Title: Relativistic quantum description of particle decays

Date: May 07, 2008 11:00 AM

URL: http://pirsa.org/08050030

Abstract: Wigner-Dirac relativistic quantum theory is applied to decay laws of an unstable particle in different reference frames. It is shown that decay slows down from the point of view of the moving observer, as expected. However, small deviations from Einstein\'s time dilation formula are also found. The origin of these deviations is discussed, as well as possibilities for their experimental detection.

Relativistic quantum description of particle decays and time dilation

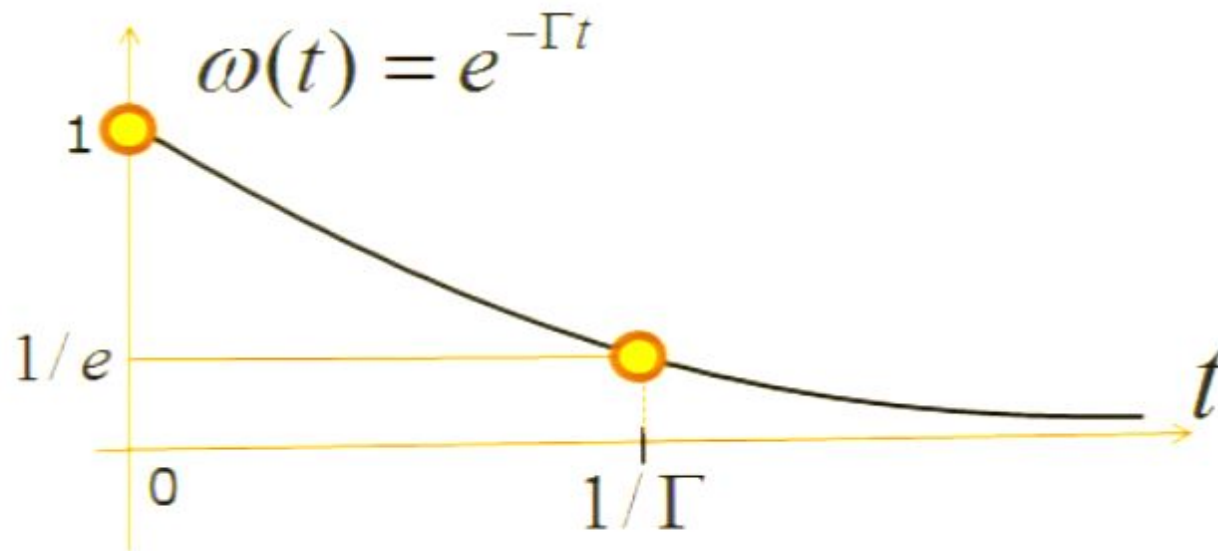Eugene V. Stefanovich

Perimeter Institute, May 7, 2008

OUTLINE

- Decay dilation in special relativity
- Relativistic quantum mechanics
- Decay law of particles at rest
- Decay law of moving particles
- Discussion

OUTLINE

- Decay dilation in special relativity

Decay from the point of view of observer at rest

$$\omega(t) = e^{-\Gamma t}$$

Two events:

$$(x_1 = 0, t_1 = 0)$$
$$(x_2 = 0, t_2 = 1/\Gamma)$$

Decay from the point of view of moving observer

$$v = c \tanh \theta \qquad \cosh \theta = 1/(1 - v^2/c^2)^{1/2} \geq 1$$
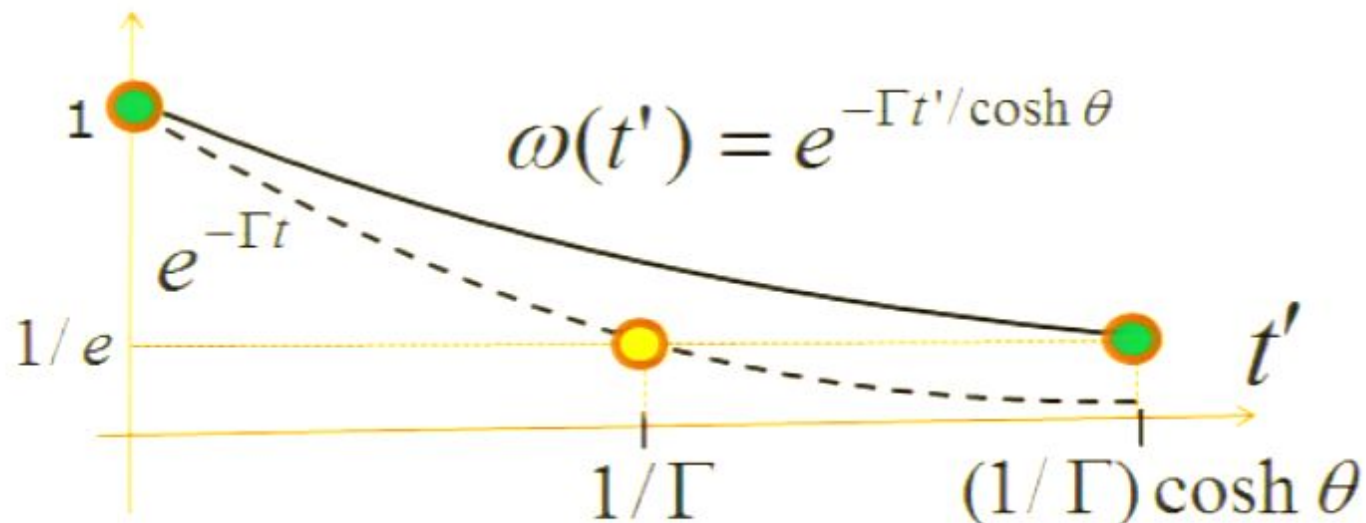
Lorentz transformations

$$t' = t \cosh \theta - \frac{x}{c} \sinh \theta$$

$$x' = x \cosh \theta - ct \sinh \theta$$

Coordinates of 2 events

$$(x'_1 = 0,\; t'_1 = 0)$$

$$\left(x'_2 = -\frac{c}{\Gamma} \sinh \theta,\; t'_2 = \frac{1}{\Gamma} \cosh \theta\right)$$

$$\omega(t') = e^{-\Gamma t'/\cosh \theta}$$

$$e^{-\Gamma t}$$

Universal slowdown of the decay law

Einstein's time dilation formula

$$\omega_\theta(t) = \omega_0(t / \cosh\theta)$$

was found correct to **0.1%** in decays of fast ($\cosh\theta = 29.3$) muons: Bailey et al. (1977), Farley (1992).

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Decay is a typical quantum process.

Idea: construct a quantum relativistic description of decay
and compare decay laws in different frames with the
special-relativistic time dilation formula

Shall we obtain the same result?

OUTLINE

- Decay dilation in special relativity
- Relativistic quantum mechanics

Wigner (1939)

- Hilbert space **H** of any quantum relativistic system carries a unitary representation of the Poincare group U_g
- Elementary inertial transformations are represented by unitary operators

Operator	Inertial transformation	Generator
e^{iHt}	time translations	total energy H
$e^{-i\vec{P}\vec{a}}$	space translations	total momentum $\vec{P}$
$e^{-i\vec{J}\vec{\phi}}$	rotations	total angular momentum $\vec{J}$
$e^{-ic\vec{K}\vec{\theta}}$	boosts	(center of energy) $\vec{K}$

- Generators of inertial transformations are Hermitian operators (total observables) that satisfy commutation relations of the Poincare Lie algebra

$$[\vec{P},H]=0 \qquad [\vec{K},H]=-i\vec{P} \qquad [K_i,P_j]=-\frac{i}{c^2}H\delta_{ij}\ldots$$

Wigner (1939)

- Elementary systems (particles) are described by irreducible unitary representations of the Poincare group.
- They are characterized by two Casimir invariants:

mass $m \geq 0$ and spin $s = 0, \dfrac{1}{2}, 1, \dfrac{3}{2}, \ldots$

N-particle systems

- Hilbert space $H = H_1 \otimes H_2 \otimes \ldots \otimes H_N$

- Which representation of the Poincare group acts in H?

$$U_g^0 = U_{g1} \otimes U_{g2} \otimes \ldots \otimes U_{gN}$$

with generators $H_0, \vec{P}_0, \vec{J}_0, \vec{K}_0$ corresponds to N non-interacting particles.

Dirac (1949)

Classification of different ways (**forms of dynamics**) to introduce interactions in quantum (and classical) multiparticle systems. **Important**: commutation relations of the Poincare Lie algebra should be preserved.

Form of dynamics	Kinematical generators	Dynamical generators
General form		$H \neq H_0, \vec{P} \neq \vec{P}_0, \vec{J} \neq \vec{J}_0, \vec{K} \neq \vec{K}_0$
Instant form	$\vec{P}_0, \vec{J}_0$	$H = H_0 + V, \vec{K} = \vec{K}_0 + \vec{Z}$
Point form	$\vec{K}_0, \vec{J}_0$	$H = H_0 + W, \vec{P} = \vec{P}_0 + \vec{Y}$
Front form	...	...

Which form of dynamics is realized in nature?
We will use the instant form of dynamics.

Systems in which the number of particles can change

Fock space $\quad H = \underbrace{|0\rangle}_{\substack{vacuum \\ no\ particles}} \oplus \underbrace{H_1 \oplus H_2}_{1-particle\ spaces} \oplus \underbrace{(H_1 \otimes H_2)}_{2-particle\ spaces} \oplus ...$
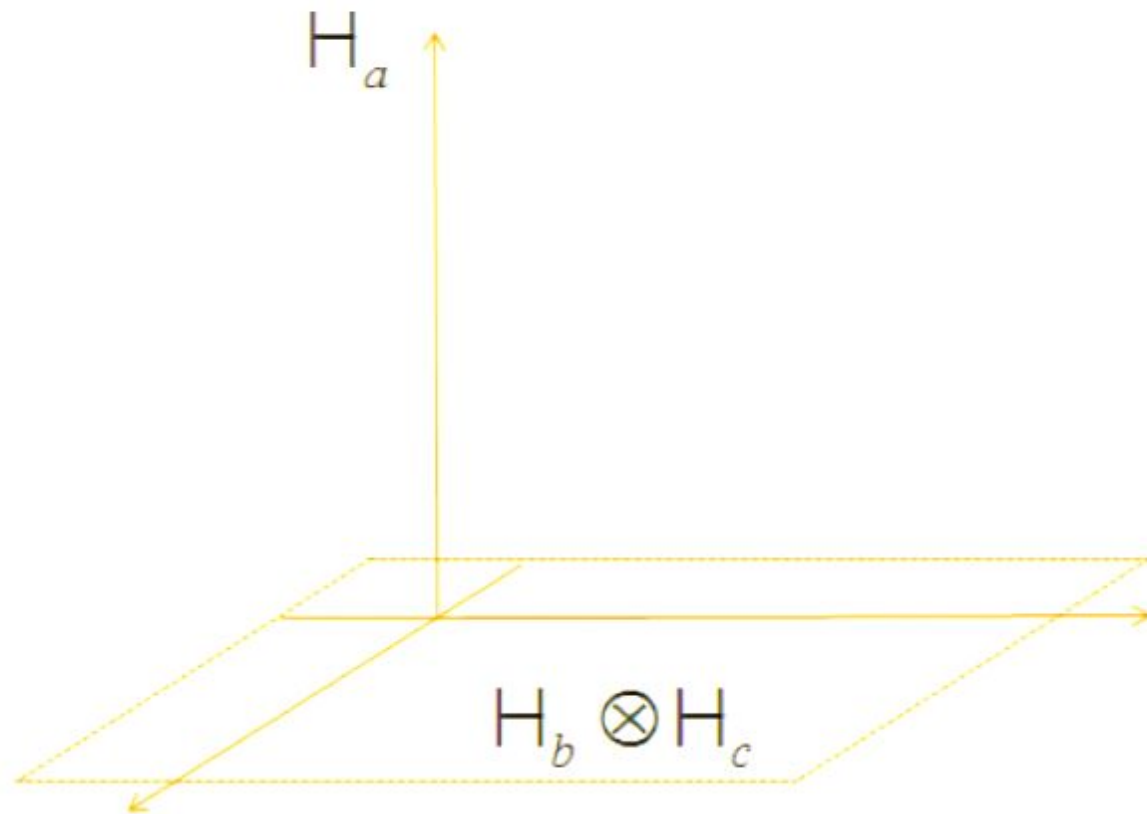
The interacting representation U_g of the Poincare group in **H** can cross borders between different sectors in the Fock space, i.e., particles can be created and annihilated via time evolution. That's what we need to describe decays.

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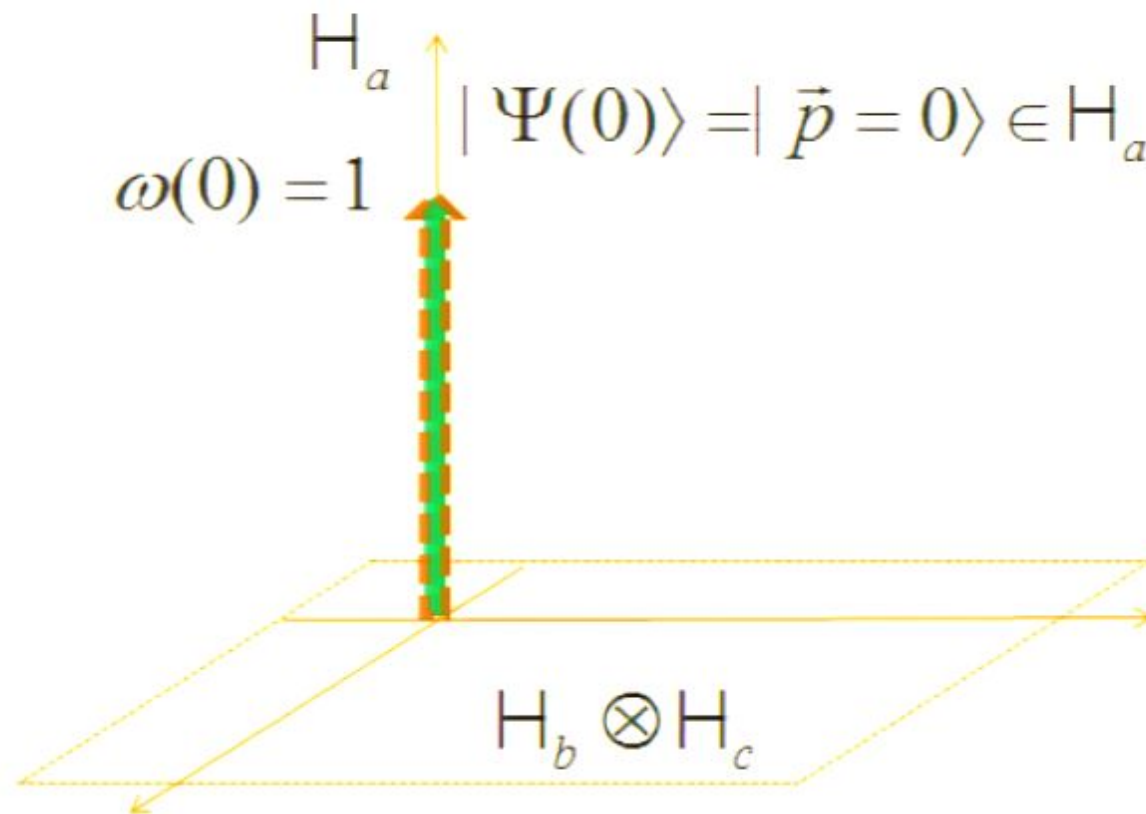- Decay law of particles at rest

Quantum description of the decay $a \to b + c$ at rest

Two relevant sectors of the Fock space $H = H_a \oplus (H_b \otimes H_c)$

H_a

$H_b \otimes H_c$

Quantum description of the decay $a \rightarrow b + c$ at rest

Two relevant sectors of the Fock space $H = H_a \oplus (H_b \otimes H_c)$

H_a

$\omega(0) = 1$

$|\Psi(0)\rangle = |\vec{p} = 0\rangle \in H_a$

$H_b \otimes H_c$

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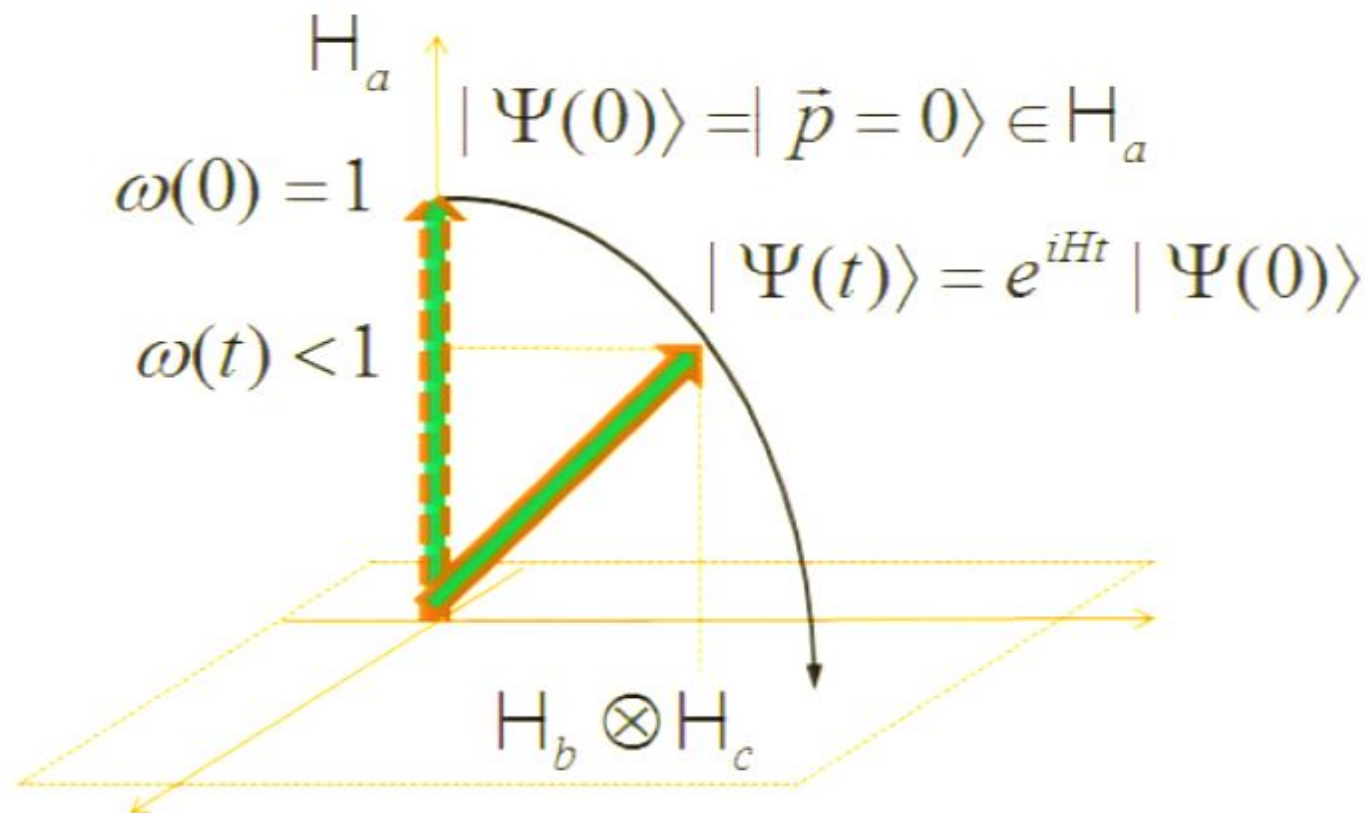
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$\omega(t) < 1$

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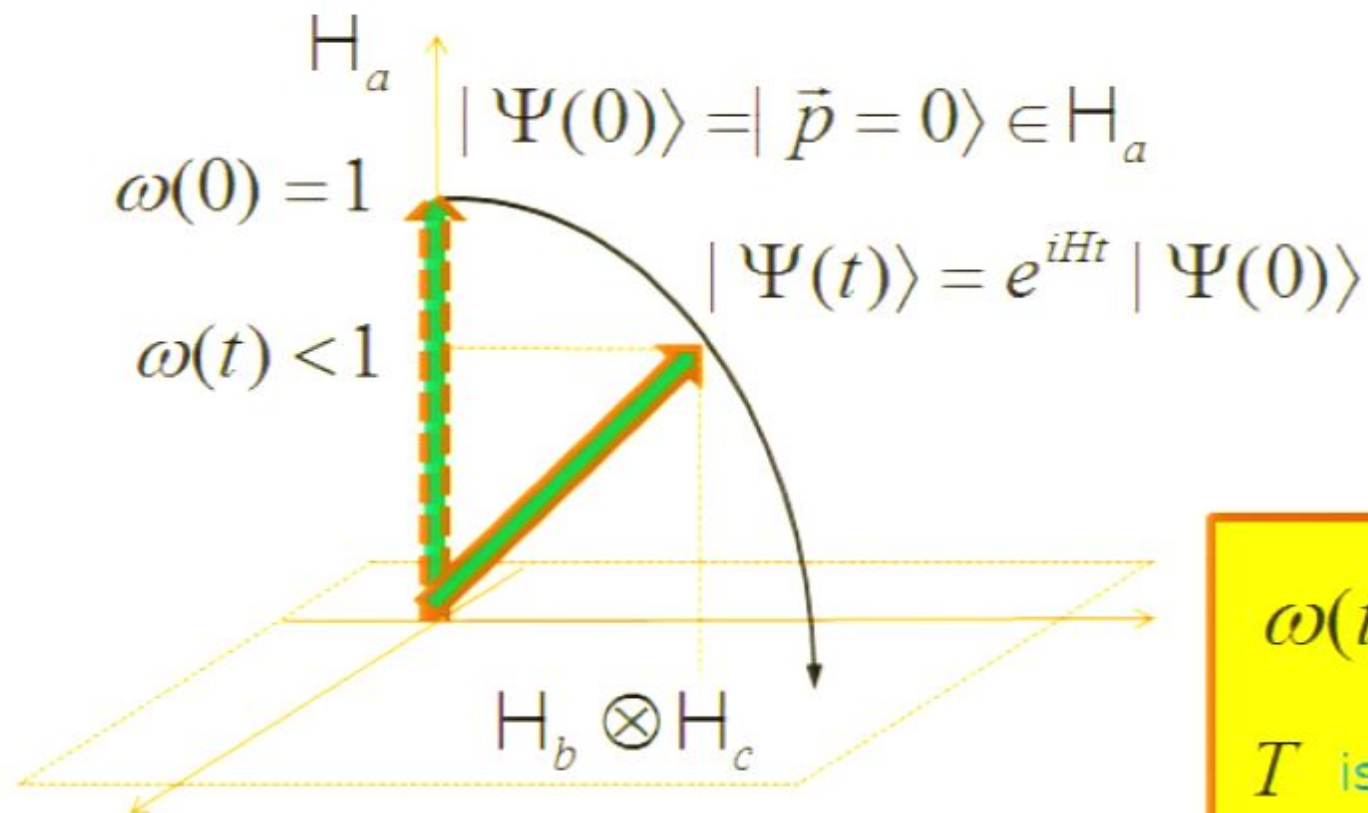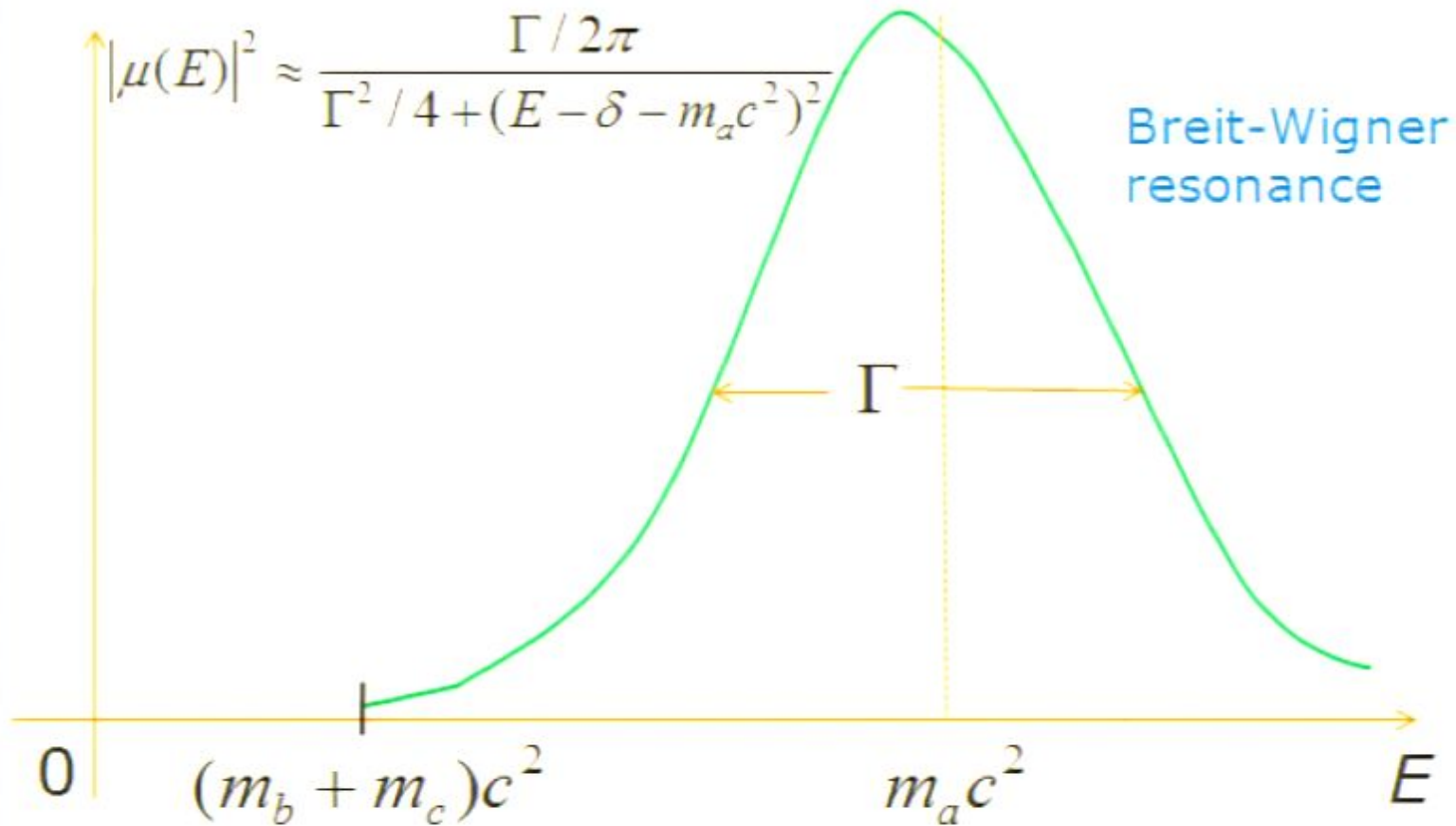Two relevant sectors of the Fock space $H = H_a \oplus (H_b \otimes H_c)$

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$|\Psi(0)\rangle = |\vec{p} = 0\rangle \in H_a$

$|\Psi(t)\rangle = e^{iHt} |\Psi(0)\rangle$

$$\omega(t) = \left\| T |\Psi(t)\rangle \right\|^2$$

T is projection on H_a

$$T = \int d\vec{p} \, |\vec{p}\rangle\langle\vec{p}|$$

$$H\,|\,E\rangle = E\,|\,E\rangle \qquad\qquad |\,\vec{p}=0\rangle = \int dE\,\mu(E)\,|\,E\rangle$$

$$|\mu(E)|^2 \approx \frac{\Gamma/2\pi}{\Gamma^2/4 + (E - \delta - m_a c^2)^2}$$

Breit-Wigner
resonance

Γ

0

$(m_b + m_c)c^2$

$m_a c^2$

E

$$\omega(t) = \left|\int dE\,|\mu(E)|^2\,e^{iEt}\right|^2 \approx e^{-\Gamma t}$$

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Decay law for the particle at rest

$$\omega_0(t) = \left| \int dE |\mu(E)|^2 e^{iEt} \right|^2 \approx e^{-\Gamma t}$$

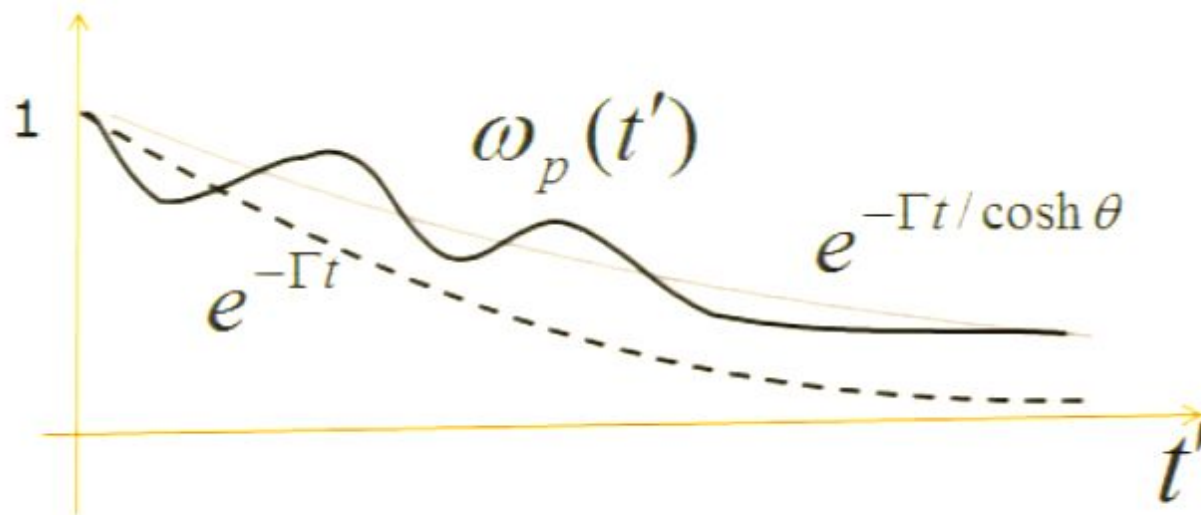How are we going to describe the decay of a moving particle?

$$| \Psi_p(0) \rangle = | \vec{p} = mc \frac{\vec{\theta}}{\theta} \sinh\theta \rangle = e^{ic\vec{K}_0\vec{\theta}} | \vec{p} = 0 \rangle \in H_a$$

$$| \Psi_p(t) \rangle = e^{iHt} | \Psi_p(0) \rangle$$

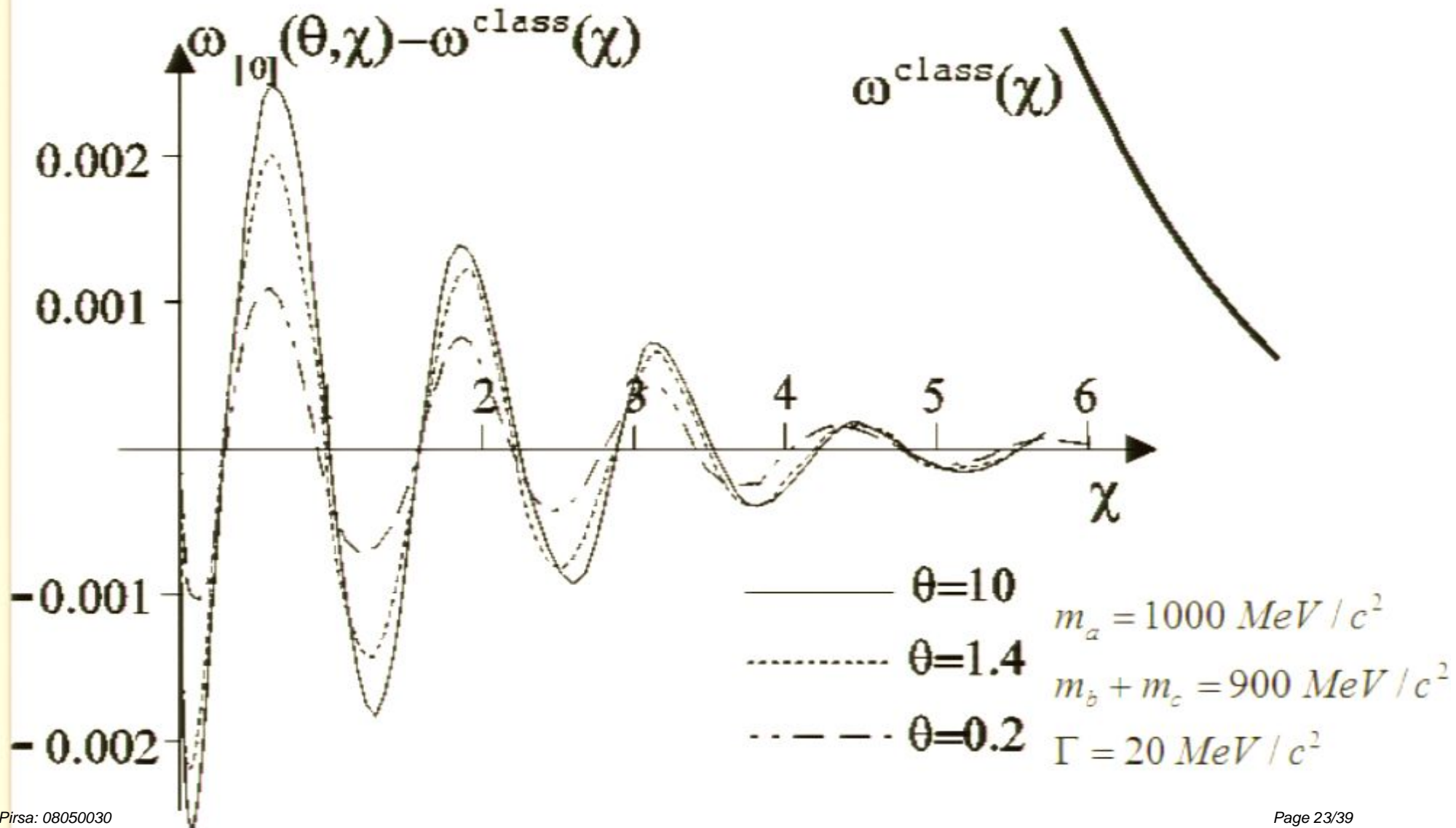$$\omega_p(t) = \left\| T e^{iHt} e^{ic\vec{K}_0\vec{\theta}} | \vec{p} = 0 \rangle \right\|^2$$

$$\omega_p(t) = \left| \int dE |\mu(E)|^2 e^{it\sqrt{E^2 + p^2 c^2}} \right|^2 \neq e^{-\Gamma t / \cosh\theta} = \omega_0(t / \cosh\theta)$$

Corrections to Einstein's time dilation formula for the decay law of a moving particle (grossly exaggerated)

$$\omega_p(t')$$

$$e^{-\Gamma t/\cosh\theta}$$

$$e^{-\Gamma t}$$

$$t'$$

Best measurements of the decay law of fast muons in Bailey et al. (1977) . The accuracy of Einstein's time dilation formula was verified to 0.1%.

Corrections to Einstein's time dilation formula for the decay law of a moving particle

$$\omega_{[0]}(\theta, \chi) - \omega^{class}(\chi)$$

$$\omega^{class}(\chi)$$

—————— $\theta=10$

············ $\theta=1.4$

·—·—·—· $\theta=0.2$

$m_a = 1000 \ MeV/c^2$

$m_b + m_c = 900 \ MeV/c^2$

$\Gamma = 20 \ MeV/c^2$

Conclusion: Einstein's time dilation formula is not an accurate description of the decay of moving particles

$$\omega_p(t) \neq \omega_0(t/\cosh\theta)$$

"Time dilation" is not universal, but depends on the strength of interaction causing the decay.

This is disturbing: we used a relativistically invariant theory and obtained results contradicting special relativity!

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What could be wrong?

- Could there be an error in calculations? Unlikely. Three independent checks: E.S. (1996), Khalfin (1997), Shirokov (2004)

- We have calculated the decay law of a moving particle rather than the decay law of a particle viewed from a moving reference frame. There are subtle differences between these two situations, but numerical quantum-mechanical results are close (E.S. physics/0603043).

One can easily prove that in the instant form of dynamics the Einstein's time dilation formula **cannot** be exactly reproduced. Suppose that this formula is true, i.e.,

$$\omega_\theta(0) = \omega_0(0/\cosh\theta) = \omega_0(0)$$

In QM this means that subspace H_a is invariant with respect to boosts

$$[T, K_x] = 0$$

Then, using Jacobi identity

$$[T, H] = ic^2[T, [P_x, K_x]] = -ic^2\underbrace{[P_x, [K_x, T]]}_{=0} - ic^2\underbrace{[K_x, [T, P_x]]}_{=0} = 0$$

which means that subspace H_a is invariant with respect to time translations. **So, there is no decay!**

What else could be wrong?

- We used Dirac's instant form of dynamics. Perhaps other forms give "better" results? No. In the point form dynamics the decay of moving particles accelerates instead of slowing down!

- The exponential decay law was used only for presentation purposes here. Corrections to the usual time dilation formula appear for arbitrary functions $|\mu(E)|^2$ Our result is general.

- Could it be that special relativity is valid only approximately?

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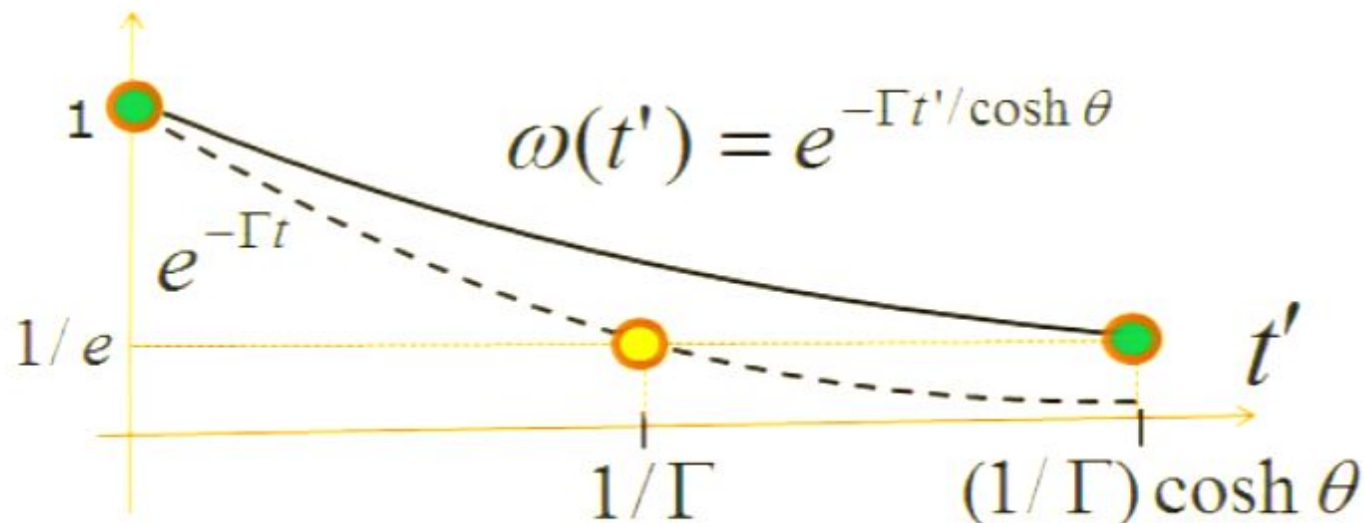$$v = c\tanh\theta \qquad \cosh\theta = 1/(1 - v^2/c^2)^{1/2} \geq 1$$

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$e^{-\Gamma t}$

1

$1/e$

$1/\Gamma$

$(1/\Gamma)\cosh\theta$

t'

Universal slowdown of the decay law
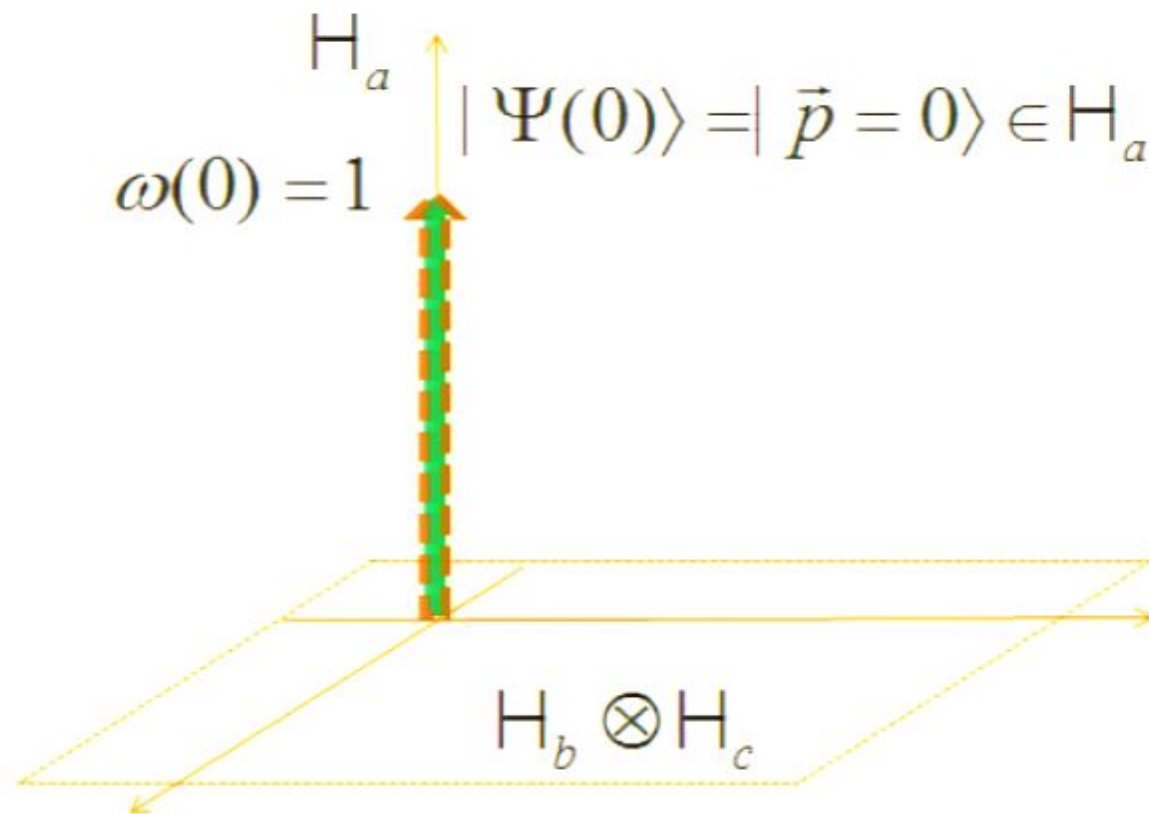
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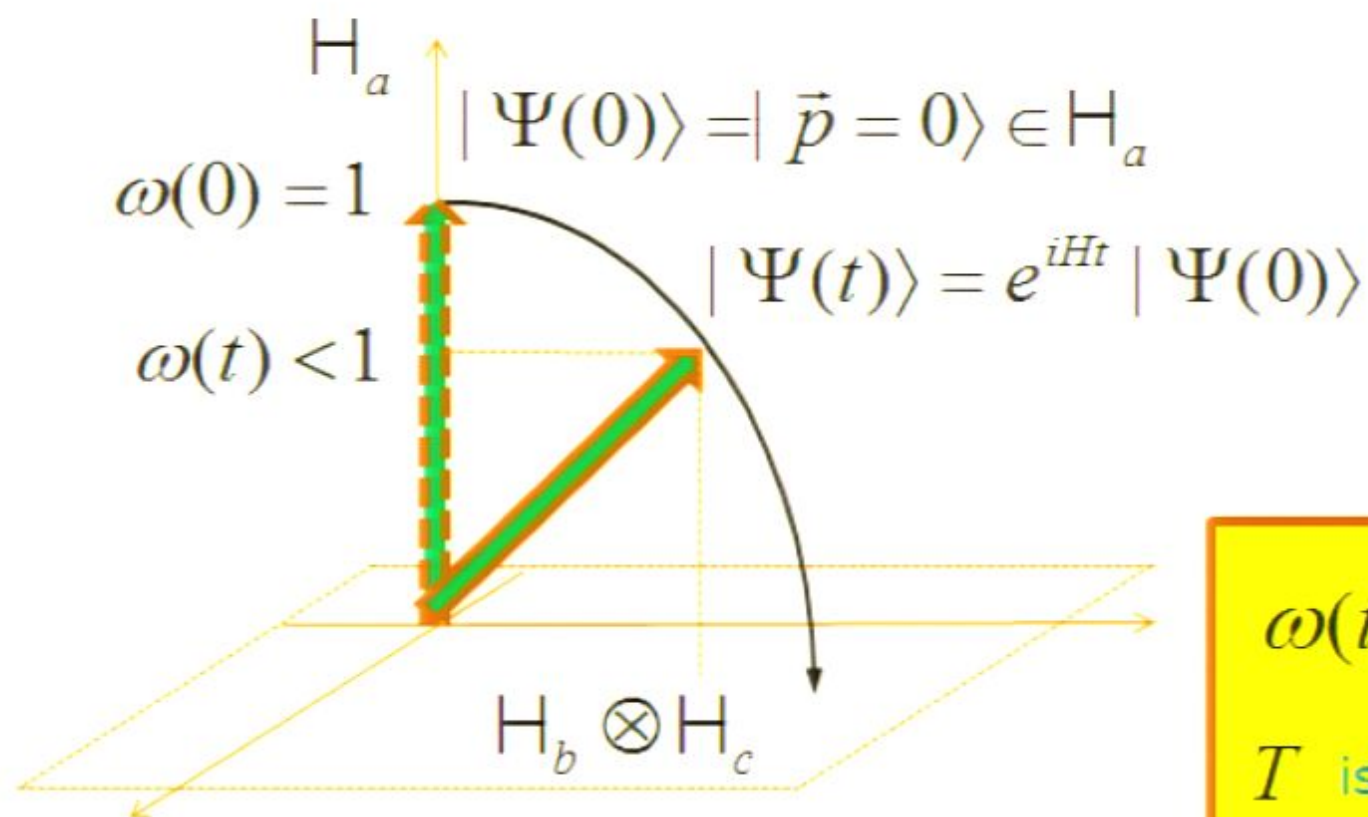Two relevant sectors of the Fock space $H = H_a \oplus (H_b \otimes H_c)$

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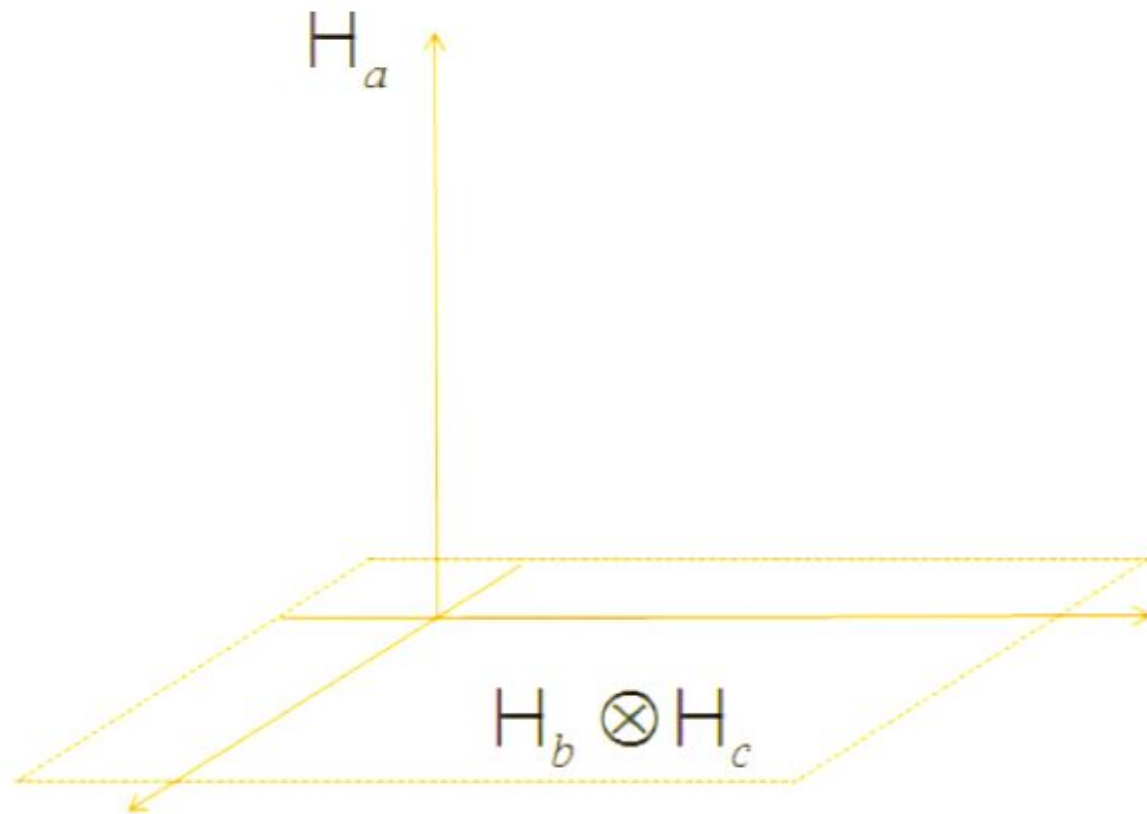
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