Title: Ground-code measurement-based quantum computer

Date: May 01, 2008 03:50 PM

URL: http://pirsa.org/08050025

Abstract: I will talk about a scheme of the ground-code measurement-based quantum computer, which enjoys two major advantages. (i) Every logical qubit is encoded in the gapped degenerate ground subspace of a spin-1 chain with nearest-neighbor two-body interactions, so that it equips built-in robustness against noise. (ii) Computation is processed by single-spin measurements along multiple chains dynamically coupled on demand, so as to keep teleporting only logical information into a gap-protected ground state of the rest chains after the interactions with spins to be measured are adiabatically turned off. Our scheme is a conceptual advance, since measurements generally create excitations in the system so that two desired properties, keeping the information in the ground state and processing the information by measurements, are not seemingly compatible. I may shortly describe implementations using trapped atoms or polar molecules in an optical lattice, where the gap is expected to be as large as 0.2 KHz or 4.8 KHz respectively. This is a joint work with G.K. Brennen.

PI workshop on QIGT, April 28-May 2 (2008)

Ground-code measurement-based quantum computer

Akimasa Miyake

in collaboration with
Gavin K. Brennen

[1]University of Innsbruck, Austria
[2]Institute for Quantum Optics & Quantum Information

PI workshop on QIGT, April 28-May 2 (2008)

Ground-code measurement-based quantum computer

Akimasa Miyake

in collaboration with
Gavin K. Brennen

[1]University of Innsbruck, Austria
[2]Institute for Quantum Optics & Quantum Information

Kindergarten Introduction

"measurement-based QC without opening a fridge !?"

our sweet ice !
(precious resource for life)

Kindergarten Introduction

"measurement-based QC without opening a fridge !?"

our sweet ice !
(precious resource for life)

WARNING !!!
it tastes best, as far as you do not open the fridge and do not touch it.

Main idea

Marriage of two appealing streams for reliable quantum computer

➢ G.K. Brennen & A. Miyake, arXiv:0803.1478

- information stored and processed in the ground state of a Hamiltonian:
  topological QC, adiabatic QC

- computation on entanglement resource by measurements:
  measurement-based QC (ex. cluster-state QC)
  Note: implicitly assume "Hamiltonian" = 0 during computation

Main idea

Marriage of two appealing streams for reliable quantum computer

➢ G.K. Brennen & A. Miyake, arXiv:0803.1478

- information stored and processed in the ground state of a Hamiltonian:
  topological QC, adiabatic QC

- computation on entanglement resource by measurements:
  measurement-based QC (ex. cluster-state QC)
  Note: implicitly assume "Hamiltonian" = 0 during computatio

...but, measurements generally create excitations, so that keeping information in the ground state and processing information by measurements are <u>seemingly</u> incompatible !?

Outline

1. Main idea

2. Motivations: issues for marriage
   list of key requirements

3. Ground-code measurement-based QC
   (i) microscopic model (Hamiltonian) of a quantum wire
   (ii) computation by local measurements (MQC)
   (iii) ground-code properties
   (iv) physical implementations by trapped atoms/molecules

4. Summary

Motivations: issues for marriage

1. fundamental (conceptual) issues ?

- Measurements in an arbitrary direction may create excitations.

→ turning off interaction with spin to be measured

→ frustration-free (two-body) Hamiltonian

Motivations: issues for marriage

1. fundamental (conceptual) issues ?

- Measurements <u>in an arbitrary direction</u> may create excitations.

  → turning off interaction with spin to be measured

  → frustration-free (two-body) Hamiltonian

- Not only the entangled resource must be a ground state, but also information must be processed in the ground state

  → two-fold energetic degeneracy per logical qubit

Motivations: issues for marriage

.. fundamental (conceptual) issues ?

- Measurements in an arbitrary direction may create excitations.

turning off interaction with spin to be measured

frustration-free (two-body) Hamiltonian

- Not only the entangled resource must be a ground state, but also information must be processed in the ground state

two-fold energetic degeneracy per logical qubit

- ground state must be gapped and persist in the thermodynamic limit to be better scalable.

spin-1 Haldane chain in stead of spin-1/2 chain

Motivations: issues for marriage

.. fundamental (conceptual) issues ?

- Measurements <u>in an arbitrary direction</u> may create excitations.

turning off interaction with spin to be measured

frustration-free (two-body) Hamiltonian

- Not only the entangled resource must be a ground state, but also information must be processed in the ground state

two-fold energetic degeneracy per logical qubit

- ground state must be gapped and persist in the thermodynamic limit to be better scalable.

spin-1 Haldane chain in stead of spin-1/2 chain

- single excitation should not immediately cause logical error

only nonlocal string order connects logical subspace

Motivations: issues for marriage

2. practical issues ?

- perturbative K-local interaction is not fully reliable (by the smaller magnitude of interaction, more complicated engineering, and higher order corrections).

two-body, nearest-neighboring

Motivations: issues for marriage

2. practical issues ?

- perturbative K-local interaction is not fully reliable (by the smaller magnitude of interaction, more complicated engineering, and higher order corrections).

  → two-body, nearest-neighboring

- no known ground state, which is universal for standard one-way QC (consuming entanglement monotonically), and satisfies all requirements above.

  → hybrid approach (use of on demand dynamical coupling). In practice, it is economical to control degeneracy

Motivations: issues for marriage

2. practical issues ?

- perturbative K-local interaction is not fully reliable (by the smaller magnitude of interaction, more complicated engineering, and higher order corrections).

→ two-body, nearest-neighboring

- no known ground state, which is universal for standard one-way QC (consuming entanglement monotonically), and satisfies <u>all</u> requirements above.

→ hybrid approach (use of on demand dynamical coupling). In practice, it is economical to control degeneracy

Note: our idea must be distinguished from other (add-on) ideas for robustness

(i) encoding every qubit into decoherence-free subspace qubi

(ii) software technique (error correction) for fault-tolerance

Previous related works

1. ground state preparation of universal resource of MQC
   - 4 or 5-local cluster Hamiltonian by perturbation of two body nearest neighboring interactions [Bartlett, Rudolph, PRA'06
   - no graph state can be an exact unique ground state of two-body spin-1/2 interactions (though degeneracy might help...) [Nielsen,'05; Van den Nest, Luttmer, Dür, Briegel, PRA'08
2. use of finitely correlated states for MQC
   - valence-bond-solid picture of cluster state computation [Verstraete, Cirac, PRA'04]
   - matrix/tensor-product-states by graphical representatio ex. modified 1D/2D AKLT-type resource [Gross, Eisert, PRL'07; Gross, Eisert, Schuch, Perez-Garcia, PRA'07

Previous related works

1. ground state preparation of universal resource of MQC
   - 4 or 5-local cluster Hamiltonian by perturbation of two body nearest neighboring interactions [Bartlett, Rudolph, PRA'06
   - no graph state can be an exact unique ground state of two-body spin-1/2 interactions (though degeneracy might help...) [Nielsen,'05; Van den Nest, Luttmer, Dür, Briegel, PRA'08
2. use of finitely correlated states for MQC
   - valence-bond-solid picture of cluster state computation [Verstraete, Cirac, PRA'04]
   - matrix/tensor-product-states by graphical representatio ex. modified 1D/2D AKLT-type resource [Gross, Eisert, PRL'07; Gross, Eisert, Schuch, Perez-Garcia, PRA'07

MQC while keeping and processing information in a ground state (i.e., "marriage") is the main result of our work.

Outline

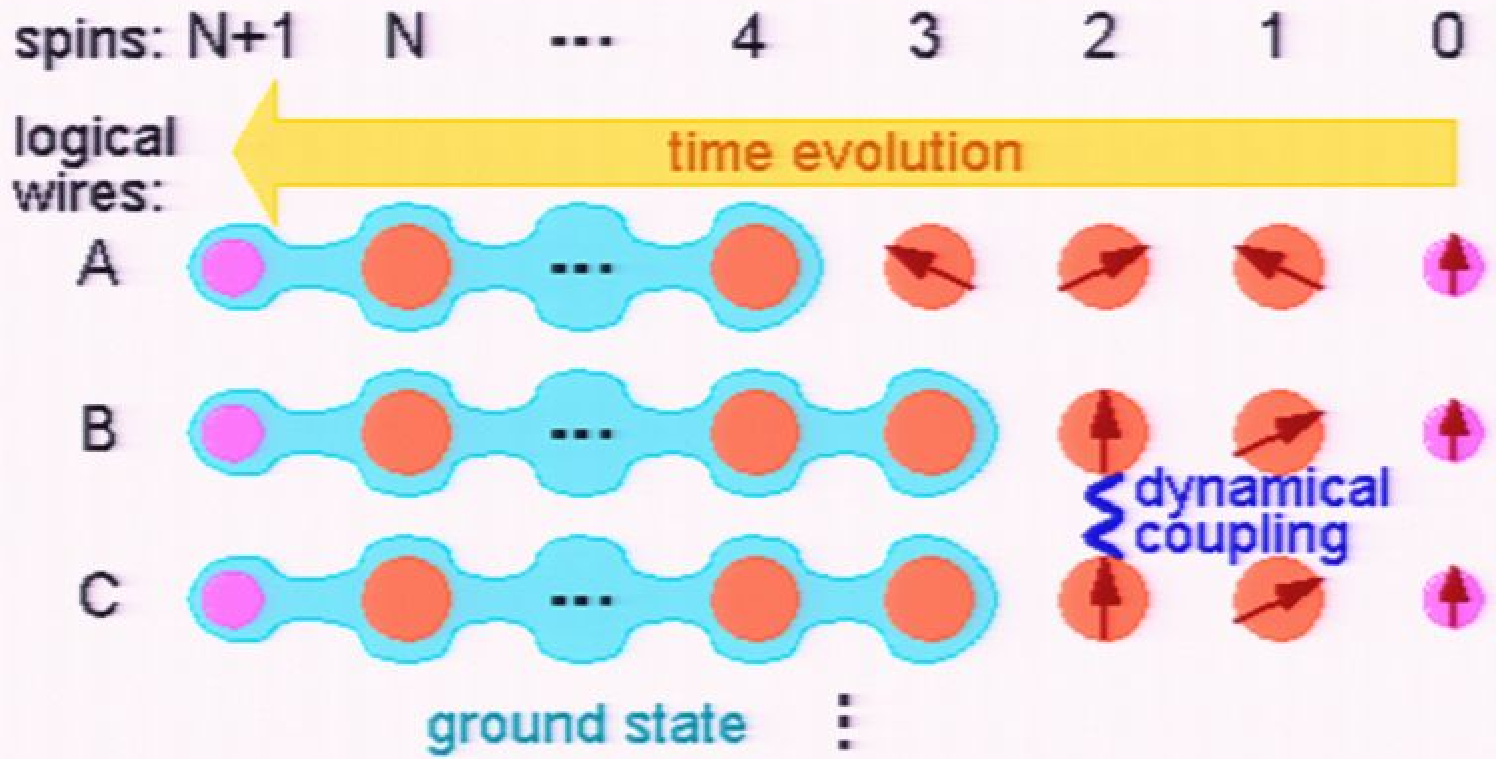
1. Main idea

2. Motivations: issues for marriage
   list of key requirements

3. Ground-code measurement-based QC
   (i) microscopic model (Hamiltonian) of a quantum wire
   (ii) computation by local measurements (MQC)
   (iii) ground-code properties
   (iv) physical implementations by trapped atoms/molecules

4. Summary

big picture of ground-code MQC



- *n* parallel spin-1 chains, dynamically coupled on demand

- computation (right to left) by single-spin measurements, after turning off two-body interaction along every chain

- time evolution of logical information is encoded in the gapped two-fold ground subspace of the residual chain

persistent gap and Haldane phase

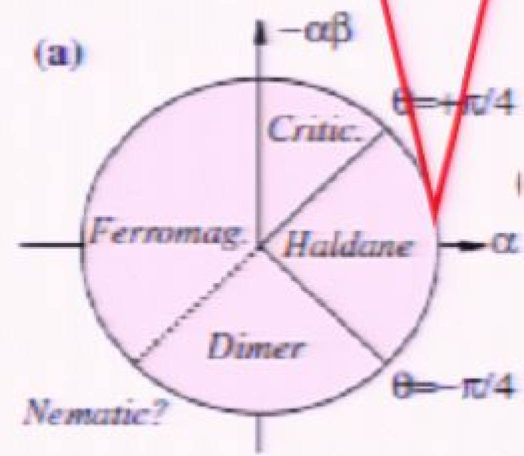
ieb-Schultz-Mattis-Haldane theorem:

Consider 1D translationally-invariant quantum system of
anti-ferromagnetic nearest-neighboring Heisenberg interactio
The ground state is

gapless (or degenerate) : half-integer spin
gapped : integer spin

AKLT:
θ= arctan(1/3)

$$H_{spin1} = \alpha \sum_{j=1}^{N-1} (\mathbf{S}_j \cdot \mathbf{S}_{j+1} - \beta (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2)$$

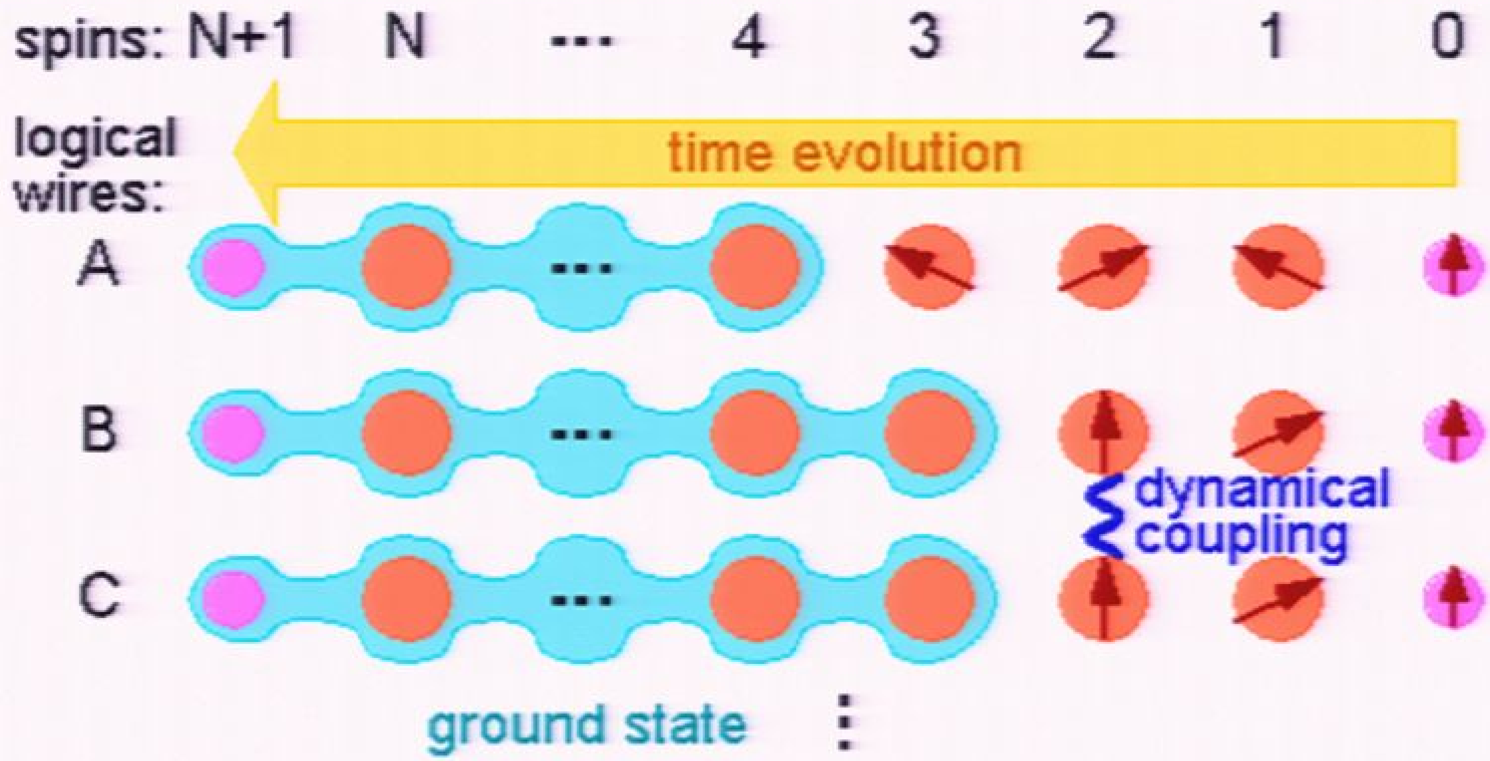


Haldane phase:

ex. Heisenberg, AKLT
(i) gap persists in thermodynamic limit
(ii) degeneracy by edge states

[from Garcia-Ripoll, Martin-Delgado, Cirac PRL'04]

big picture of ground-code MQC



- *n* parallel spin-1 chains, dynamically coupled on demand

- computation (right to left) by single-spin measurements, after turning off two-body interaction along every chain

- time evolution of logical information is encoded in the gapped two-fold ground subspace of the residual chain

persistent gap and Haldane phase

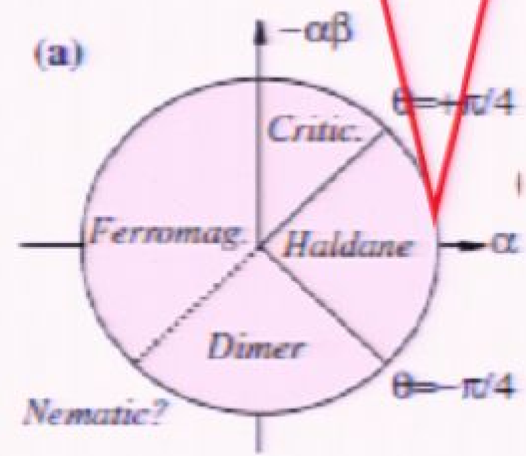
ieb-Schultz-Mattis-Haldane theorem:

Consider 1D translationally-invariant quantum system of
inti-ferromagnetic nearest-neighboring Heisenberg interactio
The ground state is

gapless (or degenerate) : half-integer spin
gapped : integer spin

AKLT:
θ= arctan(1/3)

$$H_{spin1} = \alpha \sum_{j=1}^{N-1} (\mathbf{S}_j \cdot \mathbf{S}_{j+1} - \beta (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2)$$



Haldane phase:

ex. Heisenberg, AKLT
(i) gap persists in thermodynamic limit
(ii) degeneracy by edge states

[from Garcia-Ripoll, Martin-Delgado, Cirac PRL'04]

Microscopic model of quantum wires

ffleck-Kennedy-Lieb-Tasaki (AKLT) Hamiltonian

$$H_{AKLT} = J\sum_{j=1}^{N-1}\underbrace{\left[\frac{1}{2}S_j\cdot S_{j+1}+\frac{1}{6}(S_j\cdot S_{j+1})^2+\frac{1}{3}\right]}_{P^2_{j,j+1}} + J\underbrace{\frac{2}{3}(1+s_0\cdot S_1)}_{P^{3/2}_{0,1}} + J\underbrace{\frac{2}{3}(1+S_N\cdot s_{N+}}_{P^{3/2}_{N,N+1}}$$

- spin 1 chain in a Haldane phase
- two-body, nearest-neighboring interaction
- frustration-free (total ground state minimizes the energy of every summand at the same time)
- boundary condition: two end spin ½ particles (projecting onto a global singlet)

unique ground state of AKLT Hamiltonian

- gap is constant (independent of system size) ~ 0.35 J
- unique ground state with boundary spin $\frac{1}{2}$ particles

Note: four-fold degeneracy in open boundary (without spin $\frac{1}{2}$)

- localizable entanglement is known to be maximum in arbitrary distance (if Hamiltonian is off).

unique ground state of AKLT Hamiltonian

- gap is constant (independent of system size) ~ 0.35 J
- unique ground state with boundary spin ½ particles

Note: four-fold degeneracy in open boundary (without spin ½)

- localizable entanglement is known to be maximum in arbitrary distance (if Hamiltonian is off).

atrix product state (MPS) representation

[Verstraete, Martin-Delgado, Cirac, PRL'04; Fan, Korepin, Roychowdhury, PRL'04

$$|G\rangle = \frac{1}{3^{N/2}} \sum_{\{\alpha_j\}=1}^{3} |\alpha_1\rangle \ldots |\alpha_N\rangle \left[1 \otimes \prod_{j=N}^{1} \langle \alpha_j | M_j \rangle \right] |\Psi^-_{0,N+1}\rangle$$

$$|M_j\rangle = X|1_j\rangle - iY|2_j\rangle + Z|3_j\rangle$$

$$|1_j\rangle = \tfrac{-1}{\sqrt{2}}\left(|S_j^z = 1\rangle - |S_j^z = -1\rangle\right)$$

$$|2_j\rangle = \tfrac{1}{\sqrt{2}}\left(|S_j^z = 1\rangle + |S_j^z = -1\rangle\right)$$

$$|3_j\rangle = |S_j^z = 0\rangle$$

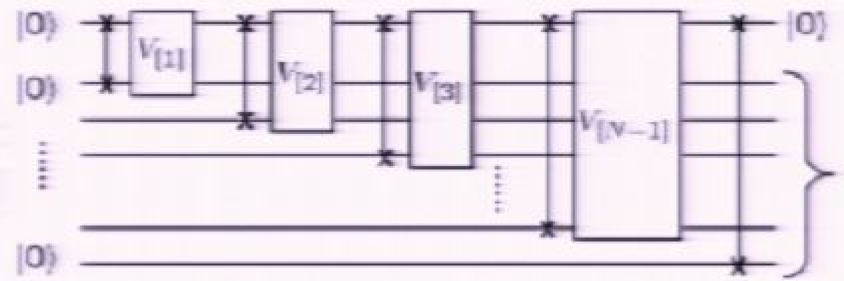
$$M[\alpha_j] = \begin{cases} M[1_j] = X \\ M[2_j] = -iY \\ M[3_j] = Z \end{cases}$$

preparation of the unique ground state

Both methods are deterministic and time-efficient.

method 1: sequential unitary gate preparation (available for matrix product states)

[Schön, Solano, Verstraete, Cirac, Wolf, PRL'05;
Schön, Hammerer, Wolf, Cirac, Solano, PRA'07]



circuit depth is N (chain length), for unitaries do not commute

cf. analogy with dynamical cluster state preparation by CZ gates, in which however circuit depth is constant.

method 2: adiabatic preparation (in optical lattice implementations by trapped atoms/molecules)

starting from Neel state (antiparallel spins) by magnetic field, we progressively decrease it and increase AKLT interactions.

constant time due to a constant gap above the ground state

unique ground state of AKLT Hamiltonian

- gap is constant (independent of system size) ~ 0.35 J
- unique ground state with boundary spin $\frac{1}{2}$ particles

Note: four-fold degeneracy in open boundary (without spin $\frac{1}{2}$)

- localizable entanglement is known to be maximum in arbitrary distance (if Hamiltonian is off).

atrix product state (MPS) representation

[Verstraete, Martin-Delgado, Cirac, PRL'04; Fan, Korepin, Roychowdhury, PRL'04

$$|G\rangle = \frac{1}{3^{N/2}} \sum_{\{\alpha_j\}=1}^{3} |\alpha_1\rangle \ldots |\alpha_N\rangle \left[1 \otimes \prod_{j=N}^{1} \langle \alpha_j | M_j \rangle \right] |\Psi^-_{0,N+1}\rangle$$

$$|M_j\rangle = X|1_j\rangle - iY|2_j\rangle + Z|3_j\rangle$$

$$|1_j\rangle = \tfrac{-1}{\sqrt{2}}\left(|S_j^z = 1\rangle - |S_j^z = -1\rangle\right)$$

$$|2_j\rangle = \tfrac{1}{\sqrt{2}}\left(|S_j^z = 1\rangle + |S_j^z = -1\rangle\right)$$

$$|3_j\rangle = |S_j^z = 0\rangle$$

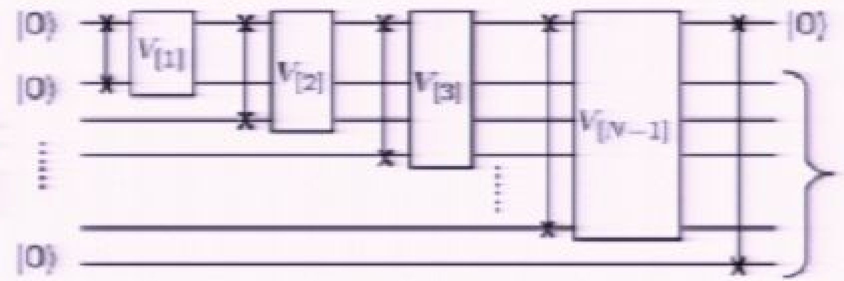
$$M[\alpha_j] = \begin{cases} M[1_j] = X \\ M[2_j] = -iY \\ M[3_j] = Z \end{cases}$$

preparation of the unique ground state

Both methods are deterministic and time-efficient.

nethod 1: sequential unitary gate preparation (available for matrix product states)

Schön, Solano, Verstraete, Cirac, Wolf, PRL'05;
Schön, Hammerer, Wolf, Cirac, Solano, PRA'07]



ircuit depth is N (chain length), for unitaries do not commute

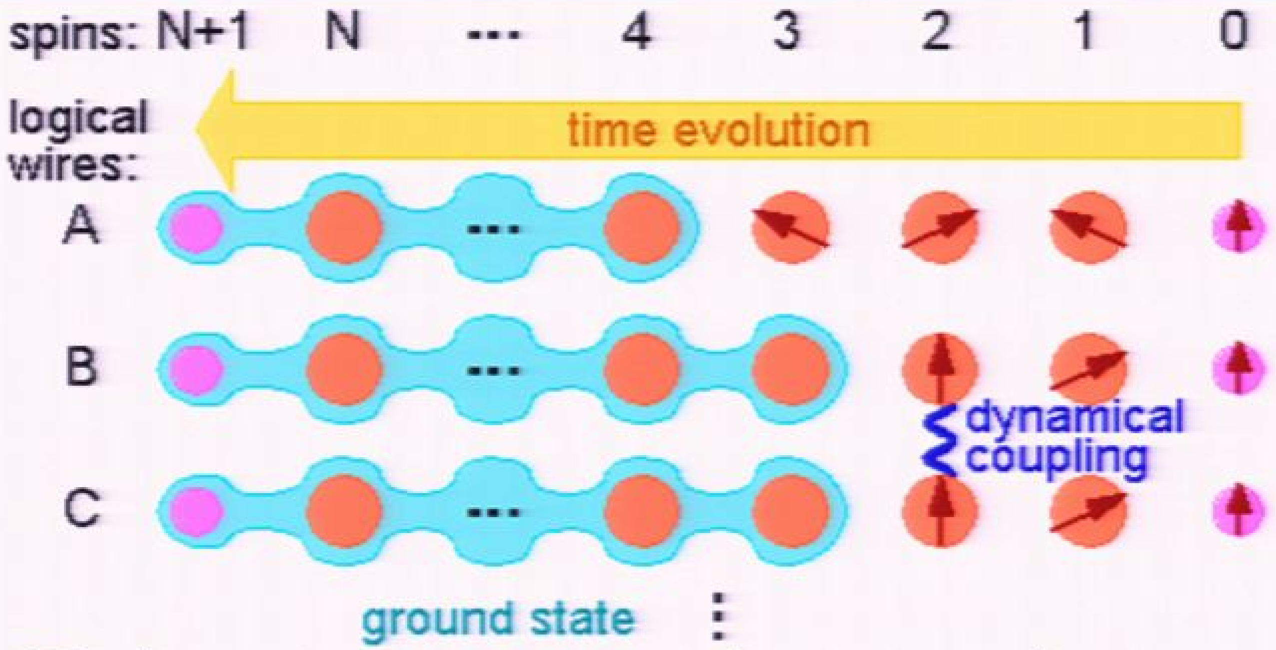
f. analogy with dynamical cluster state preparation by CZ gates, in which however circuit depth is constant.

nethod 2: adiabatic preparation (in optical lattice implementations by trapped atoms/molecules)

starting from Neel state (antiparallel spins) by magnetic field, ve progressively decrease it and increase AKLT interactions.

constant time due to a constant gap above the ground state

Computation by local measurements



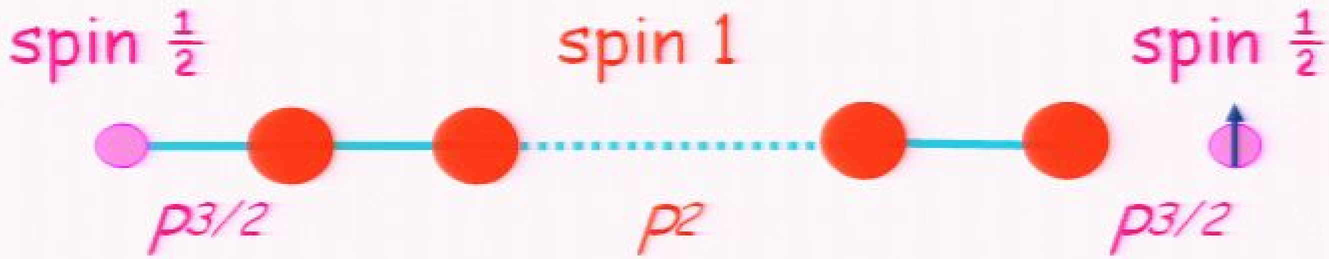
1. turning off the interaction with spin to be measured.
   ground-code properties are explained later !

2. logical single-qubit operation by single-spin measurement

3. logical two-qubit operation by single-spin measurement after dynamical coupling

4. final readout measurement (after teleportation to left end

Initialization of a quantum wire



turning off $P^{3/2}_{0,1}$ and measuring rightmost spin-½ in the z direction .

two-fold degeneracy by "one closed and one open" boundaries

$$\langle r_0|G\rangle = \frac{1}{3^{N/2}} \sum_{\{\alpha_j\}=1}^{3} |\alpha_1\rangle \ldots |\alpha_N\rangle \left[\prod_{j=N}^{1} \langle \alpha_j | M_j \rangle \right] \langle r_0 | \Psi^{-}_{0,N+1} \rangle$$

- ½ ($|1_0\rangle$) outcome: initialize logical wire to be $|0_{N+1}\rangle$

+ ½ ($|0_0\rangle$) outcome: initialize logical wire to be $X|0_{N+1}\rangle$

$\langle 1_0|G\rangle$, $\langle 0_0|G\rangle$ span the two-fold ground subspace of the residual Hamiltonian !

single-qubit gate operations

ogical single-qubit rotation: Euler angles construction

$$SU(2) = R^z(\theta_3) R^x(\theta_2) R^z(\theta_1$$

$$R^Z(\theta) = |0^L\rangle\langle 0^L| + e^{i\theta}|1^L\rangle\langle 1^L|$$

$$R^X(\theta) = |+^L\rangle\langle +^L| + e^{i\theta}|-^L\rangle\langle -^L|$$

$$|G(0)\rangle = \langle 1_0|G\rangle = \frac{1}{3^{N/2}} \sum_{\{\alpha_j\}=1}^{3} |\alpha_1\rangle \ldots |\alpha_N\rangle \left[\prod_{j=N}^{1} \langle \alpha_j | M_j \rangle\right] |0_{N+1}\rangle$$

elative state:

$$\gamma_j(\theta)|G(j-1)\rangle = \ldots \underbrace{\langle \gamma_j(\theta)| \sum_{\alpha_j=1,2,3} |\alpha_j\rangle\langle \alpha_j | M_j\rangle}_{\Upsilon' R'(\theta)} \Upsilon R |0_{N+1}\rangle$$

byproduct:
Pauli operators

adaptation of next measurement directions
from θ to $-\theta$ for non-commuting byproducts

single-qubit gate operations

$$R^Z(\theta) = |0^L\rangle\langle 0^L| + e^{i\theta}|1^L\rangle\langle 1^L|$$

$$R^X(\theta) = |+^L\rangle\langle +^L| + e^{i\theta}|-^L\rangle\langle -^L|$$

byproduct:
Pauli operators

relative state:

$$\langle\gamma_j(\theta)|G(j-1)\rangle = \ldots\underbrace{\langle\gamma_j(\theta)|\sum_{\alpha_j=1,2,3}|\alpha_j\rangle\langle\alpha_j|M_j\rangle}_{\Upsilon' R'(\theta)}\Upsilon R|0_{N+1}\rangle$$

$$|\gamma_j^Z(\theta)\rangle = \begin{cases} \tfrac{1}{2}((1+e^{-i\theta})|1_j\rangle + (1-e^{-i\theta})|2_j\rangle) : & XR^Z(\theta) \\ \tfrac{1}{2}((1-e^{-i\theta})|1_j\rangle + (1+e^{-i\theta})|2_j\rangle) : & XZR^Z(\theta) \\ |3_j\rangle : & Z \end{cases}$$

$$|\gamma_j^X(\theta)\rangle = \begin{cases} \tfrac{1}{2}((1+e^{i\theta})|2_j\rangle + (1-e^{i\theta})|3_j\rangle) : & XZR^X(\theta) \\ \tfrac{1}{2}((1-e^{i\theta})|2_j\rangle + (1+e^{i\theta})|3_j\rangle) : & ZR^X(\theta) \\ |1_j\rangle : & X \end{cases}$$

gate success
probability: 2/

two-qubit CZ gate operation

dynamical coupling of two chains A, B on demand

$$\exp[i\pi |S^z_{A_j}=1\rangle\langle S^z_{A_j}=1| \otimes |S^z_{B_j}=1\rangle\langle S^z_{B_j}=1|]$$

followed by measurements A_j and B_j in the standard basis

$$\begin{cases} |1_{A_j}1_{B_j}\rangle: & (XZ\otimes XZ)CZ_{A_jB_j} \\ |1_{A_j}2_{B_j}\rangle: & (XZ\otimes X)CZ_{A_jB_j} \\ |2_{A_j}1_{B_j}\rangle: & (X\otimes XZ)CZ_{A_jB_j} \\ |2_{A_j}2_{B_j}\rangle: & (X\otimes X)CZ_{A_jB_j} \\ |others\rangle: \Upsilon = \langle \alpha_{A_j}|M_{A_j}\rangle \otimes \langle \beta_{B_j}|M_{B_j}\rangle & \end{cases}$$

gate success probability: $(2/3)^2 = 4/9$

Note: the failure of gate attempt is heralded and harmless.

It is considered as the "identity" by byproduct propagation

single-qubit gate operations

$$R^Z(\theta) = |0^L\rangle\langle 0^L| + e^{i\theta}|1^L\rangle\langle 1^L|$$

$$R^X(\theta) = |+^L\rangle\langle +^L| + e^{i\theta}|-^L\rangle\langle -^L|$$

byproduct: Pauli operators

relative state:

$$\langle\gamma_j(\theta)|G(j-1)\rangle = \ldots\underbrace{\langle\gamma_j(\theta)|\sum_{\alpha_j=1,2,3}|\alpha_j\rangle\langle\alpha_j|M_j\rangle}_{\Upsilon' R'(\theta)}\Upsilon R|0_{N+1}\rangle$$

$$|\gamma_j^Z(\theta)\rangle = \begin{cases} \frac{1}{2}((1+e^{-i\theta})|1_j\rangle + (1-e^{-i\theta})|2_j\rangle) : & XR^Z(\theta) \\ \frac{1}{2}((1-e^{-i\theta})|1_j\rangle + (1+e^{-i\theta})|2_j\rangle) : & XZR^Z(\theta) \\ |3_j\rangle : & Z \end{cases}$$

$$|\gamma_j^X(\theta)\rangle = \begin{cases} \frac{1}{2}((1+e^{i\theta})|2_j\rangle + (1-e^{i\theta})|3_j\rangle) : & XZR^X(\theta) \\ \frac{1}{2}((1-e^{i\theta})|2_j\rangle + (1+e^{i\theta})|3_j\rangle) : & ZR^X(\theta) \\ |1_j\rangle : & X \end{cases}$$

gate success probability: 2/

two-qubit CZ gate operation

dynamical coupling of two chains A, B on demand

$$\exp[i\pi |S^z_{A_j}=1\rangle\langle S^z_{A_j}=1| \otimes |S^z_{B_j}=1\rangle\langle S^z_{B_j}=1|]$$

followed by measurements A_j and B_j in the standard basis

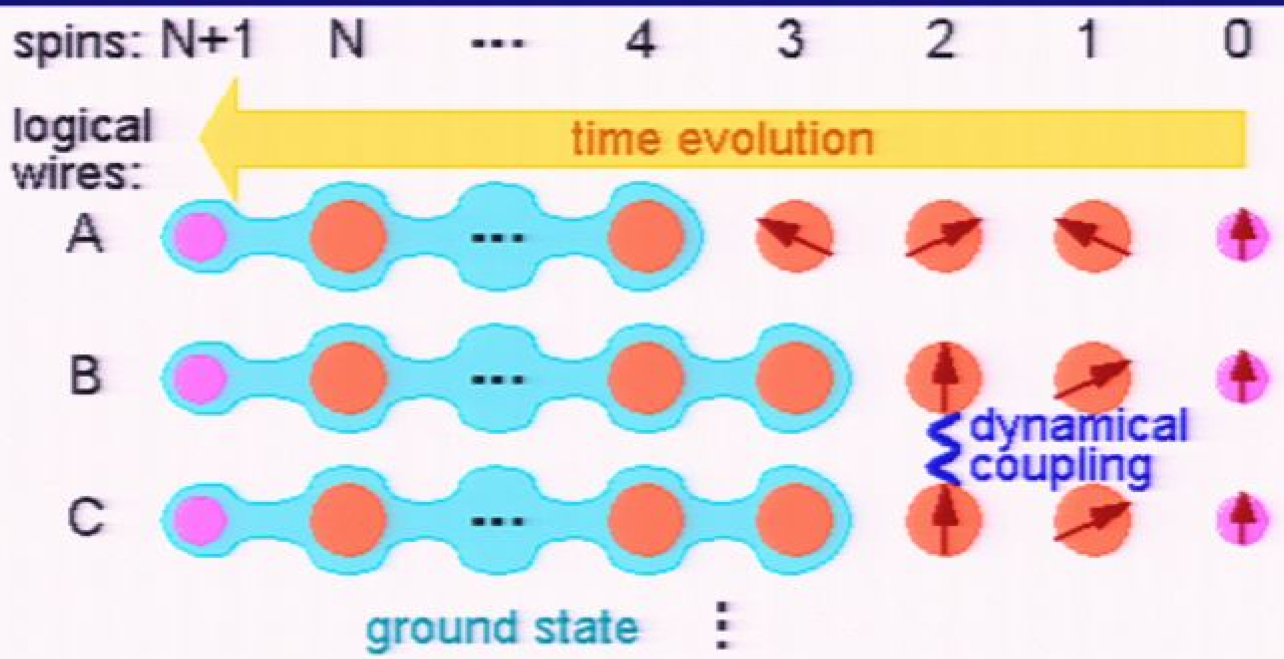
$$\begin{cases} |1_{A_j}1_{B_j}\rangle : & (XZ\otimes XZ)CZ_{A_jB_j} \\ |1_{A_j}2_{B_j}\rangle : & (XZ\otimes X)CZ_{A_jB_j} \\ |2_{A_j}1_{B_j}\rangle : & (X\otimes XZ)CZ_{A_jB_j} \\ |2_{A_j}2_{B_j}\rangle : & (X\otimes X)CZ_{A_jB_j} \\ |others\rangle : \Upsilon = \langle \alpha_{A_j}|M_{A_j}\rangle \otimes \langle \beta_{B_j}|M_{B_j}\rangle \end{cases}$$

gate success probability: $(2/3)^2 = 4/9$

Note: the failure of gate attempt is heralded and harmless.

It is considered as the "identity" by byproduct propagation

storing and reading information



After all computation is done, we deterministically teleport (by measurements in the standard basis) to the left end.

Logical information (encoded in the degenerate subspace) is now stored in the physical spin $\frac{1}{2}$, and can be read out deterministically.

Ground-code properties

ogical information is processed in the two-fold degenerate ground subspace of the residual Hamiltonian along every chain.

✓ constant gap

suppression of logical single-qubit error !

constant time for adiabatic turning off the interaction

2. string nonlocal order

3. two-body nearest-neighbor frustration-free interaction

4. single-spin measurement gate-teleports logical operation in the ground subspace of the <u>residual</u> chain

String non-local order

residual Hamiltonian at time j

(when spin j is going to be measured)

$$H(j) = J\left[\sum_{k=j}^{N-1} P^2_{k,k+1} + P^{3/2}_{N,N+1}\right]$$

string order parameters

$$\Sigma^x(j) = X_{N+1} \otimes \exp\left[i\pi \sum_{k=j}^{N} S^x_k\right], \quad \Sigma^z(j) = Z_{N+1} \otimes \exp\left[i\pi \sum_{k=j}^{N} S^z_k\right]$$

$$\left[\Sigma^\alpha(j), H(j)\right] = 0, \quad \left\{\Sigma^x(j), \Sigma^z(j)\right\}_+ = 0$$

the pair forms a representation of su(2) in ground subspaces:
("time-dependent" encoding of logical qubit)

two-fold ground subspaces are connected only by the nonlocal string order, thus environment-induced logical error is unlikely

role of frustration-free interaction

$$H(j) = J\left[\sum_{k=j}^{N-1} P^{2}_{k,k+1} + P^{3/2}_{N,N+1}\right]$$

c(t) is monotonically increasing during a constant period *T* (thanks to a constant gap)

$$H(j;t) = J\left(1 - c(t)\right)P^{2}_{j,j+1} + H(j+1)$$

Note: $P^2_{j,j+1}$ and $H(j+1)$ do not commute but they do commute in the ground state (frustration-free: total ground state also minimizes every summand in Hamiltonian.)

Note: if interaction is not two-body nearest-neighboring type, it may cause modification of the residual interactions among spins unmeasured yet in practice.

Turning off $P^2_{j,j+1}$ does not create any excitation, and a constant gap provides robustness for unwanted perturbation.

String non-local order

esidual Hamiltonian at time j

(when spin j is going to be measured)

$$H(j) = J\left[\sum_{k=j}^{N-1} P^{2}_{k,k+1} + P^{3/2}_{N,N+1}\right]$$

tring order parameters

$$\Sigma^{x}(j) = X_{N+1} \otimes \exp\left[i\pi\sum_{k=j}^{N} S^{x}_{k}\right],\ \Sigma^{z}(j) = Z_{N+1} \otimes \exp\left[i\pi\sum_{k=j}^{N} S^{z}_{k}\right]$$

$$\left[\Sigma^{\alpha}(j), H(j)\right] = 0,\ \left\{\Sigma^{x}(j), \Sigma^{z}(j)\right\}_{+} = 0$$

he pair forms a representation of su(2) in ground subspaces:
"time-dependent" encoding of logical qubit)

wo-fold ground subspaces are connected only by the nonlocal tring order, thus environment-induced logical error is unlikely

role of frustration-free interaction

$$H(j) = J\left[\sum_{k=j}^{N-1} P^2_{k,k+1} + P^{3/2}_{N,N+1}\right]$$

$c(t)$ is monotonically increasing during a constant period T (thanks to a constant gap)

$$H(j;t) = J\left(1 - c(t)\right)P^2_{j,j+1} + H(j+1)$$

Note: $P^2_{j,j+1}$ and $H(j+1)$ do not commute but they do commute in the ground state (frustration-free: total ground state also minimizes every summand in Hamiltonian.)

Note: if interaction is not two-body nearest-neighboring type, it may cause modification of the residual interactions among spins unmeasured yet in practice.

Turning off $P^2_{j,j+1}$ does not create any excitation, and a constant gap provides robustness for unwanted perturbation.

teleportation to the residual ground subspace

$$H(j+1) = J\left[\sum_{k=j+1}^{N-1} P^2_{k,k+1} + P^{3/2}_{N,N+1}\right]$$

$$|G(j+1)\rangle = \sum_{\{\alpha_k\}} |\alpha_{j+1}\rangle \dots |\alpha_N\rangle \langle r_0| \left[1 \otimes \prod_{k=N}^{j+1} \langle \alpha_k | M_k \rangle \Upsilon R\right] |\Psi^-_{0,N+1}\rangle$$

$$= \sum_{\{\alpha_k\}} |\alpha_{j+1}\rangle \dots |\alpha_N\rangle \langle r_0| \left[V^\dagger \otimes \prod_{k=N}^{j+1} \langle \alpha_k | M_k \rangle\right] |\Psi^-_{0,N+1}\rangle$$

$V = \tilde{\Upsilon}\tilde{R}$

proof:

1. start with the unique ground state of $H(j+1) + P^{3/2}_{0,j+1}$
2. turning off $P^{3/2}_{0,j+1}$, measuring 0-th spin in a rotated basis: $\{V|r_0\rangle, VX|r_0\rangle\}$
3. This is in the ground subspace of $H(j+1)$, namely

$$\alpha^* \langle 0_0 | G \rangle + \beta^* \langle 1_0 | G \rangle$$

corresponding to obtain the outcome $V|r_0\rangle = \alpha|0_0\rangle + \beta|1_0\rangle$

4. similar argument for the two-qubit CZ gate

Ground-code properties (summary)

ogical information is processed in the two-fold degenerate ground subspace of the residual Hamiltonian along every chain.

1. constant gap
   supression of logical single-qubit error !
   constant time for adiabatic turning off the interaction

2. string nonlocal order
   logical (unitary) errors by environments are non-local

3. two-body nearest-neighbor frustration-free interaction
   a rightmost interaction in Hamiltonian can turn off without disturbing the ground subspace

4. single-spin measurement gate-teleports logical information into the ground subspace of the residual chain.

Some new features as MQC

Logical times are correlated, originated from physical spatial correlations in the ground state:

- Non-trivial (non-Pauli) logical gates are probabilistic (2/3 per logical qubit), while teleportation (logical identity, up to Pauli) is deterministic. "Clock time progresses forward probabilistically", and logical depth has fluctuation.

Some new features as MQC

Logical times are correlated, originated from physical spatial correlations in the ground state:

- Non-trivial (non-Pauli) logical gates are probabilistic (2/3 per logical qubit), while teleportation (logical identity, up to Pauli) is deterministic. "Clock time progresses forward probabilistically", and logical depth has fluctuation.

Comparisons

1. Raussendorf-Briegel's cluster-state model

All gates are deterministic (up to Pauli), if byproducts are propagated through by adaptation of following measurements Logical depth is fixed.

2. Gross-Eisert's MPS/tensor network model
   ex. modified AKLT chain with Hadamard in M[3]

Byproducts must be cancelled at every logical step (by random walk without adaptation of measurements).

Logical depth has fluctuation.

Some new features as MQC

ground-code properties:

gapped ground subspaces are time-dependent logical

'encoding", and are only connected by nonlocal string operators

Some new features as MQC

ground-code properties:
gapped ground subspaces are time-dependent logical "encoding", and are only connected by nonlocal string operators

Comparisons

1. cluster state
it is not clear whether it is compatible with some nonlocal order...

2. modified AKLT chain with Hadamard in M[3]
it is not clear whether it has the desired counterpart of string order...

Implementations in a 3D optical lattice

On a bit further way in simulating the 1D AKLT spin chain !

neutral atoms
[Yip PRL'03; Garcia-Ripoll, Martin-Delgado, Cirac PRL'04]

polar molecules
[Brennen, Micheli, Zoller, NJP'07]

spin 1: F=1 hyperfine sublevels of electronic ground state of an alkali atom (ex. ^{23}Na)/ a polar molecule (ex. $^{40}Ca^{35}Cl$

physical interaction:
tunneling-induced collisions

gap ~ 0.2 kHz

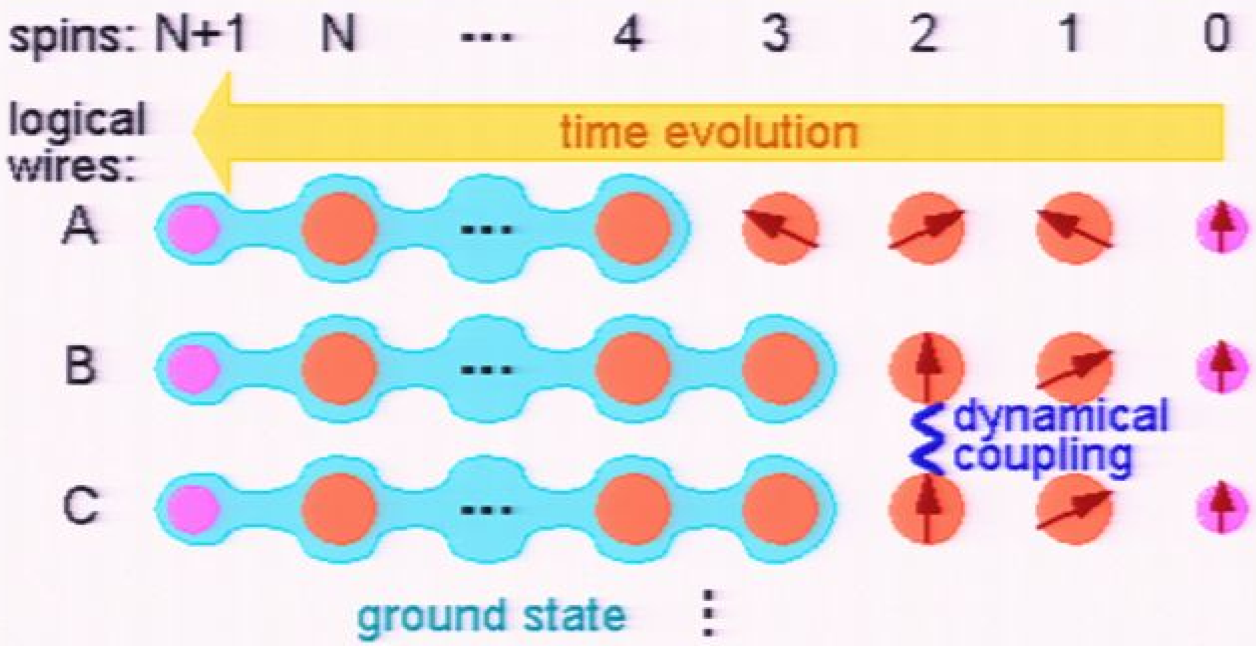
physical interaction:
dipole-dipole couplings induced by microwave fiel

gap ~ 4.8 kHz

Note: dominant noise is fluctuation of magnetic field ~ 10 Hz

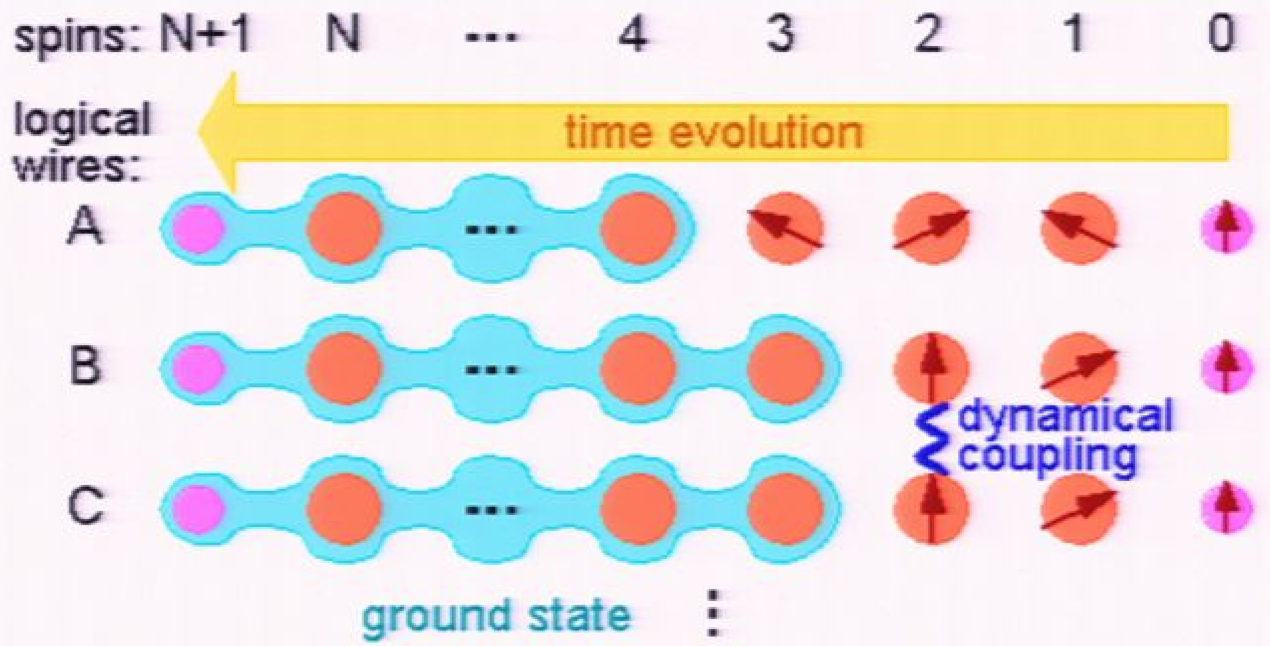
Summary

➢ G.K. Brennen & A. Miyake, arXiv:0803.1478



Summary

➢ G.K. Brennen & A. Miyake, arXiv:0803.1478



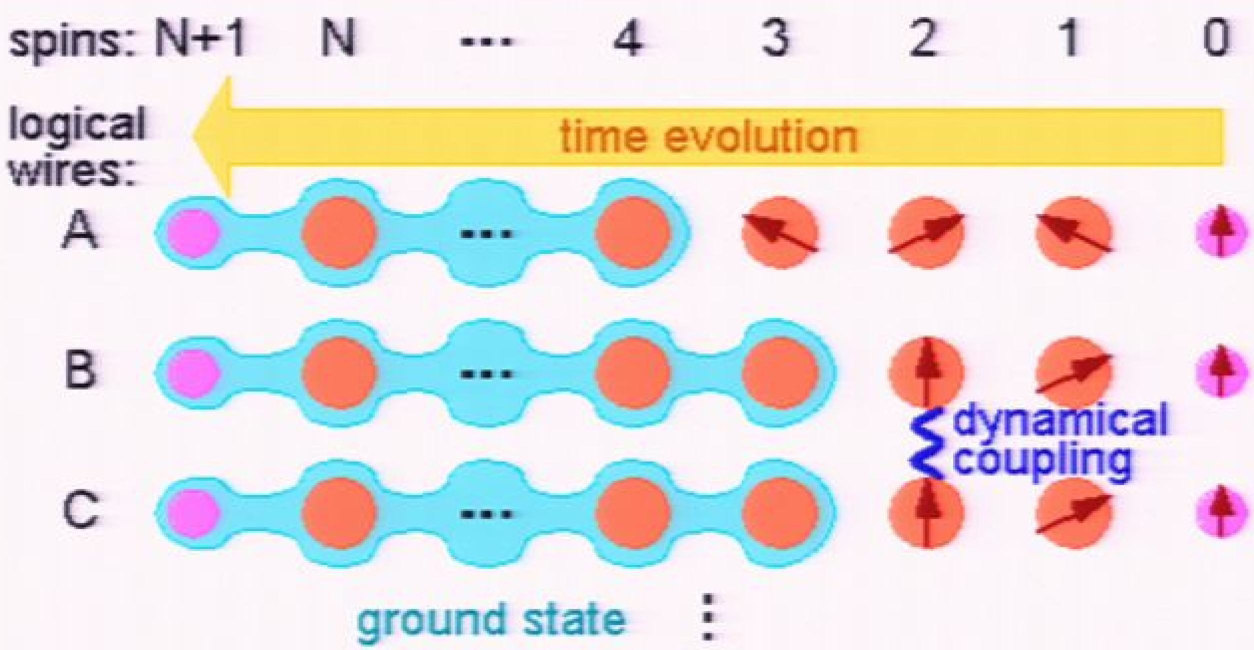
(adiabatic) turning off interaction with spin to be measured

two-body nearest-neighboring frustration-free Hamiltonian

spin-1 Haldane chain: constant gap & nonlocal string order

Summary

➢ G.K. Brennen & A. Miyake, arXiv:0803.1478



(adiabatic) turning off interaction with spin to be measured

two-body nearest-neighboring frustration-free Hamiltonian

spin-1 Haldane chain: constant gap & nonlocal string order

two-fold energetic degeneracy by one open boundary

PS ground state universal for MQC (with dynamical coupling)

teleportation to the residual ground subspace

$$H(j+1) = J\left[\sum_{k=j+1}^{N-1} P_{k,k+1}^{2} + P_{N,N+1}^{3/2}\right]$$

$$|G(j+1)\rangle = \sum_{\{\alpha_k\}} |\alpha_{j+1}\rangle \ldots |\alpha_N\rangle \langle r_0| \left[1 \otimes \prod_{k=N}^{j+1} \langle \alpha_k | M_k \rangle \Upsilon R\right] |\Psi^-_{0,N+1}\rangle$$

$$= \sum_{\{\alpha_k\}} |\alpha_{j+1}\rangle \ldots |\alpha_N\rangle \langle r_0| \left[V^\dagger \otimes \prod_{k=N}^{j+1} \langle \alpha_k | M_k \rangle\right] |\Psi^-_{0,N+1}\rangle$$

$V = \tilde{\Upsilon}\tilde{R}$

proof:

1. start with the unique ground state of $H(j+1) + P^{3/2}_{0,j+1}$
2. turning off $P^{3/2}_{0,j+1}$, measuring 0-th spin in a rotated basis: $\{V|r_0\rangle, VX|r_0\rangle\}$
3. This is in the ground subspace of $H(j+1)$, namely

$$\alpha^* \langle 0_0 | G\rangle + \beta^* \langle 1_0 | G\rangle$$

corresponding to obtain the outcome $V|r_0\rangle = \alpha|0_0\rangle + \beta|1_0\rangle$

4. similar argument for the two-qubit CZ gate

String non-local order

residual Hamiltonian at time j
(when spin j is going to be measured)

$$H(j) = J\left[\sum_{k=j}^{N-1} P_{k,k+1}^{2} + P_{N,N+1}^{3/2}\right]$$

string order parameters

$$\Sigma^{x}(j) = X_{N+1}\otimes \exp\left[i\pi\sum_{k=j}^{N} S_{k}^{x}\right],\ \Sigma^{z}(j) = Z_{N+1}\otimes \exp\left[i\pi\sum_{k=j}^{N} S_{k}^{z}\right]$$

$$\left[\Sigma^{\alpha}(j), H(j)\right] = 0,\ \left\{\Sigma^{x}(j), \Sigma^{z}(j)\right\}_{+} = 0$$

the pair forms a representation of su(2) in ground subspaces:
("time-dependent" encoding of logical qubit)

two-fold ground subspaces are connected only by the nonlocal string order, thus environment-induced logical error is unlikely

two-qubit CZ gate operation

dynamical coupling of two chains A, B on demand

$$\exp[i\pi |S^z_{A_j}=1\rangle\langle S^z_{A_j}=1| \otimes |S^z_{B_j}=1\rangle\langle S^z_{B_j}=1|]$$

followed by measurements A_j and B_j in the standard basis

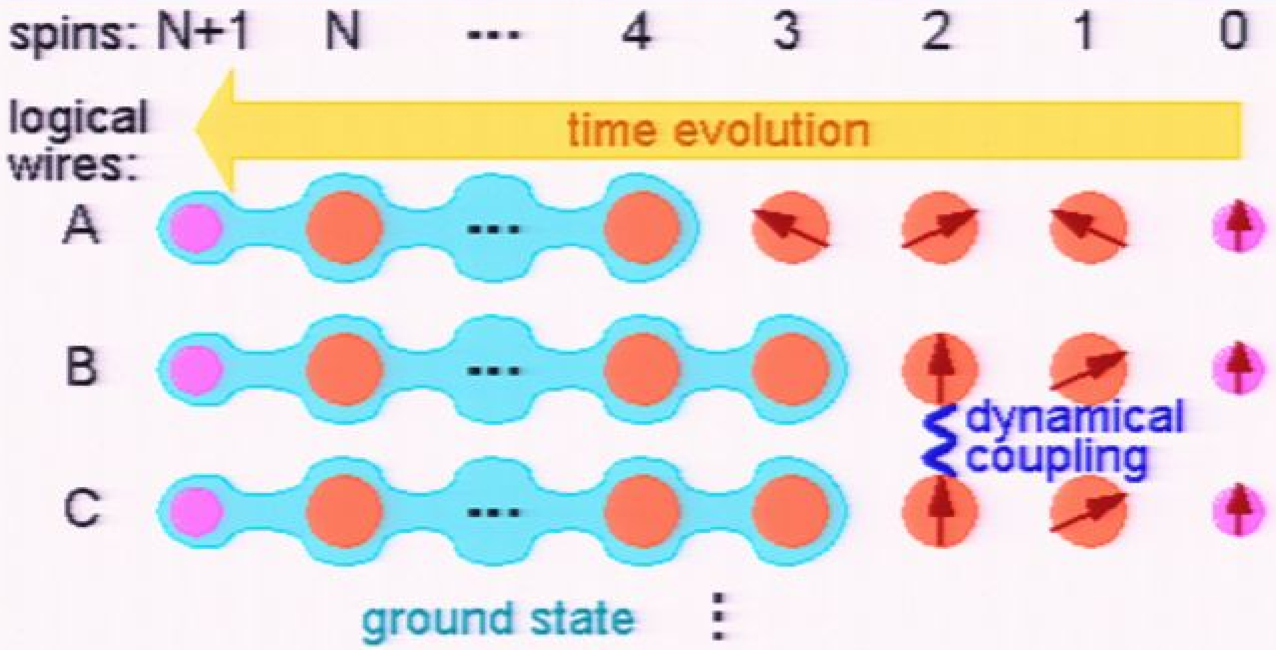
$$\begin{cases} |1_{A_j}1_{B_j}\rangle: & (XZ\otimes XZ)CZ_{A_jB_j} \\ |1_{A_j}2_{B_j}\rangle: & (XZ\otimes X)CZ_{A_jB_j} \\ |2_{A_j}1_{B_j}\rangle: & (X\otimes XZ)CZ_{A_jB_j} \\ |2_{A_j}2_{B_j}\rangle: & (X\otimes X)CZ_{A_jB_j} \\ |others\rangle: \Upsilon = \langle\alpha_{A_j}|M_{A_j}\rangle\otimes\langle\beta_{B_j}|M_{B_j}\rangle \end{cases}$$

gate success probability: $(2/3)^2 = 4/9$

Note: the failure of gate attempt is heralded and harmless.

It is considered as the "identity" by byproduct propagation

storing and reading information



After all computation is done, we deterministically teleport (by measurements in the standard basis) to the left end.

Logical information (encoded in the degenerate subspace) is now stored in the physical spin $\frac{1}{2}$, and can be read out deterministically.

Ground-code properties (summary)

ogical information is processed in the two-fold degenerate ground subspace of the residual Hamiltonian along every chain.

1. constant gap
   supression of logical single-qubit error !
   constant time for adiabatic turning off the interaction

2. string nonlocal order
   logical (unitary) errors by environments are non-local

3. two-body nearest-neighbor frustration-free interaction
   a rightmost interaction in Hamiltonian can turn off without disturbing the ground subspace

4. single-spin measurement gate-teleports logical information into the ground subspace of the residual chain.

Ground-code properties

ogical information is processed in the two-fold degenerate
ground subspace of the residual Hamiltonian along every chain.

✓ constant gap

suppression of logical single-qubit error !

constant time for adiabatic turning off the interaction

2. string nonlocal order

3. two-body nearest-neighbor frustration-free interaction

4. single-spin measurement gate-teleports logical operation
in the ground subspace of the residual chain

Ground-code properties

ogical information is processed in the two-fold degenerate
ground subspace of the residual Hamiltonian along every chain.

✓ constant gap
suppression of logical single-qubit error !
constant time for adiabatic turning off the interaction

2. string nonlocal order

3. two-body nearest-neighbor frustration-free interaction

4. single-spin measurement gate-teleports logical operation
in the ground subspace of the residual chain

Some new features as MQC

Logical times are correlated, originated from physical spatial correlations in the ground state:

- Non-trivial (non-Pauli) logical gates are probabilistic (2/3 per logical qubit), while teleportation (logical identity, up to Pauli) is deterministic. "Clock time progresses forward probabilistically", and logical depth has fluctuation.

Some new features as MQC

ground-code properties:

gapped ground subspaces are time-dependent logical

'encoding", and are only connected by nonlocal string operator:

Implementations in a 3D optical lattice

On a bit further way in simulating the 1D AKLT spin chain !

neutral atoms
[Yip PRL'03; Garcia-Ripoll, Martin-Delgado, Cirac PRL'04]

polar molecules
[Brennen, Micheli, Zoller, NJP'07]

spin 1: F=1 hyperfine sublevels of electronic ground state of an alkali atom (ex. ^{23}Na)/ a polar molecule (ex. $^{40}Ca^{35}Cl$

physical interaction:
tunneling-induced collisions

gap ~ 0.2 kHz

physical interaction:
dipole-dipole couplings
induced by microwave fiel

gap ~ 4.8 kHz

Note: dominant noise is fluctuation of magnetic field ~ 10 Hz