Title: Entanglement Dynamics of Detectors in a Relativistic Quantum Field

Date: May 06, 2008 04:00 PM

URL: http://pirsa.org/08050034

Abstract:

Entanglement Dynamics of Detectors in a Relativistic Quantum Field

Shih-Yuin Lin
National Center for Theoretical Sciences, Taiwan
in collaboration with
Chung-Hsien Chou and Bei-Lok Hu
[arXiv: 0803.3995]

May 6, 2008 @ PI

Outline

I. Introduction

II. The Model: Unruh-DeWitt Detector Theory

III. Results and Discussion

IV. Summary

I. Introduction

- Quantum entanglement plays a crucial role in EPR paradox, violation of Bell's inequality, quantum teleportation, etc.
- Entanglement states are essentially "non-local" (cannot be modeled by any local hidden-variable theory.)
- Entanglement entirely departs from classical line of thought.
- Measure of entanglement for mixed states is known only for limited cases.

- We lack in experience and intuition on entanglement.

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0,\ \gamma^2 f(t)\}$$

$C > 0$: entangled.

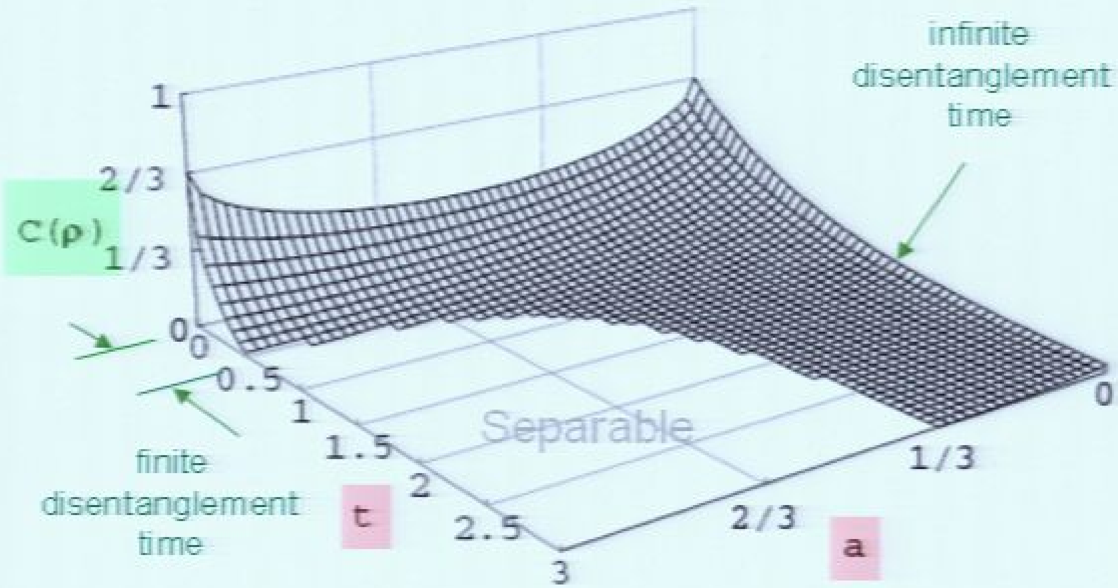
Initial state:

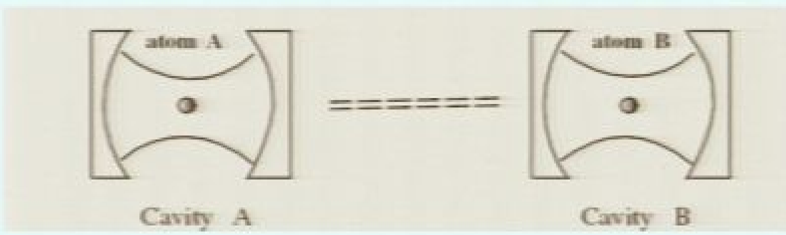
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

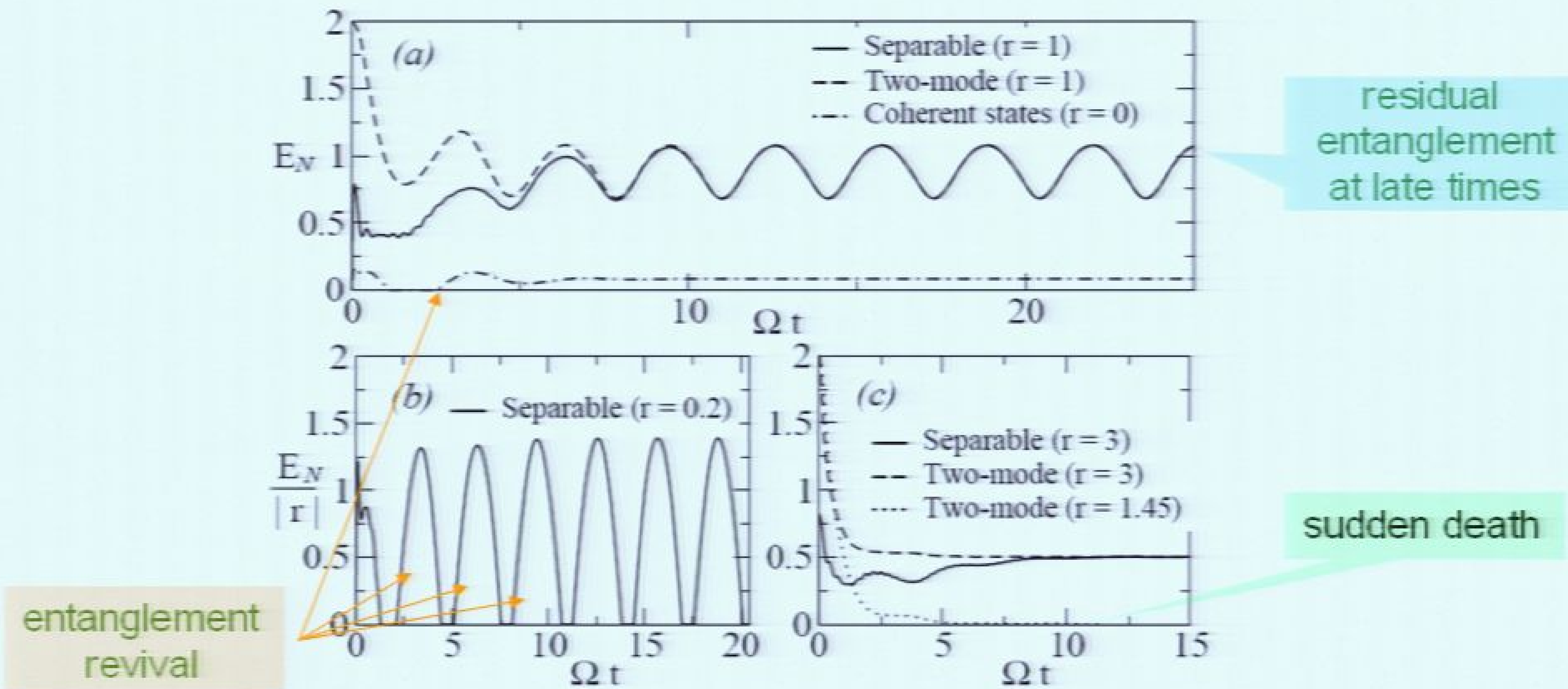
Two independent environments (cavities)

I. Introduction

- Residual entanglement, entanglement revival

Ex: 2 harmonic Oscillators located at a point in space in a quantum field

[Paz, Roncaglia, arXiv:0801.0464]



I. Introduction

- Entanglement generation (creation) [Braun 2002]

Ex: Qubits interacting separately in space, coupled with a spin chain

[Lai, Hung, Mou, Chen, arXiv:0803.0364]

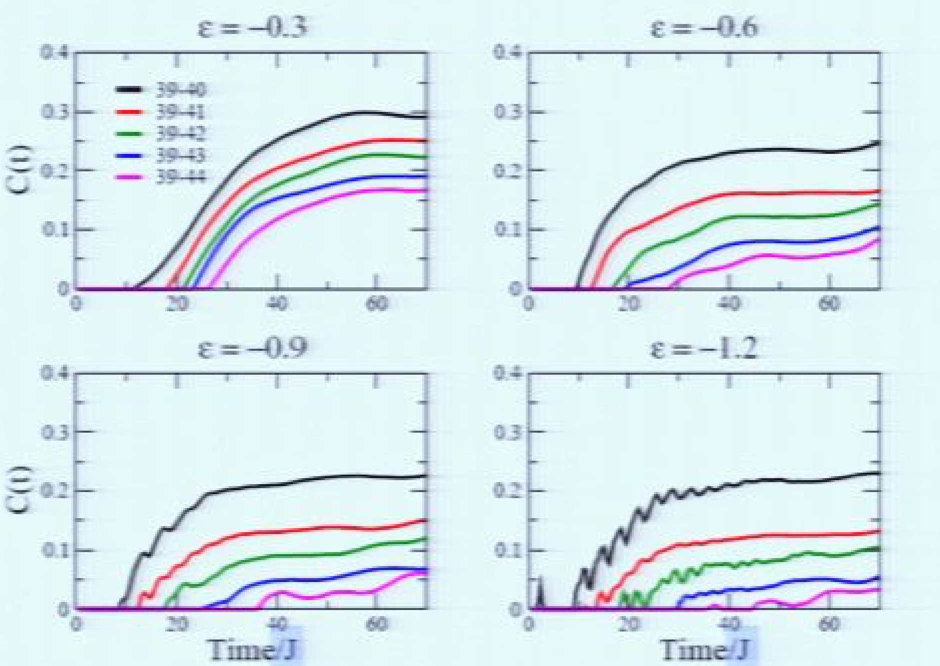


FIG. 7: Entanglement dynamics for an initially disentangled pair of qubits for the case of Heisenberg coupling. Here $\Delta = 0$ and $N = 80$. $(\epsilon_i^x = \epsilon_i^y = \epsilon_i^z \neq 0)$

"XY model"

$$\mathcal{H}_{bath} = J \sum (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z)$$

XXZ Heisenberg model

$$H_{int} = \sum_{i,\alpha} \epsilon_i^\alpha s_{A(B)}^\alpha S_i^\alpha$$

$$\epsilon_1 = \epsilon_2$$

Initial state (separable)

$$\frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

I. Introduction

Detectors/atoms in a relativistic quantum field

Issues

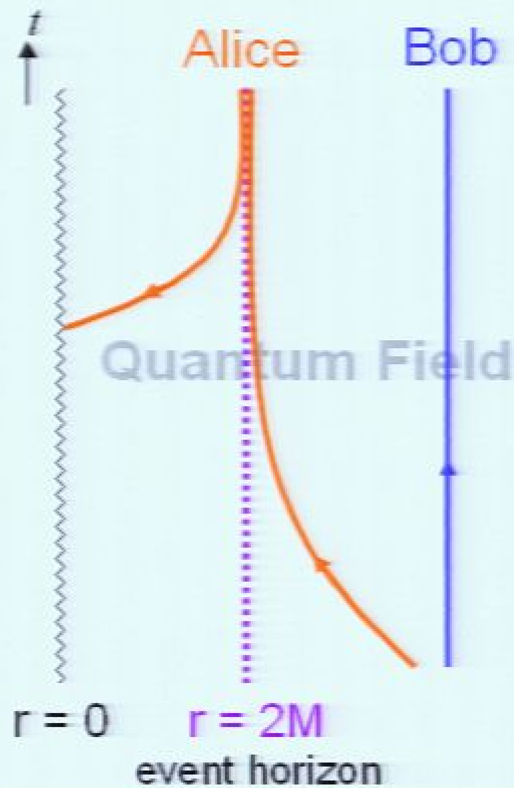
- Field states
  - infinite degrees of freedom
  - back-reaction/Interplay to the field
  - causality, retardation of mutual influences
  - spatial separation of atoms
  - Non-Markovian regime
- Coordinate (time-slicing) dependence,
  - time dilation
  - Accelerated detectors, Unruh effect
  - role of event horizon
- Entanglement dynamics of atoms
  - generation, revival, sudden death of entanglement and residual entanglement

I. Introduction

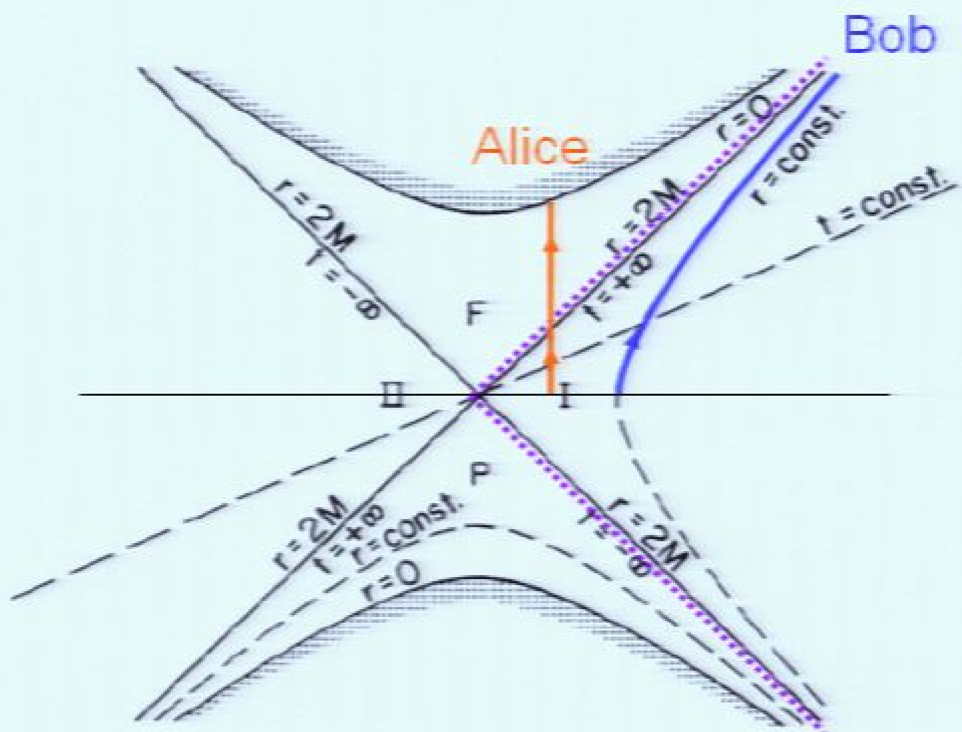
Entanglement across the event horizon

Schwarzschild black hole

Schwarzschild coordinate



Kruskal coordinate



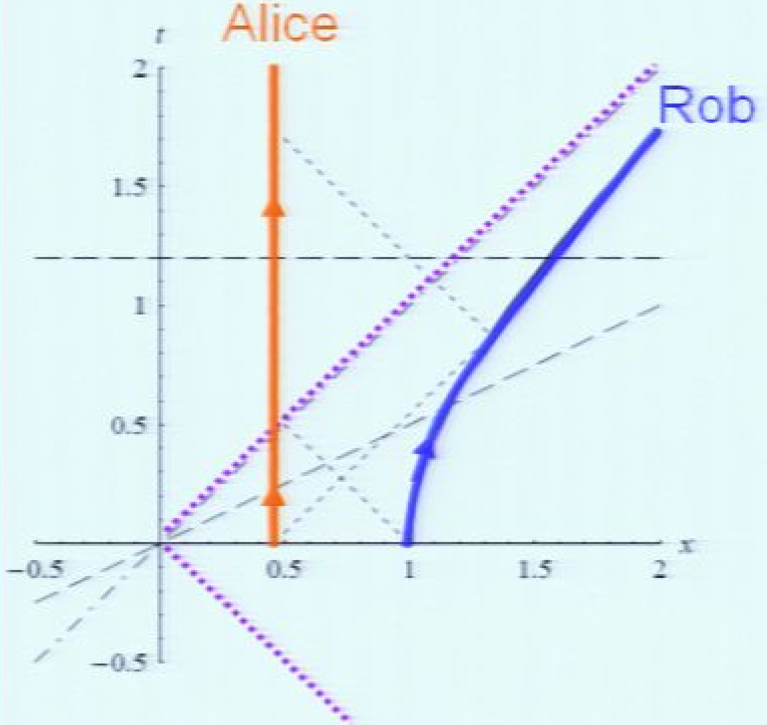
I. Introduction

Entanglement across the event horizon

Minkowski Space

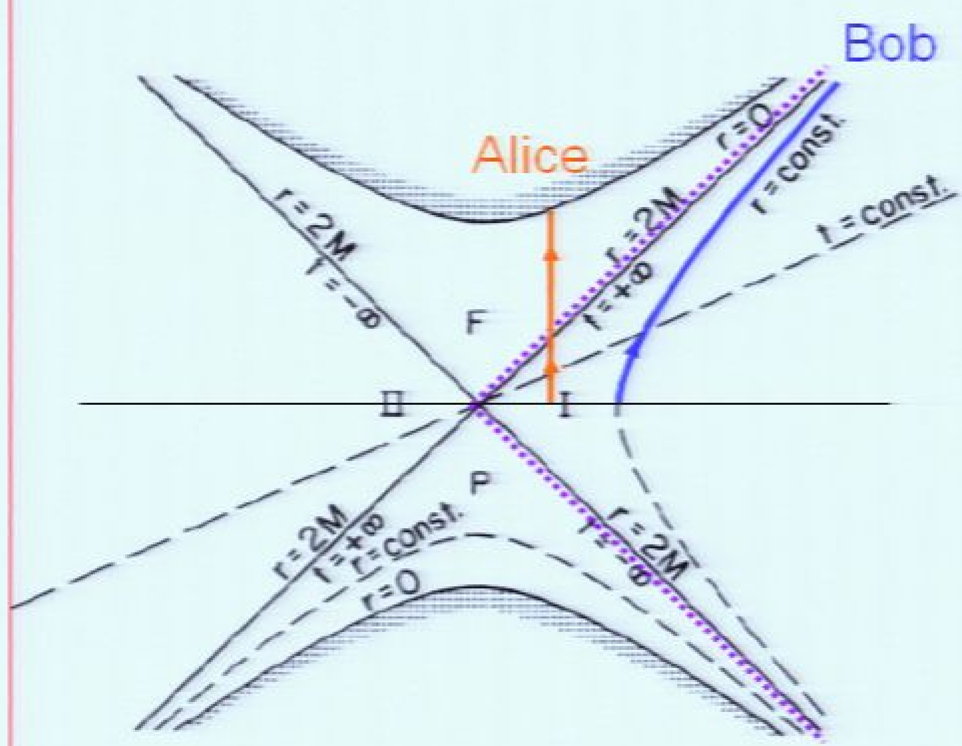
[Alsing and Milburn 2003]

[Fuentes-Schuller and Mann 2005]



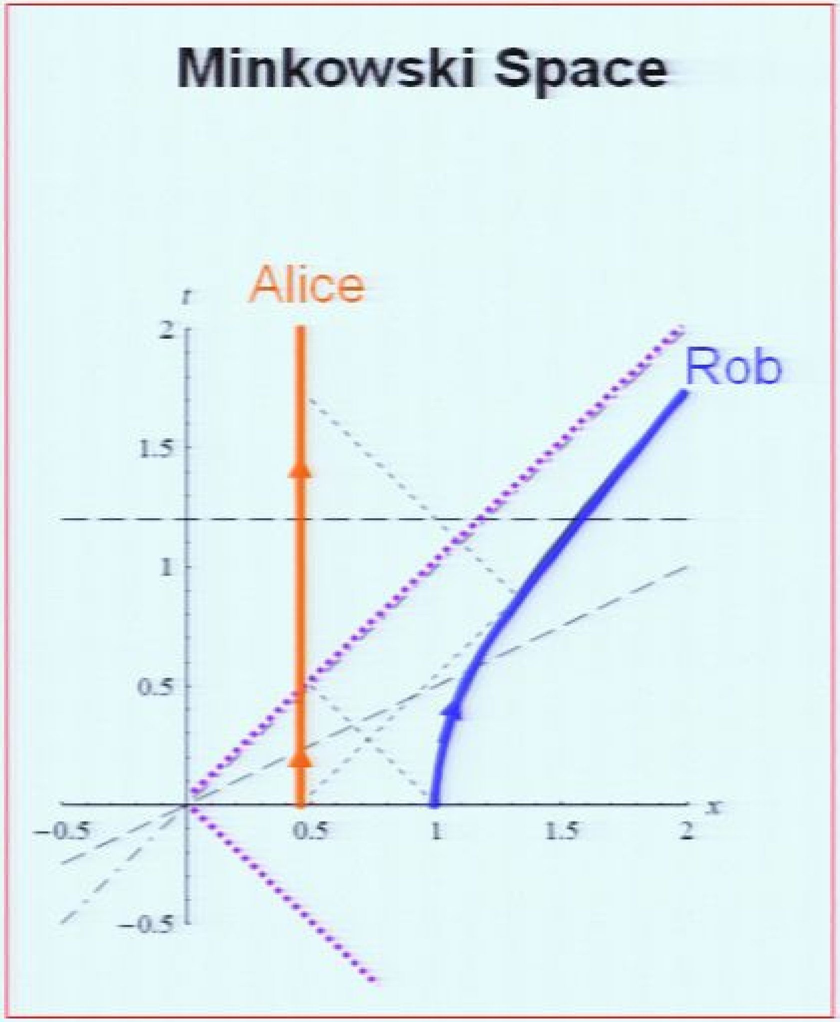
Schwarzschild black hole

Kruskal coordinate



I. Introduction

Entanglement across the event horizon



[Alsing and Milburn PRL91, 180404(2003)]:

"...our result suggests that quantum entanglement is degraded in non-inertial frames."

(See also: Comment by Schützhold and Unruh, [quant-ph/0506028].)

[Fuentes-Schuller and Mann PRL95, 120404(2005)]]:

"...a state which is maximally entangled in an inertial frame becomes less entangled if the observers are relatively accelerated....which is a consequence of the Unruh effect ..."

Unruh effect : a detector uniformly accelerated in Minkowski vacuum will experience a thermal bath at Unruh temperature $T = a/2\pi$.

II. The Model

II. The Model

- 2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space

$$S = -\int d^4x \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi$$ - massless scalar field

$$+\int d\tau_A \frac{1}{2}\left[(\partial_{\tau_A}Q_A)^2 - \Omega_0^2 Q_A^2\right] + \int d\tau_B \frac{1}{2}\left[(\partial_{\tau_B}Q_B)^2 - \Omega_0^2 Q_B^2\right]$$ - internal: HO

$$+\lambda_0 \int d^4x \Phi(x)\left[\int d\tau_A Q_A(\tau_A)\delta^4\left(x^\mu - z_A^\mu(\tau_A)\right) + \int d\tau_B Q_B(\tau_B)\delta^4\left(x^\mu - z_B^\mu(\tau_B)\right)\right]$$

- bilinear interaction [DeWitt 1979]
  Detectors A, B are point-like objects.

cf. 2 HO Quantum Brownian Motion [Chou, Yu, Hu 2007; Paz, Roncaglia 2008]

$$H_{\rm tot} = H_{\rm sys} + H_{\rm bath} + H_{\rm int}$$

$$H_{\rm sys} = \frac{P_1^2}{2M} + \frac{1}{2}M\Omega^2 x_1^2 + \frac{P_2^2}{2M} + \frac{1}{2}M\Omega^2 x_2^2 + \kappa(x_1 - x_2)^k$$

$$H_{\rm bath} = \sum_{n=1}^{N_B}\left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 q_n^2\right) \qquad H_{\rm int} = (x_1 + x_2)\sum_{n=1}^{N_B} C_n q_n$$

~ 2 inertial HOs at the same space-point

$(\, x_i \sim Q_i,\ q_n \sim \phi_k,\ C_n \sim -\lambda_0\, e^{ikz}$ with $a = \kappa = 0,\ M = 1.)$

II. The Model

- **Motion of Detectors**

Q_A : at rest, $(b>2a)$

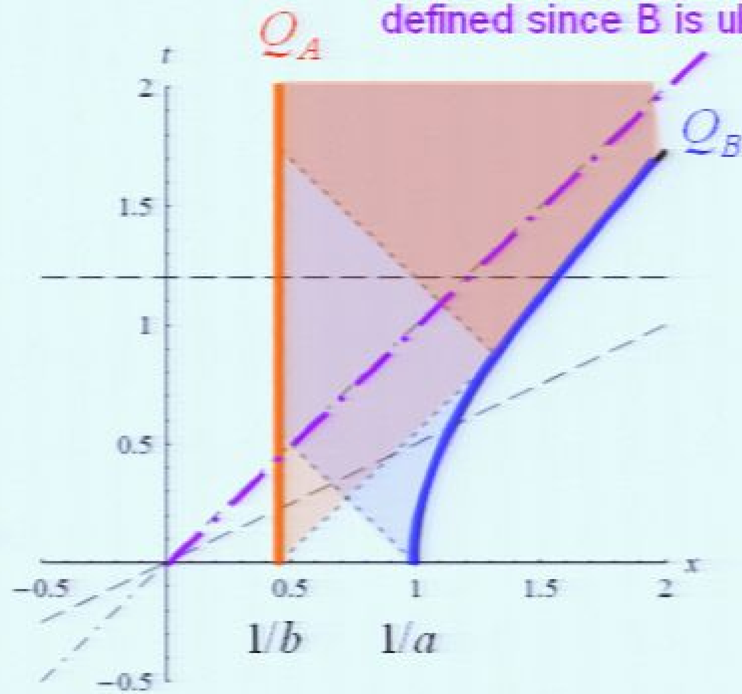
$$z_A^\mu(t) = (t, 1/b, 0, 0)$$

Q_B : uniformaly accelerated,

$$z_B^\mu(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

a: proper acceleration

Event horizon for B can be clear defined since B is ultra-localized



- **Initial state at $t = \tau = 0$,**

$$|\,\psi(0)\,\rangle = |\,q_A, q_B\,\rangle \otimes |\,0_M\,\rangle$$

$|\,0_M\,\rangle$: Minkowski vacuum

$|\,q_A, q_B\,\rangle$ ~ two-mode squeezed state, represented by Wigner function,

$$W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$$

$|\,\psi(0)\,\rangle$ is a Gaussian state!

II. The Model

- 2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space

$$S = -\int d^4x \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi$$ - massless scalar field

$$+\int d\tau_A \frac{1}{2}\left[(\partial_{\tau_A} Q_A)^2 - \Omega_0^2 Q_A^2\right] + \int d\tau_B \frac{1}{2}\left[(\partial_{\tau_B} Q_B)^2 - \Omega_0^2 Q_B^2\right]$$ - internal: HO

$$+\lambda_0 \int d^4x \Phi(x) \left[\int d\tau_A Q_A(\tau_A) \delta^4\left(x^\mu - z_A^\mu(\tau_A)\right) + \int d\tau_B Q_B(\tau_B) \delta^4\left(x^\mu - z_B^\mu(\tau_B)\right)\right]$$

- bilinear interaction [DeWitt 1979]
Detectors A, B are point-like objects.

cf. 2 HO Quantum Brownian Motion [Chou, Yu, Hu 2007; Paz, Roncaglia 2008]

$$H_{\rm tot} = H_{\rm sys} + H_{\rm bath} + H_{\rm int}$$

$$H_{\rm sys} = \frac{P_1^2}{2M} + \frac{1}{2}M\Omega^2 x_1^2 + \frac{P_2^2}{2M} + \frac{1}{2}M\Omega^2 x_2^2 + \kappa(x_1 - x_2)^k$$

$$H_{\rm bath} = \sum_{n=1}^{N_B}\left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 q_n^2\right) \qquad H_{\rm int} = (x_1 + x_2)\sum_{n=1}^{N_B} C_n q_n$$

~ 2 inertial HOs at the same space-point

$(x_i \sim Q_i,\ q_n \sim \phi_{\mathbf{k}},\ C_n \sim -\lambda_0\, e^{i k z}$ with $a = \kappa = 0,\ M = 1.)$

II. The Model

- **Motion of Detectors**

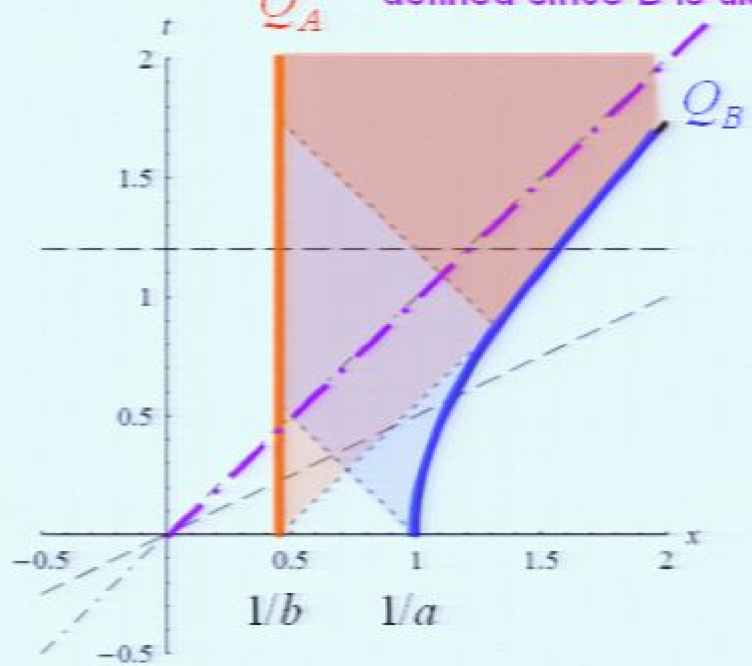
Q_A : at rest, $(b>2a)$

$$z_A^\mu(t) = (t, 1/b, 0, 0)$$

Q_B : uniformaly accelerated,

$$z_B^\mu(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

a: proper acceleration



- **Initial state at $t = \tau = 0$,**

$$|\,\psi(0)\,\rangle = |\,q_A, q_B\,\rangle \otimes |\,0_M\,\rangle$$

$|\,0_M\,\rangle$: Minkowski vacuum

$|\,q_A, q_B\,\rangle$ ~ two-mode squeezed state, represented by Wigner function,

$$W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$$

$|\,\psi(0)\,\rangle$ is a Gaussian state!

II. The Model

Criterion for separability/entanglement of A and B:

Define $$\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$$

$$\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

For Gaussian states, $\mathbf{\Sigma} < 0$ ⟺ entangled, otherwise separable [Simon 2000]

$\mathbf{V}^{PT} = \mathbf{\Lambda V \Lambda}$: Partial Transposition of $\mathbf{V}$, $\mathbf{\Lambda} = \mathrm{diag}(1, 1, 1, -1)$

$V_{\mu\nu}(t,\tau) = \langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle \equiv \frac{1}{2}\langle (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu) \rangle$: 10 symmetric two-point functions (variances) of two detectors.

$\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$

Note: This criterion is testing the property of the PT Wigner functions, thus the reduced density matrix, of the detectors:

$$\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k; \tau]$$

So t and τ in $\mathbf{\Sigma}$ must be on the same time-slice as the field's.

II. The Model

Criterion for separability/entanglement of A and B:

Define
$$\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$$

$$\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

For Gaussian states, $\Sigma < 0$ ⟺ entangled, otherwise separable [Simon 2000]

$\mathbf{V}^{PT} = \Lambda \mathbf{V} \Lambda$: Partial Transposition of $\mathbf{V}$, $\Lambda = \mathrm{diag}(1,1,1,-1)$

$V_{\mu\nu}(t,\tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu)\,\rangle$: 10 symmetric two-point functions (variances) of two detectors.

$\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$

Note: This criterion is testing the property of the PT Wigner functions, thus the reduced density matrix, of the detectors:

$$\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k; \tau]$$

So t and τ in Σ must be on the same time-slice as the field's.

Here, behavior of Σ ~ (logarithm) negativity.

II. The Model

Sketch of calculation

- Evolution of operators $Q_A, P_A, Q_B, P_B, \Phi, \Pi$ in Heisenberg picture.

$$\hat{Q}_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_r}}\sum_j \left[q_i^{(j)}(\tau_i)\hat{a}_j + q_i^{(j)*}(\tau_i)\hat{a}_j^\dagger\right] + \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[q_i^{(+)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k} + q_i^{(-)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k}^\dagger\right]$$

damped HO   damped driven HO

- Sandwiched by the initial state: 10 symmetric two-point functions

$$V_{\mu\nu}(t,\tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu\mathcal{R}_\nu + \mathcal{R}_\nu\mathcal{R}_\mu)\,\rangle \qquad \mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

- Partial Transposition: $\mathbf{V}^{PT} = \mathbf{\Lambda}\mathbf{V}\mathbf{\Lambda}$ $\mathbf{\Lambda} = \mathrm{diag}(1,1,1,-1)$

The quantity $\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$

$$\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

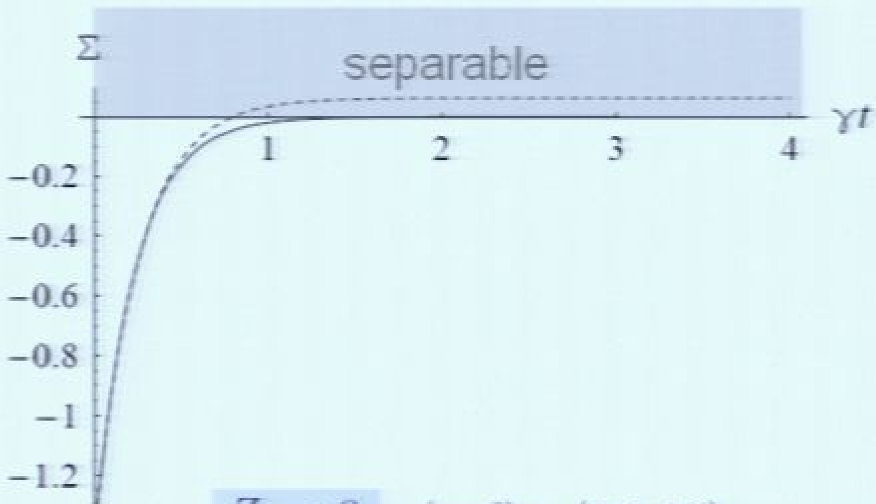
III. Results and Discussion

a. Sudden death of entanglement

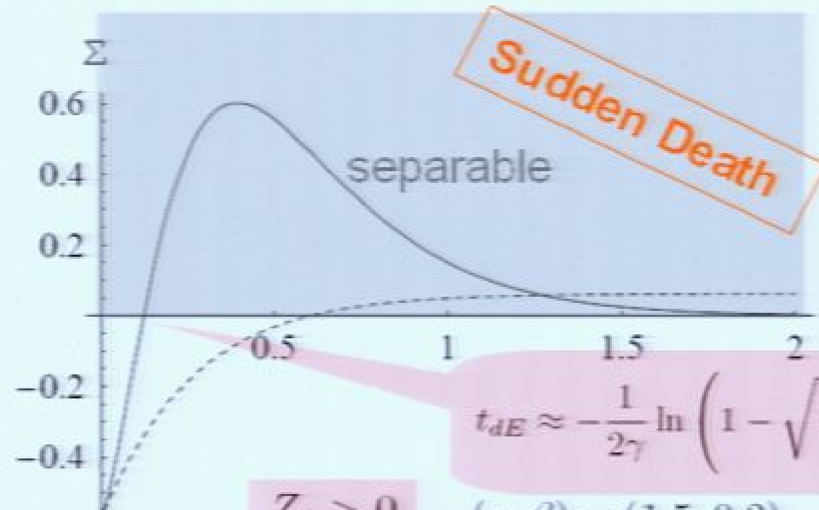
III. Results and Discussion

$\gamma \equiv \lambda_0^2/8\pi$

- Weak coupling, both at rest but well separated $\quad \Omega \gg \gamma\Lambda_1 \gg a \to 0$



$Z_4 < 0 \quad (\alpha, \beta) = (1.1, 4.5)$



$$t_{dE} \approx -\frac{1}{2\gamma} \ln\left(1 - \sqrt{1 - \frac{Z_4}{Z_8}}\right)$$

$Z_4 > 0 \quad (\alpha, \beta) = (1.5, 0.2)$

$\gamma = 10^{-5} \quad \Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right]$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2\left(\beta^2 - \hbar\Omega\right)^2,$$
$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2 + \alpha^2\Omega^2\right)\left(\hbar^2 + \alpha^2\beta^2\right).$$

Initially $\quad W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A + Q_B)^2 + \frac{1}{\alpha^2}(Q_A - Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A - P_B)^2 + \frac{1}{\beta^2}(P_A + P_B)^2\right]$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0,\ \gamma^2 f(t)\}$$

$C > 0$: entangled.

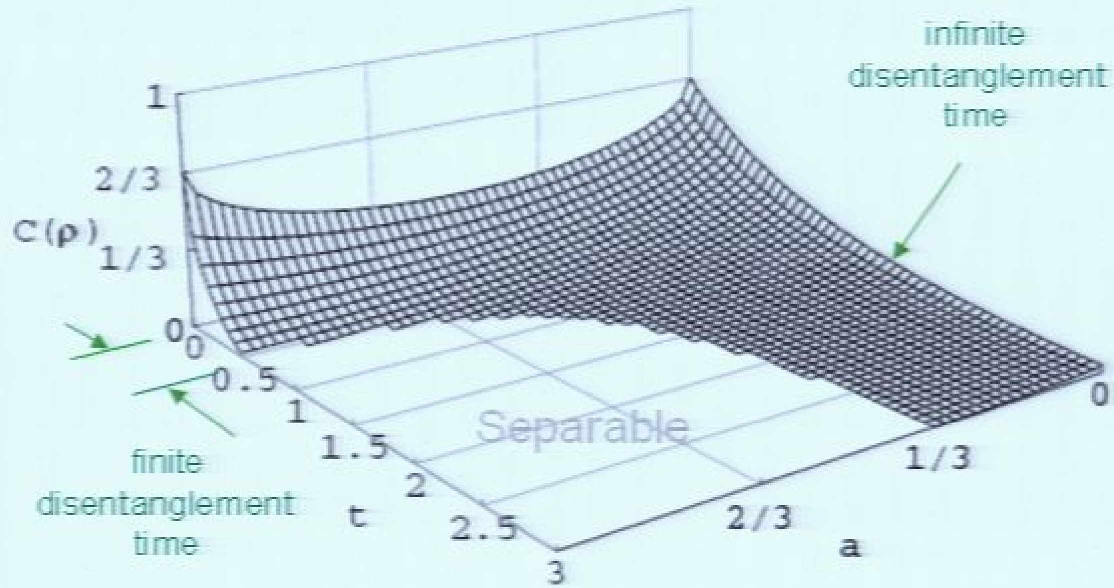
Initial state:

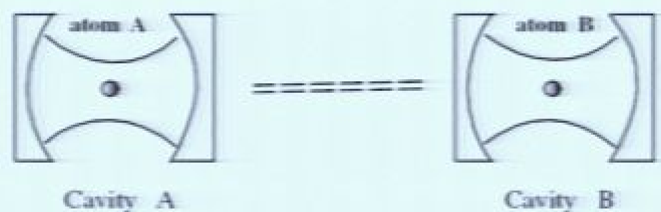
$$\rho_{\rm in} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





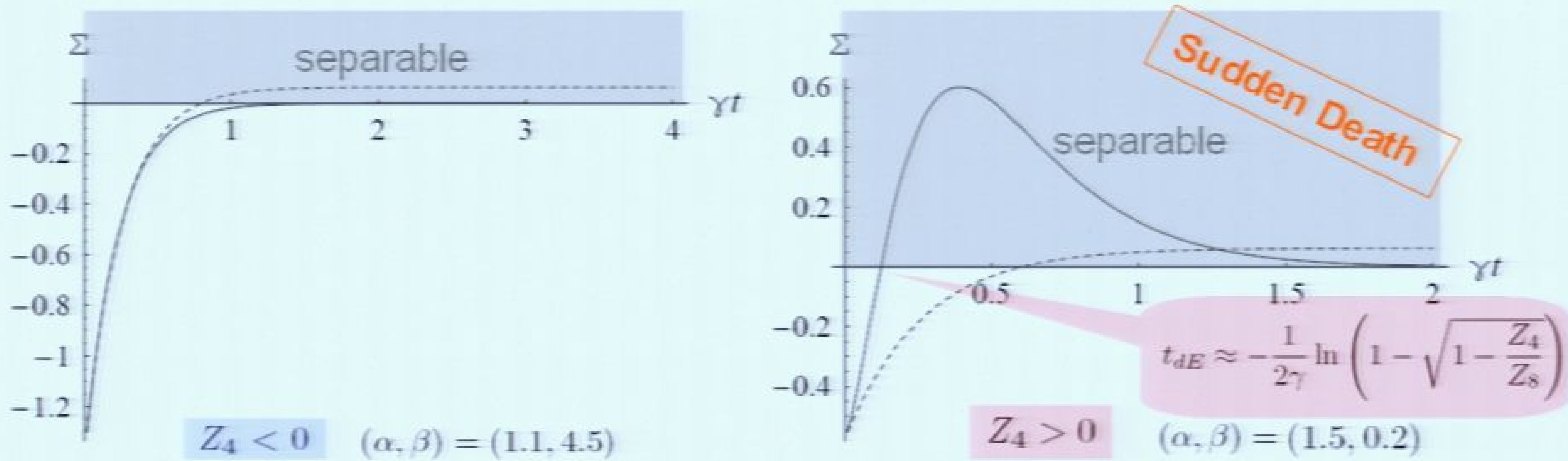
In Markovian regime

Two independent environments (cavities)

III. Results and Discussion

$\gamma \equiv \lambda_0^2/8\pi$

- Weak coupling, both at rest but well separated $\Omega \gg \gamma\Lambda_1 \gg a \to 0$



$\gamma = 10^{-5}$ $\Lambda_0 = \Lambda_1 = 100$, $\Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right]$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2\left(\beta^2 - \hbar\Omega\right)^2,$$
$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2 + \alpha^2\Omega^2\right)\left(\hbar^2 + \alpha^2\beta^2\right).$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A + Q_B)^2 + \frac{1}{\alpha^2}(Q_A - Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A - P_B)^2 + \frac{1}{\beta^2}(P_A + P_B)^2\right]$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

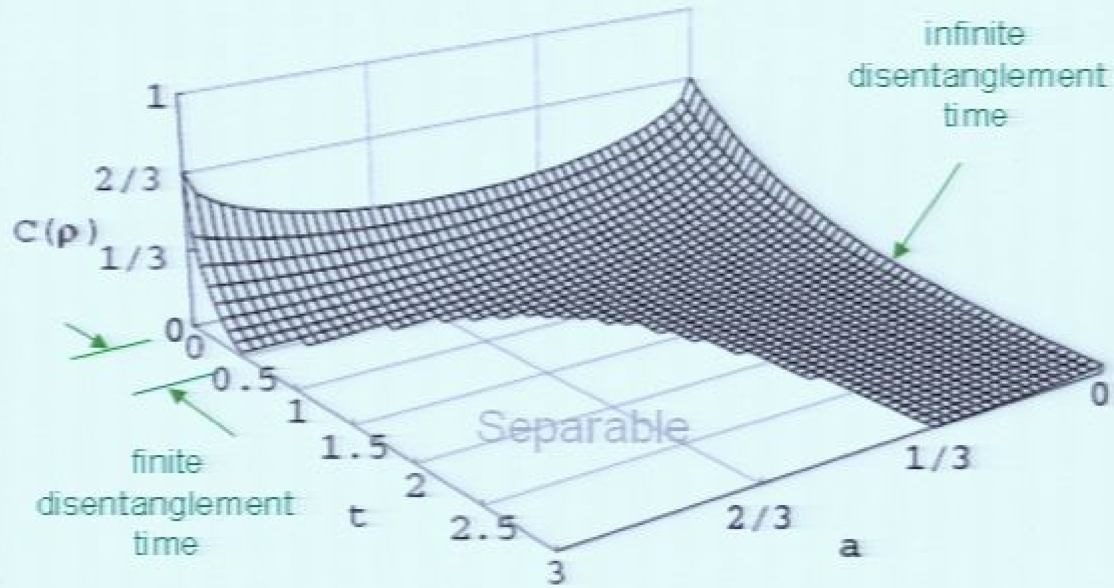
Initial state:

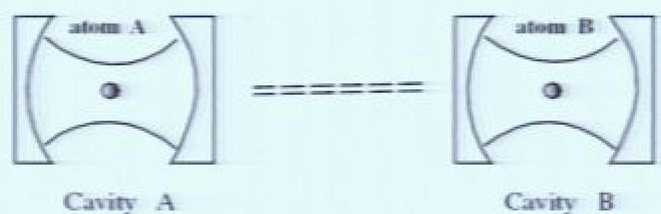
$$\rho_{\rm in} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





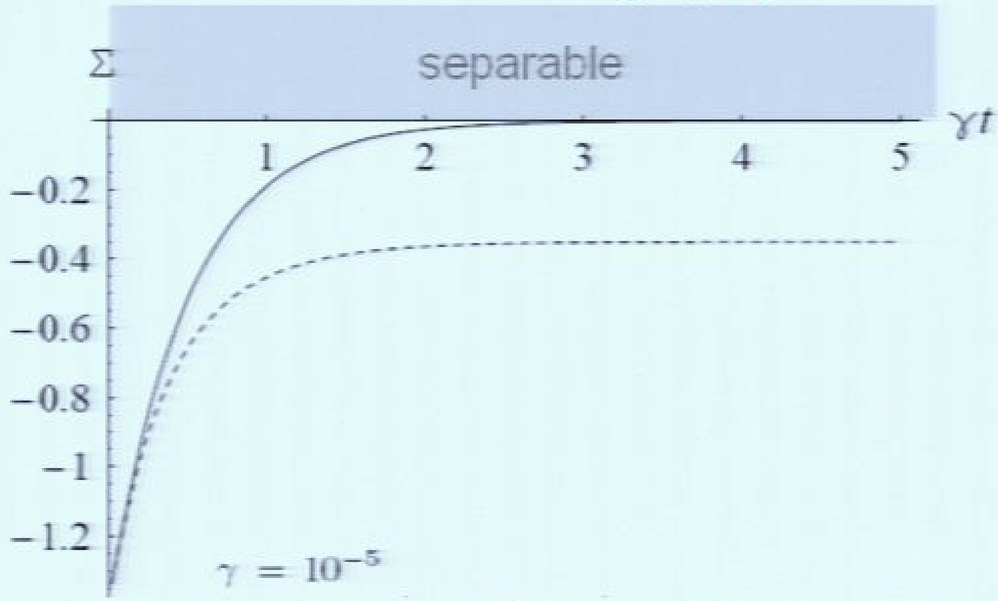
In Markovian regime

Two independent environments (cavities)

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$ $\boxed{\gamma \equiv \lambda_0^2/8\pi}$

Λ_1 ~ proper time resolution of detectors A and B



$\gamma = 10^{-5}$
$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$
$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

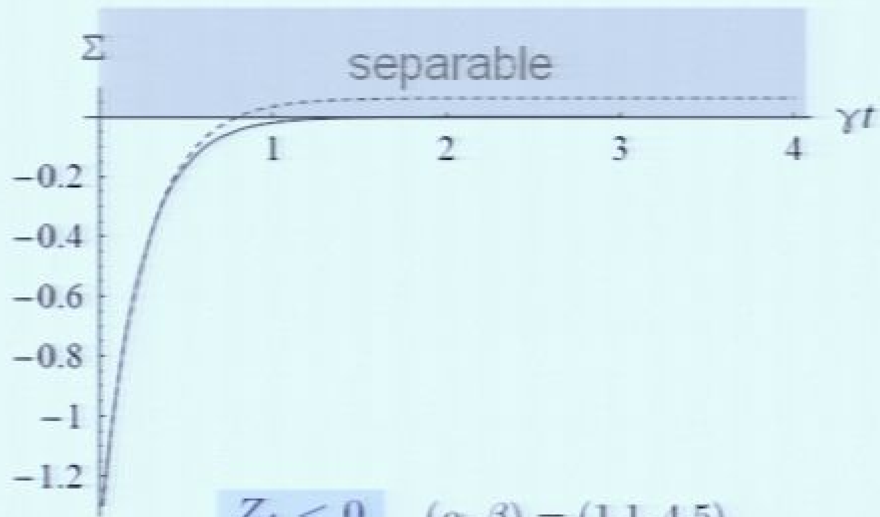
$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t}$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

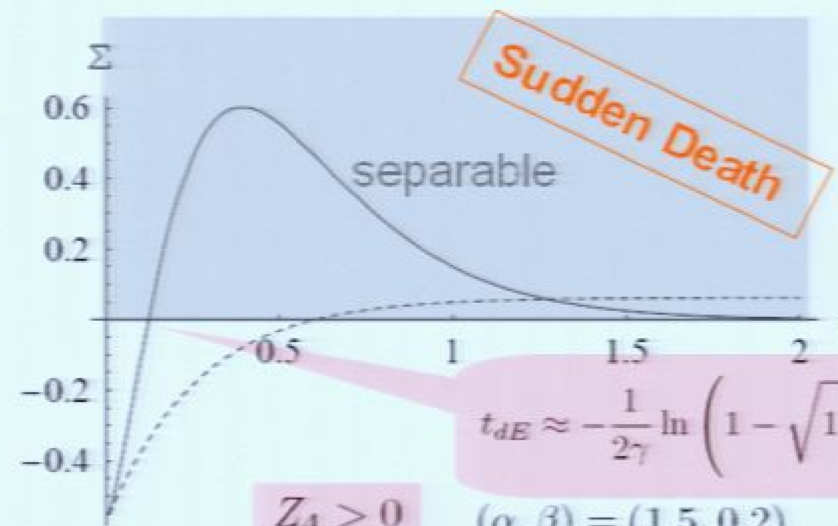
III. Results and Discussion

$\gamma \equiv \lambda_0^2/8\pi$

- Weak coupling, both at rest but well separated $\quad \Omega \gg \gamma\Lambda_1 \gg a \to 0$



$Z_4 < 0 \quad (\alpha, \beta) = (1.1, 4.5)$



$$t_{dE} \approx -\frac{1}{2\gamma} \ln\left(1 - \sqrt{1 - \frac{Z_4}{Z_8}}\right)$$

$Z_4 > 0 \quad (\alpha, \beta) = (1.5, 0.2)$

$\gamma = 10^{-5} \quad \Lambda_0 = \Lambda_1 = 100, \ \Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right]$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2\left(\beta^2 - \hbar\Omega\right)^2.$$

$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2 + \alpha^2\Omega^2\right)\left(\hbar^2 + \alpha^2\beta^2\right).$$

Initially $\quad W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A + Q_B)^2 + \frac{1}{\alpha^2}(Q_A - Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A - P_B)^2 + \frac{1}{\beta^2}(P_A + P_B)^2\right]$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

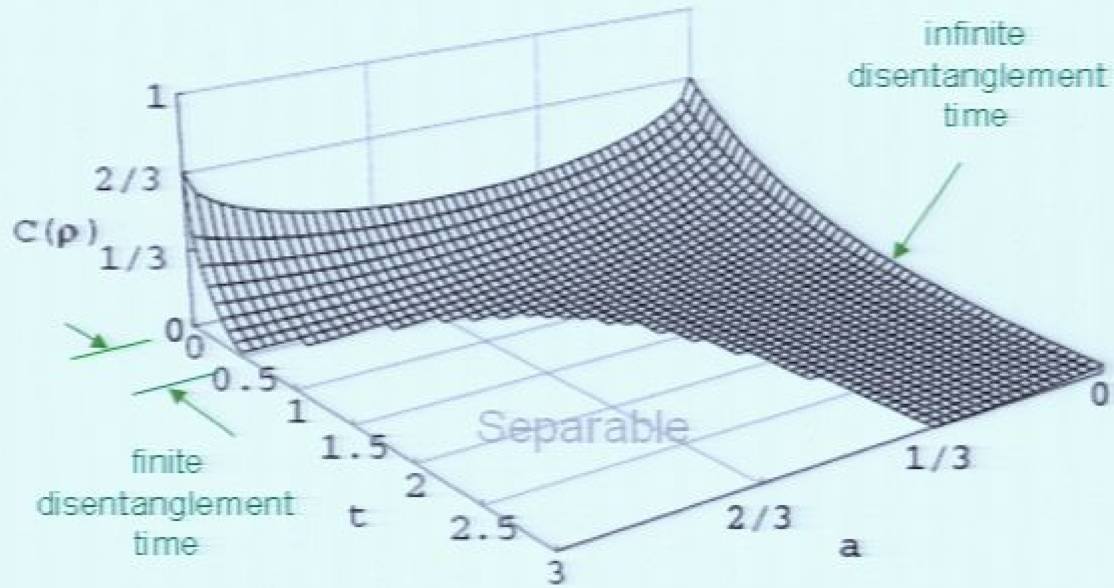
Initial state:

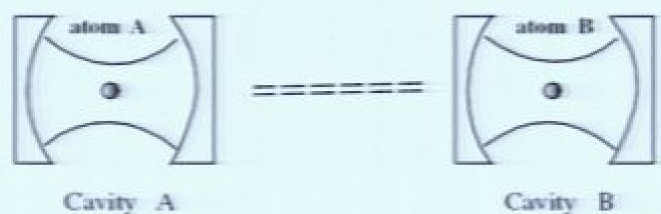
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





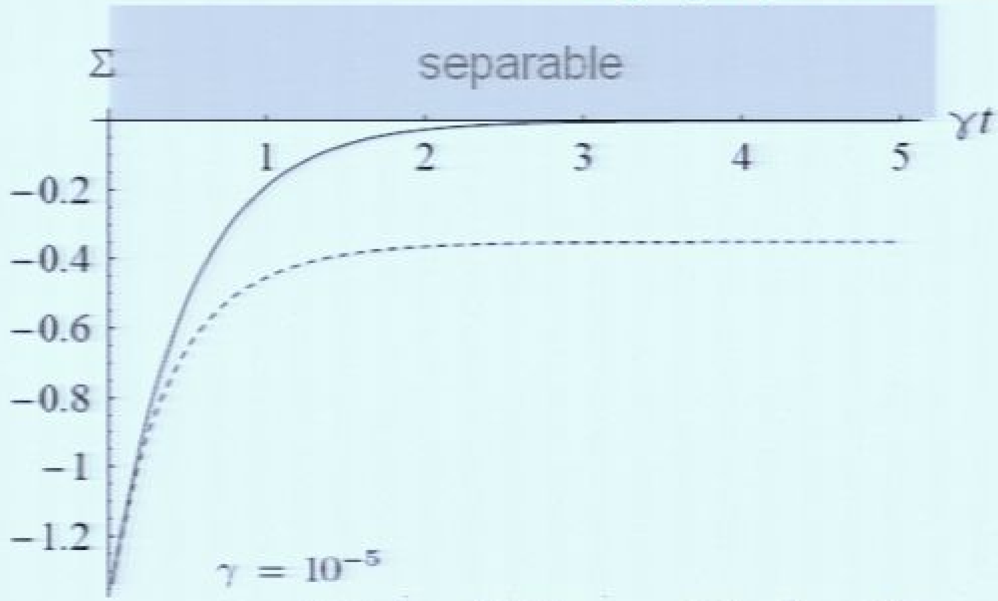
In Markovian regime

Two independent environments (cavities)

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$ $\boxed{\gamma \equiv \lambda_0^2/8\pi}$

Λ_1 ~ proper time resolution of detectors A and B



$\gamma = 10^{-5}$

$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$

$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

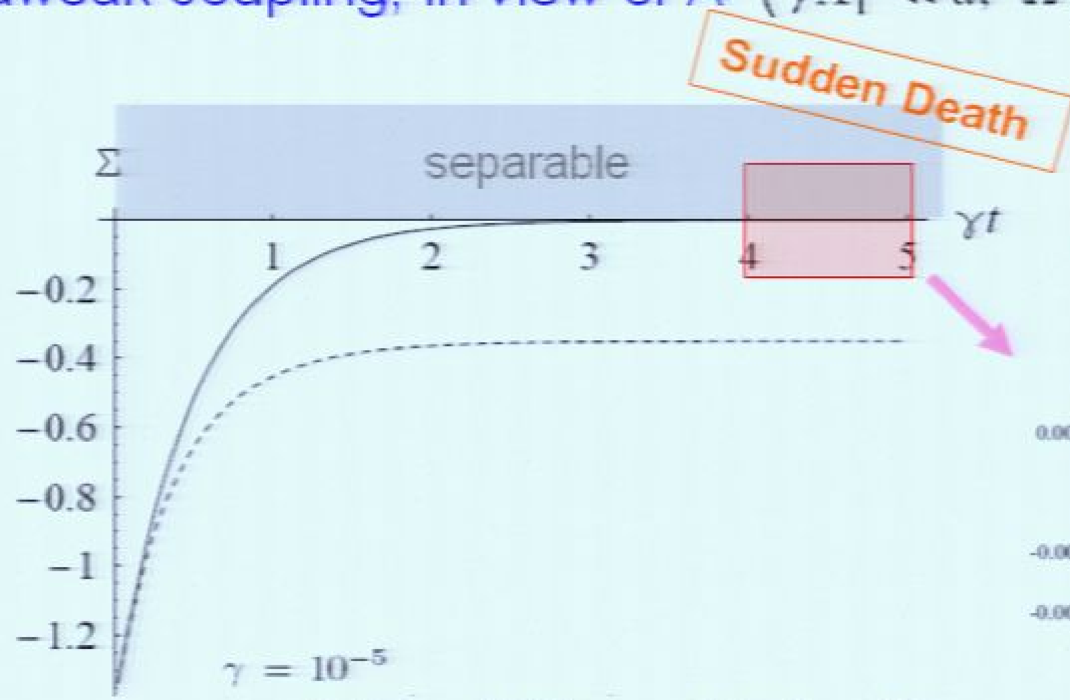
$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t}$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$

$\gamma \equiv \lambda_0^2/8\pi$



$\gamma = 10^{-5}$
$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$
$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

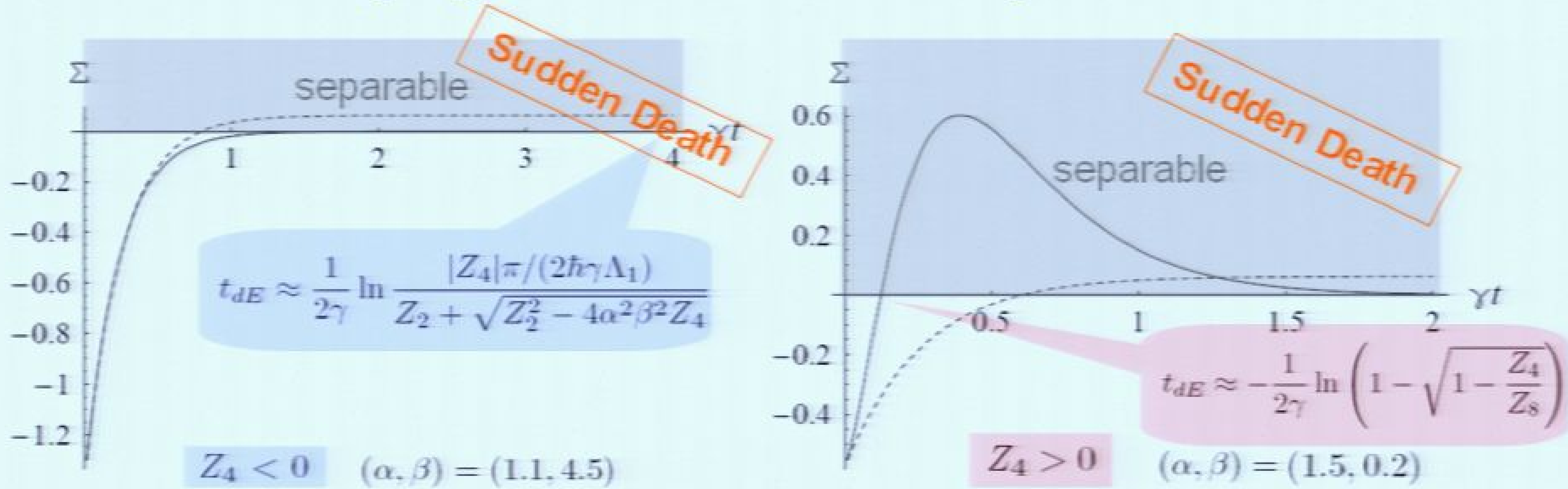
$$t_{dE} \approx \frac{1}{2\gamma}\ln\frac{\pi\Omega}{\gamma\Lambda_1}$$

$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2-\alpha^2\beta^2\right)^2 e^{-2\gamma t} + \frac{\hbar^2\gamma\Lambda_1}{16\pi\alpha^2\beta^2}\left(\hbar^2-\alpha^2\beta^2\right)^2$$

Initially $W(Q_A,P_A,Q_B,P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

III. Results and Discussion

- Ultraweak coupling, both at rest but well separated $\Omega \gg \gamma\Lambda_1 \gg a \to 0$



$\gamma = 10^{-5} \quad \Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right] + \frac{\hbar^3\gamma\Lambda_1}{4\pi\alpha^2\beta^2\Omega^2} Z_2 e^{-2\gamma t} + \frac{\hbar^4}{\pi^2\Omega^2}\gamma^2\Lambda_1^2$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2\left(\beta^2 - \hbar\Omega\right)^2,$$

$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2 + \alpha^2\Omega^2\right)\left(\hbar^2 + \alpha^2\beta^2\right).$$

$$Z_2 \equiv \alpha^2\left(\beta^2 - \hbar\Omega\right)^2 + \beta^2\left(\alpha^2\Omega - \hbar\right)^2$$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

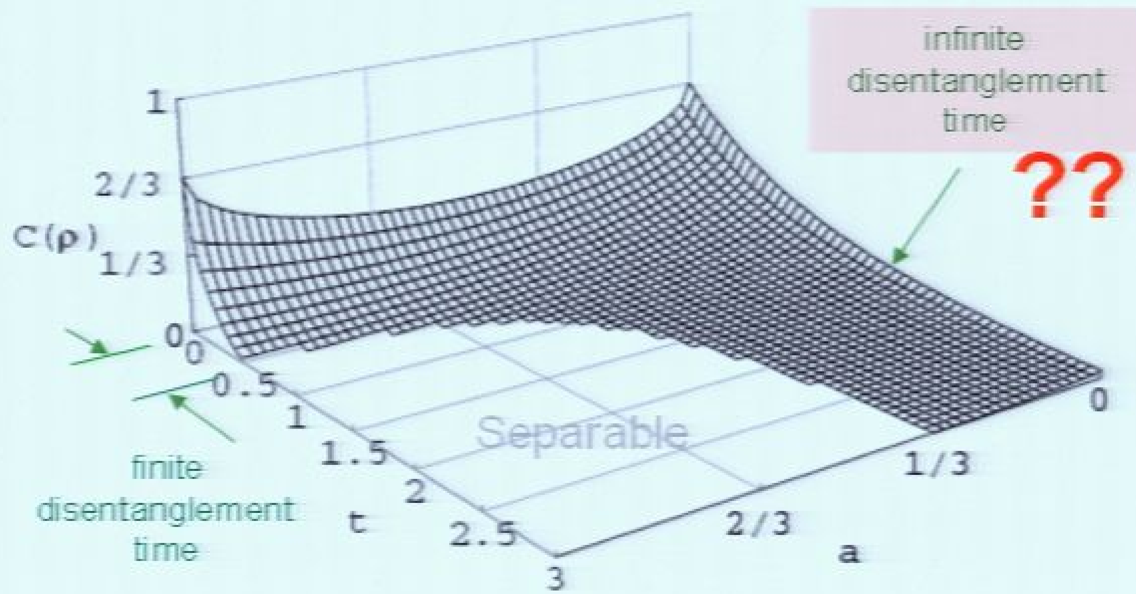
Initial state:

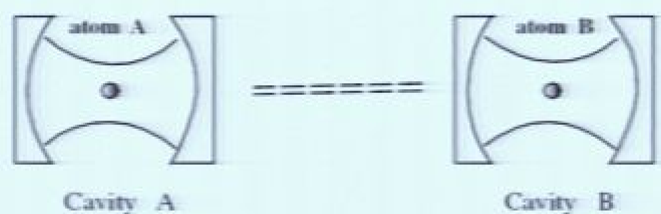
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

Two independent environments (cavities)

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

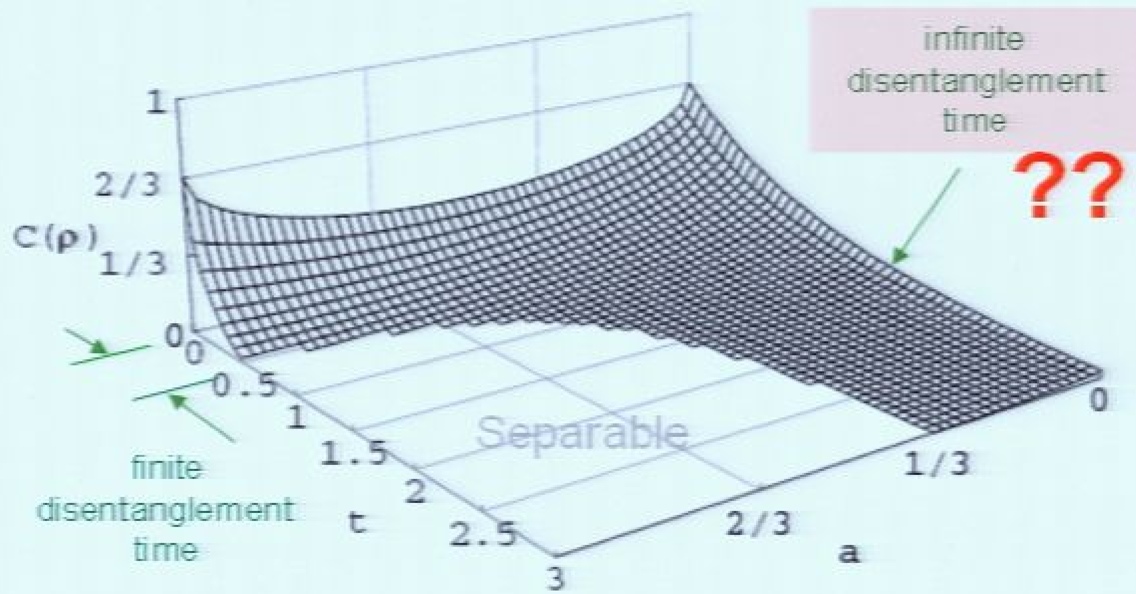
Initial state:

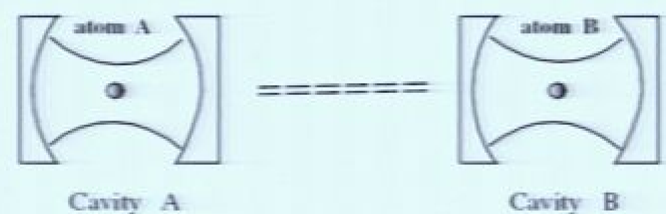
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

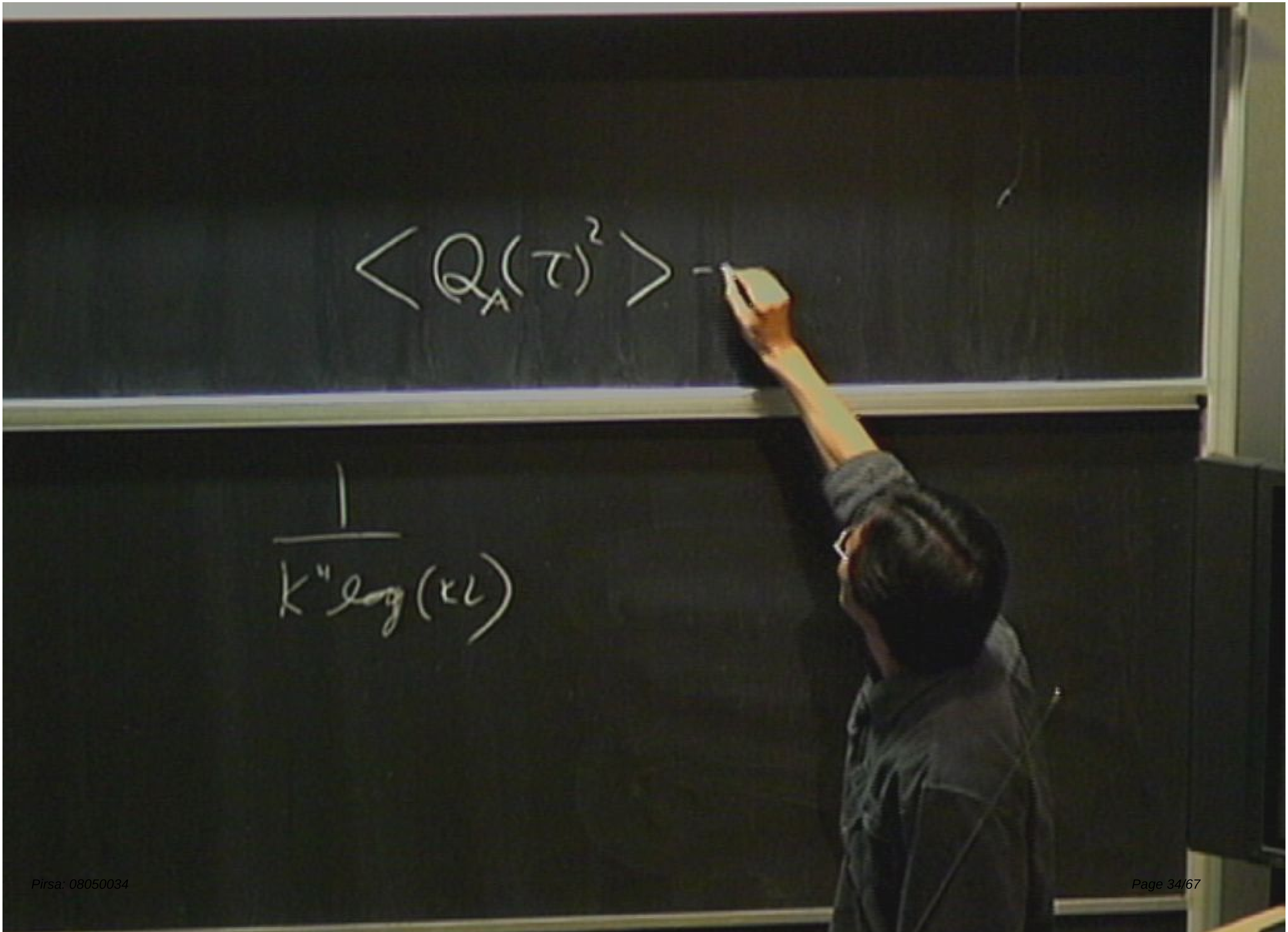
$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

Two independent environments (cavities)


$\langle Q_A(\tau)^2 \rangle -$
$\frac{1}{K'' \log(\kappa L)}$

$$\langle Q_A(\tau)^2 \rangle \leftarrow \lim_{\tau' \to \tau} \langle \{ Q_A(\tau), Q_A(\tau') \} \rangle$$

$$\frac{1}{k^4 \log(kL)}$$

$$\left\langle Q_A(\tau)^2 \right\rangle \leftarrow \lim_{\tau' \to \tau} \left\langle \{ Q_A(\tau), Q_A(\tau') \} \right\rangle$$

$$\log(\kappa L)$$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0,\ \gamma^2 f(t)\}$$

$C > 0$: entangled.

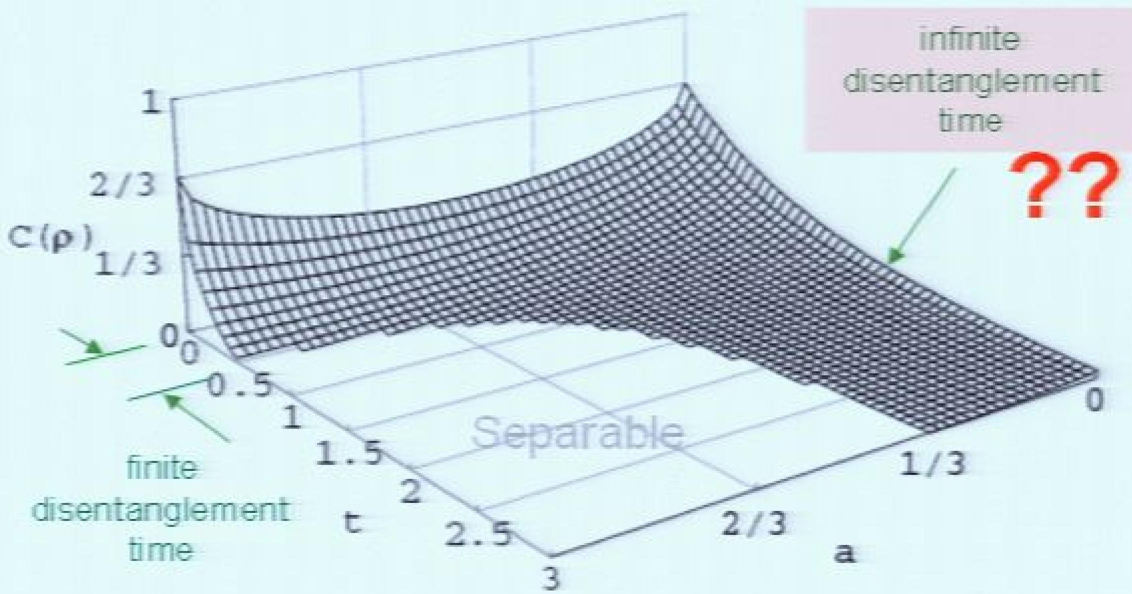
Initial state:

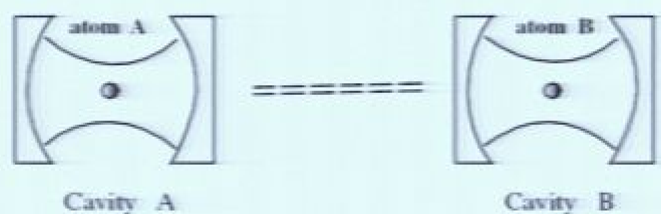
$$\rho_{\rm in} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1-a+2\omega^2+\omega^4 a)}$$

$$\omega = \sqrt{1-\exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

Two independent environments (cavities)

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0,\ \gamma^2 f(t)\}$$

$C > 0$: entangled.

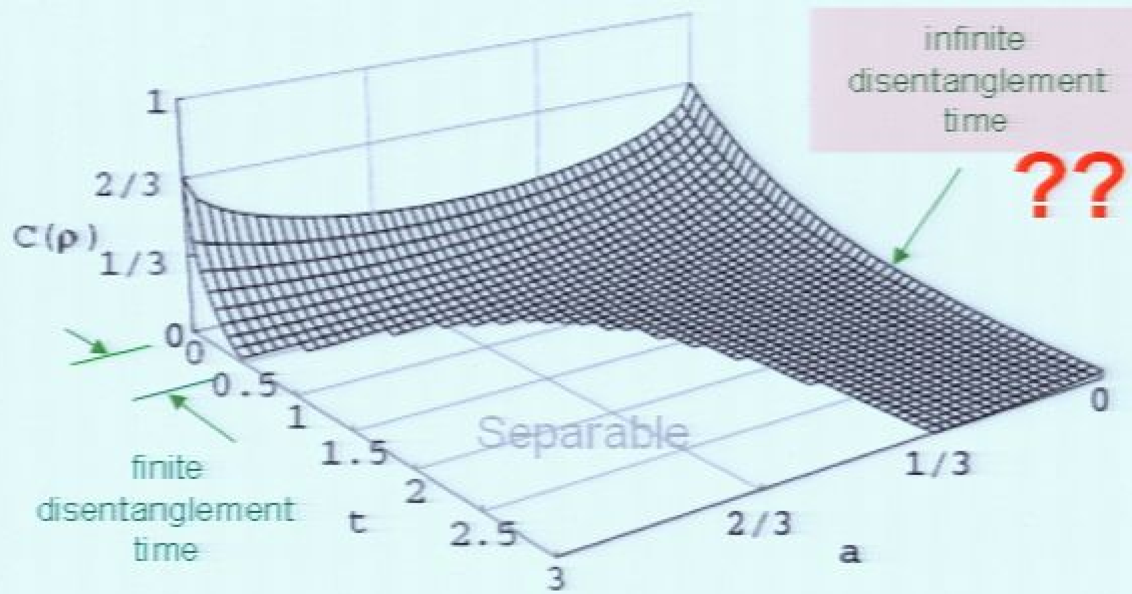
Initial state:

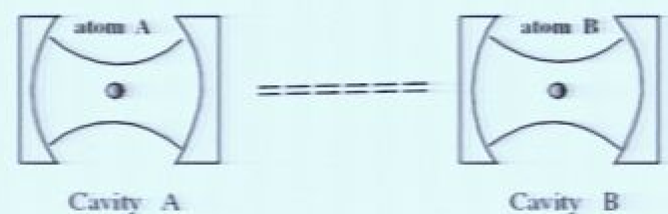
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

Two independent environments (cavities)

b. Acceleration and coordinate dependence

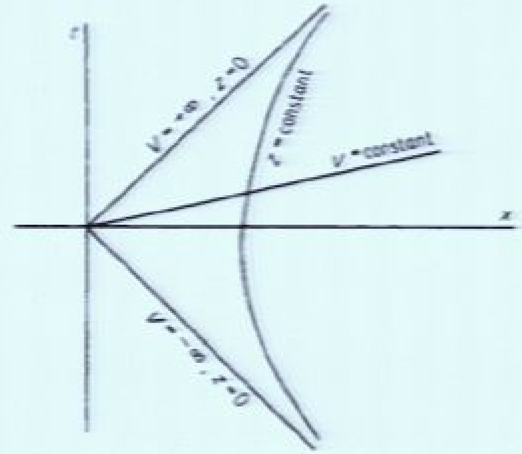
III. Results and Discussion

- Unruh effect (Unruh 1976)

  A detector uniformly accelerated in Minkowski vacuum will experience a thermal bath at **Unruh temperature**:

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

$T_U = 1\text{K}$ for $a = 2.4 \times 10^{20}$ m/s^2



* If B is initially in its ground state, in weak-coupling limit, the transition probability to its 1st excited state is

$$\rho^R_{1,1}|_{\gamma\eta\to 0} \overset{\eta \gg a^{-1}}{\longrightarrow} \frac{\lambda_0^2}{4\pi m_0}\left[\frac{\eta}{e^{2\pi\Omega_r/a}-1} + \frac{\Lambda_1 + \Lambda_0 - 2\ln(a/\Omega_r)}{2\pi\Omega_r}\right]$$

still early time

- uniform acceleration : $a_A a^A = a^2 = \text{constant}$ (a: proper acceleration)
- Minkowski vacuum: No particle (field quanta) state of the field for Minkowski observer

b. Acceleration and coordinate dependence

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

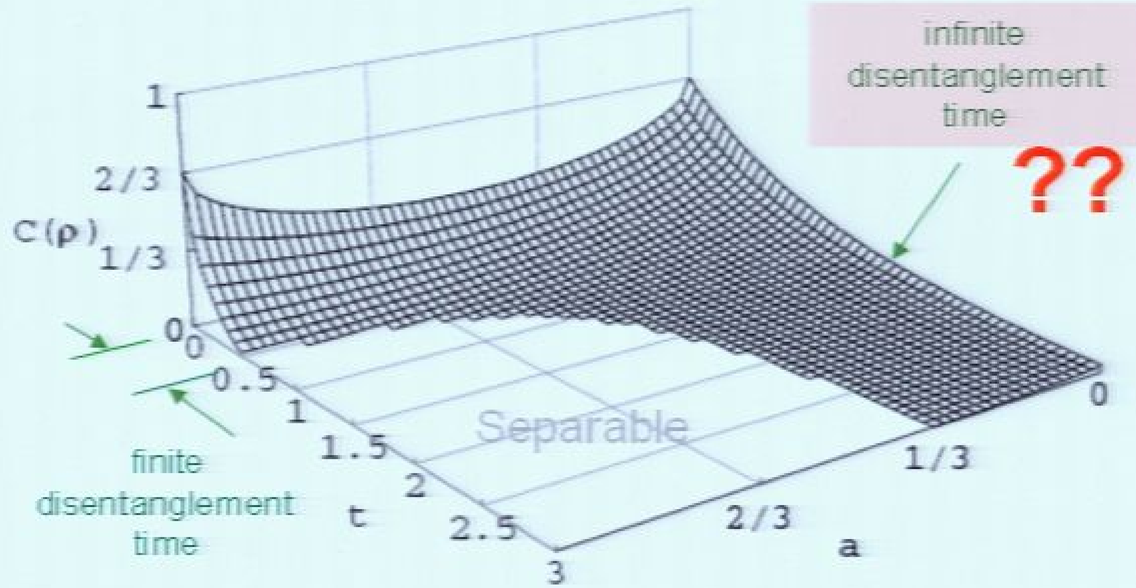
Initial state:

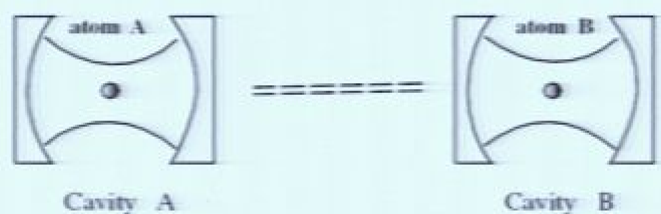
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

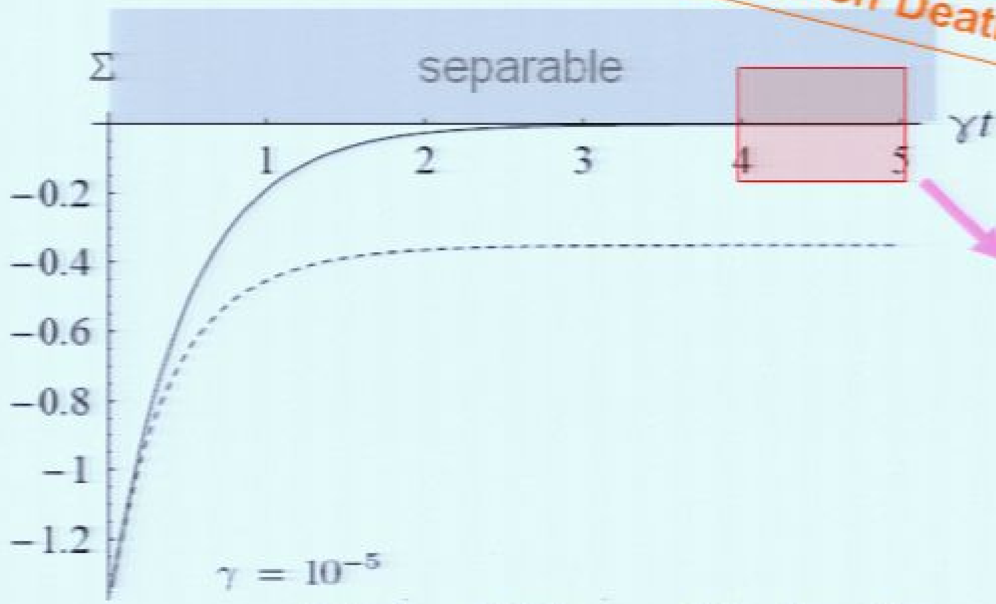
Two independent environments (cavities)

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$

$\gamma \equiv \lambda_0^2/8\pi$

Sudden Death

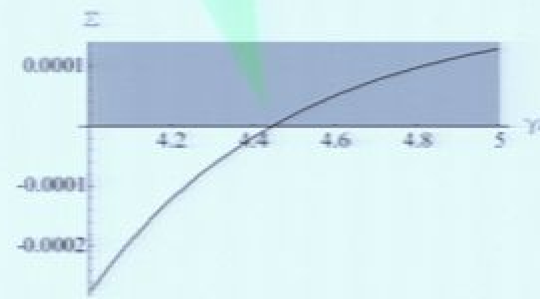


$\gamma = 10^{-5}$

$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$

$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

$$t_{dE} \approx \frac{1}{2\gamma} \ln \frac{\pi\Omega}{\gamma\Lambda_1}$$



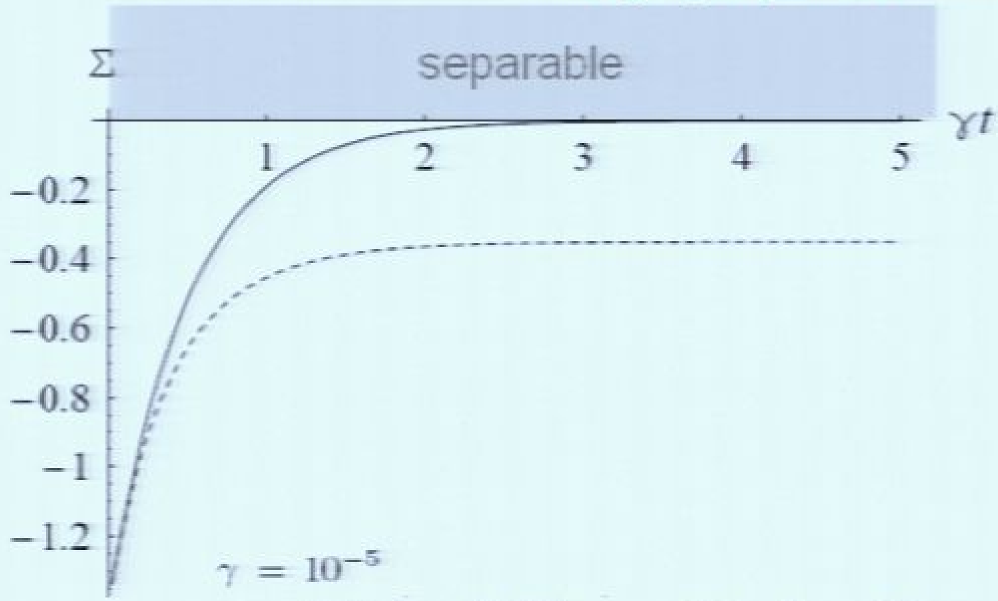
$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t} + \frac{\hbar^2\gamma\Lambda_1}{16\pi\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2} \exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A + Q_B)^2 + \frac{1}{\alpha^2}(Q_A - Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A - P_B)^2 + \frac{1}{\beta^2}(P_A + P_B)^2\right]$

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$ $\boxed{\gamma \equiv \lambda_0^2/8\pi}$

Λ_1 ~ proper time resolution of detectors A and B



$\gamma = 10^{-5}$
$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$
$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t}$$

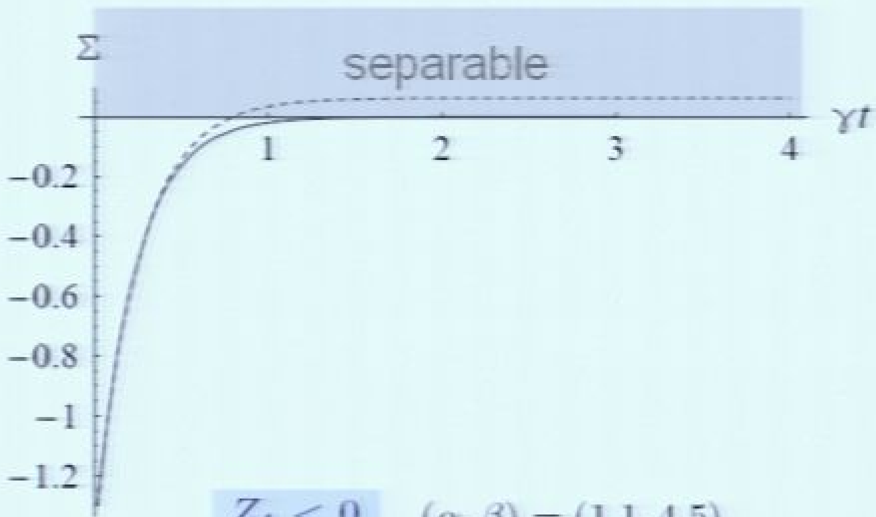
Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2} \exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

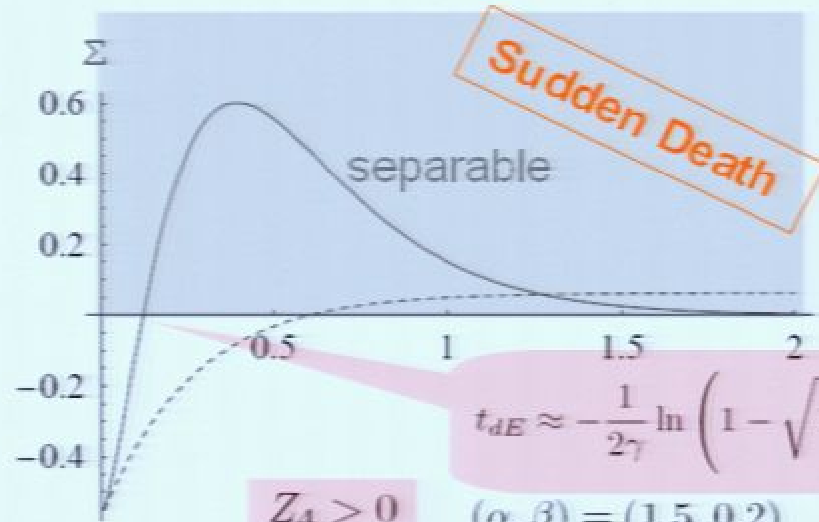
a. Sudden death of entanglement

III. Results and Discussion

$\gamma \equiv \lambda_0^2/8\pi$

- Weak coupling, both at rest but well separated $\Omega \gg \gamma\Lambda_1 \gg a \to 0$





$$t_{dE} \approx -\frac{1}{2\gamma}\ln\left(1-\sqrt{1-\frac{Z_4}{Z_8}}\right)$$

$\gamma = 10^{-5}$ $\Lambda_0 = \Lambda_1 = 100$, $\Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right]$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2\left(\beta^2 - \hbar\Omega\right)^2,$$
$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2+\alpha^2\Omega^2\right)\left(\hbar^2+\alpha^2\beta^2\right).$$

Initially $W(Q_A,P_A,Q_B,P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

I. Introduction

- A surprise: "Sudden death" of entanglement

[Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

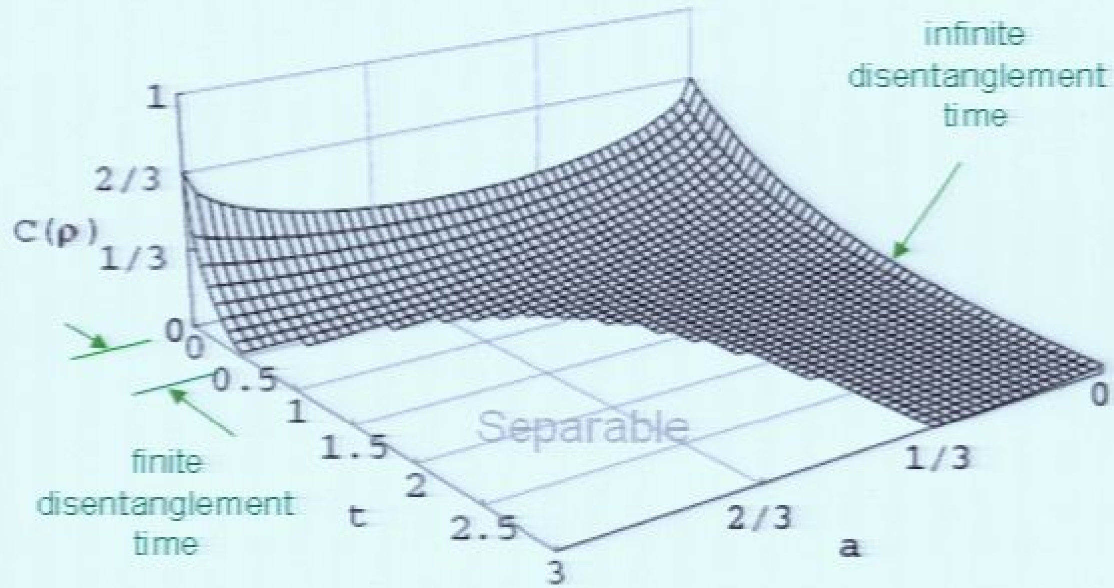
Initial state:

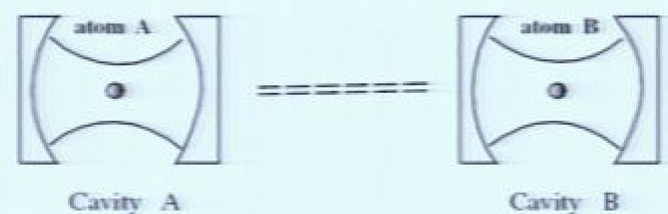
$$\rho_{\rm in} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

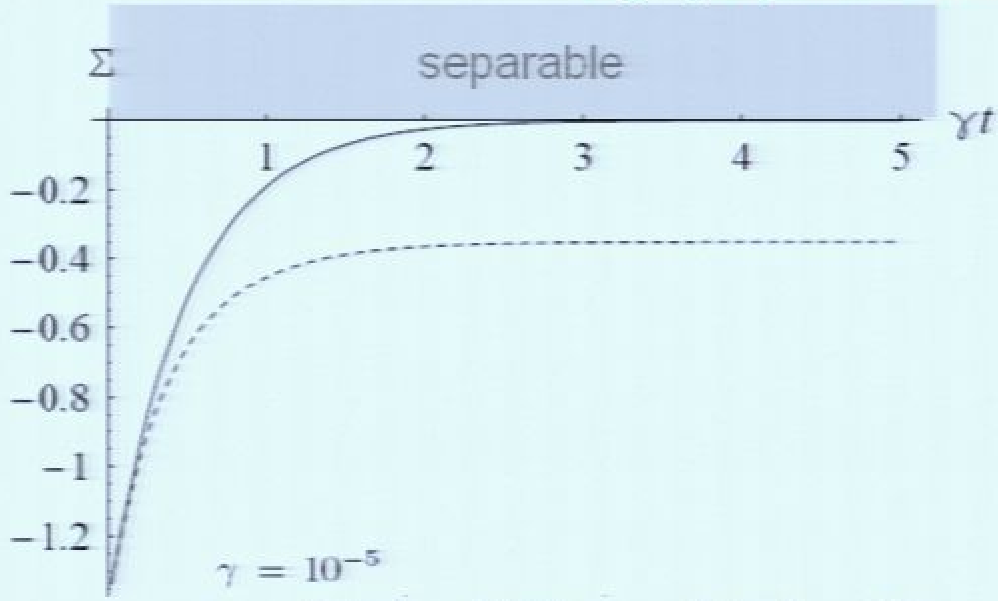
Two independent environments (cavities)

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$

$\gamma \equiv \lambda_0^2/8\pi$

$\mathbf{\Lambda_1}$ ~ proper time resolution of detectors A and B



$\gamma = 10^{-5}$

$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$

$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t}$$

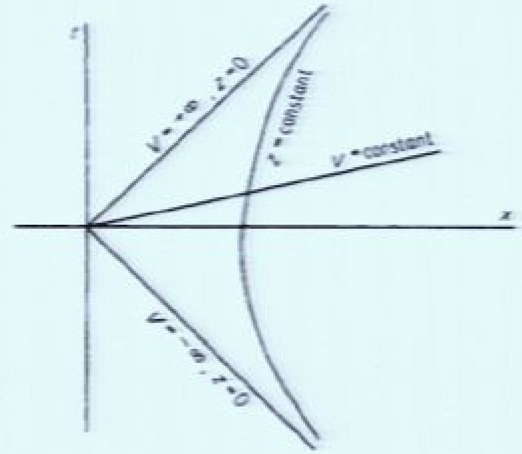
Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

III. Results and Discussion

- Unruh effect (Unruh 1976)

  A detector uniformly accelerated in Minkowski vacuum will experience a thermal bath at **Unruh temperature**:

$$T_U = \frac{\hbar a}{2\pi k_B c}$$



$T_U = 1\text{K}$ for $a = 2.4 \times 10^{20}$ m/s^2

* If B is initially in its ground state, in weak-coupling limit, the transition probability to its 1st excited state is

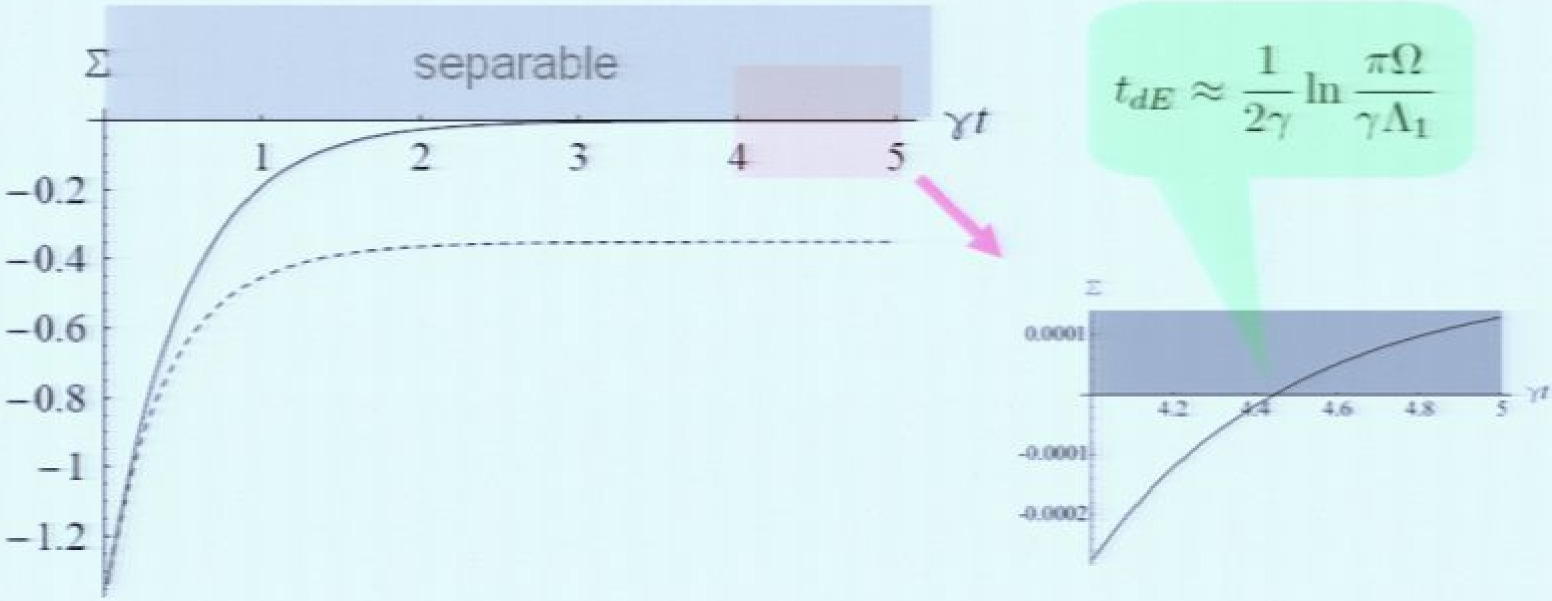
$$\rho^R_{1,1}|_{\gamma\eta\to 0} \overset{\eta\gg a^{-1}}{\longrightarrow} \frac{\lambda_0^2}{4\pi m_0}\left[\frac{\eta}{e^{2\pi\Omega_r/a}-1} + \frac{\Lambda_1+\Lambda_0-2\ln(a/\Omega_r)}{2\pi\Omega_r}\right]$$

still early time

- uniform acceleration : $a_A a^A = a^2 = \text{constant}$ (a: proper acceleration)
- Minkowski vacuum: No particle (field quanta) state of the field for Minkowski observer

III. Results and Discussion

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$



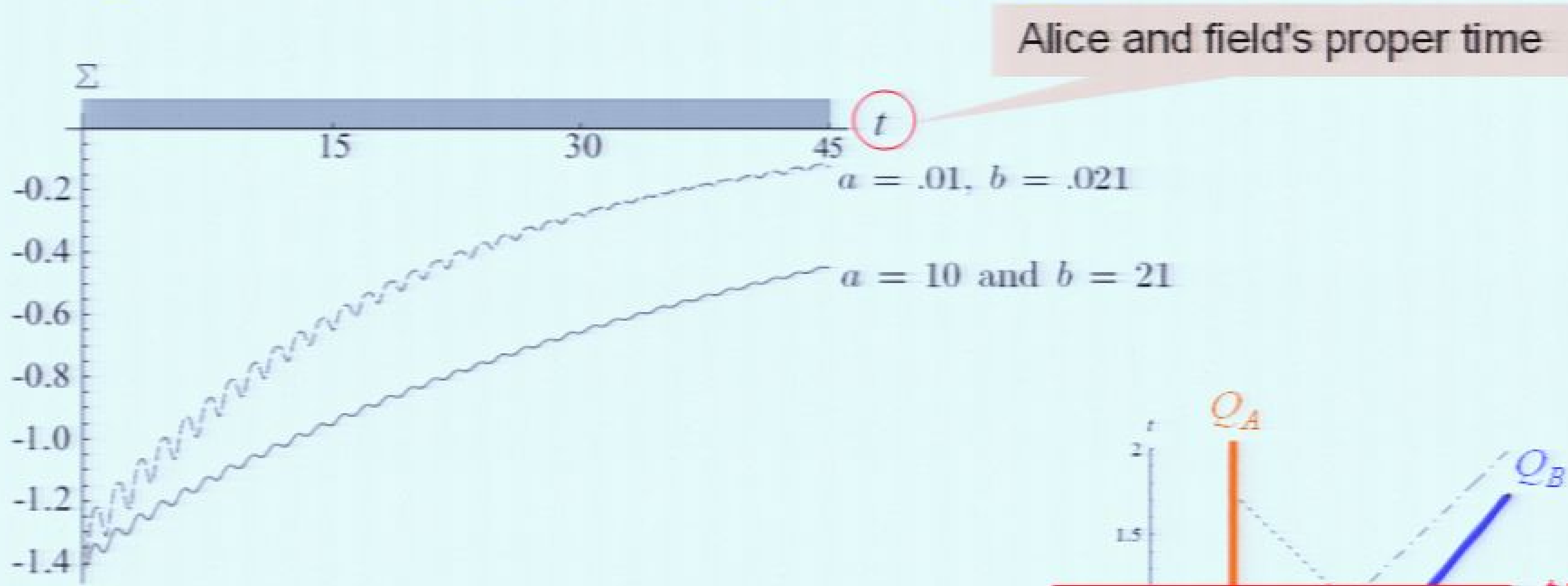
$$t_{dE} \approx \frac{1}{2\gamma}\ln\frac{\pi\Omega}{\gamma\Lambda_1}$$

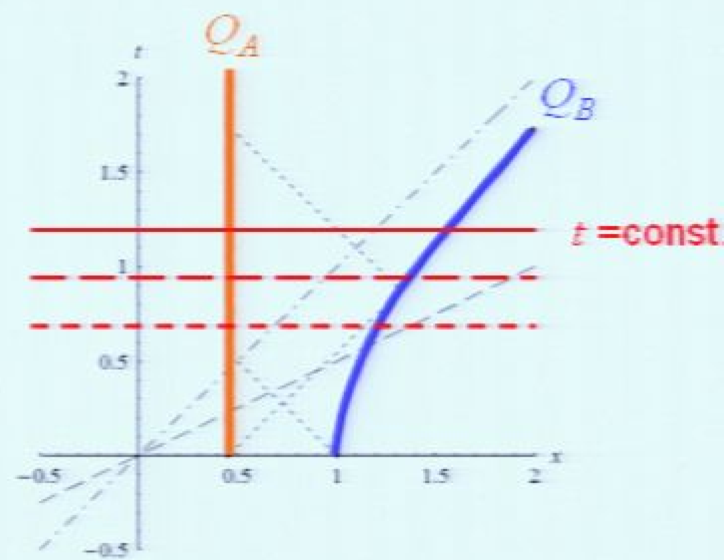
$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2-\alpha^2\beta^2\right)^2 e^{-2\gamma t}+\frac{\hbar^2\gamma\Lambda_1}{16\pi\alpha^2\beta^2}\left(\hbar^2-\alpha^2\beta^2\right)^2$$

Independent of acceleration a ...

III. Results and Discussion

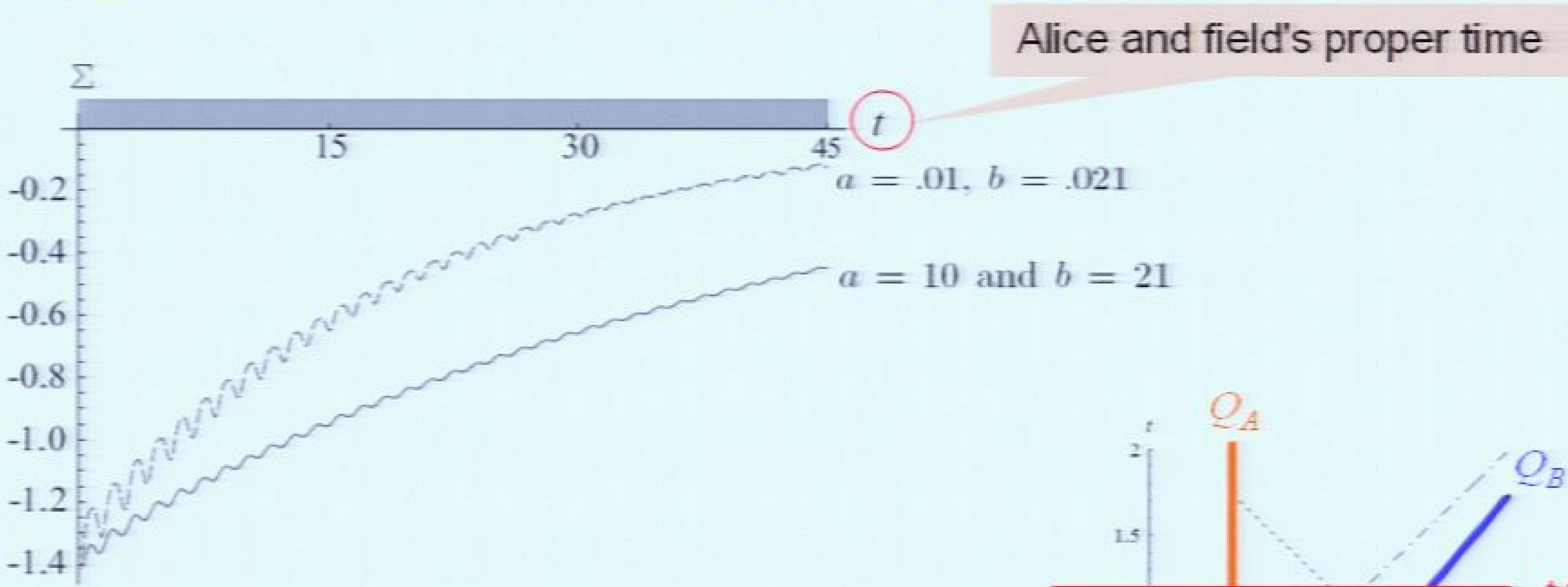
- high acceleration (Unruh temperature) regime



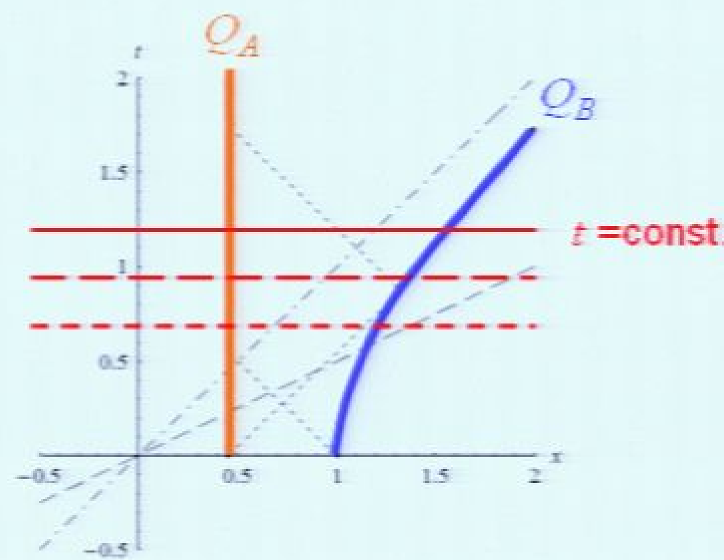


III. Results and Discussion

- high acceleration (Unruh temperature) regime



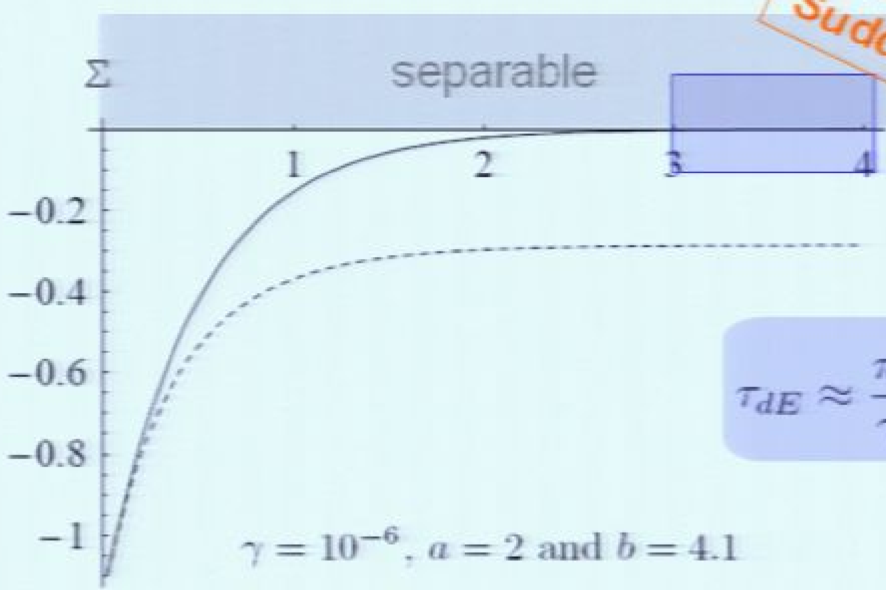
Slower climbing rate due to
larger time dilation of B in view of A



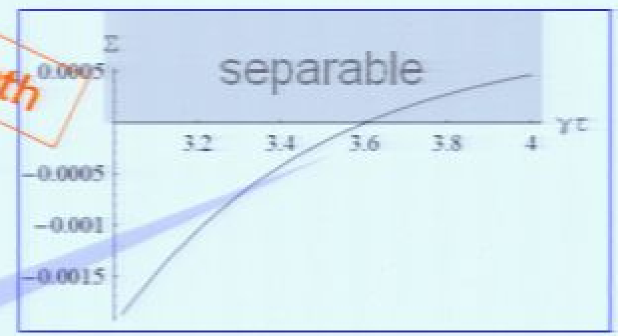
III. Results and Discussion

- Ultraweak coupling limit, in view of B

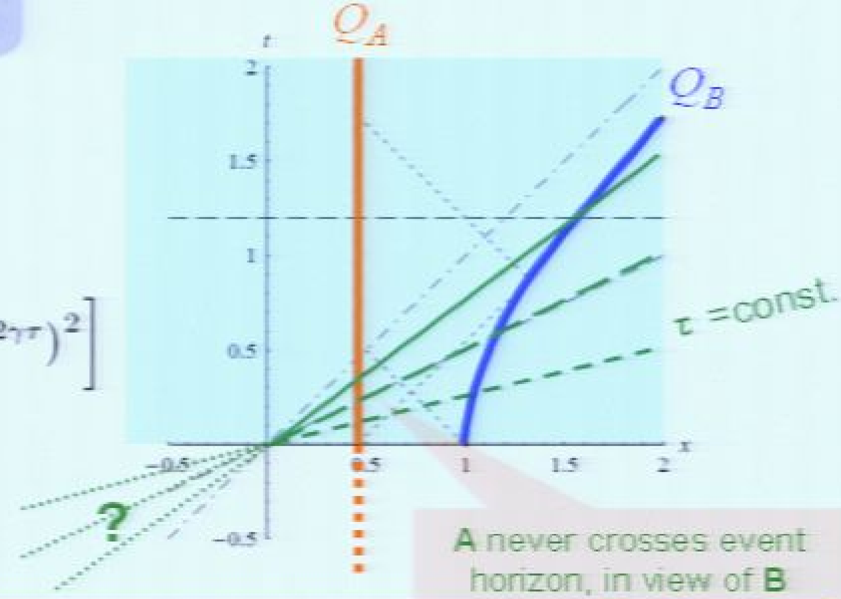
"Entanglement is degraded in non-inertial frames." (AM03)



Sudden Death



$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$



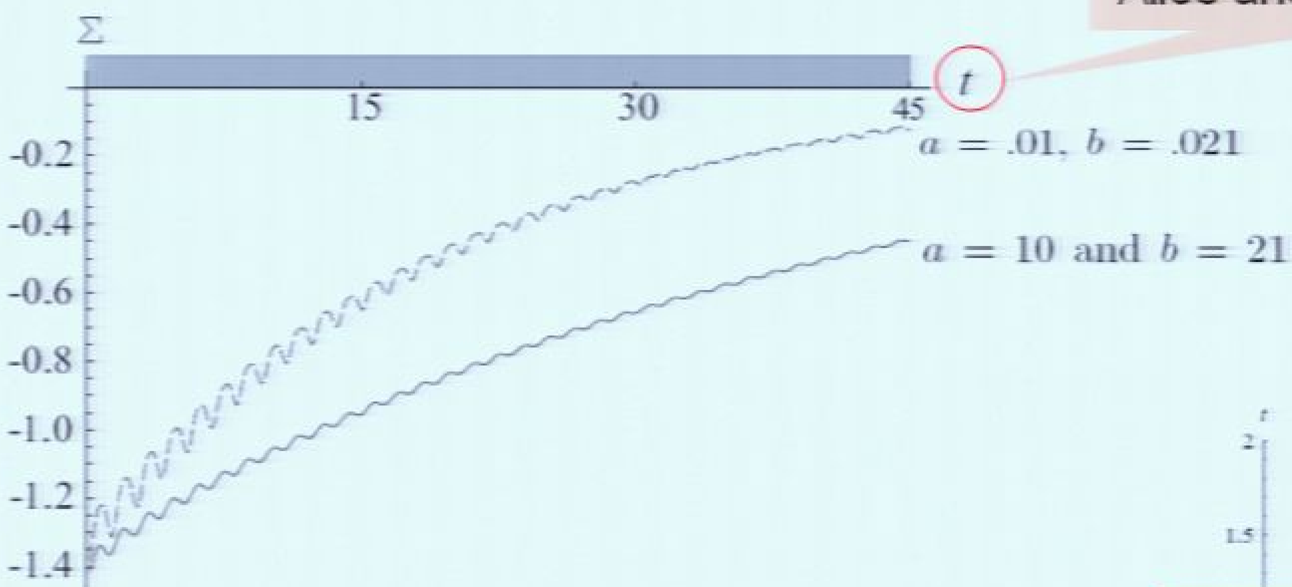
$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 \left[\left(1 - e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1 + e^{-2\gamma\tau}\right)^2\right]$$

Unruh effect

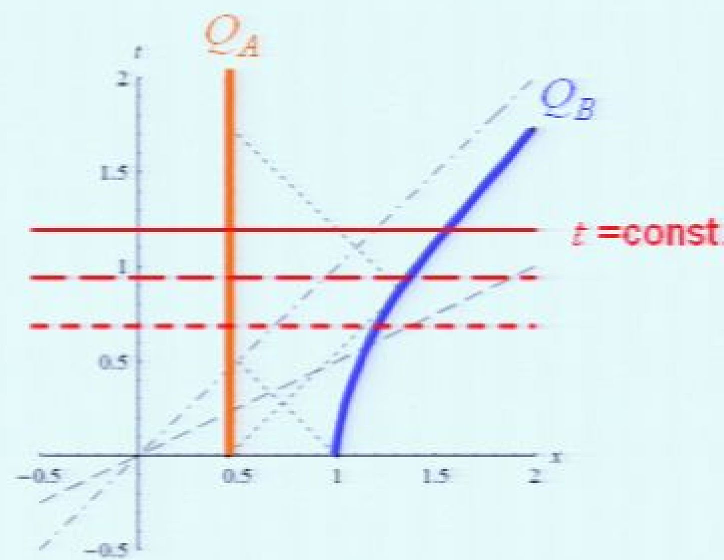
III. Results and Discussion

- high acceleration (Unruh temperature) regime

Alice and field's proper time



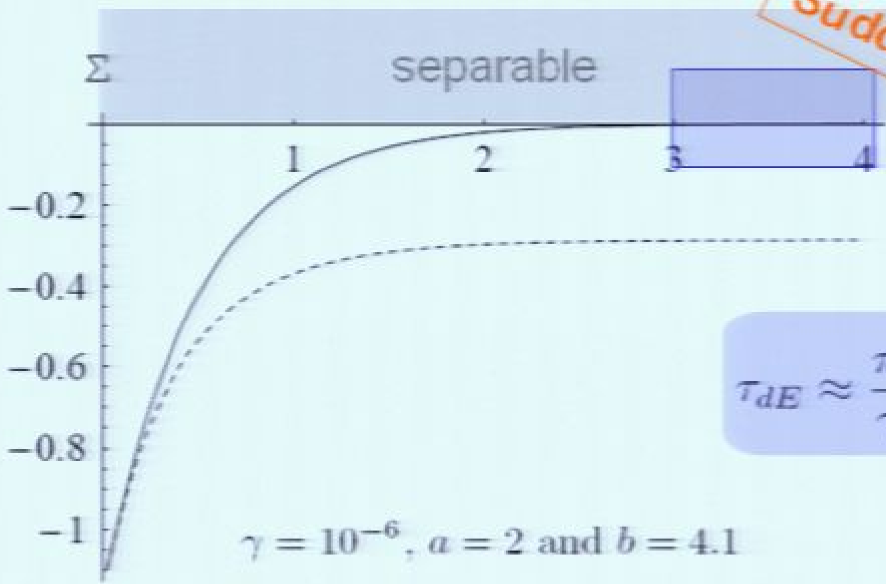
Slower climbing rate due to larger time dilation of B in view of A

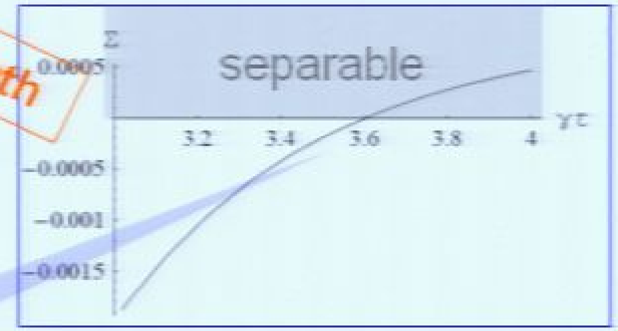


III. Results and Discussion

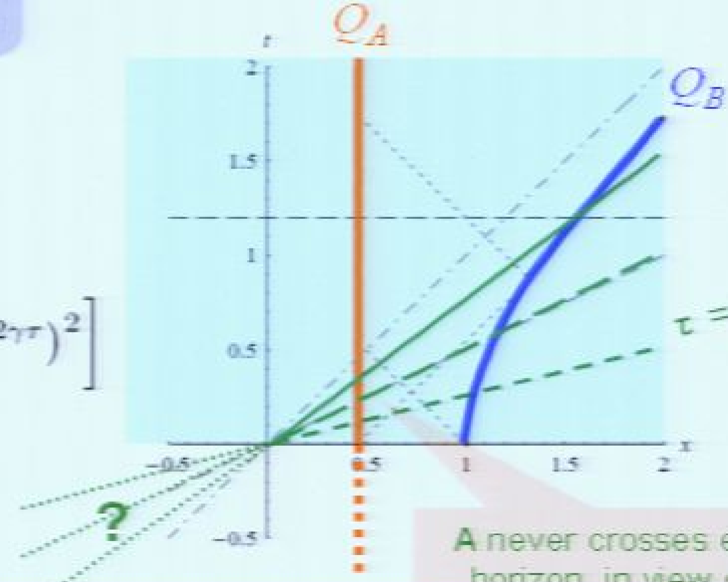
- Ultraweak coupling limit, in view of B

"Entanglement is degraded in non-inertial frames." (AM03)





$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$

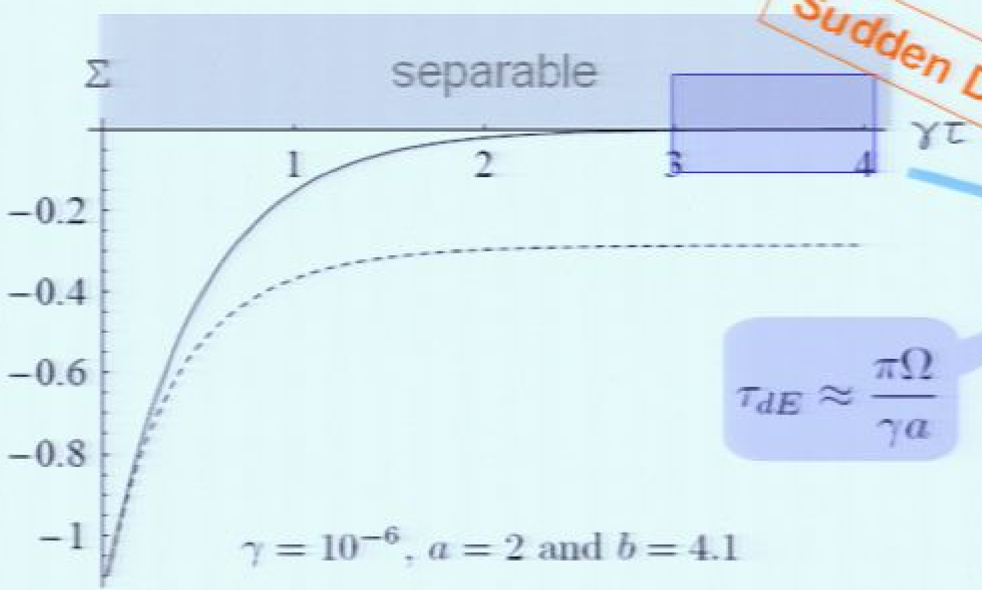


$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2-\alpha^2\beta^2\right)^2\left[\left(1-e^{-2\gamma\tau}\right)^2\coth^2\frac{\pi\Omega}{a}-\left(1+e^{-2\gamma\tau}\right)^2\right]$$

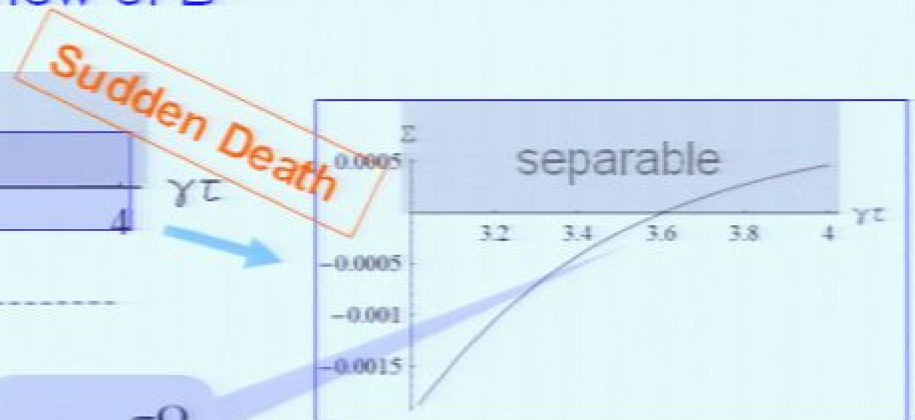
Unruh effect

III. Results and Discussion

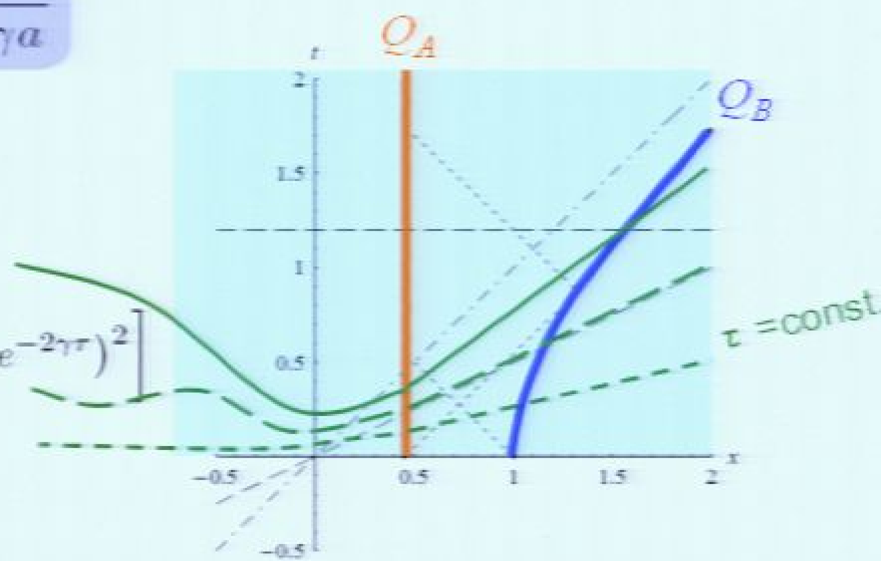
- Ultraweak coupling limit, in view of B



$\gamma = 10^{-6}$, $a = 2$ and $b = 4.1$



$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$



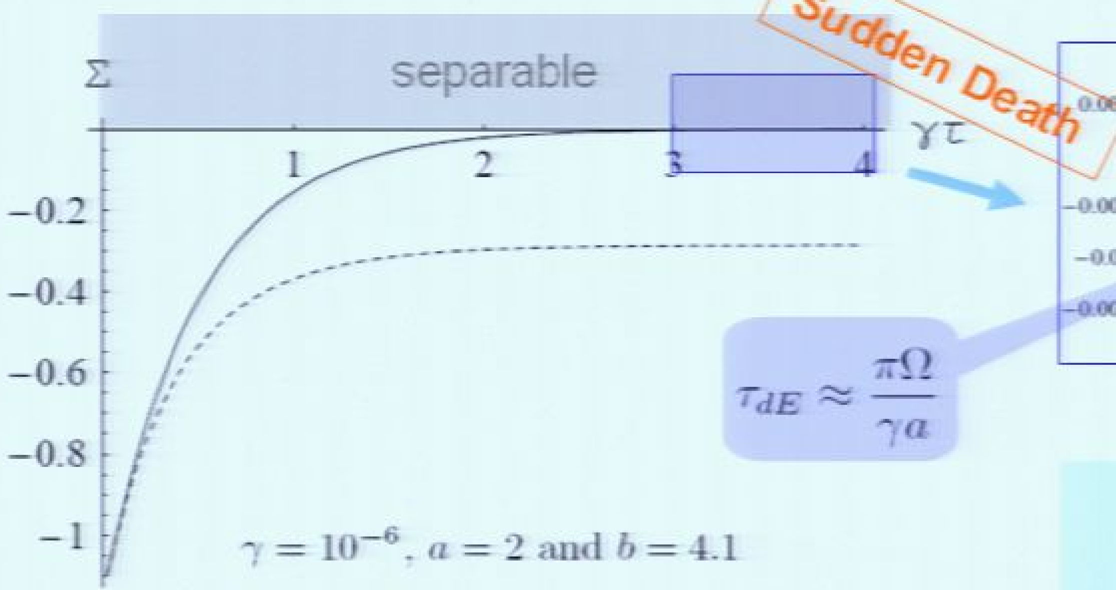
$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 \left[\left(1 - e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1 + e^{-2\gamma\tau}\right)^2\right]$$

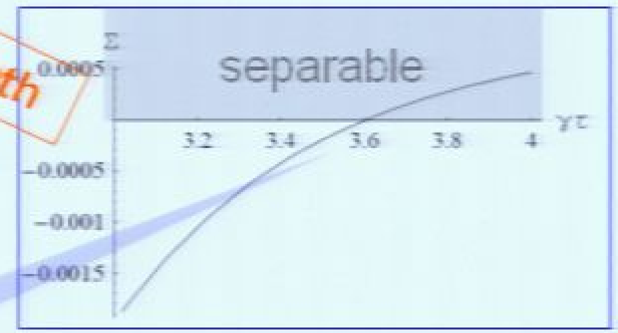
No guarantee on the Gaussianity of the field state in arbitrary time slicing.

III. Results and Discussion

- Ultraweak coupling limit, in view of B

"Entanglement is degraded in non-inertial frames." (AM03)

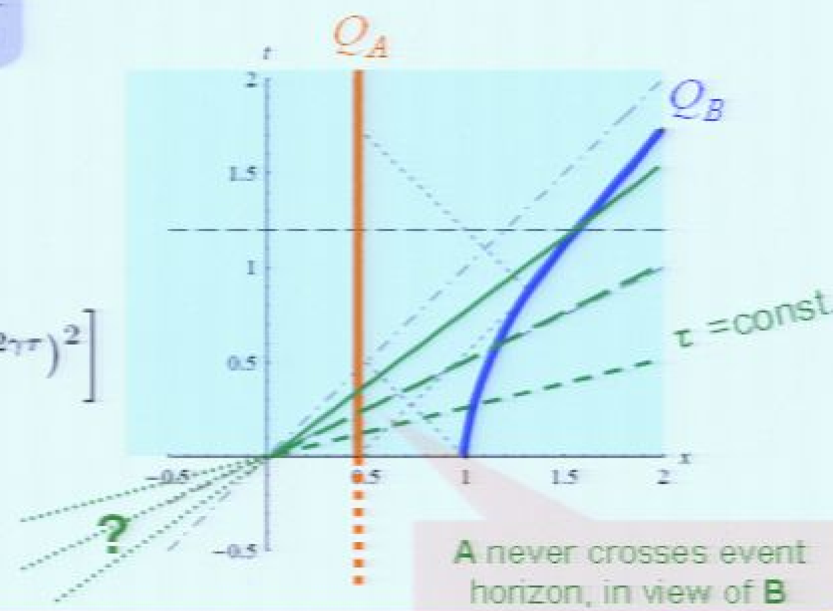




$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$

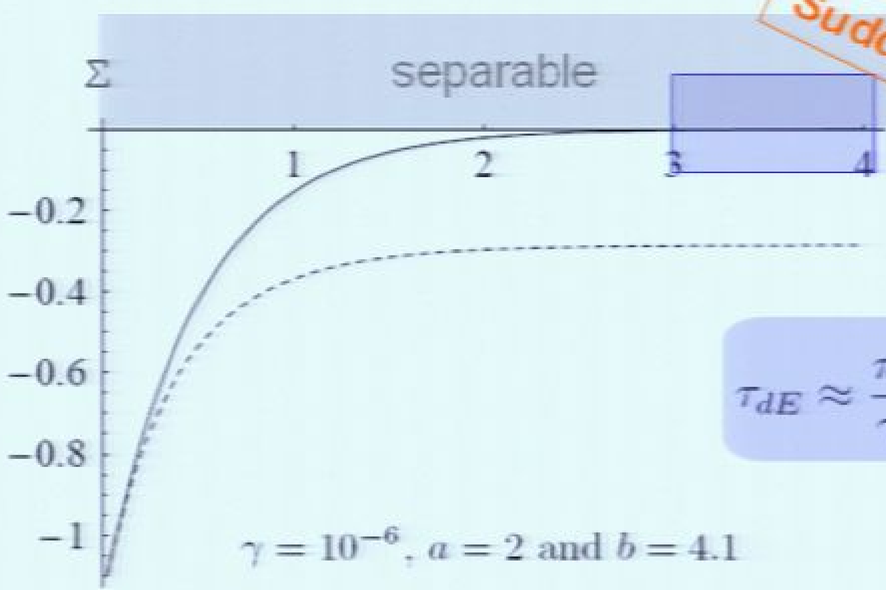
$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2\left[\left(1-e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1+e^{-2\gamma\tau}\right)^2\right]$$

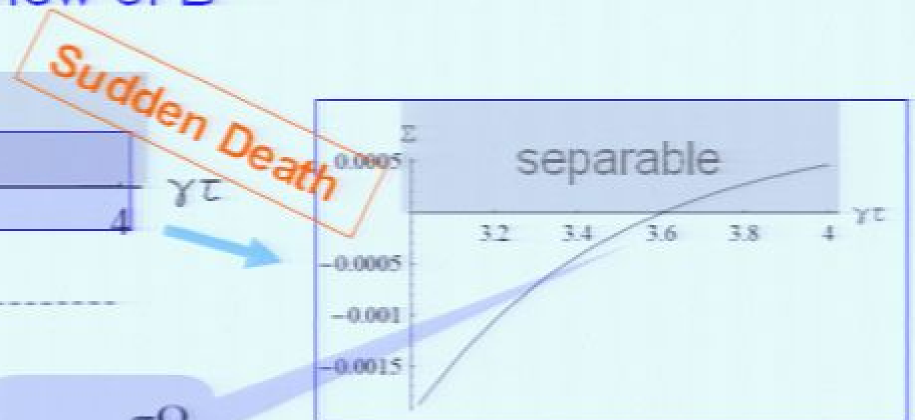
Unruh effect



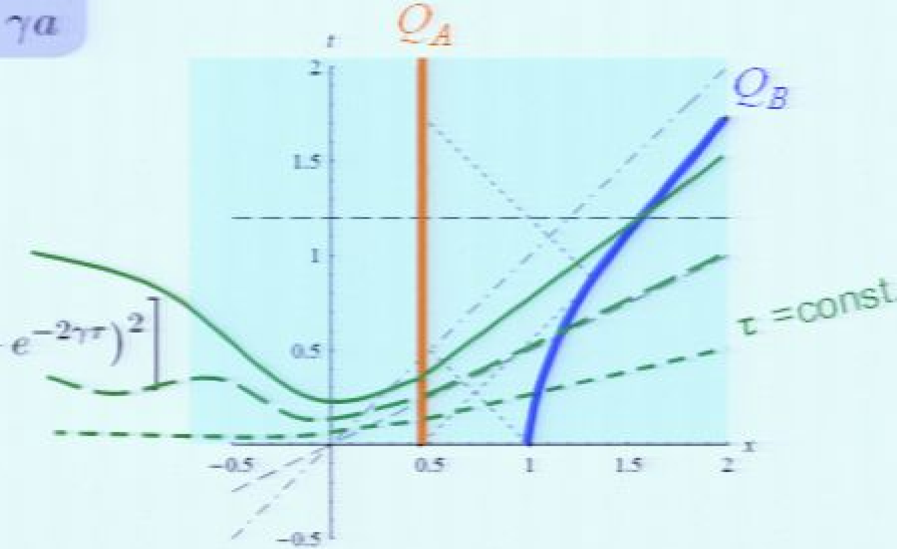
III. Results and Discussion

- Ultraweak coupling limit, in view of B





$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$



$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2\left[\left(1-e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1+e^{-2\gamma\tau}\right)^2\right]$$

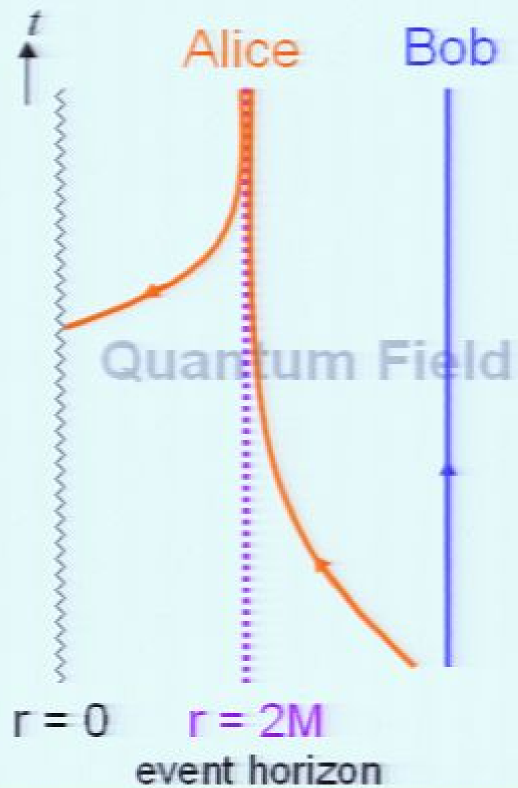
No guarantee on the Gaussianity of the field state in arbitrary time slicing.

I. Introduction

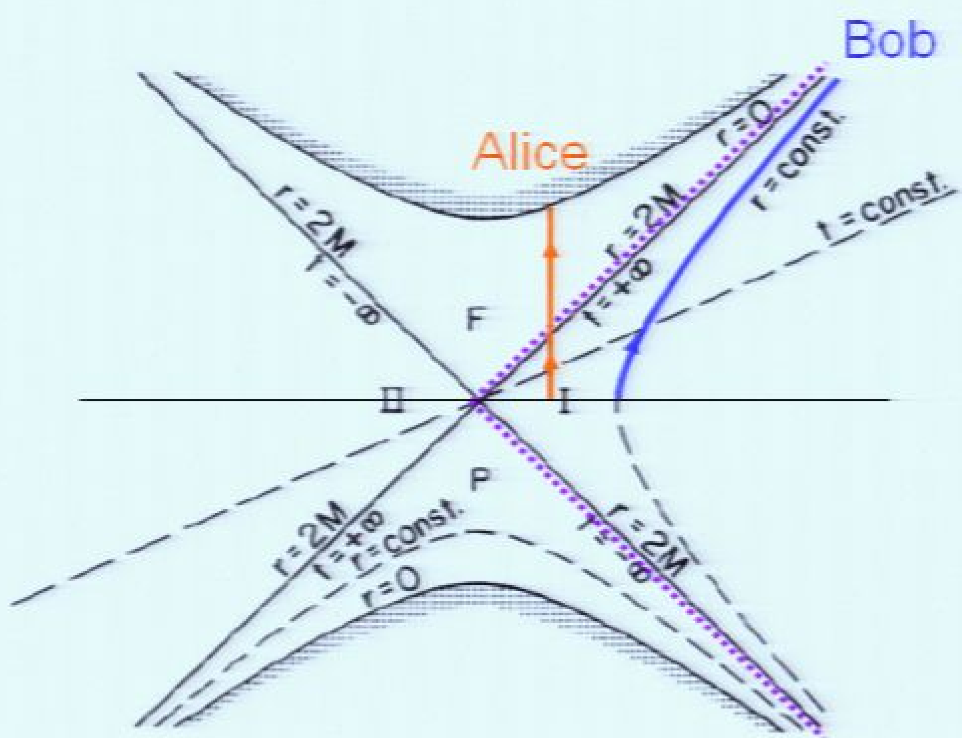
Q: Entanglement across the event horizon?

Schwarzschild black hole

Schwarzschild coordinate



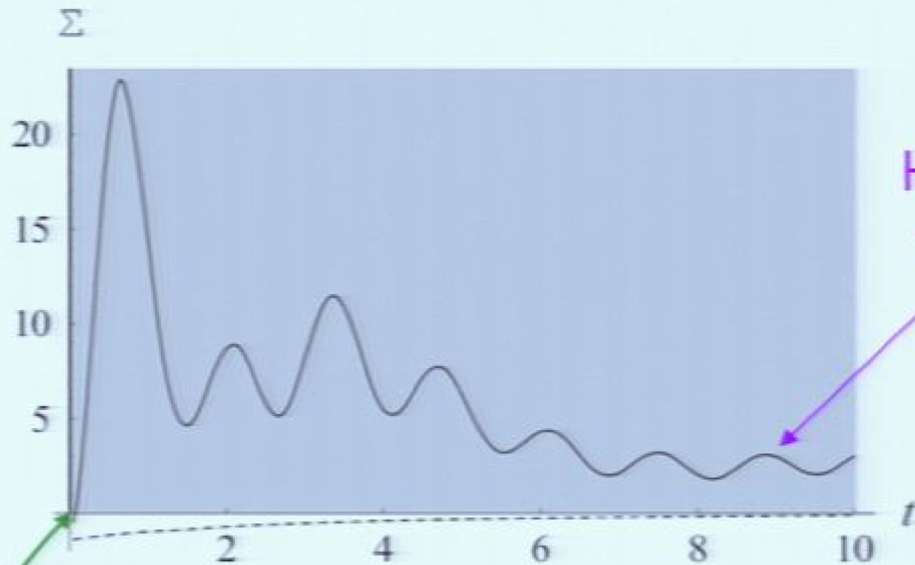
Kruskal coordinate



c. Non-Markovian regime

III. Results and Discussion

- Non-Markovian regime



$\gamma = 0.1,\ \Lambda_0 = \Lambda_1 = 50,\ a = 1,\ b = 2.01,$

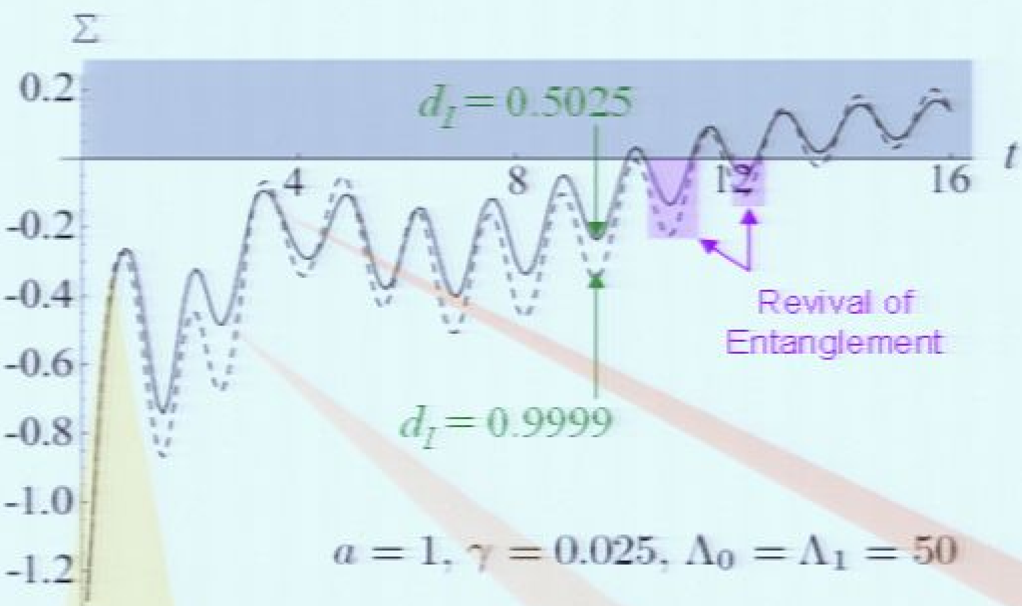
Higher order corrections from mutual influences < 2% in this plot.

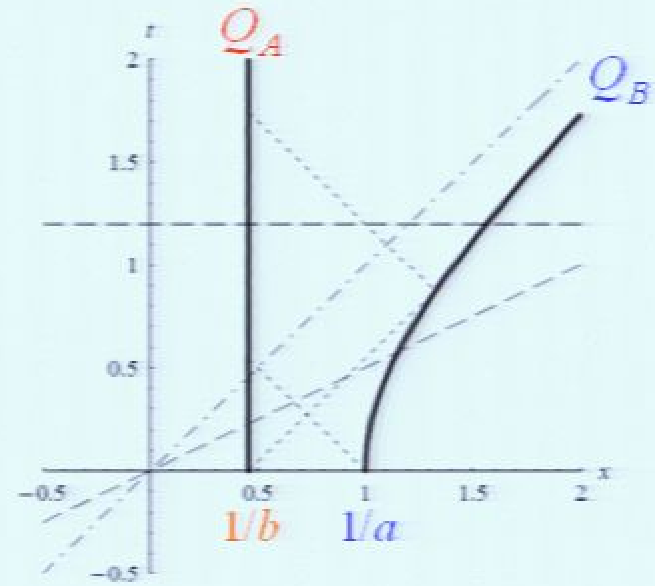
Quantum entanglement is destroyed right after the coupling is switched on.

d. Spatial separation

III. Results and Discussion

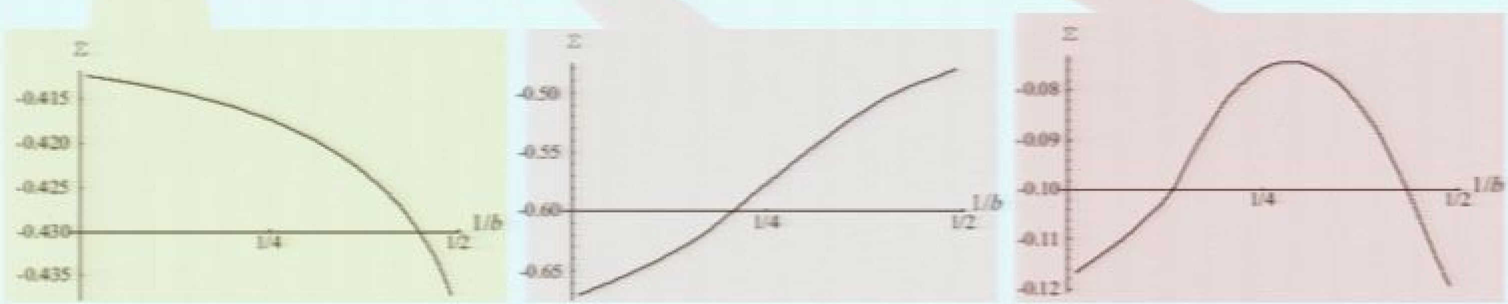
- Entanglement vs. Initial Separation





$d_I = 1/a - 1/b$

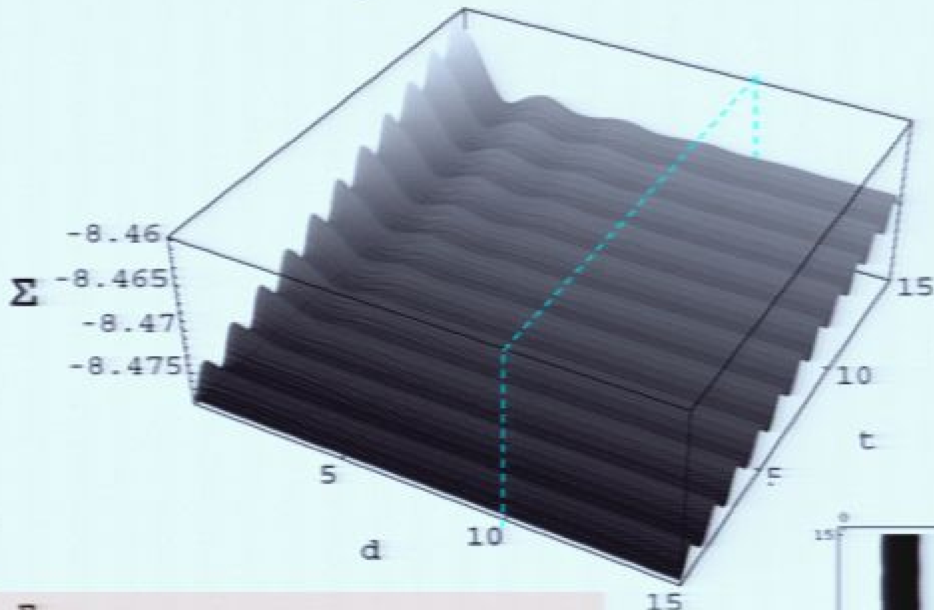
No clear relation between initial separation & entanglement

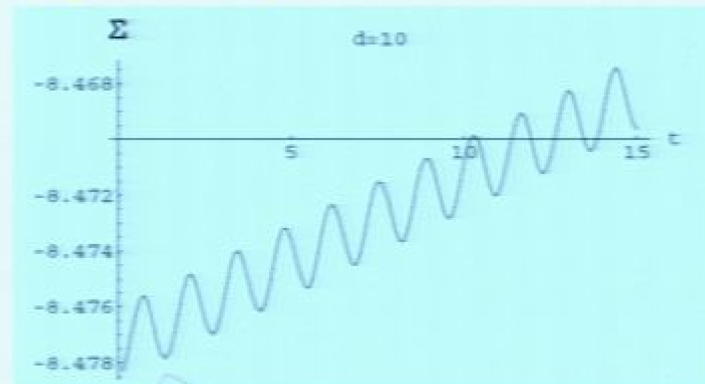


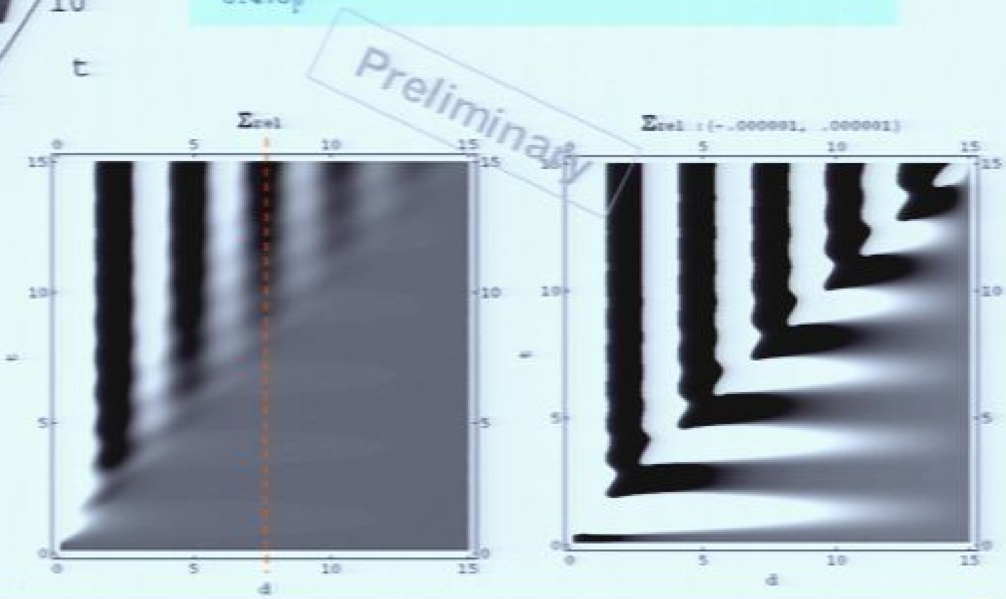
$t = 0.5$ $t = 2.5$ $t = 3.2$

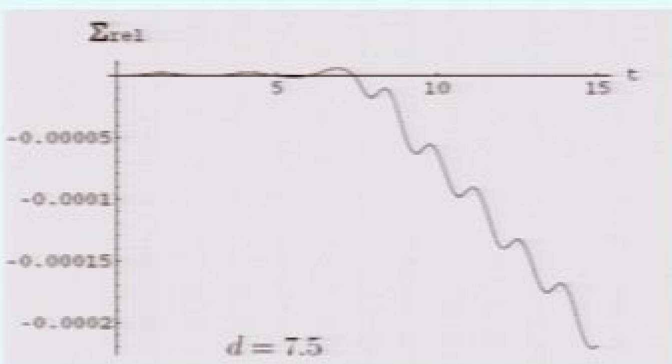
III. Results and Discussion

- Two detectors at rest with spatial separation d, weakly coupled with the field (mutual influences are neglected.)









IV. Summary

IV. Summary

- Disentanglement

Interaction with the environment (QF) does induce disentanglement.

- Disentanglement time of A and B in **all** cases we studied is **finite**; No residual A∞B (entanglement) at late times.
- No long-time ($> O(1/\Omega)$) A∞B generated.
- Strong impact kills quantum entanglement soon after the coupling is turned on in non-Markovian regime.

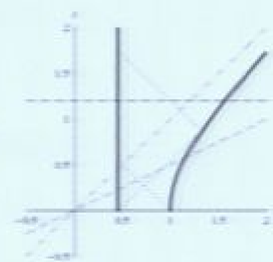
How generic are above features?

- We are considering an linearly coupling atom-field system in (3+1)D free space with no direct interaction between two spatially well-separated (and running away) atoms.

- Coordinate

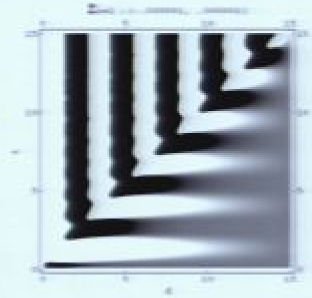
Each reduced density matrix is associated with a time-slicing scheme.

- Quantum field offers a natural choice of coordinate.
- In Minkowski time, the greater a, the longer disEnt time. (time dilation)
  In Rindler time, the greater a, the shorter disEnt time. (Unruh effect)
  the iff criterion (~ sgn Σ), however, could not be valid.



IV. Summary

- ## Spatial separation



In weak-coupling limit, with mutual influences negligible:

- Outside the light-cone, the relation between entanglement "strength" and distance between two detectors changes in time.
- Inside the light-cone, for sufficiently large separations, the disentanglement time varies in spatial separation.

- ## Role of event horizon

The event horizon helps to cut the higher order mutual influences.

- ## Outstanding issues

- strong mutual influences
- explicit time-slicing scheme dependence
- entanglement creation, residual entanglement

...