Title: Dressed particle approach in quantum field theory

Date: May 05, 2008 02:00 PM

URL: http://pirsa.org/08050029

Abstract: I will review an old (Greenberg and Schweber, 1958) and undeservedly forgotten idea in quantum field theory. This idea allows one to reformulate QFT as a Hamiltonian theory of physical (rather than bare) particles and their direct interactions. The dressed particle approach is scattering-equivalent to the traditional one, however it doesn\'t require renormalization and may provide a valuable tool for calculations of wave functions of bound states and time evolution.

Dressed particle approach in Quantum Field Theory

Eugene V. Stefanovich

Perimeter Institute, May 5, 2008

OUTLINE

- Scattering equivalence of Hamiltonians
- Quantum electrodynamics. Renormalization
- Dressed particle approach
- Questions for discussion

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Hamiltonian

$$H = H_0 + V$$

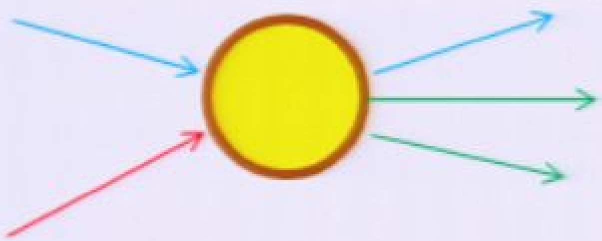
Time evolution operator

$$U(t' \leftarrow t) = e^{iH(t'-t)}$$

$$|\Psi(t')\rangle = U(t' \leftarrow t)|\Psi(t)\rangle$$

$$U^{-1}(t' \leftarrow t) = U(t \leftarrow t')$$

Scattering



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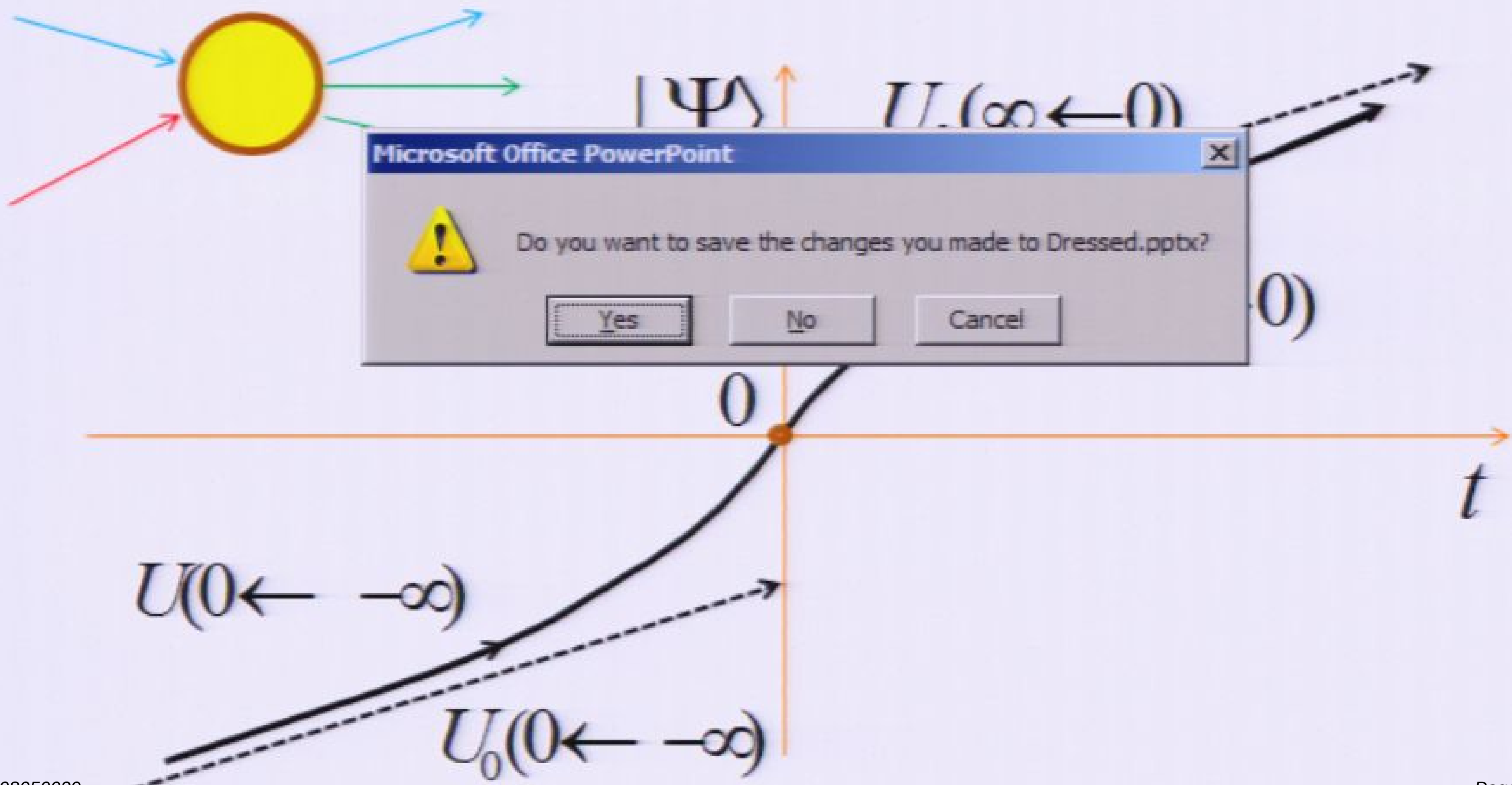

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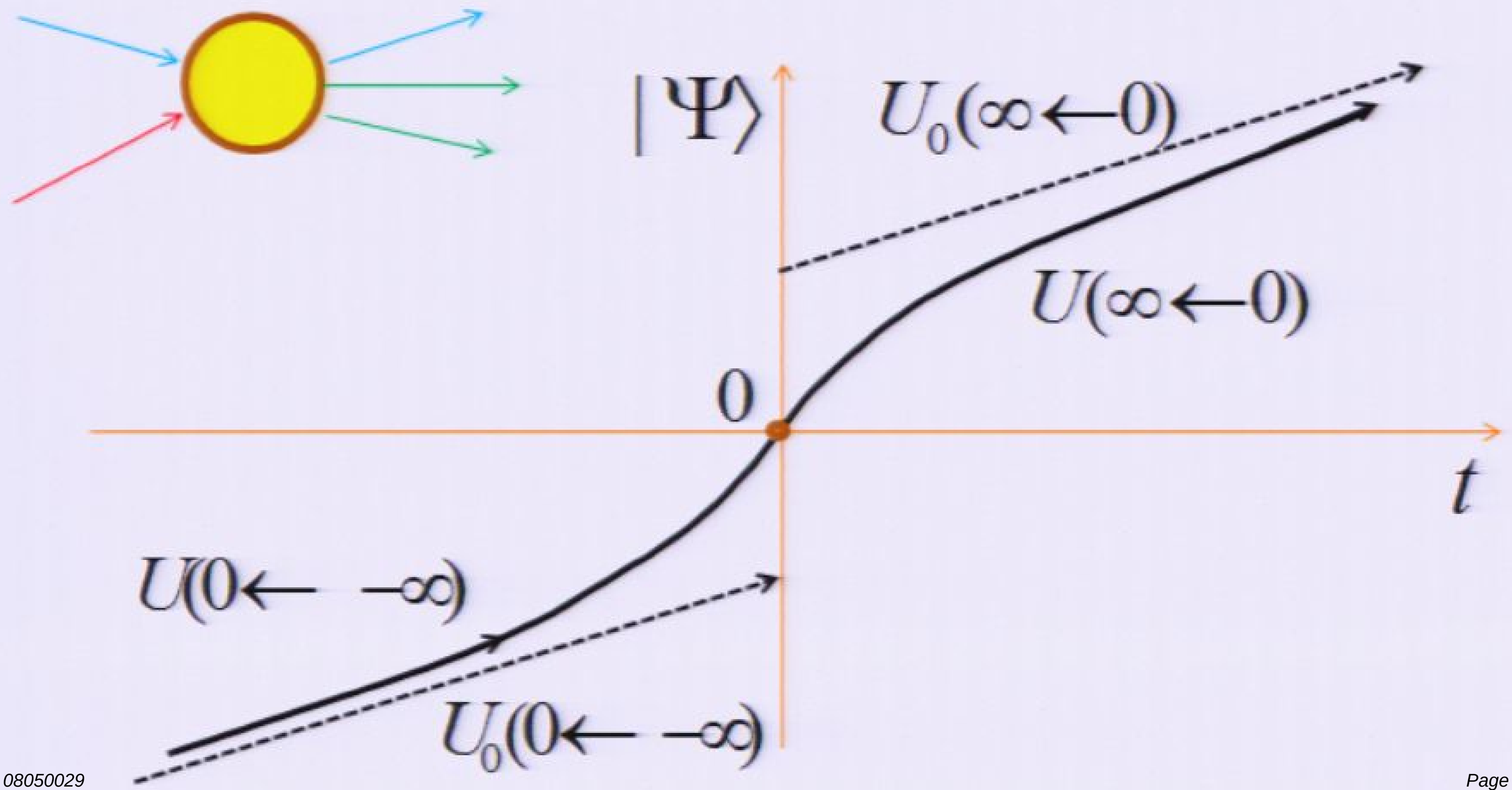
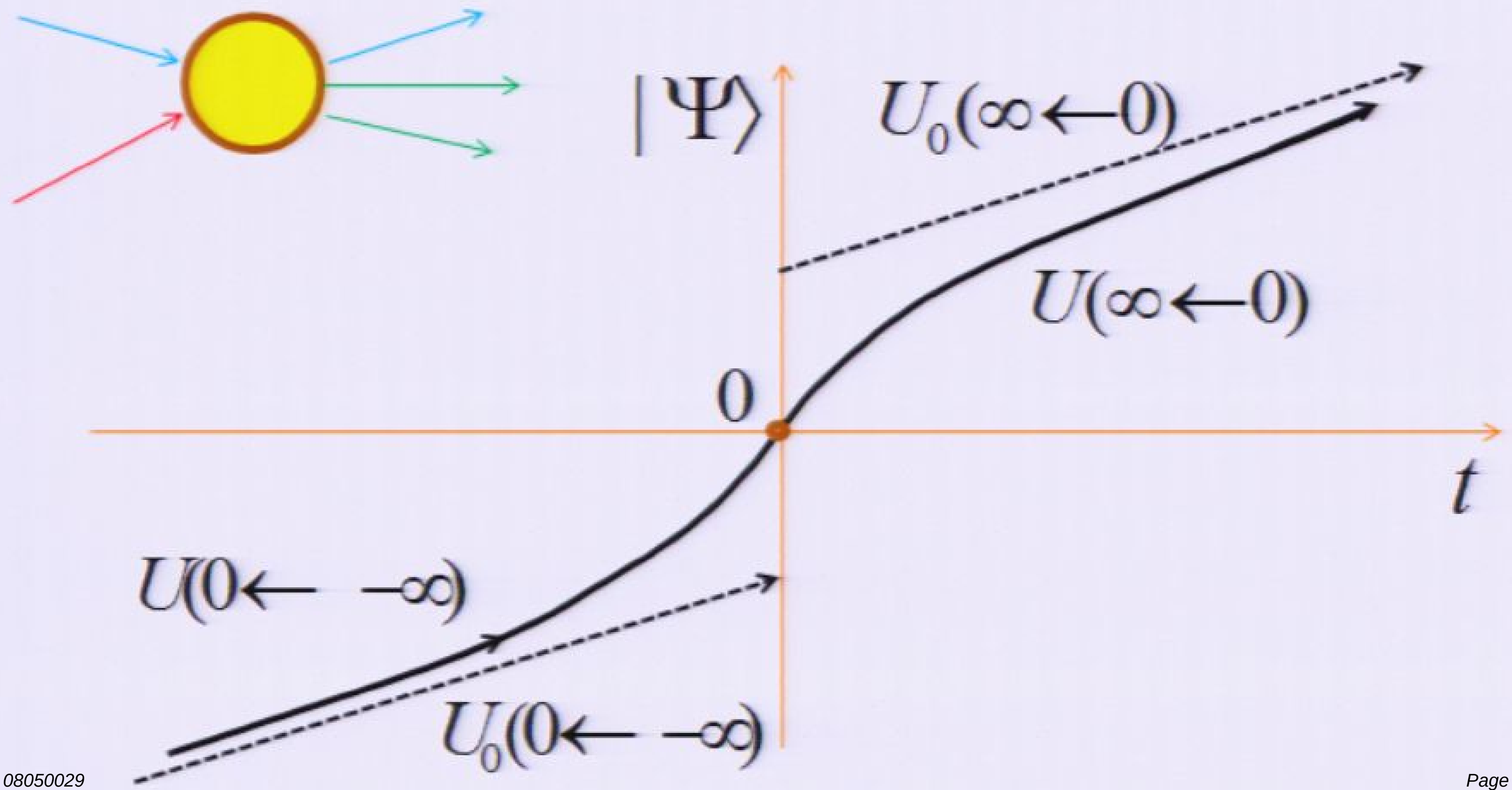
Scattering



$|\Psi\rangle$
$U_0(\infty \leftarrow 0)$
$U(\infty \leftarrow 0)$
0
t
$U(0 \leftarrow -\infty)$
$U_0(0 \leftarrow -\infty)$

Scattering



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0
t
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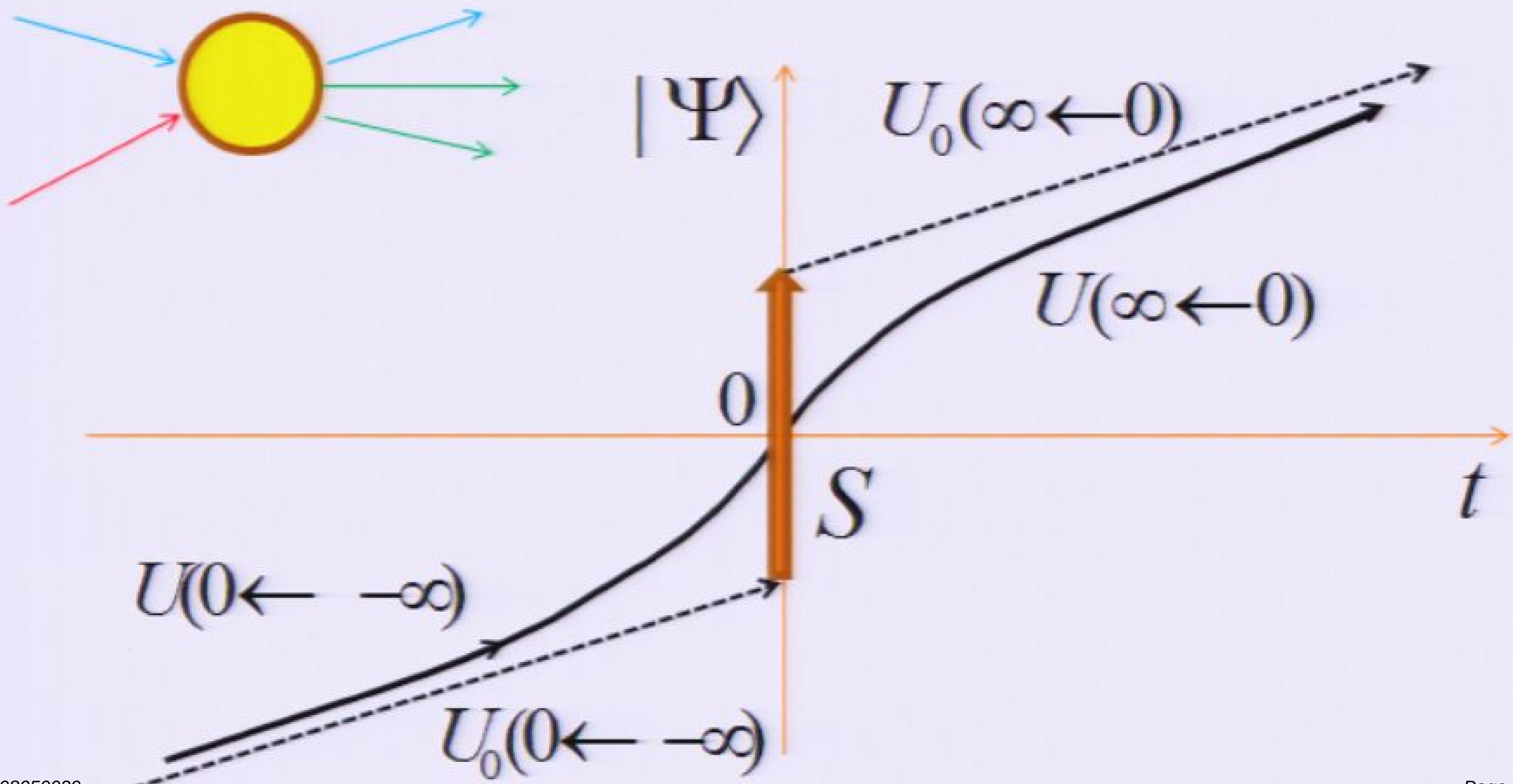
Scattering Operator

$$S = U_0(0 \leftarrow \infty) U(\infty \leftarrow 0) U(0 \leftarrow -\infty) U_0(-\infty \leftarrow 0)$$



Feynman-Dyson perturbation theory (1949)

$$S = 1 + i\int_{-\infty}^{\infty} dt V(t) - \frac{1}{2!}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' T\{V(t)V(t')\} + \ldots$$

$$V(t) = e^{-iH_0 t} V e^{iH_0 t}$$

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Magnus perturbation theory (1954)

$$S = \exp[i\int_{-\infty}^{\infty} dt F(t)]$$

S is explicitly unitary

$$F(t) = V(t) - \frac{i}{2}\int_{-\infty}^{t} dt' [V(t'), V(t)] + \ldots$$

Q: Suppose that we know the exact S-operator in all orders of the perturbation theory. Is this information sufficient to restore the Hamiltonian $H = H_0 + V$?

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A: No.

Scattering equivalence of Hamiltonians. Ekstein (1960)

H and H' yield the same S-operator if they are related by a unitary transformation $H' = e^{i\Phi} H e^{-i\Phi}$ where Φ is a "smooth" operator.

Example: H and H' have identical energies of bound states.

Important: Scattering (unitary) equivalence of two Hamiltonians does not imply full physical equivalence of the two theories.

Theory/experiment correspondence in microworld

Experiment	Theory
Scattering cross-sections (easy)	S-matrix elements
Lifetimes/decay rates (easy)	S-matrix poles
Energies of bound states (easy)	S-matrix poles
Wave functions of bound states (difficult)	Eigenvectors of H
Time evolution (difficult in microworld)	e^{iHt}
Decay law (relatively easy)	e^{iHt}

Infinite number of (scattering equivalent) Hamiltonians, which yield (almost) the same observable physics!

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The Hamiltonian of quantum electrodynamics (Coulomb gauge, interaction picture)

$$H_0 = \int dp\,\omega_p a_p^+ a_p + \int dp\,\omega_p b_p^+ b_p + \int dp\,|p|\,c_p^+ c_p$$

$$V(t) = \int d\vec{x}\,\vec{j}(\vec{x},t)\vec{A}(\vec{x},t) + \frac{1}{2}\int d\vec{x}\int d\vec{y}\,\frac{j_0(\vec{x},t)\,j_0(\vec{y},t)}{|\vec{x}-\vec{y}|}$$

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Expand $V(t)$ in creation-annihilation operators:
a^+, a electrons, b^+, b positrons, c^+, c photons

Three types of (normally ordered) operators in H

Operators	Type	Time dependence e^{iEt}
$a^+a, \quad b^+b, \quad c^+c$	"renorm"	$E \equiv 0$
$a^+a^+aa \quad b^+c^+c^+bc\ldots$	"phys"	$E = 0$ on the energy shell
$a^+ac, \quad a^+c^+a, \quad a^+b^+c^+\ldots$	"unphys"	$E \neq 0$

Let us try to calculate S-operator in the 2nd order

$$V(t) = a^{+}ac + a^{+}c^{+}a + a^{+}b^{+}c^{+} \ldots$$

$$S_2 = \frac{1}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' [V(t), V(t')]$$

$$= \int_{-\infty}^{\infty} dt (e^{iEt} \underbrace{a^{+}a^{+}aa}_{phys} + e^{iE't} \underbrace{a^{+}a^{+}b^{+}a}_{unphys} + \underbrace{a^{+}a}_{renorm} + \ldots)$$

$$= 2\pi\delta(E) a^{+}a^{+}aa + \underbrace{\int_{-\infty}^{\infty} dt a^{+}a}_{infinite\ !} + \ldots$$

Solution: Mass renormalization

Add to the Hamiltonian counterterms, which cancel out all "renorm" terms in the scattering operator (in each order n)

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$$H^c = \underbrace{H_0 + V}_{original\ H} + \underbrace{A_2 a^+ a + B_2 b^+ b}_{\substack{mass\ renormalization \\ counterterms}} + \ldots + \underbrace{C_3 a^+ a c + D_3 a^+ c^+ a}_{\substack{charge\ renormalization \\ counterterms}} + \ldots$$

A_n, B_n, C_n, D_n are divergent integrals.

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$$= a^+ |0\rangle + a^+ c^+ |0\rangle + \ldots$$

$$|\Psi(0)\rangle = a^+ |0\rangle$$

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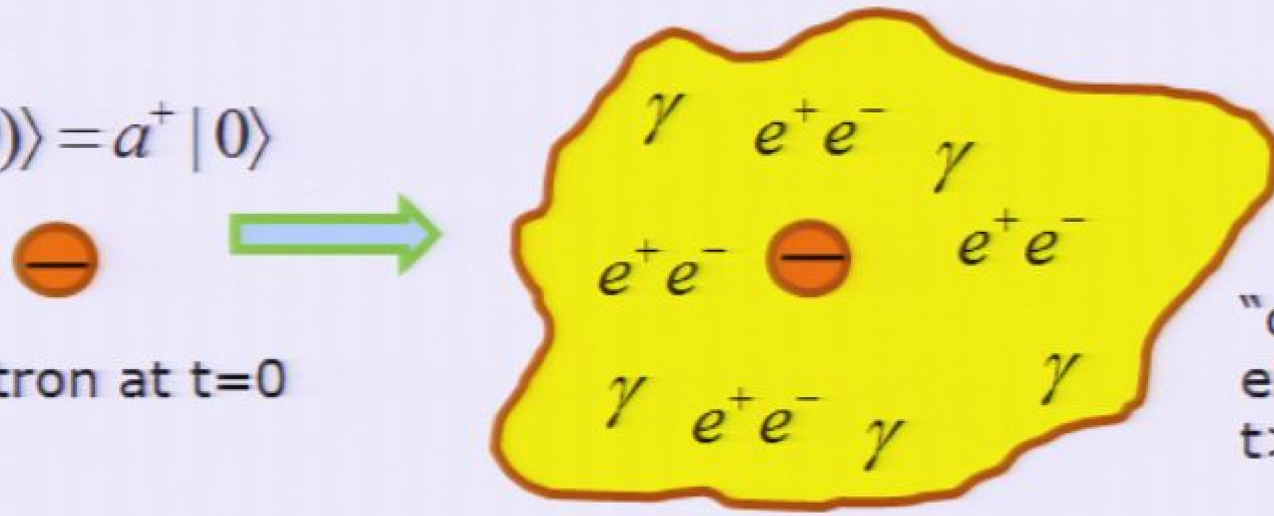

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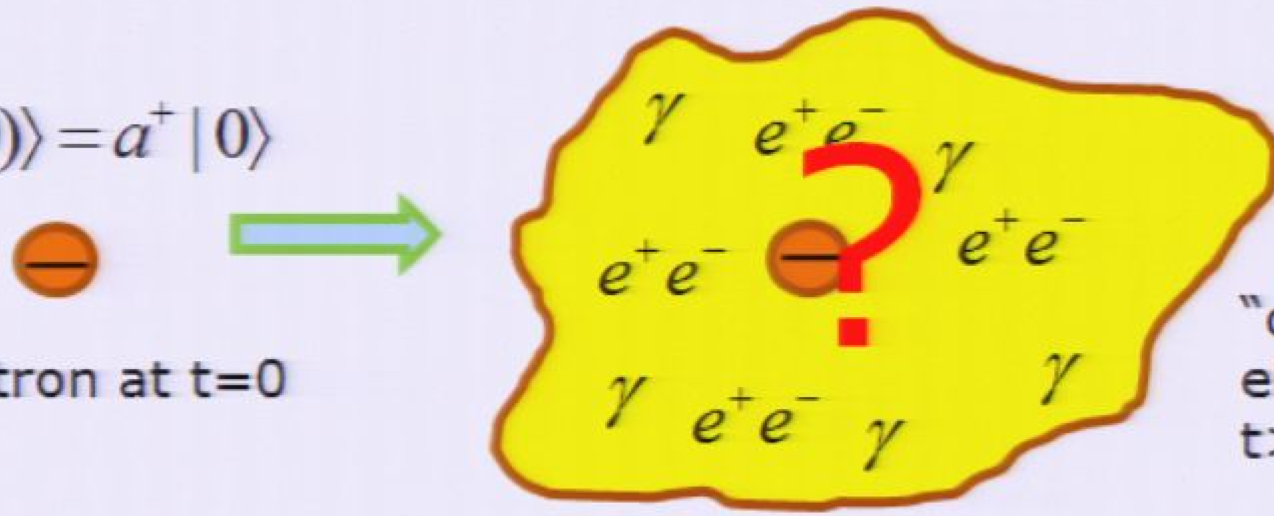

electron at t=0

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Operators a^+, a refer to fictitious "bare" electrons rather than physical electrons. Why should we care about "bare" electrons? Let us reformulate the theory in terms of physical or "dressed" particles.

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Idea: find a new Hamiltonian H^d of QED with nice properties:

- yields the same S-matrix as the traditional renormalized QED
- finite
- interaction is "phys"

$$H^d = H_0 + a^+a^+aa + a^+b^+ab + a^+a^+c^+aa + \ldots$$

It can be proven that $[phys, phys] = phys$

Therefore there can be no "renorm" terms in the S-operator

$$S = \exp\left[i\int_{-\infty}^{\infty} dt V^d(t) - \frac{i}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' [V^d(t'), V^d(t)] + \ldots \right]$$

and mass (charge) renormalization is not needed

Time evolution:

Dressed interaction yields zero when acting on the vacuum and 1-particle states

$$V^d = a^+ a^+ aa + a^+ b^+ ab + a^+ a^+ c^+ aa + \ldots$$

$$V^d \,|\, 0\rangle = 0$$

$$V^d a^+ \,|\, 0\rangle = 0$$

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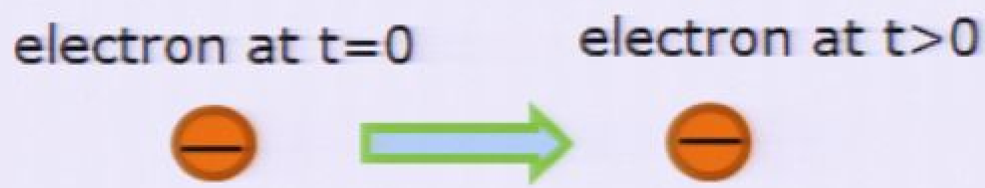
No self-interaction!

$$V^d a^+ | 0 \rangle = 0$$



$$V^d b^+ | 0 \rangle = 0$$

$$V^d c^+ | 0 \rangle = 0$$

Vacuum and 1-particle states are eigenvectors of the full interacting Hamiltonian.

Three methods to find the dressed particle Hamiltonian H^d

- Unitary dressing transformation $H^d = e^{i\Phi} H^c e^{-i\Phi}$

  Greenberg & Schweber (1958), Faddeev (1963), Shebeko & Shirokov (2001)

- Use Gell-Mann – Feynman theorem. Weber et al. (1999)
- Directly fit order-by-order to the known S-matrix of the renormalized QED. E.S. (2001)

All methods cannot yield the unique Hamiltonian H^d , which is not surprising in view of the many-to-one correspondence between Hamiltonians and *S-operators*.

Dressed particle Hamiltonian for the electron-positron interaction in the 2nd perturbation order near the energy shell (= Darwin-Breit instantaneous potential)

$$H^d = \sqrt{m^2 + p_1^2} + \sqrt{m^2 + p_2^2} + V_2^d + \ldots$$

$$V_2^d = -\frac{e^2}{4\pi r}$$ Coulomb

$$+\frac{e^2}{4m^2}\delta(\vec{r})$$ contact

$$+\frac{e^2}{8\pi m^2}\left(\frac{(\vec{p}_1 \cdot \vec{p}_2)}{r} + \frac{(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})}{r^3}\right)$$ Darwin

$$+\frac{e^2[\vec{r} \times \vec{p}_1] \cdot \vec{s}_1}{8\pi m^2 r^3} - \frac{e^2[\vec{r} \times \vec{p}_2] \cdot \vec{s}_2}{8\pi m^2 r^3} - \frac{e^2[\vec{r} \times \vec{p}_2] \cdot \vec{s}_1}{4\pi m^2 r^3} + \frac{e^2[\vec{r} \times \vec{p}_1] \cdot \vec{s}_2}{4\pi m^2 r^3}$$ spin-orbit

$$+\frac{e^2}{m^2}\left(-\frac{(\vec{s}_1 \cdot \vec{s}_2)}{4\pi r^3} + \frac{3(\vec{s}_1 \cdot \vec{r})(\vec{s}_2 \cdot \vec{r})}{4\pi r^5} + \frac{2}{3}(\vec{s}_1 \cdot \vec{s}_2)\delta(\vec{r})\right)$$ spin-spin

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This Hamiltonian describes the fine and hyperfine structures of positronium, most of classical electrodynamics, the Aharonov-Bohm effect (E.S. arXiv:0803.1326)

Further goal: obtain interaction operators in higher perturbation orders. For example, V_4^d may give us Lamb shift corrections to energies and wave functions of the positronium.

Hypothesis: Dressed particle Hamiltonian

$$H^d = H_0 + a^+a^+aa + a^+b^+ab + a^+a^+c^+aa + \dots$$

can be derived in all perturbation orders

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a^+b^+ab	electron-positron potential	**2**,4,6,...
a^+c^+ac	electron-photon potential	**2**,4,6,...
$a^+a^+a^+aaa$	3-electron potential	4,6,...
...	...	...
	Inelastic potentials	
a^+b^+cc	electron-positron pair creation	**2**,4,6,...
c^+c^+ab	electron-positron annihilation	**2**,4,6,...
$a^+b^+c^+ab$	electron-positron bremsstrahlung	**3**,5,...
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The dressed particle Hamiltonian H^d has a number of advantages as compared to the Hamiltonian H^c of renormalized QED

- H^d is finite.
- S-matrix calculations with H^d do not involve divergent integrals. Renormalization is not needed.
- Time evolution of states and observables can be obtained by usual quantum-mechanical formulas.
- Wave functions and energies of bound states can be obtained by diagonalization of H^d
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Q: Do electromagnetic interactions between charges propagate instantaneously?

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What about this quote from Weinberg?

...the idea of quantum field theory is that quantum fields are the basic ingredients of the universe, and particles are just bundles of energy and momentum of the fields.

A: Quantum fields play rather technical role in QFT. They are essential in derivations of relativistically invariant interactions using the local gauge invariance principle. Unfortunately, no substitute for this principle exists in the dressed particle approach.

Q: Do electromagnetic interactions between charges propagate instantaneously?

A: Why not? Giakos & Ishii (1991), Enders & Nimtz (1993), Steinberg & Kwiat & Chiao (1993), Ranfagni & Mugnai (1996). In dressed particle theory the full interaction consists of two components: direct instantaneous potentials (Coulomb, magnetic, etc) and indirect interaction due to real photons (an accelerated charge emits a bremsstrahlung photon and another charge either absorbs or scatters this photon).

Dressed particle Hamiltonian for the electron-positron interaction in the 2nd perturbation order near the energy shell (= Darwin-Breit instantaneous potential)

$$H^d = \sqrt{m^2 + p_1^2} + \sqrt{m^2 + p_2^2} + V_2^d + \ldots$$

$$V_2^d = -\frac{e^2}{4\pi r}$$ Coulomb

$$+\frac{e^2}{4m^2}\delta(\vec{r})$$ contact

$$+\frac{e^2}{8\pi m^2}\left(\frac{(\vec{p}_1 \cdot \vec{p}_2)}{r} + \frac{(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})}{r^3}\right)$$ Darwin

$$+\frac{e^2[\vec{r}\times\vec{p}_1]\cdot\vec{s}_1}{8\pi m^2 r^3} - \frac{e^2[\vec{r}\times\vec{p}_2]\cdot\vec{s}_2}{8\pi m^2 r^3} - \frac{e^2[\vec{r}\times\vec{p}_2]\cdot\vec{s}_1}{4\pi m^2 r^3} + \frac{e^2[\vec{r}\times\vec{p}_1]\cdot\vec{s}_2}{4\pi m^2 r^3}$$ spin-orbit

$$+\frac{e^2}{m^2}\left(-\frac{(\vec{s}_1 \cdot \vec{s}_2)}{4\pi r^3} + \frac{3(\vec{s}_1 \cdot \vec{r})(\vec{s}_2 \cdot \vec{r})}{4\pi r^5} + \frac{2}{3}(\vec{s}_1 \cdot \vec{s}_2)\delta(\vec{r})\right)$$ spin-spin

This Hamiltonian describes the fine and hyperfine structures of positronium, most of classical electrodynamics, the Aharonov-Bohm effect (E.S. arXiv:0803.1326)

Further goal: obtain interaction operators in higher perturbation orders. For example, V_4^d may give us Lamb shift corrections to energies and wave functions of the positronium.

Q: Do we actually need quantum fields in QFT?
What about this quote from Weinberg?

...the idea of quantum field theory is that quantum fields are the basic ingredients of the universe, and particles are just bundles of energy and momentum of the fields.

References:

Van Hove (1955-1956) persistent self-interactions
Greenberg and Schweber (1958) dressing transformation
Nuovo Cim. 8 (1958), 378
Faddeev (1963) perturbation theory
Visinescu and Shirokov (1974) renormalization, bound states
Kobayashi, Sato, Ohtsubo (1997) nuclear forces

Reviews:

Shebeko and Shirokov (2001) nucl-th/0102037
E.S. (2005) physics/0504062

The End

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