Title: Non-abelian topological phases and unconventional criticality in a model of interacting anyons

Date: May 02, 2008 10:00 AM

URL: http://pirsa.org/08050028

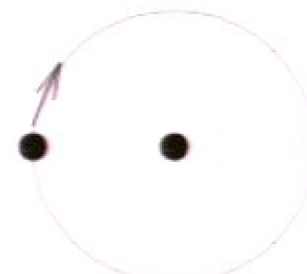
Abstract:

Quantum statistics in (2+1)D: anyons

- Consider two identical (quasi-)particles in three spatial dimensions:
  - ▷ Perform two clockwise adiabatic exchanges, resulting in a change of phase of 2θ
  - ▷ Deform path into a single point $\Longrightarrow \psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i2\theta}\psi(\mathbf{r}_1, \mathbf{r}_2)$, and hence $\theta = 0$ (bosons) or $\theta = \pi$ (fermions)

- In two spatial dimensions:
  - ▷ Cannot deform the trajectory to a point
  - ▷ The wavefunction may change by a phase factor

$$\psi(\mathbf{r}_1, \mathbf{r}_2) \rightarrow e^{i2\theta}\psi(\mathbf{r}_1, \mathbf{r}_2)$$



  - ▷ For values of $\theta \neq 0, \pi$: anyons

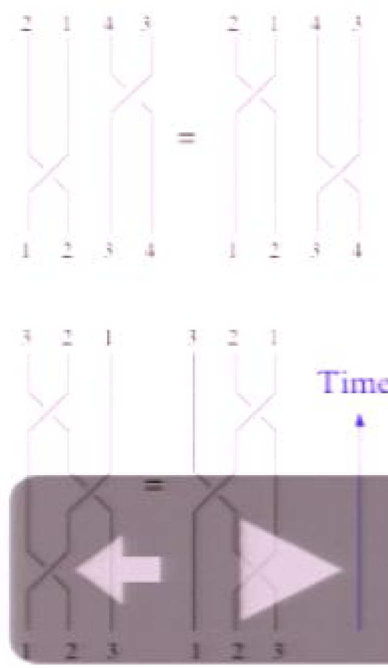
The braid group

- Consider N anyons in (2+1)D: trajectories take particle positions $R_1, R_2, ...R_N$ at time t_i to positions $R_{\pi(1)}, R_{\pi(2)}, ...R_{\pi(N)}$ at time t_f

- Each equivalence class of such worldlines that are invariant under smooth deformations is a "braid"

- Different braids correspond to different elements of the Braid group, where counter-clockwise exchanges of particles i and $i+1$, generated by σ_i, obey

$$\sigma_i\sigma_j = \sigma_j\sigma_i \text{ for } |i-j| \geq 2$$
$$\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \text{ for } 1 \leq i \leq N-1$$



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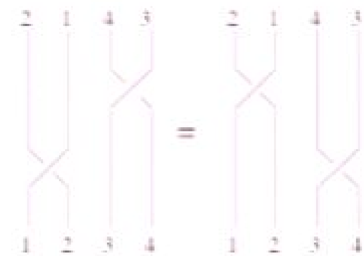

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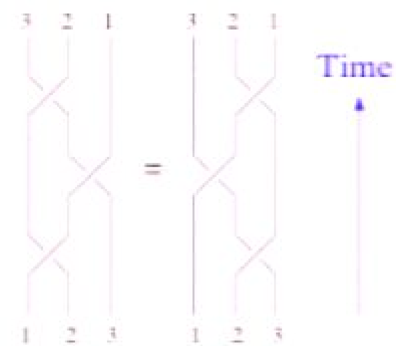
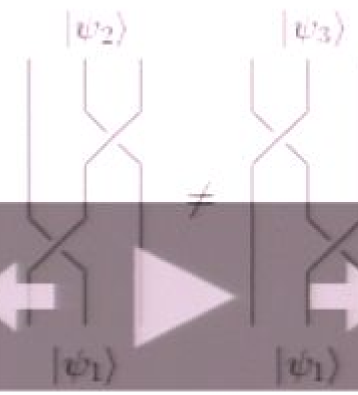


Representations of the braid group

- For abelian (1D) representations of the braid group, the order of braiding is irrelevant (exchange generators are phases)

- Higher-dimensional representations may be non-abelian:
  - ▷ Given a set of n degenerate orthonormal basis states $|\psi_k\rangle$, $k = 1, ...n$, with N identical anyons at fixed positions, the exchange of particles i and $i+1$ is represented by a $n \times n$ unitary matrix $M(\sigma_i)$
  - ▷ Non-abelian representation if

$$M(\sigma_i)\, M(\sigma_{i+1}) \neq M(\sigma_{i+1})\, M(\sigma_i)$$



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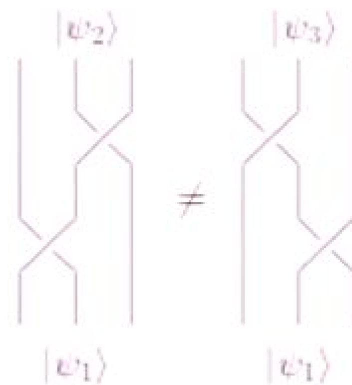

Topological quantum computation[1]

- Hilbert space $\mathcal{H}$ spanned by degenerate states $\psi_k(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)$ with non-abelian anyons at positions $\mathbf{r}_1, ..., \mathbf{r}_N$
- Unitary evolution on this space by braiding anyons
  - ▷ For most classes of non-abelian anyons, it is possible to generate all possible unitary transformation by only braiding
- Requires a system in a gapped phase with non-abelian quasiparticle excitations (a so-called "non-abelian topological phase")

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Non-abelian topological phases (NATPs)

- System that potentially are non-abelian topological phases (NATP): chiral p-wave superconductors, fractional quantum Hall (FQH) states with filling fractions $\nu = 12/5$ and $\nu = 5/2$
  - ▷ Quasiparticle excitations have statistical properties of Ising anyons ($\nu = 5/2$, $p + ip$ SC), and Fibonacci anyons ($\nu = 12/5$)
  - ▷ For the FQH with $\nu = 5/2$, the fractional charge of $e/4$ was recently measured in a shot-noise experiment[1]
- The field-theoretical description of topological phases is well understood (topological quantum field theories)
- Not much is known about microscopic models that give rise to NATPs[2,3]
- Possible realization of such models on optical lattices or in magnetic systems ? Understanding models of interacting anyons ?

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Fibonacci anyons

Spin-$1/2$ $SU(2)_\infty$

- Species: $j = 0, \frac{1}{2}, 1, ..., \infty$
- Fusion rules:

$$j \times j' = \sum_{j''=|j-j'|}^{j+j'} j''$$

Non-abelian anyons $SU(2)_k$

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- Fibonacci anyons are capable of universal topological quantum computation
- Species 1 $(j = 0)$ and τ $(j = 1)$, fusion rules: $\tau \times \tau = 1 + \tau$

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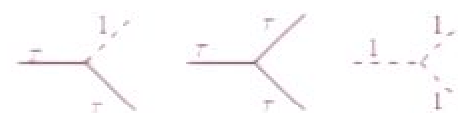

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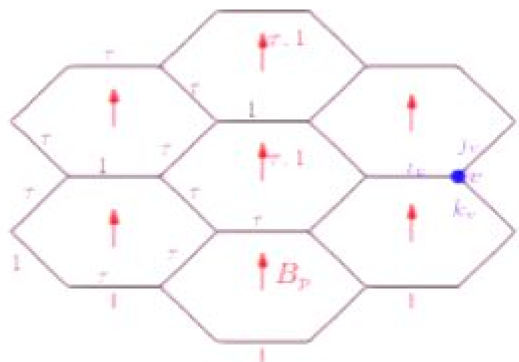
Levin-Wen model



- Spin-$1/2$ d.o.f. on links of honeycomb lattice (associate $\tau = \uparrow$, $1 = \downarrow$)

  $$H = H_v + H_p = -J_v \sum_v \delta_{i_v, j_v, k_v} - J_p \sum_p B_p$$

  where $\delta_{\uparrow\uparrow\uparrow} = \delta_{\uparrow\uparrow\downarrow} = \delta_{\downarrow\downarrow\downarrow} = 1$

- H_v-term enforces fusion rules of Fibonacci anyons

  H_p-term is a projector to anyon d.o.f. through a plaquette (zero if $\delta_{i_v, j_v, k_v} = 0$ for a $v \in p$)

- Two types of quasiparticle excitations that are Fibonacci anyons: plaquette-type (from H_p) and vertex-type (from H_v)

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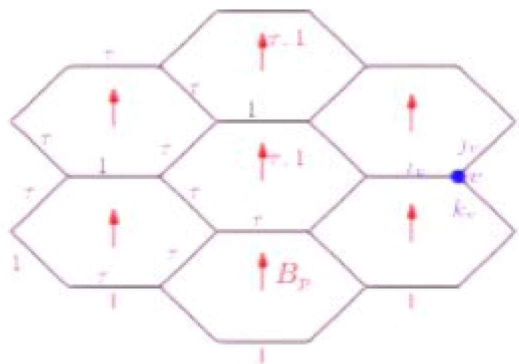

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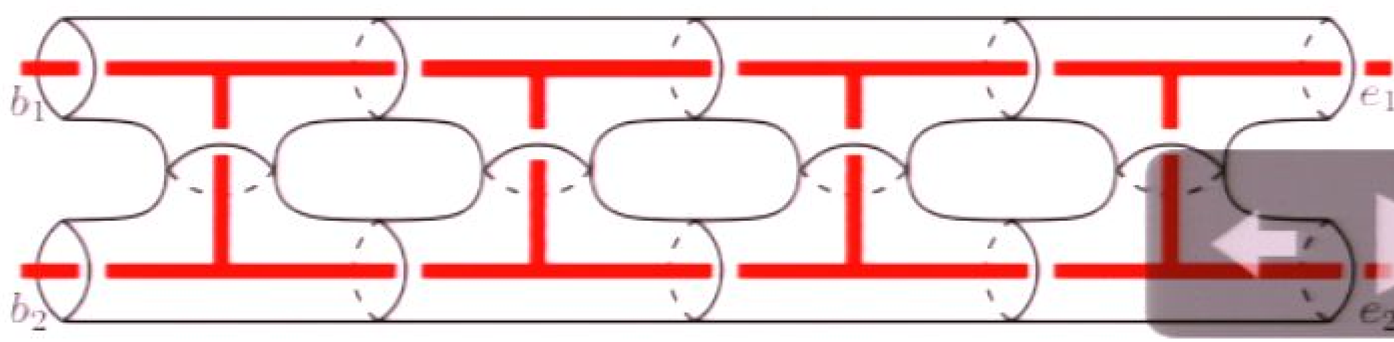
Model: Hilbert space

- Hilbert space of a multi-anyon system is defined by a certain topology
  Anyons are associated (*e.g.*, in TQFT context) with punctures of a surface $\longrightarrow$ decomposition into $3-$punctured spheres yields basis
- High-genus sphere with periodic boundary conditions ($b_1 = e_1$, $b_2 = e_2$) $\rightarrow$ one possible basis is ladder
- Fibonacci anyon degrees of freedom: 1's and τ's on links of ladder, with fusion rules being obeyed at all vertices



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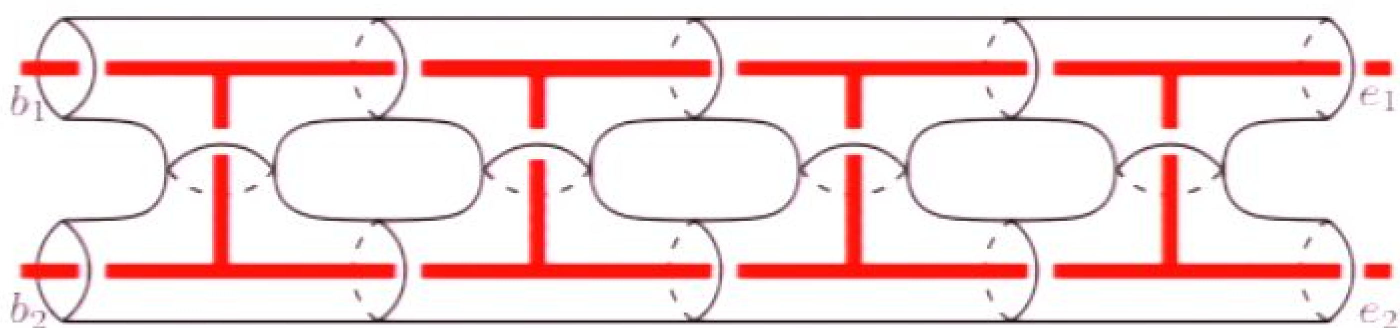

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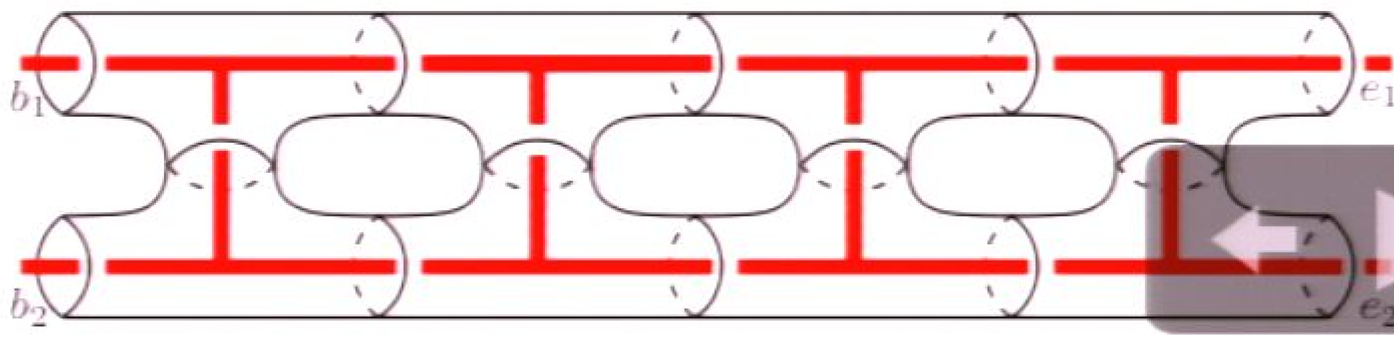

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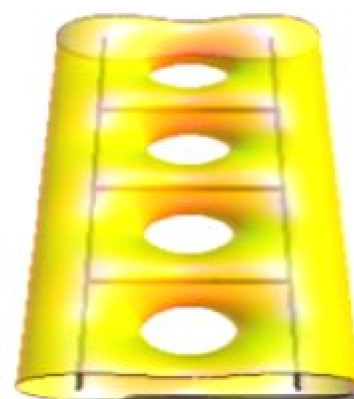

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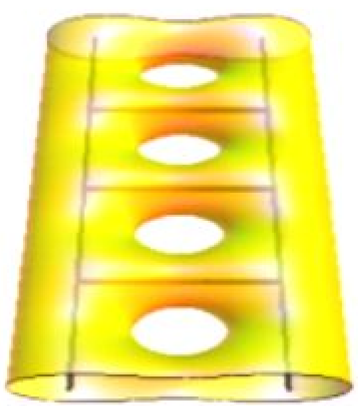

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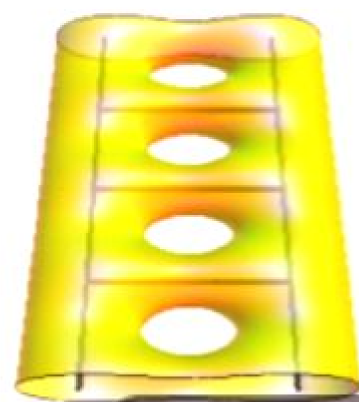

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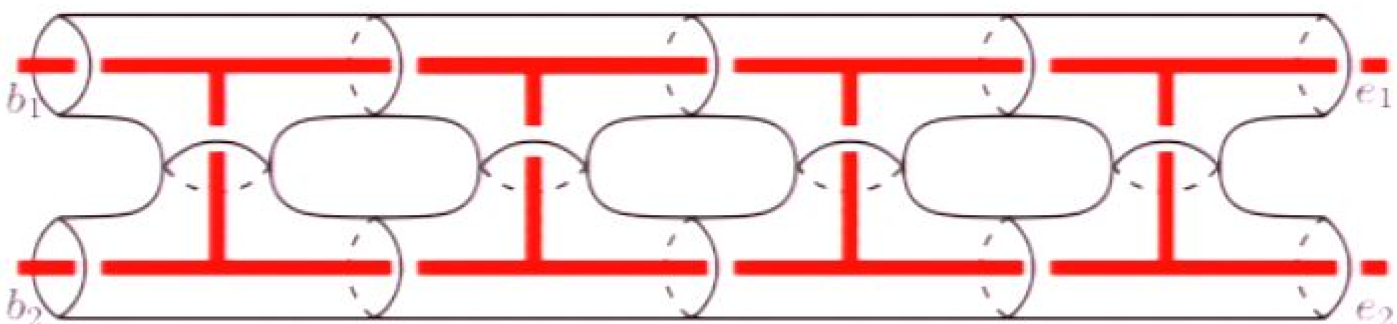

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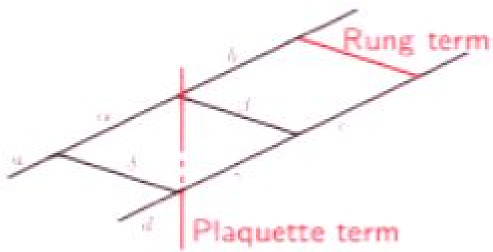

- Non-commuting plaquette and rung terms:

$$H = -\cos(\theta)\sum_p P_p - \sin(\theta)\sum_r R_r$$

- Rung term $R_r = \delta_{1,r}$: energy gain if no τ-anyon on rung
- Plaquette term[1,2] favors the absence of τ-anyon through plaquette:

$$P_p \left| \begin{array}{ccc} a & \alpha & b \\ \delta & & \beta \\ d & \gamma & c \end{array} \right\rangle = \sum_{i=1,\tau} \frac{d_i}{D^2} \sum_{\substack{\alpha',\beta',\\ \gamma',\delta'}} (F^i_{\delta a \alpha'})^{\alpha}_{\delta'} (F^i_{\alpha b \beta'})^{\beta}_{\alpha'} (F^i_{\beta c \gamma'})^{\gamma}_{\beta'} (F^i_{\gamma d \delta'})^{\delta}_{\gamma'} \left| \begin{array}{ccc} a & \alpha' & b \\ \delta' & & \beta' \\ d & \gamma' & c \end{array} \right\rangle$$

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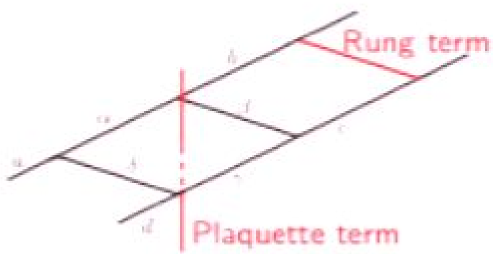

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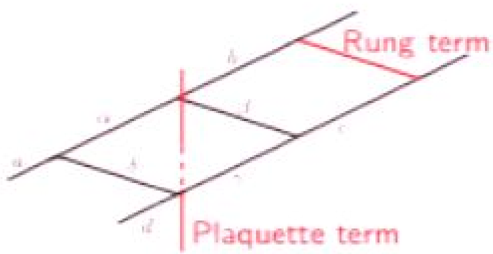

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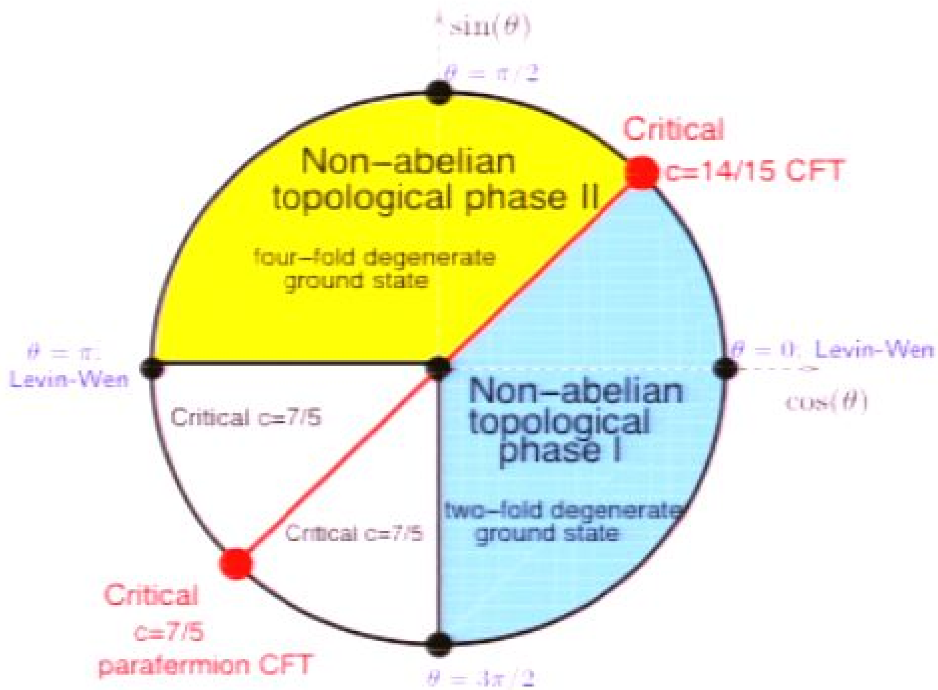
Phase diagram



- Exact diagonalization yields energy spectra (energy dispersion $E(k_x, k_y)$ by making use of translation and reflection symmetries)
- Almost duality (up to degeneracies) for periodic boundaries

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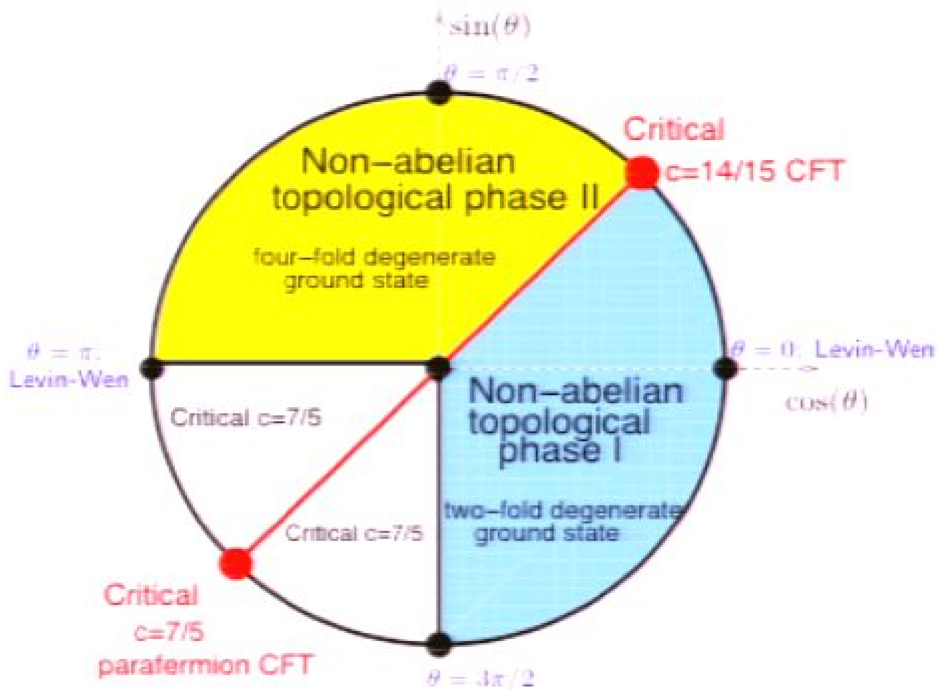

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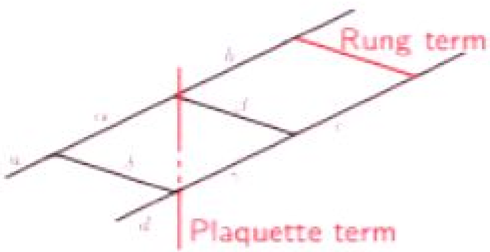

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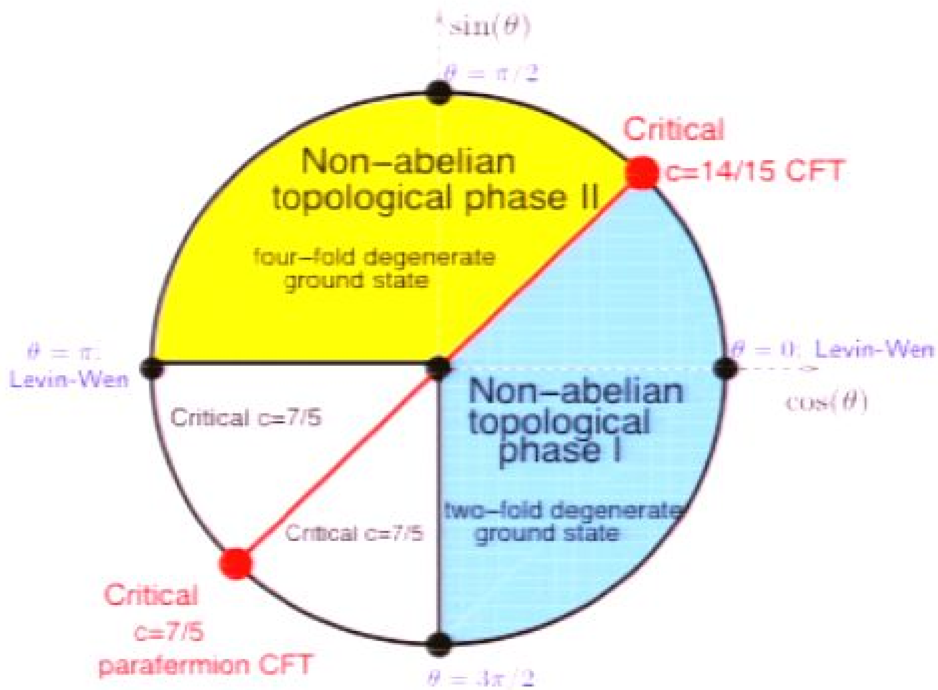

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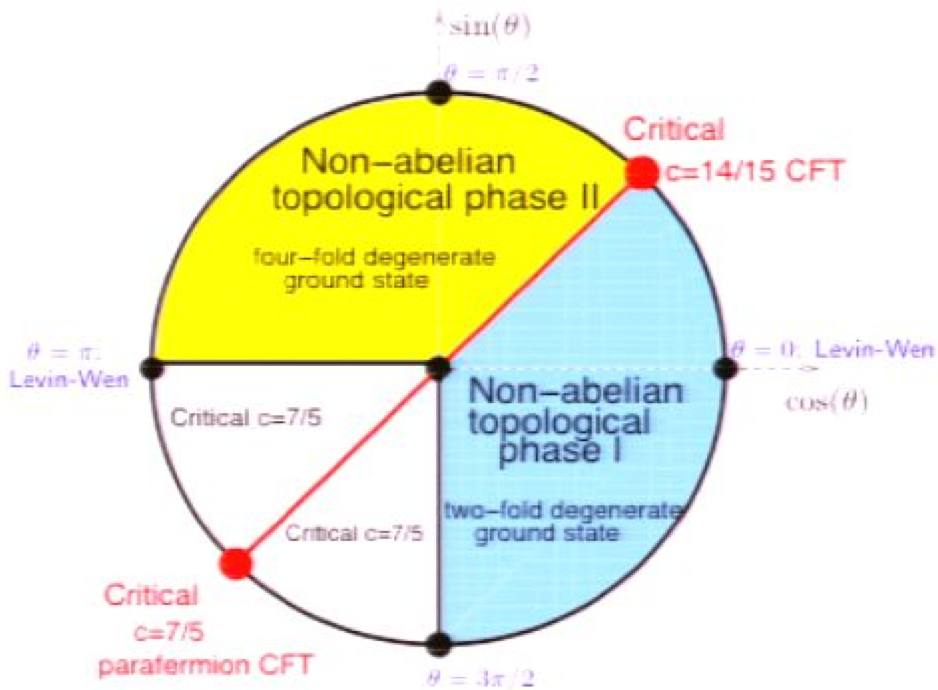

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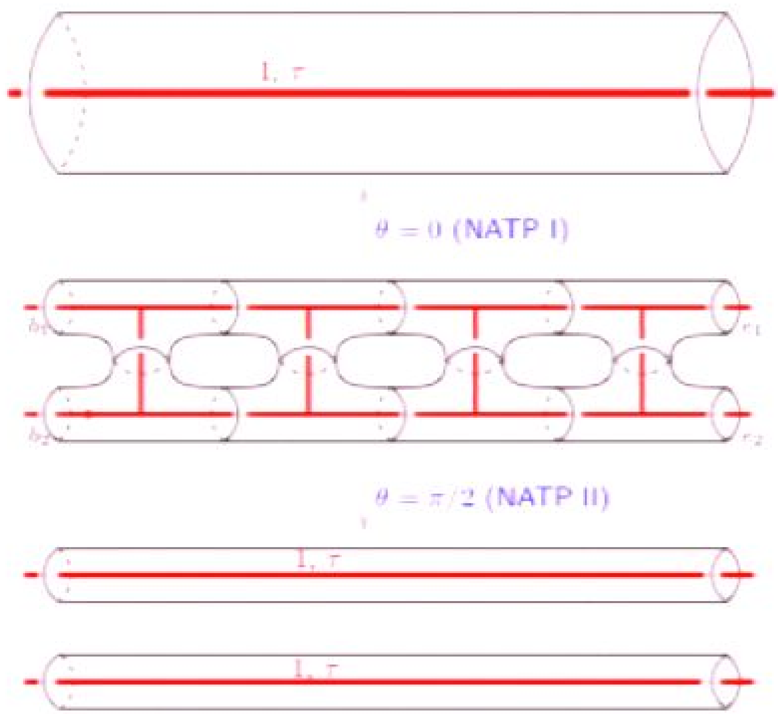
NATPs: ground state degeneracy



- Essential properties of NATPs from "effective topologies"
- Ground state (GS)
  - ◇ NATP I: no plaquette fluxes $\rightarrow$ close holes in sphere $\rightarrow$ becomes a torus $\rightarrow$ two-fold GS degeneracy
  - ◇ NATP II: no rung-fluxes $\rightarrow$ remove rungs $\rightarrow$ two tori $\rightarrow$ four-fold GS degeneracy

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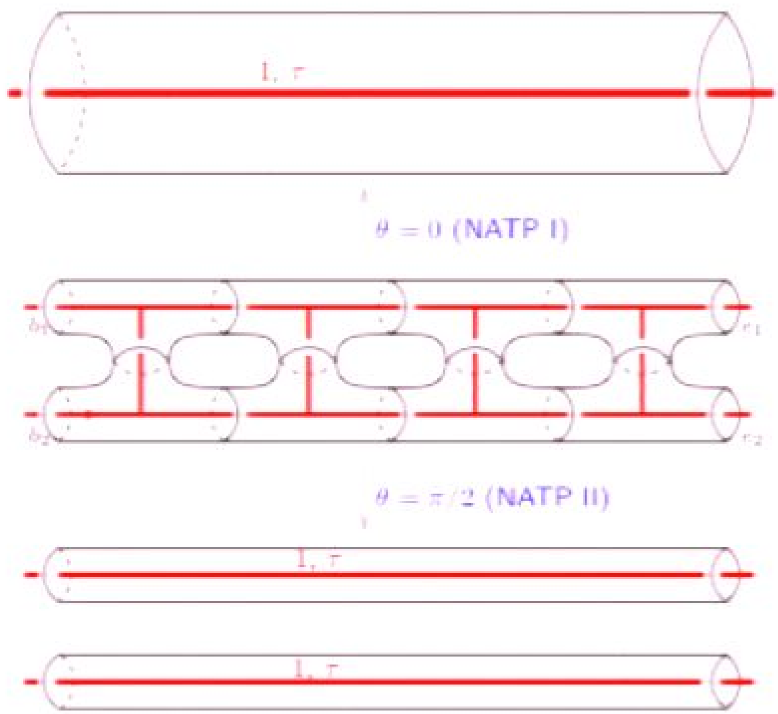

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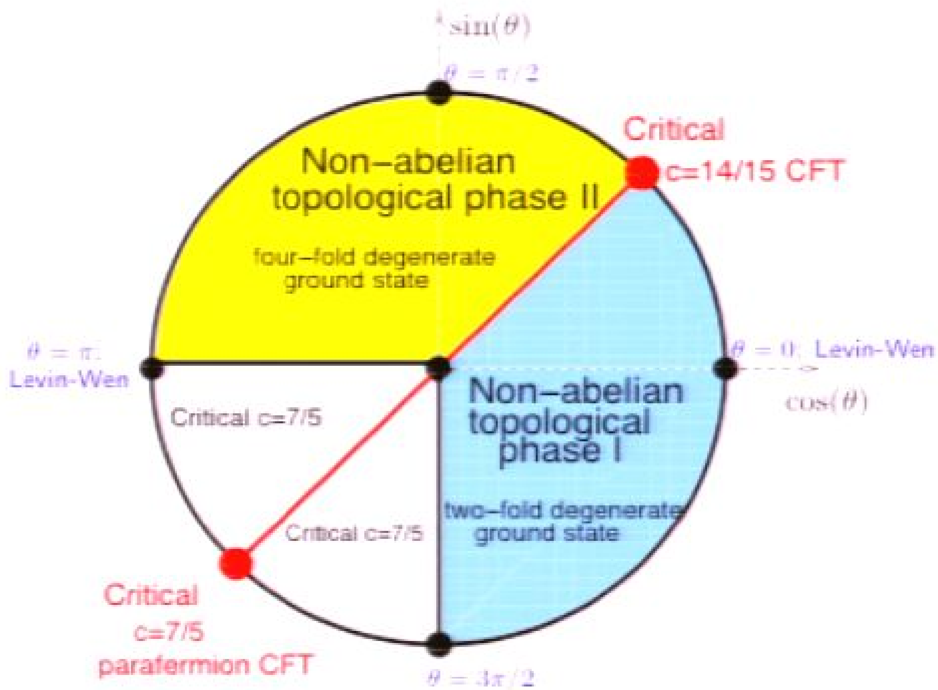

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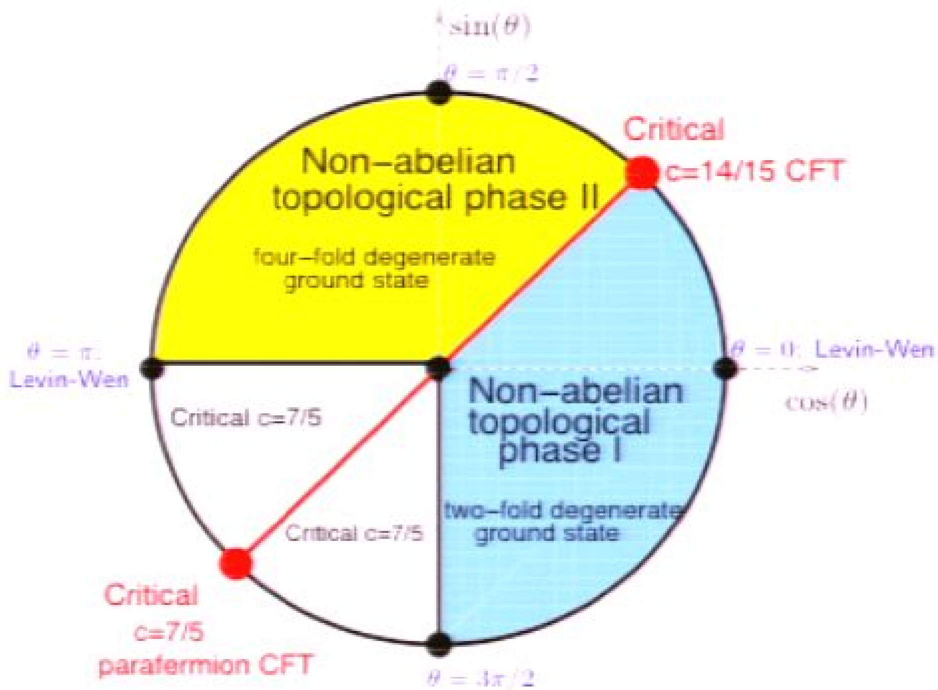

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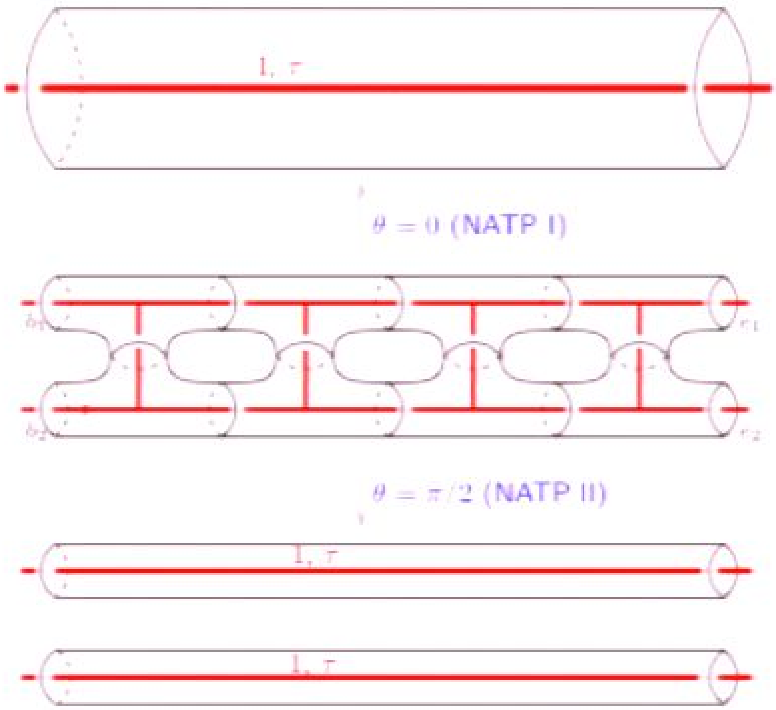
NATPs: ground state degeneracy



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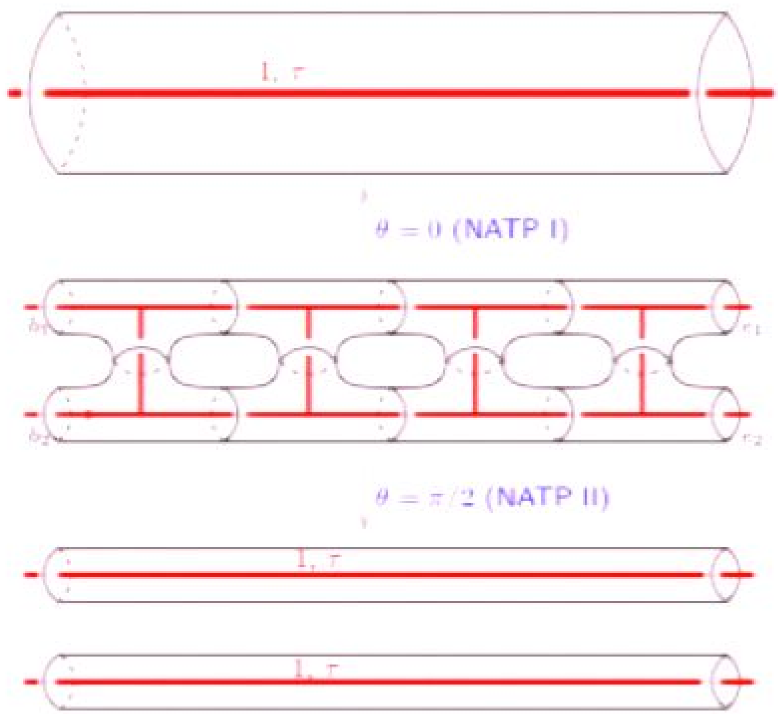

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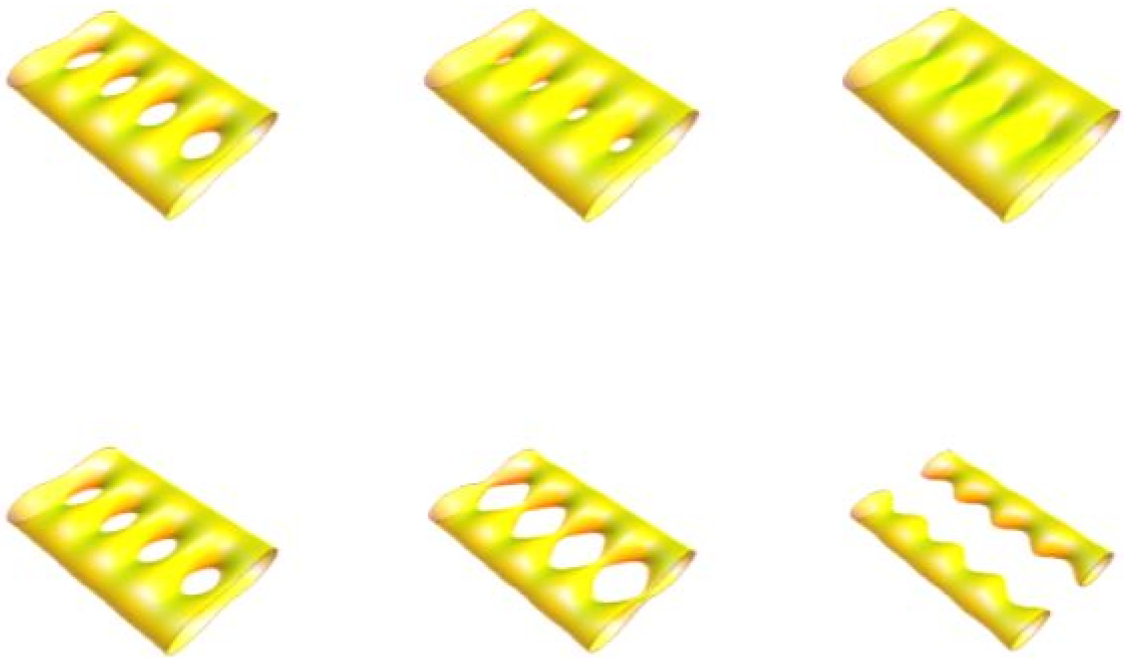

NATP I

NATP II

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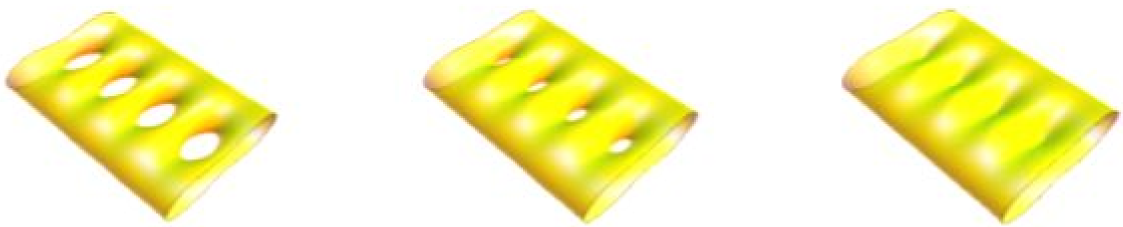

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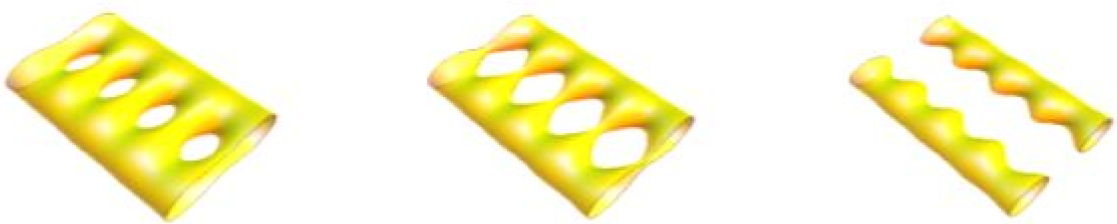

NATP II

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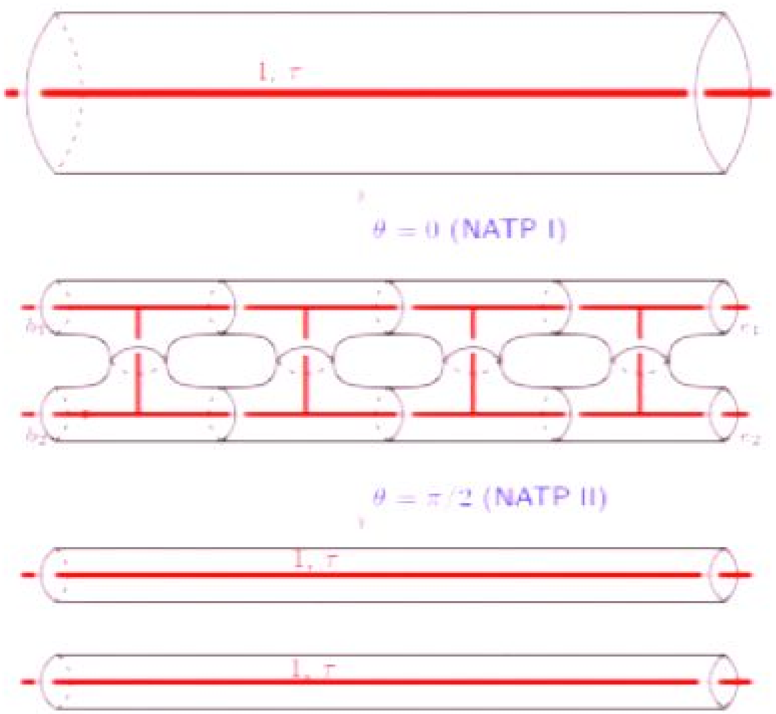

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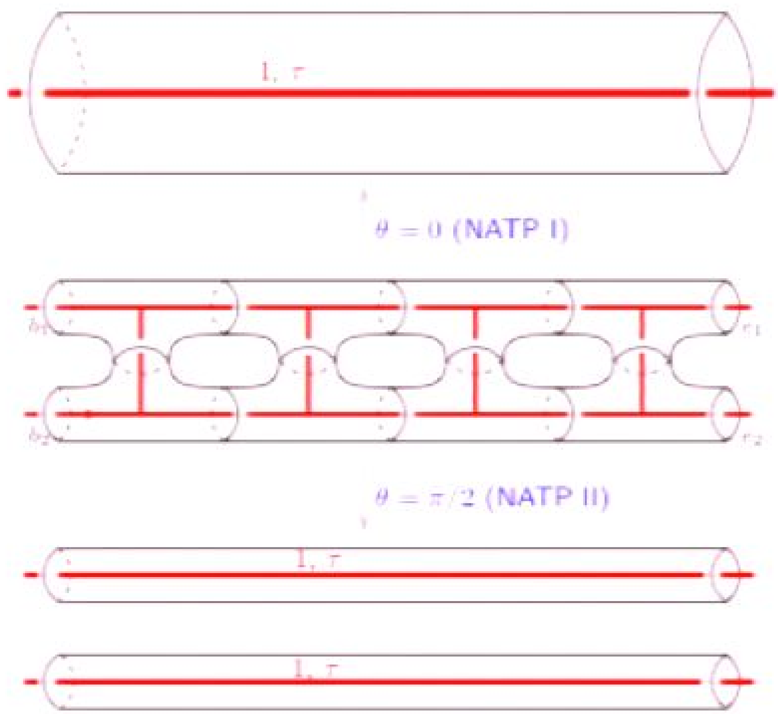

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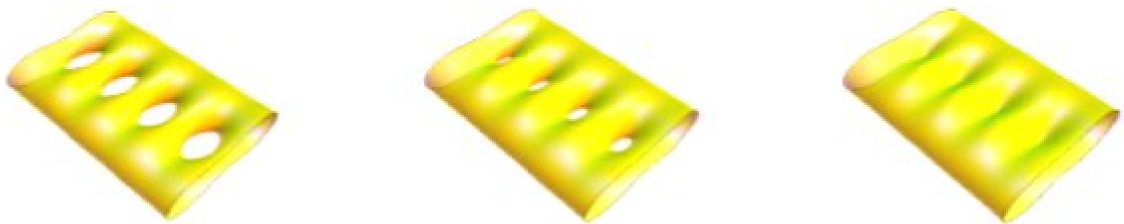

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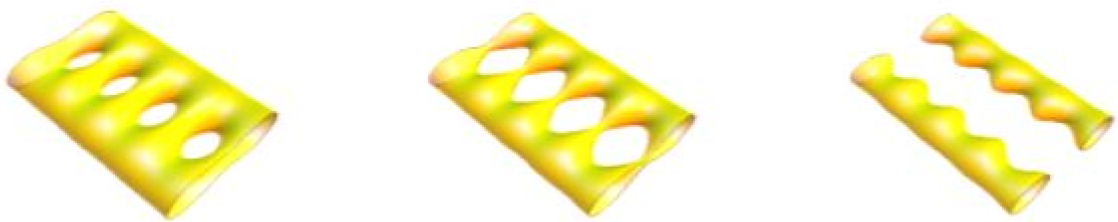

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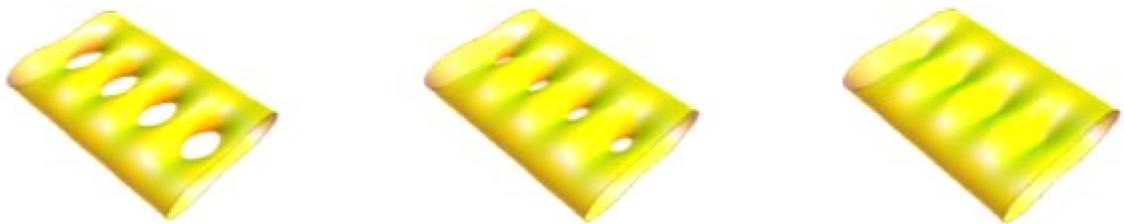

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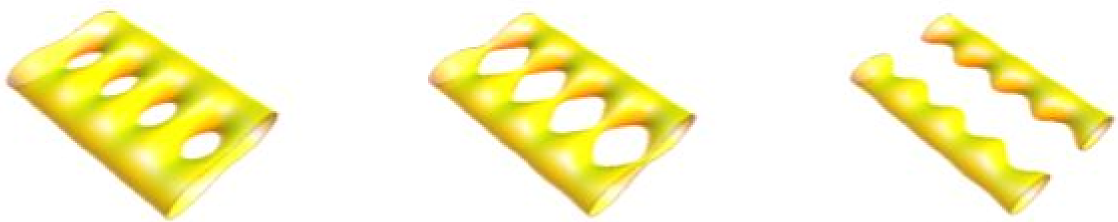

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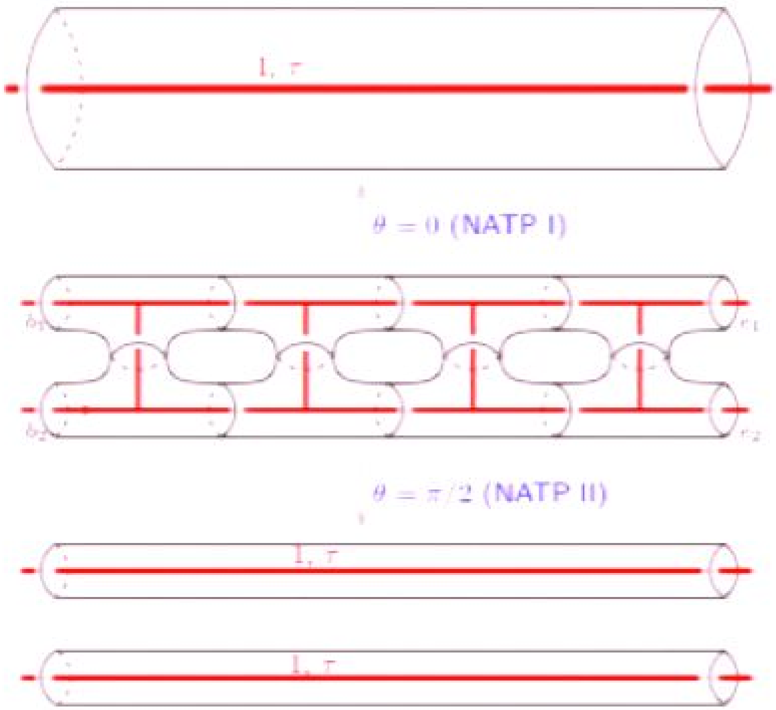

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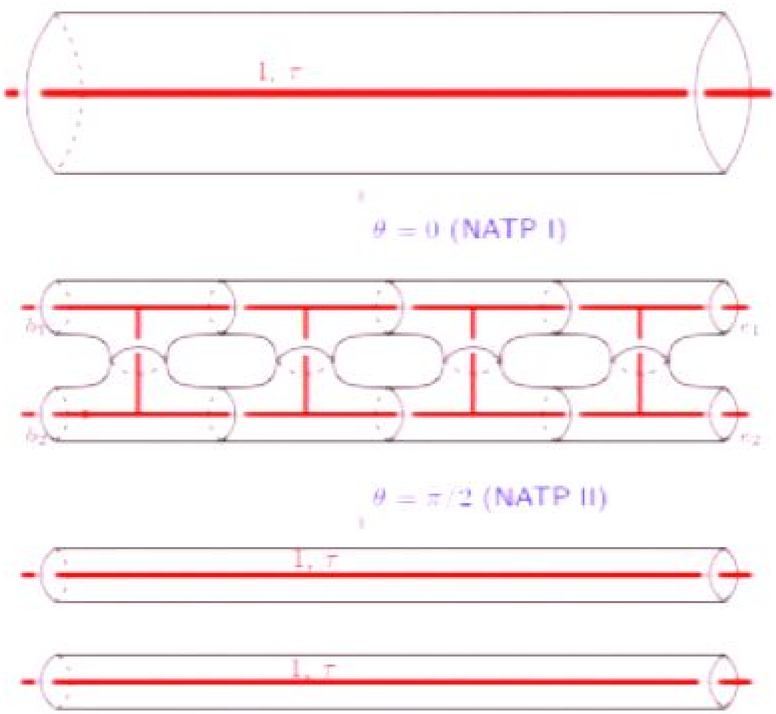

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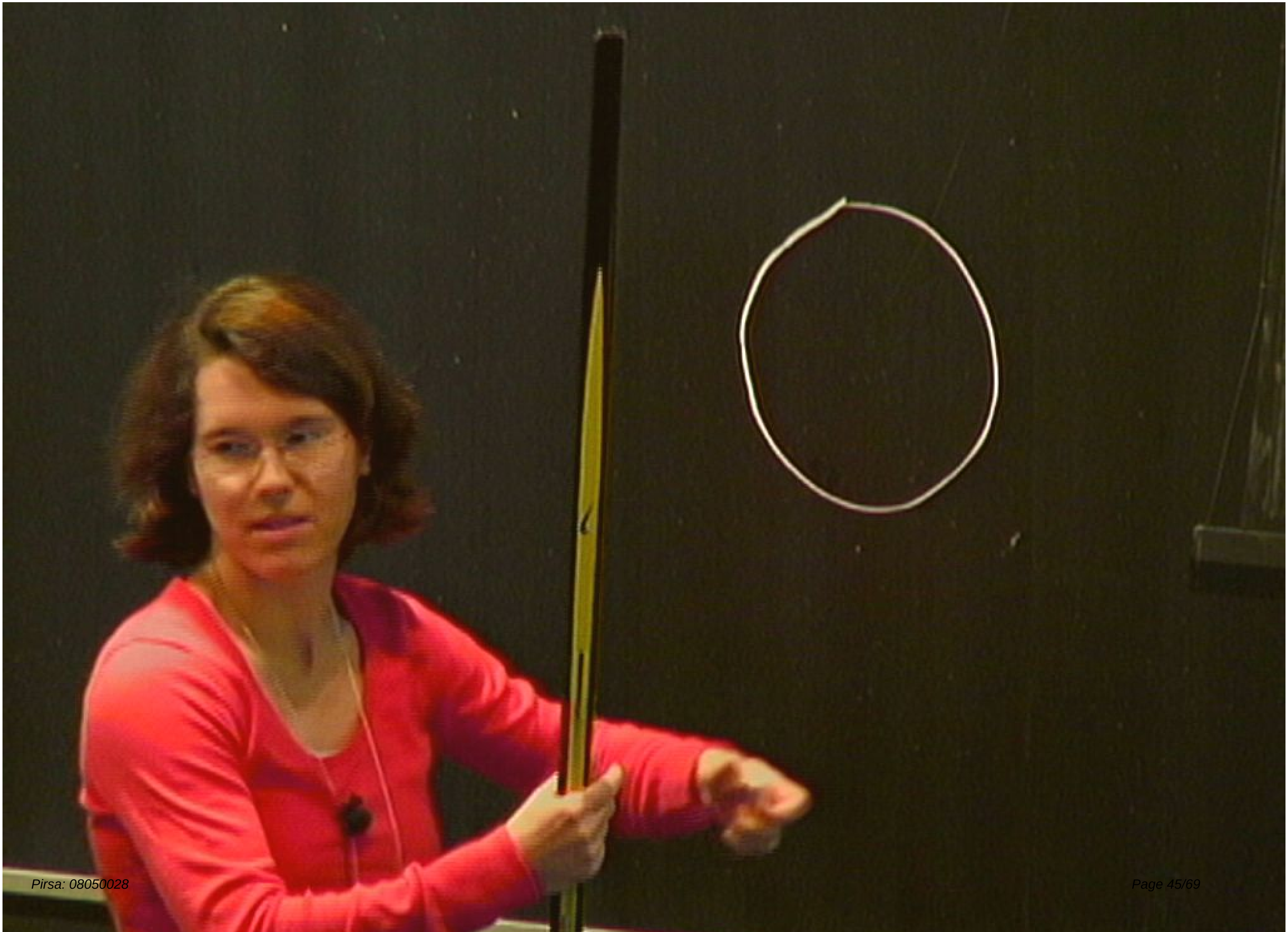
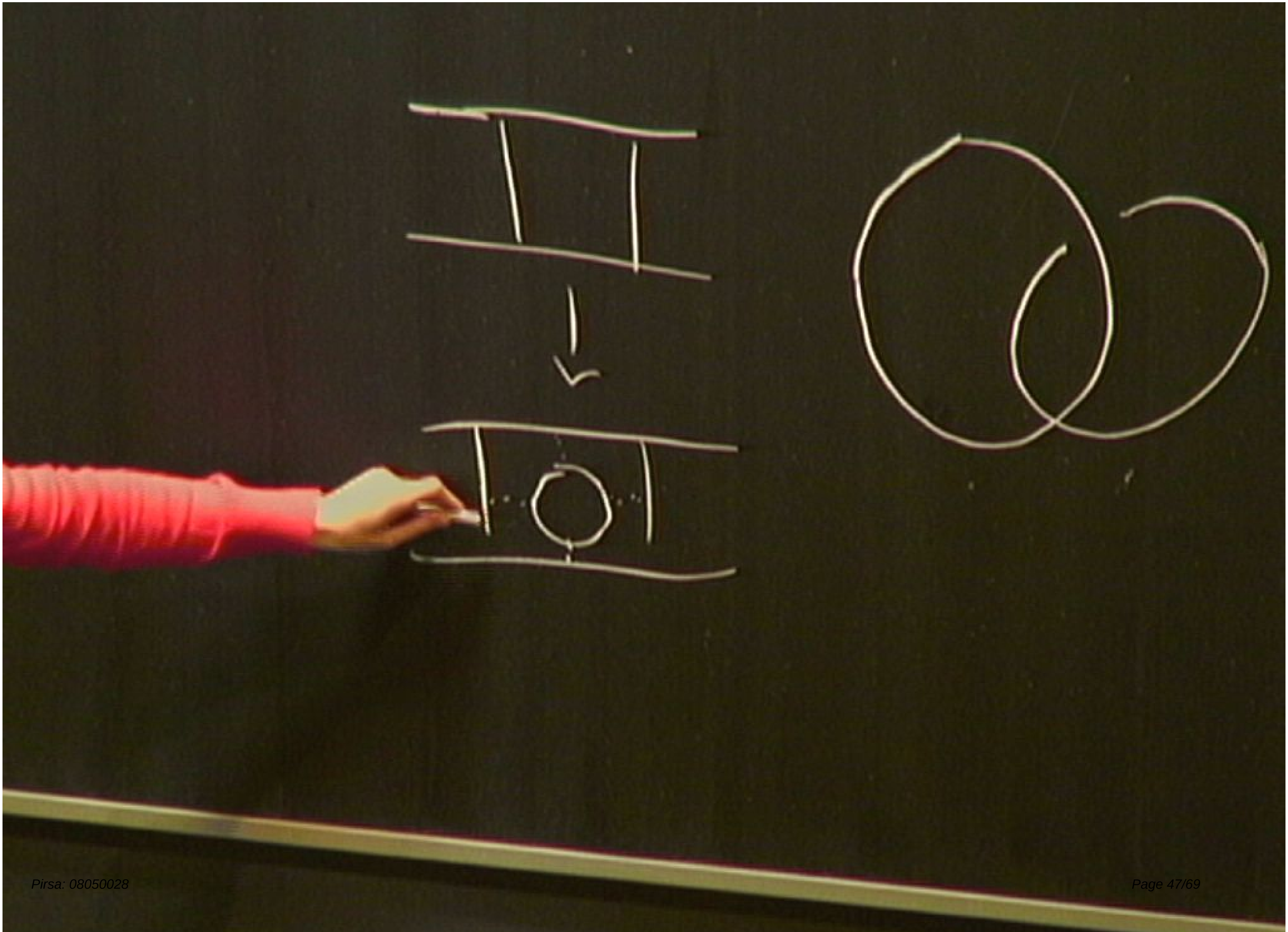
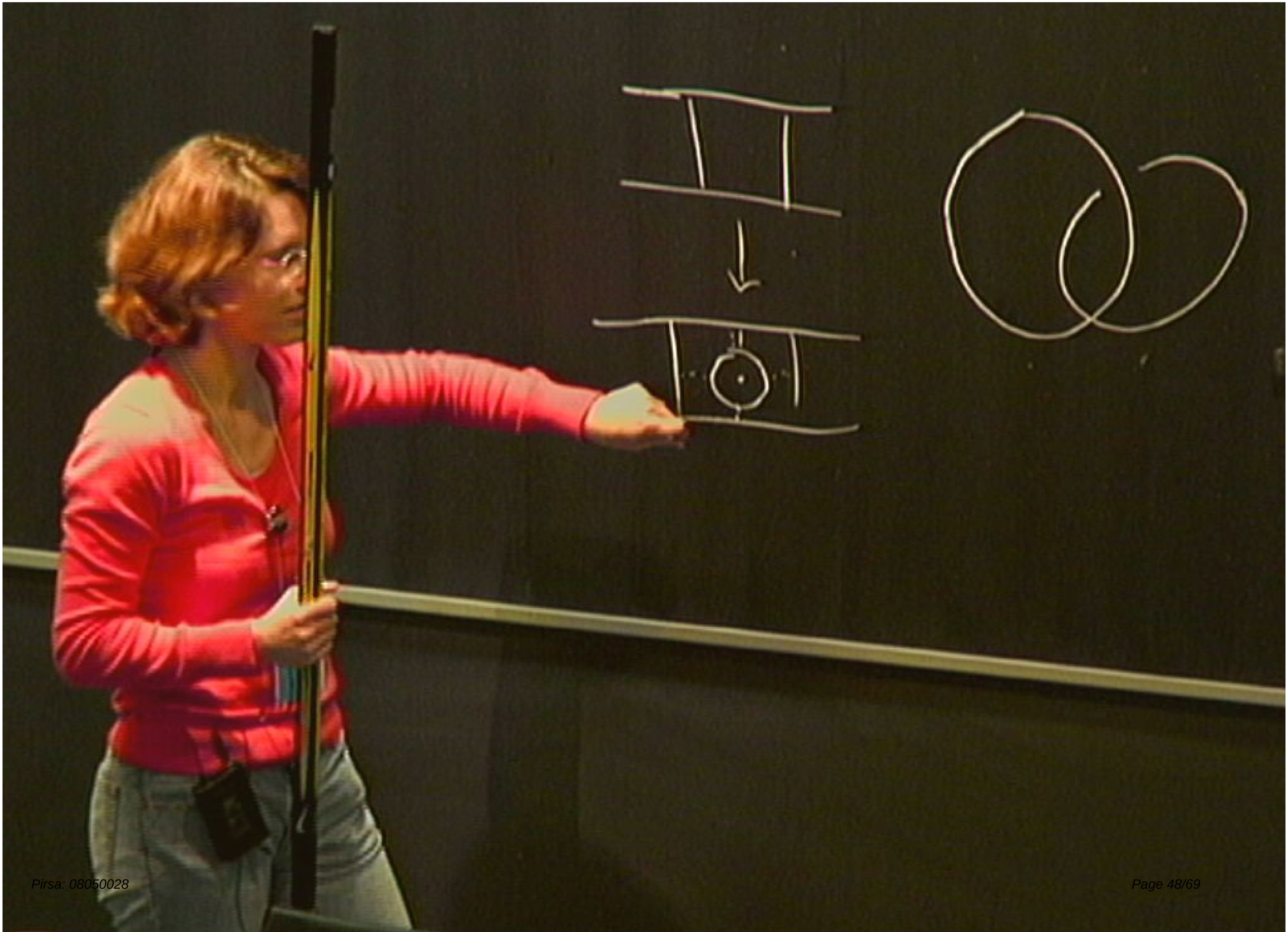
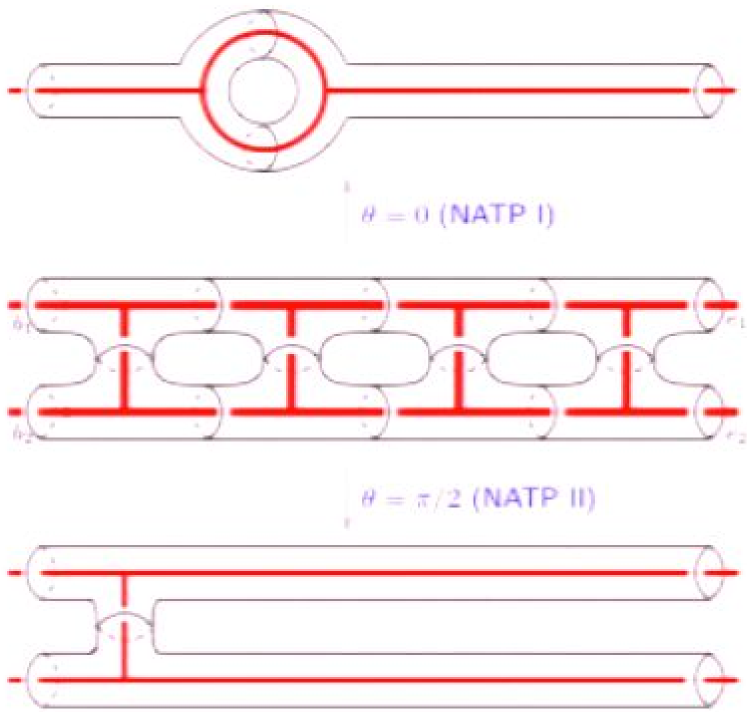






NATPs: first excited state



- ◇ NATP I: plaquette-type excitation, $3L$-fold degenerate
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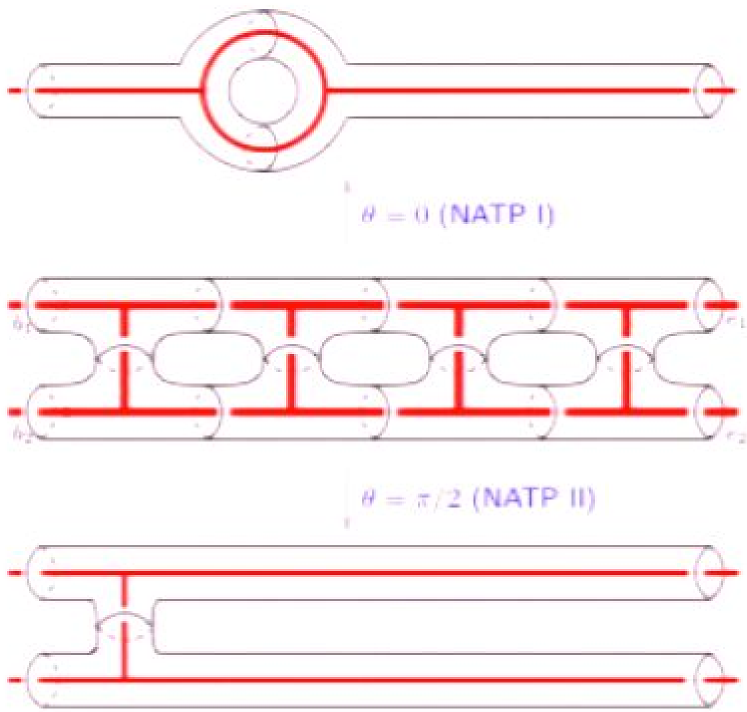

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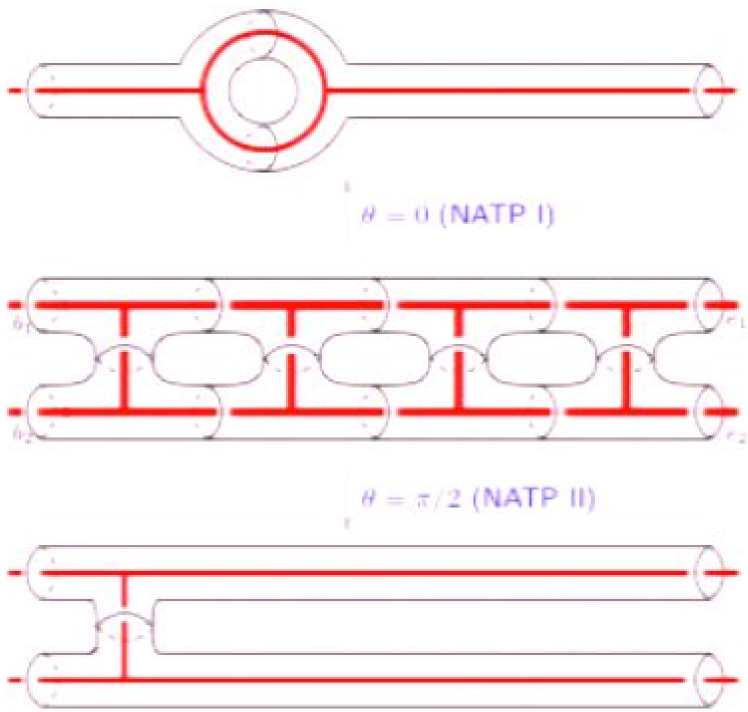

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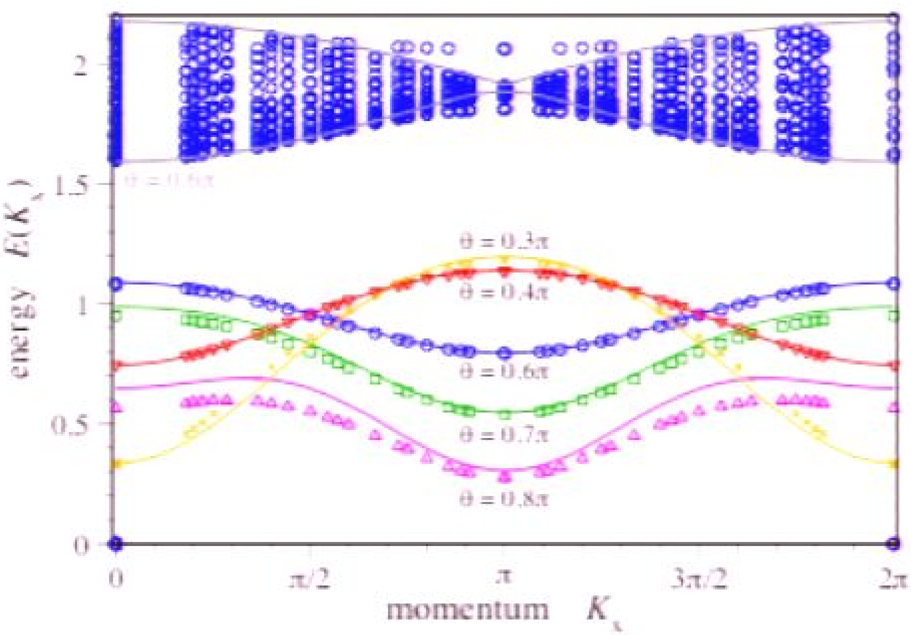

- Finite-size scaling analysis: gap and GS degeneracy are preserved for $\theta \in (\frac{\pi}{4}, \pi)$ and $\theta \in (-\frac{\pi}{2}, \frac{\pi}{4})$
- Lines: perturbation theory
- (Almost) duality
  $\Rightarrow$ perturbative results also apply close to Levin-Wen point $(\theta = 0)$
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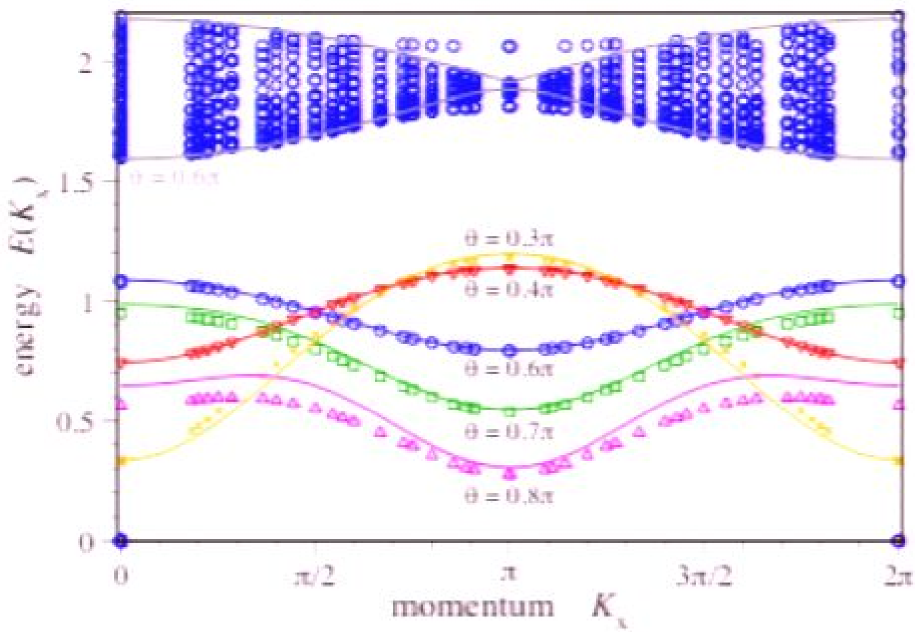

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Critical points

- At $\theta = \pi/4$: energy spectrum matches 2D CFT with central charge $c = 14/15$, where

$$E(k_x) = E_1 L + \frac{a}{L}\left(-\frac{c}{12} + h_L + h_R\right),$$

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  - $k = 8$ restricted-solid-on-solid (RSOS) model
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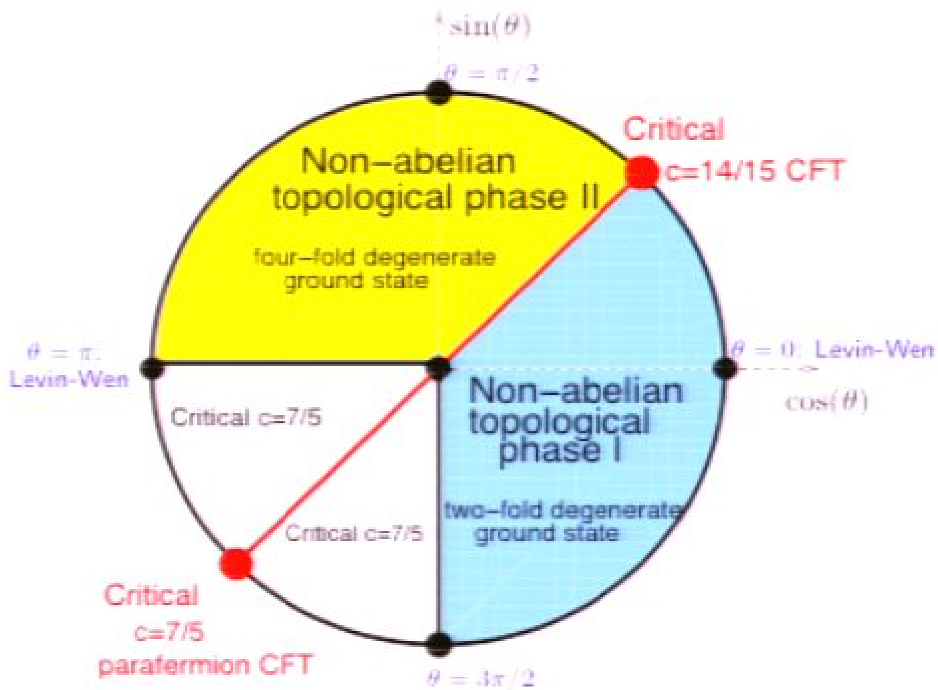
Phase diagram



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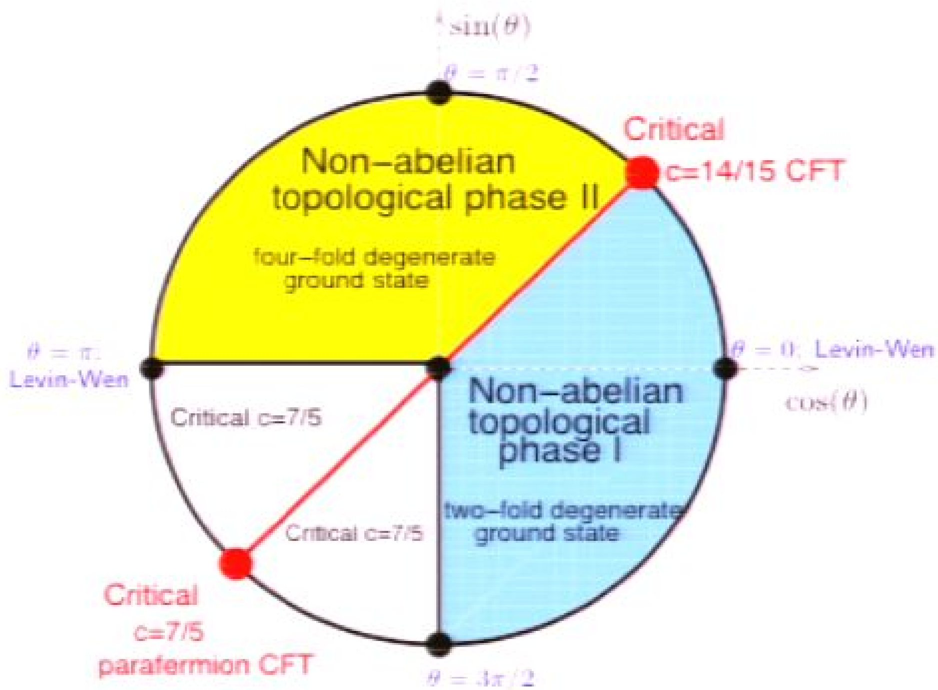

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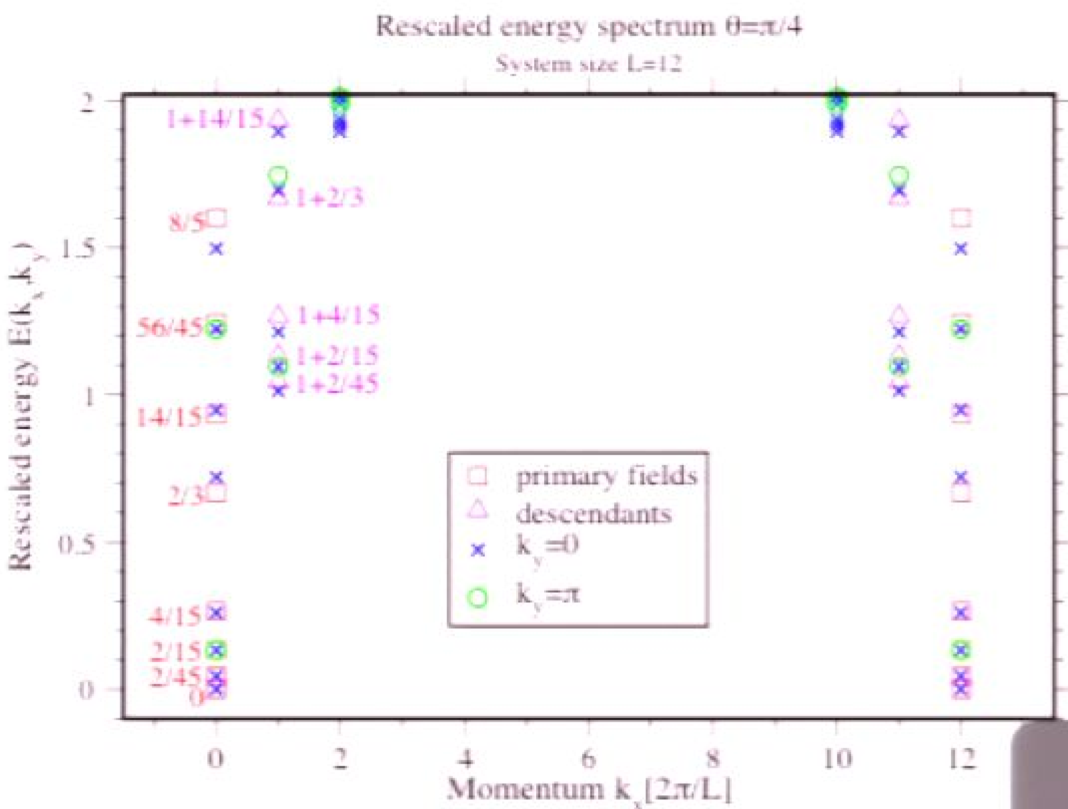

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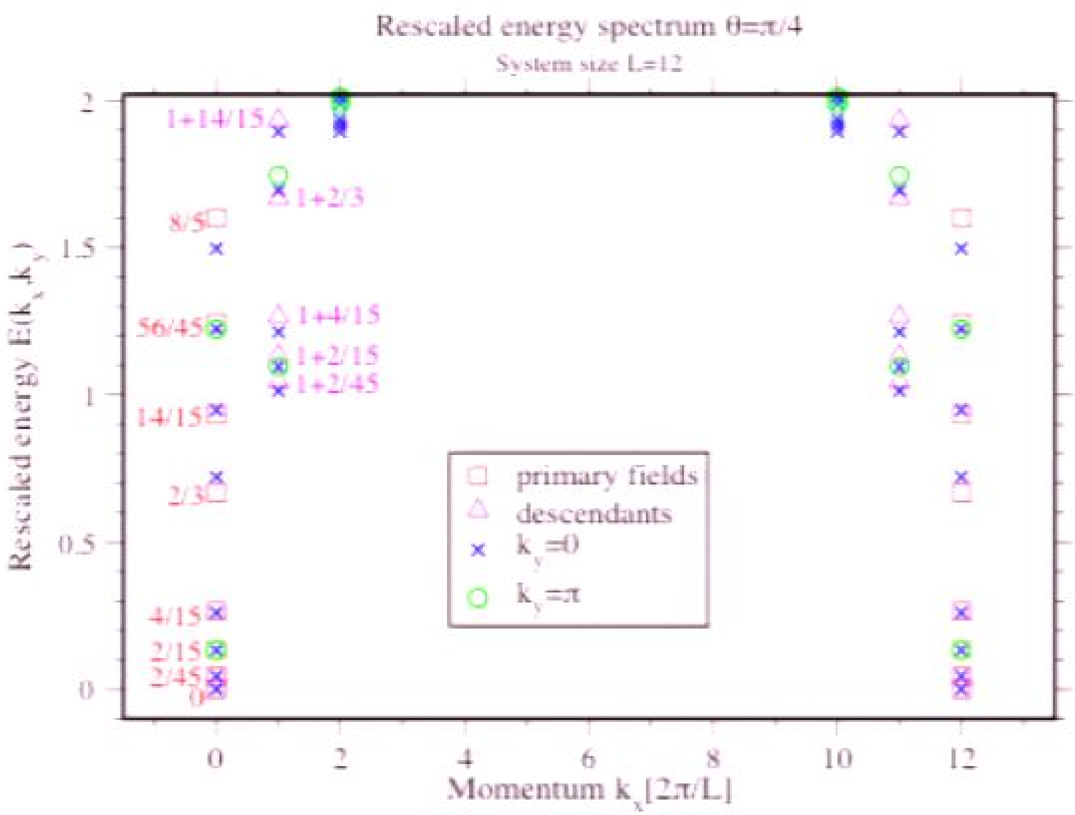

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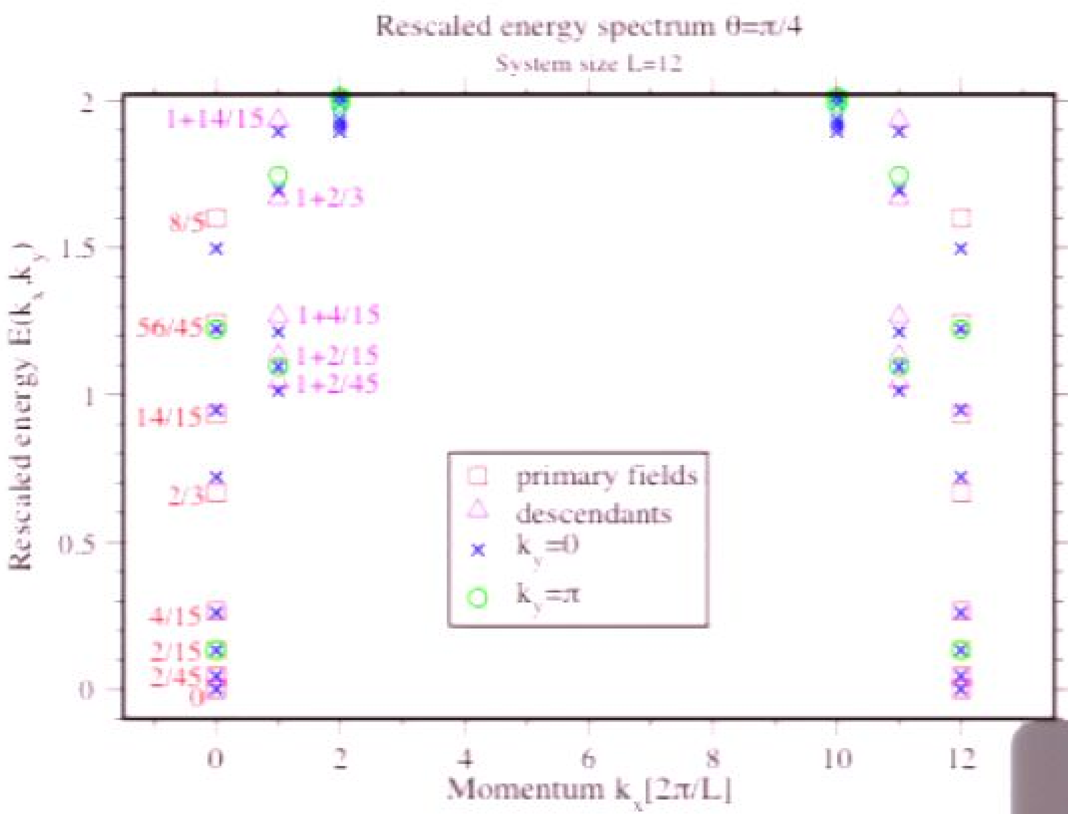

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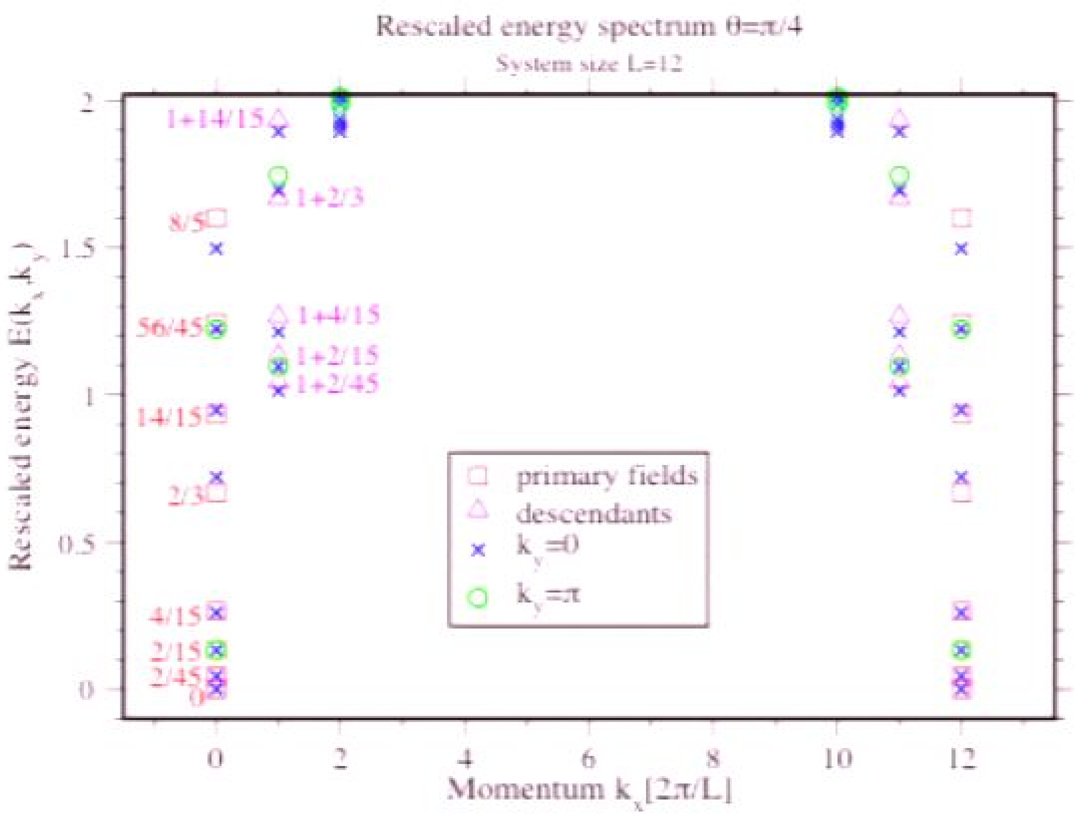

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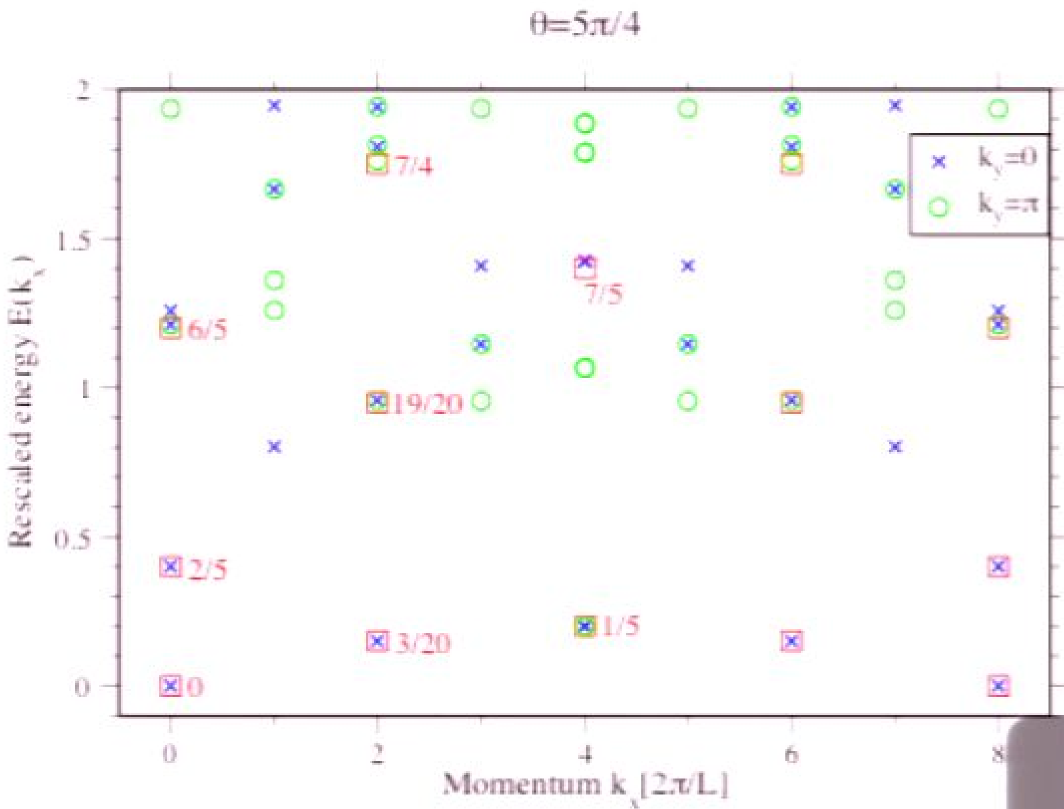

Conclusions

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- Stability of non-abelian topological phases
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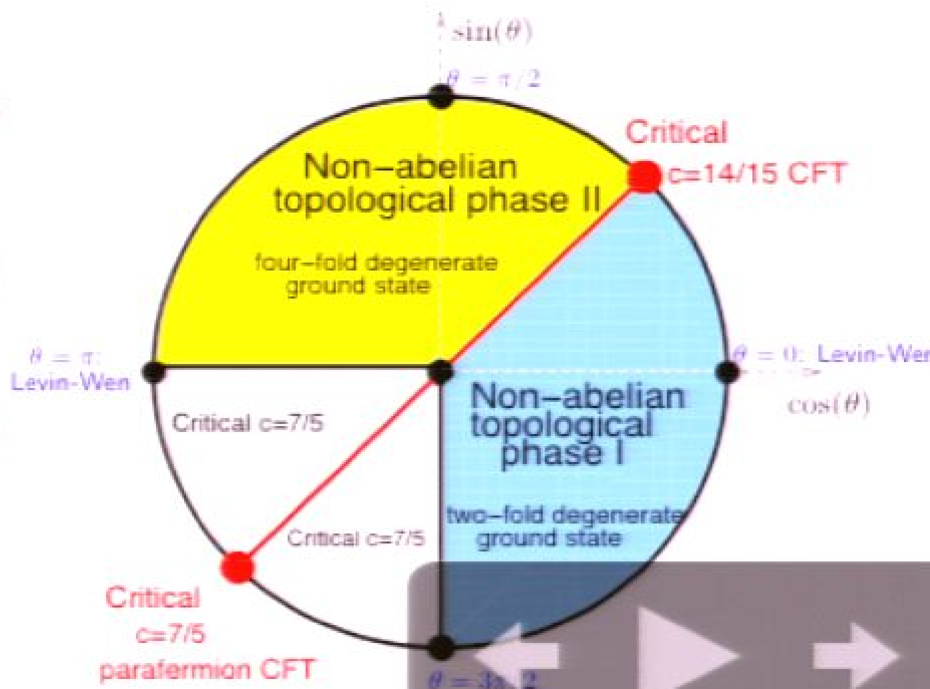

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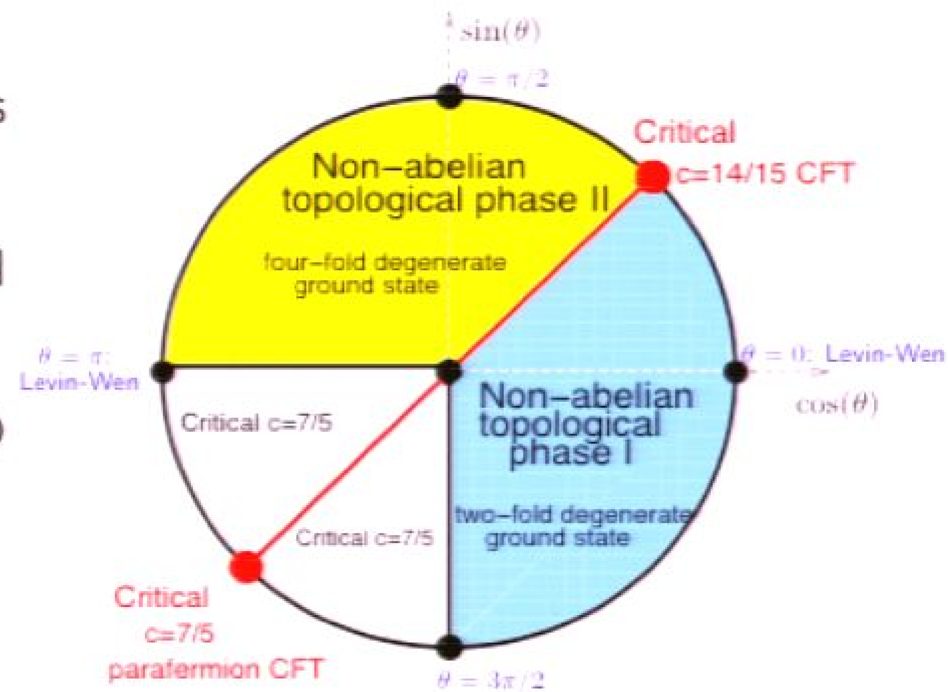

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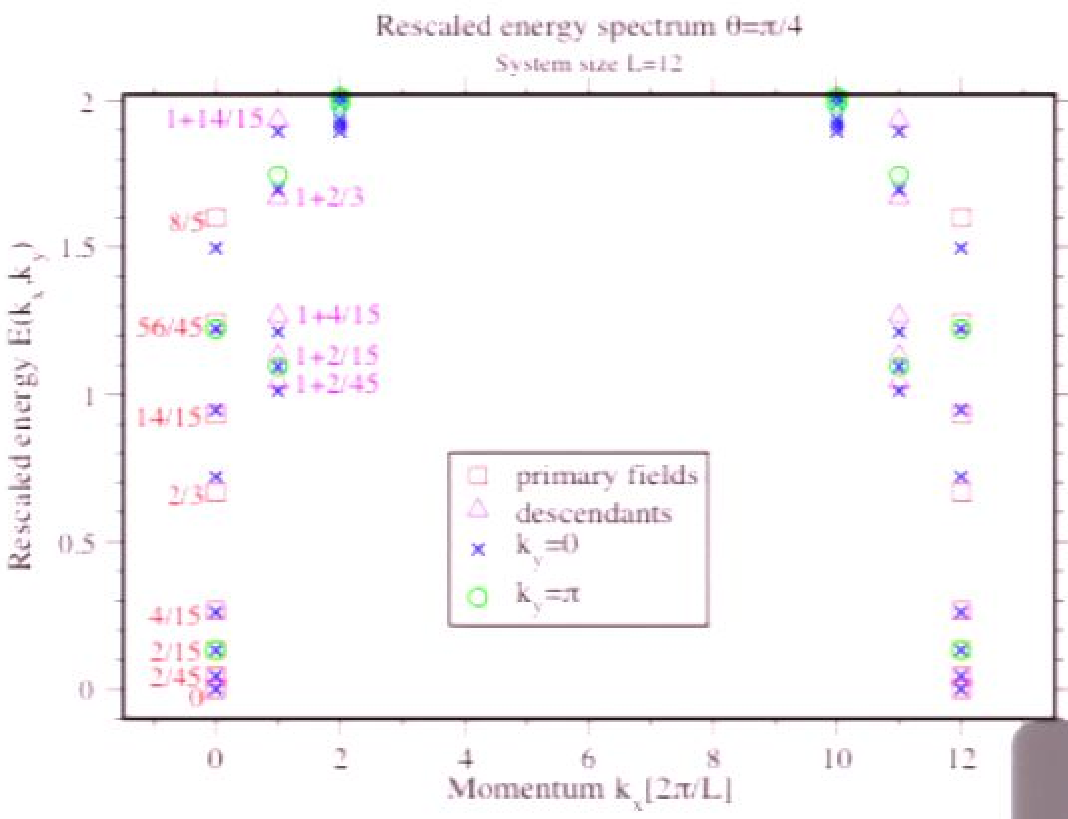

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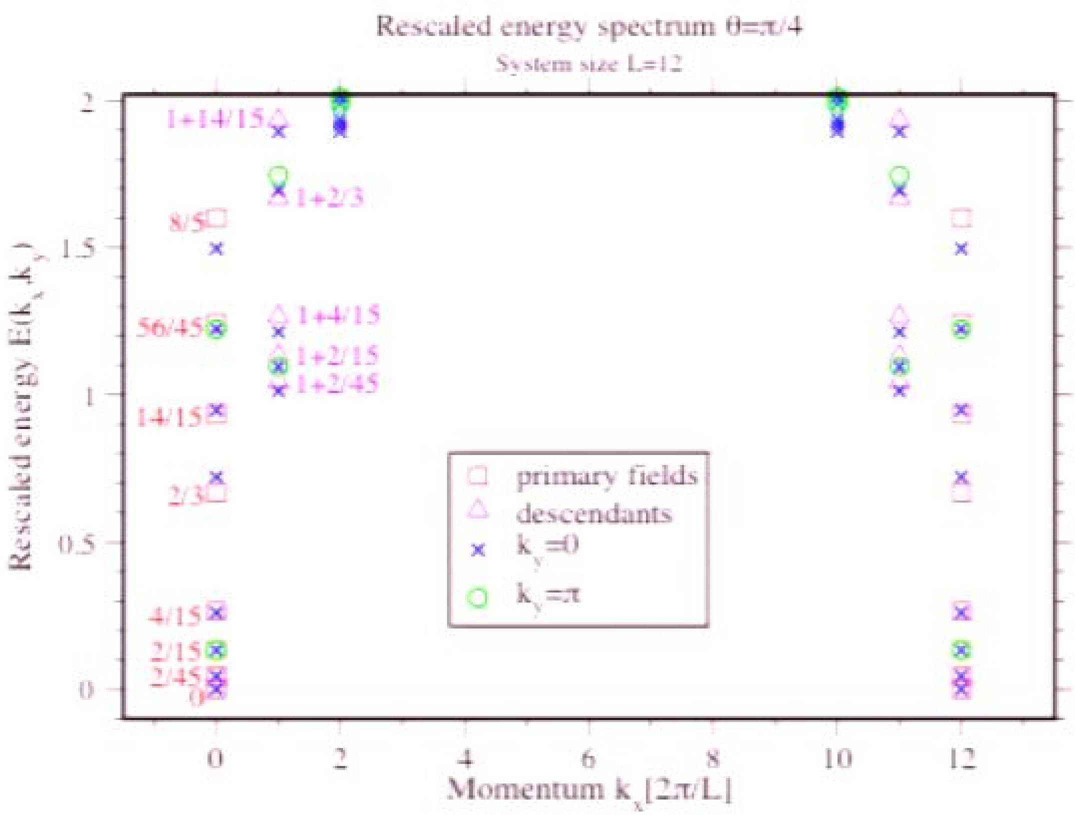

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