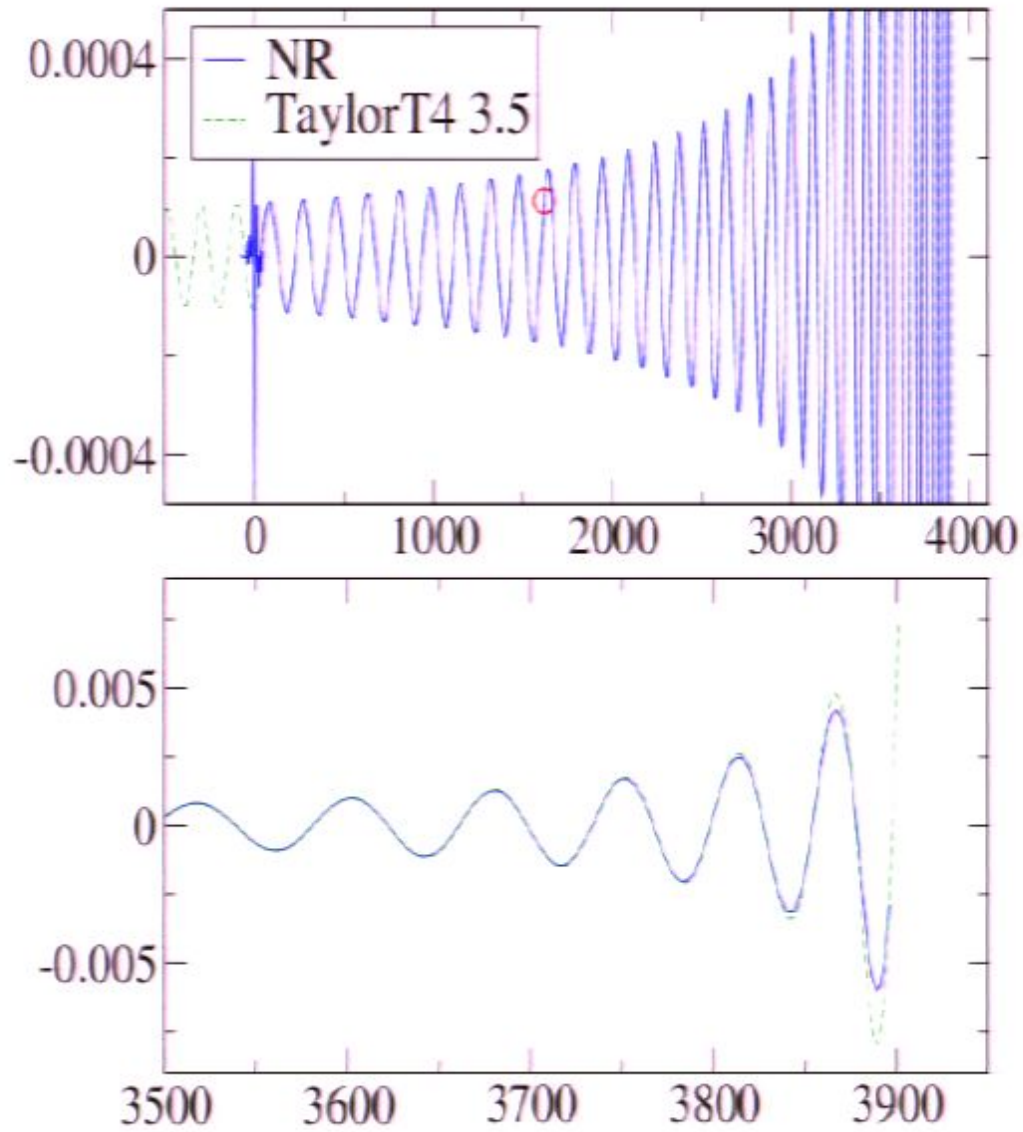


Numerical relativity vs. post-Newtonian

Boyle et al., 2007

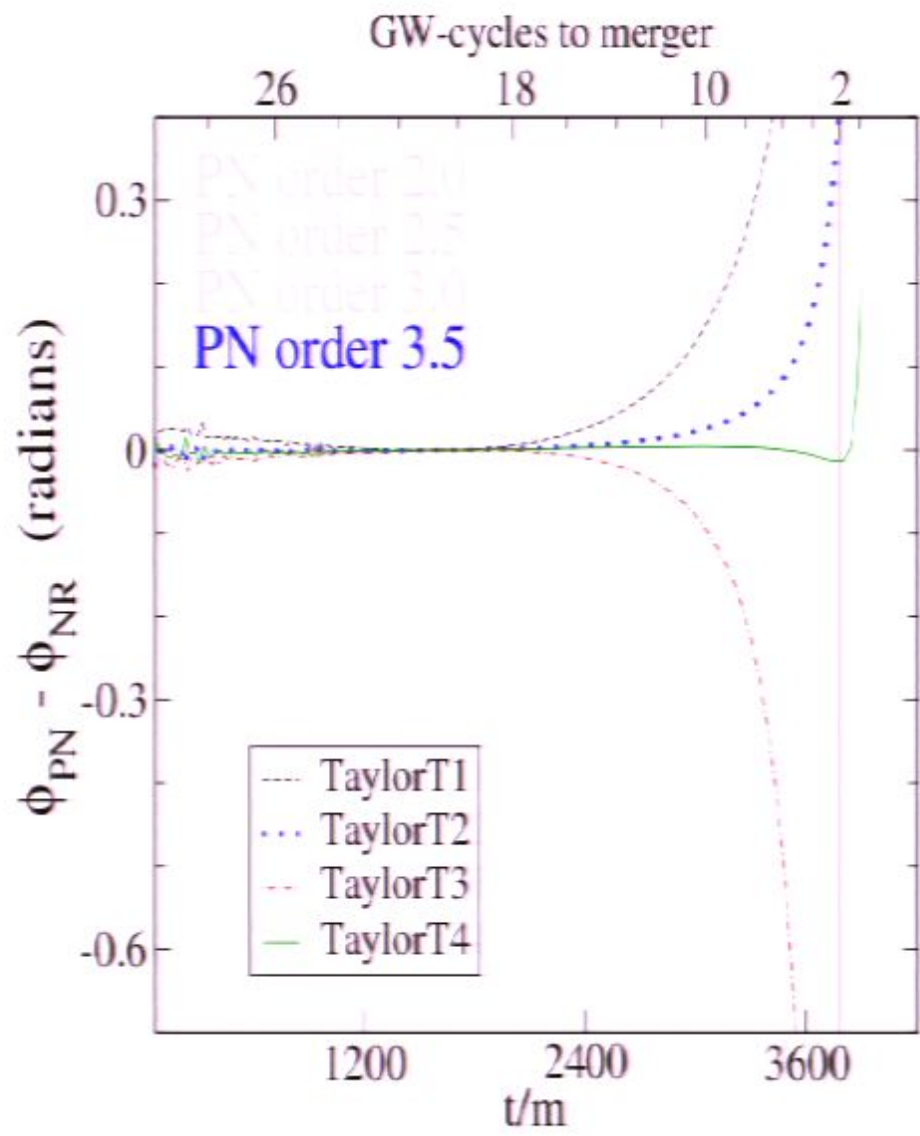


Comparing Waveforms

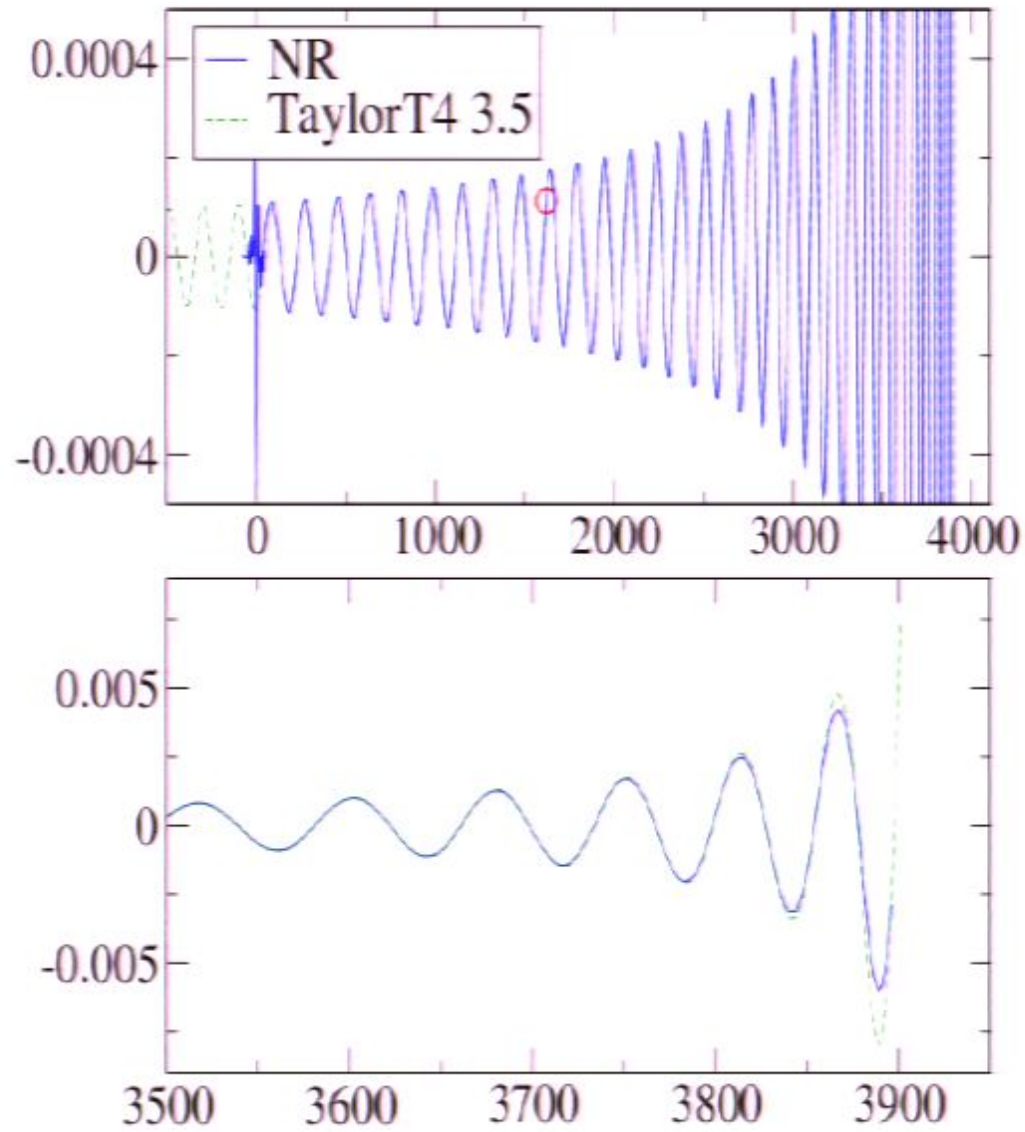


Numerical relativity vs. post-Newtonian

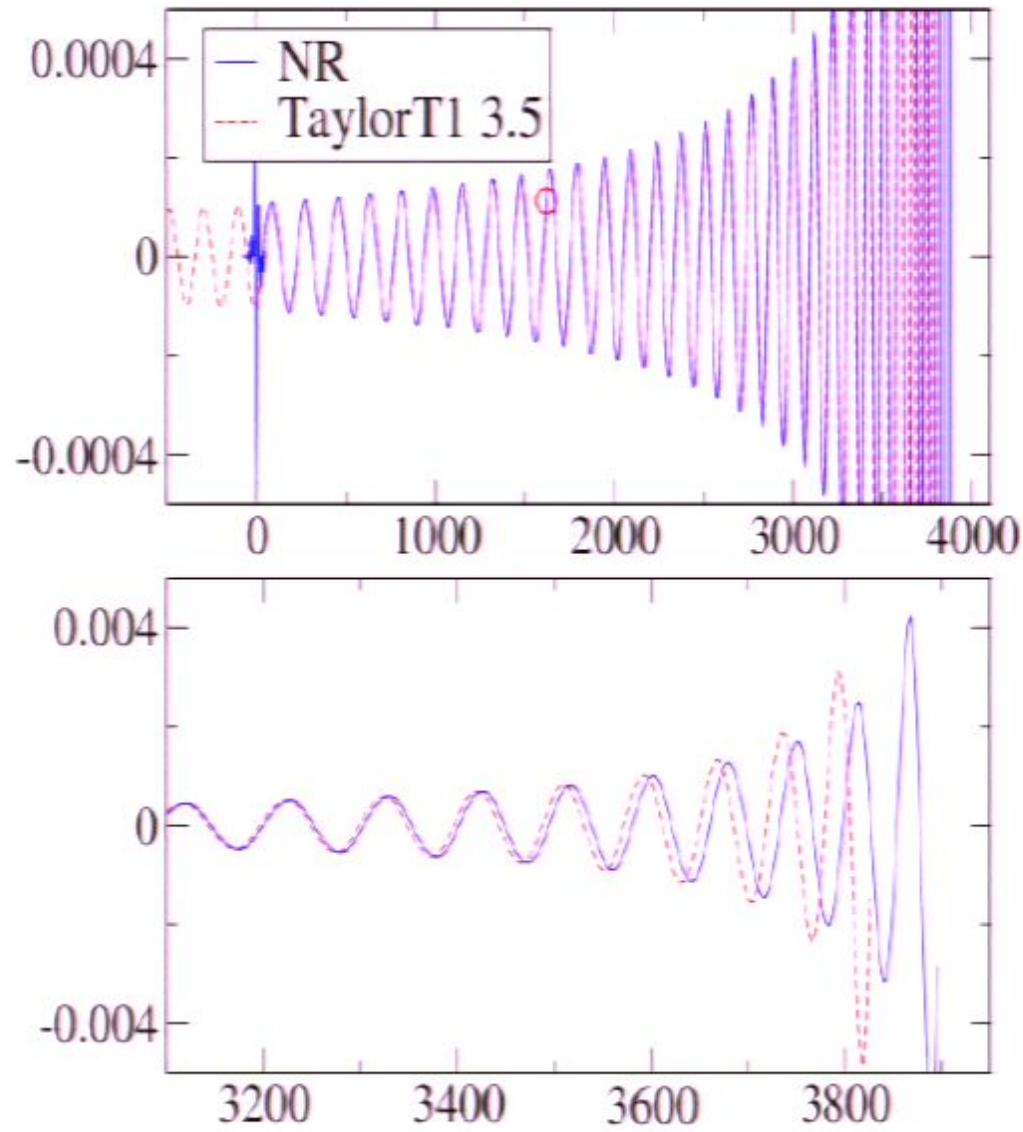
Boyle et al., 2007



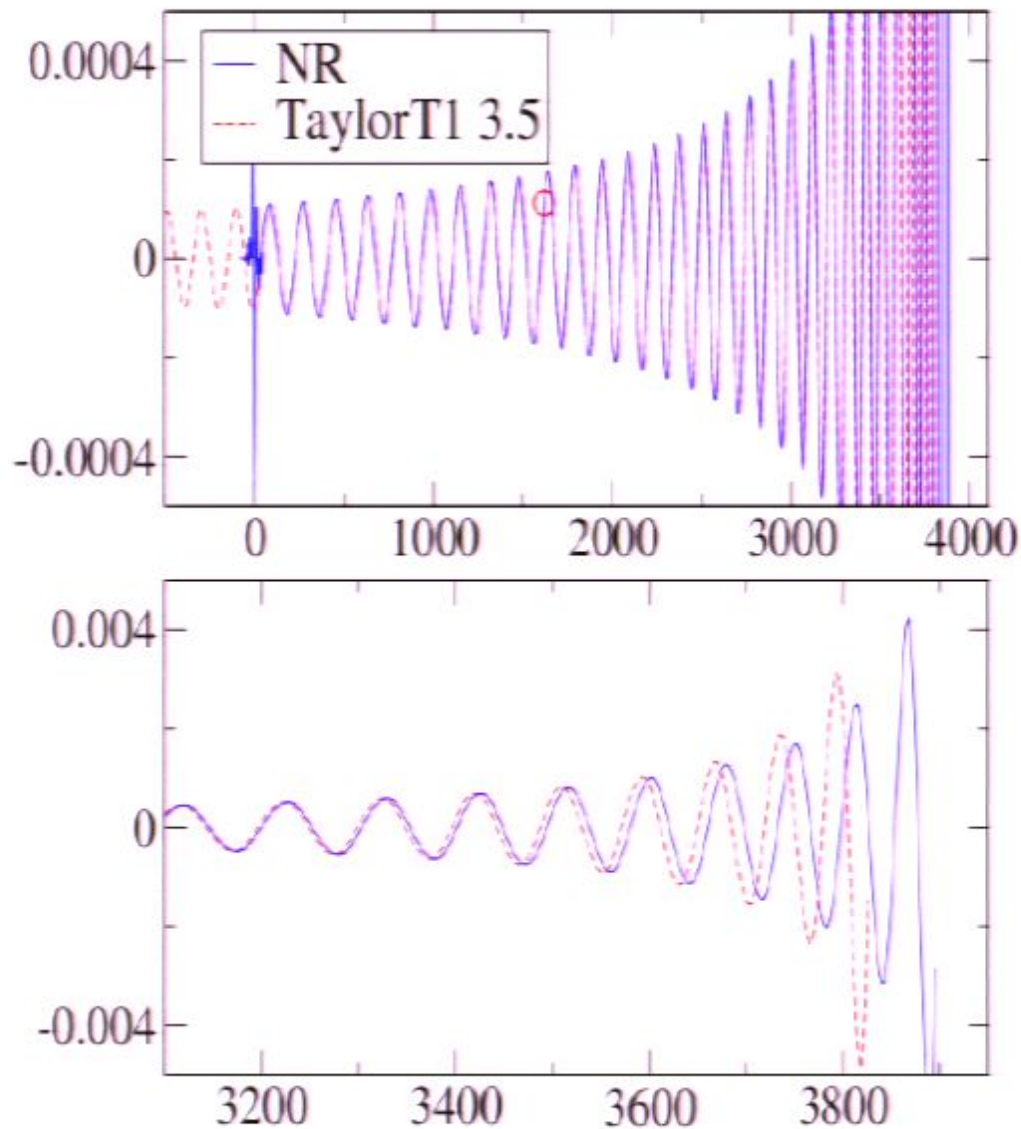
Comparing Waveforms



Comparing Waveforms

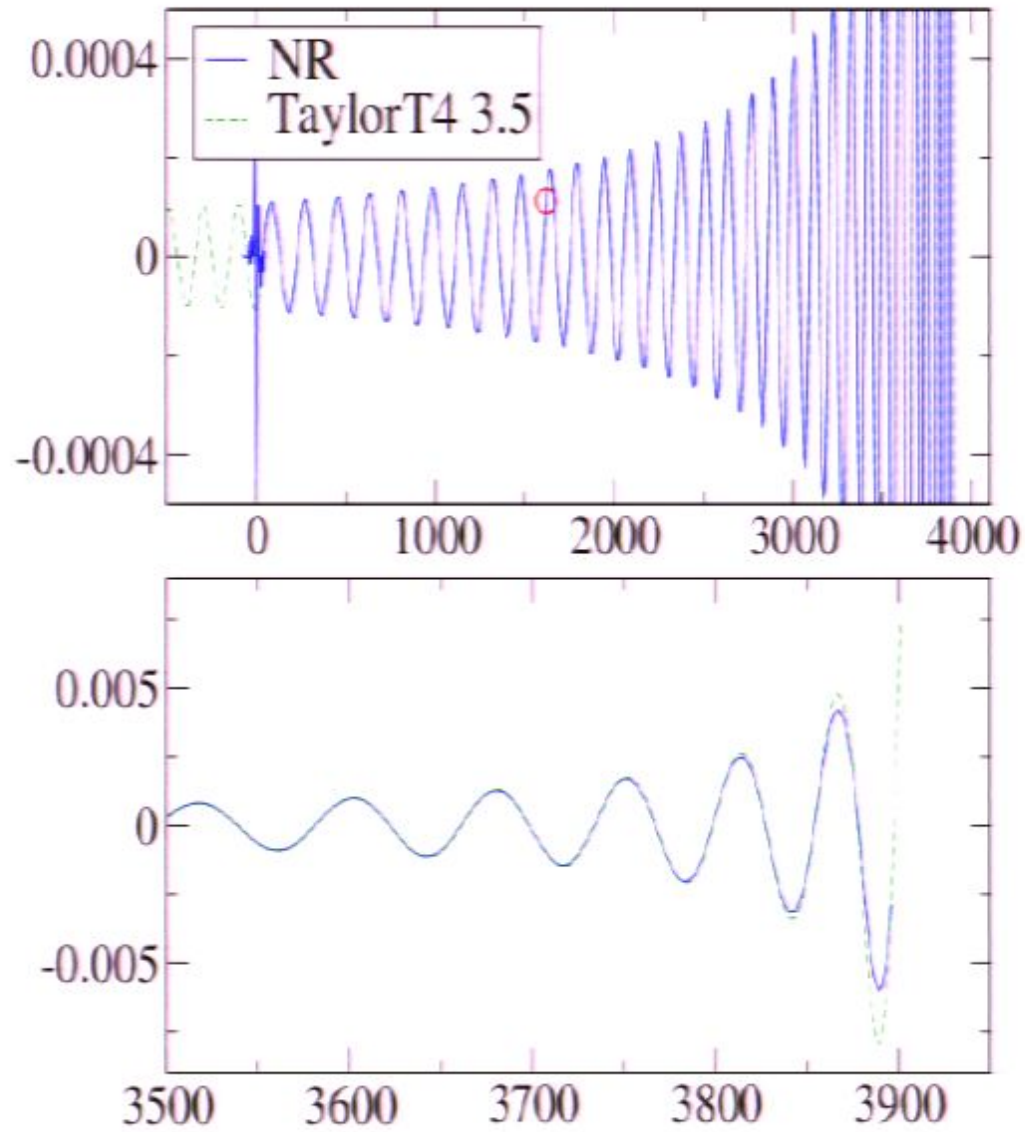


Comparing Waveforms

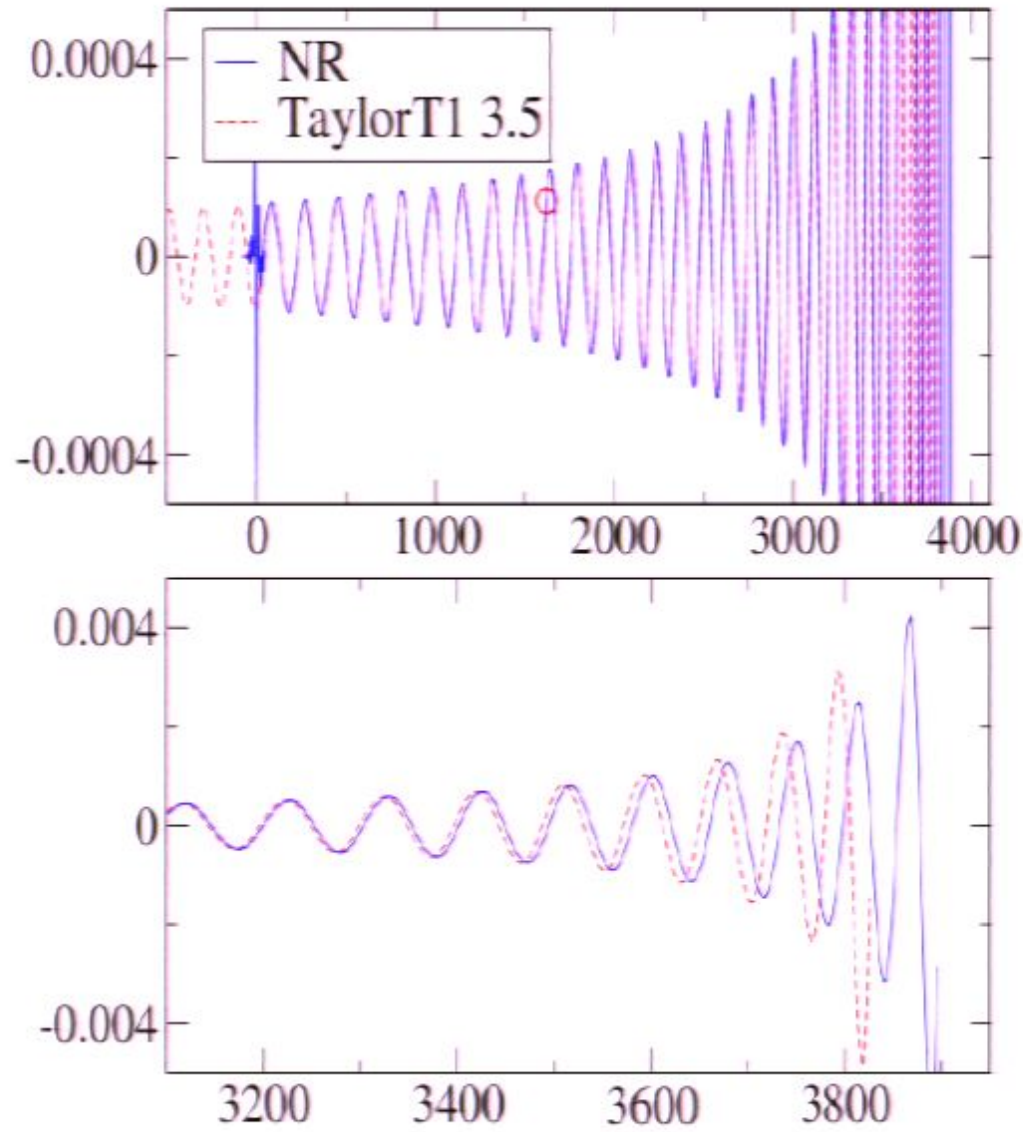


- *Some* PN-approximants match extremely well.
- No a priori knowledge; **NR must tell which one.**

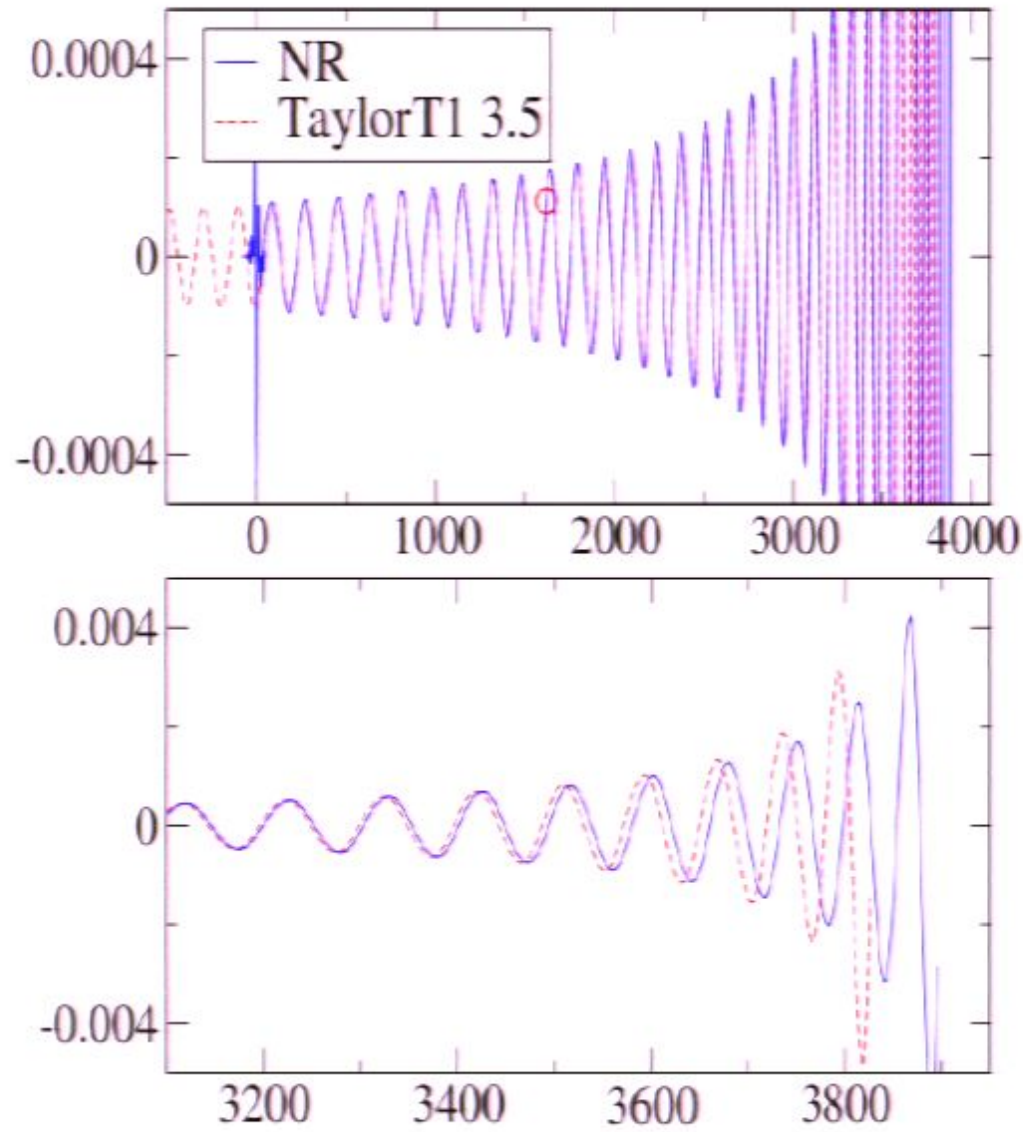
Comparing Waveforms



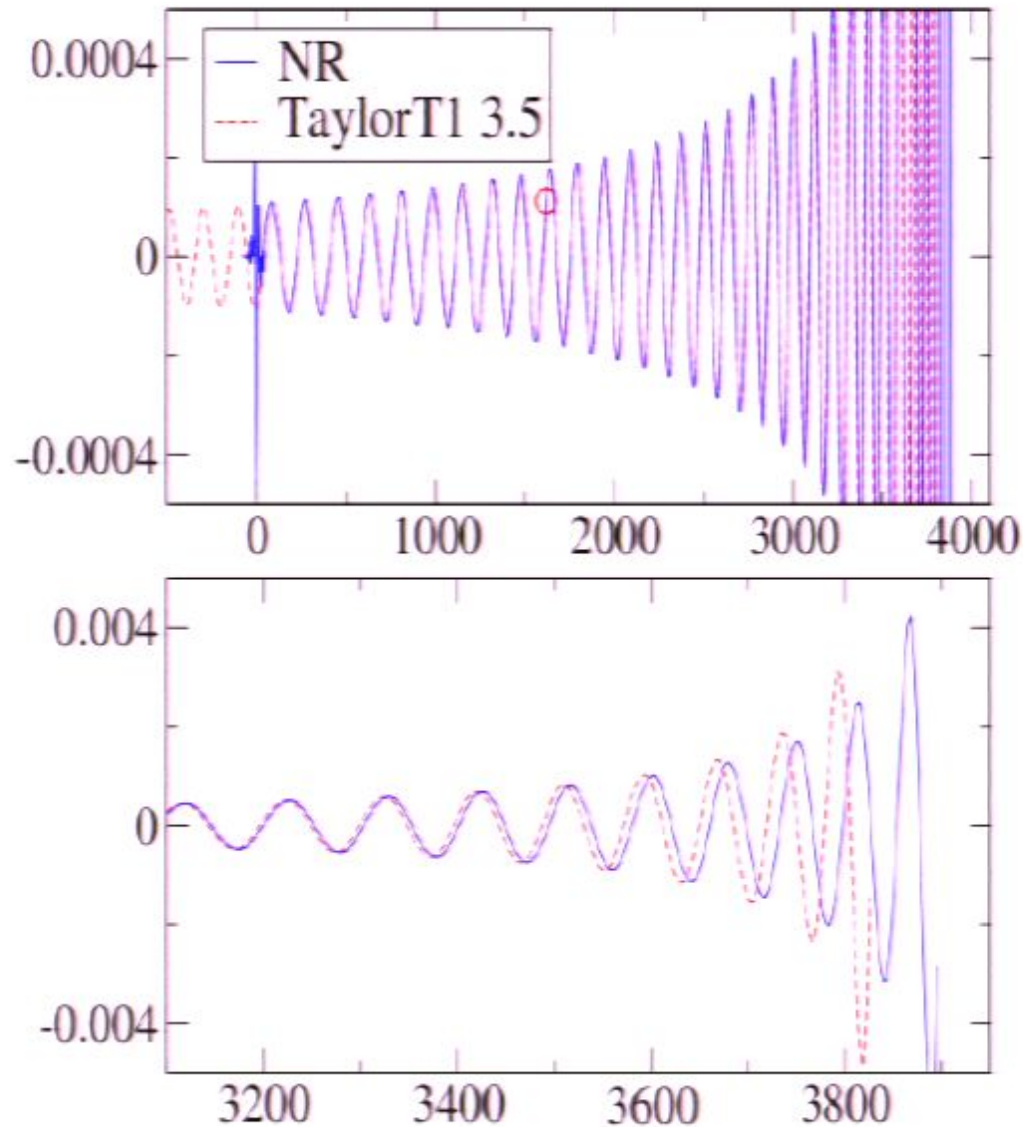
Comparing Waveforms



Comparing Waveforms



Comparing Waveforms

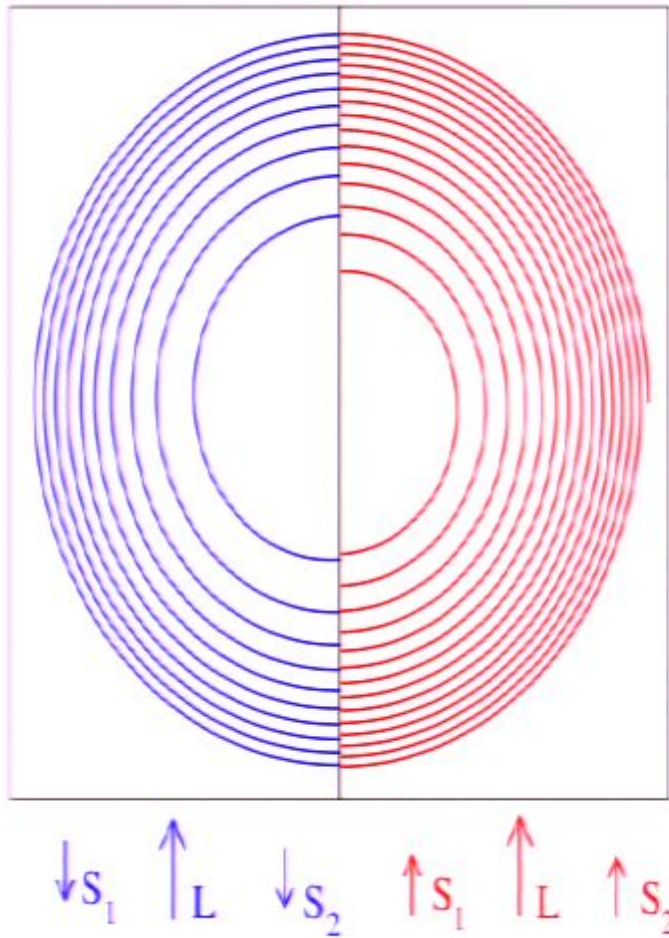


- *Some* PN-approximants match extremely well.
- No a priori knowledge; **NR must tell which one.**
- Equal mass, no spin most favorable for PN:
 - ▶ Spinning PN only known to lower order.
 - ▶ Non-equal mass binary has more cycles in strong-field regime.
- PN has great potential as basis for fitting formulae.

In the pipeline

Spinning BHs

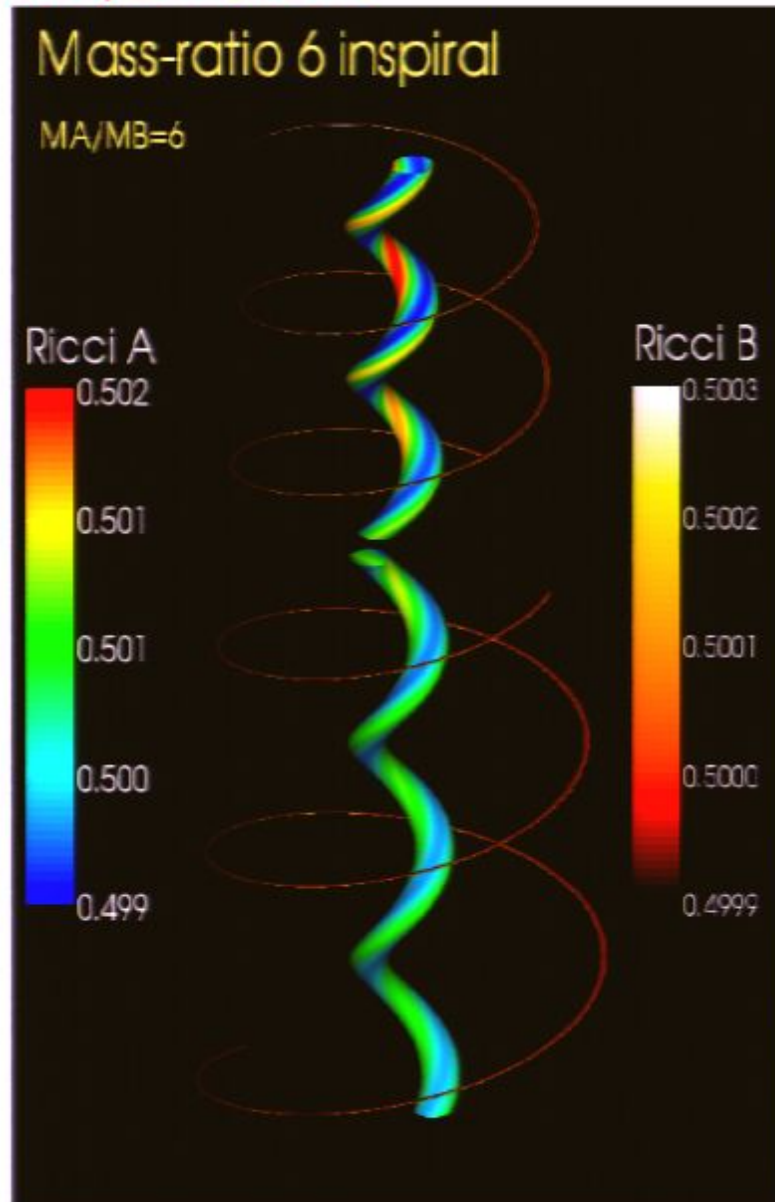
(Trajectory of BH A)



Unequal mass BHs

Mass-ratio 6 inspiral

$M_A/M_B=6$

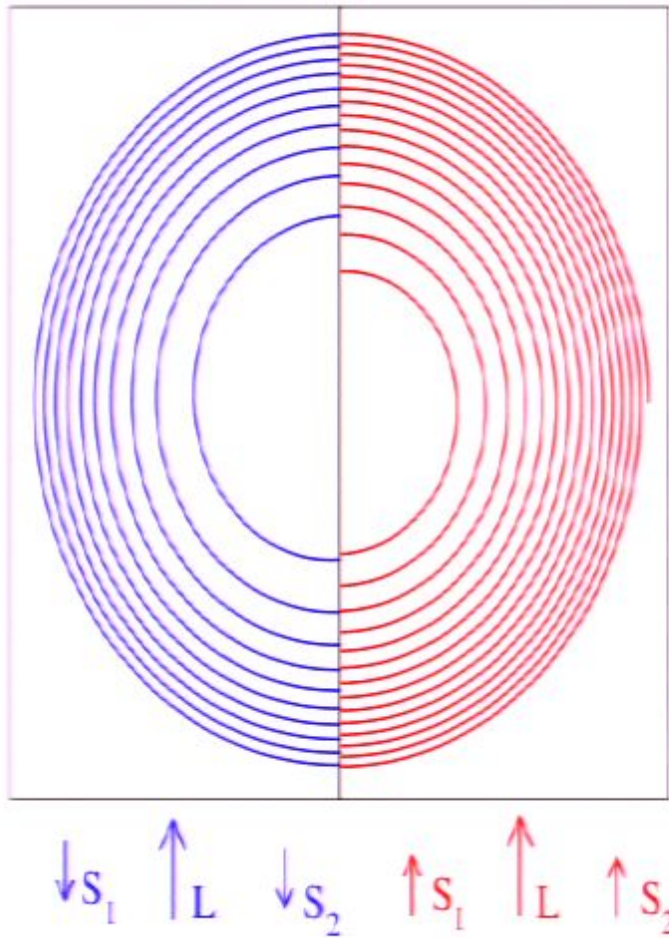


Properties of Einstein's Equations

In the pipeline

Spinning BHs

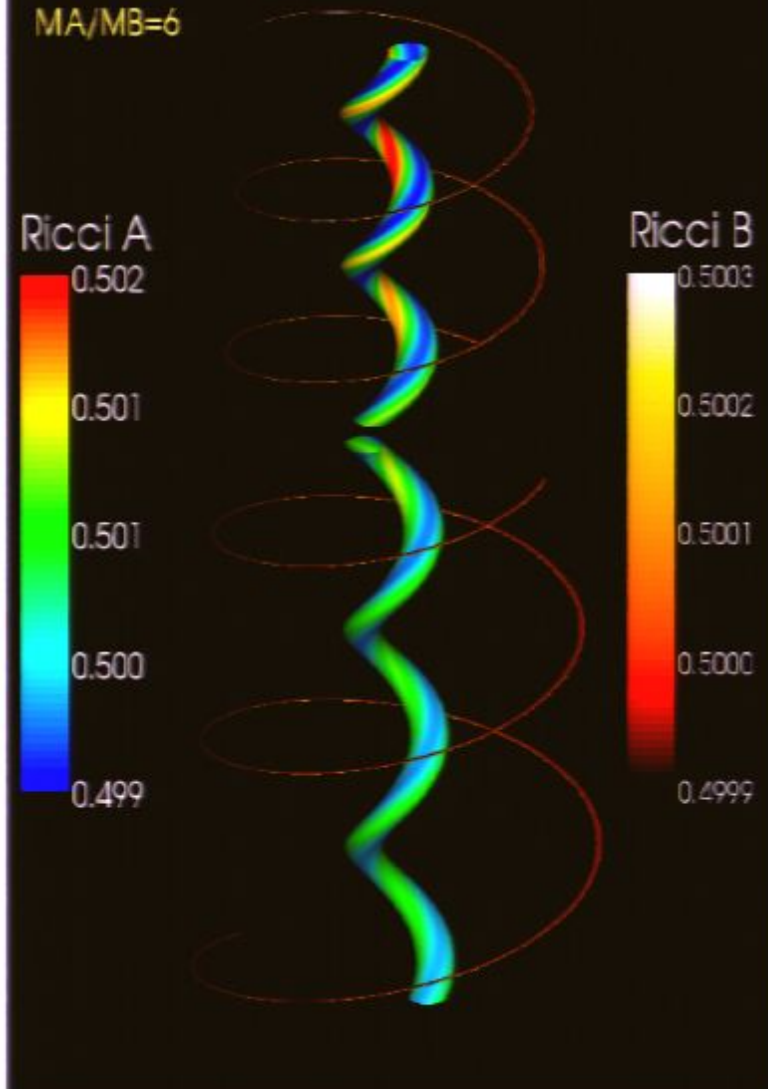
(Trajectory of BH A)



Unequal mass BHs

Mass-ratio 6 inspiral

$M_A/M_B=6$

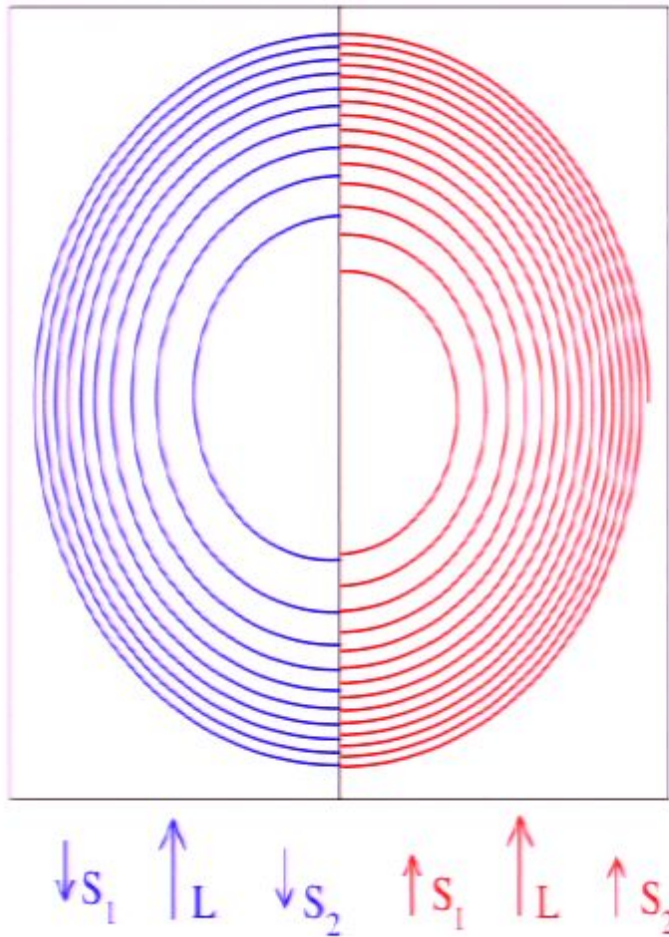


Properties of Einstein's Equations

In the pipeline

Spinning BHs

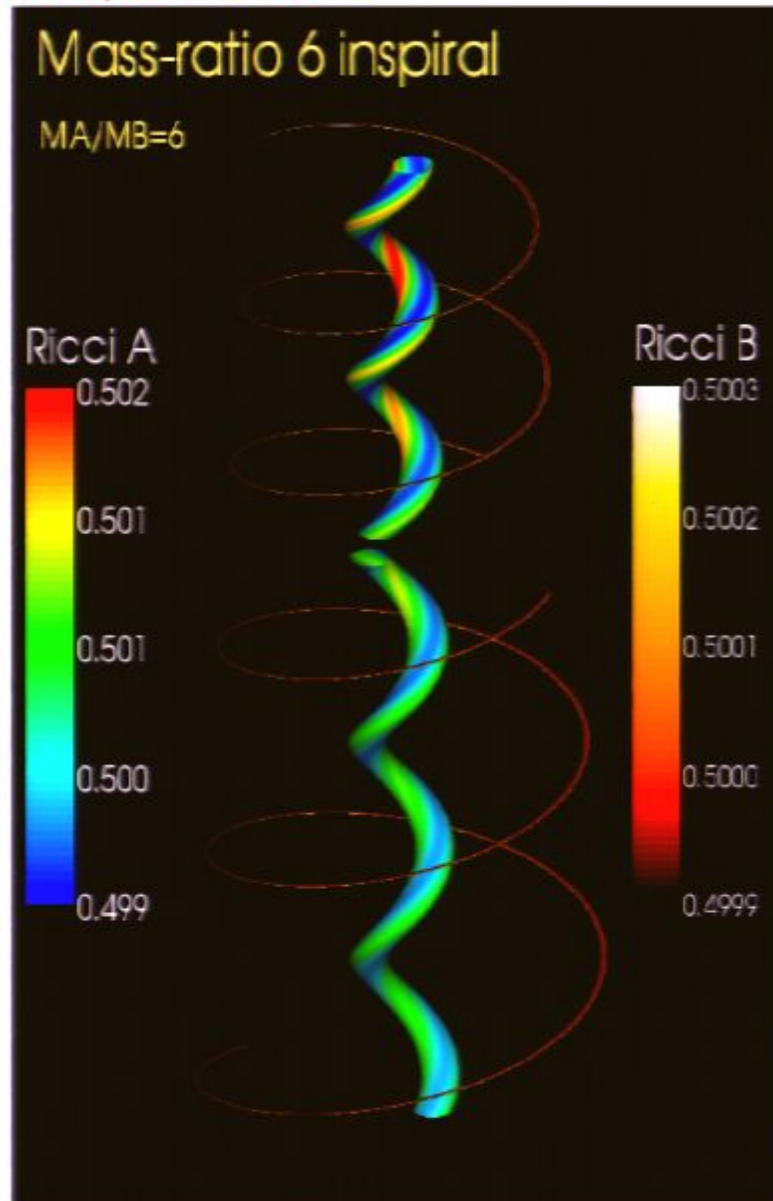
(Trajectory of BH A)



Unequal mass BHs

Mass-ratio 6 inspiral

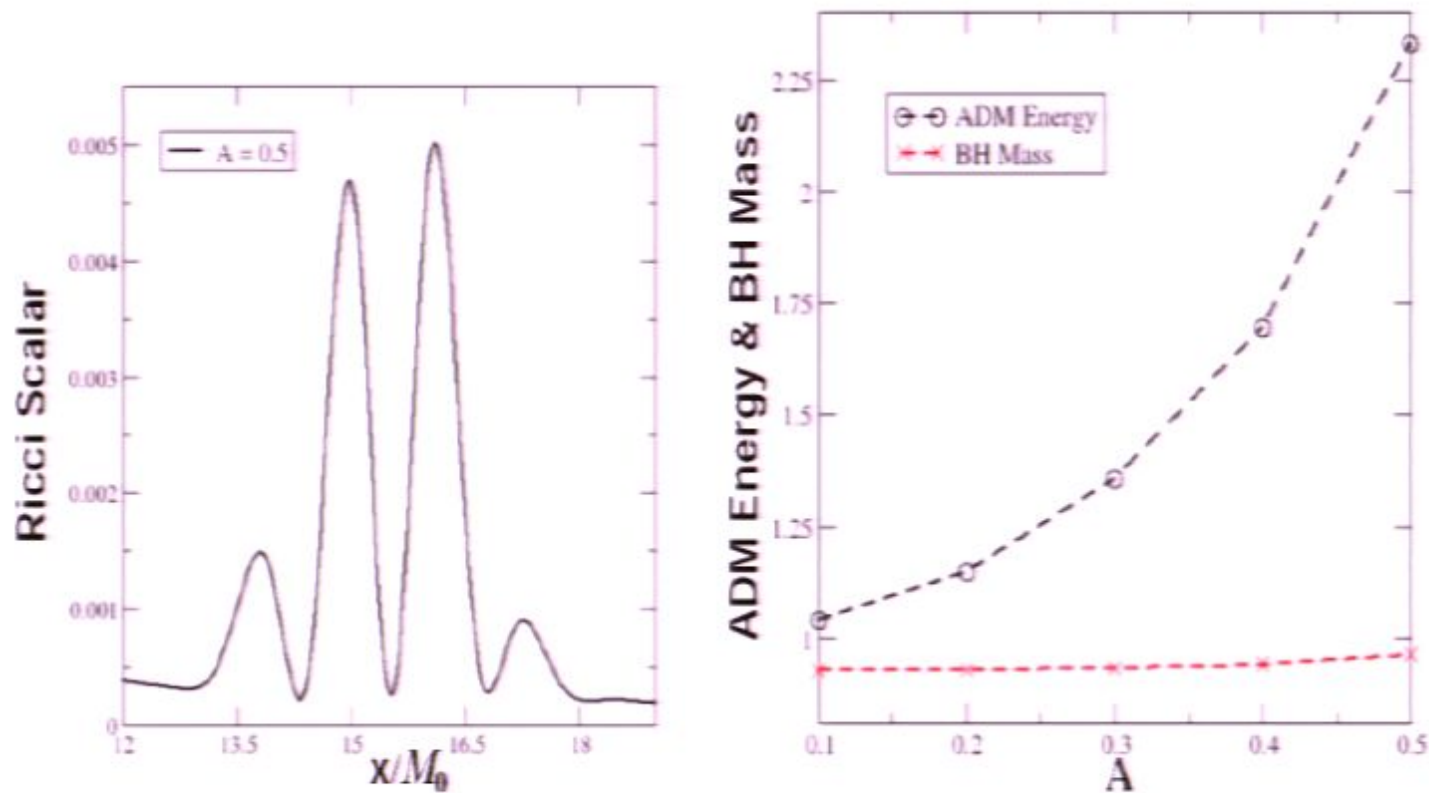
$M_A/M_B=6$



Properties of Einstein's Equations

Distorted Kerr black holes

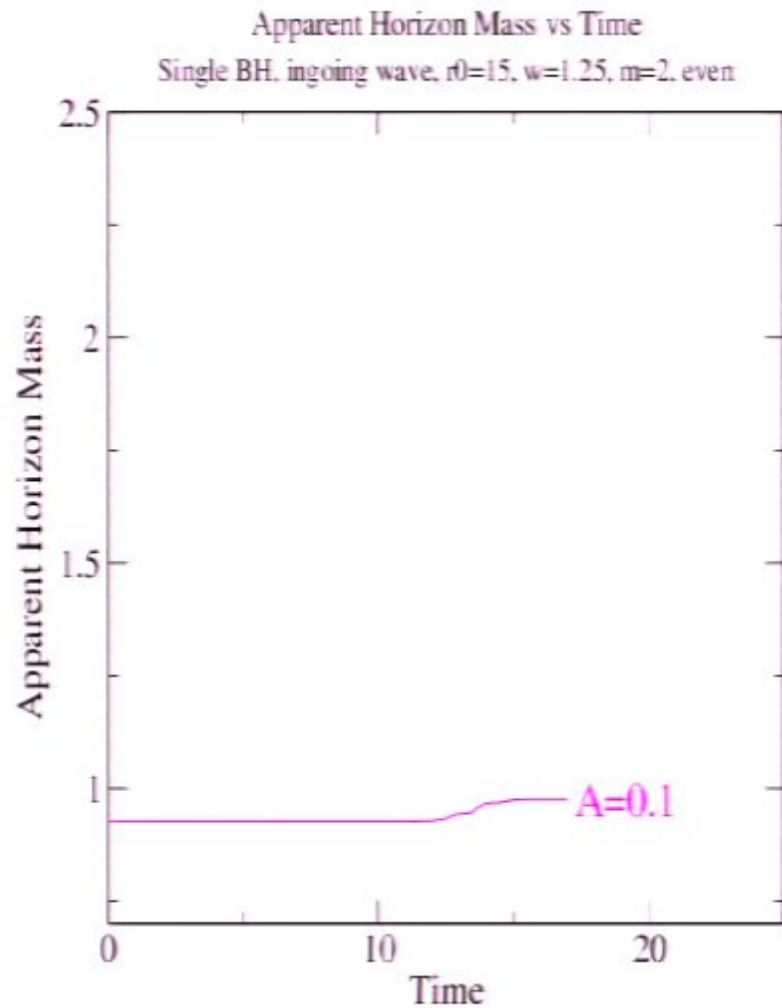
- With Tony Chu & Mike Cohen (Caltech grad students)
- Initial data: **Kerr black hole with incoming spherical gravitational wave** ($r=15M$, width 1.25, even parity, $m=2$).



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

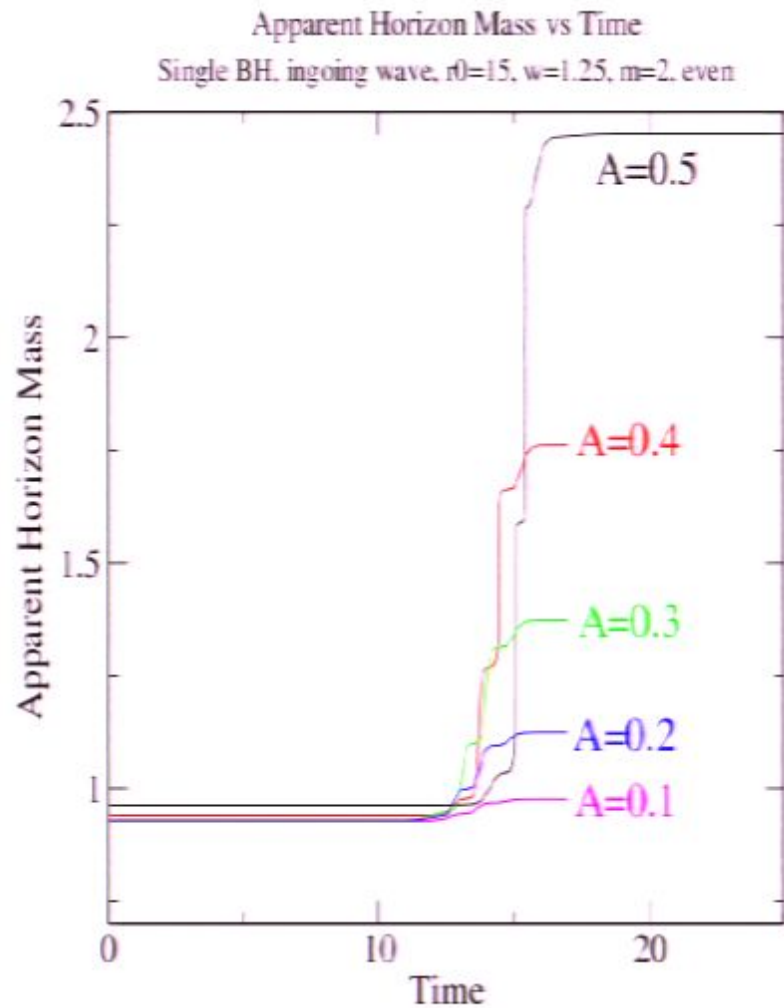
$$M \equiv \sqrt{A/16\pi}$$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

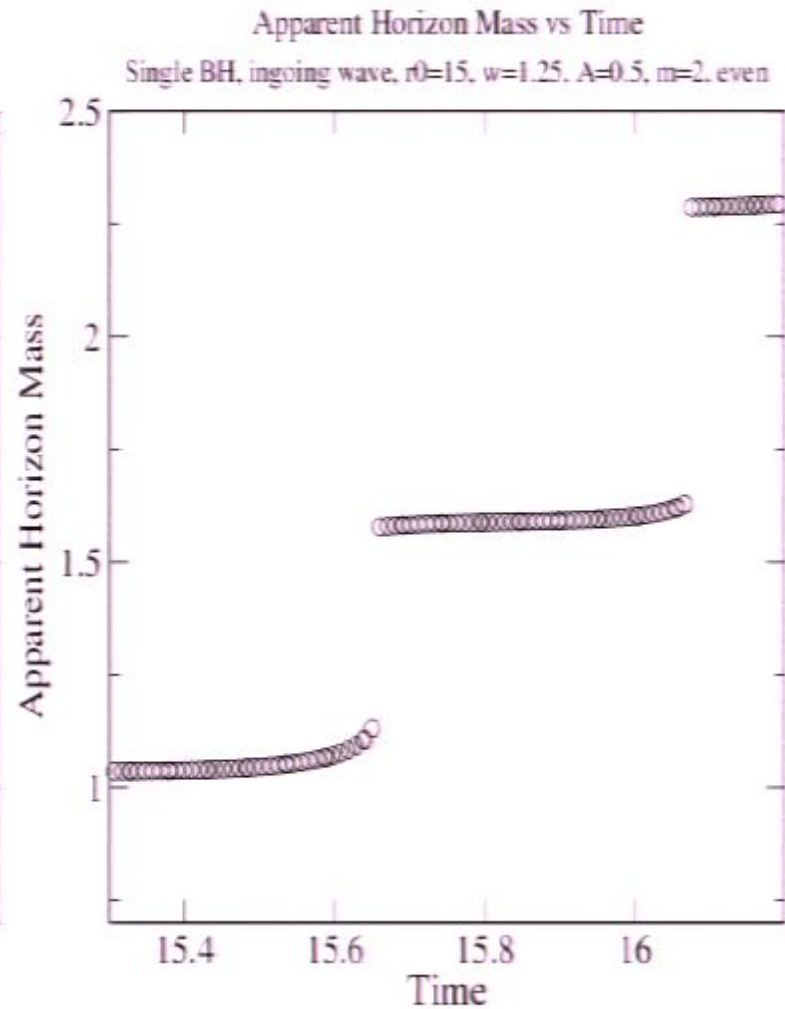
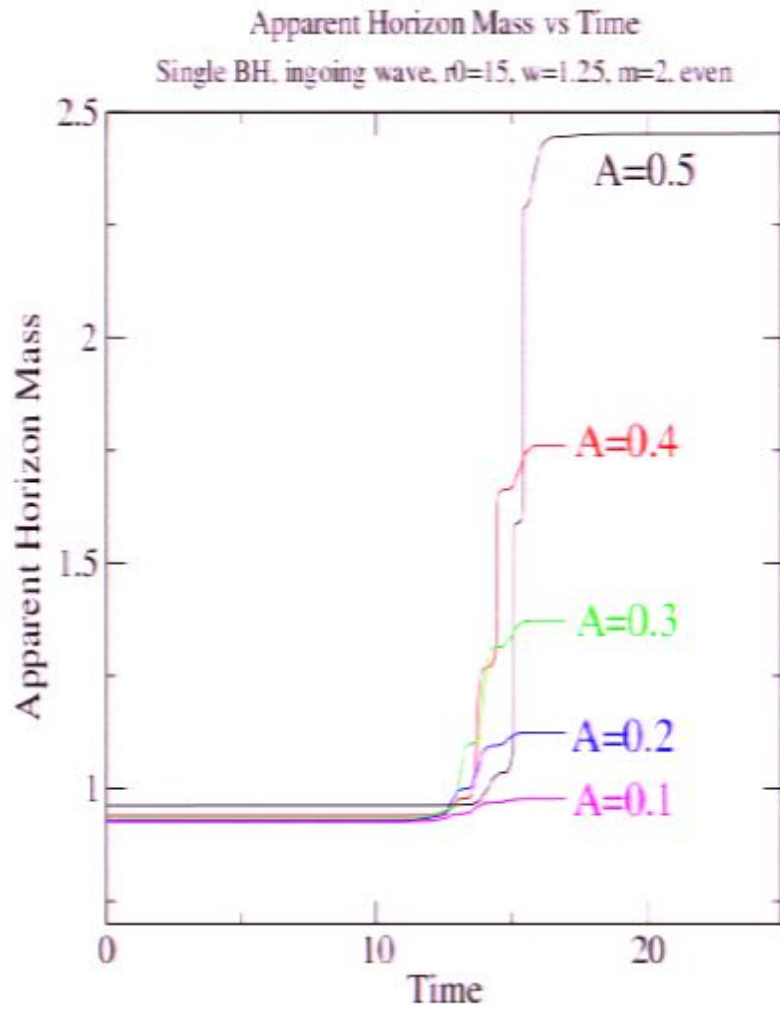
$$M \equiv \sqrt{A/16\pi}$$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

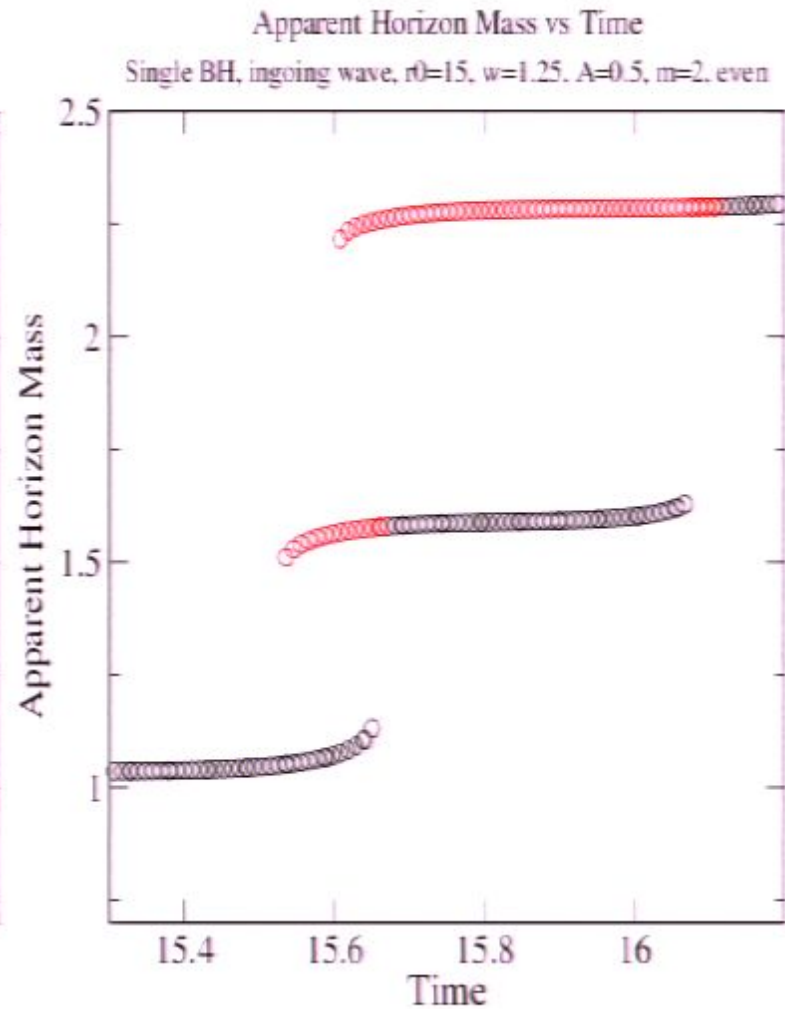
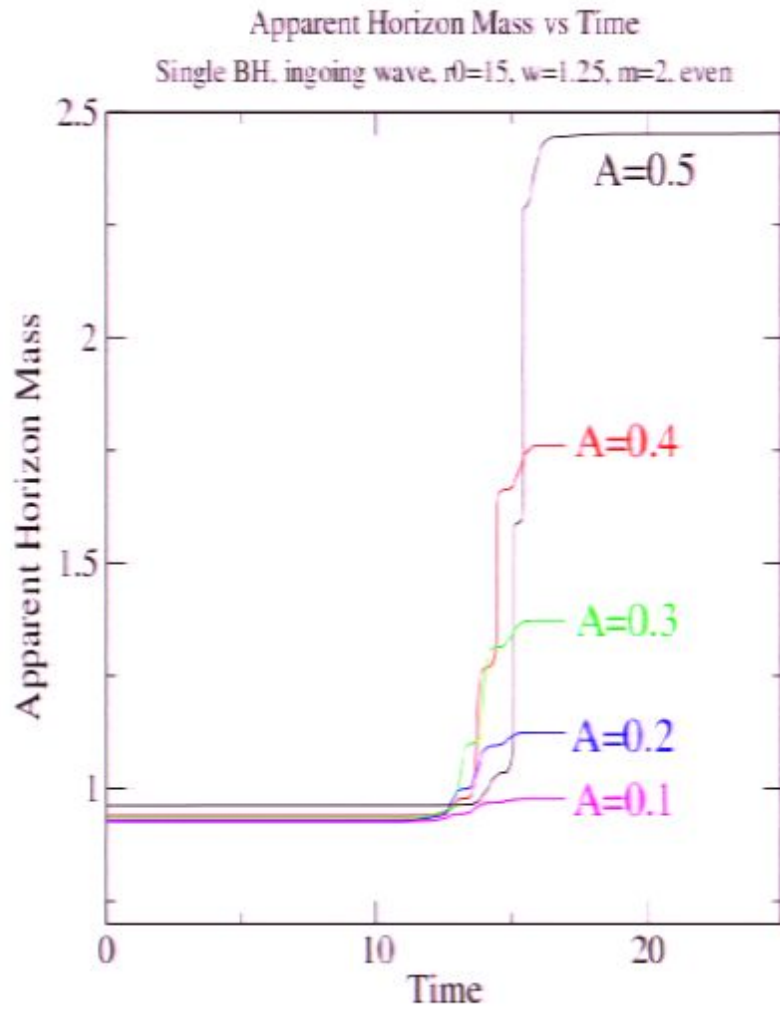
$$M \equiv \sqrt{A/16\pi}$$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

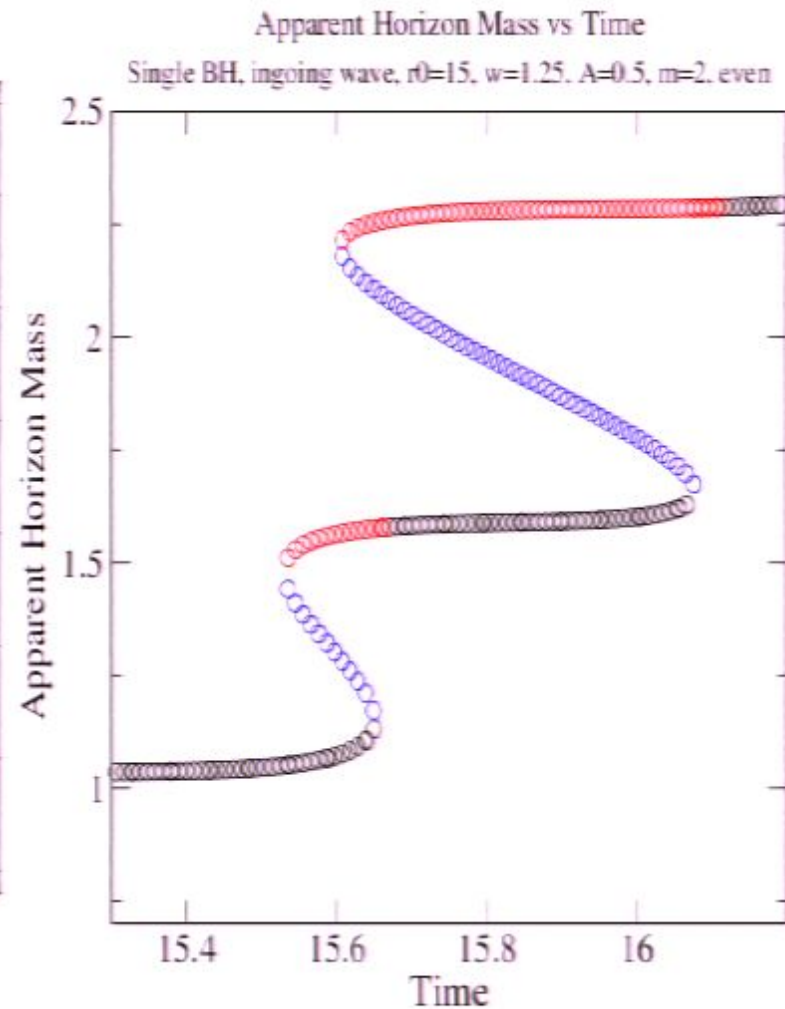
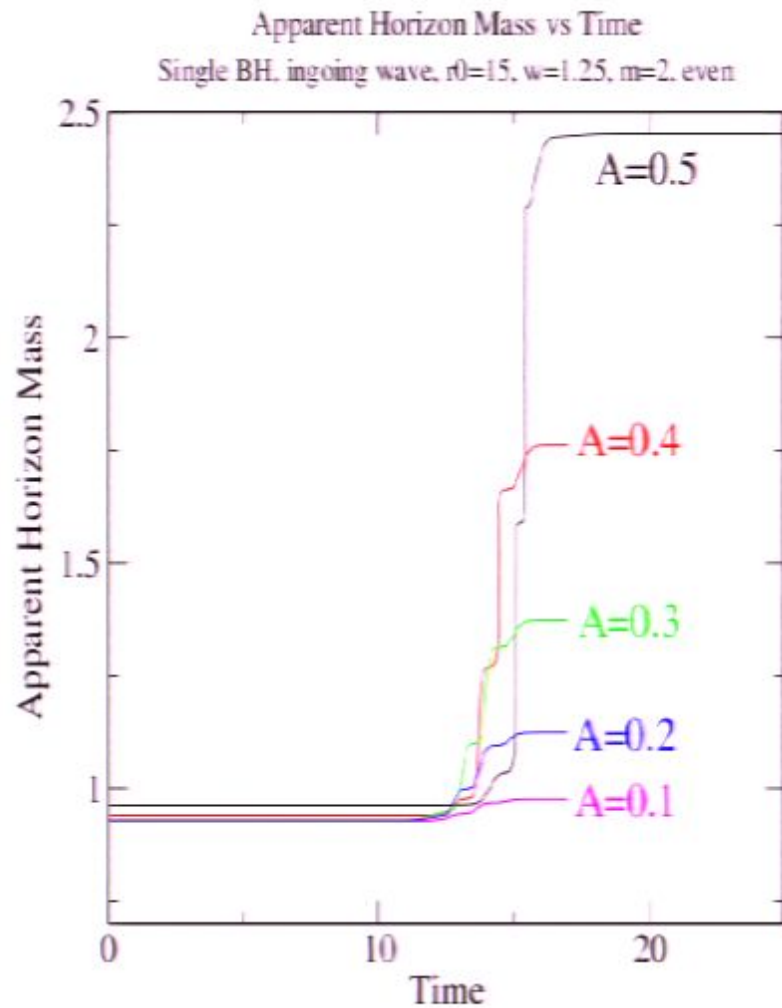
$$M \equiv \sqrt{A/16\pi}$$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

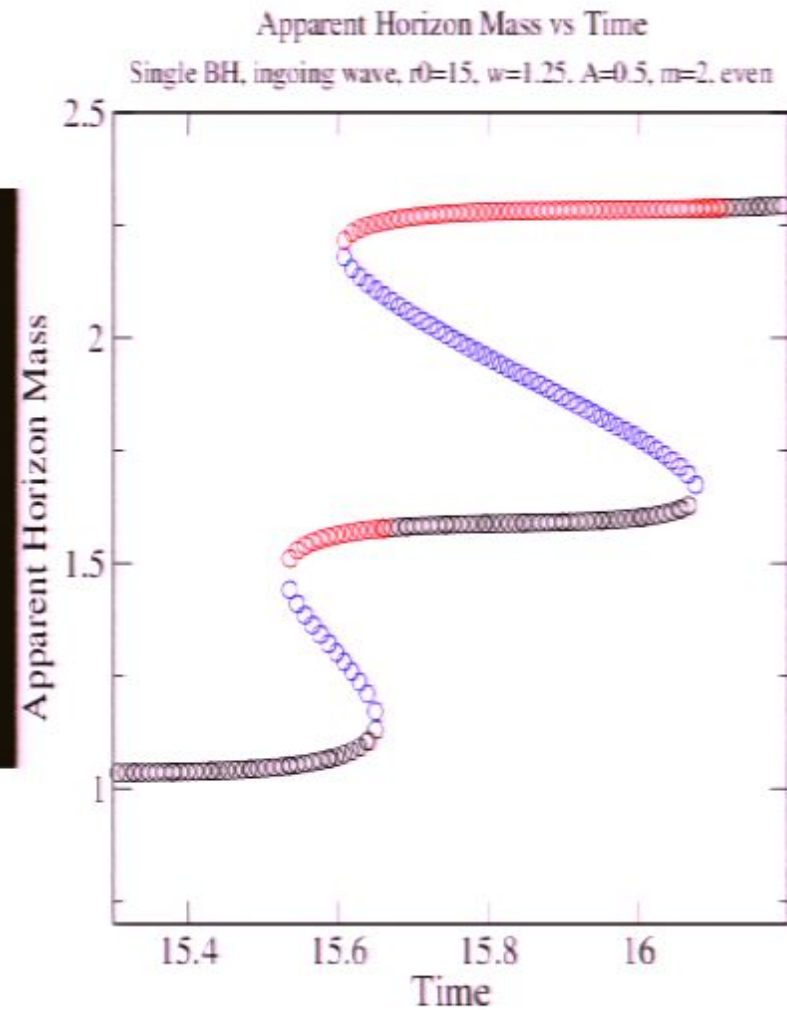
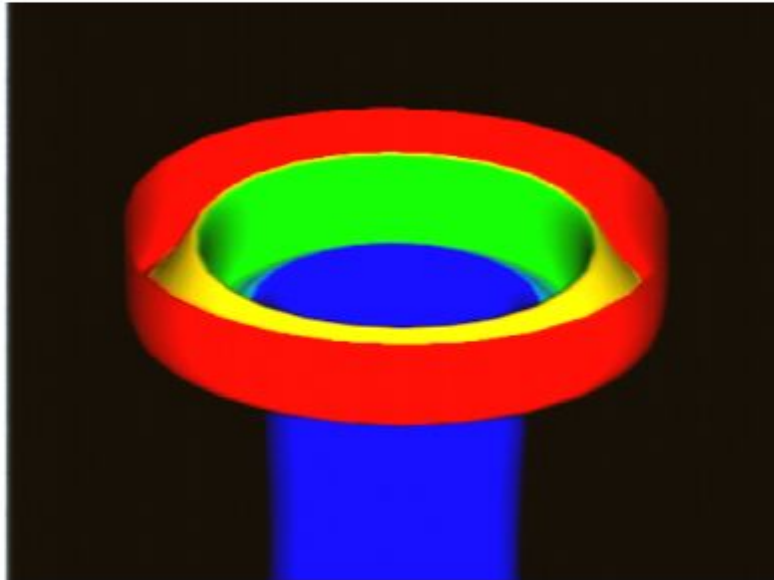
$$M \equiv \sqrt{A/16\pi}$$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

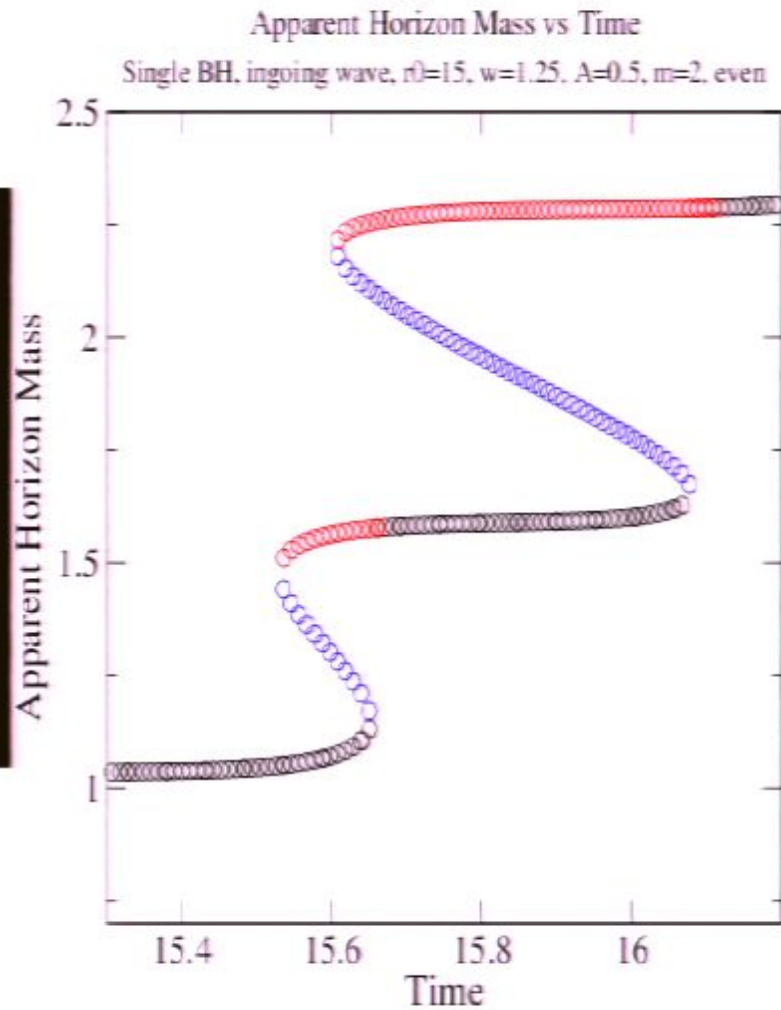
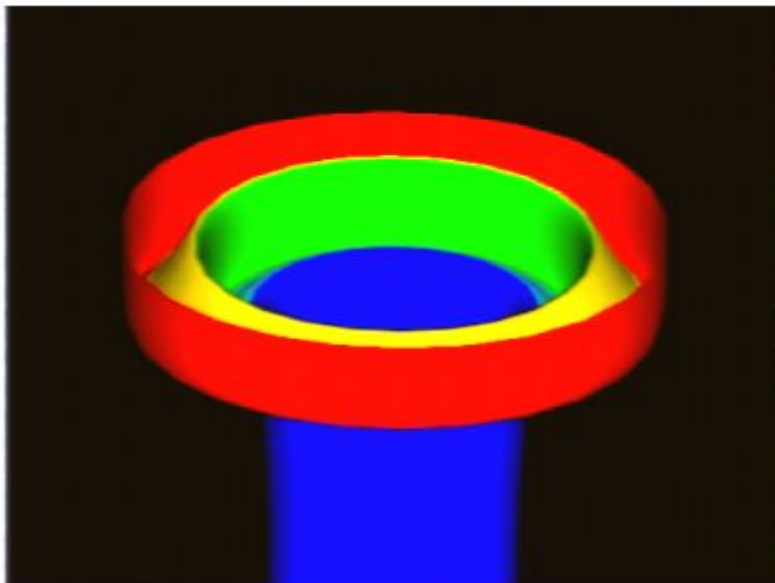
$$M \equiv \sqrt{A/16\pi}$$

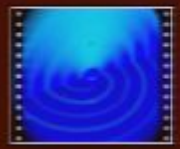


Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M \equiv \sqrt{A/16\pi}$$





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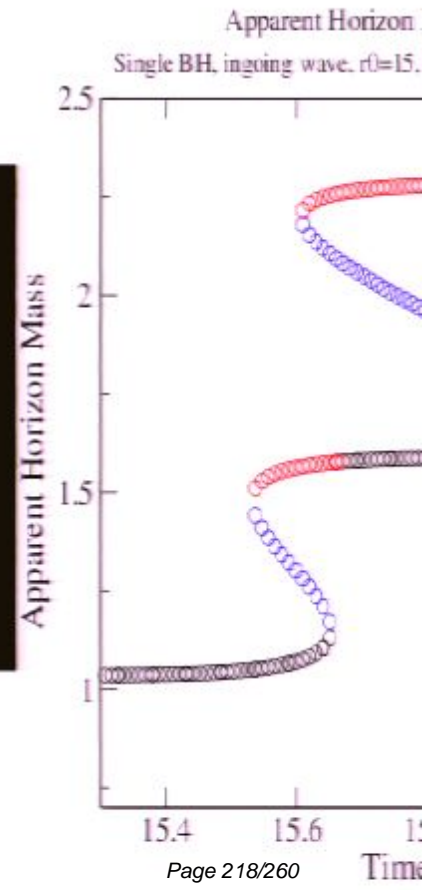
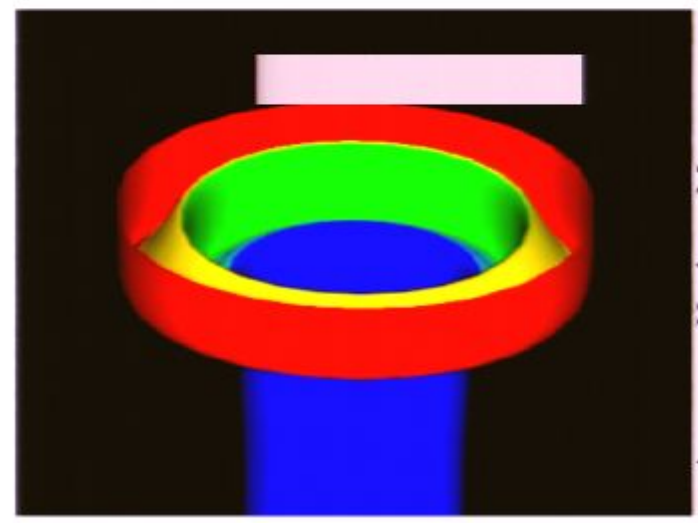
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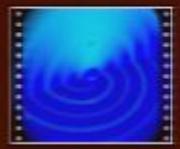
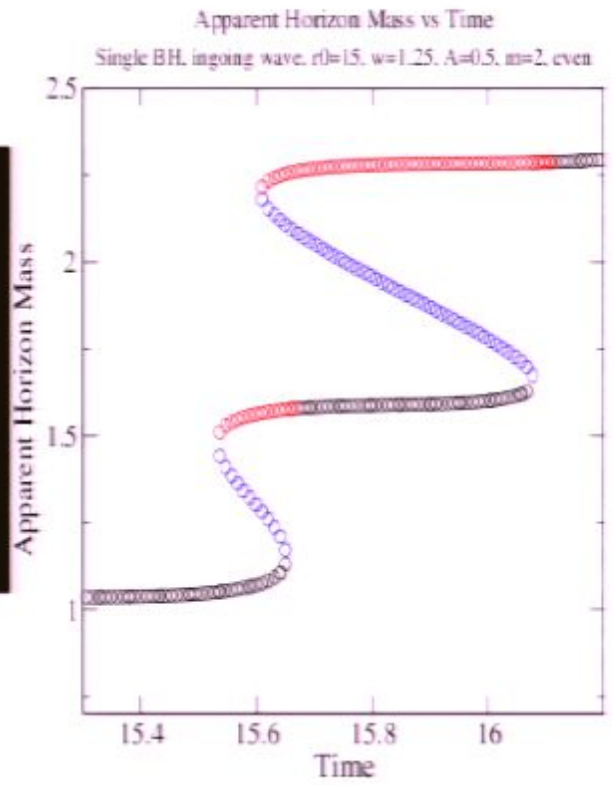
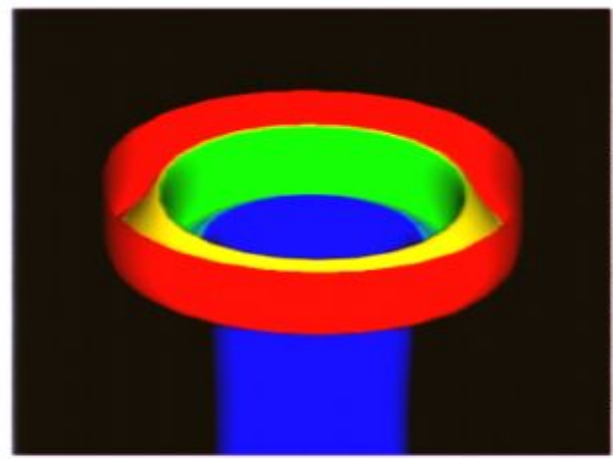
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-g
- $M \equiv \sqrt{A/16\pi}$



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics
- $M \equiv \sqrt{A/16\pi}$



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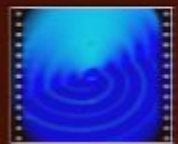
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harald@Neville:AA~/research/Talks/08Ma





0093g_800x600.mpeg



ORSHOLE7.jpg



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Numerical study of black hole spacetimes

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Previous Next 51 of 67 Best Fit

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M \equiv \sqrt{A/16\pi}$$

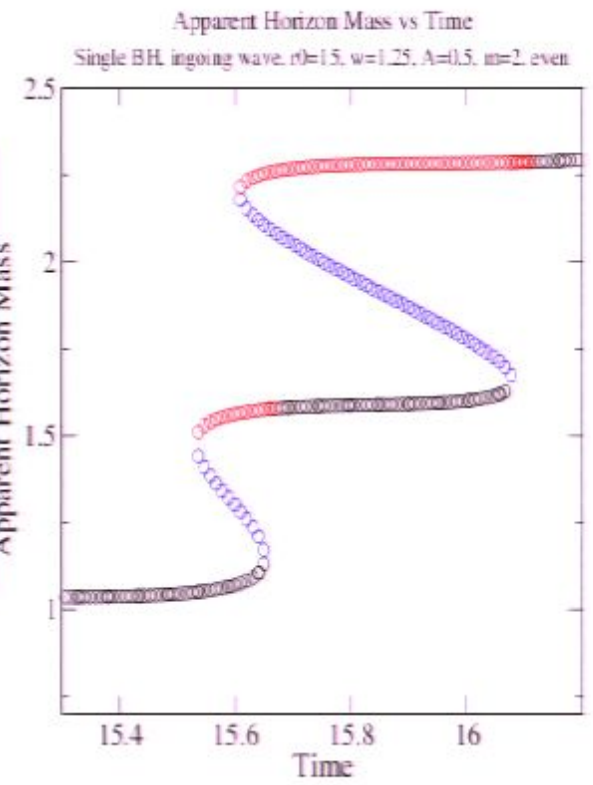
harald@Neville:AA~/research/Talks/D8May_Perimeter/Movies

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harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$ ls *mpg
Embedding_0093r_Large.mpg      ProlateCriticalCollapse.mpg
Embedding_0093r_Medium.mpg    RePsi4_0093r.mpg
Embedding_0093r_Small.mpg     Shibata_APR-135-165.mpg
Embedding_0093r.mpg           Shibata_APR-140-140.mpg
EmbeddingFast_0093r.mpg       TrKBlowup.mpg
EmbeddingSmall_0093r.mpg      viz_shiftingall_21.mpg
EmbeddingSmallFast_0093r.mpg  viz_shiftingall_ImPsi4_20.mpg
foris_qc_0093r_psi4r_z_log.mpg
harald@Neville Movies]$

```



Numerical study of black hole spacetimes

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← 51 of 67 → Best Fit

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

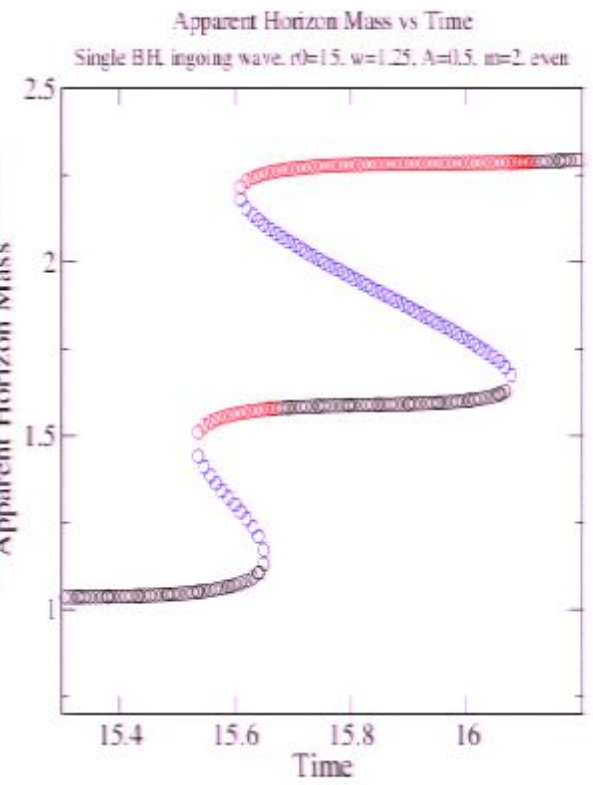
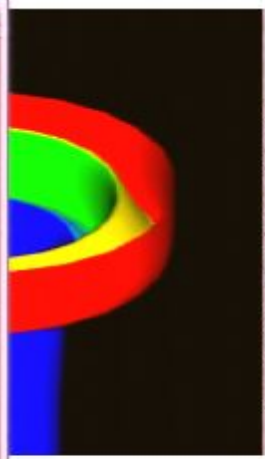
$$M \equiv \sqrt{A/16\pi}$$

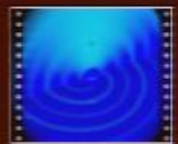

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harald@Neville:AA~/research/Talks/08May_Perimeter
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harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$ ls *mpg
| Embedding_0093r_Large.mpg          ProlateCriticalCollapse.mpg
| Embedding_0093r_Medium.mpg        RePsi4_0093r.mpg
| Embedding_0093r_Small.mpg         Shibata_APR-135-165.mpg
| edding_0093r.mpg                  Shibata_APR-140-140.mpg
| eddingFast_0093r.mpg              TrKBlowup.mpg
| eddingSmall_0093r.mpg             viz_shiftingall_21.mpg
| eddingSmallFast_0093r.mpg         viz_shiftingall_ImPsi4_20.mpg
| torius_qe_19_lm1_psi4r_z_log.mpg
harald@Neville Movies]$ cd ..
harald@Neville 08May_Perimeter]$ ls *mpg

```





0093g_800x600.mpeg



ORSHOLE7.jpg



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Numerical study of black hole spacetimes

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← 51 of 67 → Best Fit

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

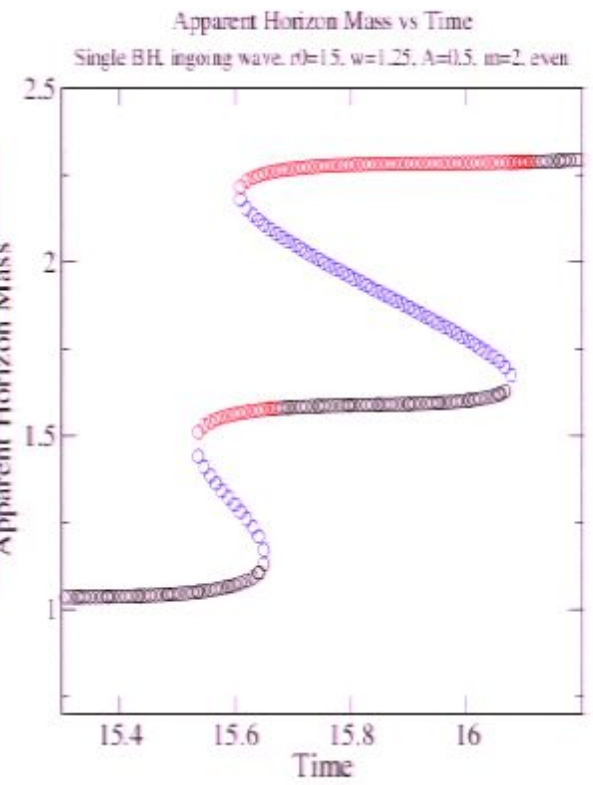
$$M \equiv \sqrt{A/16\pi}$$

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harald@Neville:AA~/research/Talks/08May_Perimeter
Edit View Terminal Tabs Help
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harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$
harald@Neville Movies]$ ls *mpg
Embedding_0093r_Large.mpg      ProlateCriticalCollapse.mpg
Embedding_0093r_Medium.mpg    RePsi4_0093r.mpg
Embedding_0093r_Small.mpg     Shibata_APR-135-165.mpg
Embedding_0093r.mpg           Shibata_APR-140-140.mpg
EmbeddingFast_0093r.mpg       TrKBlowup.mpg
EmbeddingSmall_0093r.mpg      viz_shiftingall_21.mpg
EmbeddingSmallFast_0093r.mpg  viz_shiftingall_ImPsi4_20.mpg
torius_qe_19_Lml_psi4r_z_log.mpg
harald@Neville Movies]$ cd ..
harald@Neville 08May_Perimeter]$ ls *mpg
*mpg: No such file or directory
harald@Neville 08May_Perimeter]$

```



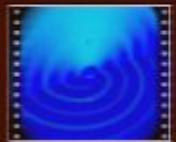
Numerical study of black hole spacetimes

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← 51 of 67 Best Fit

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M \equiv \sqrt{A/16\pi}$$


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harald@Neville:AA~/research/Talks/08May_Perimeter

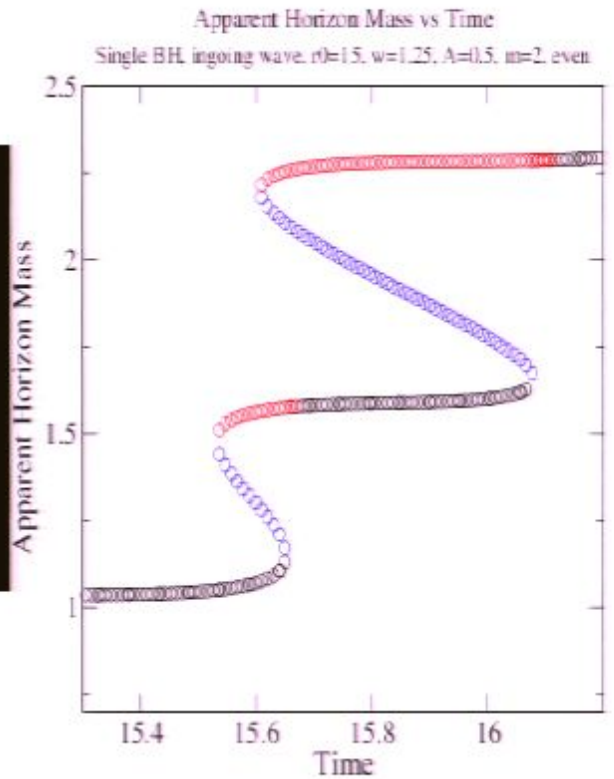
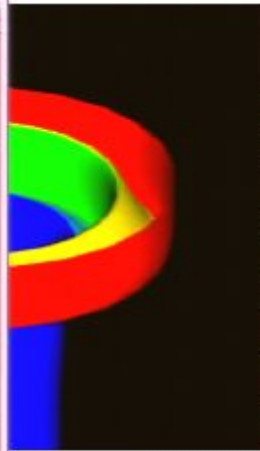
Edit View Terminal Tabs Help

no context error: NULL pointer
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harald@Neville Movies]$  
harald@Neville Movies]$  
harald@Neville Movies]$  
harald@Neville Movies]$ ls *mpg  
_Embedding_0093r_Large.mpg      ProlateCriticalCollapse.mpg  
_Embedding_0093r_Medium.mpg    RePsi4_0093r.mpg  
_Embedding_0093r_Small.mpg     Shibata_APR-135-165.mpg  
_Embedding_0093r.mpg           Shibata_APR-140-140.mpg  
_EmbeddingFast_0093r.mpg       TrKBlowup.mpg  
_EmbeddingSmall_0093r.mpg      viz_shiftingall_21.mpg  
_EmbeddingSmallFast_0093r.mpg  viz_shiftingall_ImPsi4_20.mpg  
torius_qe_19_LmI_psi4r_z_log.mpg
```

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harald@Neville Movies]$ cd ..  
harald@Neville 08May_Perimeter]$ ls *mpg  
*mpg: No such file or directory  
harald@Neville 08May_Perimeter]$ ls *mpeg  
viz_shifting.mpeg  
harald@Neville 08May_Perimeter]$
```

ProlateCriticalCollapse.mpg
RePsi4_0093r.mpg
Shibata_APR-135-165.mpg
Shibata_APR-140-140.mpg
TrKBlowup.mpg
viz_shiftingall_21.mpg
viz_shiftingall_ImPsi4_20.mpg



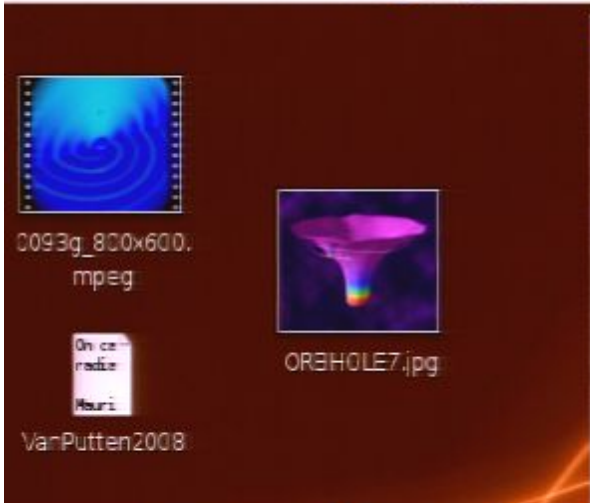
Numerical study of black hole spacetimes

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← 51 of 67 Best Fit

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

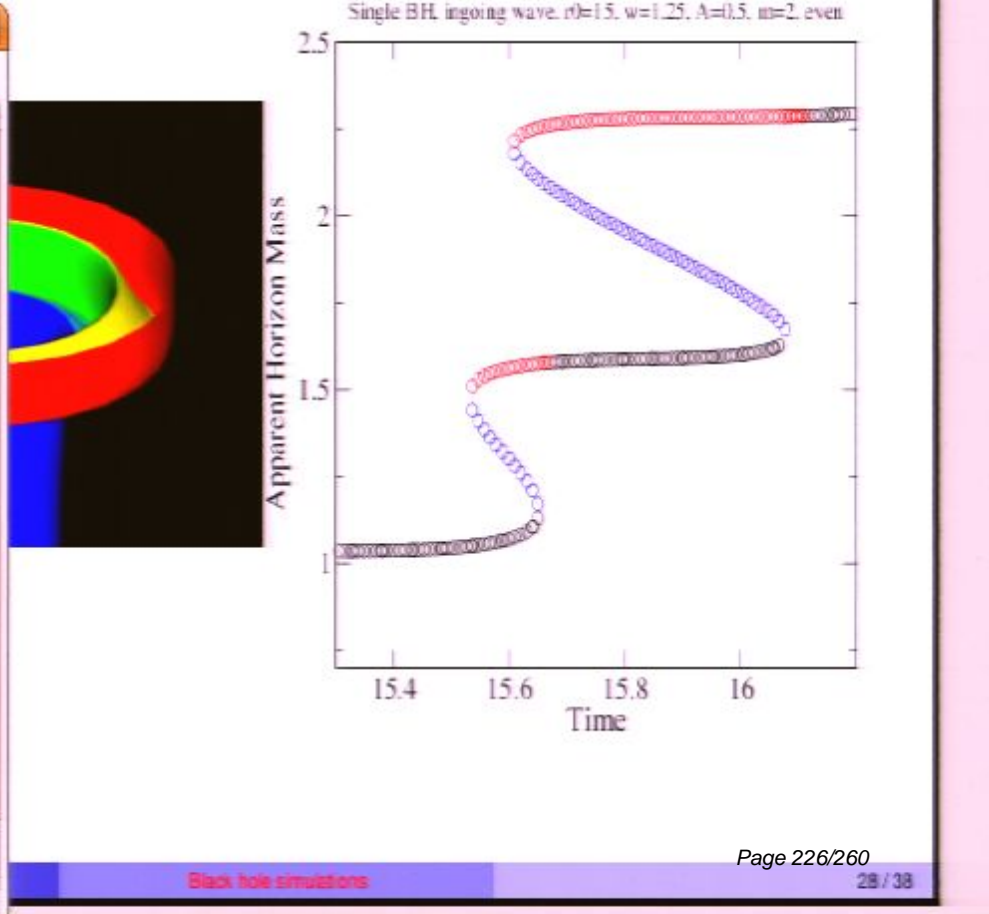
$$M \equiv \sqrt{A/16\pi}$$


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harald@Neville:AA~/research/Talks/08May_Perimeter
Edit View Terminal Tabs Help
no context error: NULL pointer
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harald@Neville Movies]$
harald@Neville Movies]$
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harald@Neville Movies]$ ls *mpg
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Embedding_0093r_Small.mpg     Shibata_APR-135-165.mpg
Embedding_0093r.mpg           Shibata_APR-140-140.mpg
EmbeddingFast_0093r.mpg       TrKBlowup.mpg
EmbeddingSmall_0093r.mpg      viz_shiftingall_21.mpg
EmbeddingSmallFast_0093r.mpg  viz_shiftingall_ImPsi4_20.mpg
torius_qe_19_Lml_psi4r_z_log.mpg
harald@Neville Movies]$ cd ..
harald@Neville 08May_Perimeter]$ ls *mpg
*mpg: No such file or directory
harald@Neville 08May_Perimeter]$ ls *mpeg
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harald@Neville 08May_Perimeter]$ mpeg_play 5Ho
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harald@Neville 08May_Perimeter]$ mpeg_play 5HorizonSlicing.

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Numerical study of black hole spacetimes

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Previous Next

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

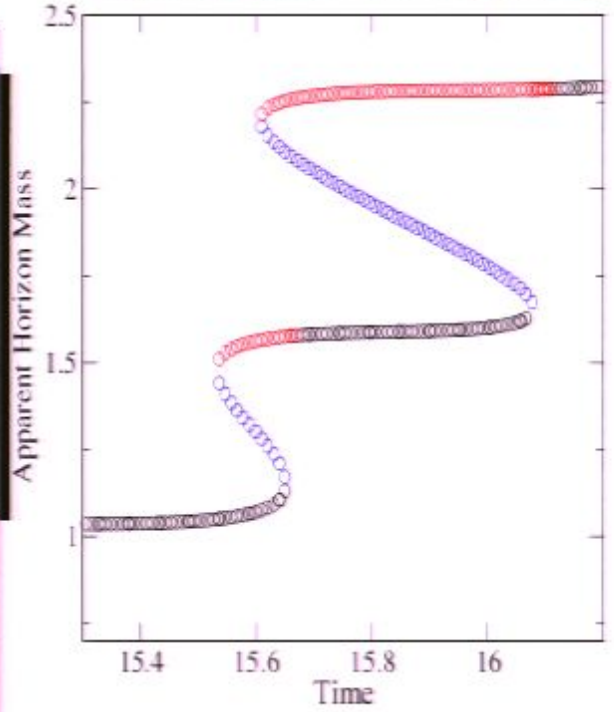
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 22/25.6 Rewind Pause Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5$, $w=1.25$, $A=0.5$, $m=2$, even



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harald@Neville Movies]$
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Numerical study of black hole spacetimes

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← Previous 51 of 67 Next → Best Fit

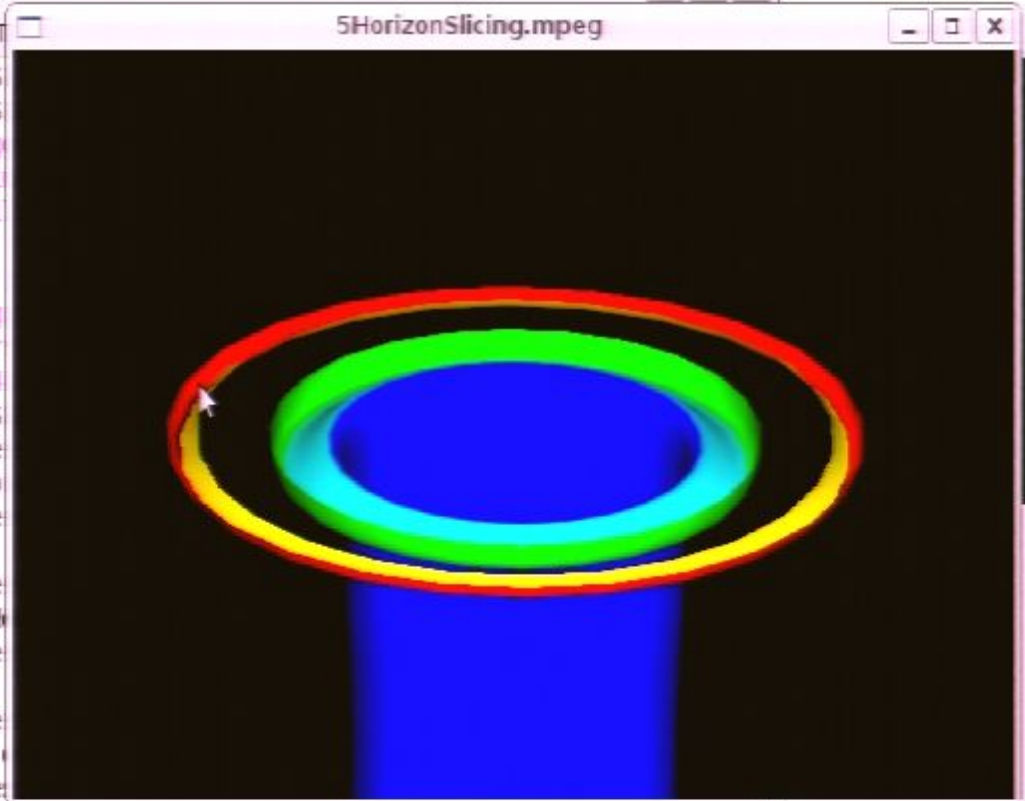
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

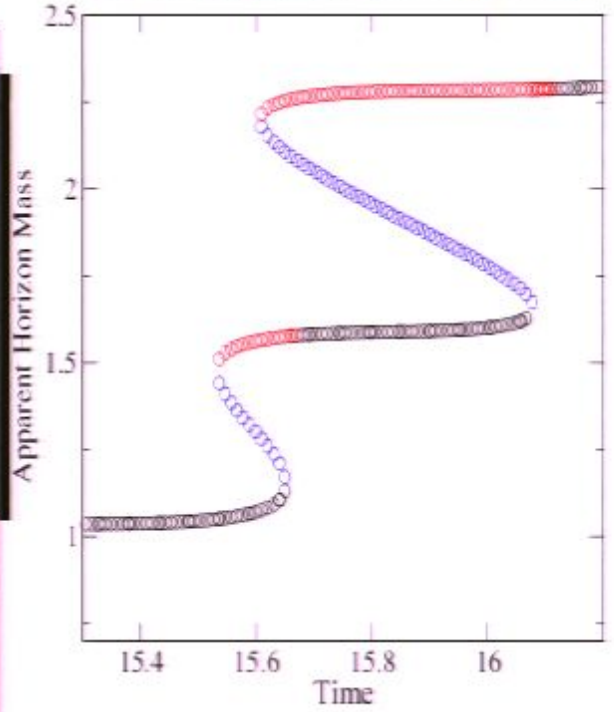
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 102/25.1 | Rewind | Pause | Step | Play | Loop OFF | Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r=1.5$, $w=1.25$, $A=1.5$, $m=2$, even



0093g_800x600.mpeg



ORSHOLE7.jpg

VanPutten2008

harald@Nev

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harald@Neville Movies]$
harald@Neville Movies]$
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harald@Neville Movies]$
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24 bit displays: use
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Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Apparent Horizon Area vs. Time

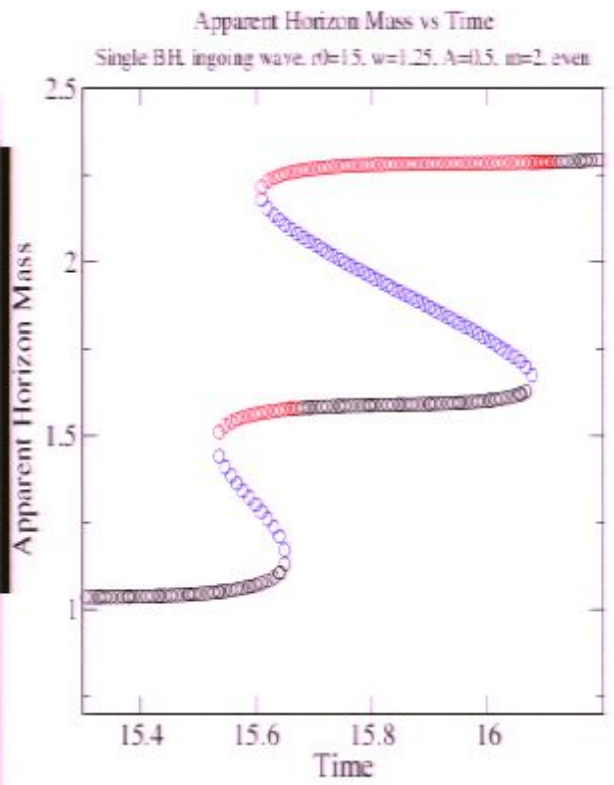
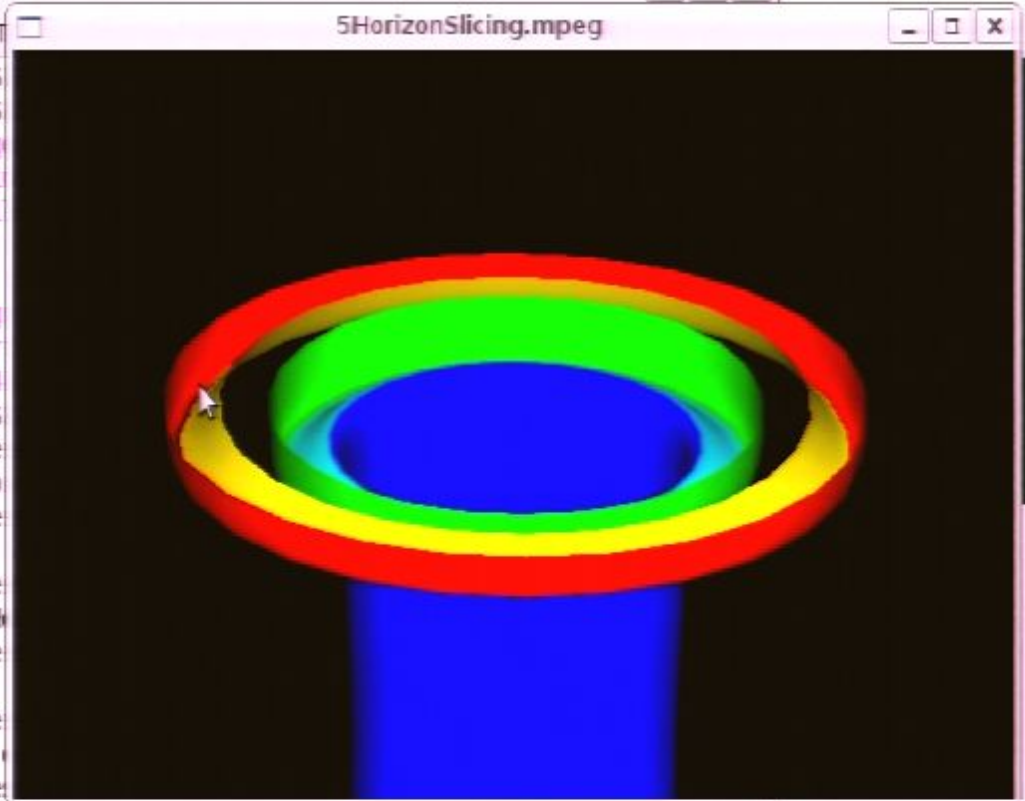
- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M = \sqrt{A/16}$$


ORSHOLE7.jpg

MPEG Player Controls

Frame/Rate 168/25.1 | Rewind | Pause | Step | Play | Loop OFF | Exit



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harald@Nev
Edit View Terminal
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24 bit displays: use
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Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

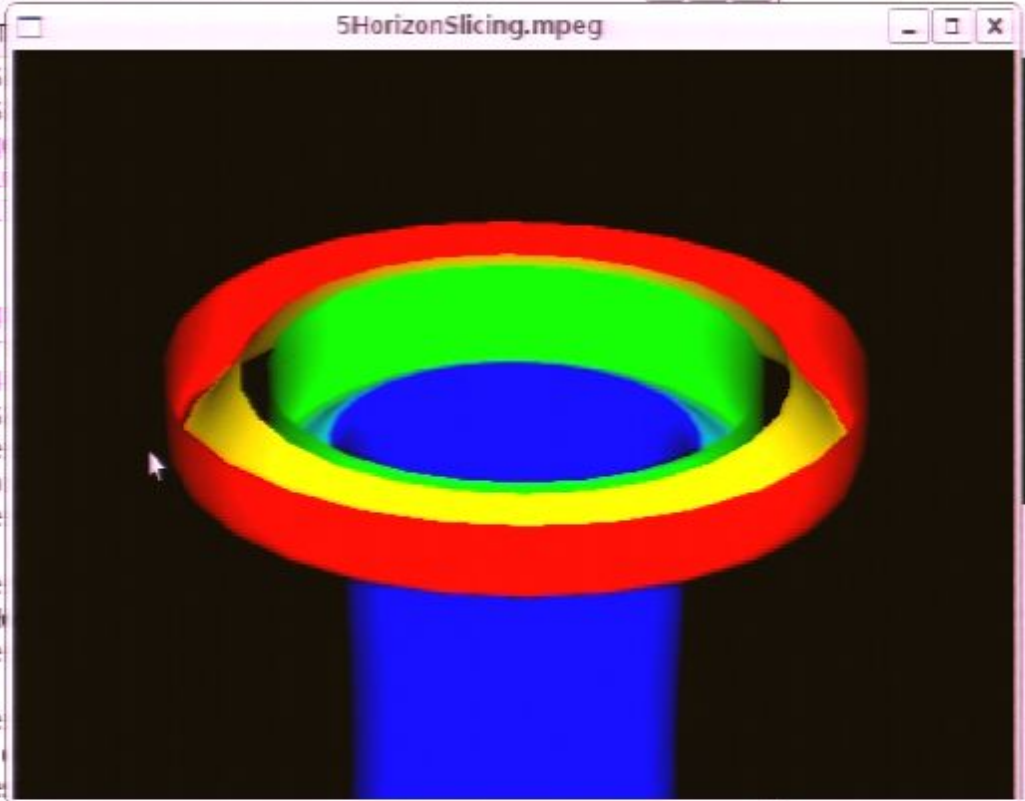
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

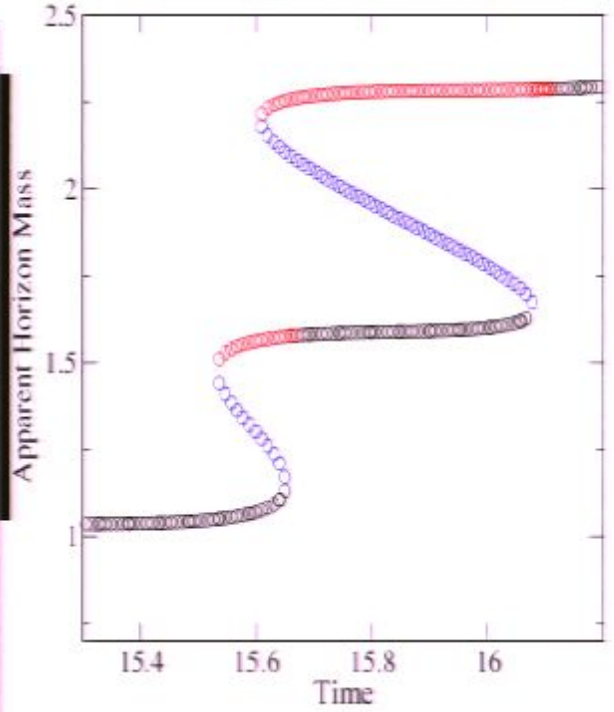
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 232/25.0 Revind Pause Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5, w=1.25, A=0.5, m=2, \text{even}$



0093g_800x600.mpeg



ORSHOLE7.jpg

VanPutten2008

harald@Nev

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harald@Neville Movies]$
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*mpg: No such file o
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24 bit displays: use
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Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

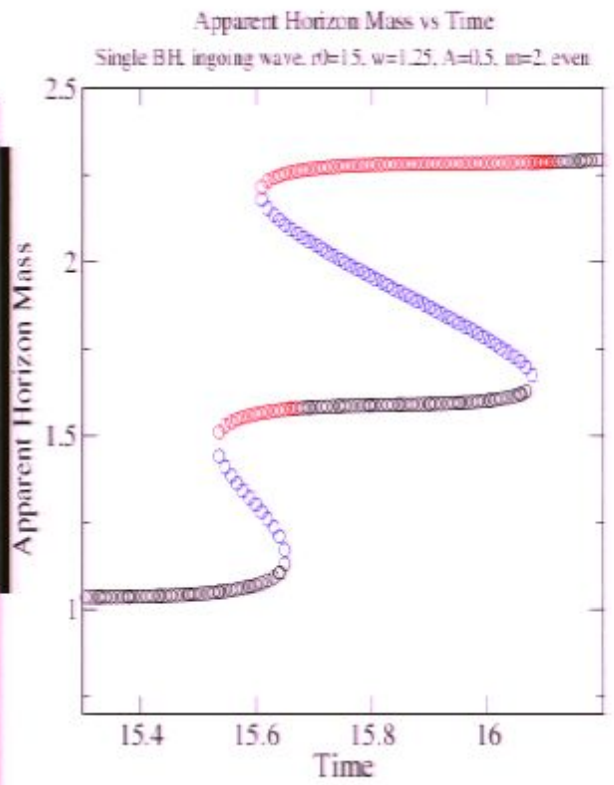
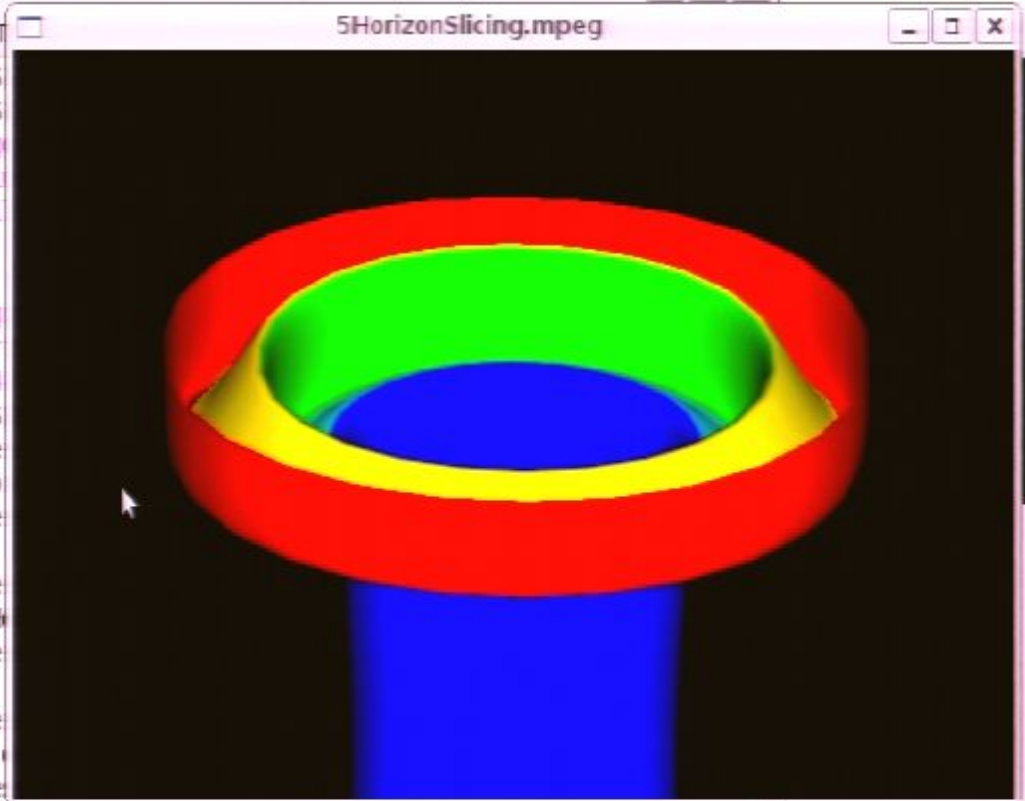
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M = \sqrt{A/16}$$


MPEG Player Controls

Frame/Rate 282/25.0 Revind Pause Step Play Loop OFF Exit



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24 bit displays: use
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File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

Apparent Horizon Area vs. Time

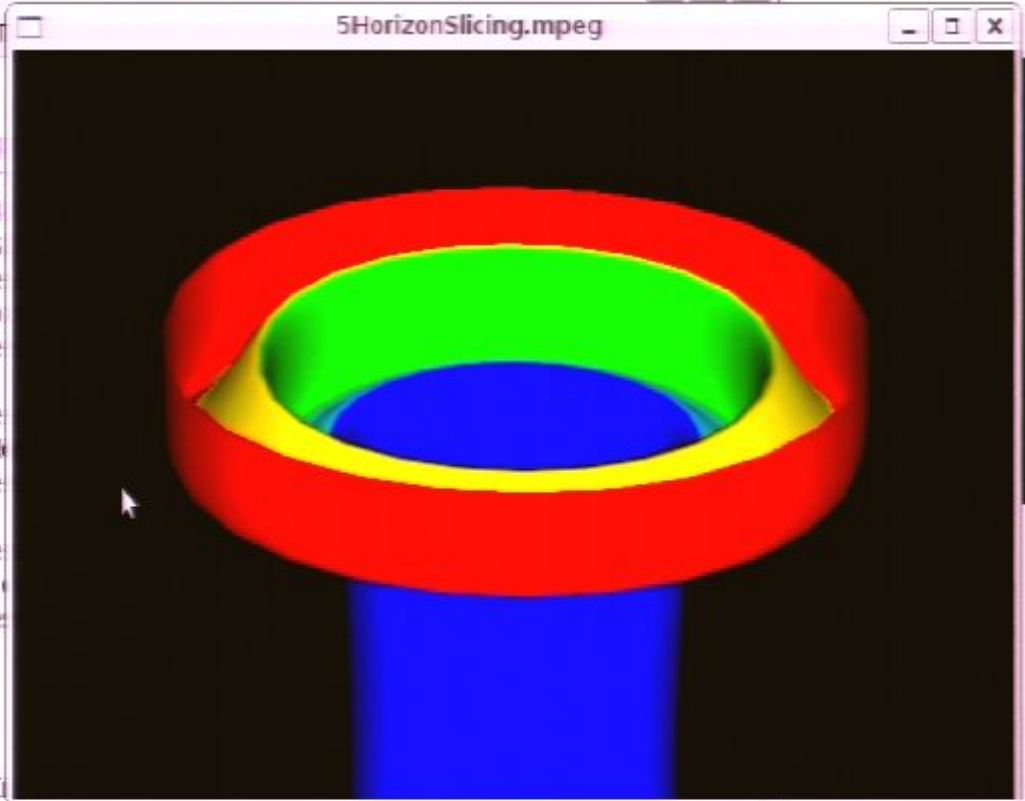
• Apparent horizon: Surface w/ zero expansion of outgoing null-geodesics

$$M = \sqrt{A/16}$$

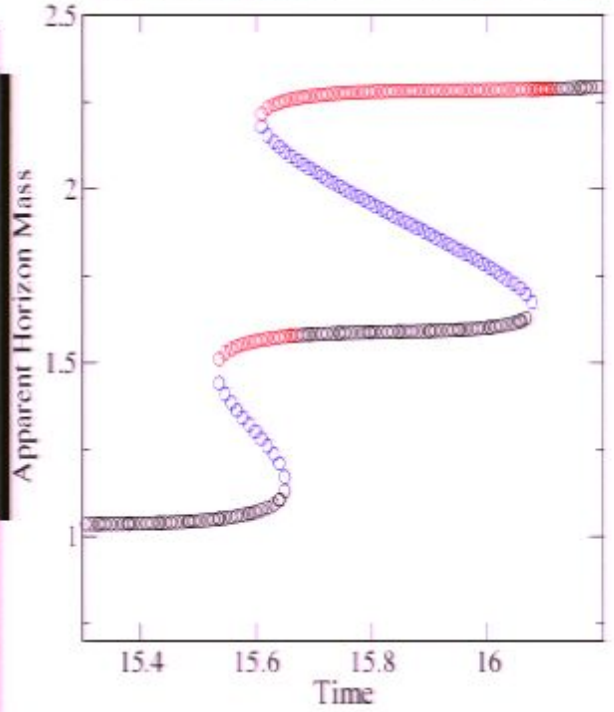
MPEG Player Controls

Frame/Rate 301/25.0

Rewind End Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5$, $w=1.25$, $A=0.5$, $m=2$, even



0093g_800x600.mpeg



ORSHOLE7.jpg

VanPutten2008

harald@Neville

Edit View Terminal

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rald@Neville 08May Pe
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24 bit displays: use
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Time Spent (After I

Frames/Sec: 25.018544

Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

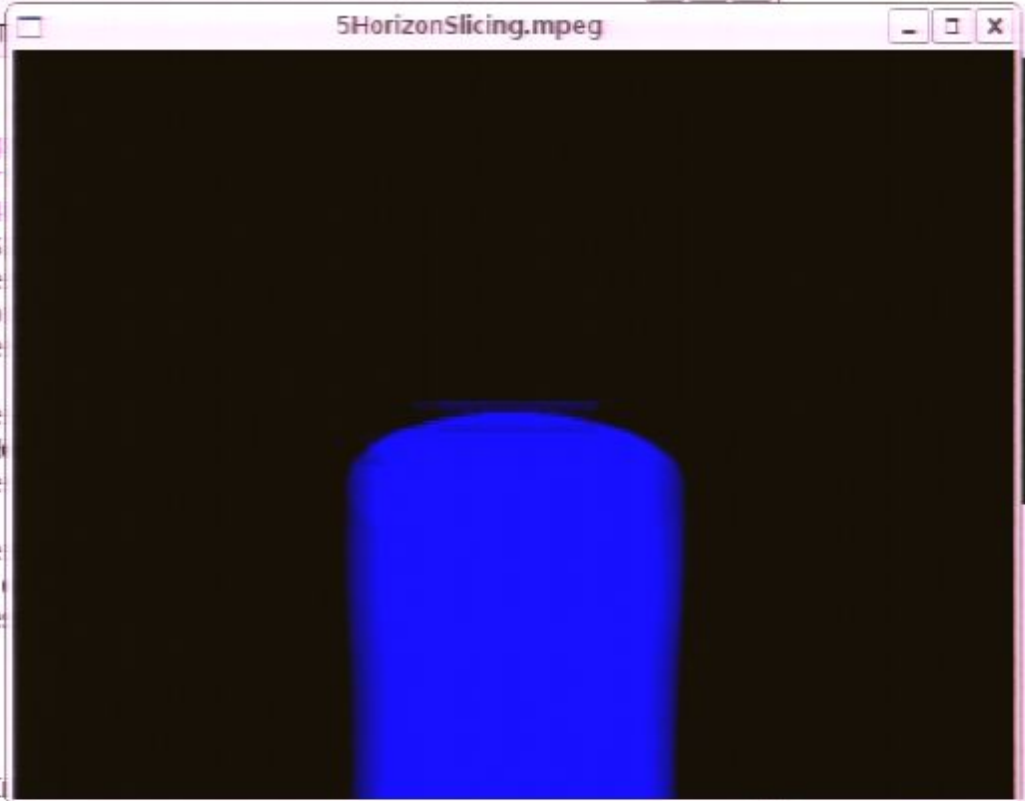
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

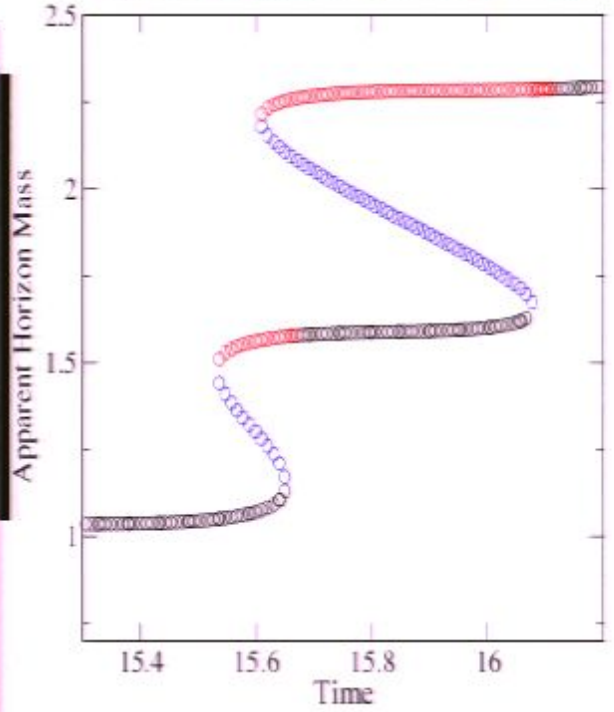
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 1/25.2 Rev Pause Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5$, $w=1.25$, $A=0.5$, $m=2$, even



Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

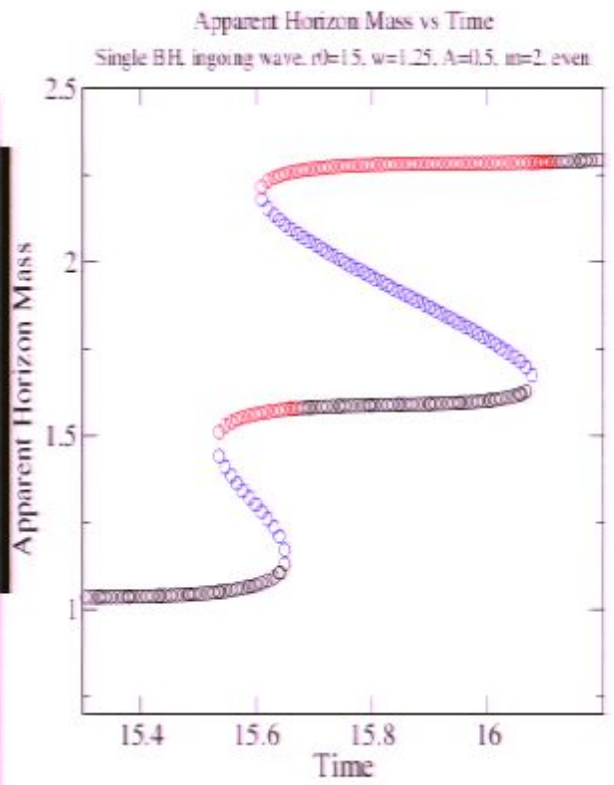
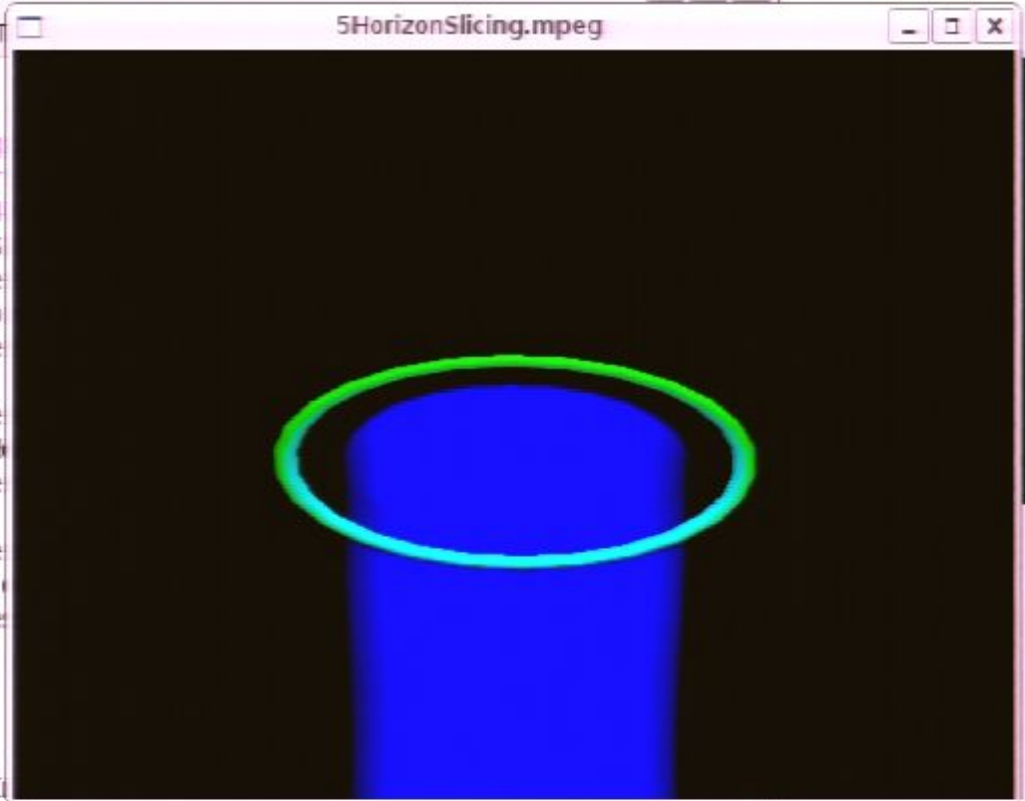
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M = \sqrt{A/16}$$


MPEG Player Controls

Frame/Rate 54/25.2 Rewind Pause Step Play Loop OFF Exit



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harald@Nev
Edit View Terminal
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Frames/Sec: 25.018544

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File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

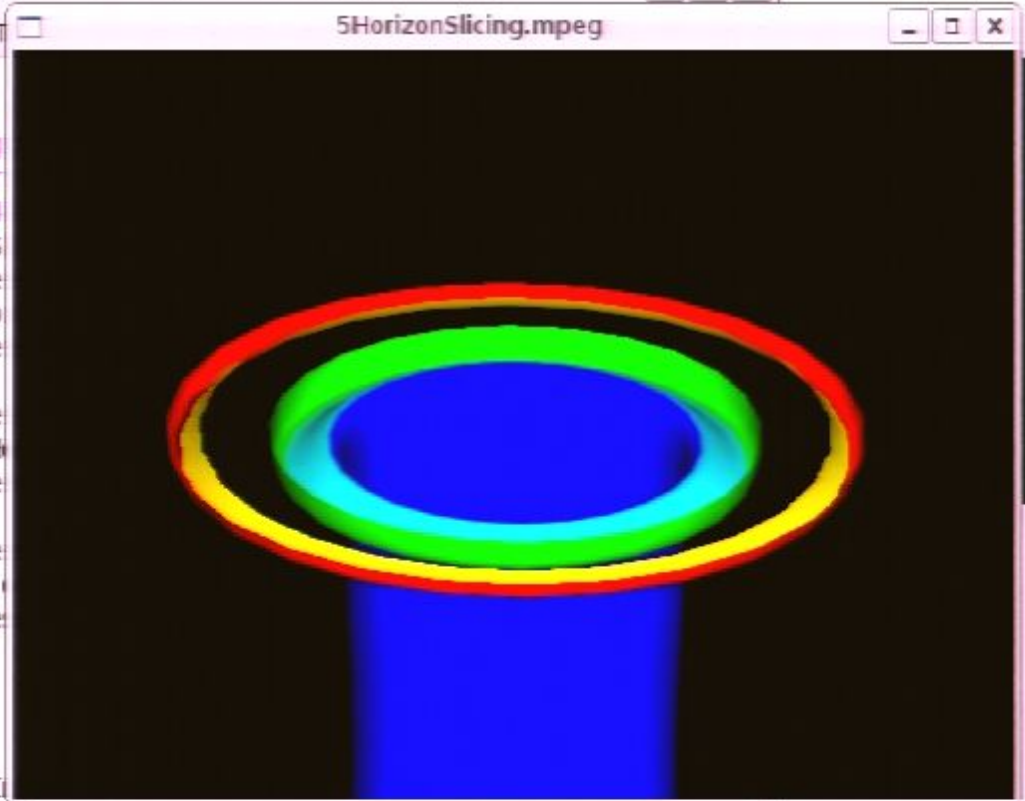
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

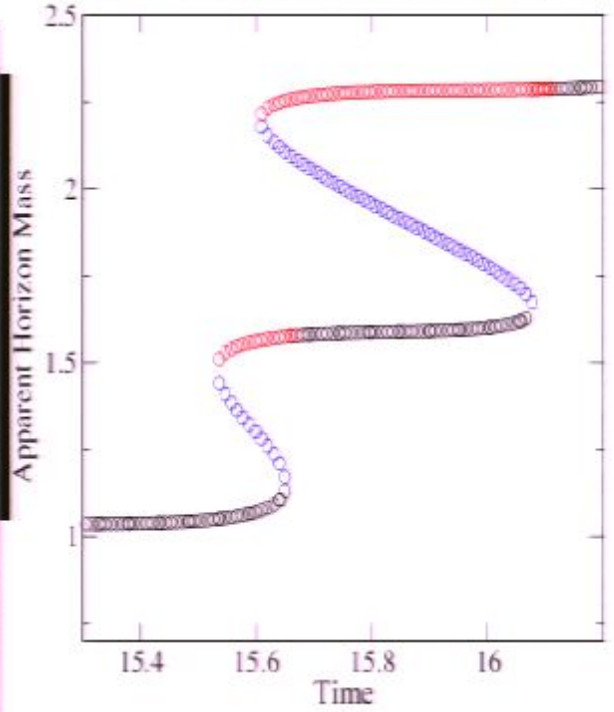
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 109/25.3 Rewind Pause Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave, $r_0=1.5$, $w=1.25$, $A=0.5$, $m=2$, even



0093g_800x600.mpeg



ORHOLE7.jpg

VanPutten2008

harald@Neville

Edit View Terminal

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beddingSmall 0093r.mpg
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torius_qe_19_Lml_psi4
rald@Neville Movies]$
rald@Neville 08May Pe
*mpg: No such file o
rald@Neville 08May Pe
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rald@Neville 08May Pe

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24 bit displays: use
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Time Spent (After I

Frames/Sec: 25.018544

Pirsa: 08050036

Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Apparent Horizon Area vs. Time

- Apparent horizon: Surface w/ zero expansion of outgoing null-geodesics

$$M = \sqrt{A/16}$$


MPEG Player Controls

FrameRate 179/25.2 Rewind Pause Step Play Loop OFF Exit

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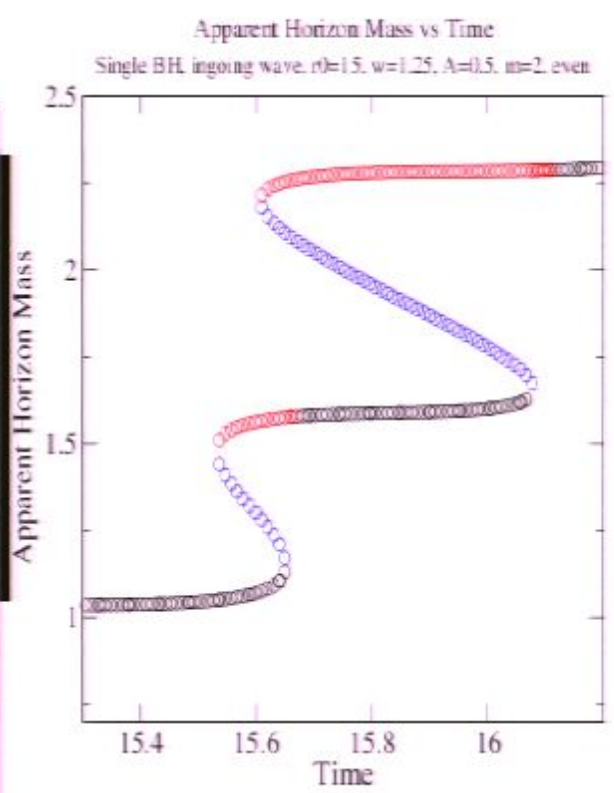
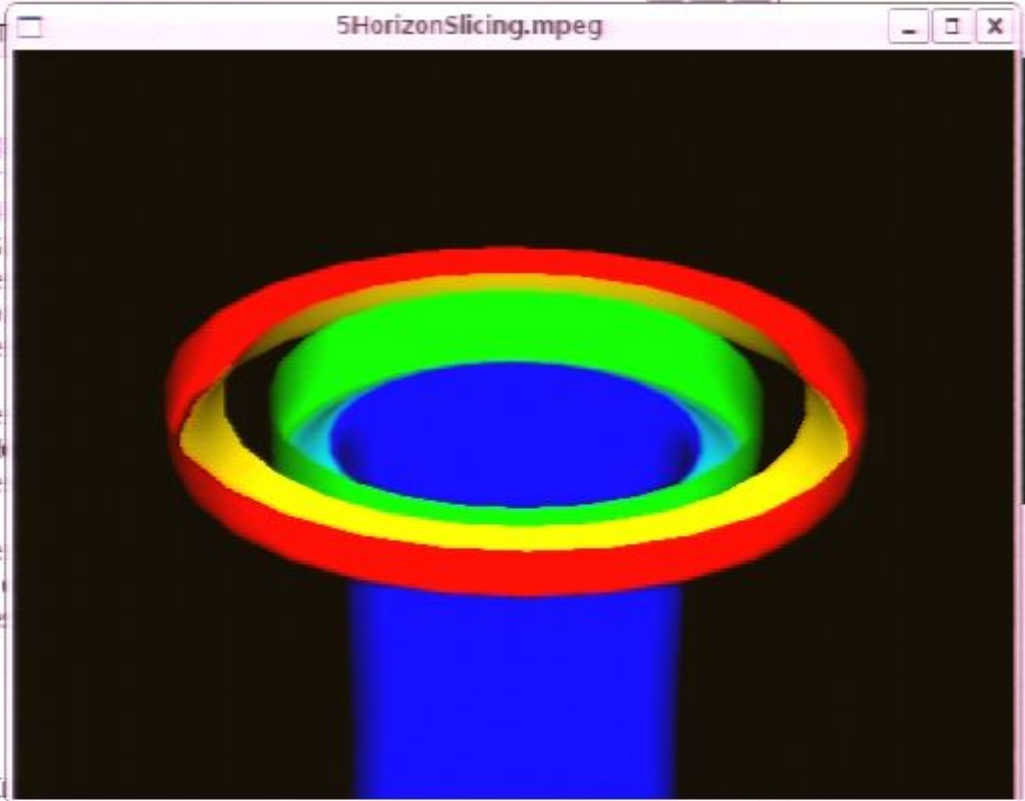
harald@Nev
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l. Frames/Sec: 25.018544

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Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

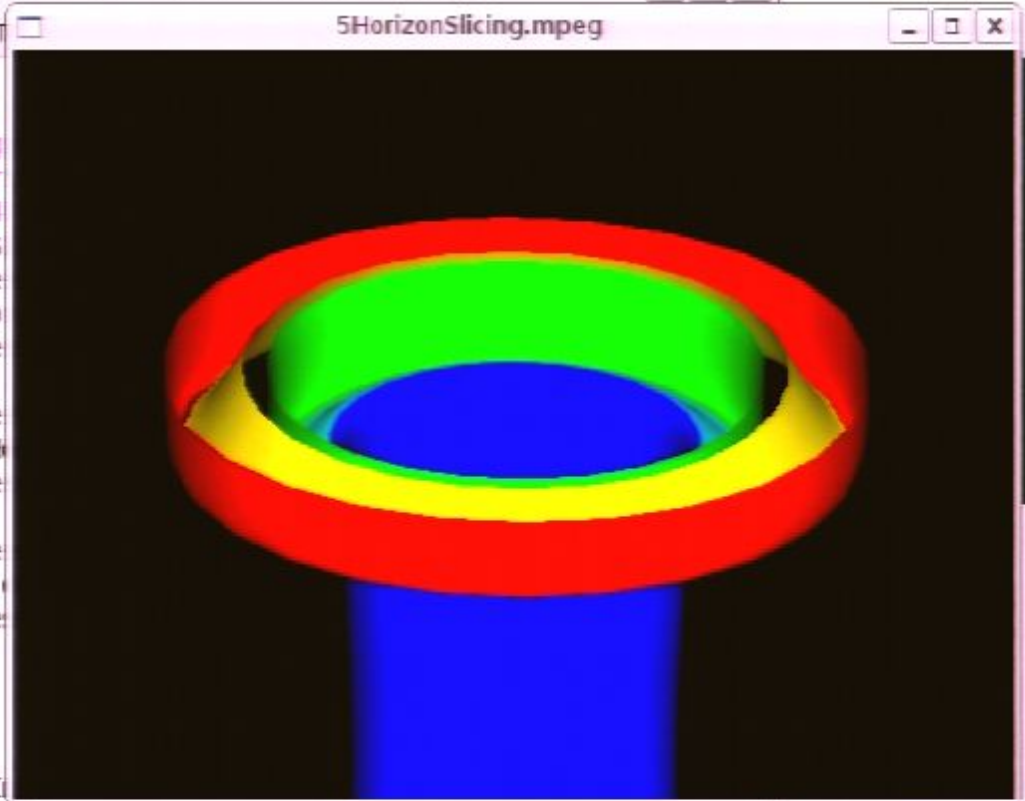
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

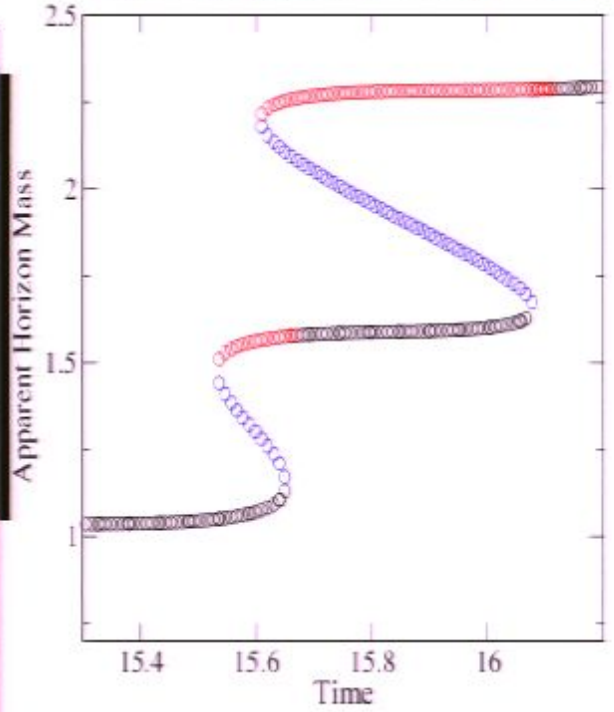
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 240/25.2 Rewind Pause Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5$, $w=1.25$, $A=0.5$, $m=2$, even



0093g_800x600.mpeg



ORSHOLE7.jpg

VarPutten2008

harald@Neville

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Time Spent (After I

Frames/Sec: 25.018544

Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

Previous Next

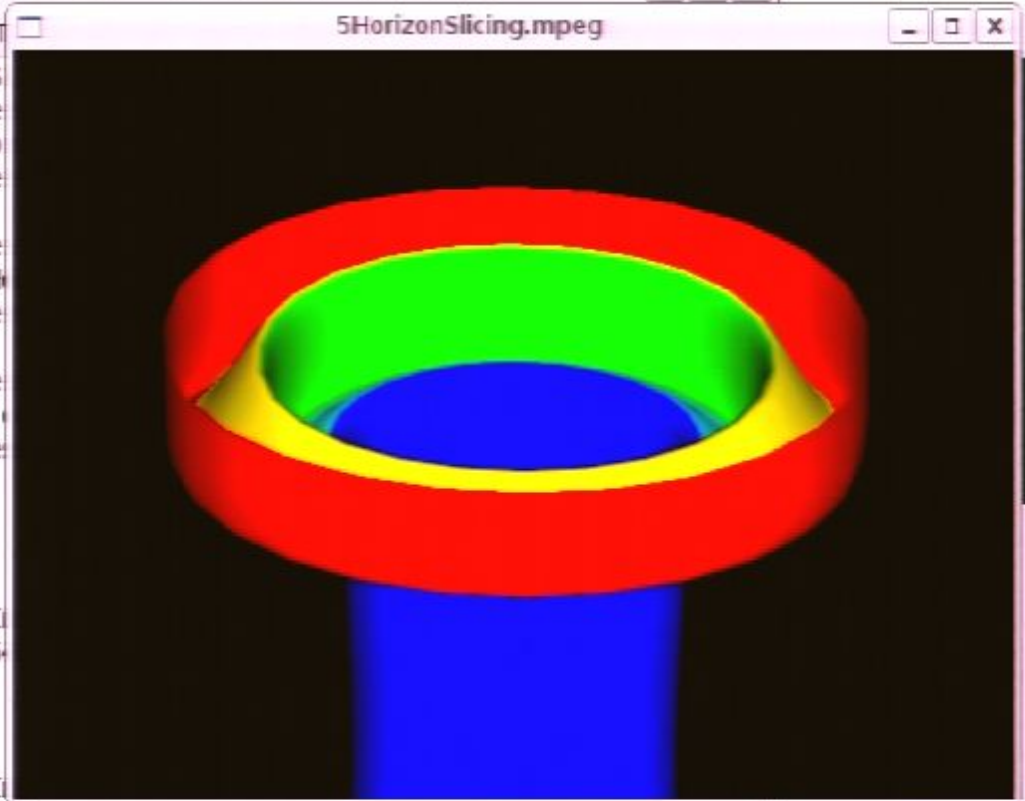
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

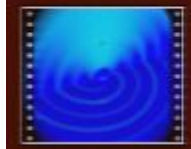
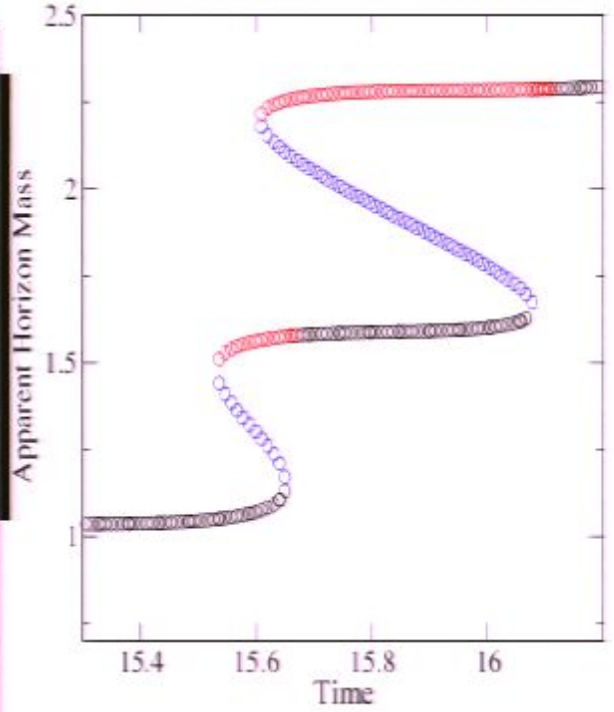
$$M = \sqrt{A/16}$$

MPEG Player Controls

Frame/Rate 301/25.2 Rewind End Step Play Loop OFF Exit



Apparent Horizon Mass vs Time
Single BH ingoing wave. $r_0=1.5$, $w=1.25$, $A=1.5$, $m=2$, even



0093g_800x600.mpeg



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VanPutten2008

harald@Nev

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Time Spent (After I
Frames/Sec: 25.0185

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Time Spent (After I
Frames/Sec: 12.559307

```


Numerical study of black hole spacetimes

File Edit View Go Help

← 51 of 67 Best Fit

→

Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics
- $$M \equiv \sqrt{A/16\pi}$$



```

harald@Neville:AA~/research/Talks/08May_Perimeter
Edit View Terminal Tabs Help
harald@Neville [Movies]$ cd ..
harald@Neville [08May_Perimeter]$ ls *mpg
*mpg: No such file or directory
harald@Neville [08May_Perimeter]$ ls *mpeg
horizonSlicing.mpeg
harald@Neville [08May_Perimeter]$ mpeg_play 5Ho
horizonSlicing.mpeg 5HorizonSlicing.png
harald@Neville [08May_Perimeter]$ mpeg_play 5HorizonSlicing.mpeg

24 bit displays: use -dither color to get full color
ordered dither is the default.
modified dither requires 8 bit display
adding -dither color

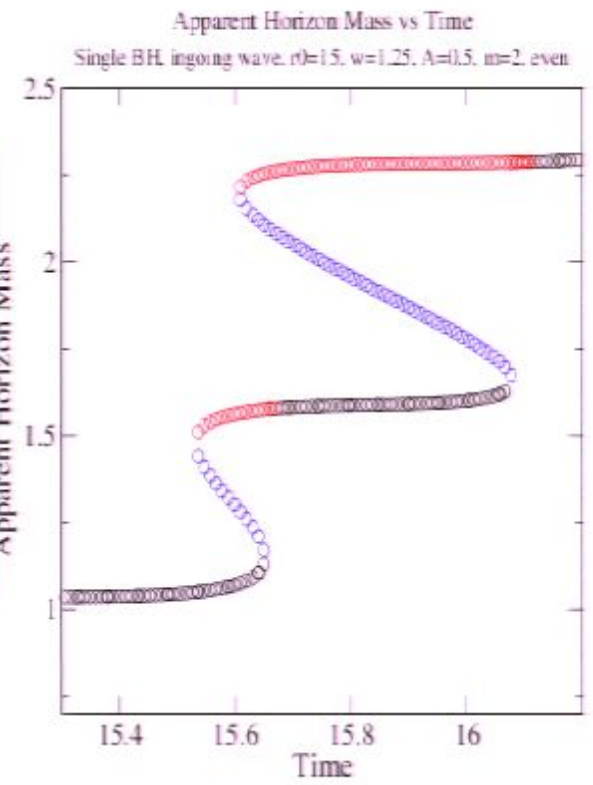
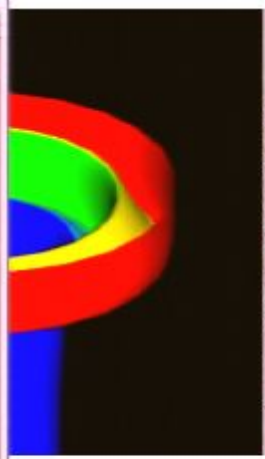
e!

Time Spent (After Initializations): 12.031076 secs.
Frames/Sec: 25.018544

e!

Time Spent (After Initializations): 23.966291 secs.
Frames/Sec: 12.559307
harald@Neville [08May_Perimeter]$

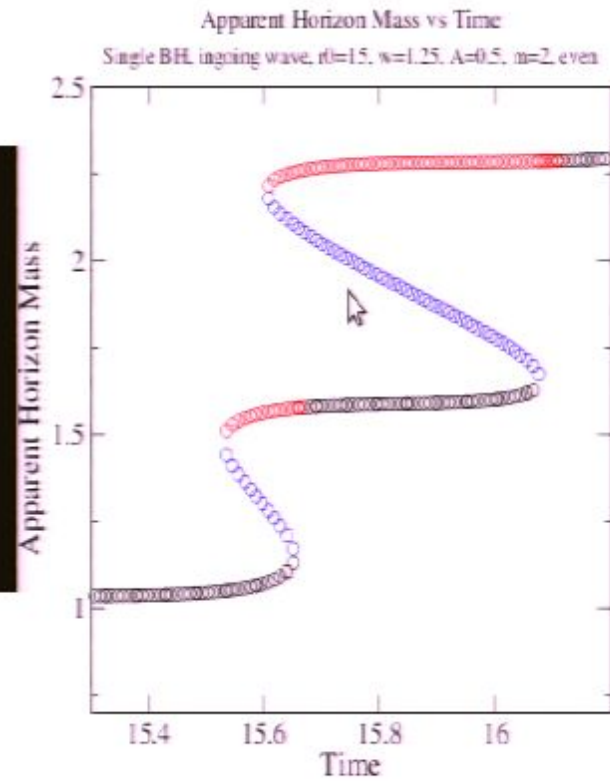
```



Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M \equiv \sqrt{A/16\pi}$$



Black hole simulations

28 / 38

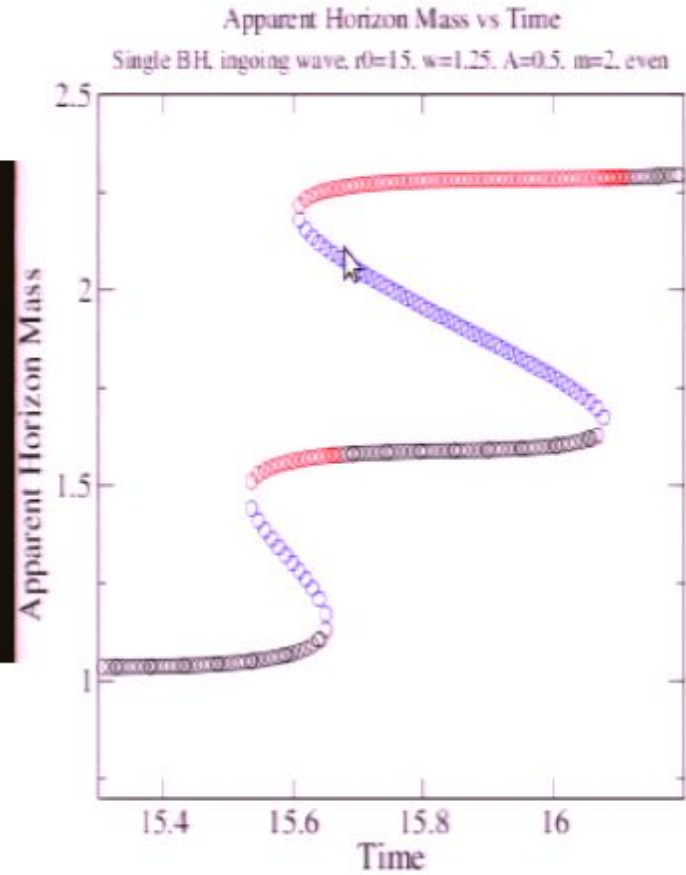
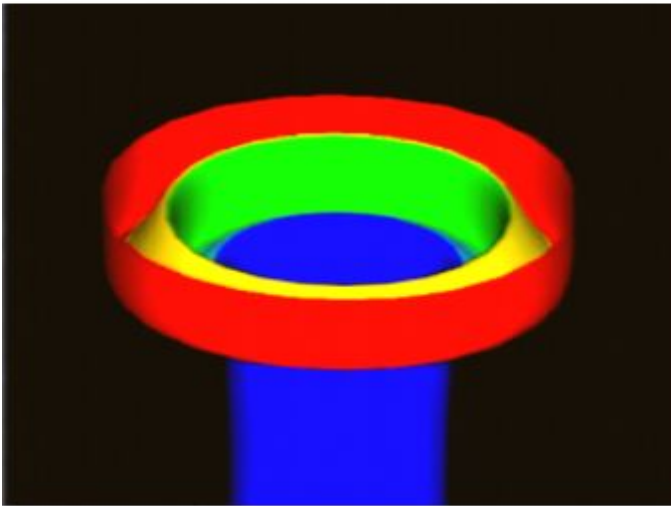
Black hole simulations

28 / 38

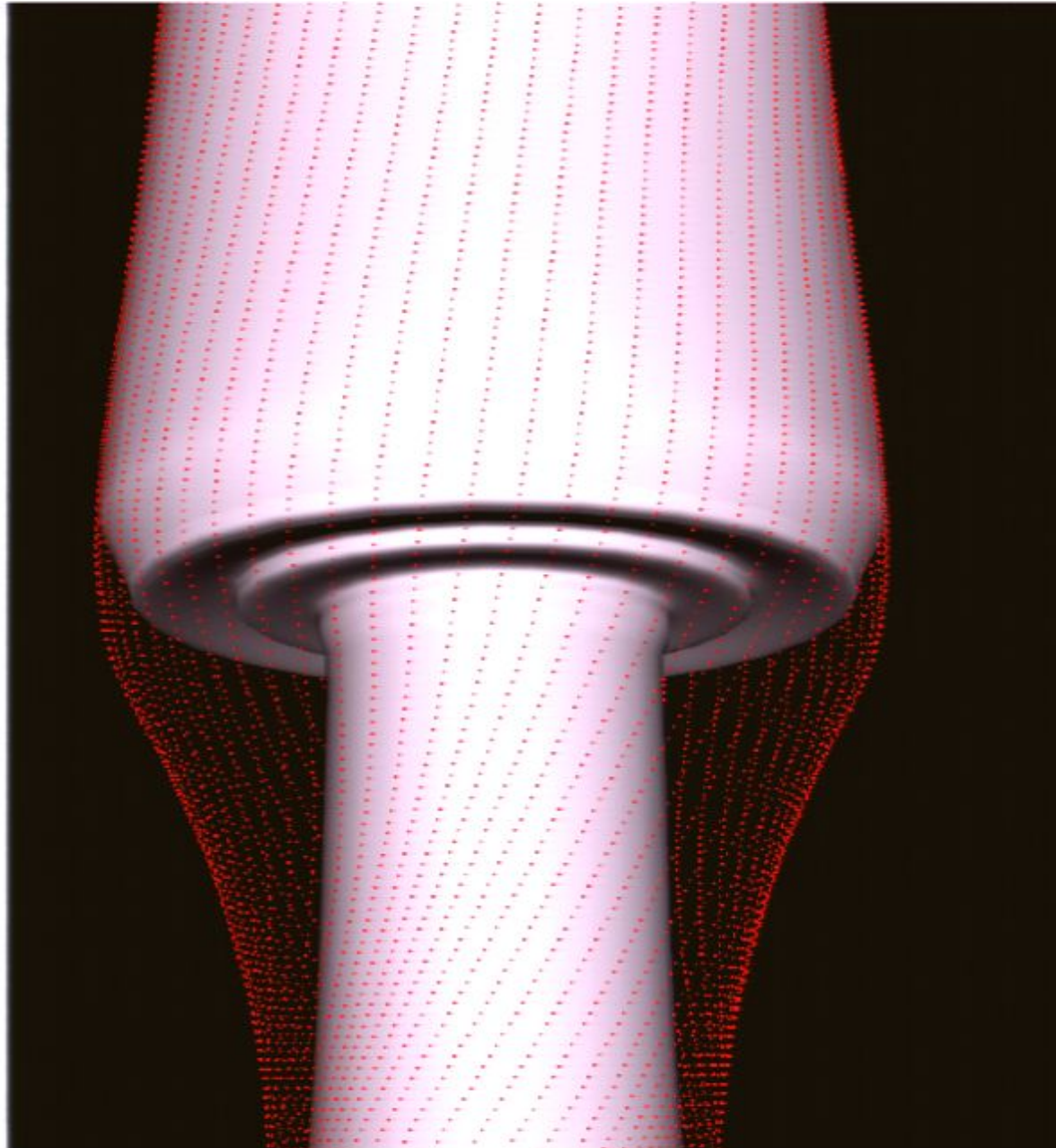
Apparent Horizon Area vs. Time

- **Apparent horizon:** Surface w/ zero expansion of outgoing null-geodesics

$$M \equiv \sqrt{A/16\pi}$$

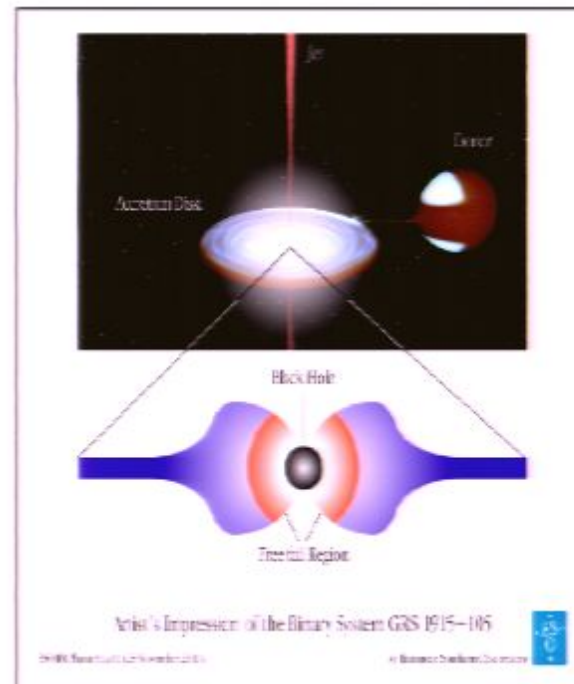


Event horizon



Black holes with near-extremal spins

- GRS 1915+105: $S/M^2 \gtrsim 0.98$ (McClintock et al, 2006)

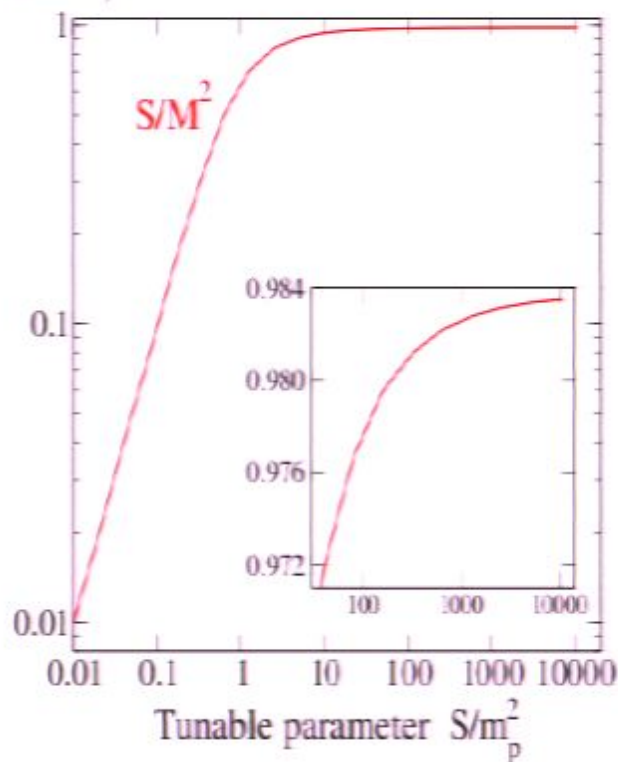


- Constraints non-linear, cannot just superpose analytical Kerr metrics to get high spin BBH.
- With **Geoffrey Lovelace**, **Rob Owen** (Caltech grads, now Cornell post-docs) & **Tony Chu** (Caltech grad)

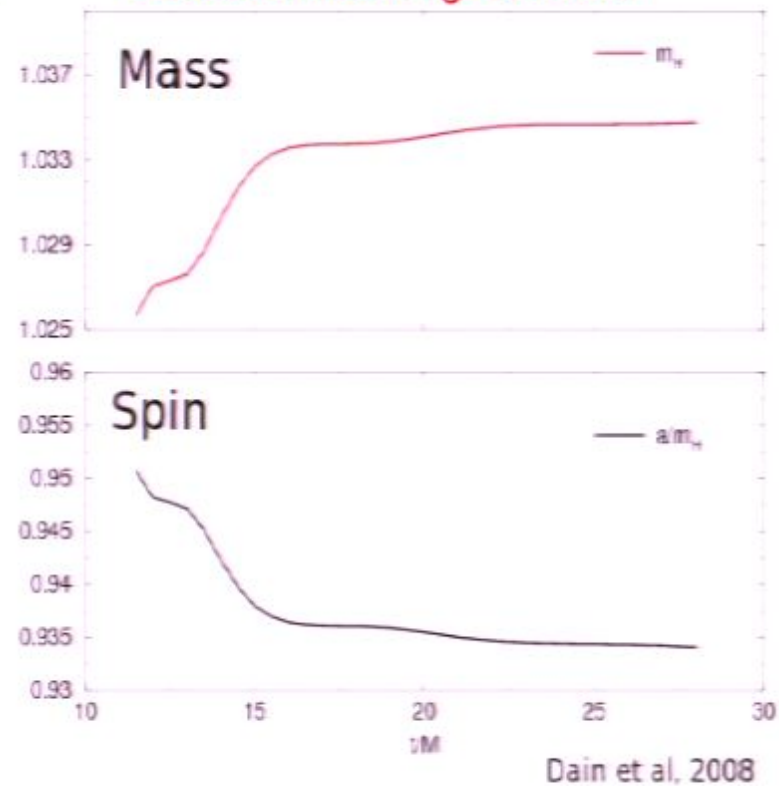
First attempt: Puncture initial data

- Most widely used BBH initial data: Just one elliptic PDE

Spin at initial time (Cook & York, 1990)



Relaxation during evolution

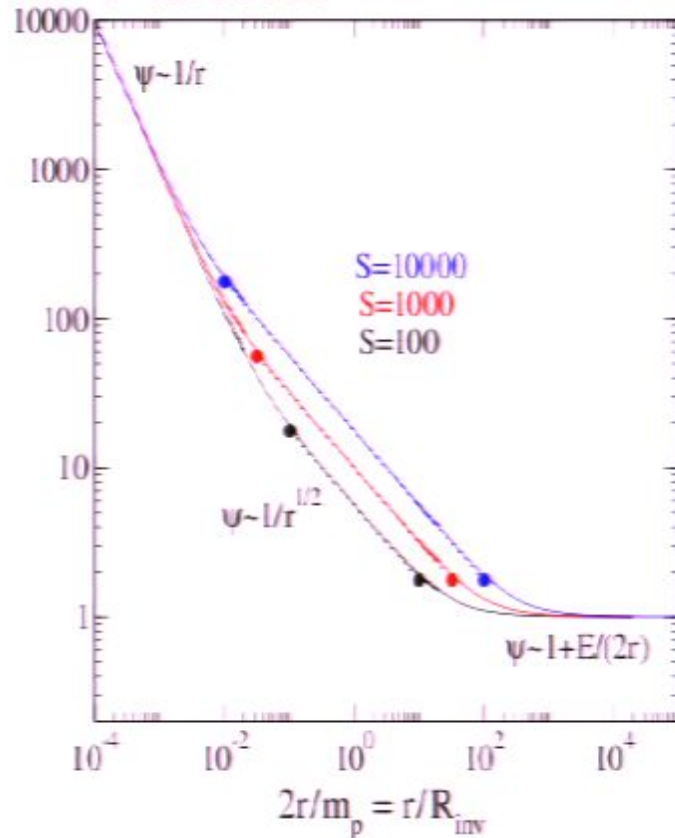


- Maximal *relaxed* spin ~ 0.93 — not large enough.

But intriguing properties of initial data geometry

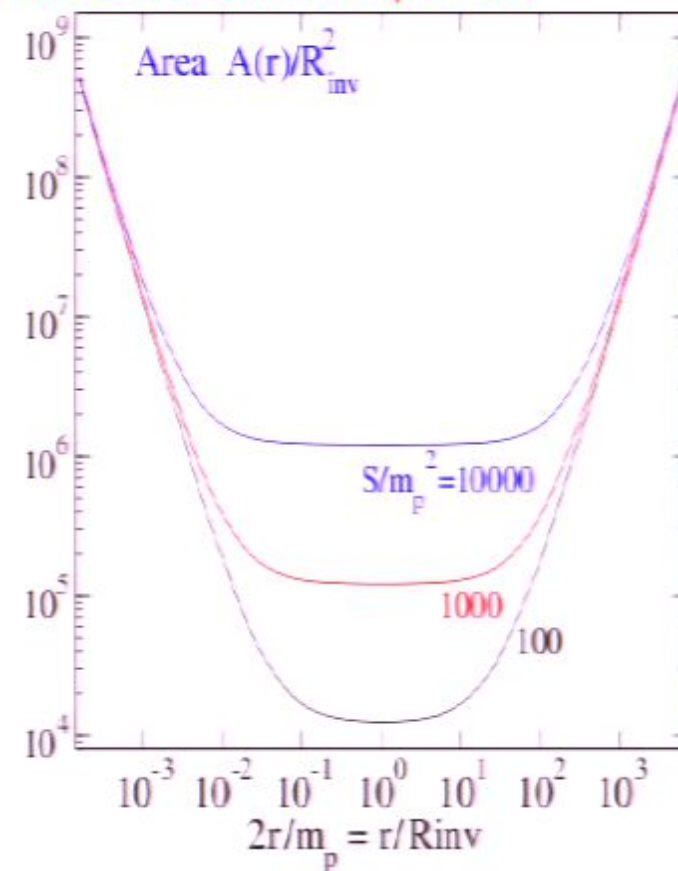
$$g_{ij} = \Psi^4 \delta_{ij}, \quad \nabla^2 \Psi + \frac{9}{4} \frac{S^2 \cos^2 \theta}{r^6} \Psi^{-7} = 0, \quad \text{BCs: } \Psi(r = \infty) = 1, \quad \Psi \rightarrow \frac{m_p}{2r} \text{ as } r \rightarrow 0$$

Numerical solution



Four spherical shells
cover radii $10^{-4} < r < 10^9$

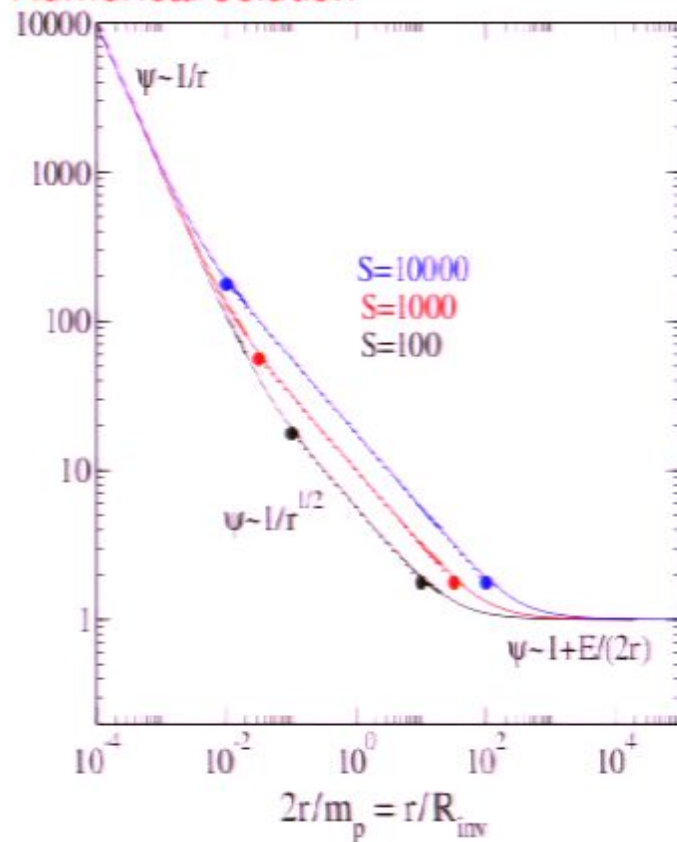
Area of coordinate spheres



But intriguing properties of initial data geometry

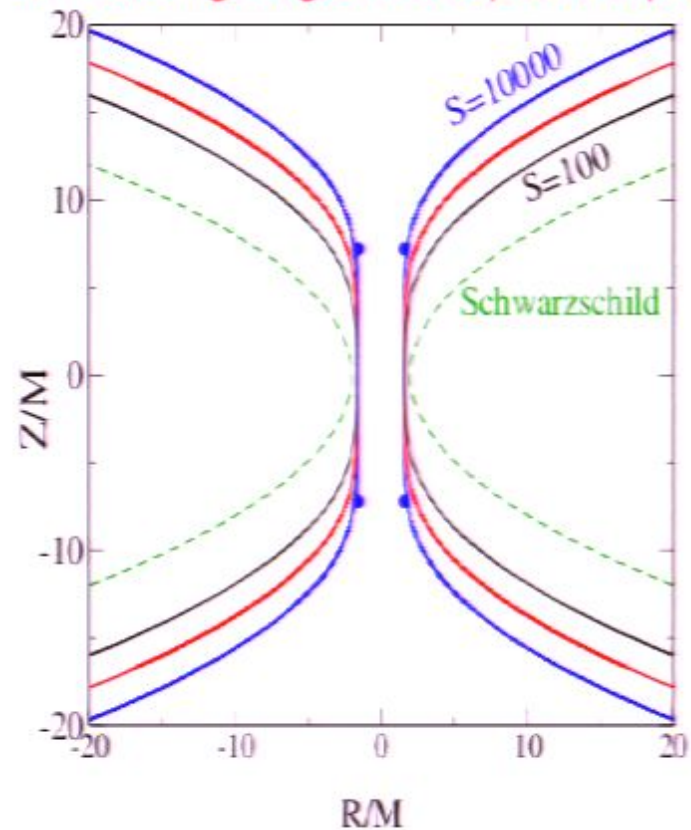
$$g_{ij} = \Psi^4 \delta_{ij}, \quad \nabla^2 \Psi + \frac{9}{4} \frac{S^2 \cos^2 \theta}{r^6} \Psi^{-7} = 0, \quad \text{BCs: } \Psi(r = \infty) = 1, \quad \Psi \rightarrow \frac{m_p}{2r} \text{ as } r \rightarrow 0$$

Numerical solution



Four spherical shells
cover radii $10^{-4} < r < 10^9$

Embedding diagram of equatorial plane

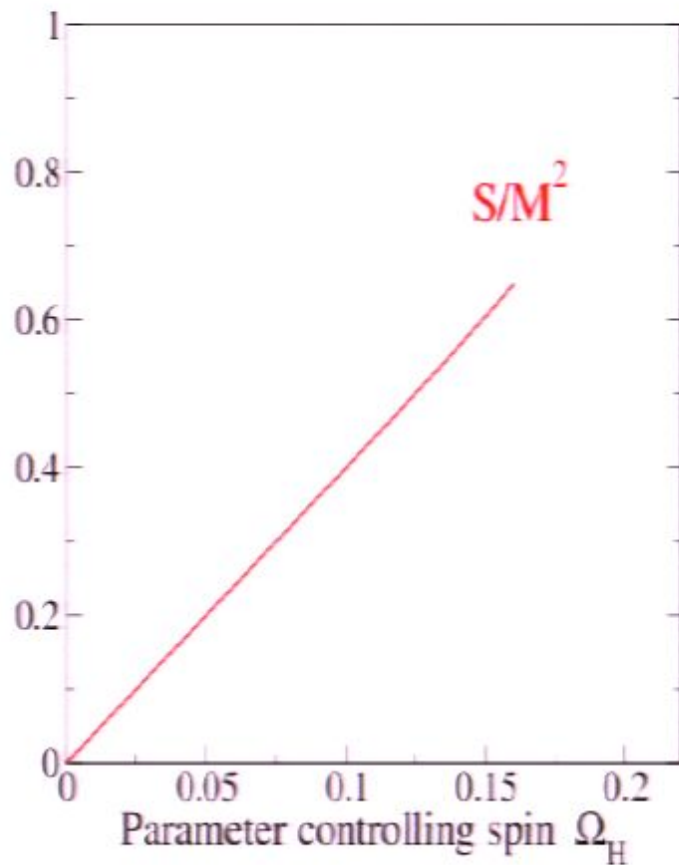


Lengthening cylindrical throat
Analytic approximation of throat region
matches well

Second attempt: Quasi-equilibrium initial data (Cook, HP, 2002-2006)

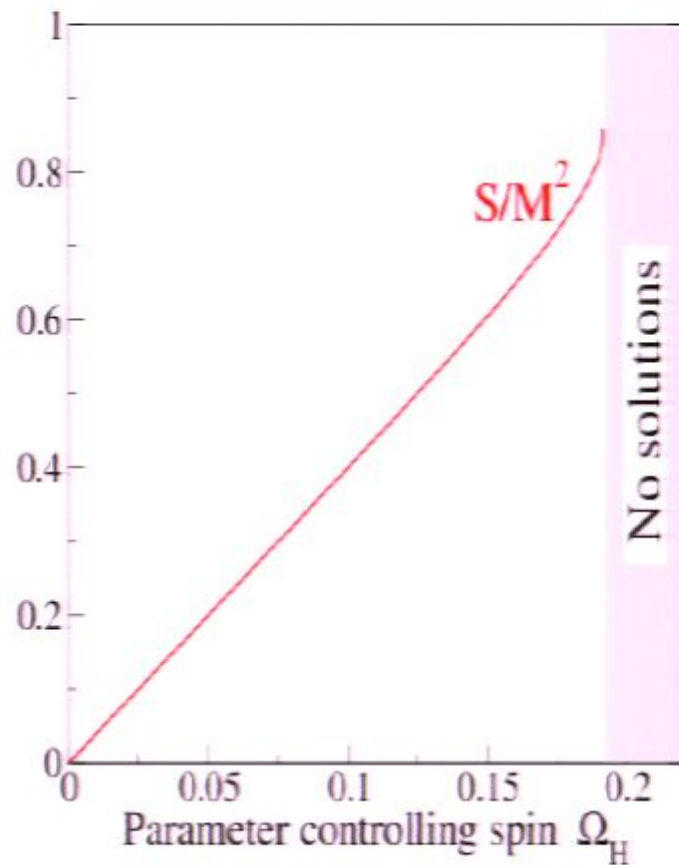
• Based on:

- 1) **time-independence** \Rightarrow Five coupled elliptic PDE's
- 2) **BH's in equilibrium** [vanishing shear of horizon] \Rightarrow BCs at excision surfaces
- 3) **Simplicity** (conformal flatness, maximal slicing)



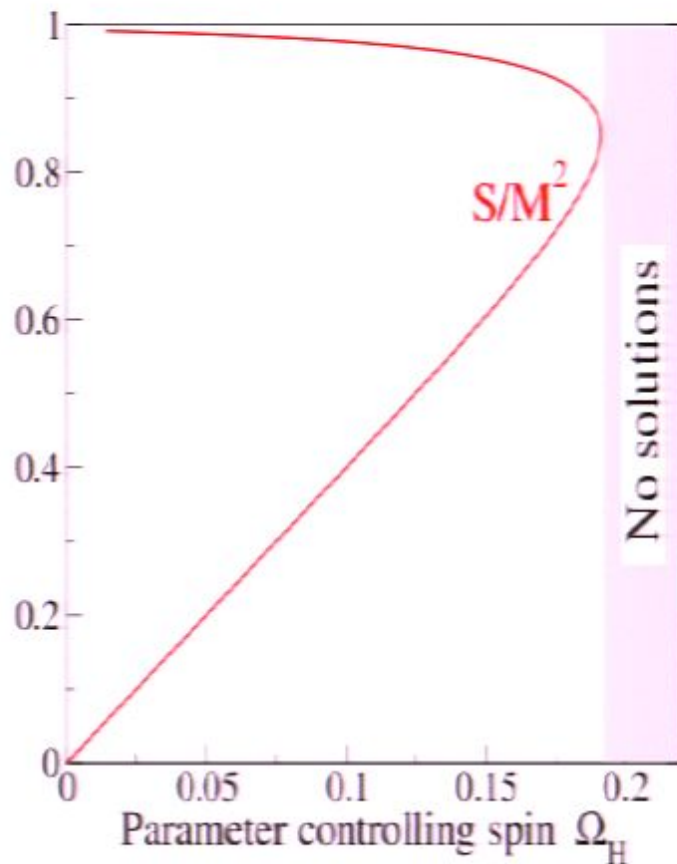
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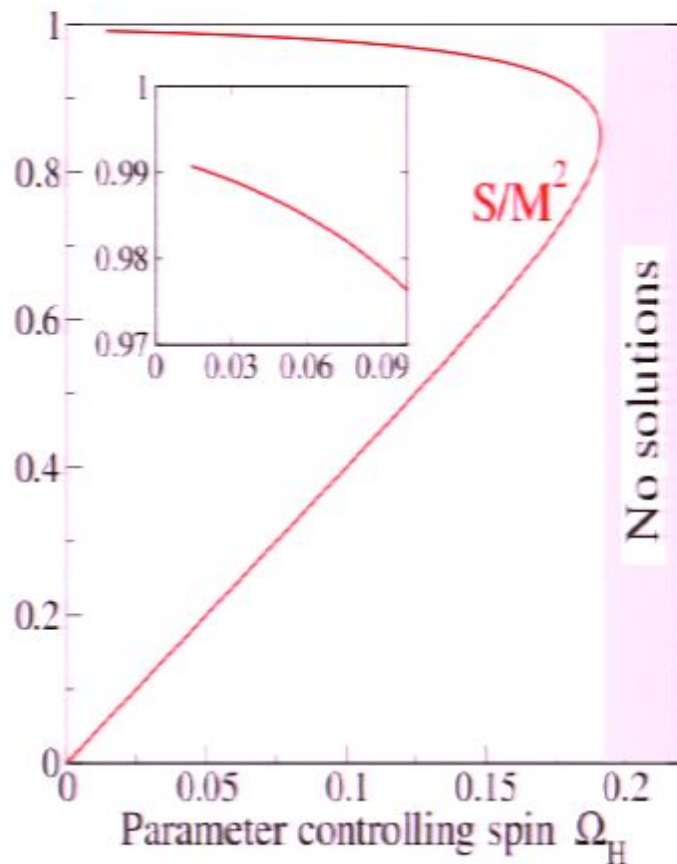
- Based on:
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- **Two** solutions for **same** Ω_H (HP, York, 2005).
- Novel discovery, previously always unique solution of constraints.
- With model-problem, behavior can be traced to a term which makes the linearized operator indefinite (Walsh, 2007; Baumgarte, O'Murchadha, HP, 2007.)

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- Novel discovery, previously always unique solution of constraints.
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- Maximal *initial* spin ~ 0.99 , but expect spin to relax to ~ 0.94 .

Third attempt

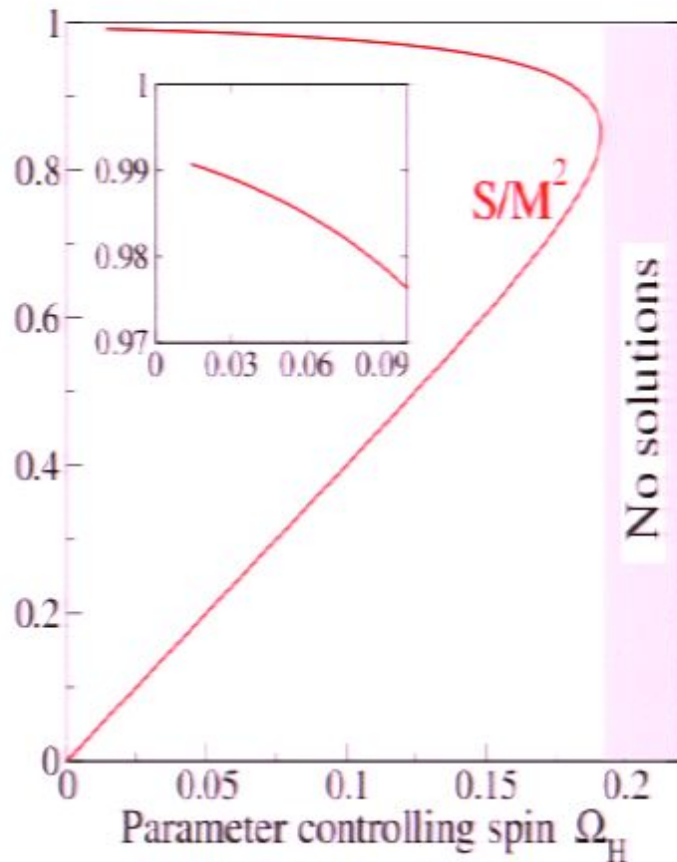
- Same quasi-equilibrium formalism as in 2nd attempt
- Adapted remaining choices:

$$\tilde{g}_{ij} = \delta_{ij} + \left(g_{ij}^{\text{Kerr,A}} - \delta_{ij} \right) + \left(g_{ij}^{\text{Kerr,B}} - \delta_{ij} \right)$$

$$K = K^{\text{Kerr,A}} + K^{\text{Kerr,B}}$$

Second attempt: Quasi-equilibrium initial data (Cook, HP, 2002-2006)

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 - 1) **time-independence** \Rightarrow Five coupled elliptic PDE's
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- With model-problem, behavior can be traced to a term which makes the linearized operator indefinite (Walsh, 2007; Baumgarte, O'Murchadha, HP, 2007.)
- Maximal *initial* spin ~ 0.99 , but expect spin to relax to ~ 0.94 .

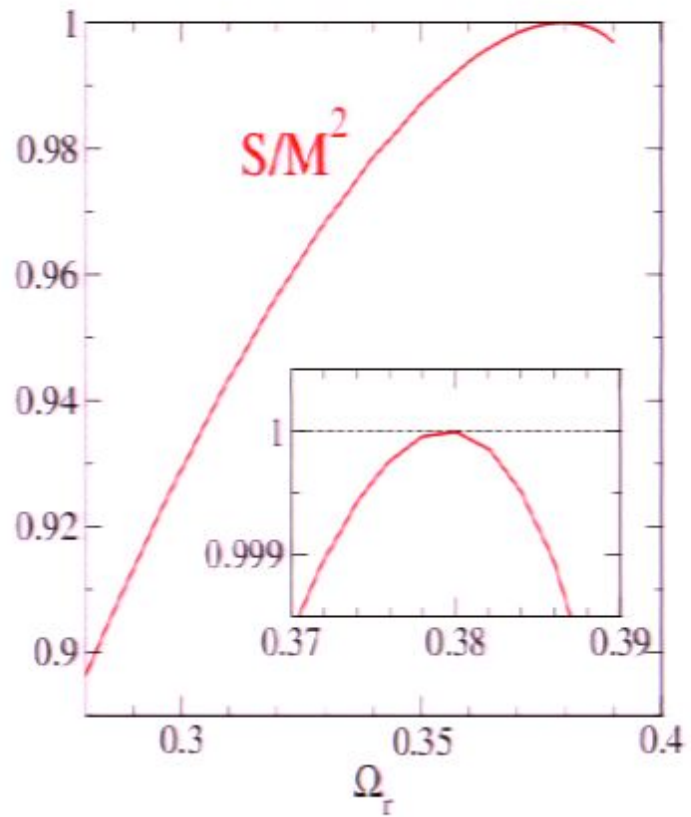
Third attempt

- Same quasi-equilibrium formalism as in 2nd attempt
- Adapted remaining choices:

$$\tilde{g}_{ij} = \delta_{ij} + \left(g_{ij}^{\text{Kerr,A}} - \delta_{ij} \right) + \left(g_{ij}^{\text{Kerr,B}} - \delta_{ij} \right)$$

$$K = K^{\text{Kerr,A}} + K^{\text{Kerr,B}}$$

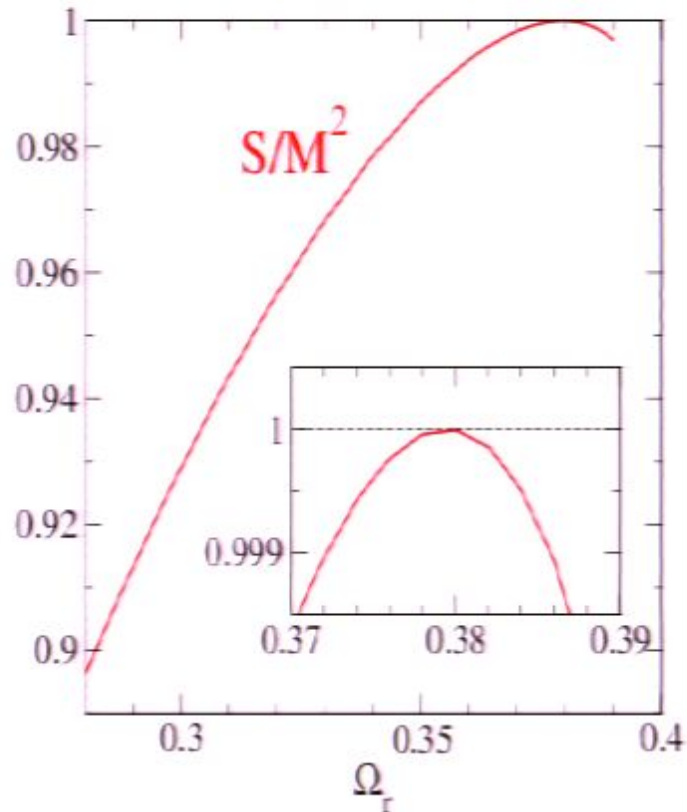
Third attempt



$$M^2 = M_{\text{irr}}^2 + \frac{S^2}{4M_{\text{irr}}^2}$$

$$M_{\text{irr}} = \sqrt{A_{\text{AH}}/16\pi}$$

Third attempt



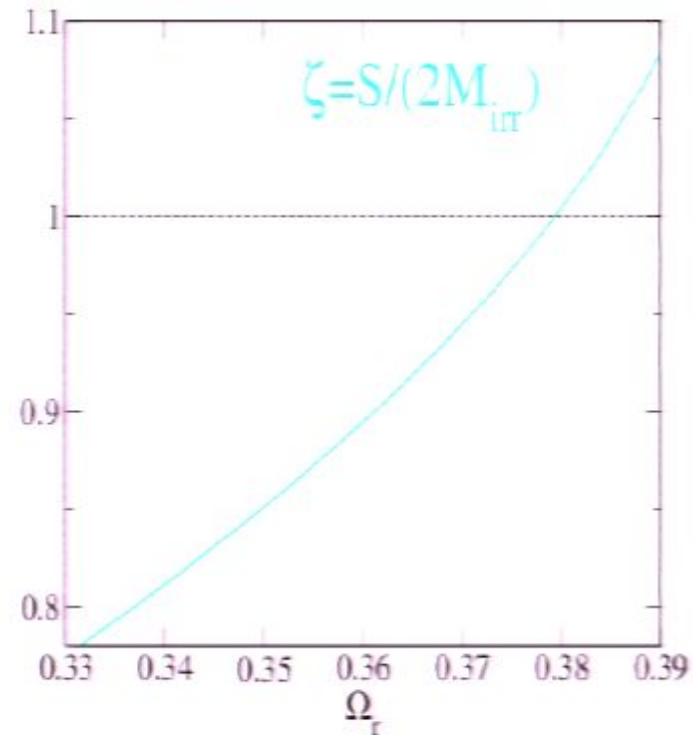
$$M^2 = M_{\text{irr}}^2 + \frac{S^2}{4M_{\text{irr}}^2}$$

$$M_{\text{irr}} = \sqrt{A_{\text{AH}}/16\pi}$$

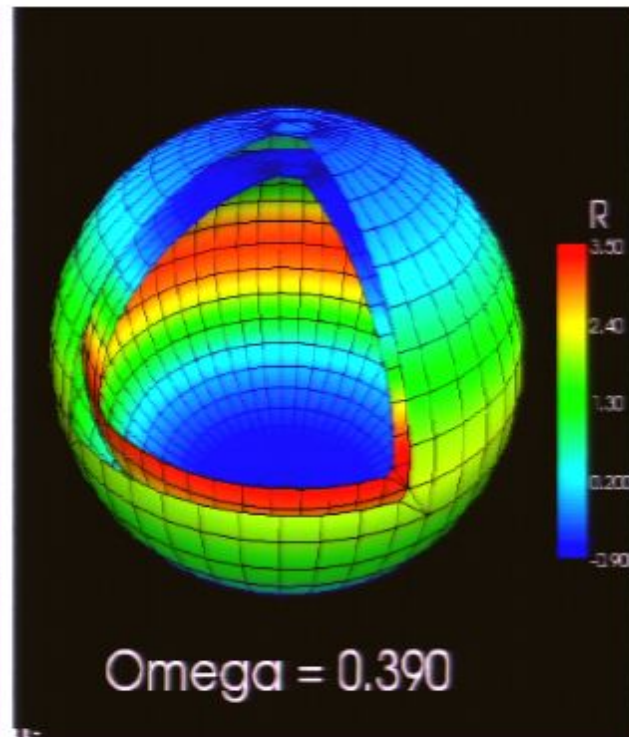
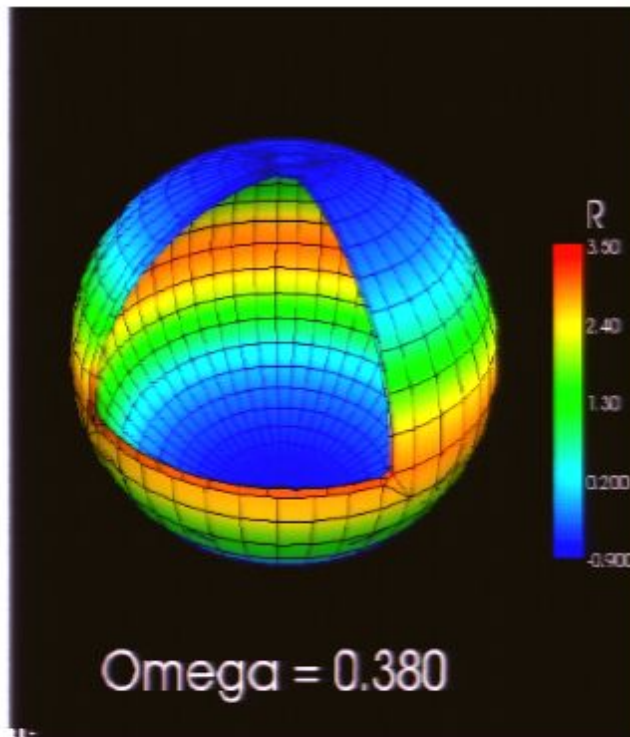
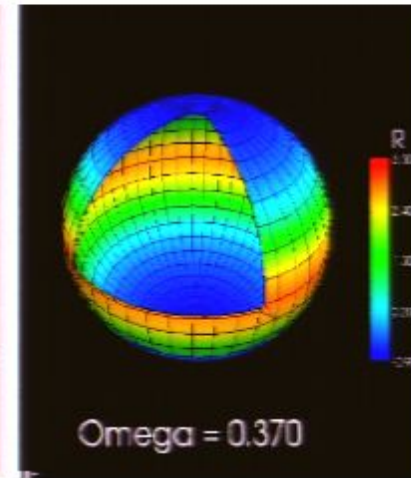
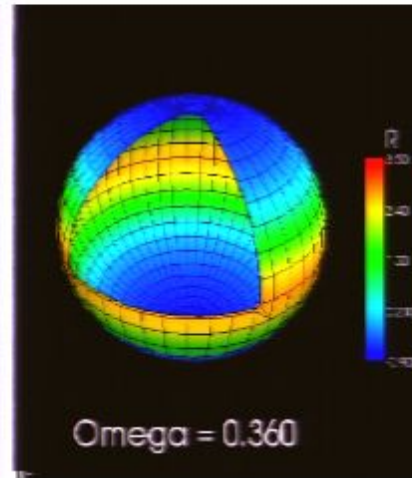
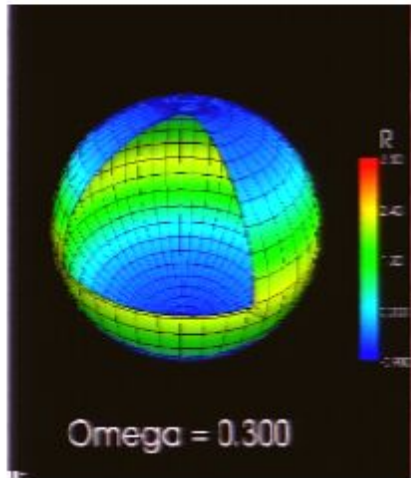
- Extremality parameter $\zeta = \frac{S}{2M_{\text{irr}}}$

$$\frac{S}{M^2} = 1 - \frac{(1 - \zeta)^2}{1 + \zeta^2} \leq 1$$

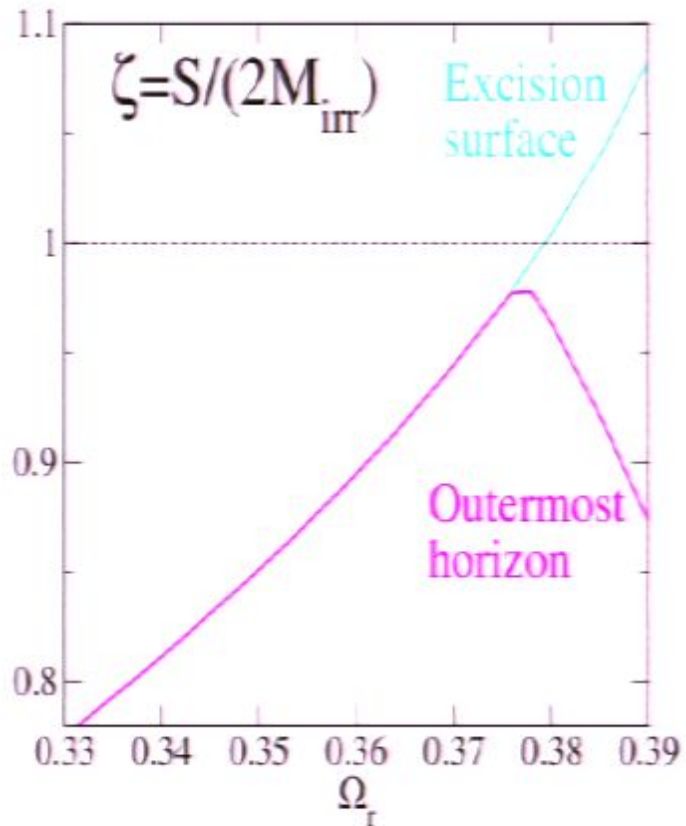
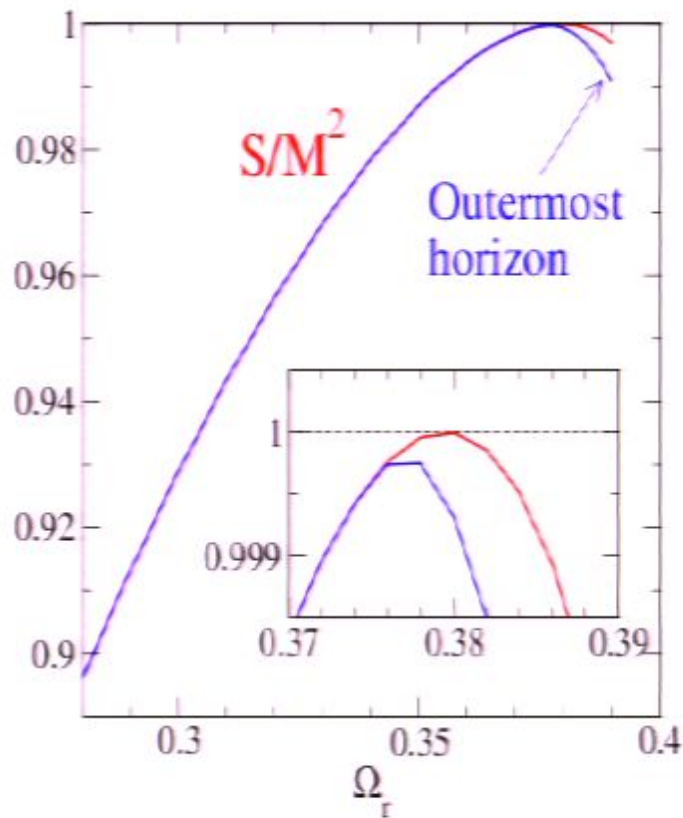
- Kerr: $\zeta \leq 1$ w/ equality for extremal BH



A new horizon outside the excision boundary



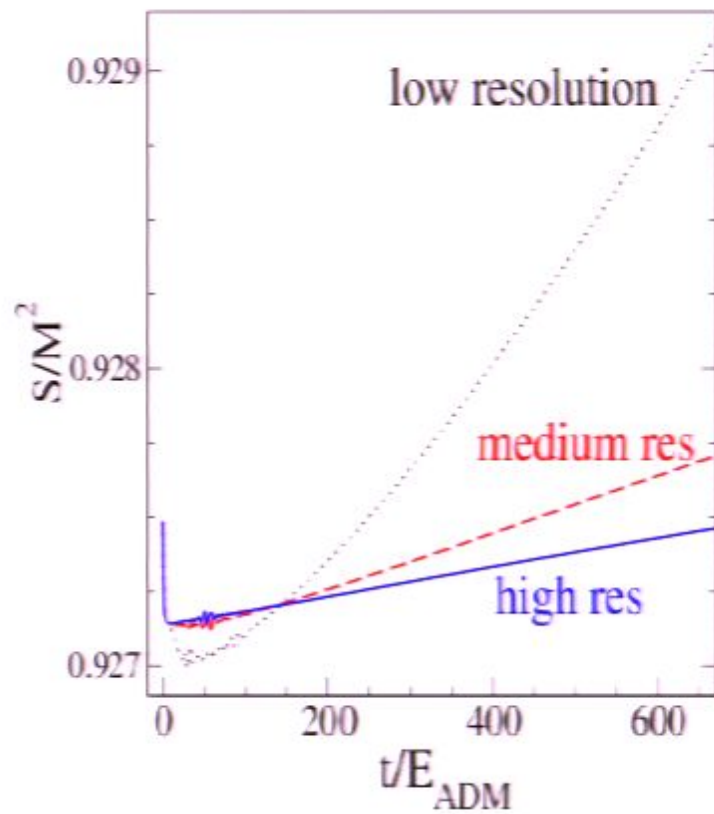
Super-extremal black holes?



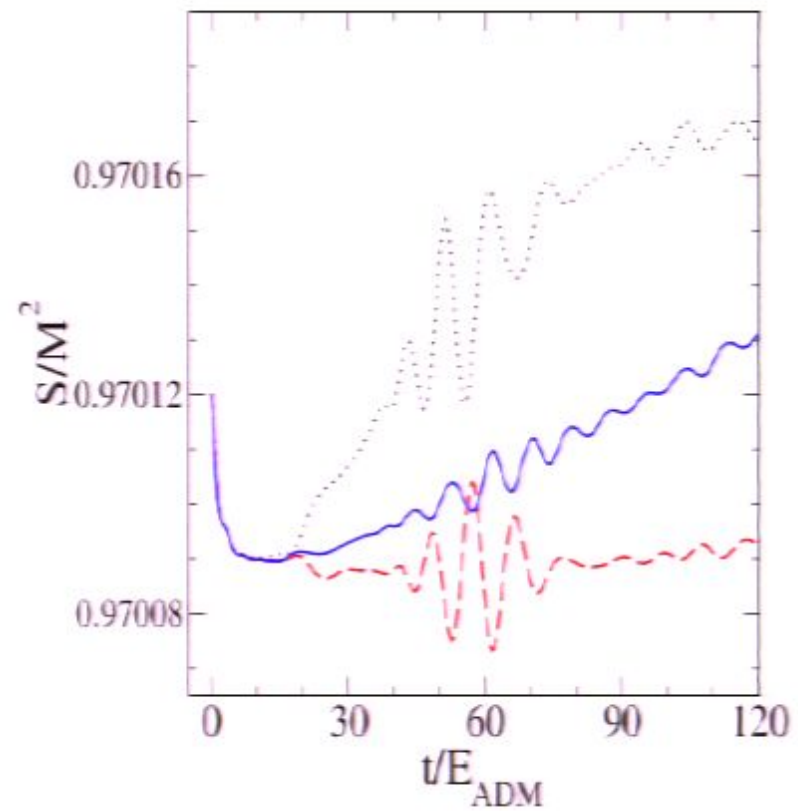
- No super-extremal black holes.
- But nearly-extremal spin, $S/M^2 \approx 0.9998$!!

Virtually no spin relaxation

Orbiting binary black hole



Head-On collision from rest



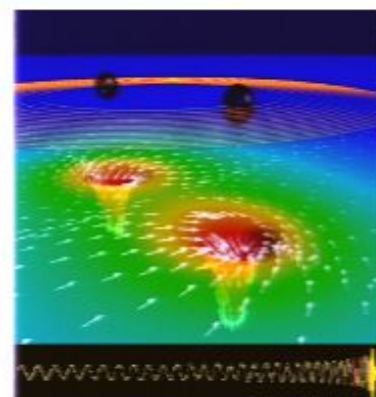
Summary

- GW-detectors require accurate **templates**



- **Spectral Einstein Code *SpEC***

- ▶ Elliptic & hyperbolic
- ▶ Efficient & flexible domain-decomposition
- ▶ 15 orbits & merger, $\delta\phi \lesssim \text{few} \times 10^{-2}$ radians



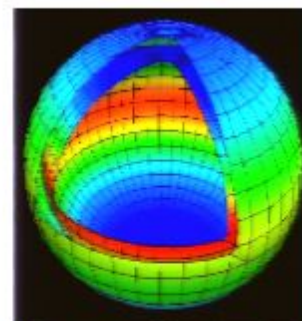
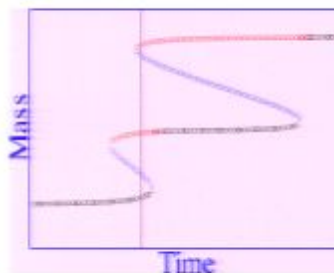
- **PN-NR comparison**

- ▶ Large PN-truncation error in last 20 GW-cycles
- ▶ Only simulations can find good PN approximants
- ▶ Non-equal masses, spins underway

- **BHs with multiple apparent horizons**

- **BBH with nearly-extremal spins**

- ▶ $S/M^2 = 0.9998$; plenty surprises



- **Collaborators:** Mike Boyle, Tony Chu, Lee Lindblom, Oliver Rinne, Mark Scheel (Caltech); Larry Kidder, Geoffrey Lovelace, Abdul Mroue, Rob Owen, Saul Teukolsky (Cornell); Duncan Brown (Syracuse), Greg Cook (Wake Forest)