Title: Renormalization in Noncommutative Quantum Field Theory

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Abstract: This talk presents some recent results in renormalizable noncommutative quantum field theory. After introducing the renormalization group approach in the commutative setting I will procede to its generalization to the simplest noncommutative model, $phi_4^{star 4}$ on the Moyal space. The well known phenomenon of ultraviolet/infrared mixing is cured by adding a harmonic potential term to the free action. Under the new renormalization group, adapted to the noncommutative geometry, this model turns out to be renormalizable to all orders in perturbation theory. Moreover it is { f asymptotically safe} at all orders in perturbation theory. The consequences of this results are discussed.

Renormalization in Noncommutative Quantum Field Theory

Răzvan Gurău

Perimeter Institute, May 2008

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As experimental data lack, we need to deepen our understanding of the available theories in order to access them.

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We will discuss the second problem in detail.

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- a the "wave function constant", usually fixed to 1;
- Z_0 is a normalization;
- $D\phi$ is an ill defined product $\prod_{x \in \mathbb{R}^d} d\phi(x)$.

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- A power counting. This allow to classify graphs into convergent (finite amplitude) and divergent (infinite amplitude) ones.
- A locality principle. Any divergent graph with high internal scales must look in some sense like a point. This allows the infinite amplitude of the graph to be reabsorbed in a redefinition of the initial parameters of the action.

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We cut the propagator into a sequence of slices

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A propagator at the scale i contributes a factor M^{2i}. At scale i, $\|x-y\| \approx M^{-i}$.
Thus an integral $\int d^4x$ gives a M^{-4i} factor.

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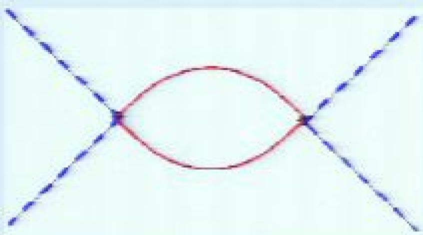

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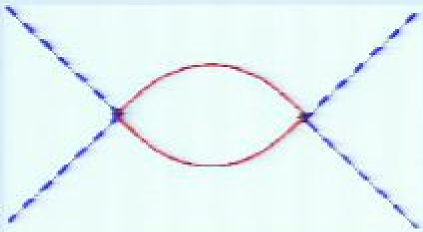

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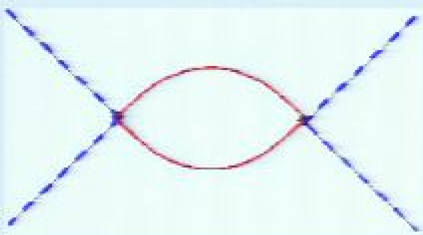

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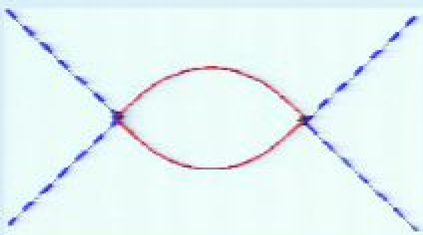

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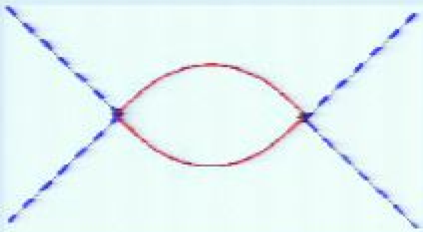

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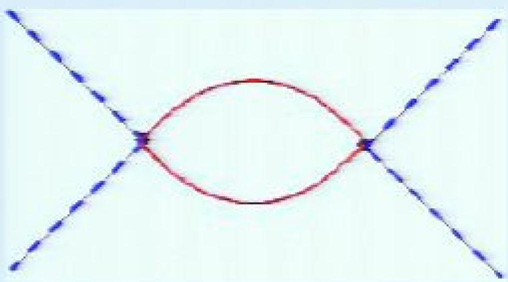

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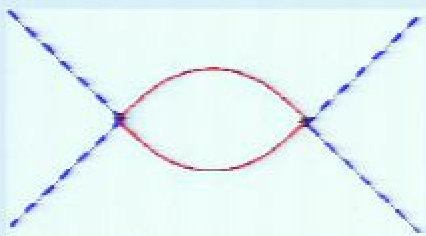

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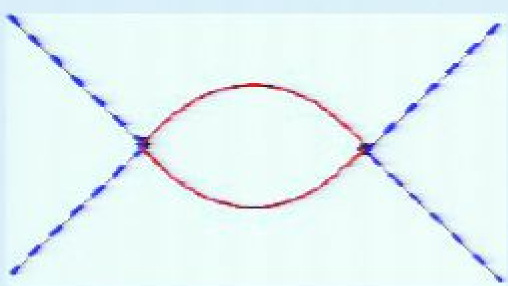

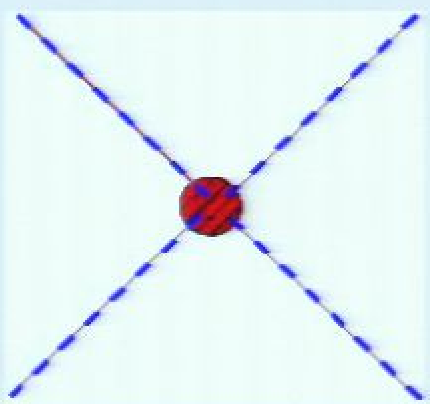

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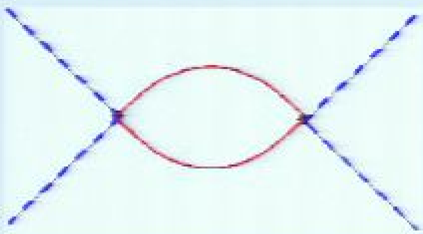

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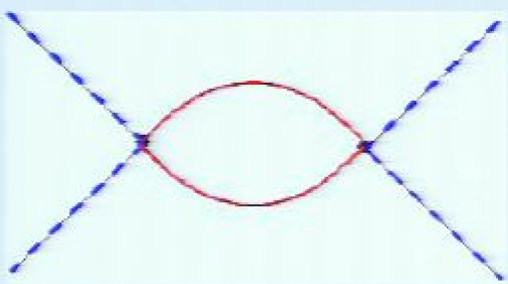

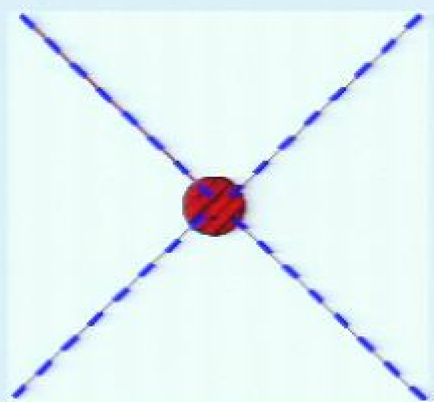

The leading contribution is divergent, but due to locality, it can be reabsorbed in a redefinition of the coupling constant λ

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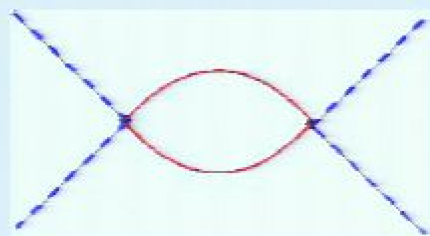

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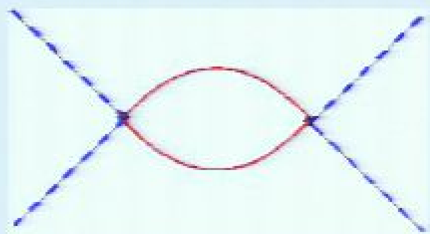

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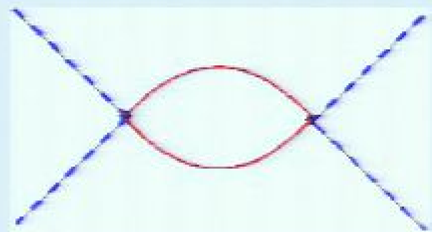

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- Around the same time Wilson developed the renormalization group.

Asymptotic Freedom

In fact field theory and renormalization made in the early 70's a spectacular *comeback*:

- Weinberg and Salam unified the weak and electromagnetic interactions into the formalism of Yang and Mills of non-Abelian gauge theories, which are based on an internal non commutative symmetry.
- 't Hooft and Veltmann showed that these theories are renormalizable.
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- But the price to pay is that the infrared limit is nonperturbative!

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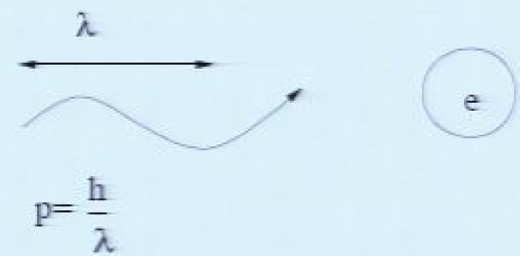

Space-time itself could be of this type; for instance, at a certain scale, new uncertainty relations could appear between length and width at the Planck length: $\Delta x \Delta y \approx l_P^2$.

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- '06-present: Asymptotic safeness, dimensional renormalization, etc. mark the advent of the noncommutative renormalization group

The Moyal space $\mathbb{R}^4_\theta$

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Defined by the algebra of Schwartz class functions endowed with the Moyal $\star$ product on $\mathbb{R}^D$

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with θ an antisymmetric constant matrix.

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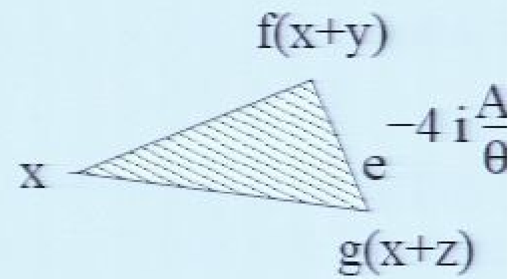

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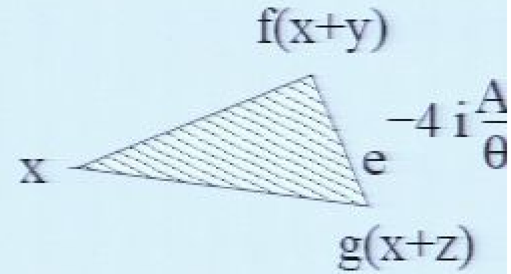

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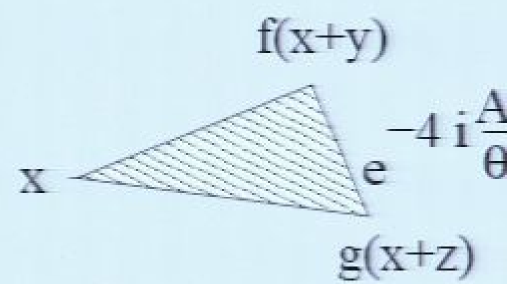

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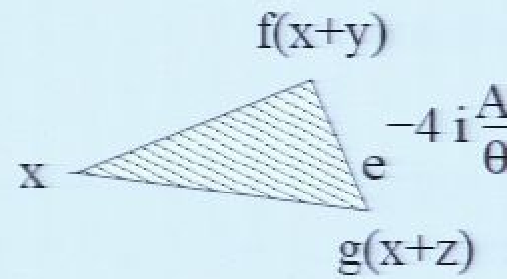

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We recover the commutative product as θ goes to 0.

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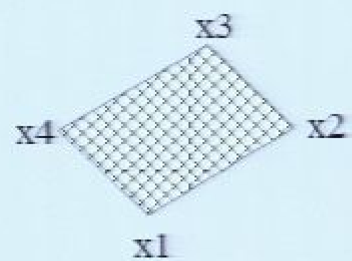

Feynman graph G: N vertices, L propagators, F faces, B faces broken by N_e external legs and genus g.

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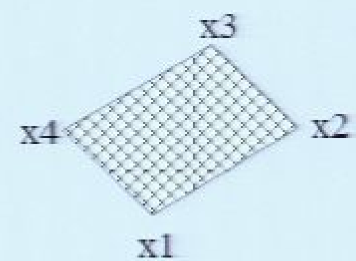

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$2 - 2g = N - L + F \qquad 4N - N_e = 2L$

Examples of Feynman graphs

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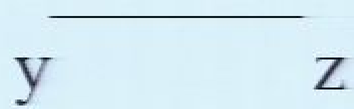

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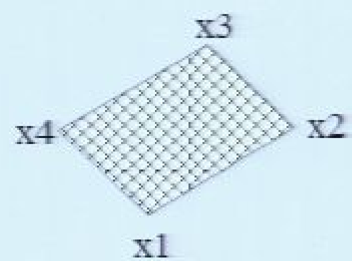

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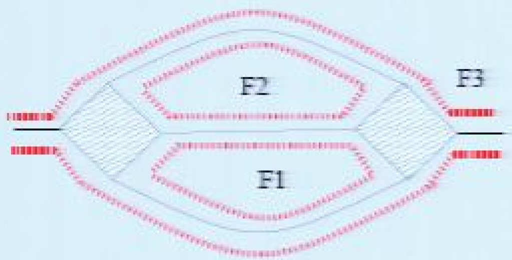

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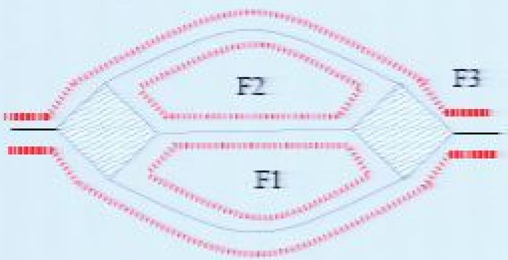

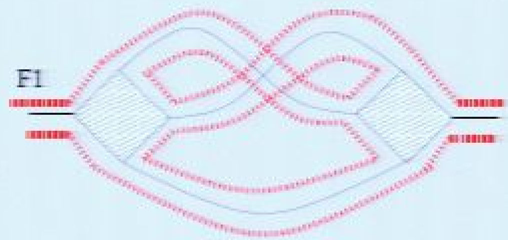

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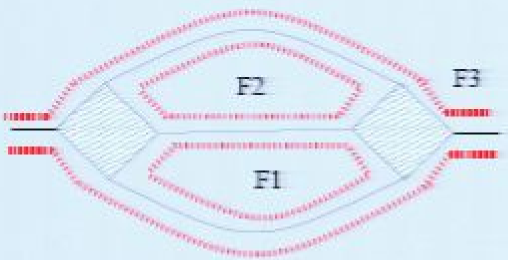

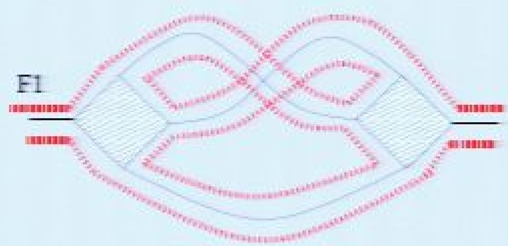

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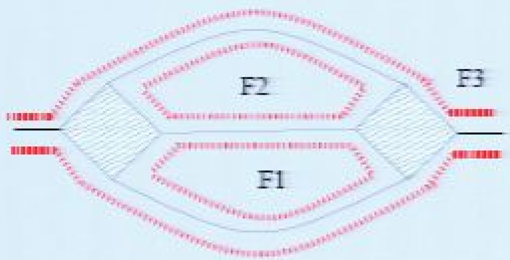

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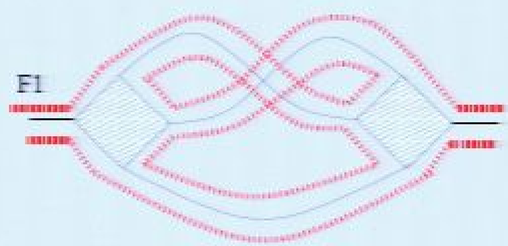

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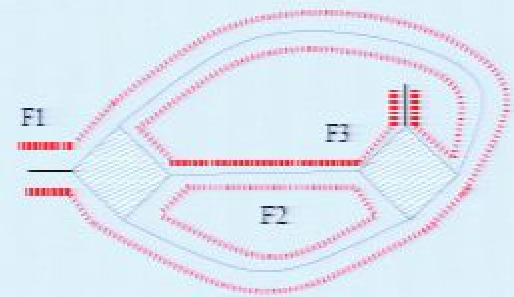

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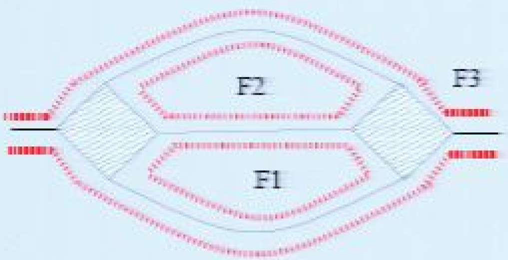

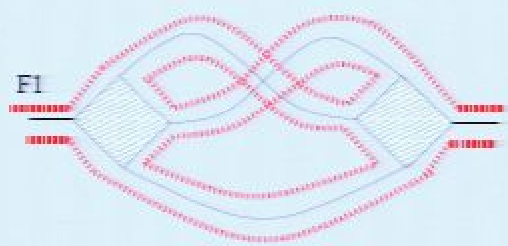

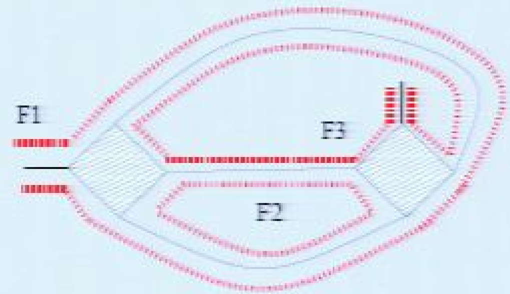

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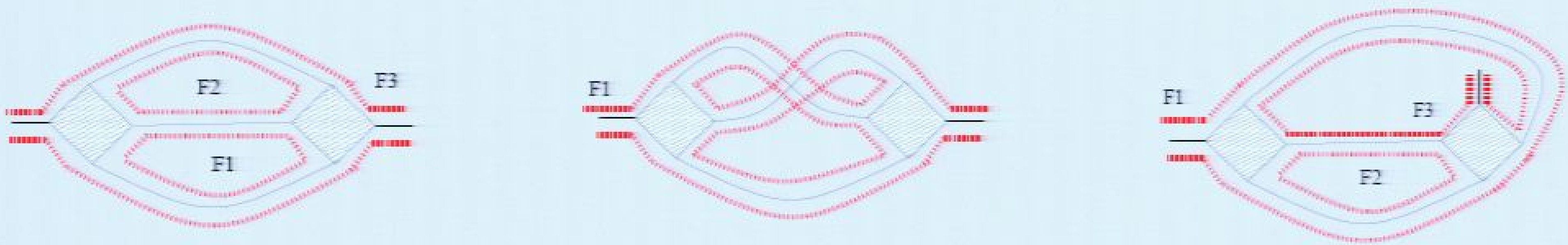

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Only planar ($g=0$) graphs with one broken face ($B=1$) are divergent.

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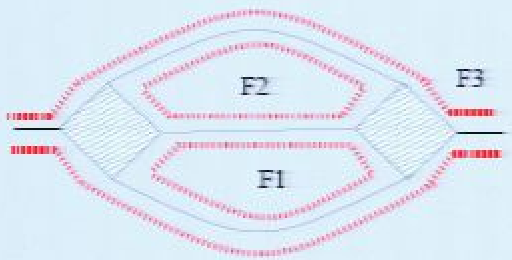

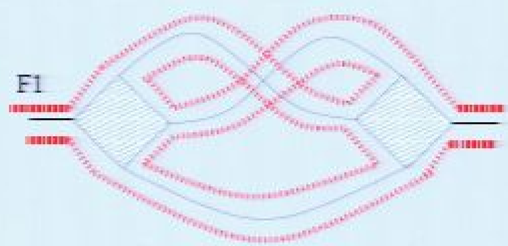

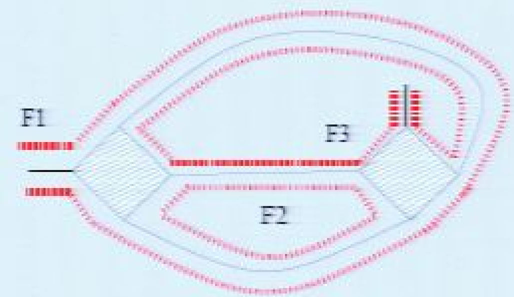

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They have the form of the initial Lagrangian, thus the theory seems renormalizable.

UV/IR mixing

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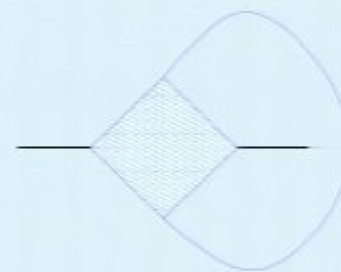

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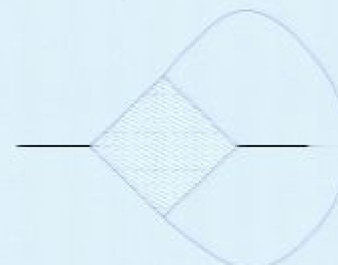

It's amplitude is $A = \int d^4 p \frac{e^{\imath k \theta p}}{p^2 + m^2} \approx \frac{1}{k^2}$.

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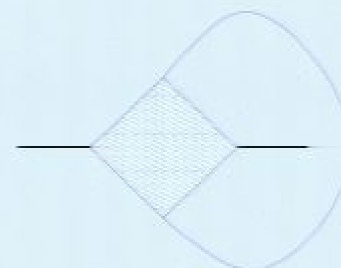

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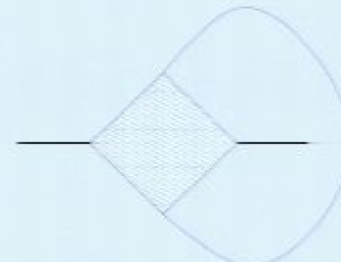

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The model is non renormalizable. Although a lot of effort has been put into finding a cure for this problem, the solution was not easy to find.

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For all lines l going from x to y we set

- $u_l = x - y$, the UV-short variable
- $v_l = x + y$, the UV-long variable

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Careful analysis: only $N_e = 2$ and $N_e = 4$ point graphs may diverge.

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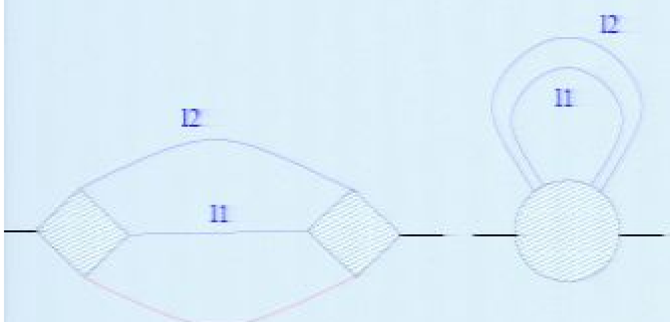

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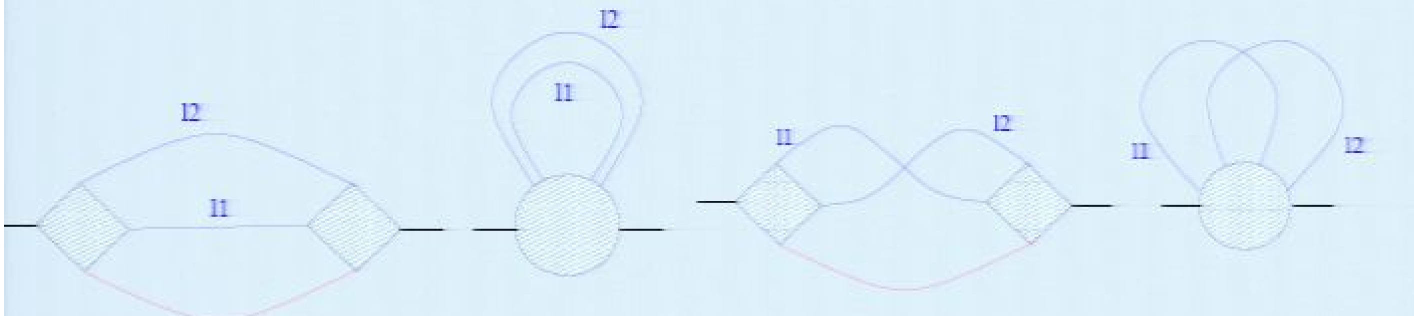

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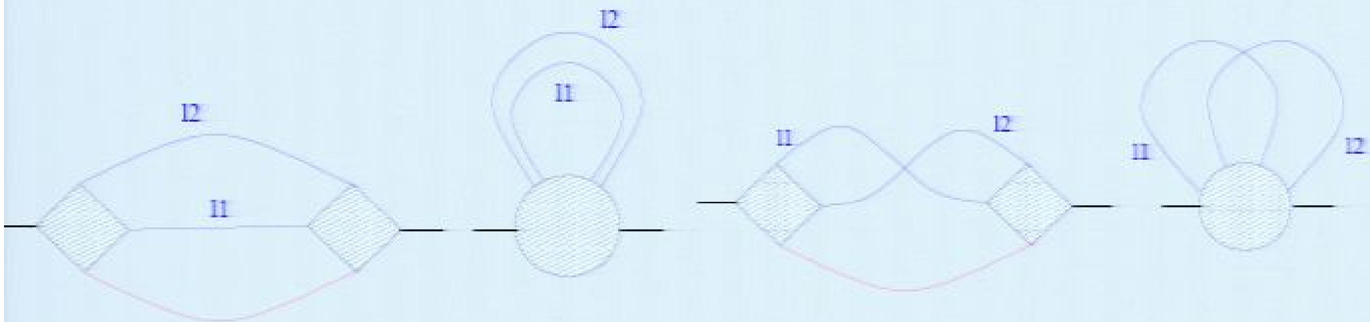

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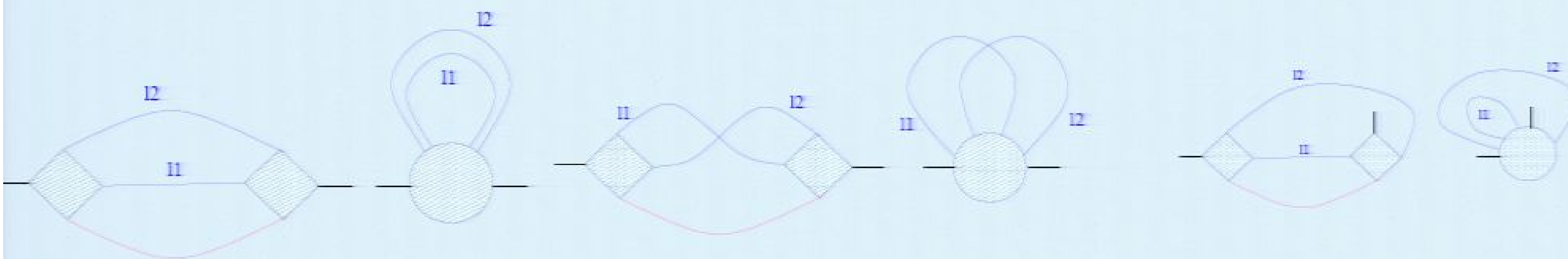

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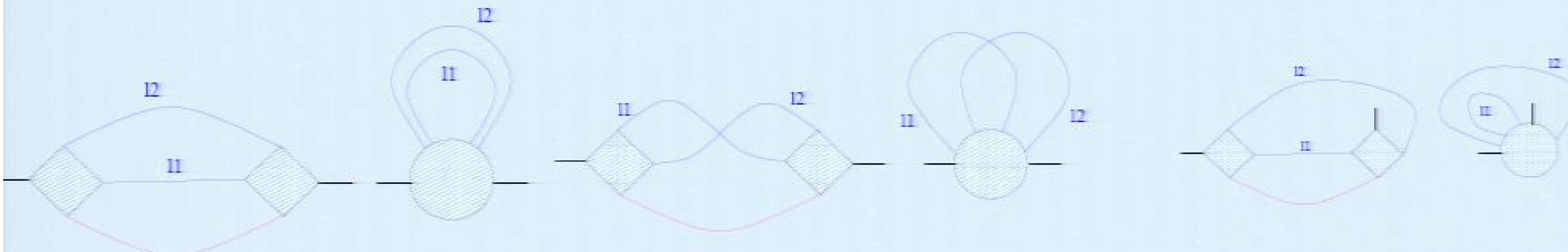

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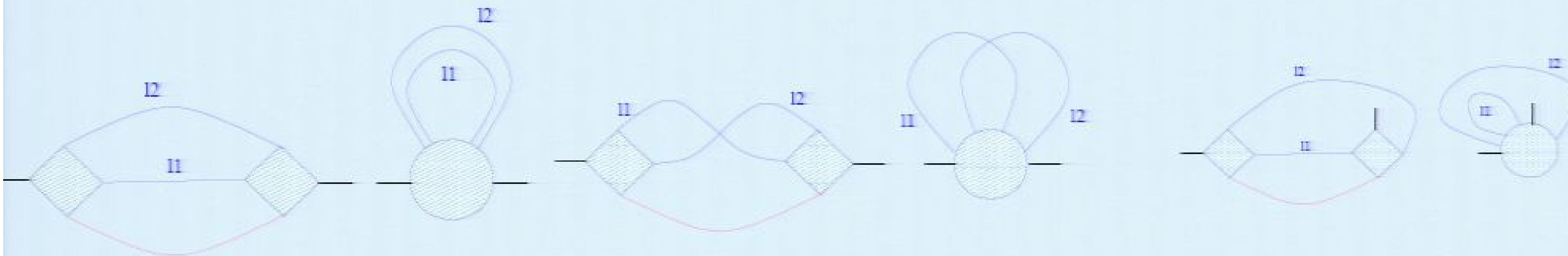

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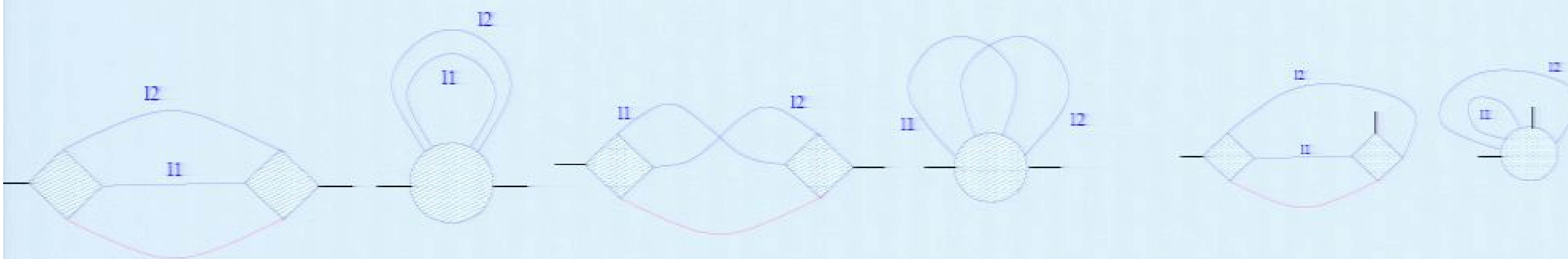

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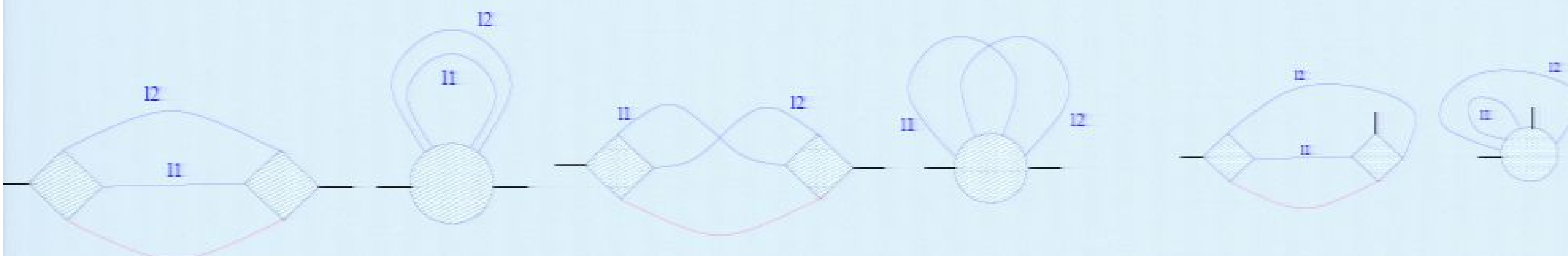

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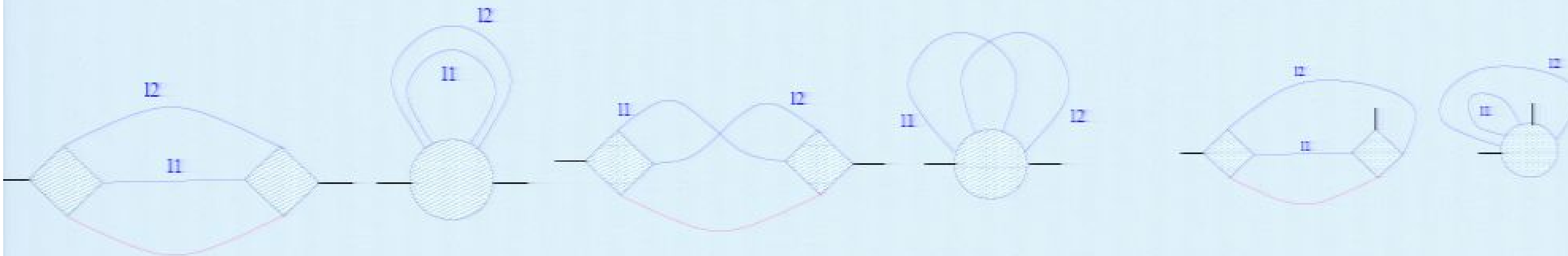

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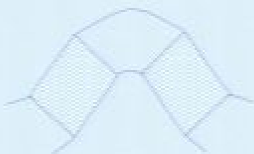

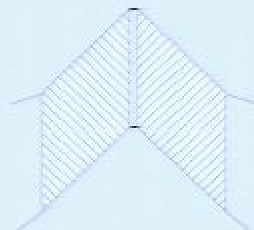

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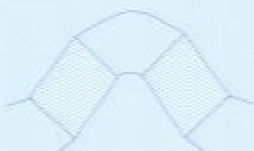

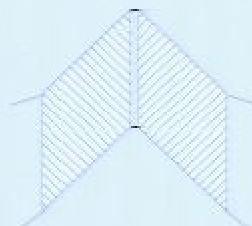

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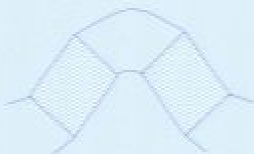

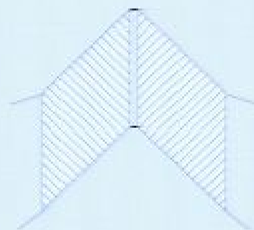

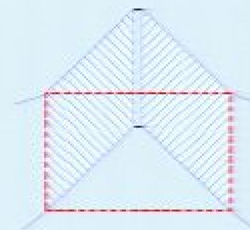

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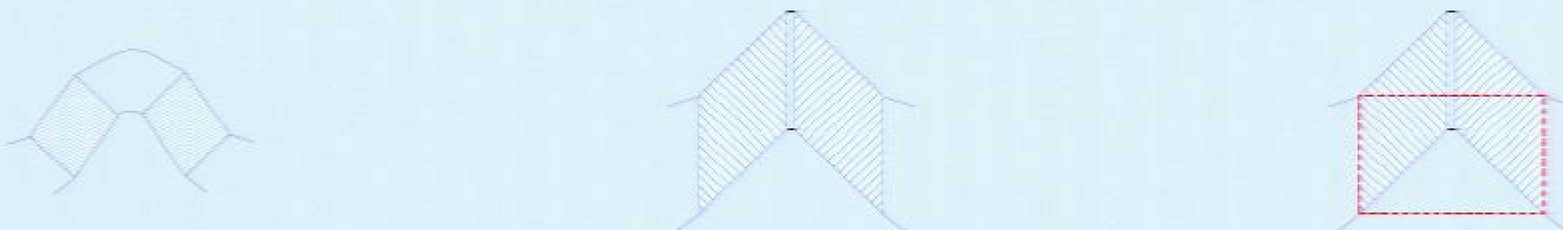

The vertex contribution for a planar regular graph is exactly:

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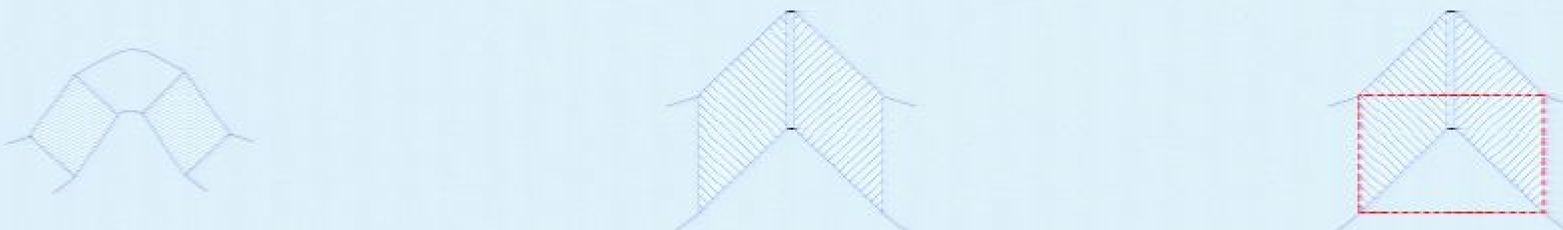

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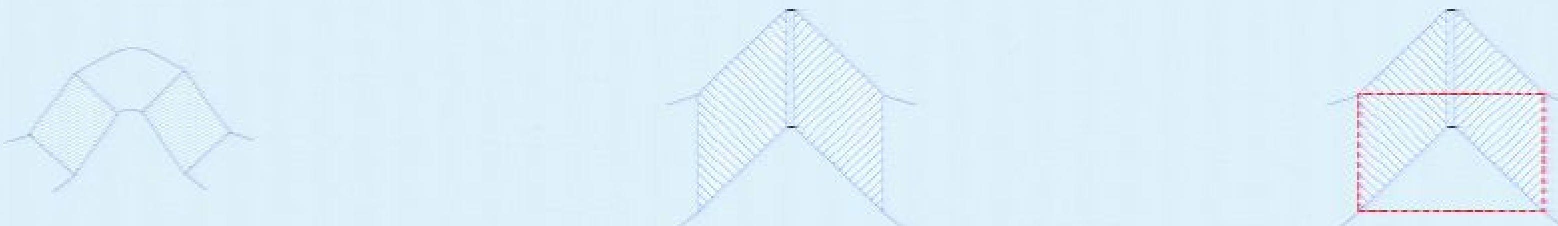
Moyality Versus Locality: The Four point function

Subtract graphs with UV internal scales.



The vertex contribution for a planar regular graph is exactly:

$$\delta\Big(\sum_i (-1)^{i+1} x_i + \sum_{l\in T\cup\mathcal{L}} u_l\Big) e^{\imath \sum_{i,j} (-1)^{i+j+1} x_i \theta^{-1} x_j}$$
$$e^{\imath \sum_{l\in T\cup\mathcal{L},\, l\prec j} u_l \theta^{-1} (-1)^j x_j + \imath \sum_{l\in T\cup\mathcal{L},\, l\succ j} (-1)^j x_j \theta^{-1} u_l}$$
$$e^{-\imath \sum_{l,l'\in T\cup\mathcal{L},\, l\prec l'} u_l \theta^{-1} u_{l'} - \imath \sum_{l\in T} \frac{u_l \theta^{-1} v_l}{2} \varepsilon(l) - \imath \sum_{l\in\mathcal{L}} \frac{u_l \theta^{-1} w_l}{2} \varepsilon(l)}$$
$$e^{-\imath \sum_{l\in\mathcal{L},\, l'\in\mathcal{L}\cup T;\, l'\subset l} u_{l'} \theta^{-1} w_l \varepsilon(l)}$$

UV: the first line decouples. We obtain a Moyal kernel times a divergent integral. The divergence can be reabsorbed into a redefinition of the coupling constant.

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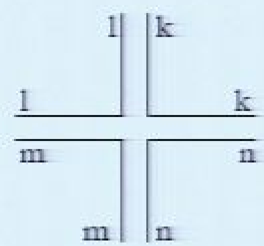

vertex

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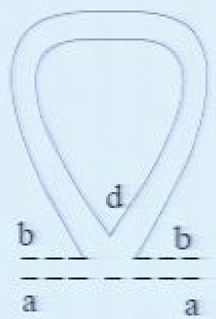

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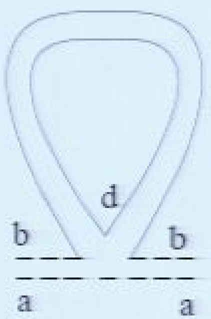

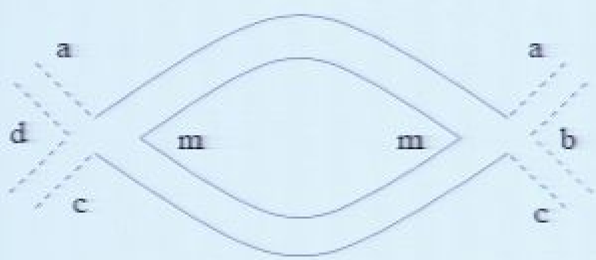

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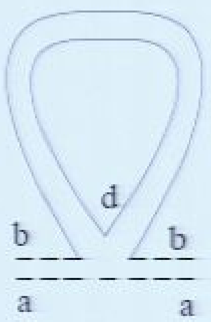

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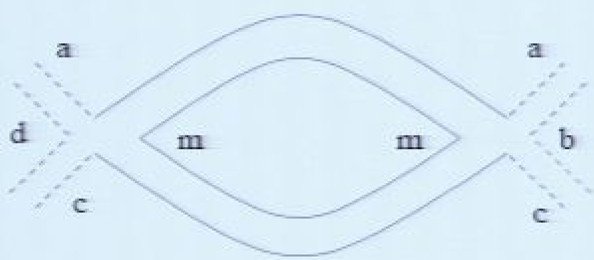

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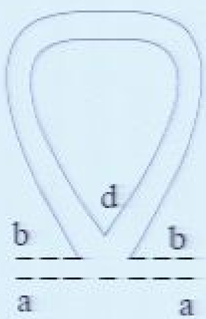

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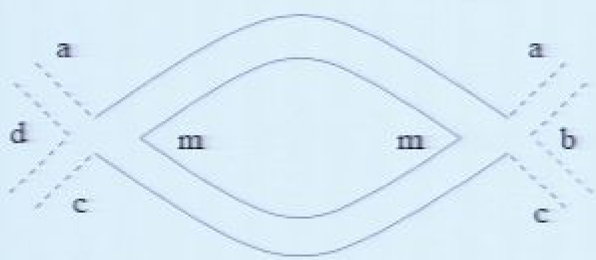

$$A^{bubble}(a,b,c,d) = \frac{1}{2}\frac{\lambda^2}{2^2}2\cdot 4\cdot 2$$

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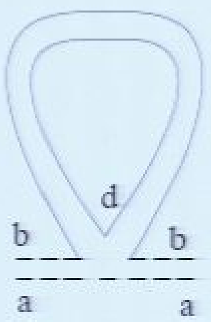

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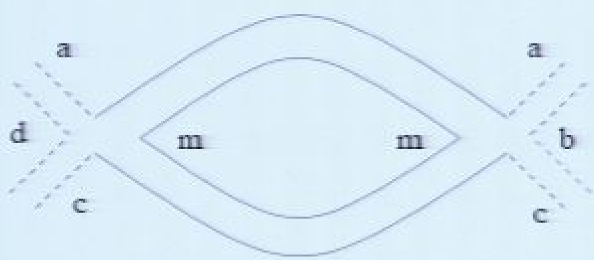

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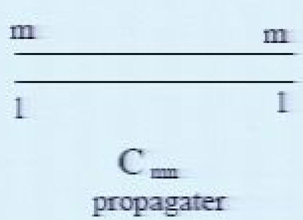

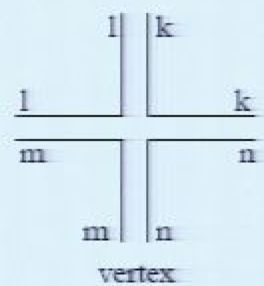

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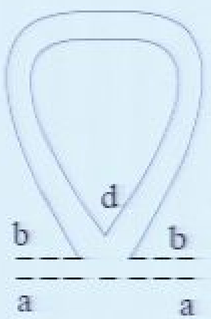

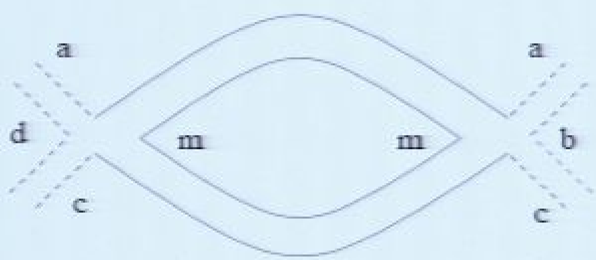

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$$\lambda^{effective} = -\frac{\Gamma^4}{Z^2}$$

where Γ^4 and Z^2 are infinite power series in λ.

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The proof relies on the Dyson equation and a set of Ward identities associated to the underlying Area Preserving Diffeomorphisms covariance of the theory.

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- Renormalization group flows are indeed modified when there is non commutativity of space-time, thus predictions can be tested.

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Nevertheless spectral scales could be defined for LQG. A diffeomorphism invariant renormalization group can exist!