Title: Lorentz transformations in relativistic quantum theory?

Date: May 08, 2008 04:00 PM

URL: http://pirsa.org/08050031

Abstract: I will review relativistic quantum theory that is based on Wigner\'s unitary representations of the Poincare group, Dirac\'s forms of dynamics, and Newton-Wigner\'s definition of the position operator. Formulas will be derived that transform particle observables between different inertial reference frames. In the absence of interactions, these formulas coincide with Lorentz transformations from special relativity. However, when interaction is turned on, some deviations appear. The relationship of this result to the Currie-Jordan-Sudarshan \'no-interaction\' theorem will be mentioned, and the status of Lorentz transformations within quantum and classical theories of interacting particles will be discussed.

OUTLINE

- Relativistic quantum mechanics
- Non-interacting multiparticle systems
- Hamiltonian theory of interactions
- "Dynamical boost" hypothesis
- Interactions: instantaneous vs. retarded

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Wigner (1939)

- Hilbert space H of any quantum relativistic system carries a unitary representation of the Poincare group U_g
- Elementary inertial transformations are represented by unitary operators

Operator	Inertial transformation	Generator
e^{iHt}	time translations	total energy H
$e^{-i\vec{P}\vec{a}}$	space translations	total momentum $\vec{P}$
$e^{-i\vec{J}\vec{\phi}}$	rotations	total angular momentum $\vec{J}$
$e^{-ic\vec{K}\vec{\theta}}$	boosts	(center of energy) $\vec{K}$

- Generators of inertial transformations are Hermitian operators (total observables) that satisfy commutation relations of the Poincare Lie algebra

$$[\vec{P},H]=0 \qquad [\vec{K},H]=-i\vec{P} \qquad [K_i,P_j]=-\frac{i}{c^2}H\delta_{ij}\ldots$$

Wigner (1939)

- Elementary systems (particles) are described by irreducible unitary representations of the Poincare group.
- They are characterized by two Casimir invariants:

mass $m \geq 0$ and spin $s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

Newton-Wigner (1949)

Operator of (center of mass) position

$$\vec{R} = -\frac{c^2}{2}(H^{-1}\vec{K} + \vec{K}H^{-1}) - \frac{c^2\left[\vec{P}\times\vec{W}\right]}{MH(Mc^2+H)}$$

$$\vec{W} = \frac{1}{c}H\vec{J} - c\left[\vec{P}\times\vec{K}\right]$$ Pauli-Lubanski vector

Important commutators:

$$[J_i, R_j] = i\sum_{k=1}^{3}\varepsilon_{ijk}R_k \qquad [R_i, P_j] = i\delta_{ij}$$

$$[\vec{R}, \vec{S}] = 0 \qquad [R_i, R_j] = 0$$

$$[\vec{R}, H] = i\frac{c^2\vec{P}}{H} = i\vec{V}$$

$$\vec{R}(t) = e^{iHt}\vec{R}(0)e^{-iHt} = \vec{R}(0) + \vec{V}t$$

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N-particle systems

- Hilbert space $\mathrm{H} = \mathrm{H}_1 \otimes \mathrm{H}_2 \otimes \ldots \otimes \mathrm{H}_N$
- Which representation of the Poincare group acts in H?
- Representation

$$U_g^0 = U_{g1} \otimes U_{g2} \otimes \ldots \otimes U_{gN}$$

with generators

$$H_0 = h_1 + h_2 + \ldots + h_N$$

$$\vec{P}_0 = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_N$$

$$\vec{J}_0 = \vec{j}_1 + \vec{j}_2 + \ldots + \vec{j}_N$$

$$\vec{K}_0 = \vec{k}_1 + \vec{k}_2 + \ldots + \vec{k}_N$$

corresponds to N non-interacting particles.

Particle observables (properties) seen by two observers

Property	Observer at rest	Moving observer
Position	x_i	$x'_i = e^{-icK_{0x}\theta} x_i e^{icK_{0x}\theta}$
Velocity	v_i	$v'_i = e^{-icK_{0x}\theta} v_i e^{icK_{0x}\theta}$
Hamiltonian	H_0	$H'_0 = e^{-icK_{0x}\theta} H_0 e^{icK_{0x}\theta}$
trajectory	$x_i(t) = e^{iH_0 t} x_i e^{-iH_0 t} = x_i + v_i t$	$x'_i(t') = e^{iH'_0 t'} x'_i e^{-iH'_0 t'} = x'_i + v'_i t'$

Is it true that $x(t)$ and $x'(t')$ are related by Lorentz transformation?
YES! (E.S. Found. Phys. **32** (2002), 673)

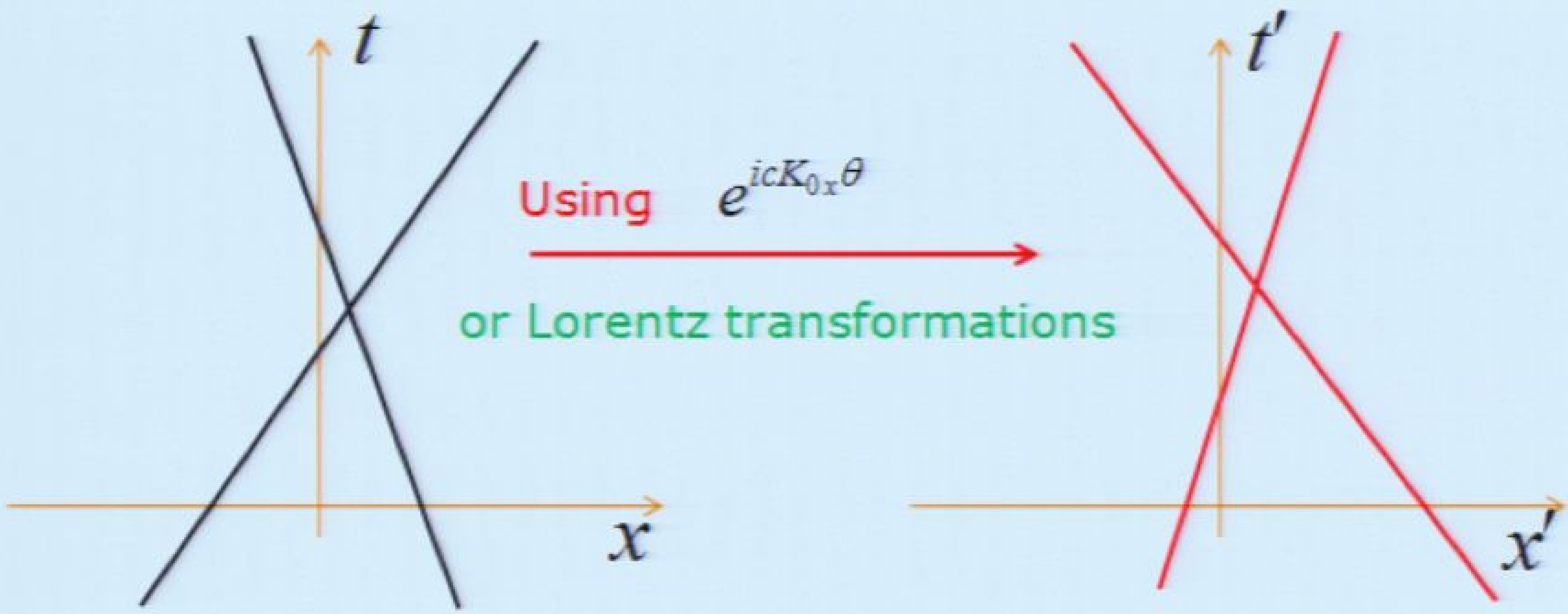
$$t' = t\cosh\theta - \frac{x}{c}\sinh\theta$$

$$x' = x\cosh\theta - ct\sinh\theta$$

Note that non-interacting operators H_0 and K_{0x} have been used in this derivation!

Trajectories of two non-interacting particles

In the reference frame at rest | In the moving reference frame



- Observables of particles $\vec{r}_i, \vec{p}_i, \vec{v}_i, \vec{s}_i, \ldots$ in a non-interacting system transform by Lorentz formulas.

What do I mean by Lorentz transformations?

Special-relativistic formulas that connect observables of physical systems (=collections of particles) viewed from two different reference frames in relative movement:

- Lorentz transformations for space and time coordinates of events (e.g., collisions) in multiparticle systems
- Lorentz transformations for the energy-momentum 4-vector
- Relativistic formula for the "addition" of velocities
- Length contraction formula
- "Time dilation" formula
- ...

What do I mean by boost transformations?

Transformation formulas obtained by applying boost operators $\exp\left(ic\vec{K}\vec{\theta}\right)$ from the Poincare group representation to particle observables.

For non-interacting particle systems
Lorentz transformations = boost transformations
What about interacting particle systems?

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Dirac (1949)

Classification of different ways (forms of dynamics) to introduce interactions in quantum (and classical) multiparticle systems. Important: commutation relations of the Poincare Lie algebra should be preserved.

Form of dynamics	Kinematical generators	Dynamical generators
General form		$H \neq H_0, \vec{P} \neq \vec{P}_0, \vec{J} \neq \vec{J}_0, \vec{K} \neq \vec{K}_0$
Instant form	$\vec{P}_0, \vec{J}_0$	$H = H_0 + V, \vec{K} = \vec{K}_0 + \vec{Z}$
Point form	$\vec{K}_0, \vec{J}_0$	$H = H_0 + W, \vec{P} = \vec{P}_0 + \vec{Y}$
Front form	...	...

Which form of dynamics is realized in nature?
We will use the instant form of dynamics.

Two methods for obtaining time-position coordinates in the moving reference frame

Using boost transformations	**Using Lorentz transformations**
$x_i'(t') = e^{iH't'} x_i' e^{-iH't'}$ $= e^{iH't'} e^{-icK_x\theta} x_i e^{icK_x\theta} e^{-iH't'}$ $= e^{-icK_x\theta} e^{iHt'} x_i e^{-iHt'} e^{icK_x\theta}$ $\neq x_i' + v_i' t'$	$t' = t\cosh\theta - \frac{x}{c}\sinh\theta$ $x' = x\cosh\theta - ct\sinh\theta$

Interacting operators H and K_x are used in this derivation!

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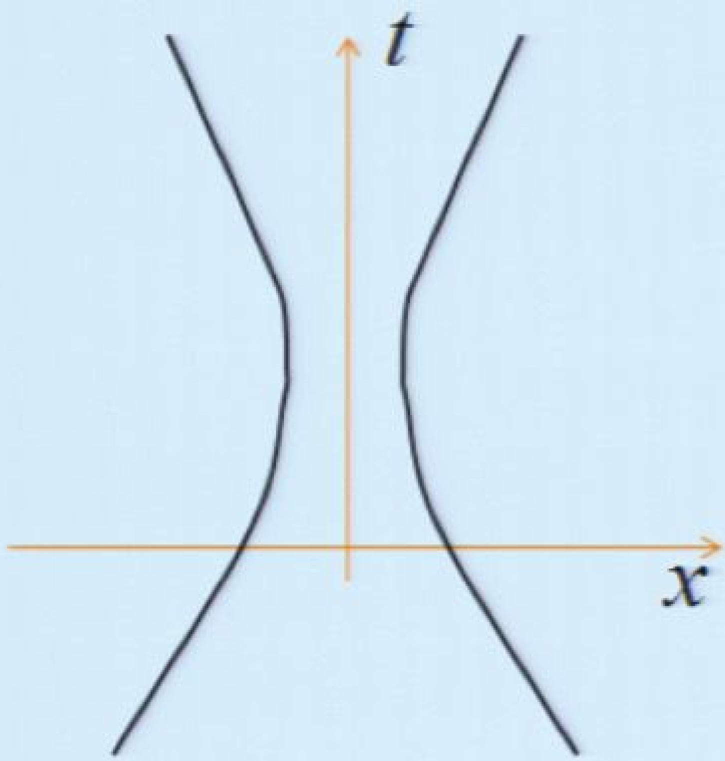
Currie, Jordan, Sudarshan (1963)
Trajectories of two interacting particles

In the reference frame at rest



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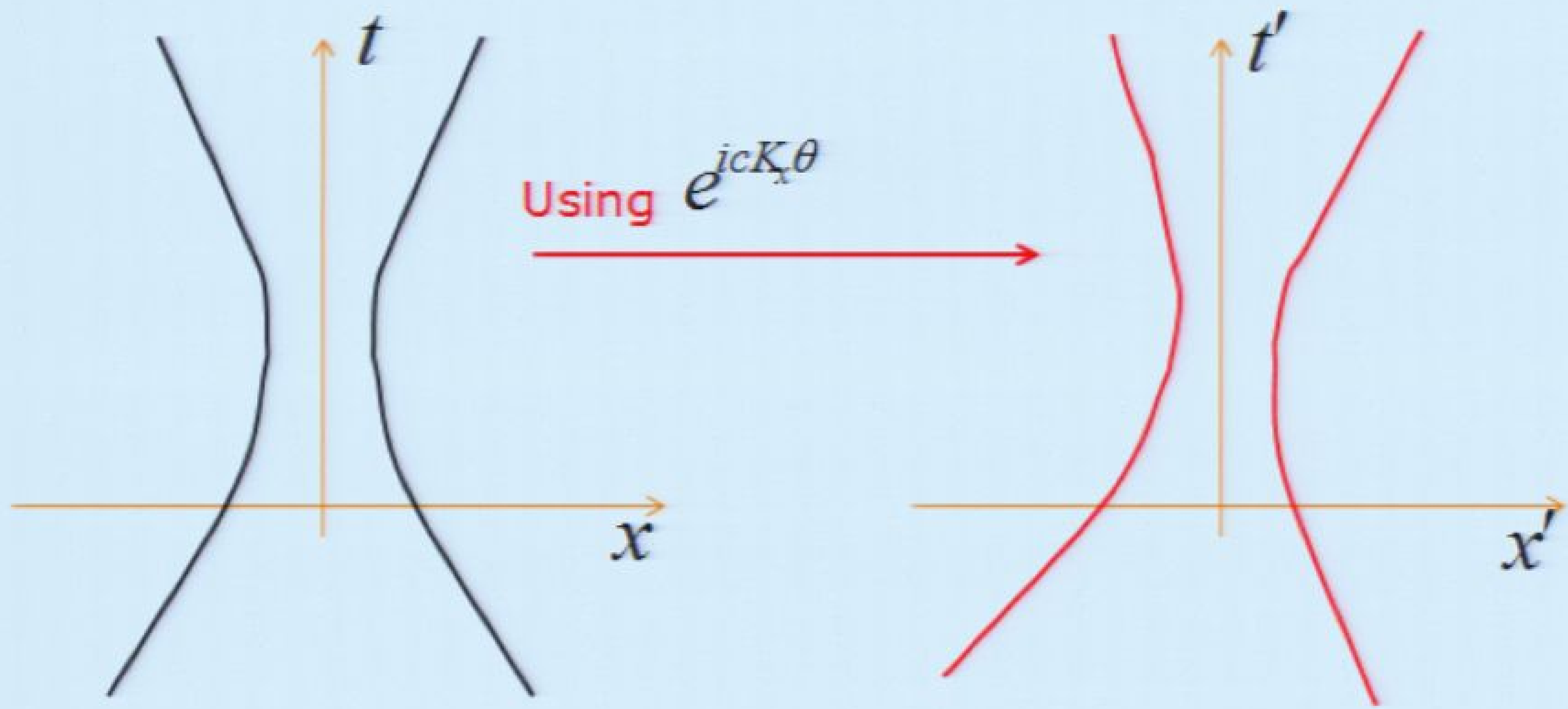
In the reference frame at rest | In the moving reference frame



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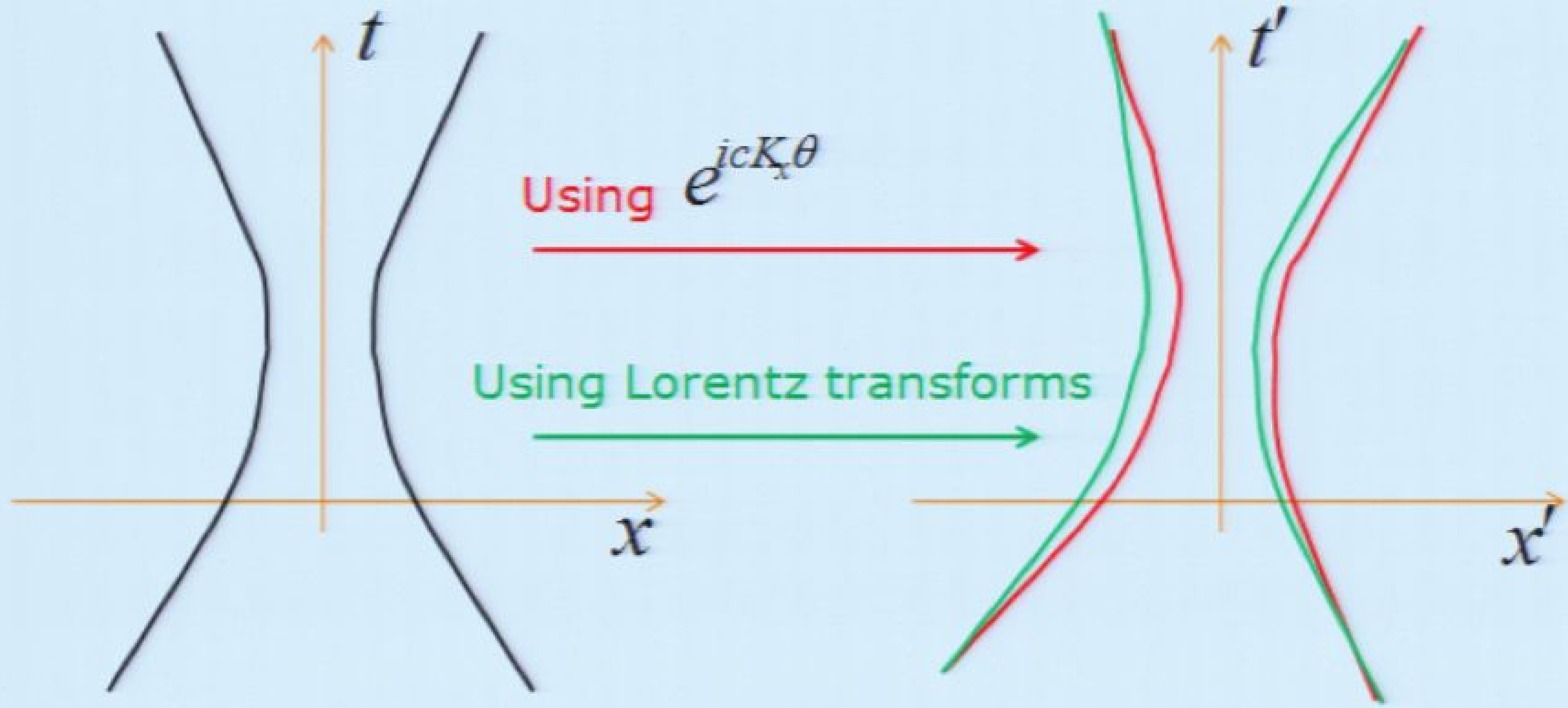
In the reference frame at rest — In the moving reference frame



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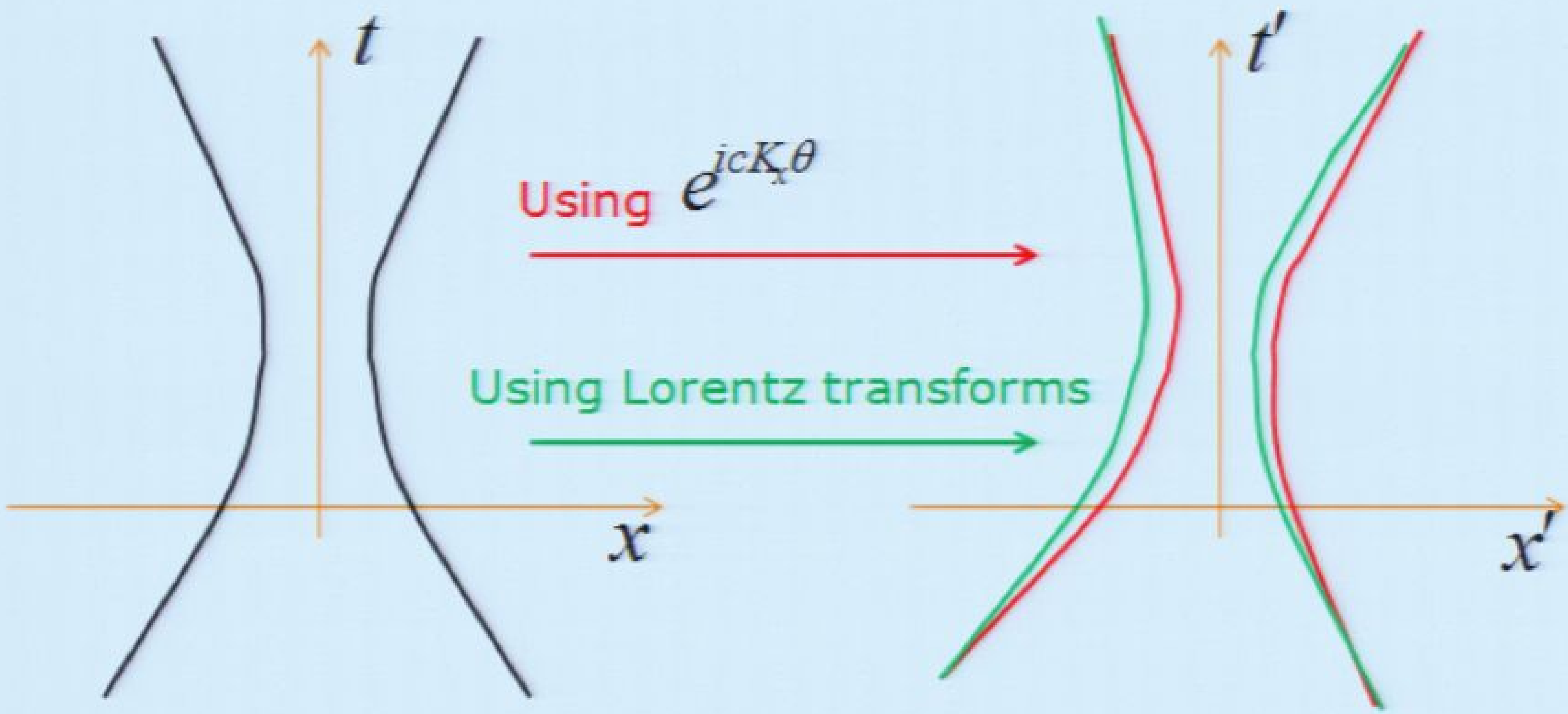

"No-interaction" theorem: trajectories of particles obey Lorentz transformation formulas only if the particles are non-interacting.

$$\vec{K} = h_1 \vec{r}_1 + h_2 \vec{r}_2 + \ldots$$

$$H = h_1 + h_2 + \ldots$$

$$h = \sqrt{m^2 + p^2}$$

$$\vec{K} = h_1 \vec{r}_1 + h_2 \vec{r}_2 - t\,\vec{P}$$

$$H = h_1 + h_2 + V$$

$$h = \sqrt{m^2 + p^2}$$

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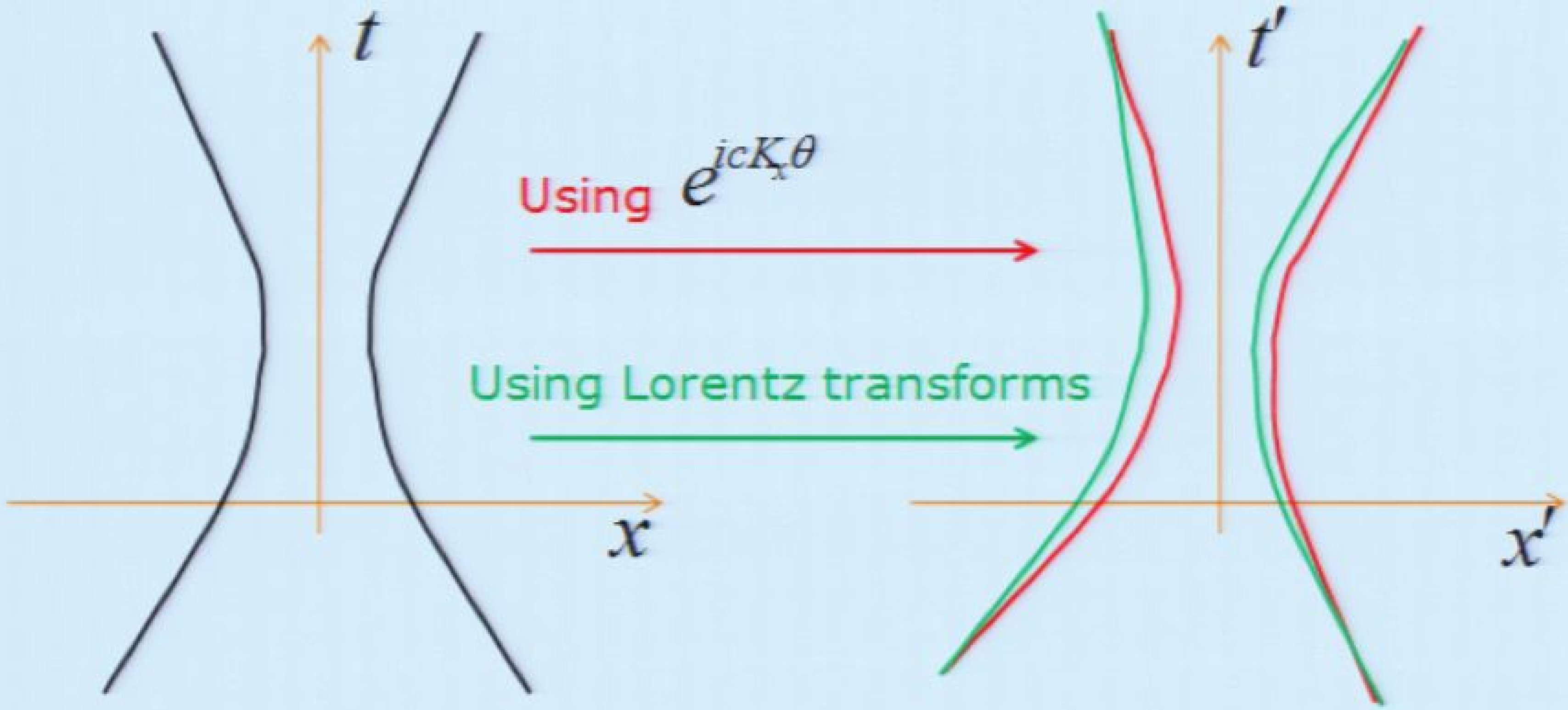
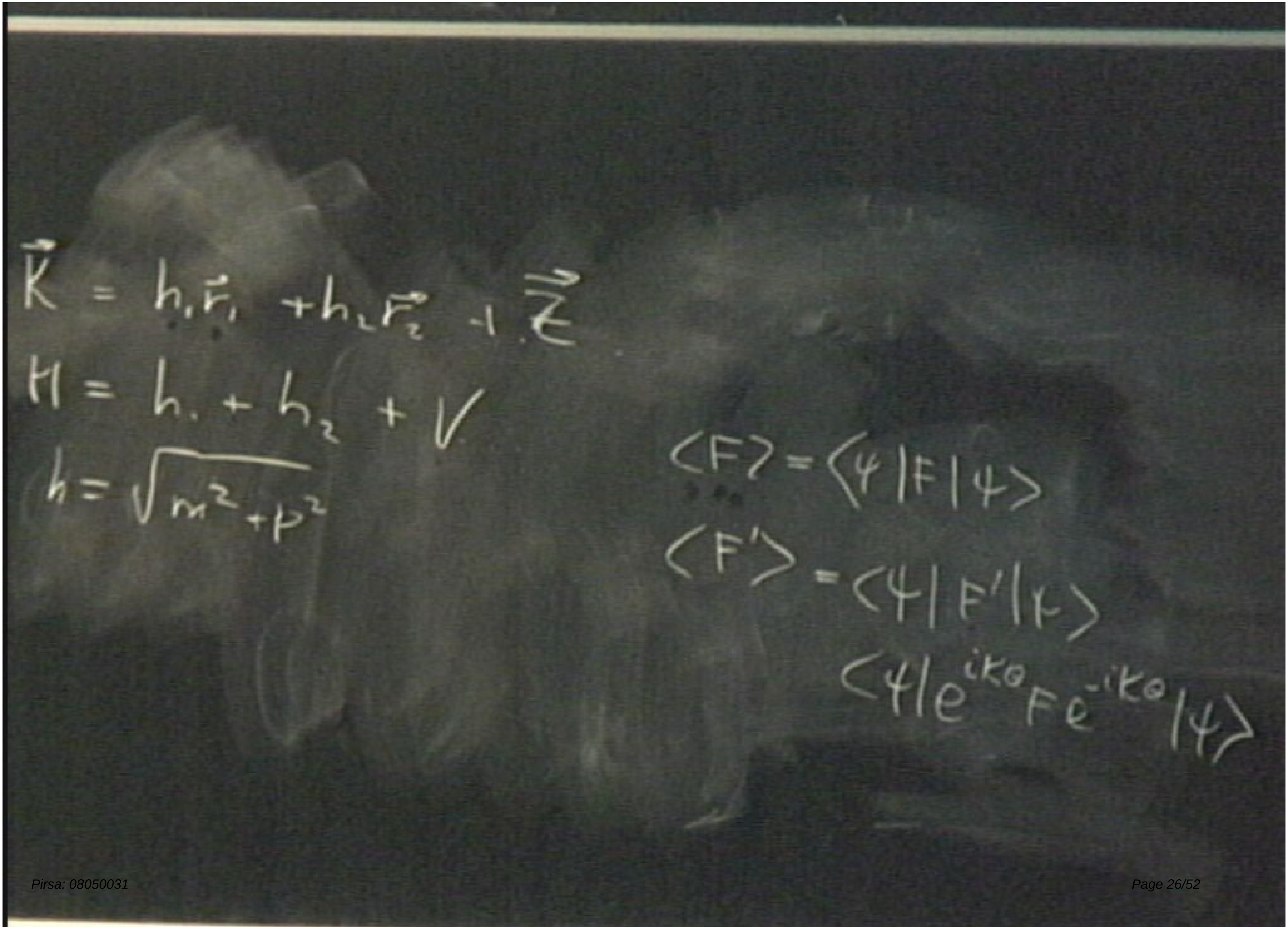


"No-interaction" theorem: trajectories of particles obey Lorentz transformation formulas only if the particles are non-interacting.



$\vec{K} = h_1 \vec{r}_1 + h_2 \vec{r}_2 + \vec{Z}$
$H = h_1 + h_2 + V$
$h = \sqrt{m^2 + p^2}$
$\langle F \rangle = \langle \psi | F | \psi \rangle$
$\langle F' \rangle = \langle \psi | F' | \psi \rangle$
$\langle \psi | e^{ik\theta} F e^{-ik\theta} | \psi \rangle$

Existing ideas about how to deal with the "no-interaction" paradox:

- Particles and their observables are not fundamental entities. Fields rule.
- Various versions of non-Hamiltonian dynamics. "Constraint dynamics".

A different idea:

"Dynamical boost" hypothesis: Lorentz transformations can be applied to observables of interacting particles only approximately. Exact transformations of observables are given by applying the interacting boost operator $\exp\left(ic\vec{K}\vec{\theta}\right)$

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Is there a good reason to trust Lorentz transformation formulas when applied to observables of individual particles in a multiparticle interacting system?

- All existing derivations of Lorentz transformations use (tacit) assumption about the absence of interactions.
- Lorentz transformations are assumed to be universal and kinematical, i.e., they do not depend on the nature of the physical system and on interactions acting there. This assumption contradicts the Poincare group properties.

How inertial transformations of observers affect observables of particles in multiparticle systems?

Inertial transformation	Non-interacting system		Interacting system	
	Generator	Type	Generator	Type
Space translation	$\vec{P}_0$	kinematical	$\vec{P}_0$	kinematical
Rotation	$\vec{J}_0$	kinematical	$\vec{J}_0$	kinematical
Time translation	H_0	kinematical	$H = H_0 + V$	dynamical
Boost	$\vec{K}_0$	kinematical	$\vec{K} = \vec{K}_0 + \vec{Z}$	dynamical?

- Dynamical time translations means that time evolution of particle observables is interaction-dependent and is not universal.
- Dynamical boost means that transformations of particle observables to the moving reference frame are interaction-dependent and do not follow universal Lorentz formulas.

Lorentz.pptx - Microsoft PowerPoint

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Click to add notes

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- Observables of free particles (e.g., photons) transform by Lorentz formulas.
- This is consistent with all existing experimental tests of special relativity.

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What are consequences of the "dynamical boost" hypothesis?

- Total observables of multiparticle systems $\vec{R}, H, \vec{P}, \vec{V}, \vec{S}, \ldots$ transform exactly by Lorentz formulas independent on interactions acting in the system.
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Michelson-Morley	speed of photons
Light speed from moving sources	speed of photons
Relativistic energy-momentum relationship	total energy and momentum of isolated systems or free particles
Slowdown of decays of moving particles or rates of moving clocks	interacting time evolution

What are consequences of the "dynamical boost" hypothesis?

- Boost transformations of observables of individual particles (or subsystems) in the interacting system are interaction-dependent. Generally they are different from Lorentz formulas.
- Moving rods made of different materials (wood and tungsten) may contract differently.
- Moving clocks of different designs may slow-down by different amounts (see yesterday's seminar on particle decays).
- Deviations from Lorentz formulas in interacting systems are too small to be observed in existing experiments.

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What are consequences of the "dynamical boost" hypothesis?

- Since there is no universality in transformations of time and length intervals, Minkowski space-time becomes an approximate concept.
- There is no ban on superluminal (even instantaneous) propagation of interactions.

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- Relativistic quantum mechanics
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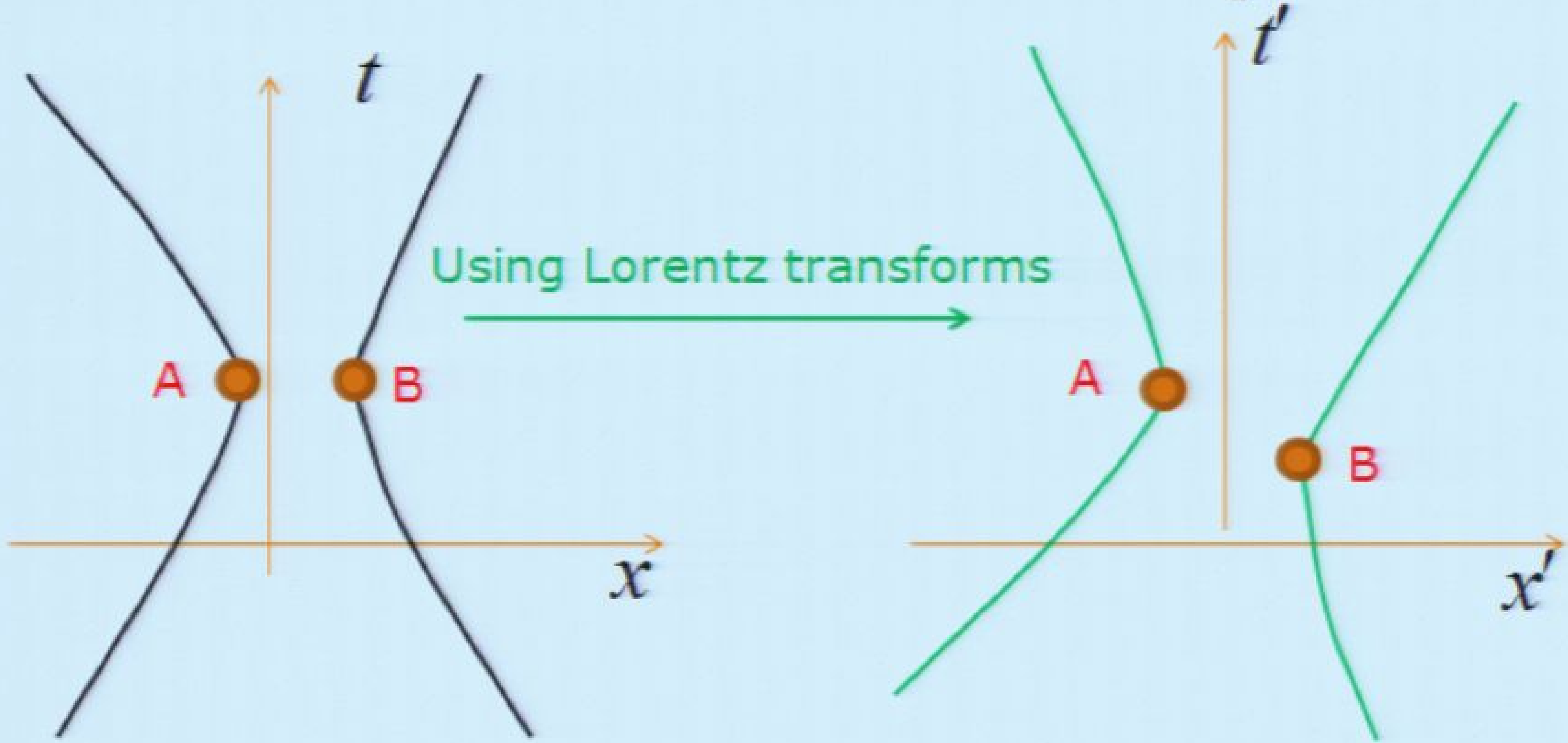
Special relativity

Event A is cause. Event B is effect. Interaction is instantaneous.

In the reference frame at rest | In the moving reference frame



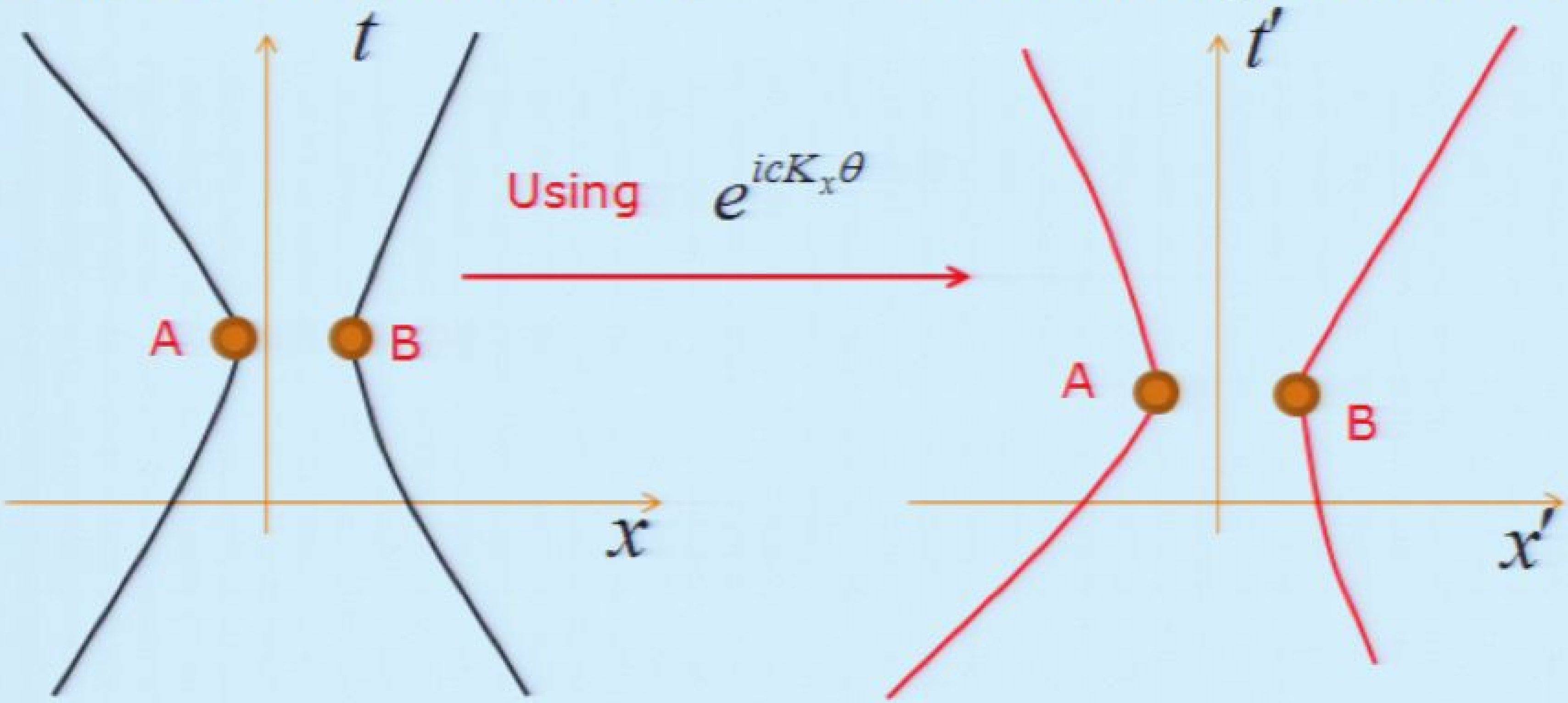
The effect B occurs earlier than the cause A

Superluminal propagation of interactions is forbidden!

Relativistic Hamiltonian dynamics

Event A is cause. Event B is effect. Interaction propagates instantaneously.

In the reference frame at rest In the moving reference frame



Due to dependence of the transformation law on interaction, events (A,B) simultaneous in the reference frame at rest remain simultaneous in the moving frame. Causality is not violated. Instantaneous propagation of interactions is not forbidden.

- There are no experiments that clearly demonstrate retarded character of interactions (electromagnetic and gravitational). (Radio transmission is propagation of particles – photons - rather than direct interaction of charges in the emitter and the receiver. So, it doesn't count as an evidence of retardation.)
- There are numerous experiments which can be interpreted as evidence of superluminal propagation of electromagnetic interactions. Giakos & Ishii (1991), Enders & Nimtz (1993), Steinberg & Kwiat & Chiao (1993), Ranfagni & Mugnai (1996),...

If the "dynamical boost" hypothesis is correct and the Minkowski space-time is not fundamental, then a significant portion of modern theoretical physics must be re-evaluated:

- Classical electrodynamics
  (field-less approach: E.S. arXiv:0803.1326)
- Relativistic (quantum) theory of gravity without curved space-time
  (direct gravitational potentials: E.S. physics/0612019)
- Quantum field theory
  (dressed particle approach: E.S. physics/0504062)

Which experiments can confirm/reject the "dynamical boost" hypothesis?

Sincere thanks
to everybody at

$$\pi$$

for your warm hospitality!

The End

System of two particles $(\vec{r}_1, \vec{p}_1)$ and $(\vec{r}_2, \vec{p}_2)$ interacting via instantaneous force in the reference frame at rest.

Force acting on the particle 1 at time t is a function of observables of both particles taken at the same time instant t

$$\vec{f}_1(t) = \frac{d}{dt}\vec{p}_1(t) = \{\vec{p}_1(t), H\} = \vec{f}_1(\vec{r}_1(t), \vec{p}_1(t), \vec{r}_2(t), \vec{p}_2(t))$$

Let us find the force acting on the particle 1 in the moving reference frame at time t′ measured by this frame's clock.

$$\vec{f}_1'(t') = \frac{d}{dt'}\vec{p}_1'(t') = \{\vec{p}_1'(t'), H'\}$$

where $H' = e^{-ic\vec{K}\vec{\theta}} H e^{ic\vec{K}\vec{\theta}}$ is the Hamiltonian in the moving frame

$$\vec{p}_1'(t') = e^{iH't'} e^{-ic\vec{K}\vec{\theta}} \vec{p}_1 e^{ic\vec{K}\vec{\theta}} e^{-iH't'} = e^{-ic\vec{K}\vec{\theta}} e^{iHt'} \vec{p}_1 e^{-iHt'} e^{ic\vec{K}\vec{\theta}}$$

$$\vec{r}_1'(t') = e^{-ic\vec{K}\vec{\theta}} e^{iHt'} \vec{r}_1 e^{-iHt'} e^{ic\vec{K}\vec{\theta}}$$

$$\vec{p}_2'(t') = e^{-ic\vec{K}\vec{\theta}} e^{iHt'} \vec{p}_2 e^{-iHt'} e^{ic\vec{K}\vec{\theta}}$$

$$\vec{r}_2'(t') = e^{-ic\vec{K}\vec{\theta}} e^{iHt'} \vec{r}_2 e^{-iHt'} e^{ic\vec{K}\vec{\theta}}$$

$$\vec{f}_1'(t') = e^{-ic\vec{K}\vec{\theta}} \left\{ e^{iHt'} \vec{p}_1 e^{-iHt'}, H \right\} e^{ic\vec{K}\vec{\theta}}$$

$$= e^{-ic\vec{K}\vec{\theta}} \left\{ \vec{p}_1(t'), H \right\} e^{ic\vec{K}\vec{\theta}}$$

$$= e^{-ic\vec{K}\vec{\theta}} \vec{f}_1(\vec{r}_1(t'), \vec{p}_1(t'), \vec{r}_2(t'), \vec{p}_2(t')) e^{ic\vec{K}\vec{\theta}}$$

$$= e^{-ic\vec{K}\vec{\theta}} e^{iHt'} \vec{f}_1(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2) e^{-iHt'} e^{ic\vec{K}\vec{\theta}}$$

$$= \vec{f}_1(\vec{r}_1'(t'), \vec{p}_1'(t'), \vec{r}_2'(t'), \vec{p}_2'(t'))$$

The force acting on the particle 1 in the moving frame is a function of observables of the two particles taken at the same time instant t′.

Interaction that was instantaneous in the reference frame at rest remains instantaneous in the moving reference frame.

The End