Title: Spin graphs for quantum communication and ground state entanglement

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Abstract: In this talk, two specific directions of research in quantum information are presented which could potentially gain from graph theory. The first is the study of quantum communication using systems of perpetually interacting qubits (or spins) as a databus. After introducing the topic through the simplest examples of linear chains of spins as transmitters of quantum information, we briefly mention existing work on quantum communication through more general graphs of spins. We then explain why the transmission of quantum information between vertices of a graph in the case of an isotropic Heisenberg interaction (between the spins placed at these vertices) depends on the Laplacian of the graph. How the quality of communication varies when starts cutting edges after starting a fully connected graph will be discussed *. Another direction is related to the entanglement naturally present in the ground state of a graph of perpetually interacting spins: various specific examples --- fully connected, star and tree will be discussed. Some easily solvable interactions obtained by putting higher spins in specific vertices of simple graphs will also be discussed. In the end we also present an example where one can get a \'graph independent\' ground state by placing qudits with exchange interactions on an arbitrary graph. (* The first part of the talk is based on ongoing work with Simone Severini, Stefano Mancini and Andrea Casaccino, while the second part is based on work with Vladimir Korepin and Christopher Hadley)

Quantum Communications Through & Entanglement in Spin Graphs

Sougato Bose

University College London (UCL)

Perimeter Institute, 2008

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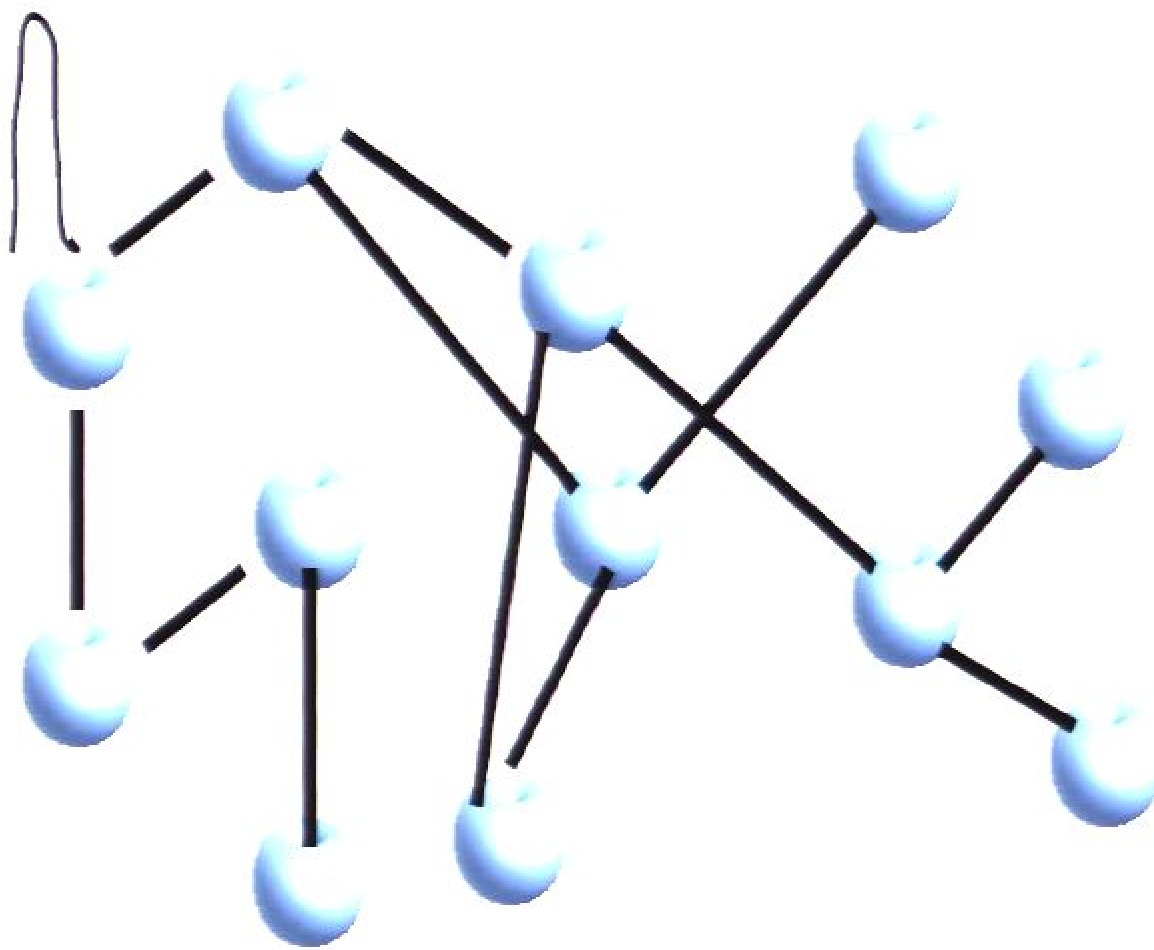
Quantum Information at UCL:

Part A: State Transfer Through the *Dynamics* of Graphs of Permanently Interacting Quantum Systems

Step 1: Placement of the state



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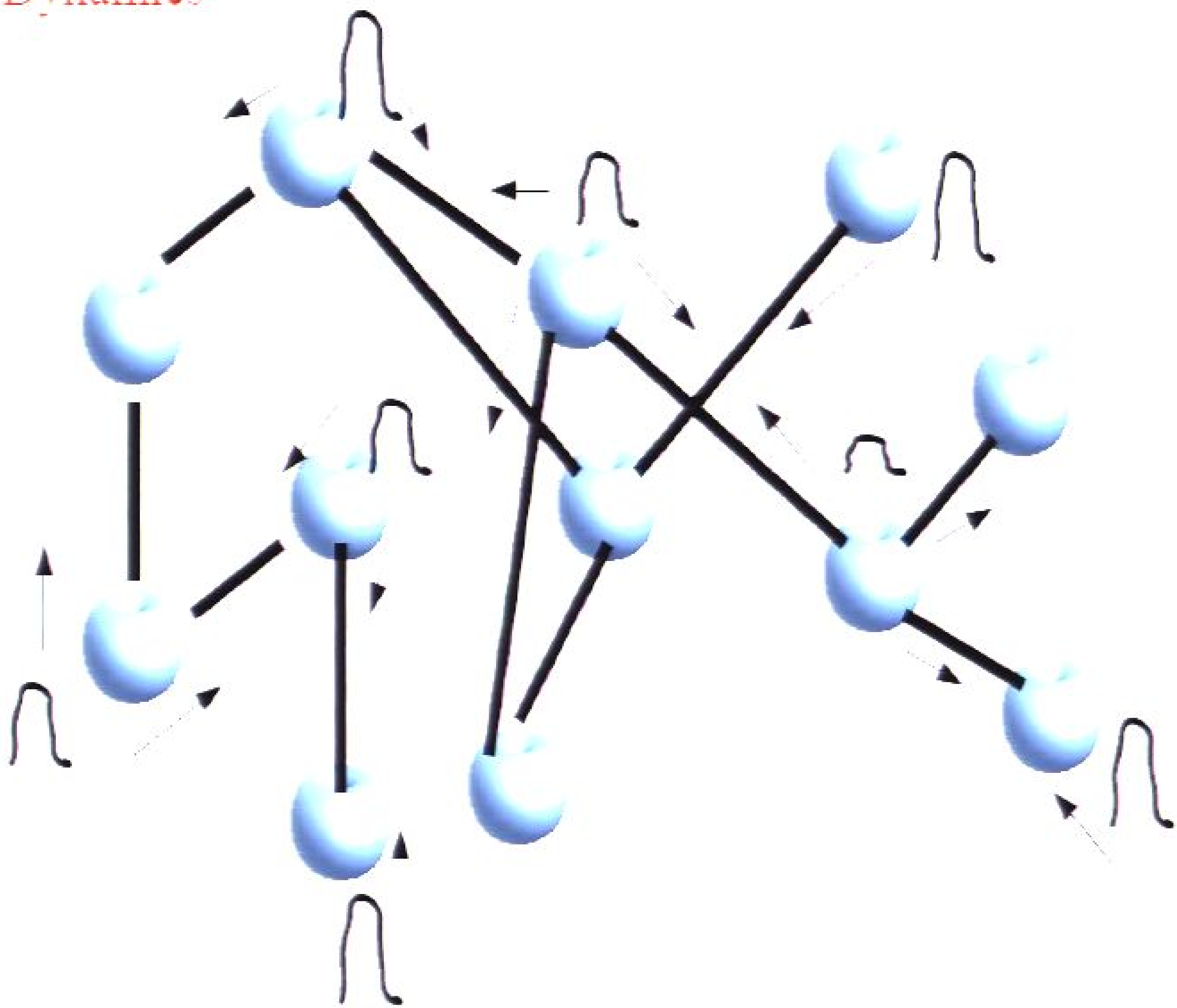
Step 2: Dynamics



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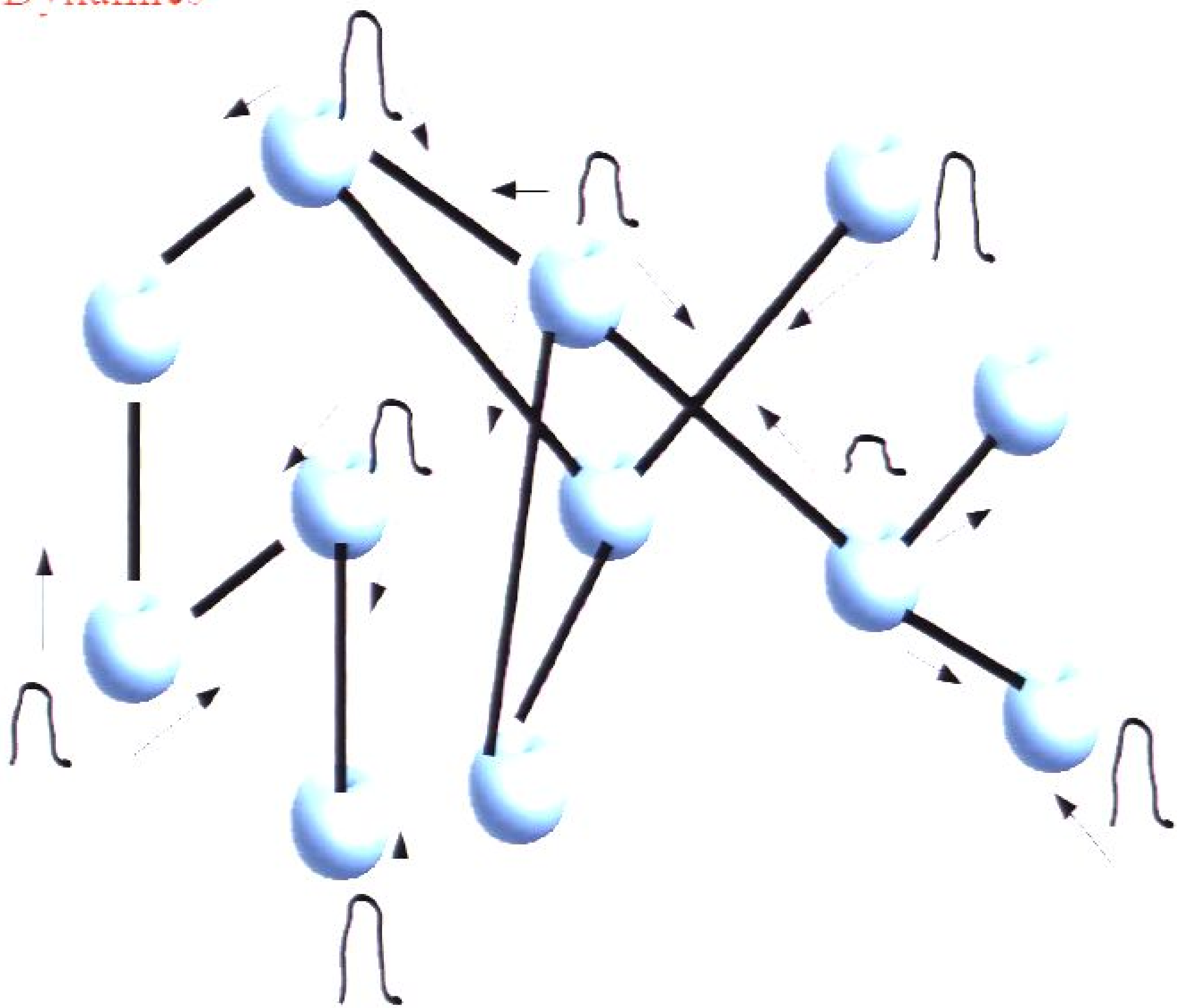

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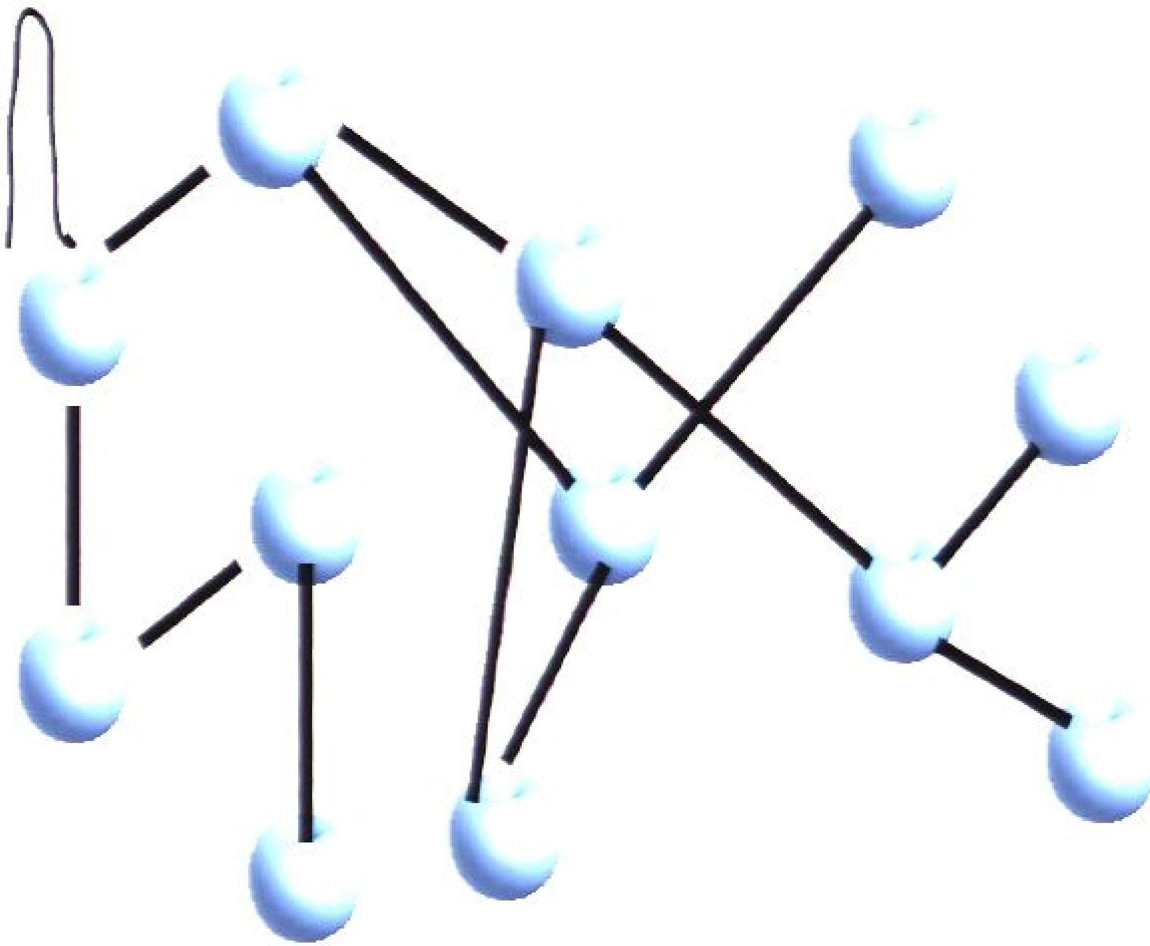

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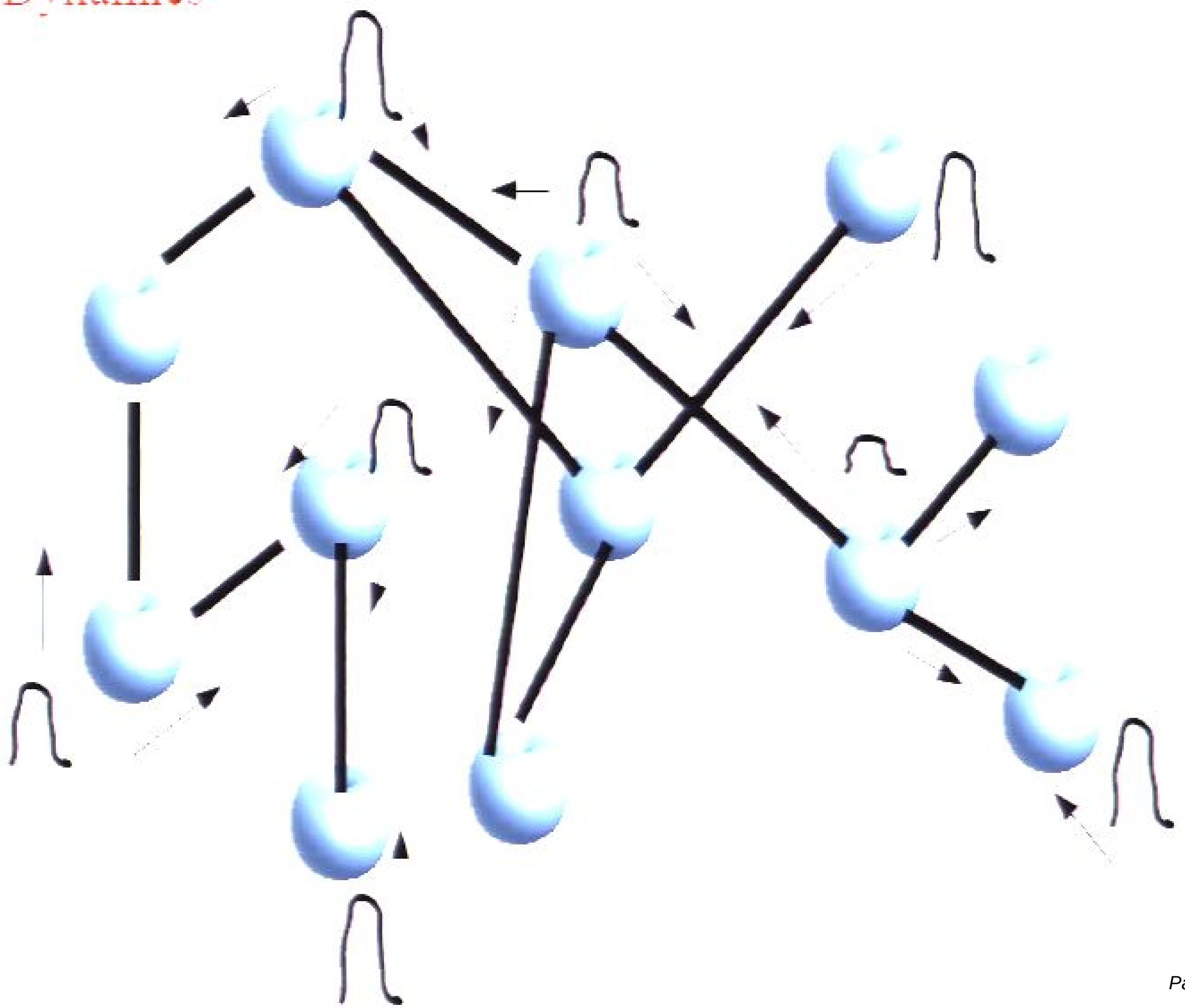

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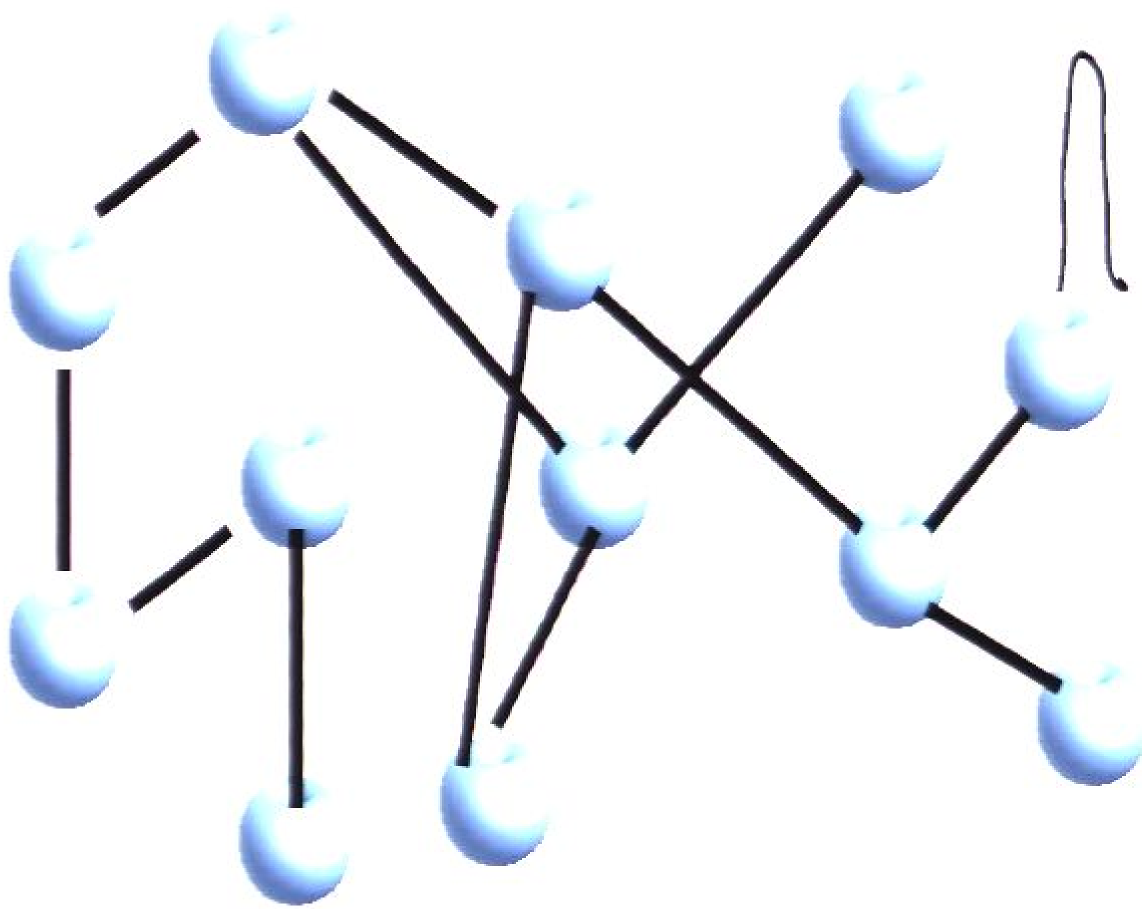
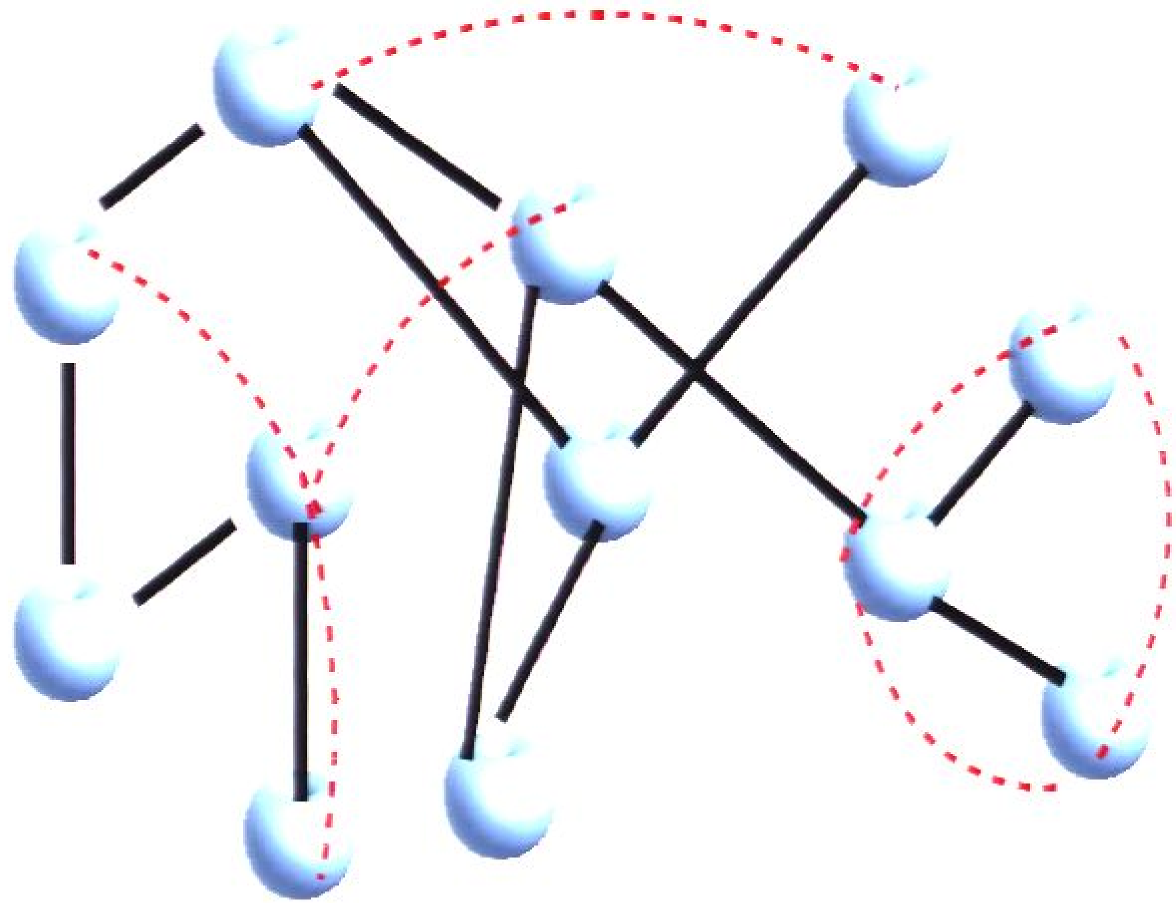
Step 1: Reception of the state

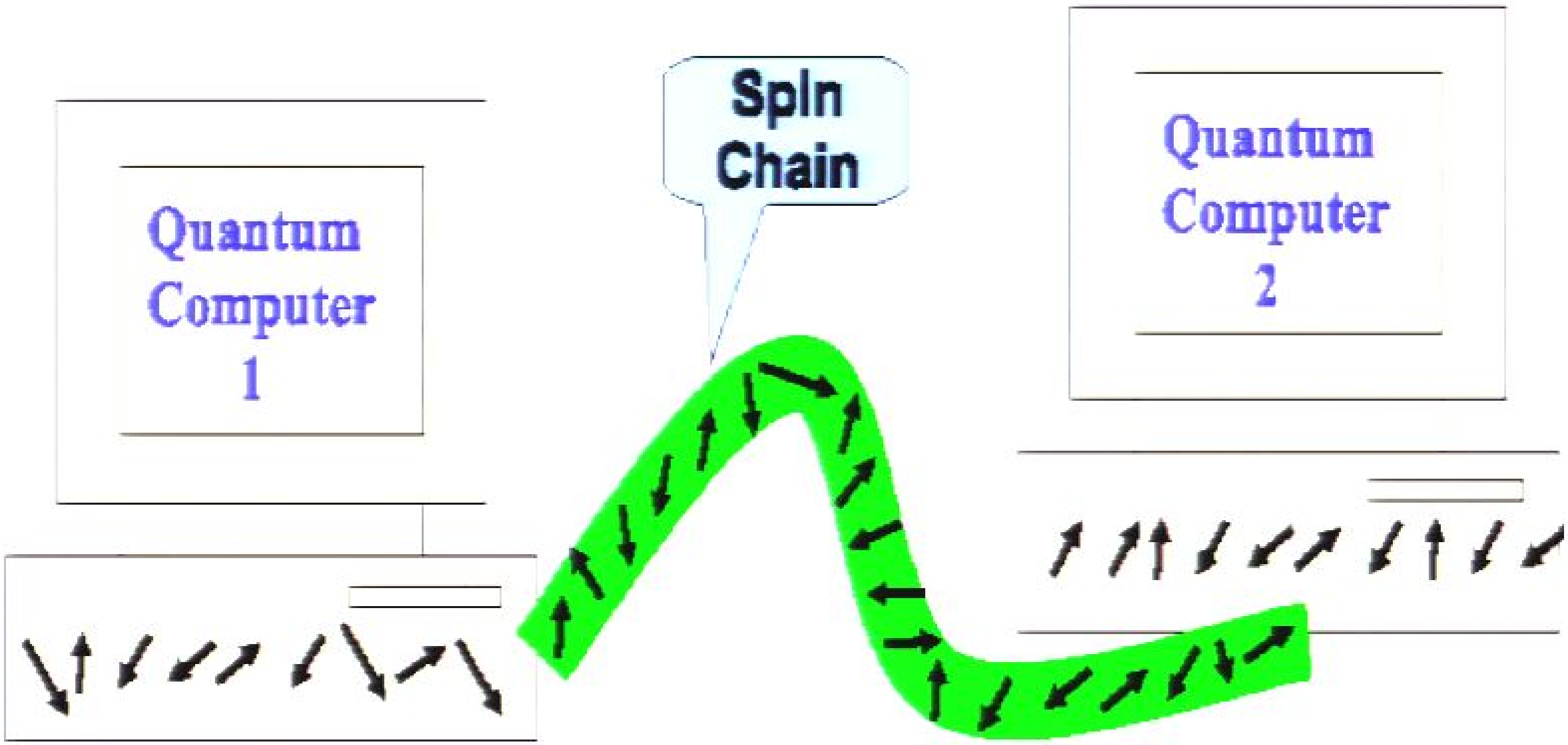


Part B: Entanglement naturally present in the ground states (the lowest energy eigenstates) of spin graphs

Could be bipartite, multipartite. Could altogether be a different graph

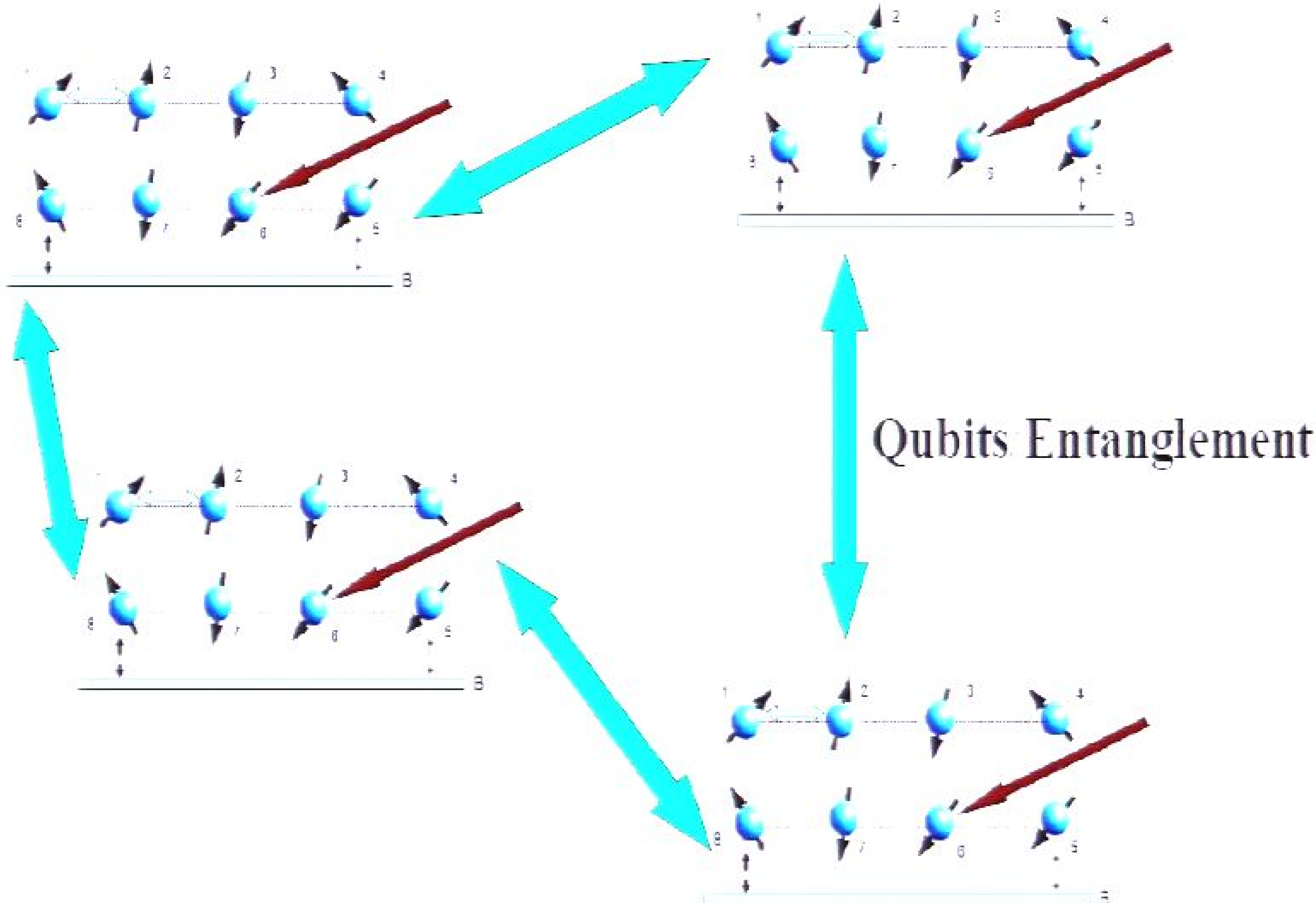


We will investigate whether measuring some spins project the rest to any interesting (known) multipartite entangled state

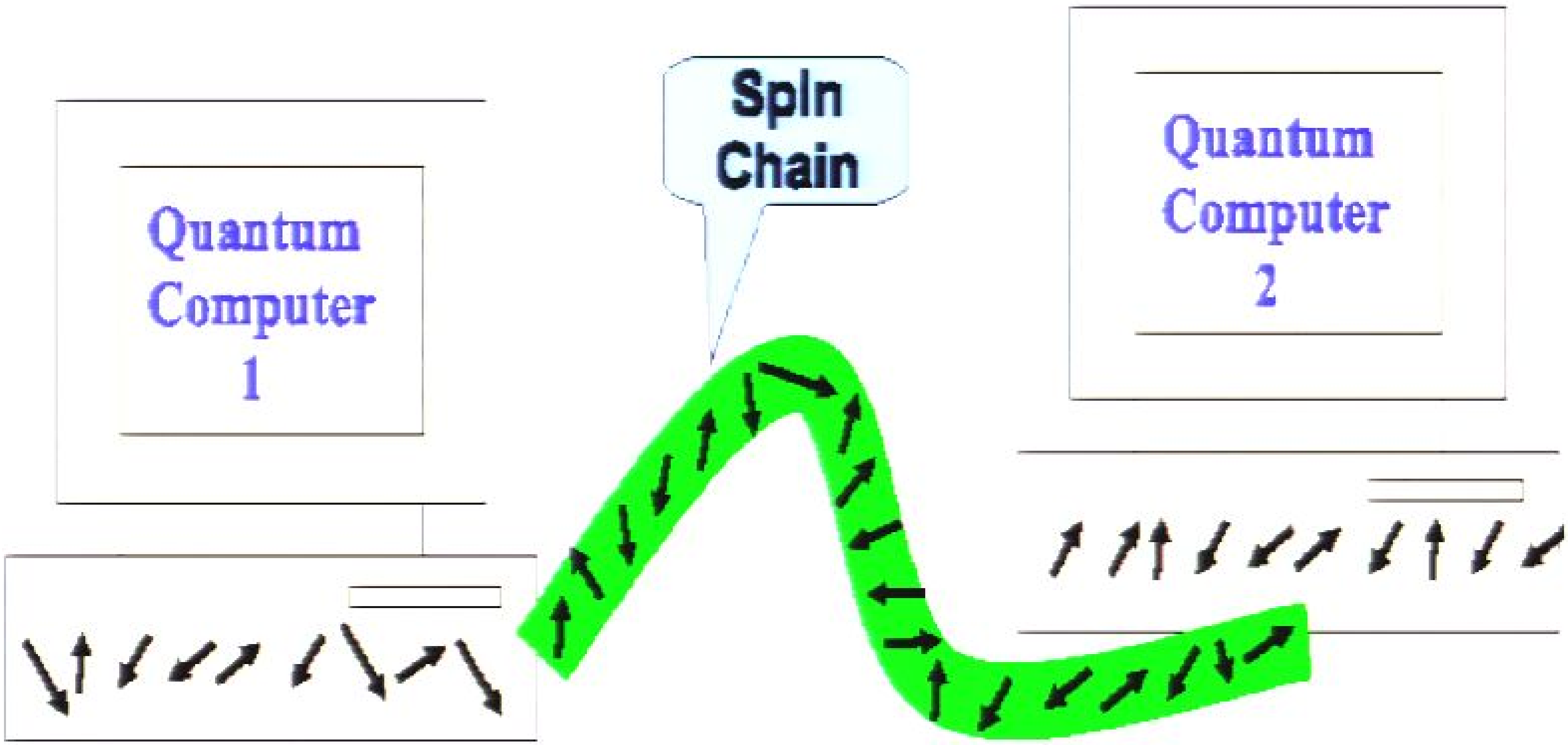


- Motivation 1:
  Avoiding inter-conversion between solid state qubits and photons for linking quantum registers.

Quantum processors in the near future will be small!
--- bus mode, fabrication, efficiency



Usually the use of photons are envisaged for making the links



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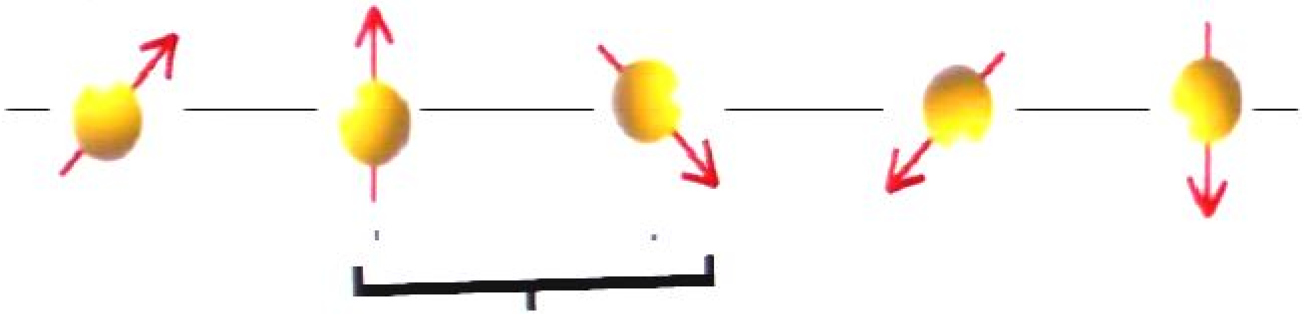
What are spin chains?

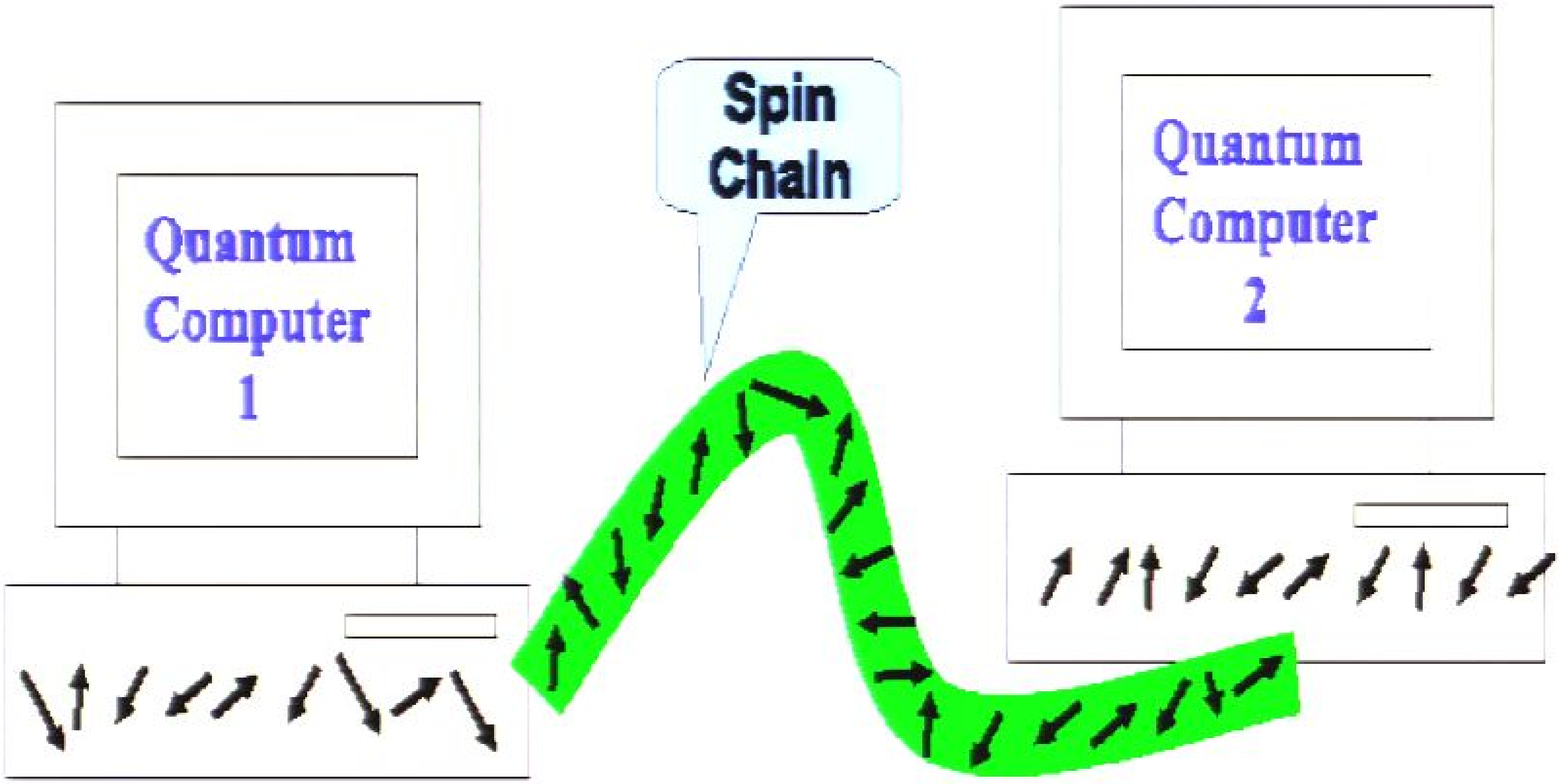
One dimensional array of spins (qubits, qutrits etc.)



Perpetually interacting

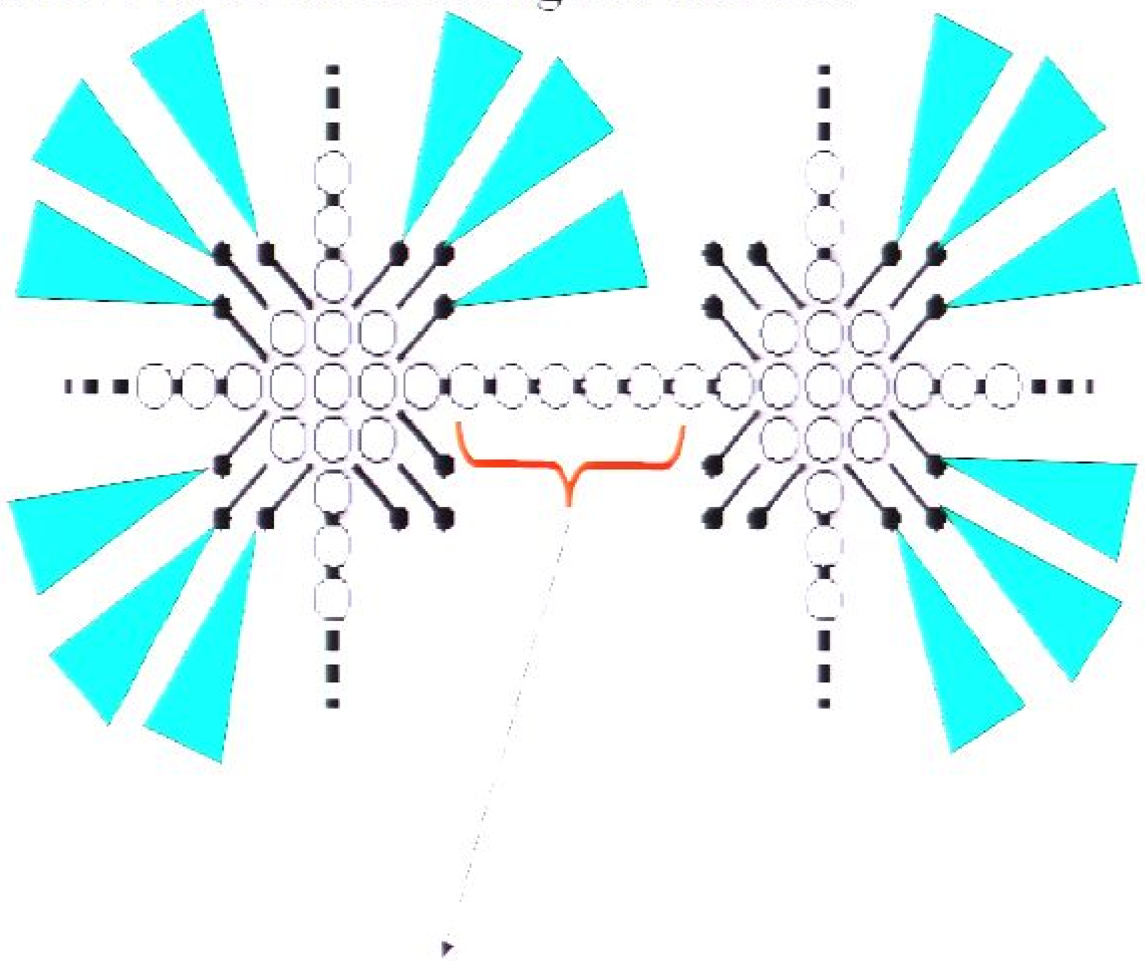
Generally, interactions fall off with the separation between spins

Examples: Quasi 1D magnets, Josephson junction arrays, quantum dot arrays, optical lattices, coupled cavity arrays etc.



Motivation 2: A quantum information perspective on a canonical condensed matter system (how various types of chains transmit information) --- perfect fidelity not required for this study.

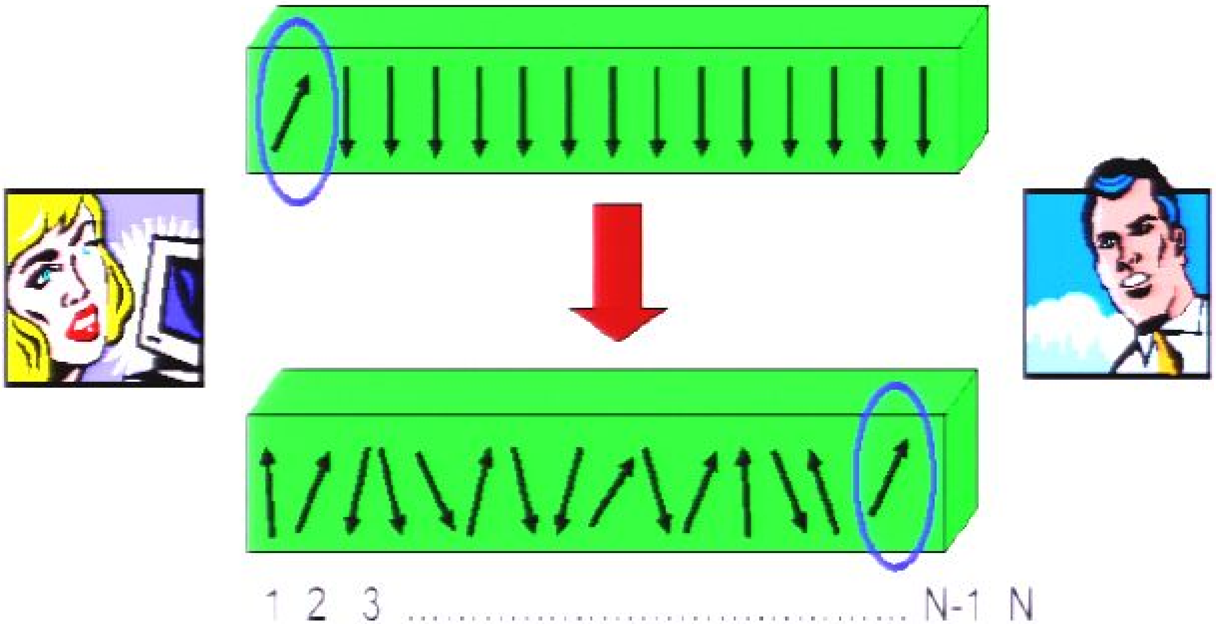
Macroscopic Control Gates cannot control too many qubits, which have to be close enough to interact



Low or no control region (equivalent to a spin chain)

Motivation 3: Spin chains may naturally arise in low control parts of a quantum computer.

The simplest spin chain protocol one can imagine:



Path Graph

$$H = J\sum_i \left(X_i \otimes X_{i+1} - Y_i \otimes Y_{i+1} - Z_i \otimes Z_{i+1} \right)$$

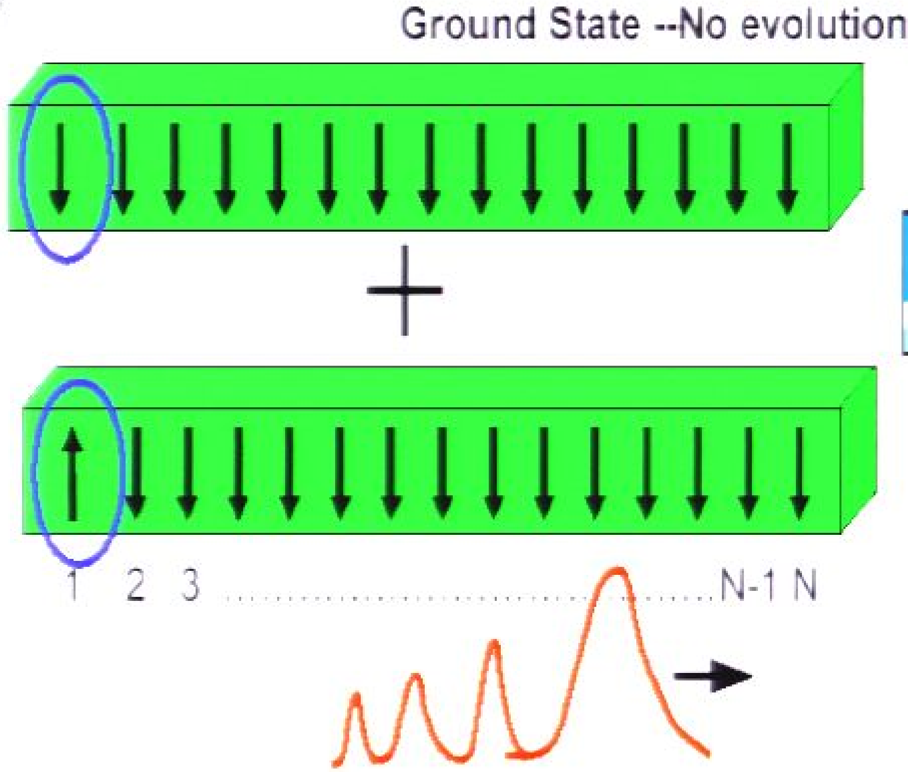
- Isotropic
- Uniform
- Ferromagnetic.
- Nearest Neighbor

S. Bose. Phys. Rev. Lett. 91, 207901 (2003).

Alice's input state: $|\phi\rangle_1 = \cos\frac{\theta}{2}|0\rangle_1 + e^{i\phi}\sin\frac{\theta}{2}|1\rangle_1$

$\downarrow \equiv |0\rangle \qquad \uparrow \equiv |1\rangle$

$\cos\frac{\theta}{2}$



$e^{i\phi}\sin\frac{\theta}{2}$

Optimize

Everything depends *only* on: $f_{N,1}(t) = \langle \mathbf{N}|e^{-iHt}|\mathbf{1}\rangle$

$|\mathbf{1}\rangle \equiv |100...00\rangle$

$|\mathbf{N}\rangle \equiv |000...01\rangle$

Let the transition amplitude: $f_{N,1}(t) = \langle \mathbf{N} | e^{-iHt} | \mathbf{1} \rangle$

$|\mathbf{1}\rangle \equiv |100\ldots00\rangle$

$|\mathbf{N}\rangle \equiv |000\ldots01\rangle$

At any general time t the output density matrix is:

$$\rho_N(t) = P(t)|\psi(t)\rangle\langle\psi(t)|_N + (1 - P(t))|0\rangle\langle 0|_N$$

Unavoidable mixing due to dispersion.

with $P(t) = \cos^2\frac{\theta}{2} + |f_{N,1}(t)|^2 \sin^2\frac{\theta}{2}$

$$|\psi(t)\rangle_N = \frac{1}{\sqrt{P(t)}}\left(\cos\frac{\theta}{2}|0\rangle_N + e^{i\varphi}|f_{N,1}(t)|\sin\frac{\theta}{2}|1\rangle_N\right)$$

Correctable phase factor

Damping factor due to dispersion

At any time t it behaves *as* an amplitude damping channel

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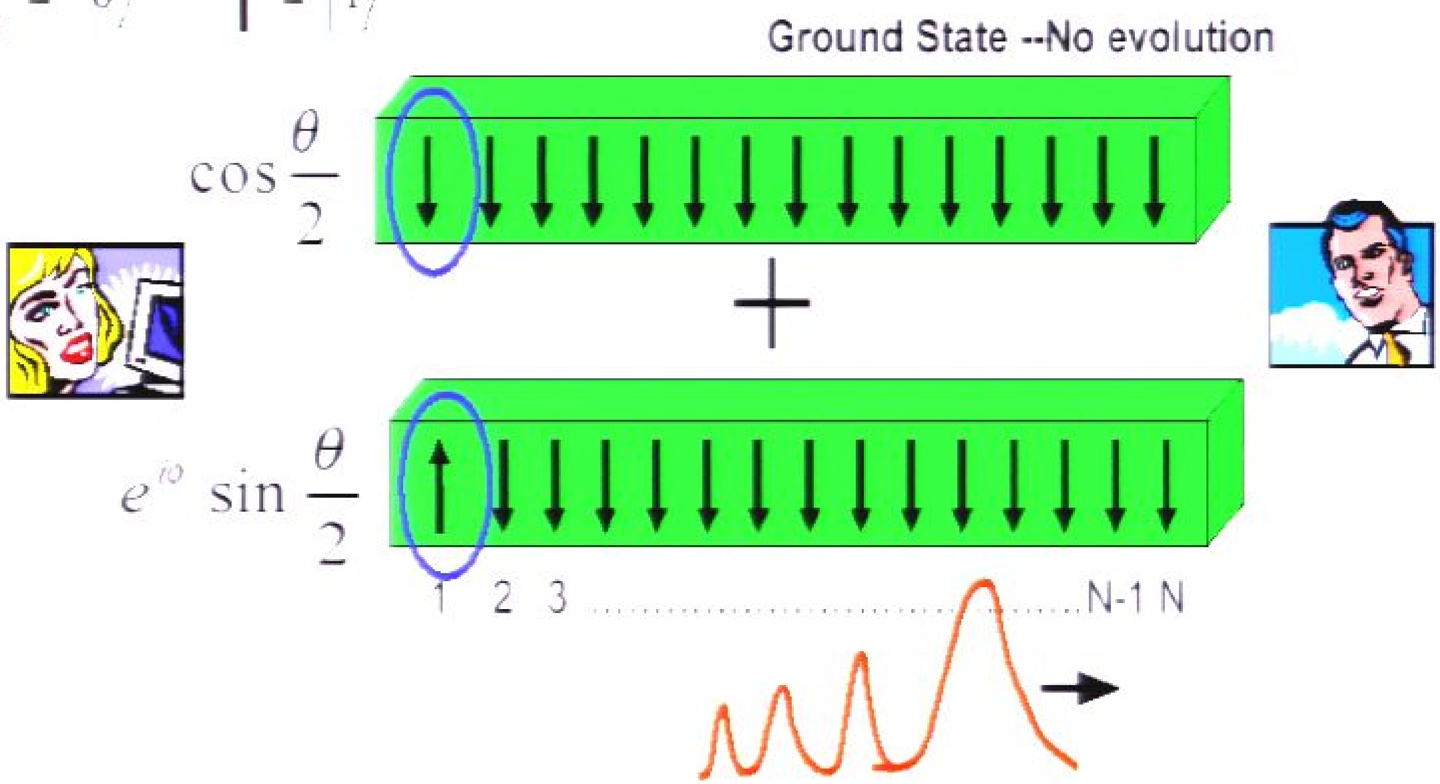

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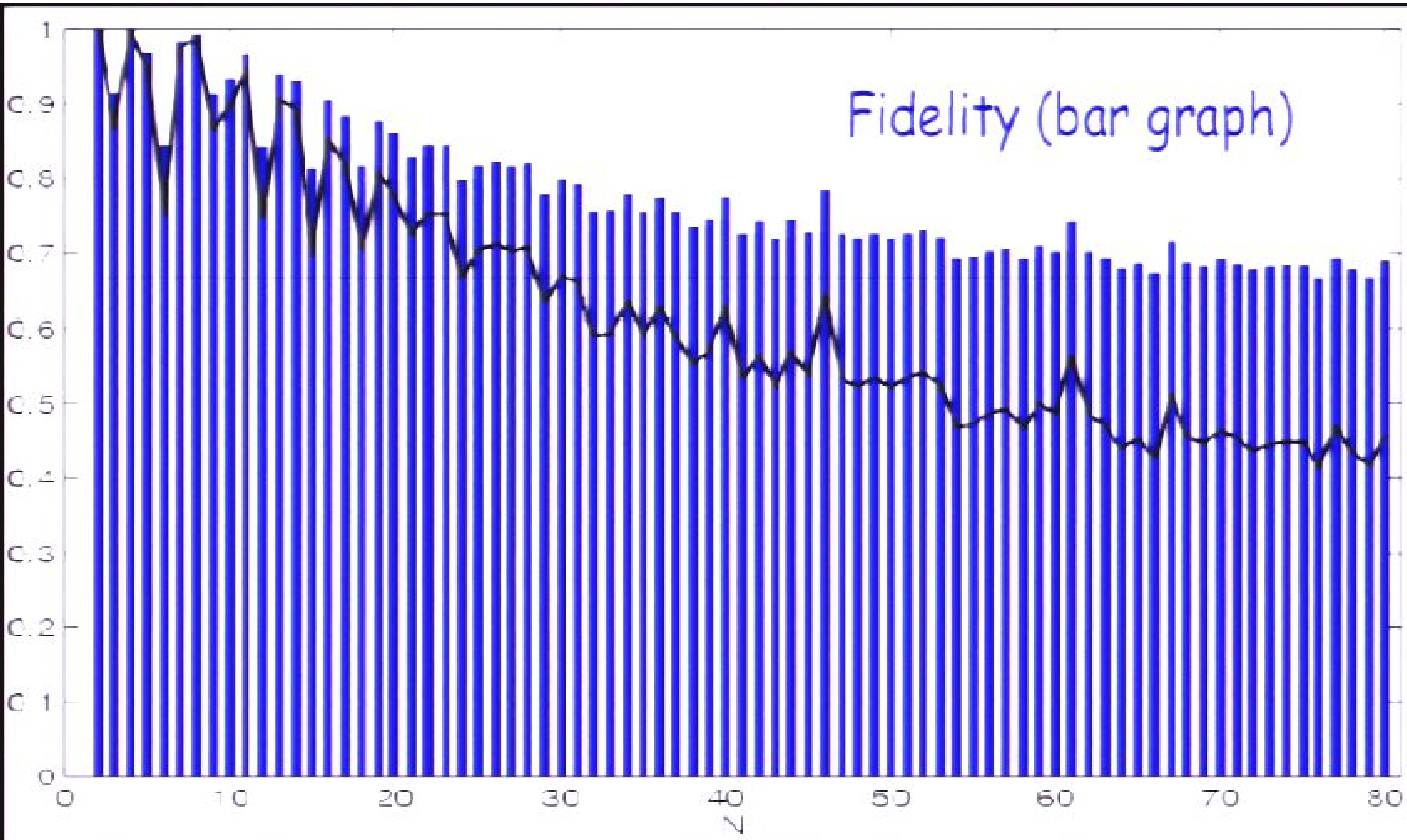
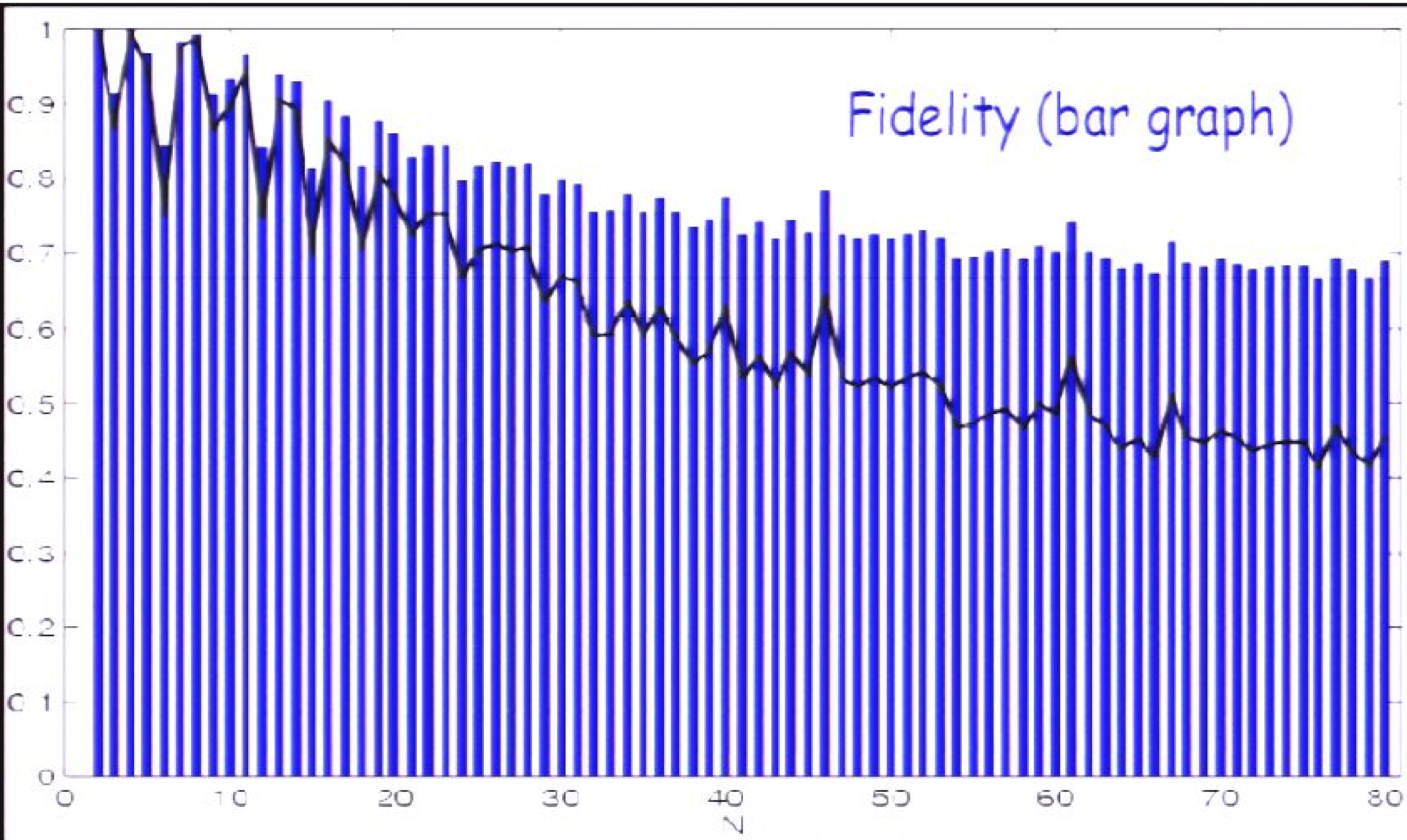


Fidelity:

- function of time and imperfect in general
- near perfect for N=4 & 8

The longer you wait, the better it gets, but no promises!



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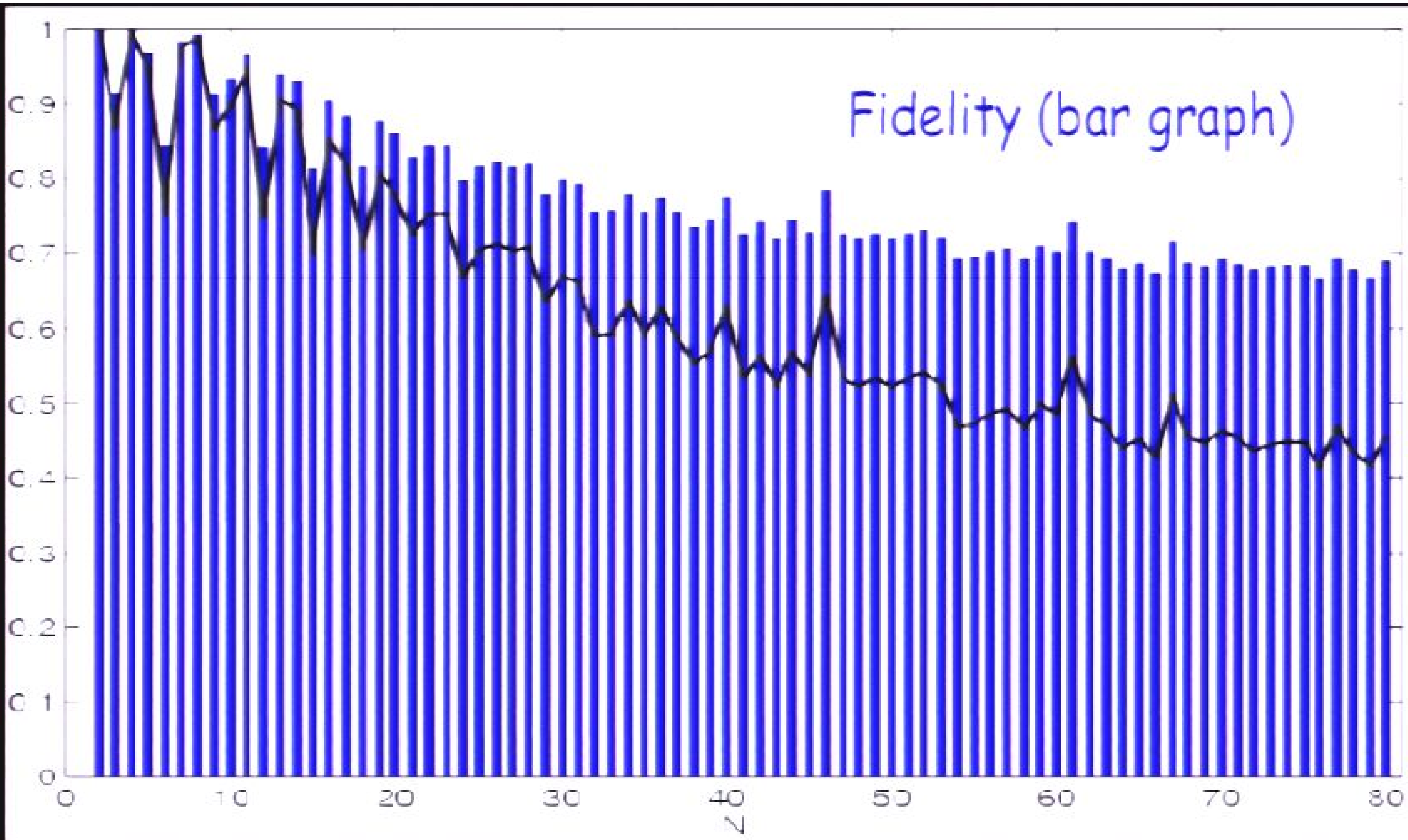

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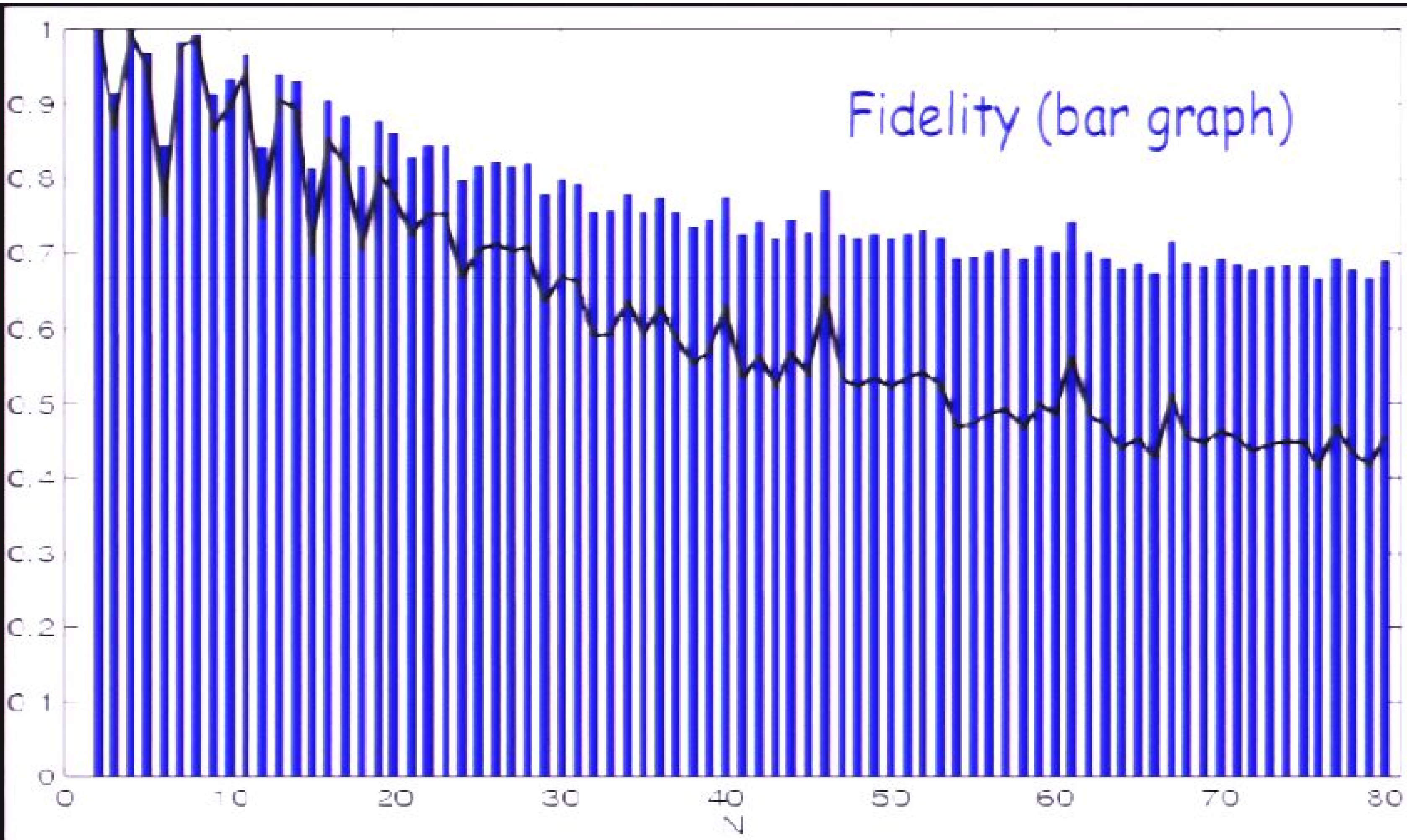

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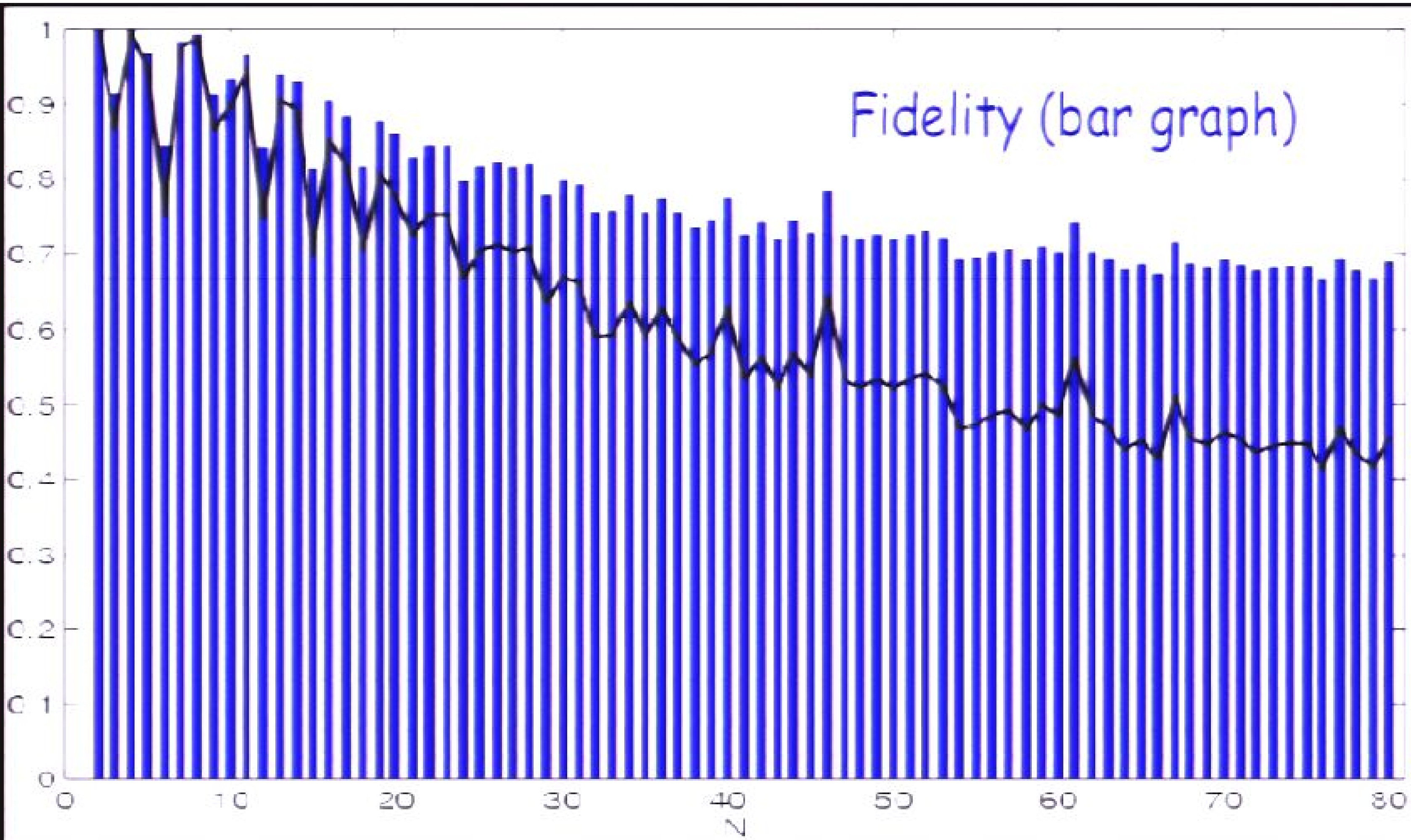

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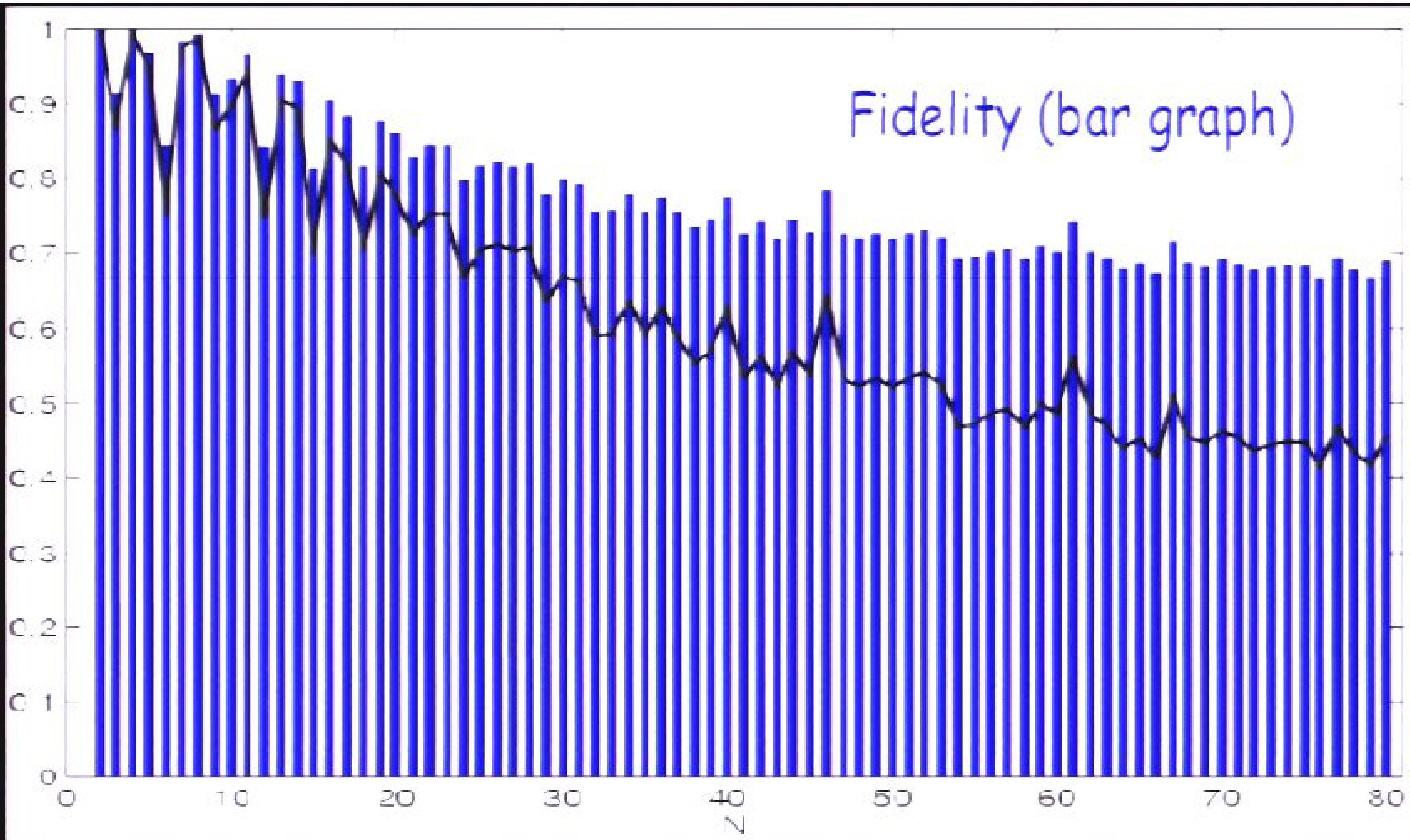

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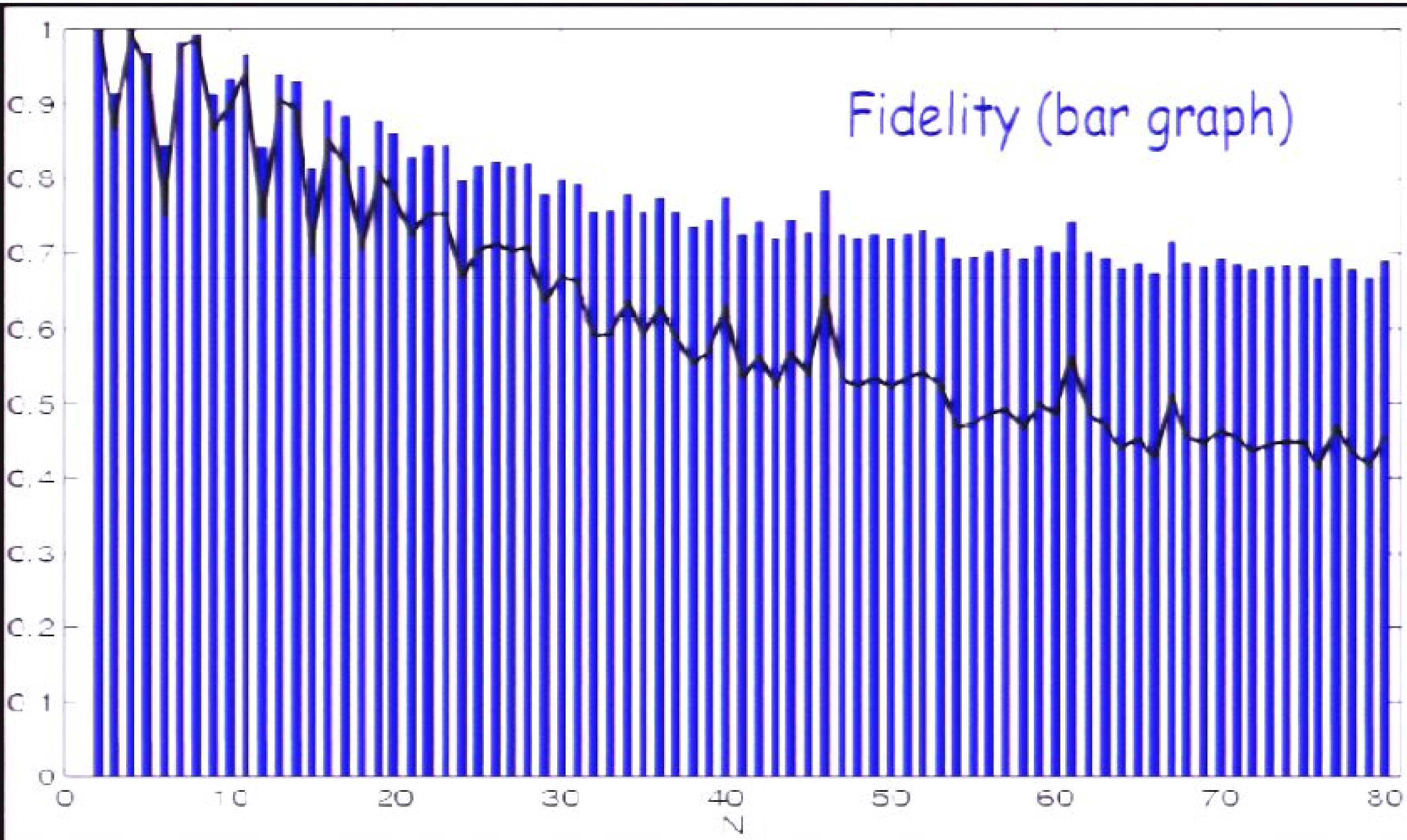

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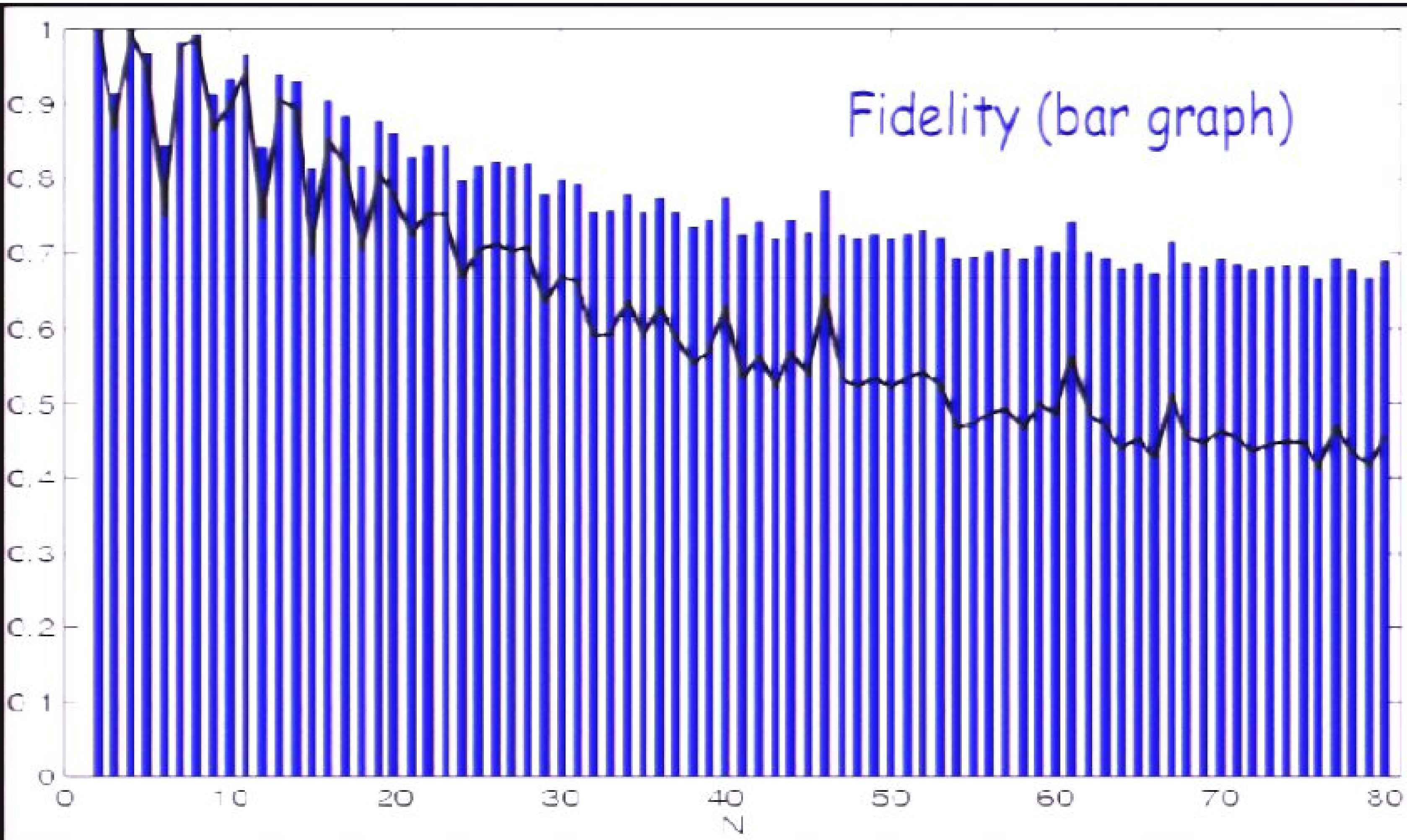

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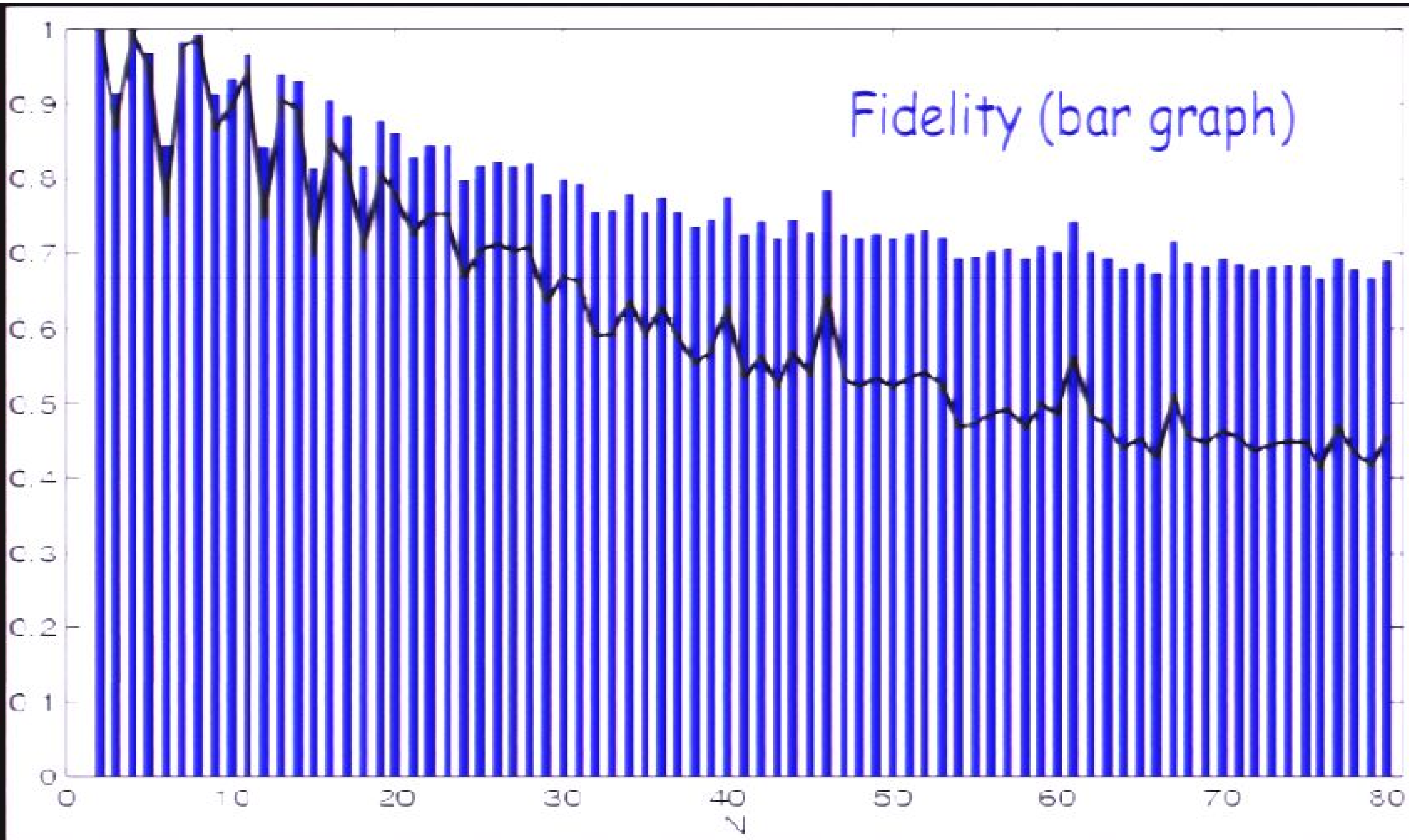

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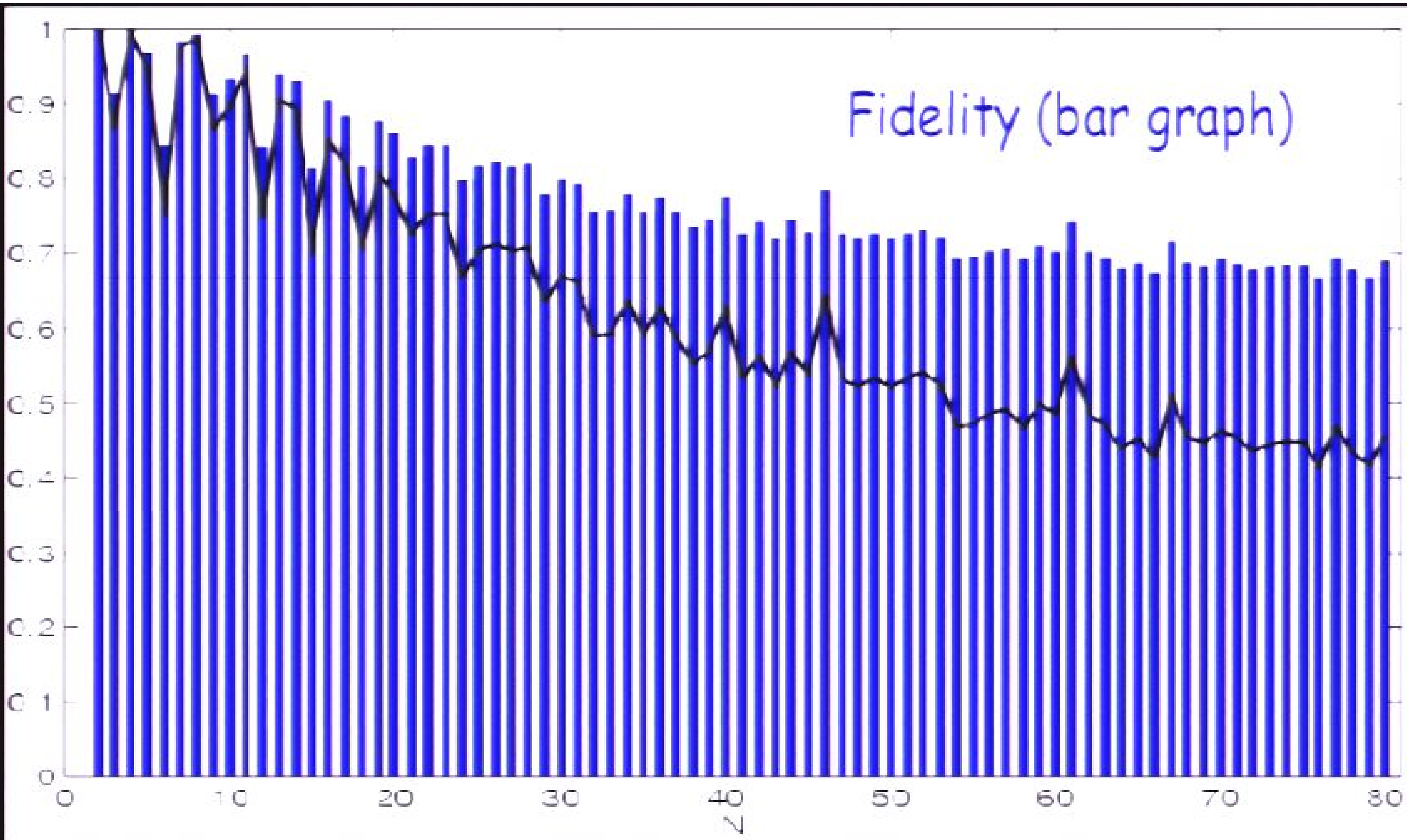

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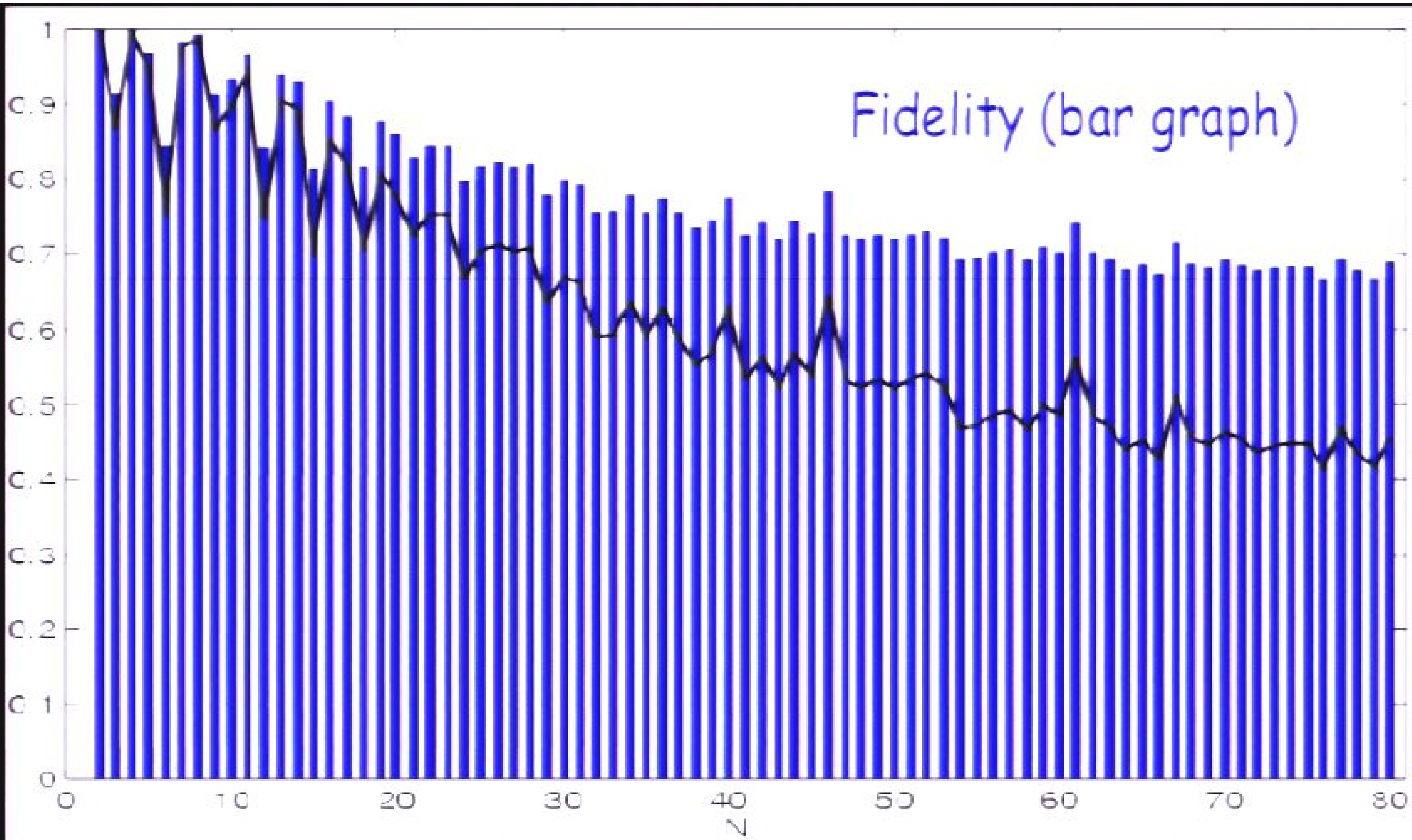

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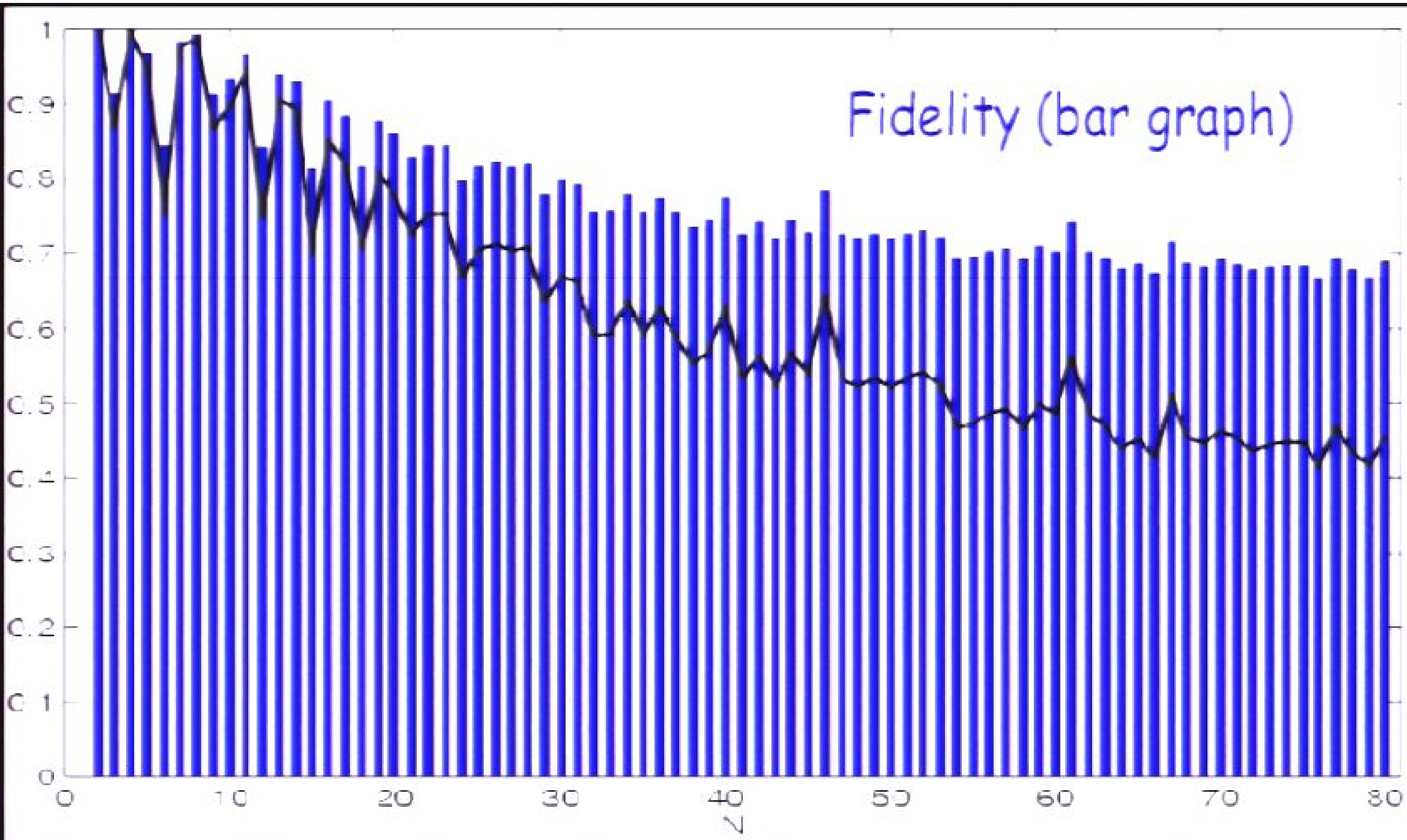

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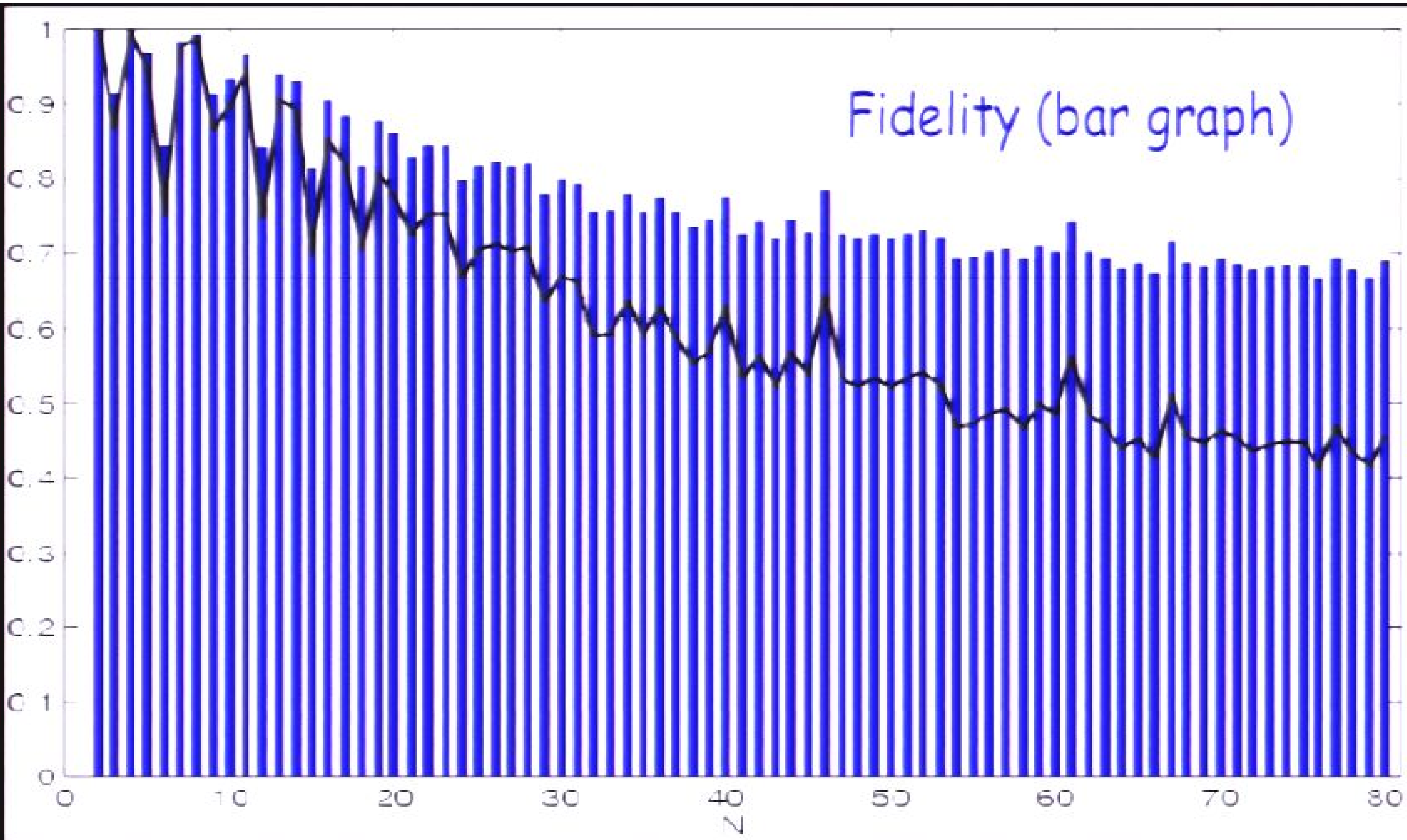

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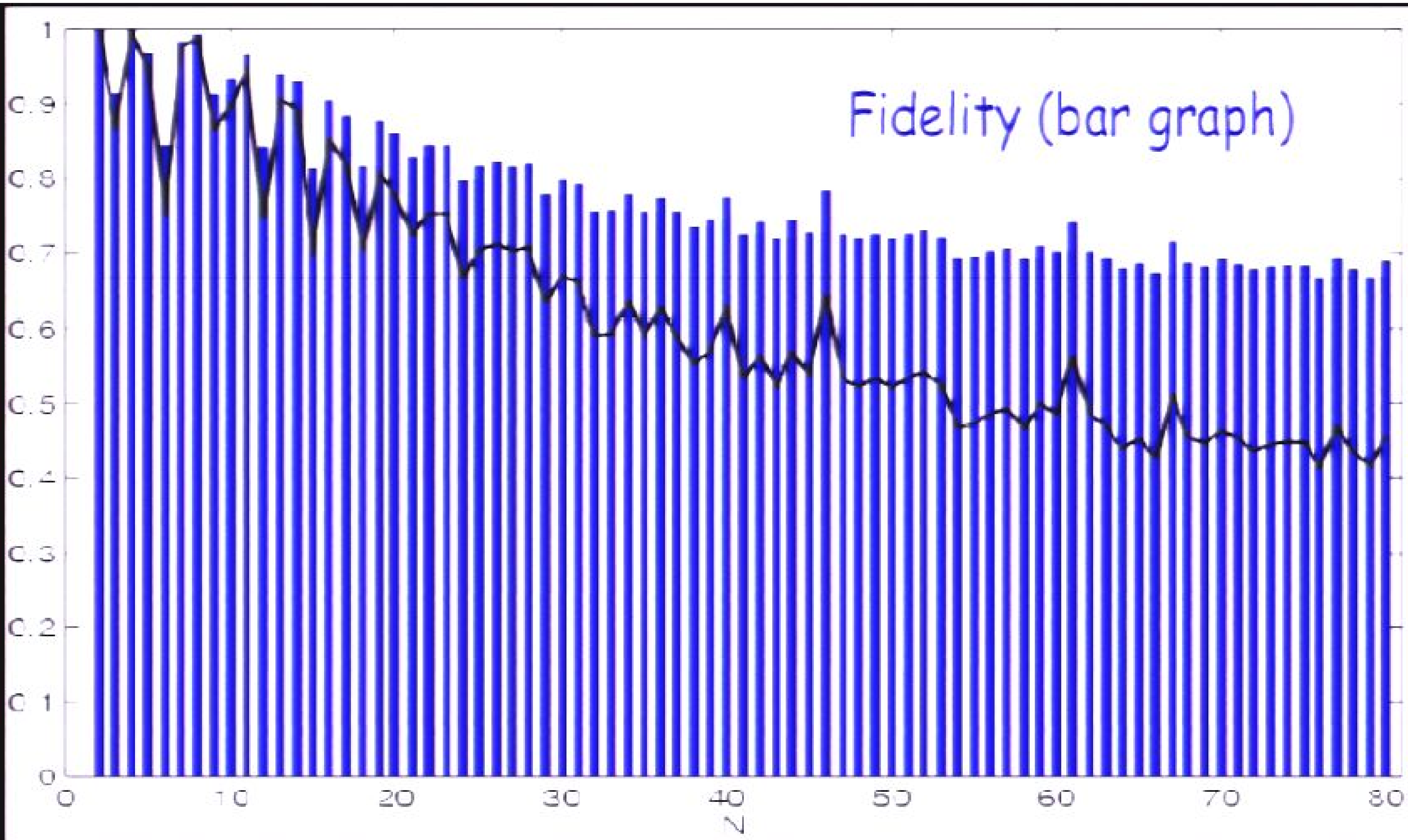

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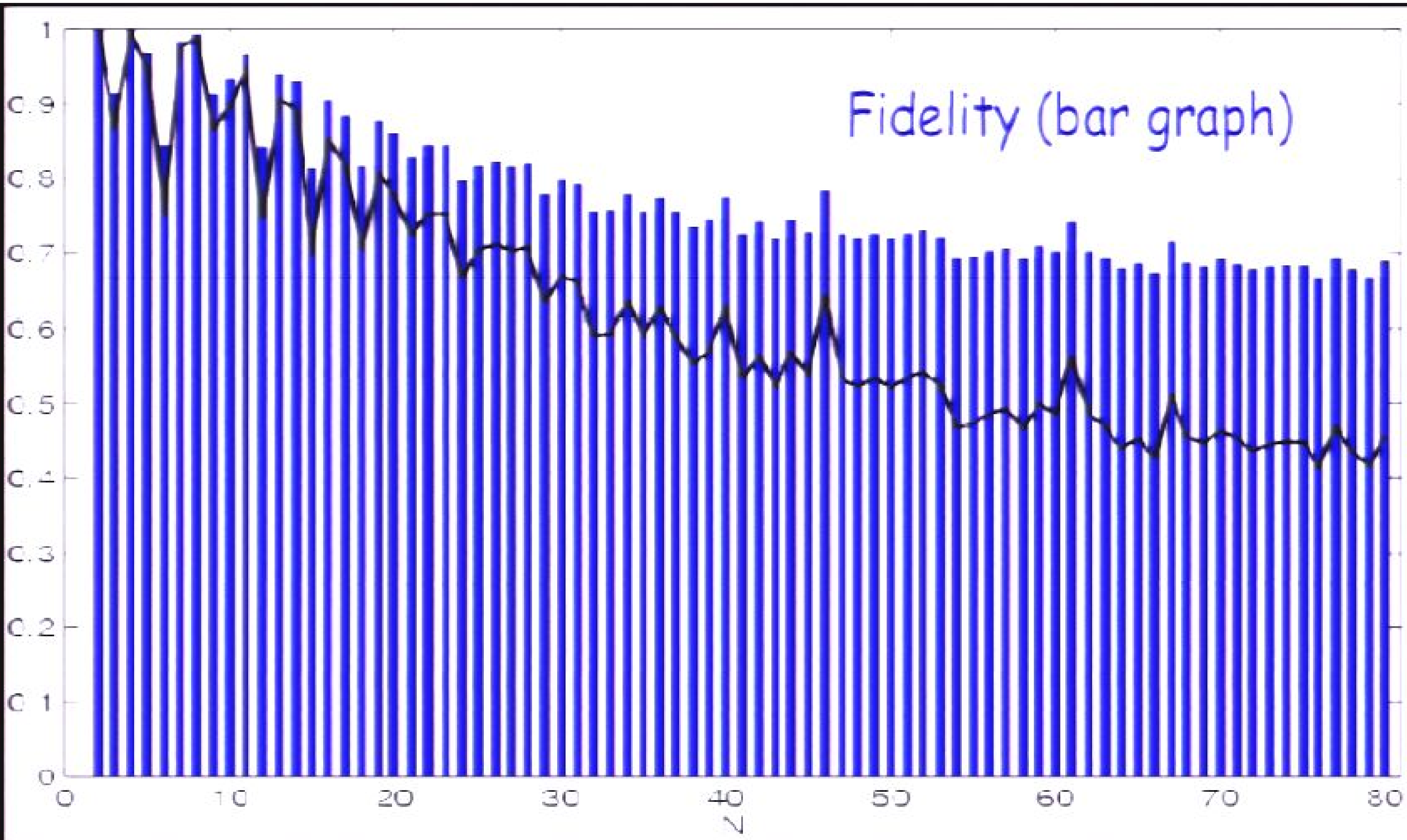

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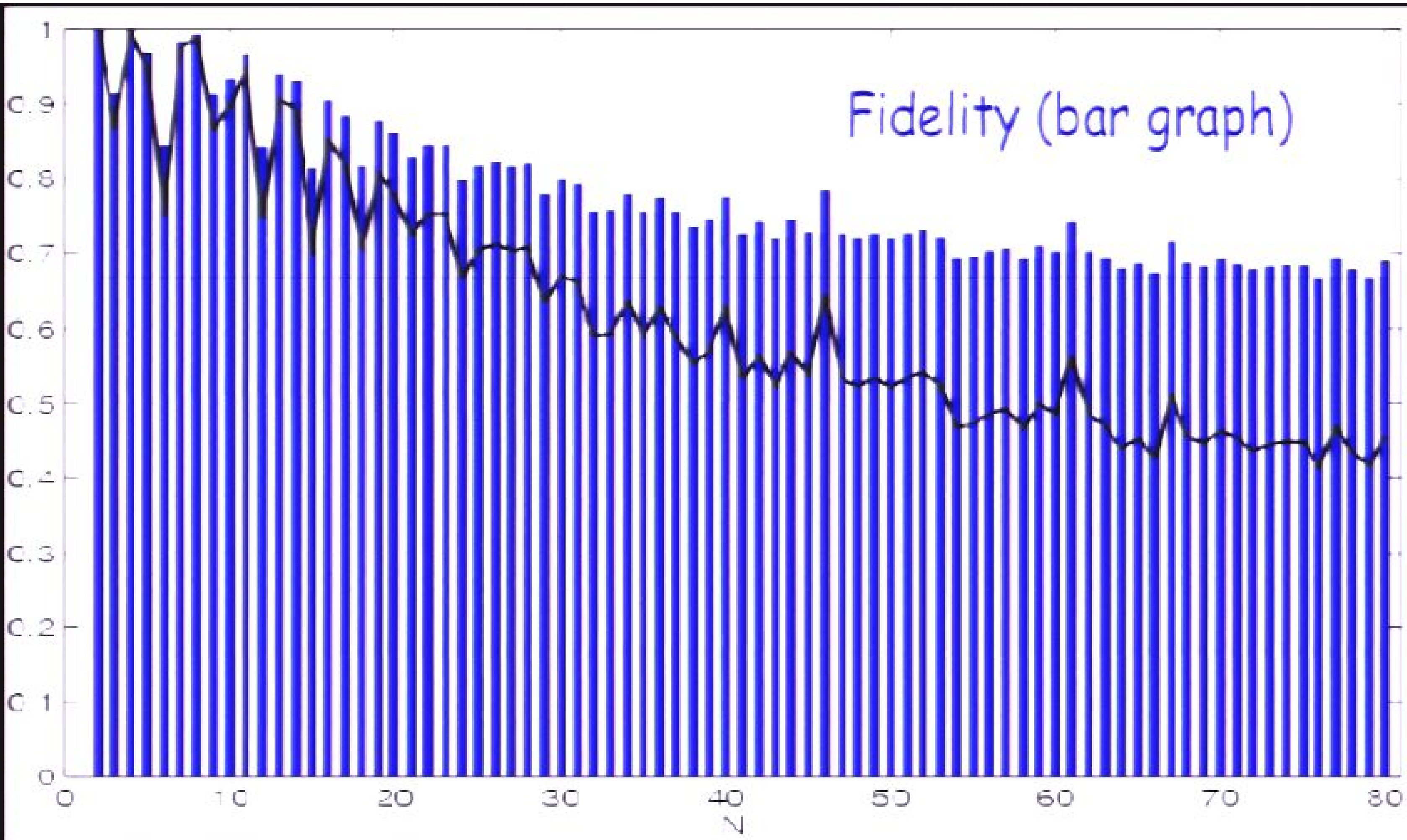

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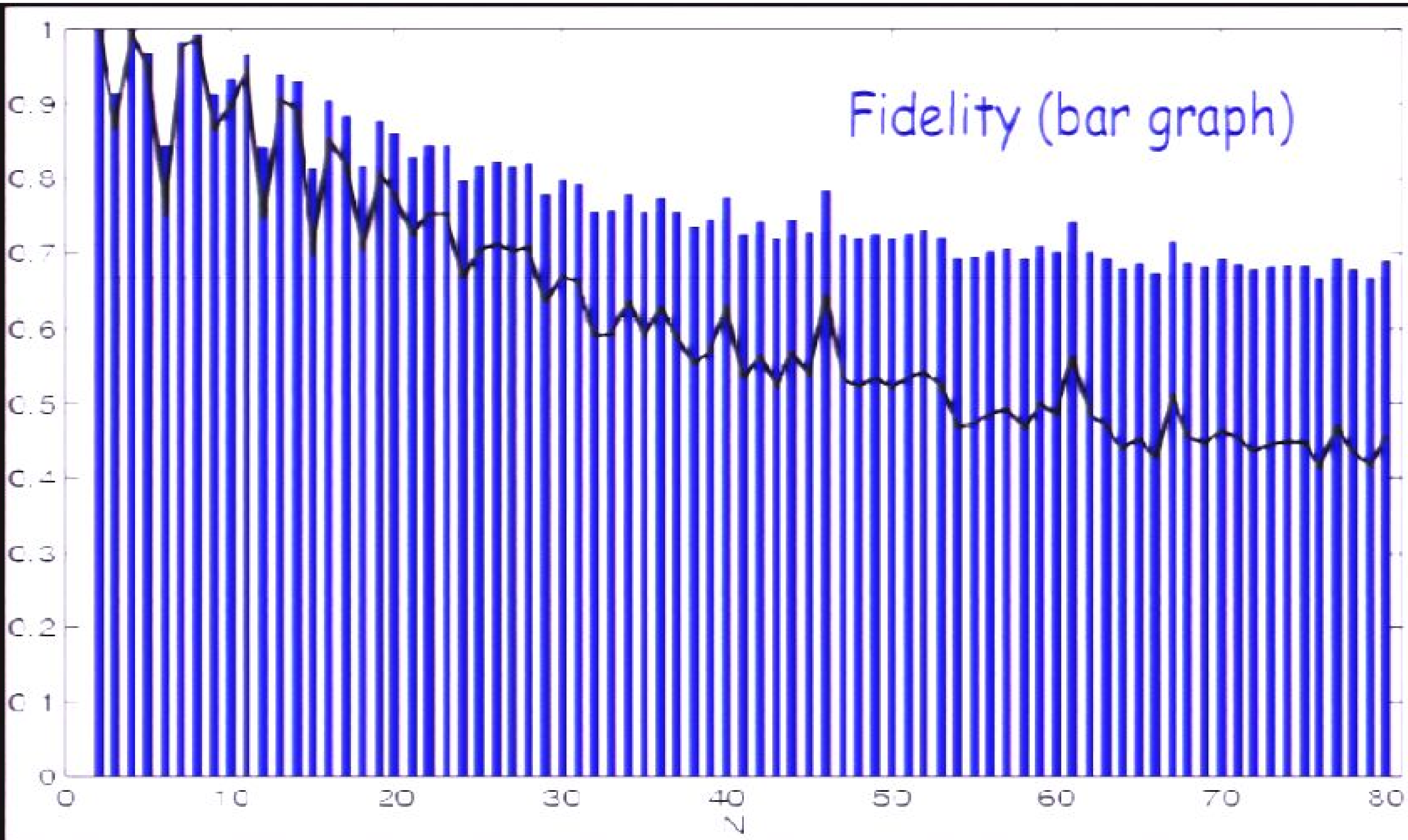

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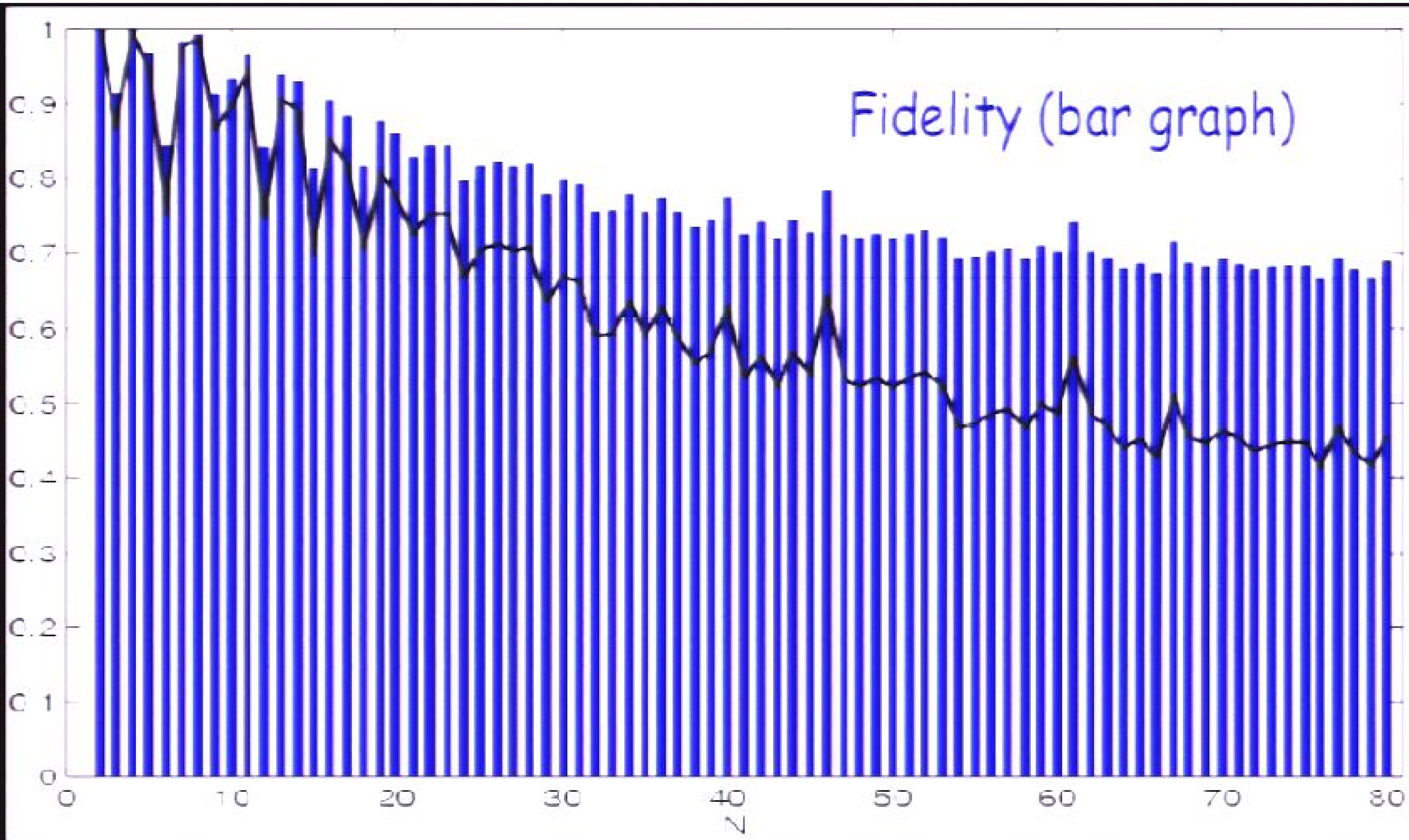

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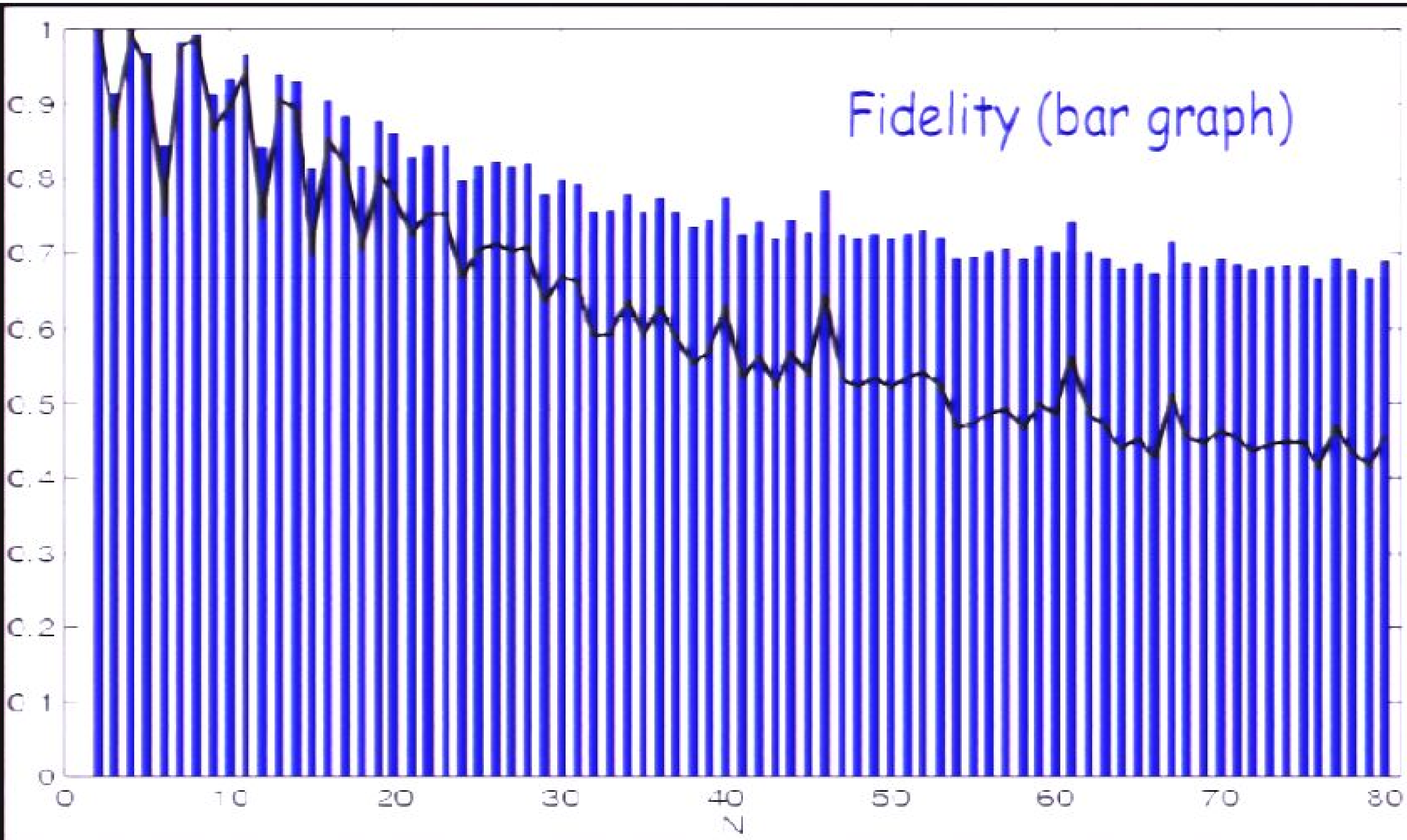

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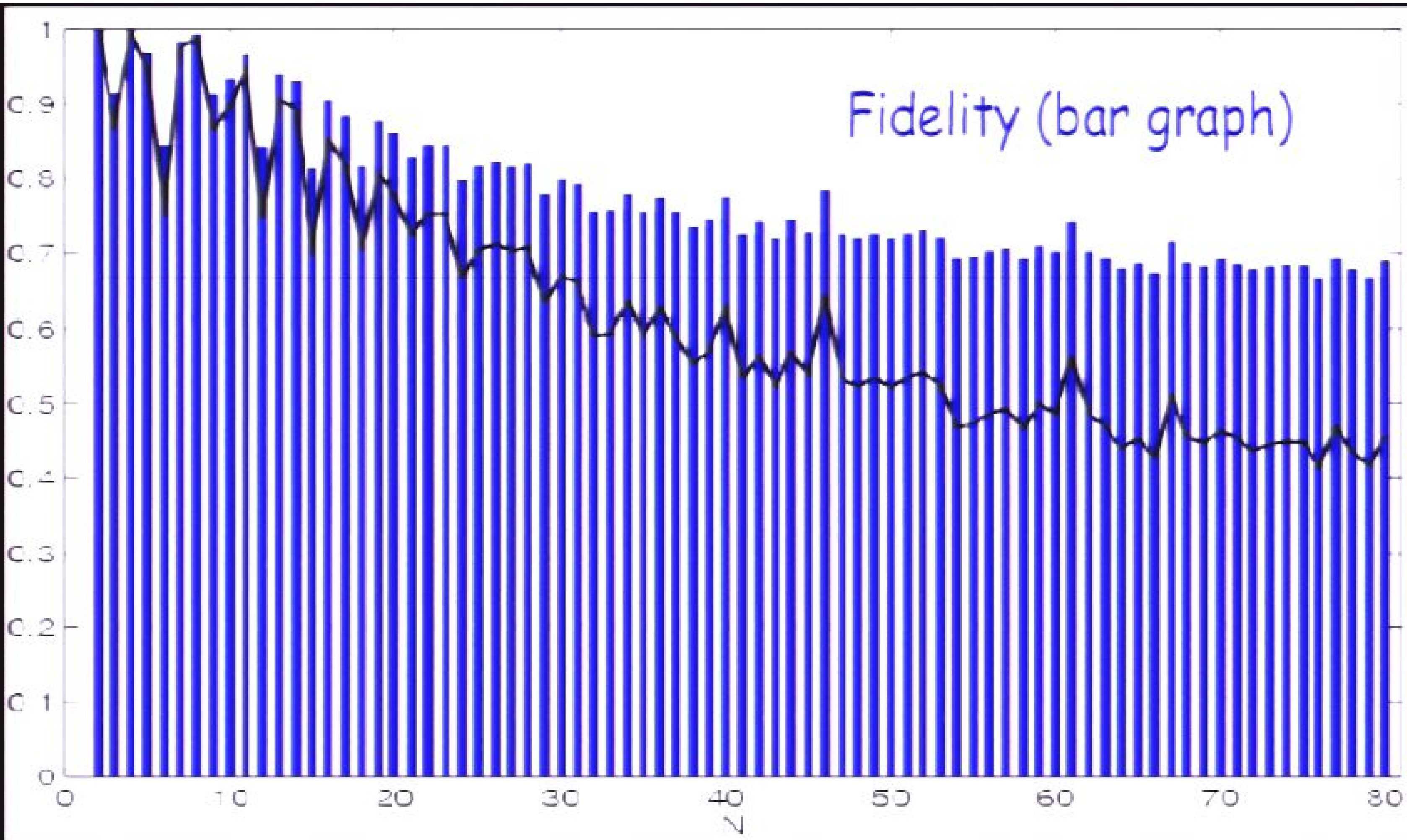

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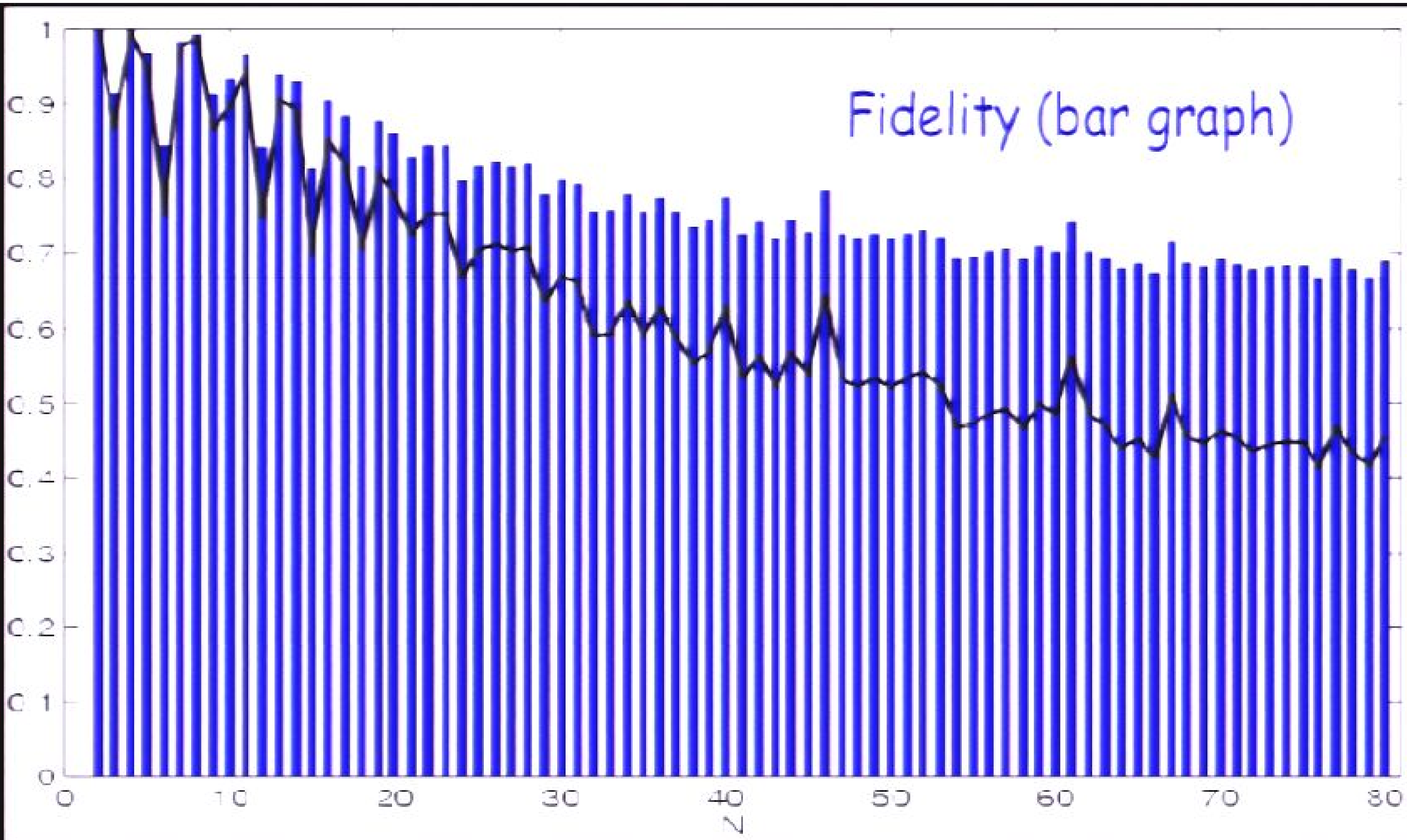

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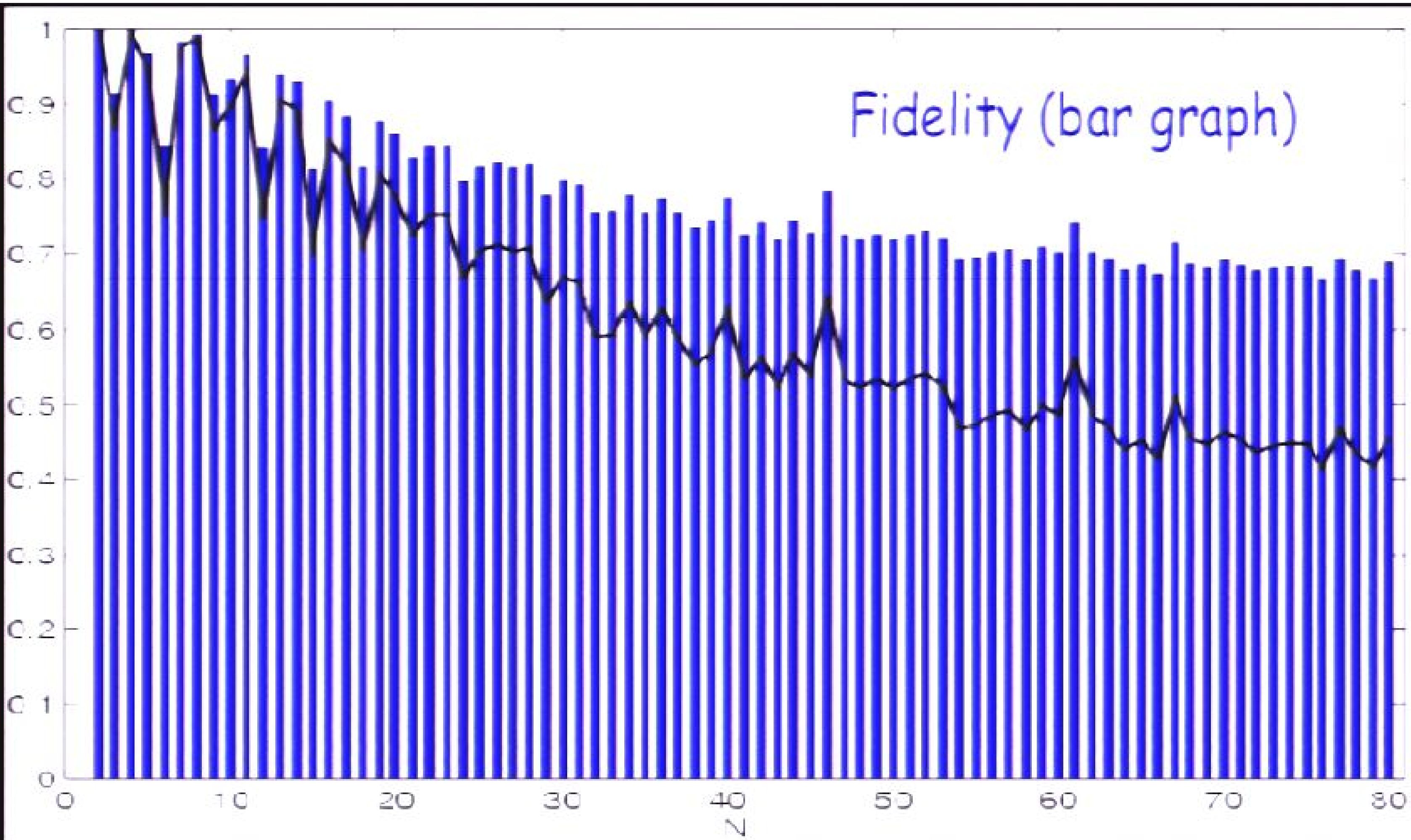

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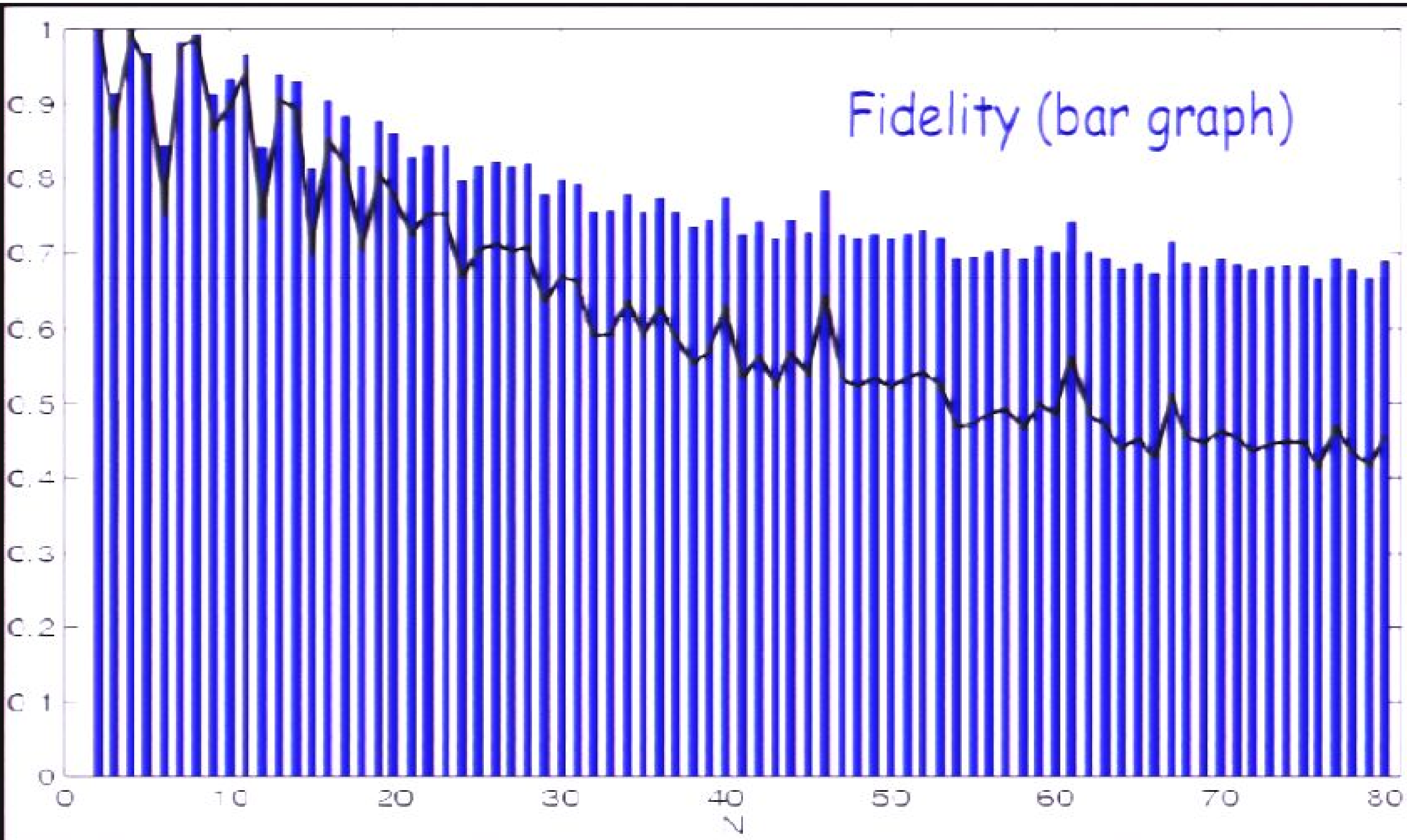

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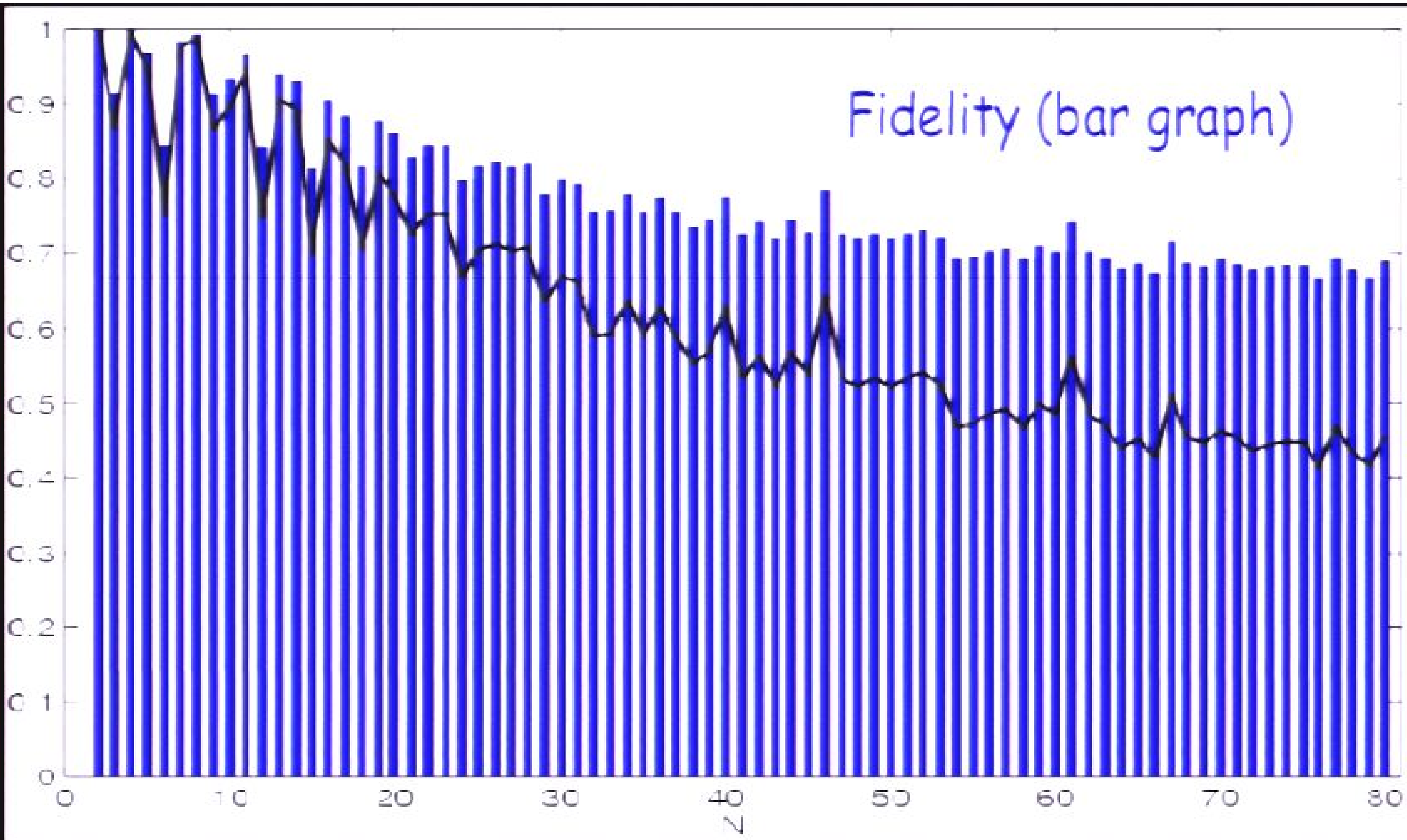

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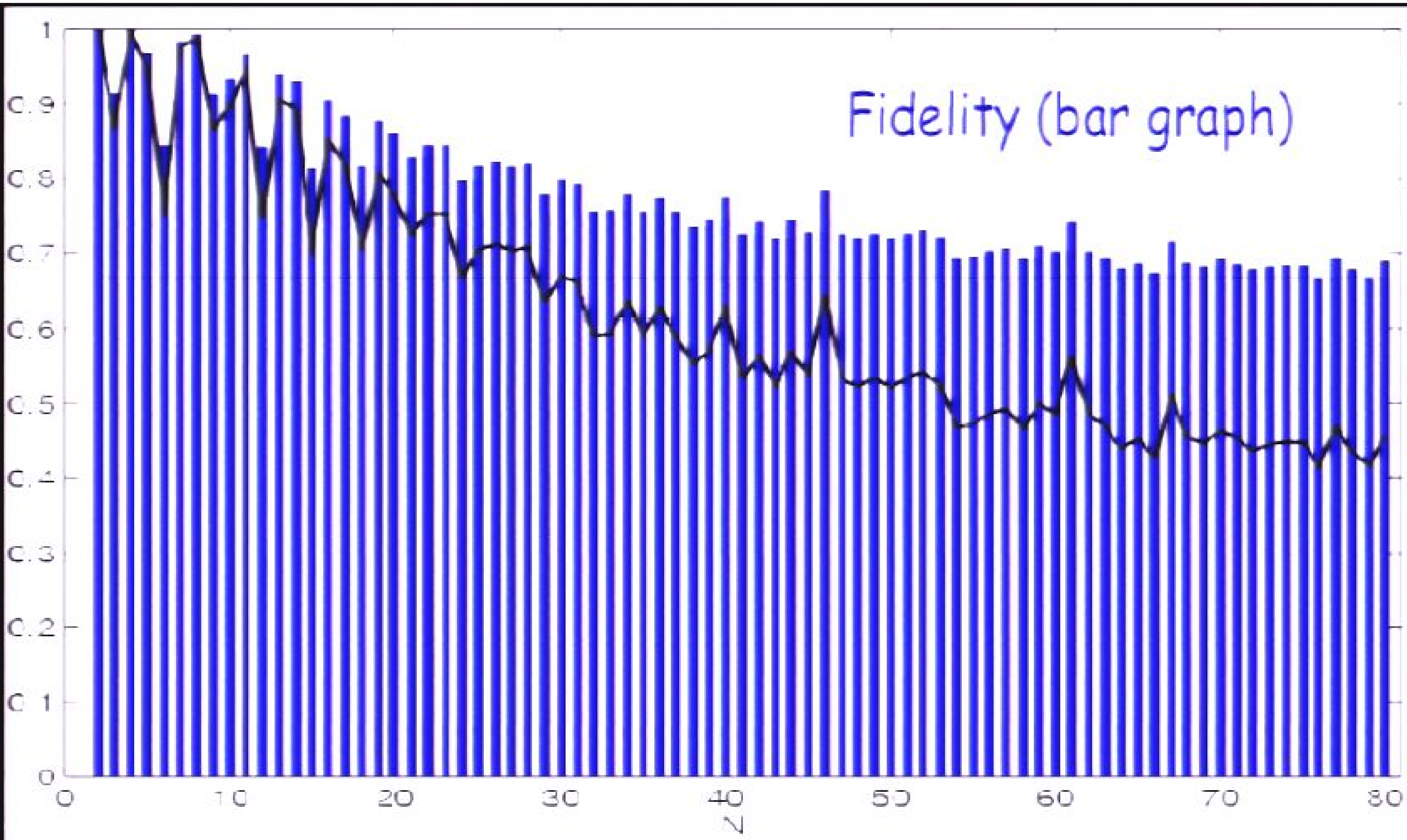

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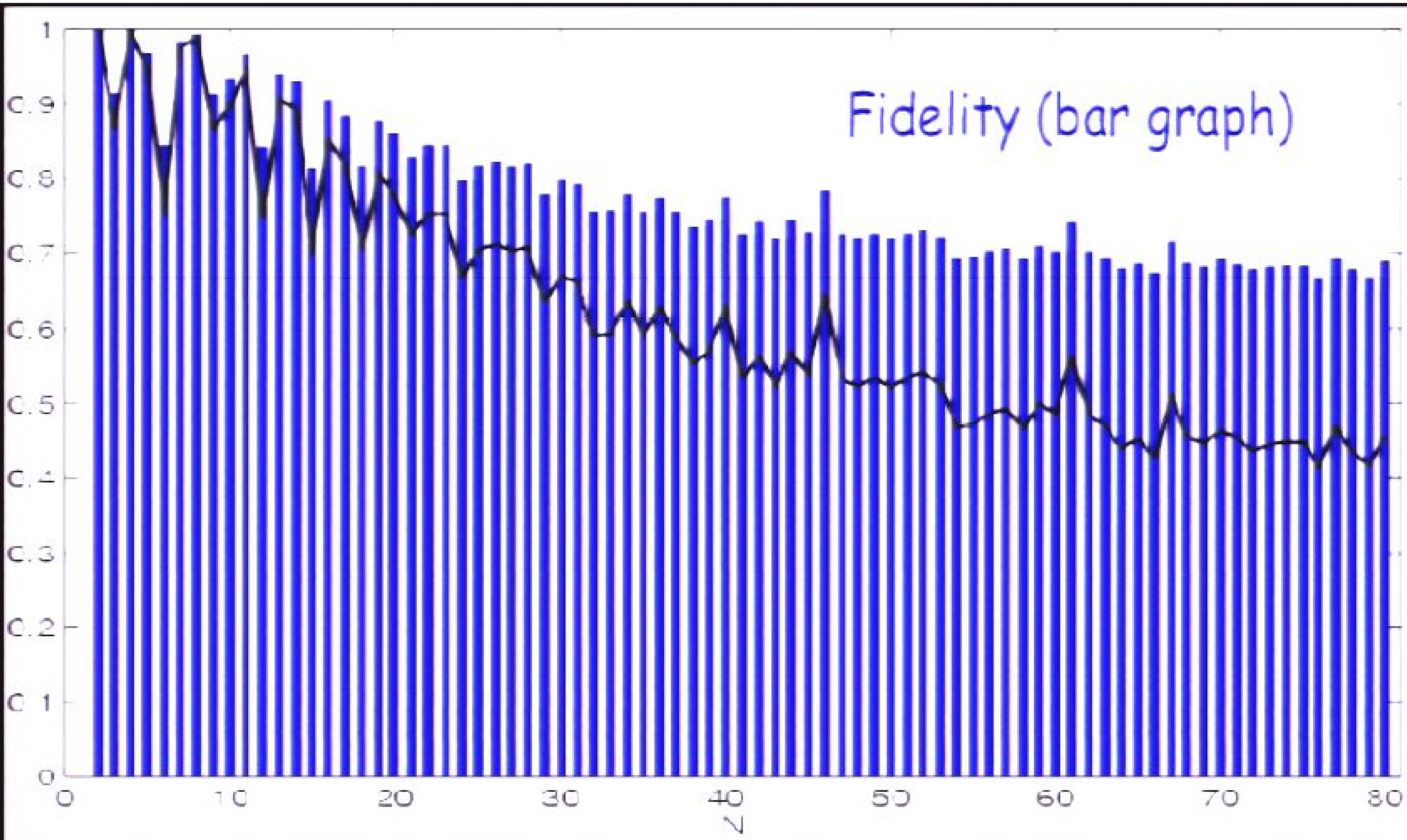

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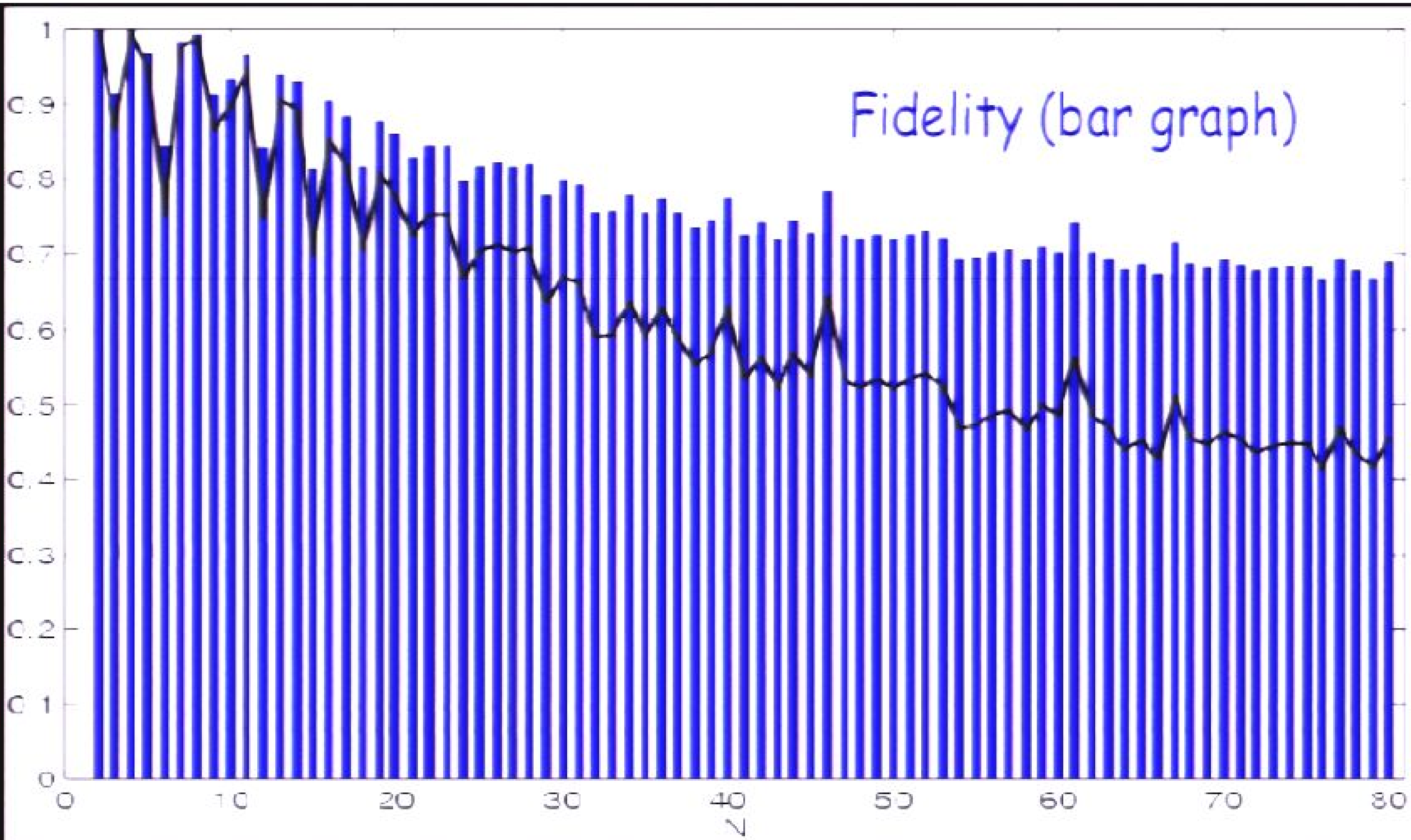

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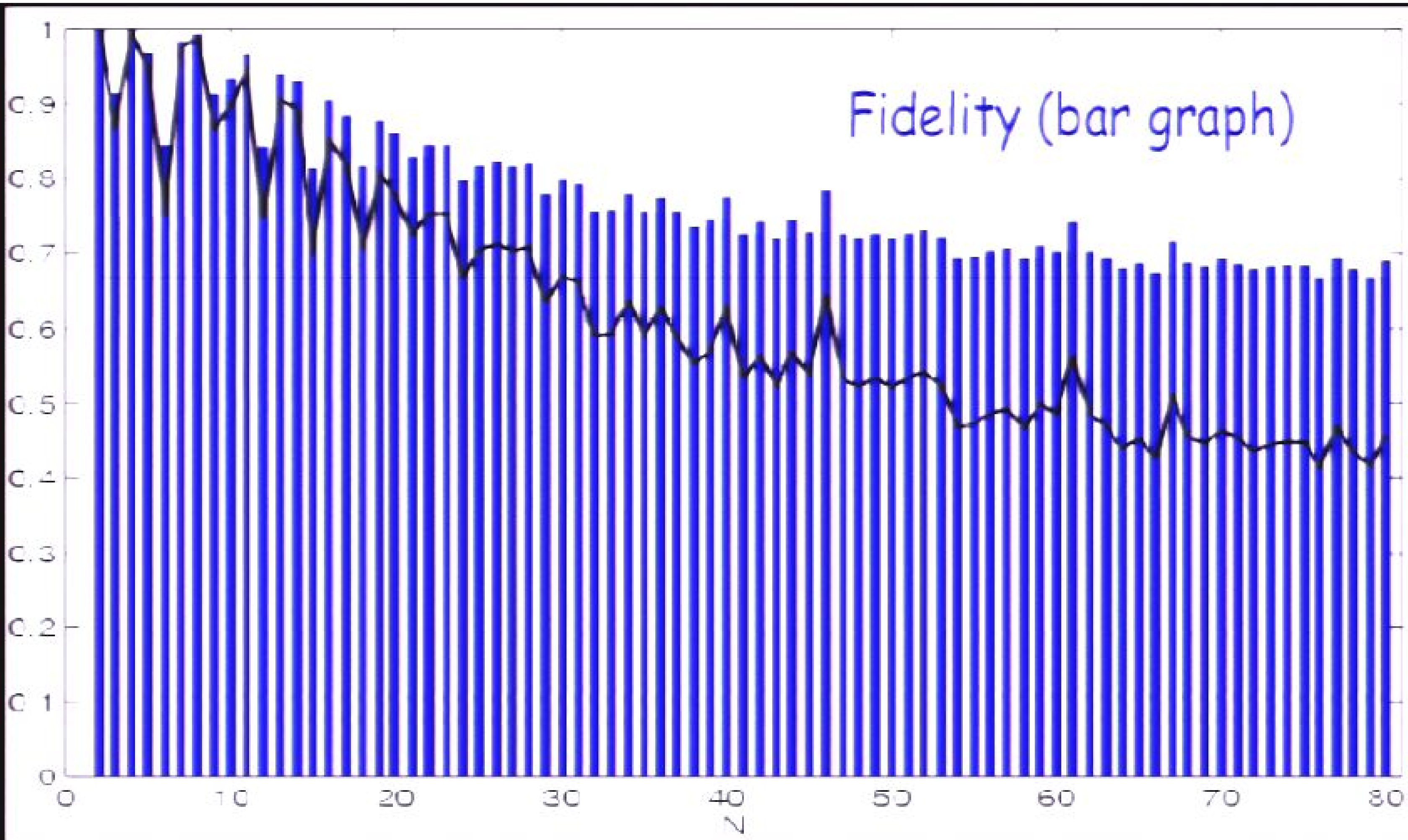

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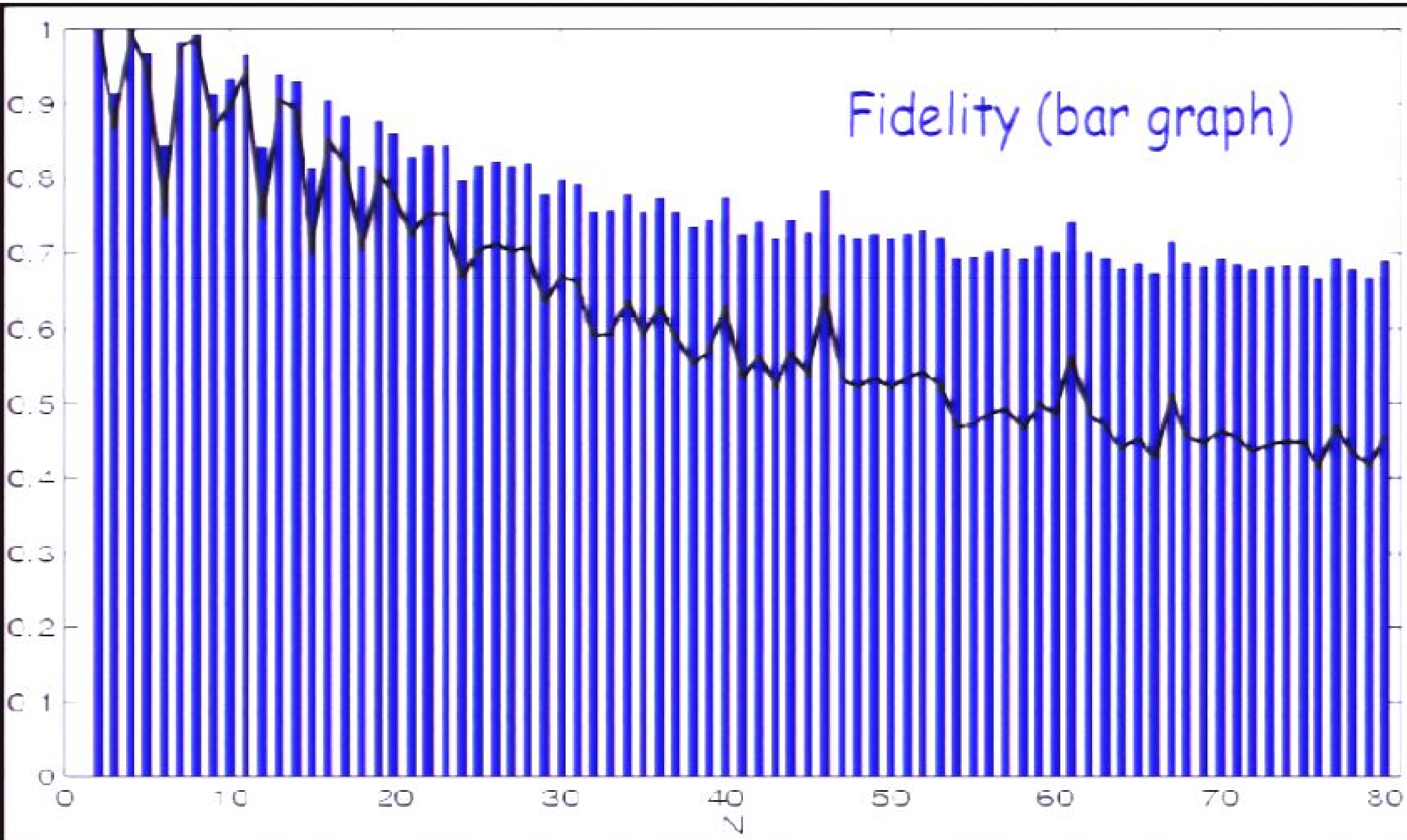

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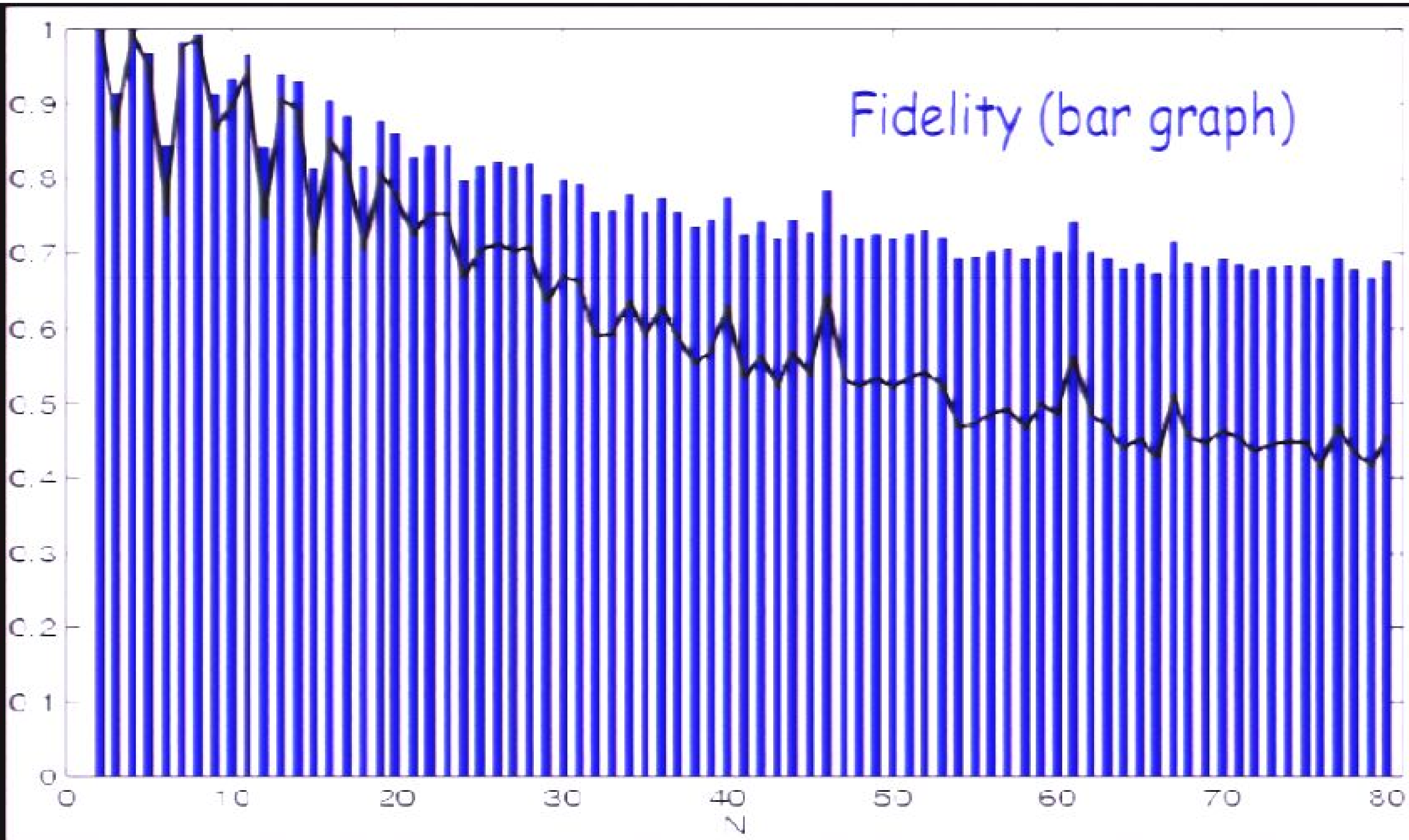

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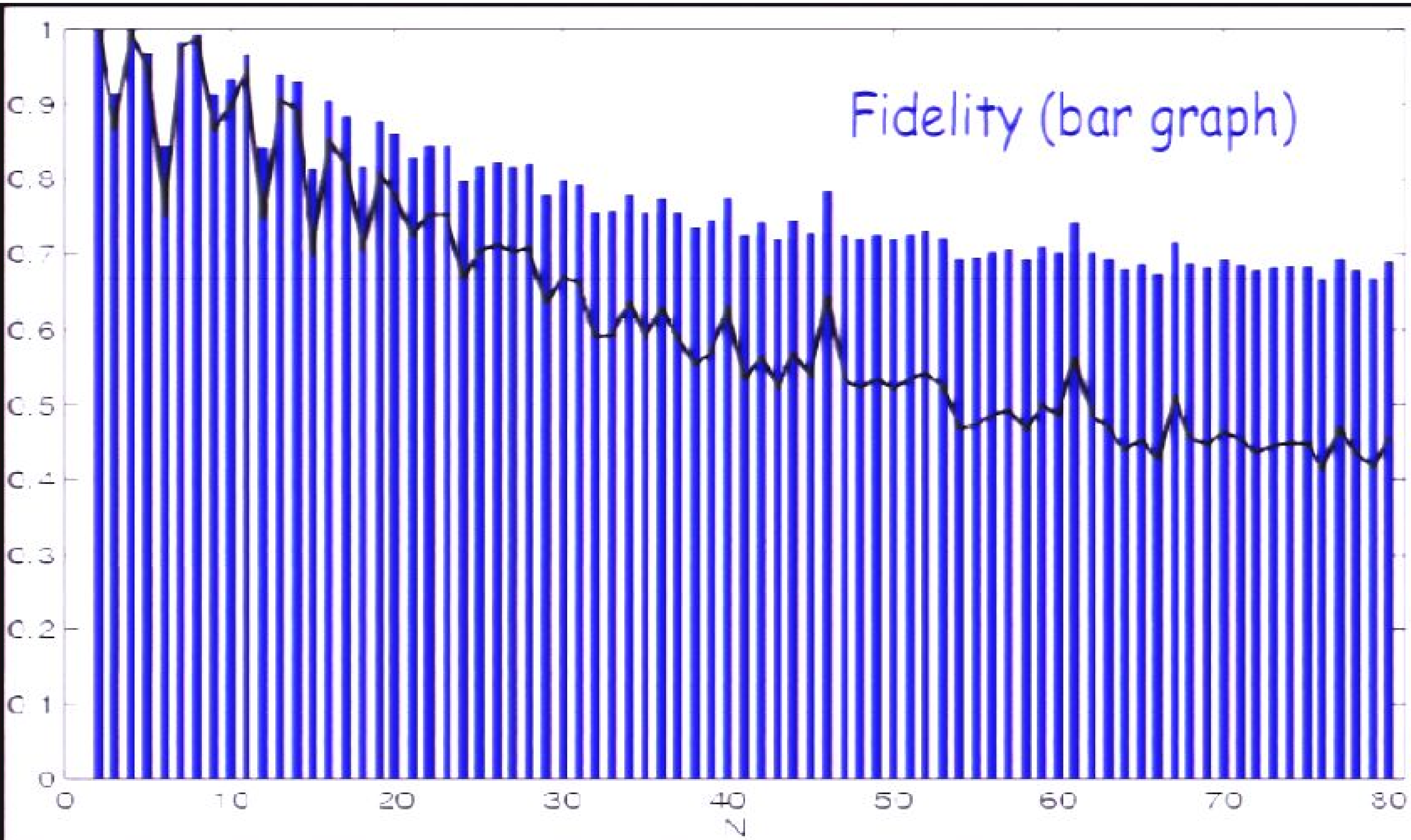

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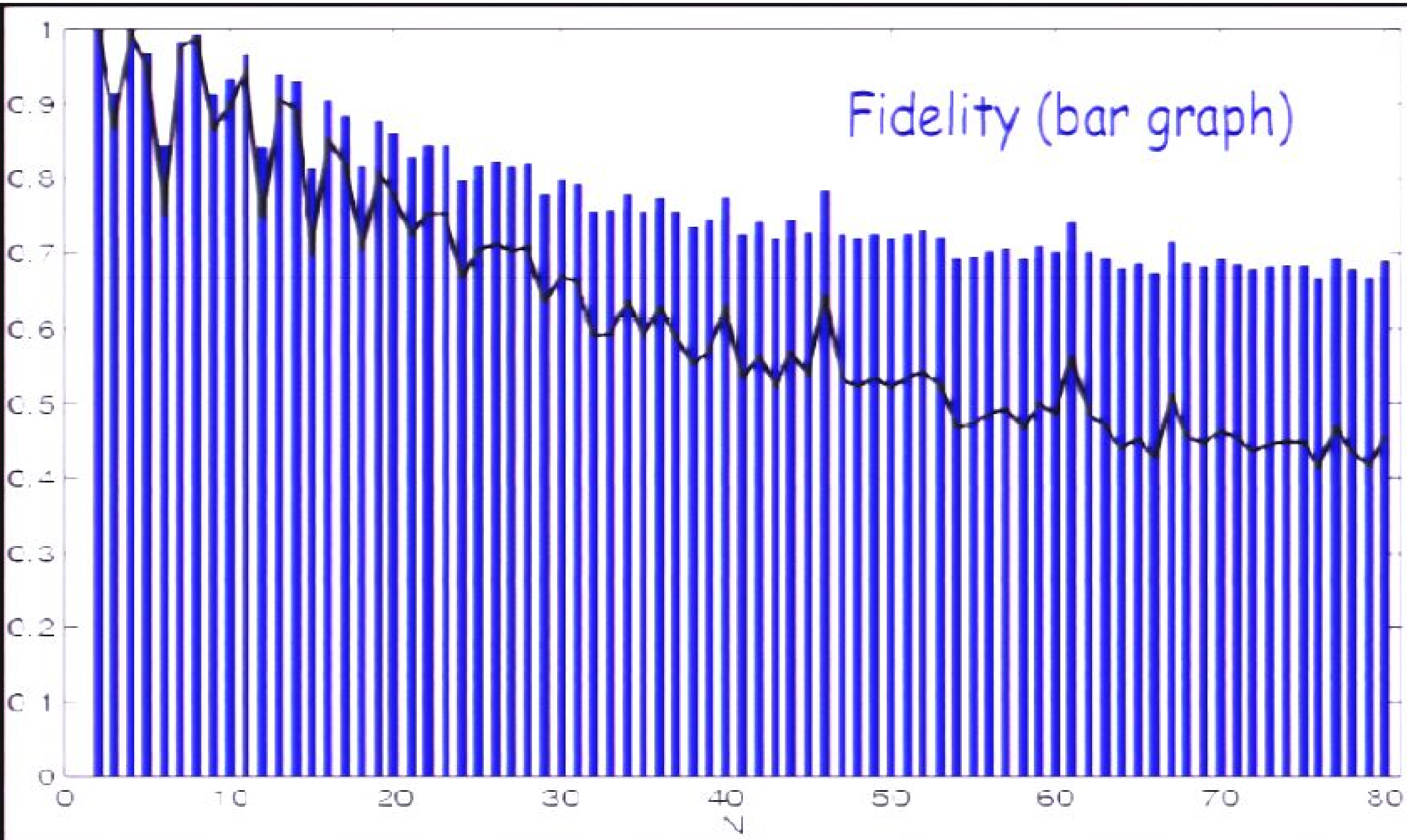

Fidelity:

- function of time and imperfect in general
- near perfect for N=4 & 8

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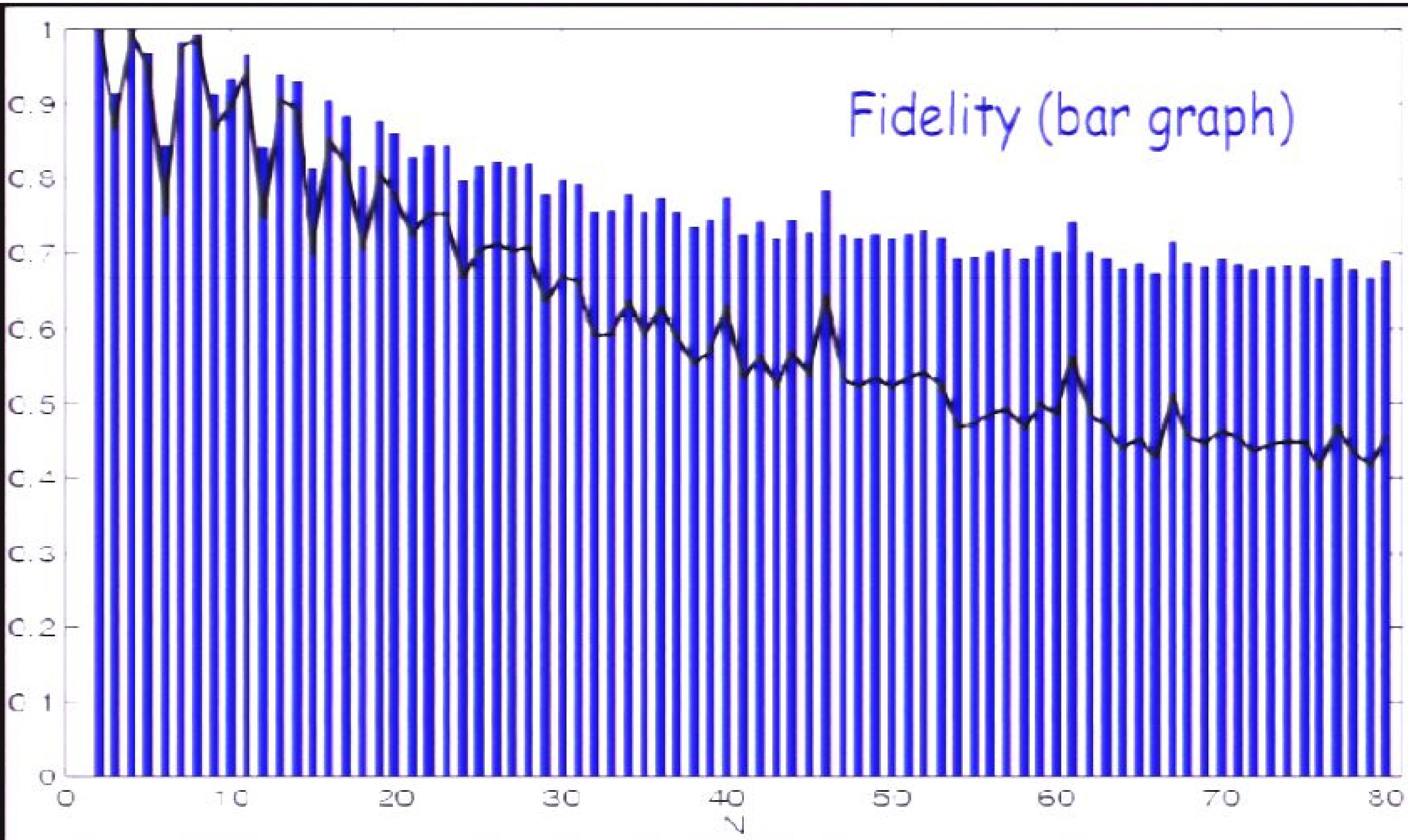
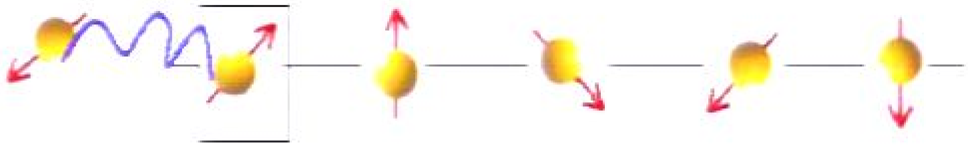


Fidelity:

- function of time and imperfect in general
- near perfect for N=4 & 8

The longer you wait, the better it gets, but no promises!

Entanglement transfer through arbitrarily long chains:



$$E_C \approx f_{N,1}(t)$$

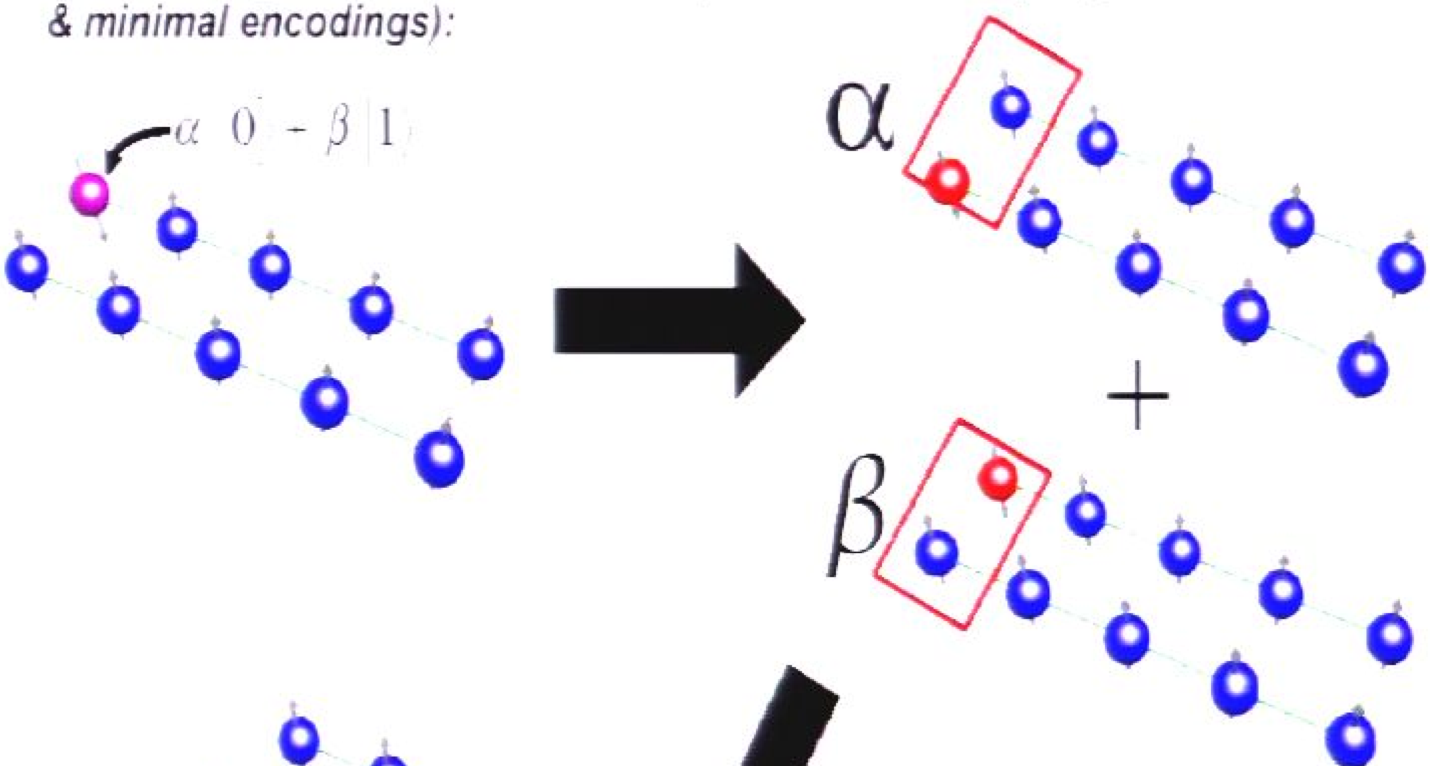
$$\text{At} \quad t \approx O\left(\frac{N}{J}\right), \quad E_C \approx \frac{1}{N^{1/3}}$$

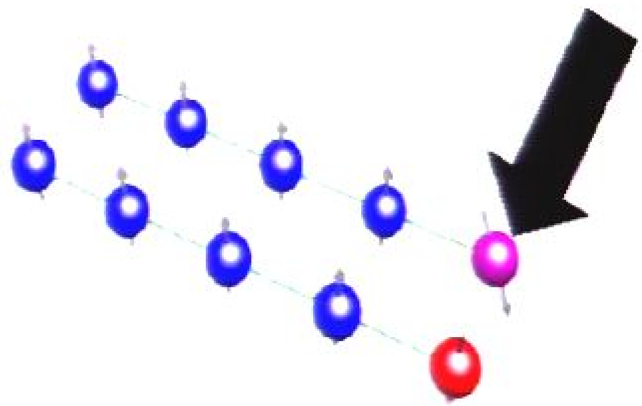
This proves that a spin chain of any length can behave as a quantum channel

Practicality: *Several transmissions followed by entanglement distillation needed even for transmitting one ebit of entanglement !!!*

S.Bose, PRL 91, 207901 (2003).

Conclusively Perfect State Transfer (with uniform couplings & minimal encodings):

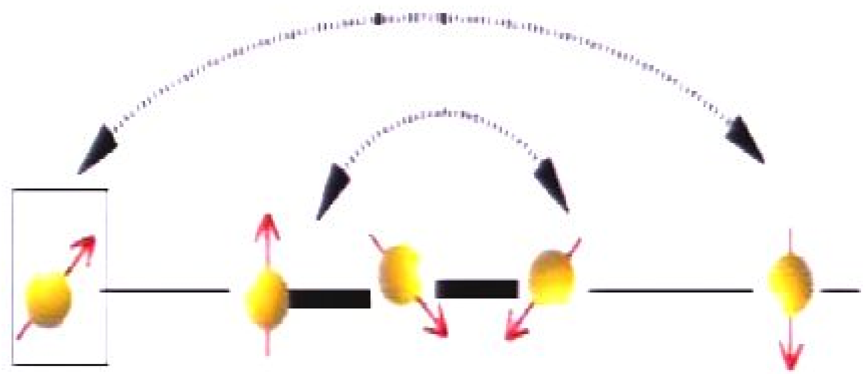




CNOT at receiving end (can be thought of as a parity meas + decoding)

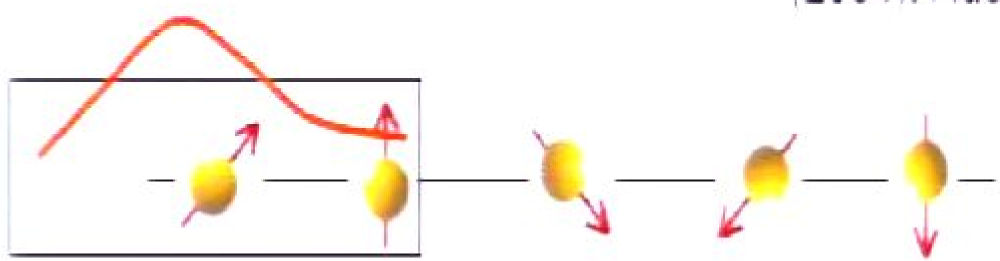
D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

Engineered Couplings: Christandl, Datta, Ekert, Landahl, PRL **92**, 187902 (2004), Albanese, Christandl, Datta, Ekert, PRL **93**, 230502 (2004).



Could we do without engineering?

Encoding a qubit in a large number of spins: Osborne & Linden, PRA **69**, 052315 (2004), Haselgrove, PRA (2005).



Could we do with coding on a lower number of spins?

At least two other broad classes of ideas:

Gapped system (L. *et. al.*, PRA (2005), Plenio & Semiao, NJP (2005), Wojcik, PRA, '05)

Modulated Ising Chain (Raussendorf, PRA (2005), Fitzsimons & Twamley, PRL(2006).)

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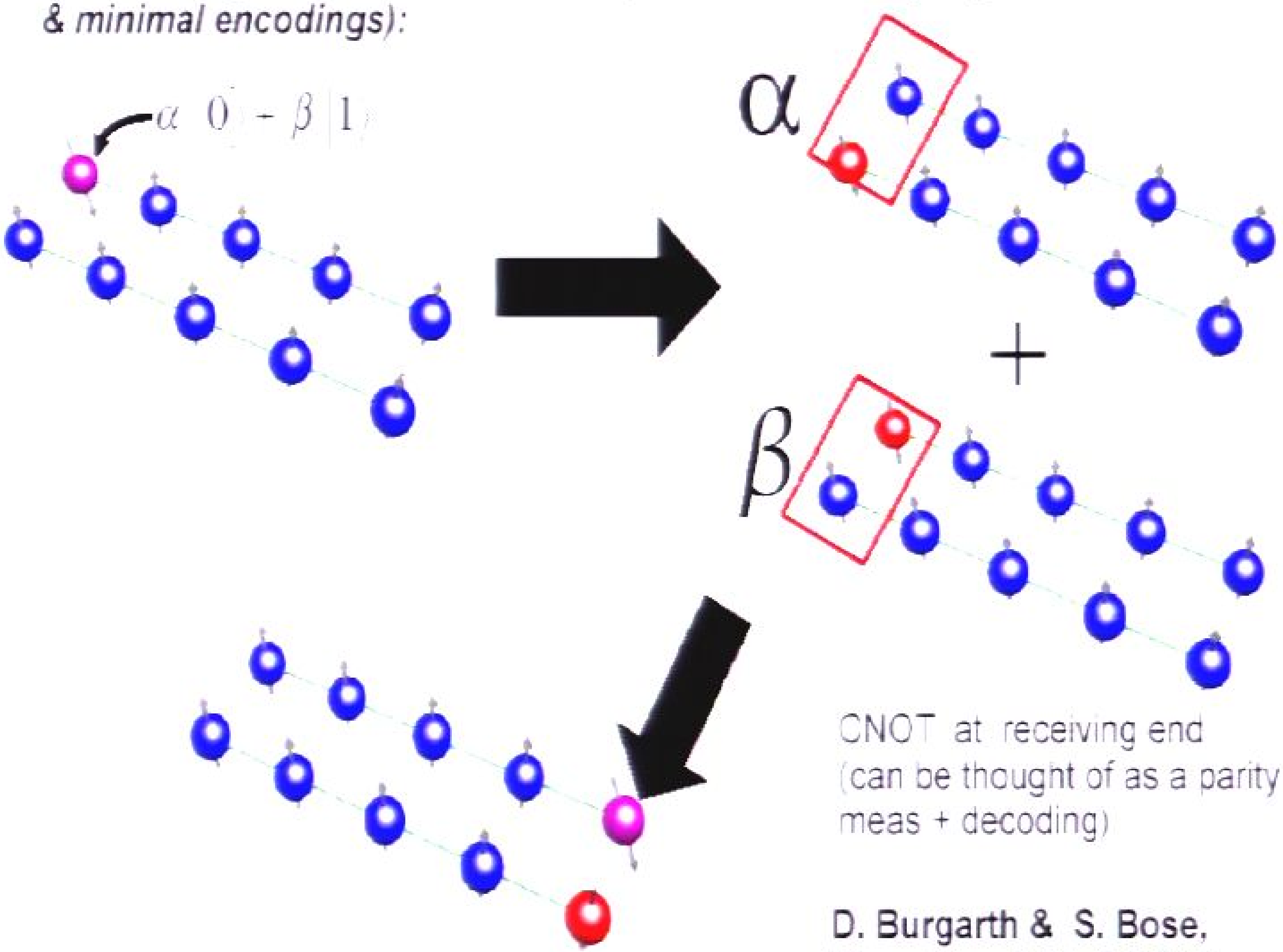
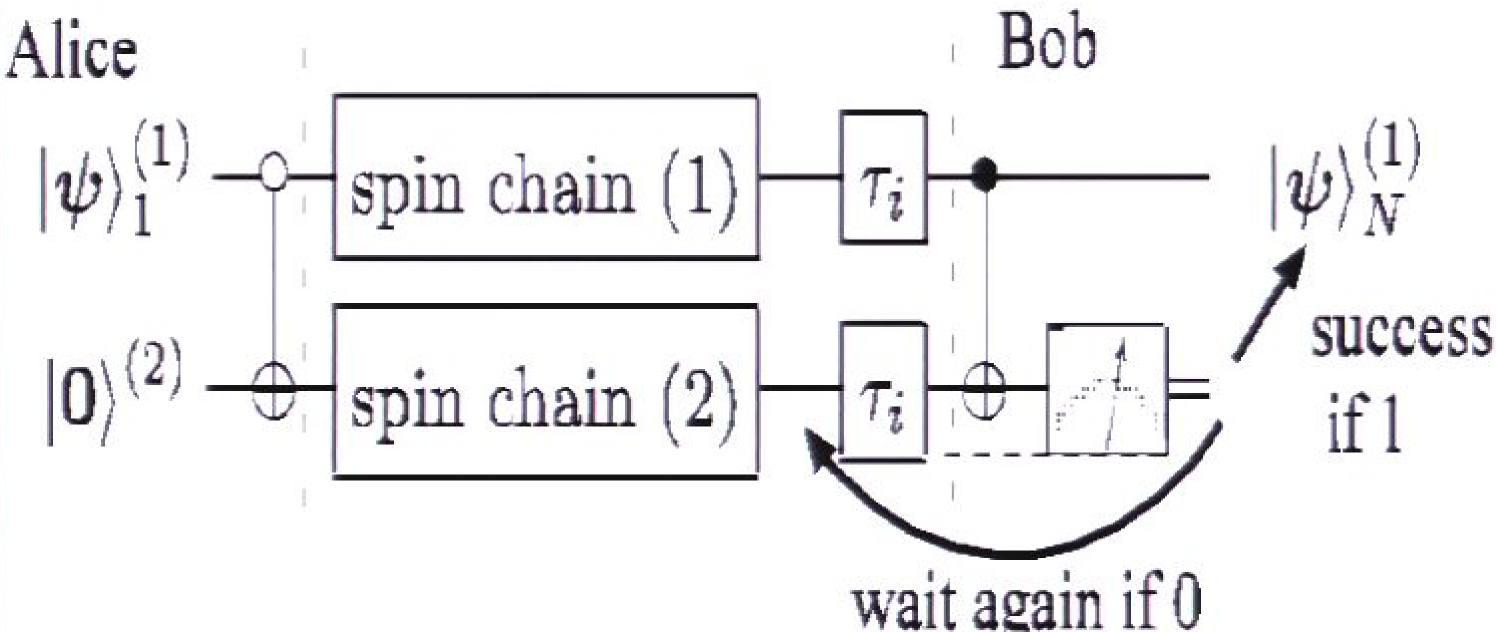


CNOT at receiving end (can be thought of as a parity meas + decoding)

D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

Arbitrarily Perfect State Transfer
Parallel Spin Chains for an amplitude delaying channel!



The initial and final gates are not even needed if the quantum computers being connected are themselves using dual rail encoding (such as double dot systems).

D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

Time-scale from heuristic argument:

Choose first measurement at $\tau_{max} \approx \dfrac{N}{2J}$ so that the maxima of the

first Bessel wave $|f_N(\tau_{max})|^2 \approx \dfrac{1.35}{N^{2/3}}$ reaches the N th site.

Then wait for $O(\tau_{return}) \approx \dfrac{N}{J}$ (for the wave packet to be reflected back from Alice's end and returned again to Bob)

and perform the second measurement, and so on ...

To make the joint probability of failure lower than δ we require time

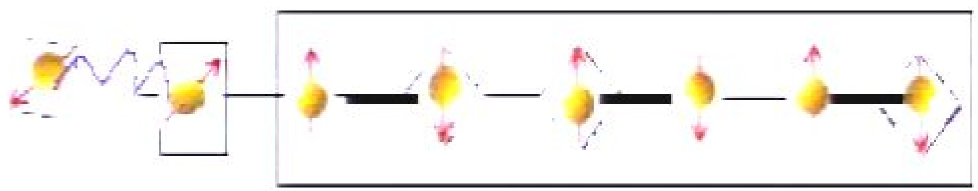
$$\tau(\delta) \approx 0.51 \frac{N^{5/3}}{J} |\ln \delta|$$

The optimized scheme performs better, of course!

Now let us start the channel in a different initial state

Antiferromagnetic (AFM) ground state

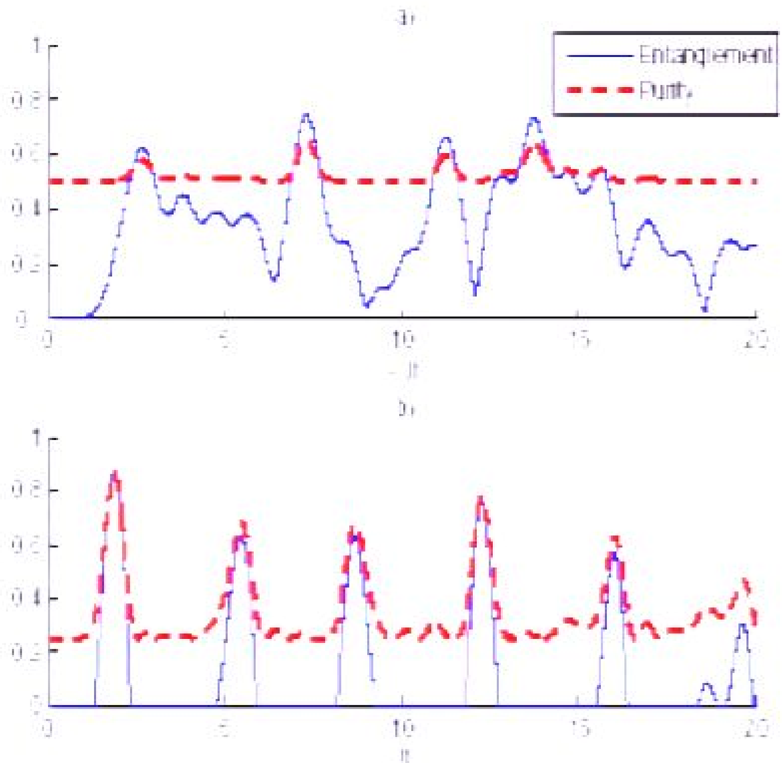
- We append one member of a singlet to one end of an open AFM chain in its ground state.



- Motivation 1: Approx half spins facing oppositely
- Motivation 2: Already entanglement inside
- Motivation 3: Symmetry: Depolarizing channel

Bayat & Bose, arXiv:0706.4176 (2007)

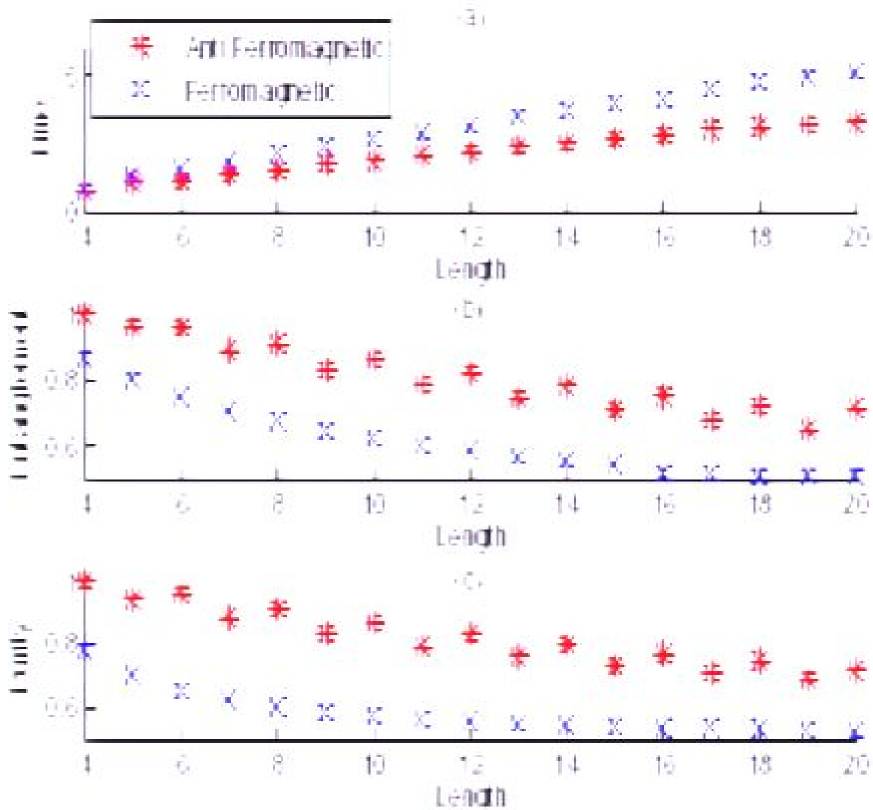
Entanglement through FM vs AFM



- Reason: Depolarizing channel. Werner state

entangled for

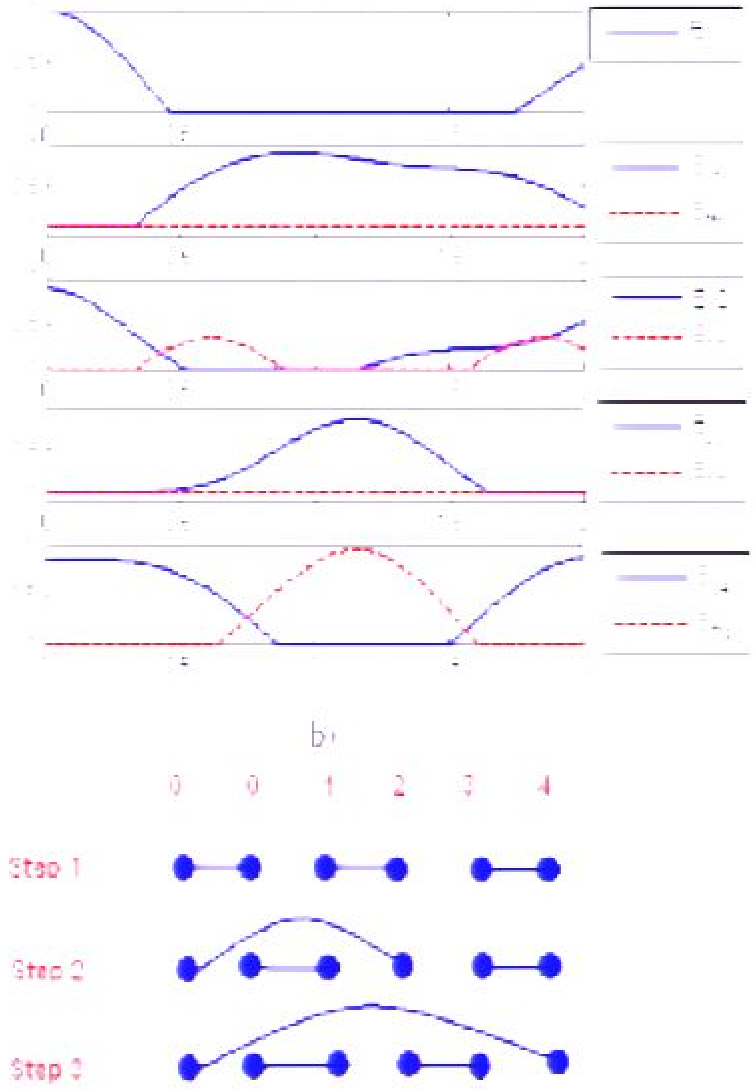
AFM vs FM for various Lengths



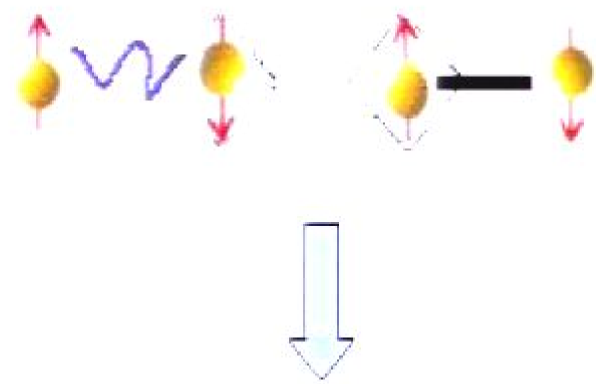
Still important to study imperfect fidelity cases: this is because **quantum error correction can be efficiently used** for spin chains

Allcock & Linden, arXiv 0801.4867v1

Mode of entanglement propagation



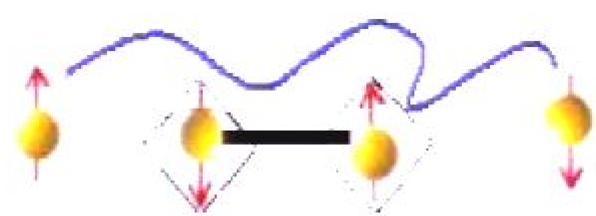
Simplest 2+2 case



Cos 2Jt

+

Sin 2Jt



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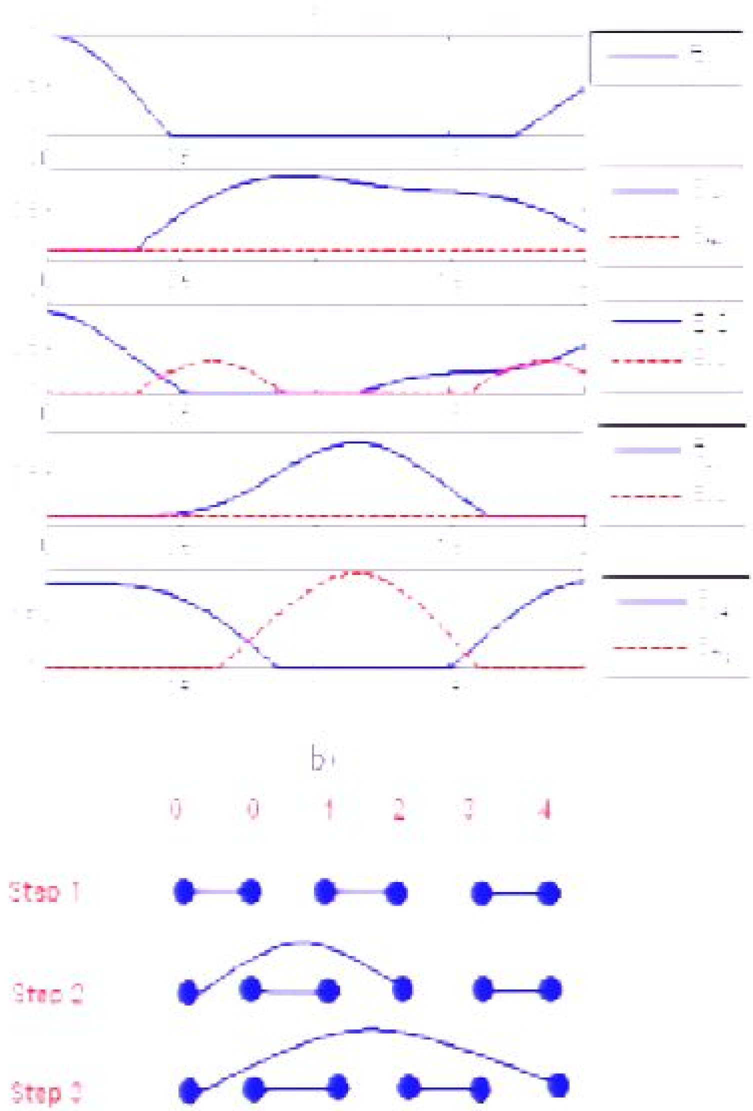


What about State Transfer through Spin Graphs?

Most known results are for the XY model:

$$H = \sum_{(i,j)\in \text{Edges}} \left(X_i \otimes X_j + Y_i \otimes Y_j \right)$$

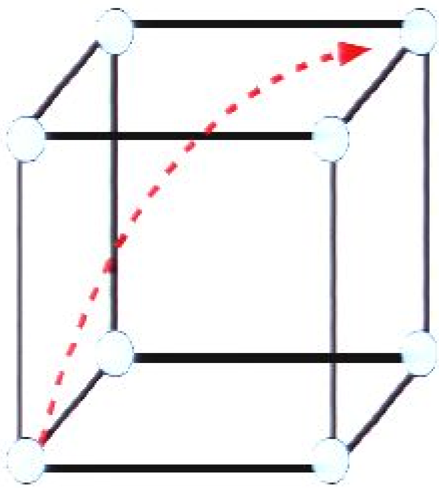
In this case, in the single excitation sector,

$$H = 2A$$

Where A is the adjacency matrix of the graph

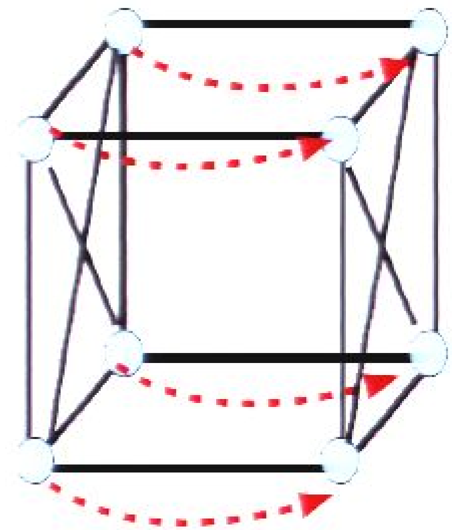
Perfect State Transfer

Hypercubes



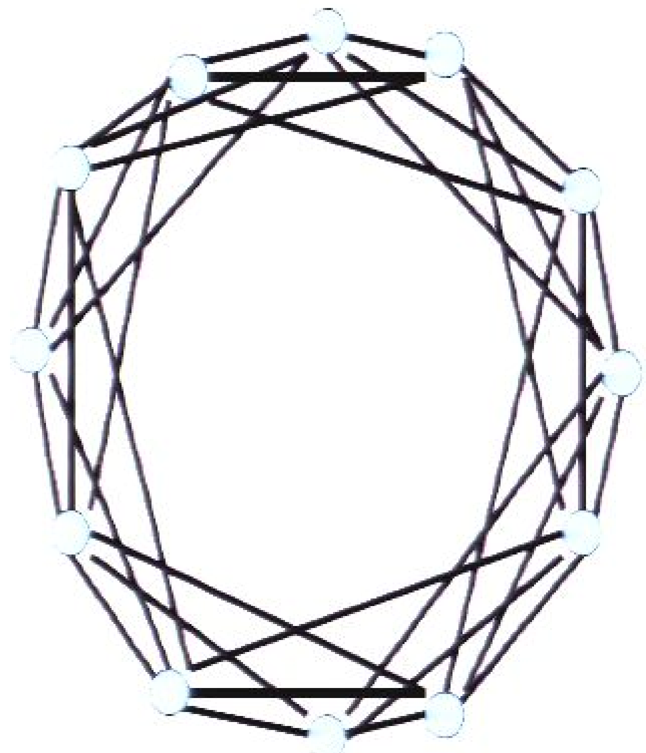
Christandl, Datta, Dorlas, Ekert, Kay, Landahl, PRA **71**, 03212 (2005)

Hypercubes with extra edges



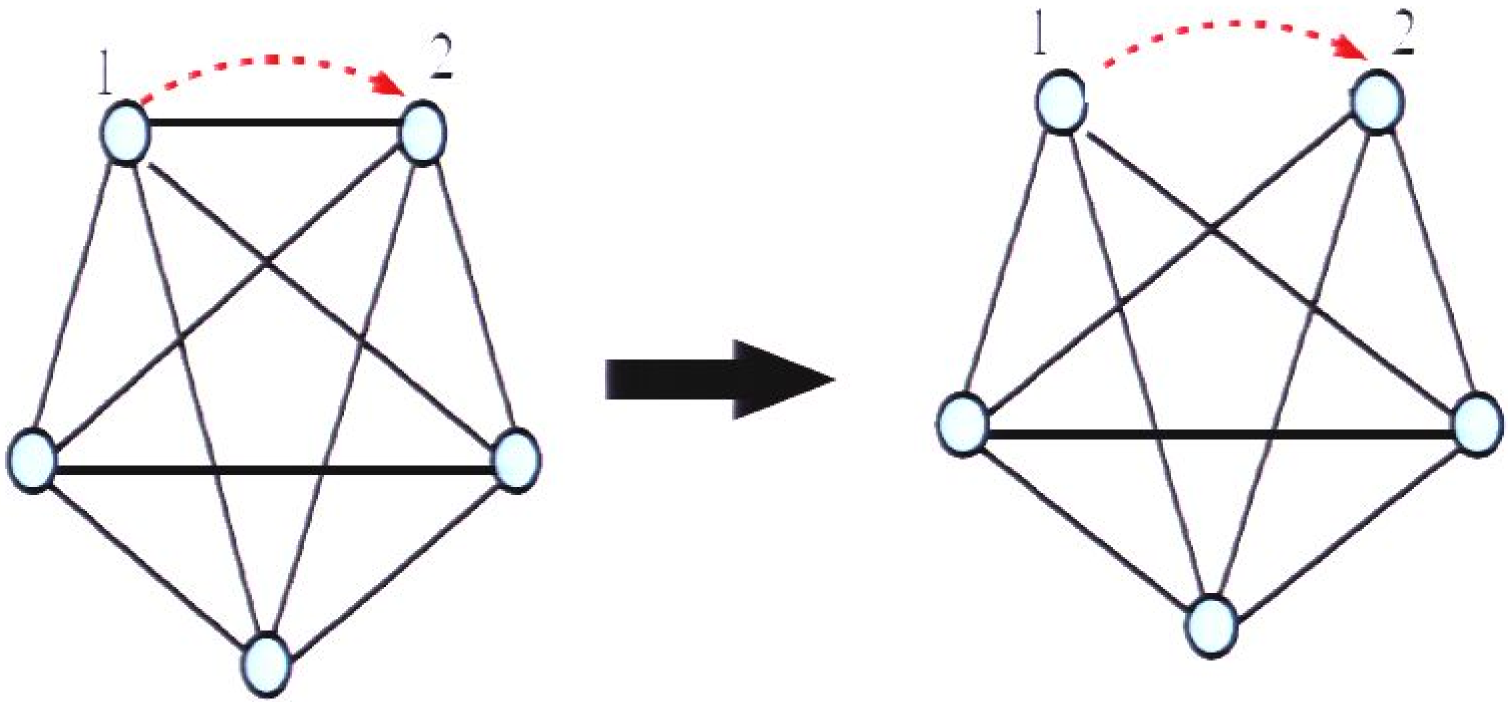
Facer, Twamley and Cresser, Phys. Rev. A 77, 012334 (2008)

Circulant graphs: (no perfect state transfer for odd number of vertices)



Saxena, Severini, Shparlinski, quant-ph 0703236

We consider the deletion of edges starting from a fully connected graph $H = \sum_{<i,j> \in \text{Edges}} \left(X_i \otimes X_j + Y_i \otimes Y_j + Z_i \otimes Z_j \right)$



$\max |f_{12}(t)| = 2/n$

$\max |f_{12}(t)| = 1$ for certain n, high always

With Simone Severini, Stefano Mancini & Andrea Casaccino

Connection between graph theory and state transfer when the couplings are "isotropic Heisenberg"

$$H = \sum_{\langle i,j\rangle \in \text{Edges}} \left(X_i \otimes X_j - Y_i \otimes Y_j - Z_i \otimes Z_j \right)$$

In the single excitation sector, for a degree matrix D

$$H = 2(A - D) = \mathbf{L}$$

Where $\mathbf{L}$ is the *Laplacian* of the graph

State transfer is different for XY and isotropic Heisenberg for *non-symmetric* graphs

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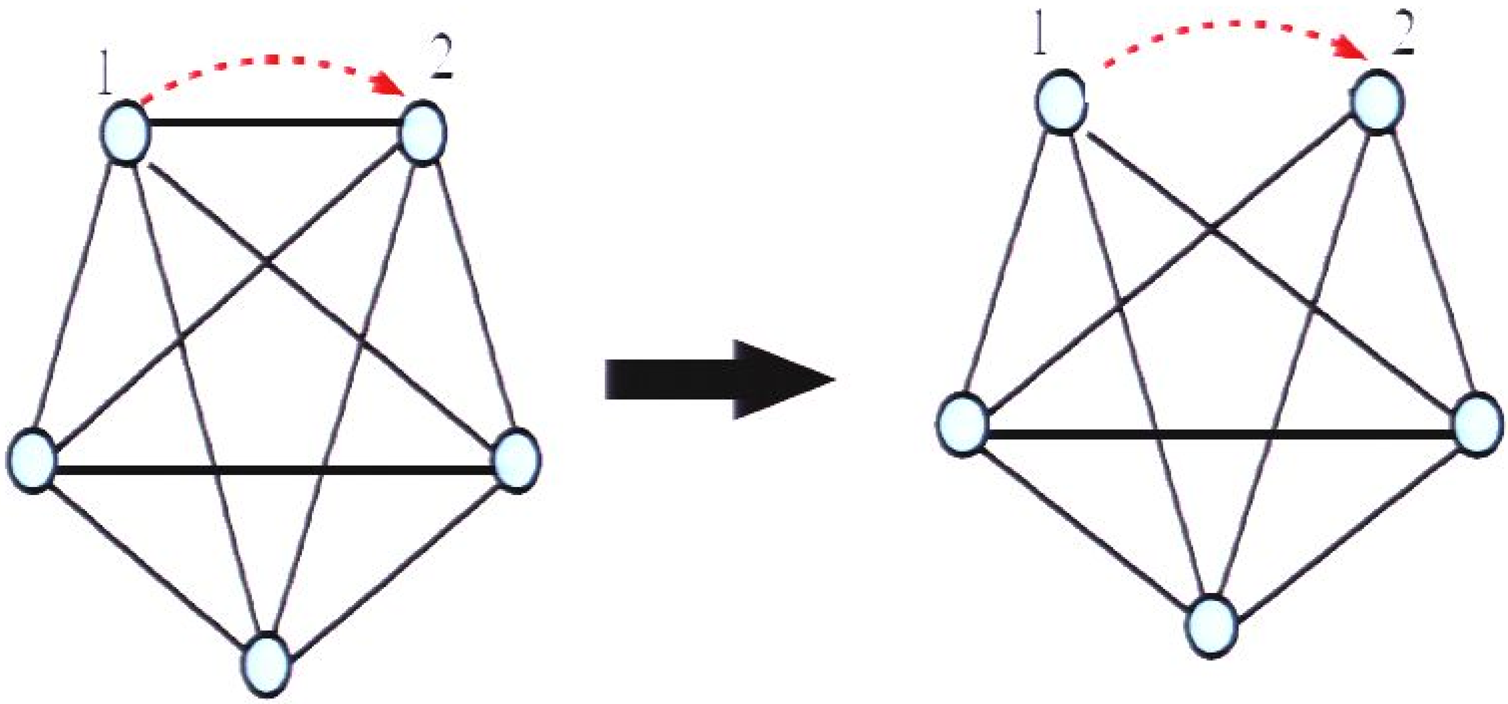

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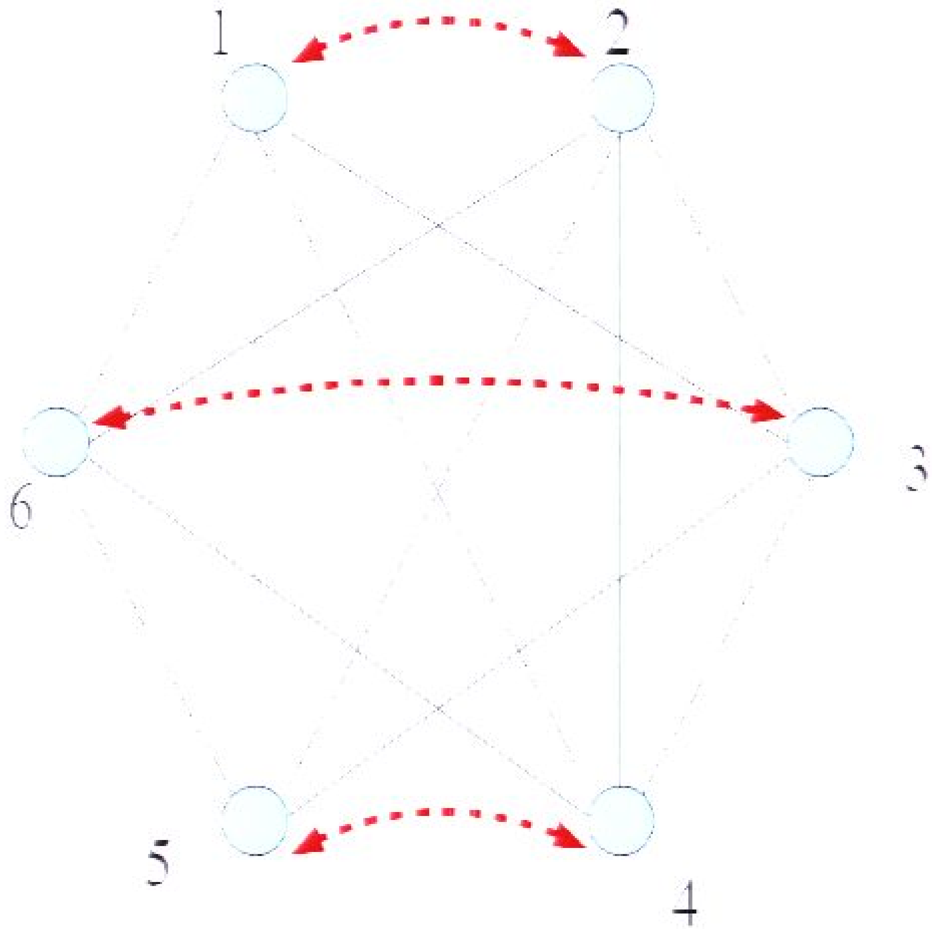

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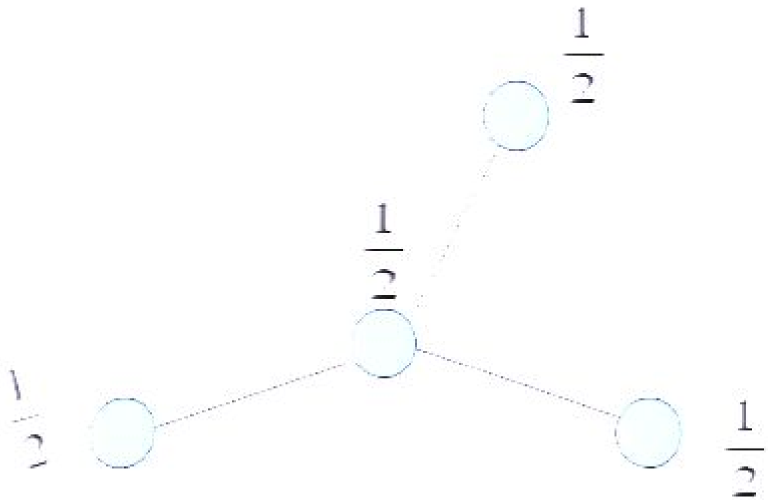
With Simone Severini, Stefano Mancini & Andrea Casaccino

Now we start speaking about entanglement in ground states of spin graphs:

Spin Star:



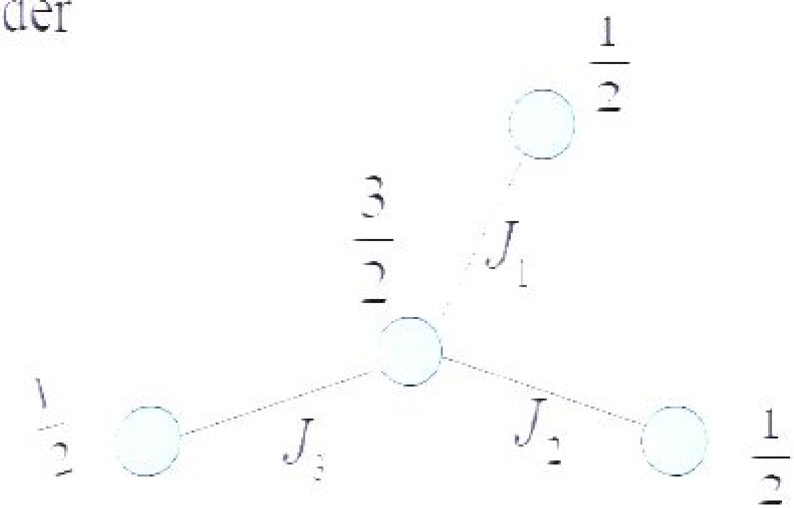
$$|0\rangle = |\uparrow\rangle,\ |1\rangle = |\downarrow\rangle$$

$$\frac{1}{\sqrt{2}}\left[|0\rangle\frac{1}{\sqrt{3}}(|011\rangle + |110\rangle + |101\rangle) - |1\rangle\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)\right]$$

You can project the outer spins to a W state by measuring the central spin

A. Hutton & S.Bose, Phys. Rev. A 69, 042312 (2004);

Now consider



$$\sqrt{6}\left(|\Uparrow\rangle|\downarrow\downarrow\downarrow\rangle + |\Downarrow\rangle|\uparrow\uparrow\uparrow\rangle\right) - \sqrt{2}\Big[|\Uparrow\rangle\{|\uparrow\downarrow\downarrow\rangle - |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle\} -$$
$$+ |\Downarrow\rangle\{|\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle\}\Big]$$

Now you can produce a GHZ of the outer spins (with a certain probability of success) as well by measuring the central spin

With *Vladimir Korepin*

Similar for larger stars – enhance central spin

If one generalized the interactions, then one could as easily write down the ground states of any graph:

$$H = \sum_{j,k} \prod_{s_j + s_k} (j,k)$$

where $\prod_{s_j + s_k} (j,k)$ is the projection of the spins at site j and k to the maximum value

Generalization of AKLT (1987,88) to arbitrary graphs by Kirillov & Korepin (1989).

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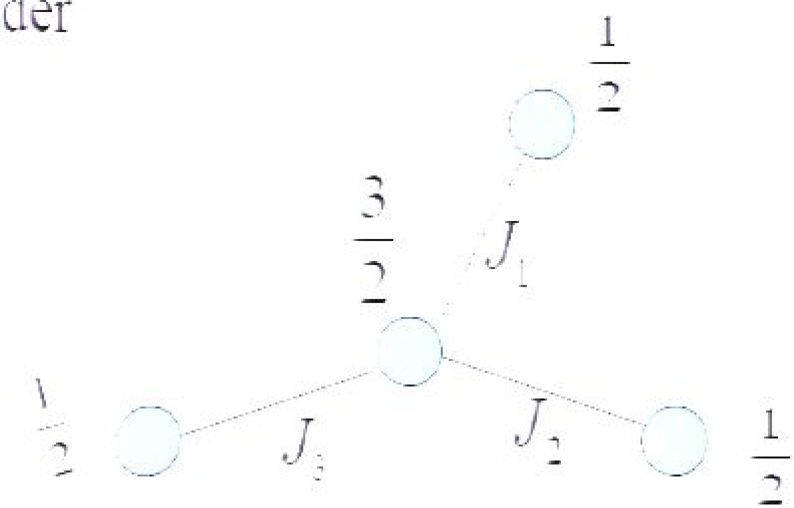

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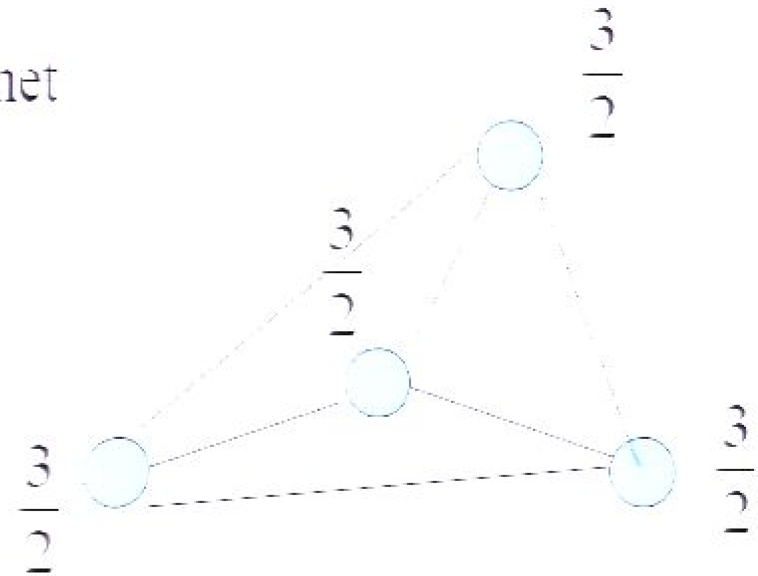
Quantum Internet



$$
\left(|\uparrow\rangle_1|\Downarrow\rangle_2 - |\Downarrow\rangle_1|\Uparrow\rangle_2\right)\left(|\uparrow\rangle_3|\downarrow\rangle_4 - |\downarrow\rangle_3|\uparrow\rangle_4\right)
$$

$$
+\left(|\uparrow\rangle_1|\Downarrow\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_3\right)\left(|\uparrow\rangle_4|\downarrow\rangle_2 - |\downarrow\rangle_4|\uparrow\rangle_2\right)
$$

$$
+\left(|\uparrow\rangle_3|\downarrow\rangle_2 - |\Downarrow\rangle_3|\Uparrow\rangle_2\right)\left(|\uparrow\rangle_4|\downarrow\rangle_1 - |\downarrow\rangle_1|\uparrow\rangle_4\right)
$$

$$
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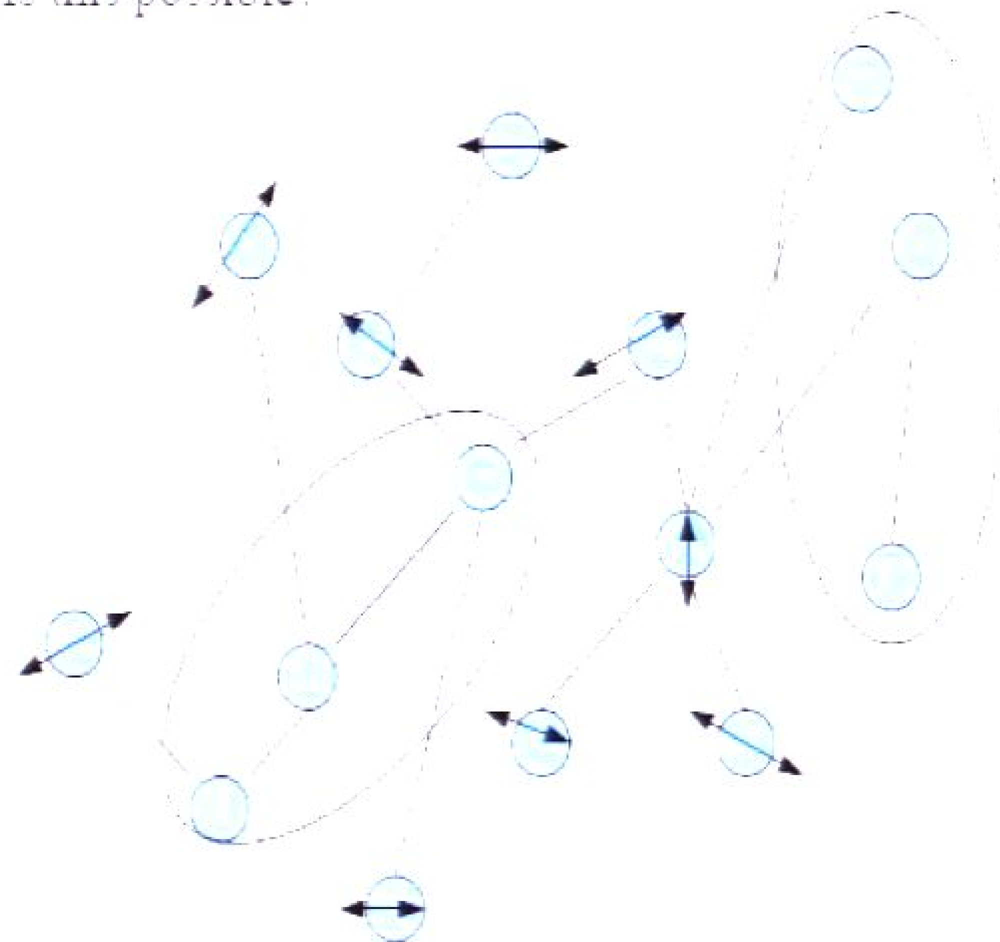
$$

$$
+\left(|\uparrow\rangle_4|\downarrow\rangle_2 - |\downarrow\rangle_4|\uparrow\rangle_2\right)\left(|\uparrow\rangle_1|\downarrow\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_3\right)
$$

$$
+\left(|\uparrow\rangle_4|\downarrow\rangle_1 - |\downarrow\rangle_4|\Uparrow\rangle_1\right)\left(|\uparrow\rangle_3|\downarrow\rangle_2 - |\downarrow\rangle_3|\uparrow\rangle_2\right)
$$

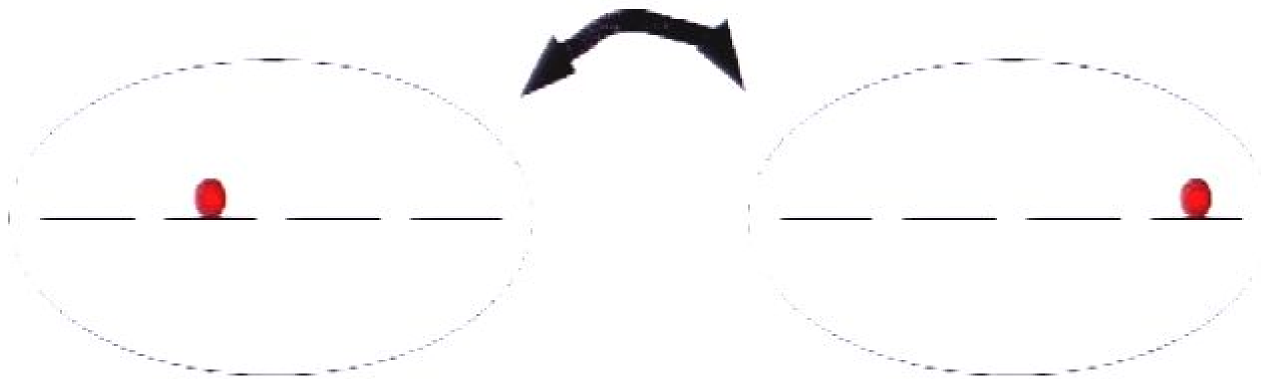
With Korepin

Is this possible?



Yes indeed, if you have qudits with $d=N$

If interactions are of the following form:



$$P_{ij}|\psi\rangle_i|\phi\rangle_j = |\phi\rangle_i|\psi\rangle_j$$

Hubbard model with strong repulsion

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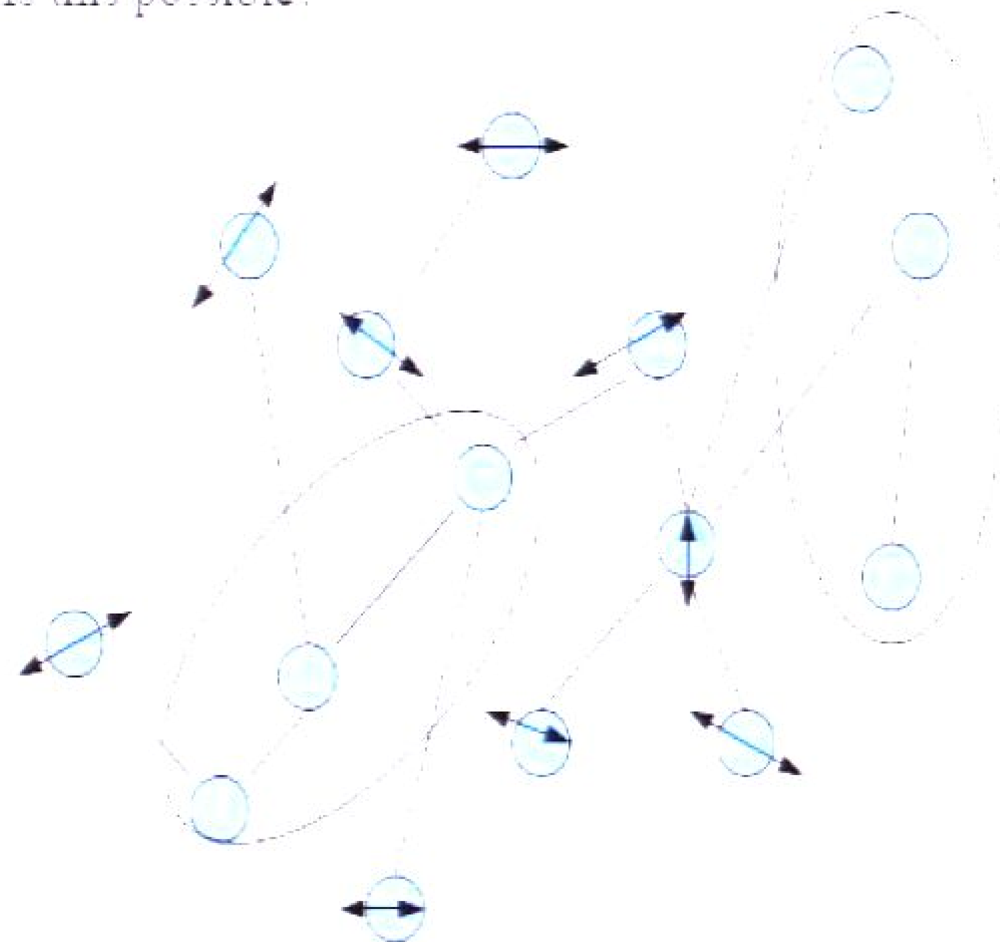

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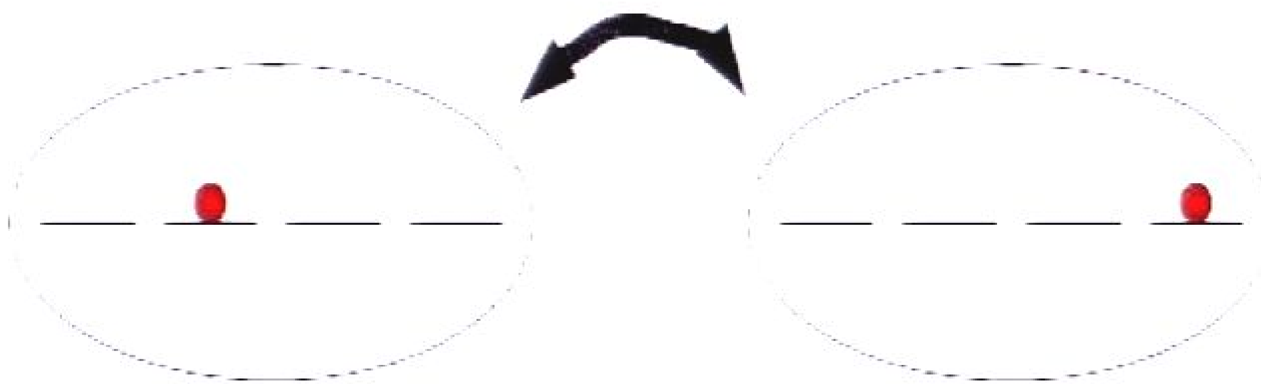

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Hubbard model with strong repulsion

$$H = \sum_{ij} P_{ij}$$

Note that

- Eigenvalues of $P_{ij} = \pm 1$
- A state with $P_{ij} = -1$ for all connected i and j implies that $P_{kl} = -1$ for any pair of vertices of the graph

$$|S_3\rangle = \frac{1}{\sqrt{3!}}\big(|123\rangle - |132\rangle + |312\rangle - |321\rangle + |231\rangle - |213\rangle\big)$$

C. Hadley & S. Bose, PRA (to appear, rapid comm), 2008

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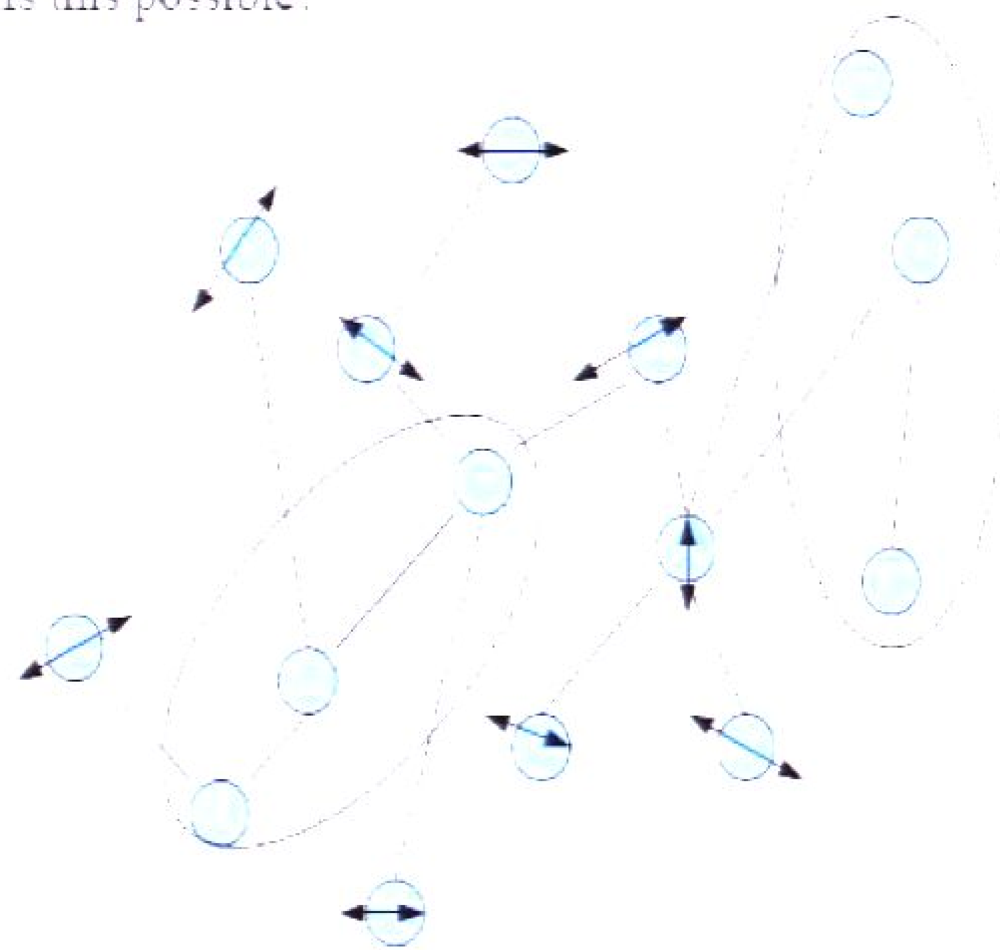

Thank You

Acknowledgements: EPSRC, QIPIRC,
Royal Society, Wolfson Foundation

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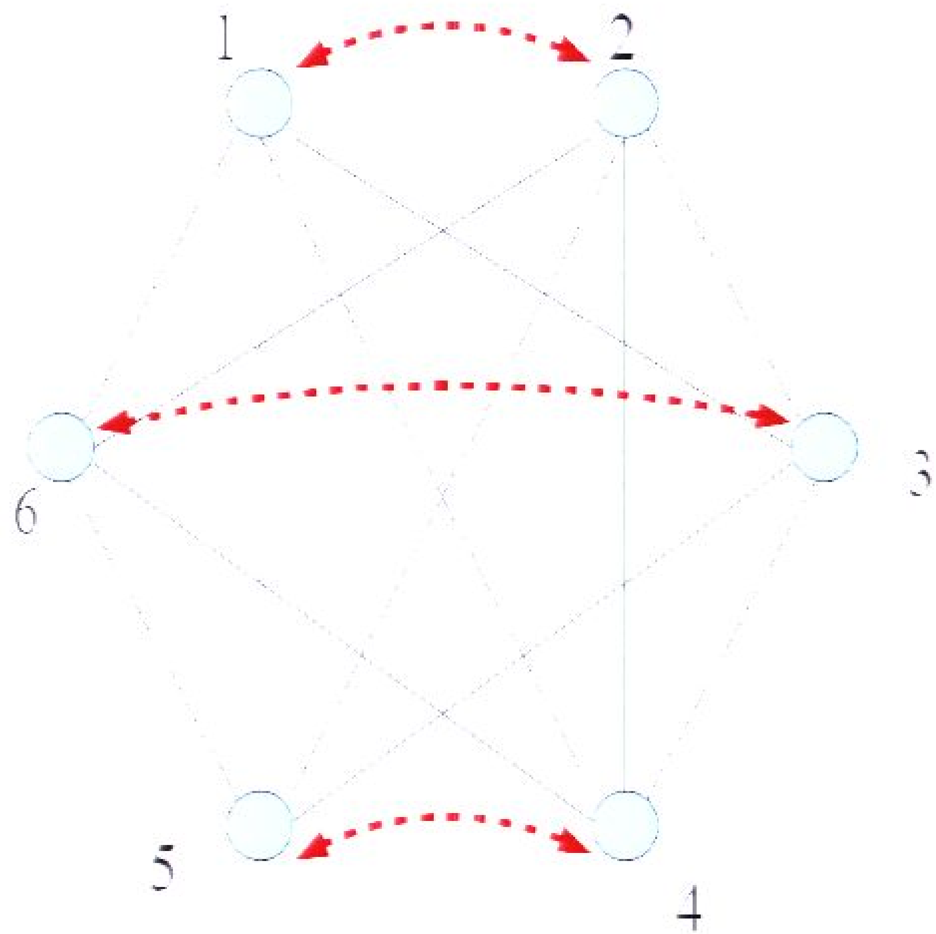

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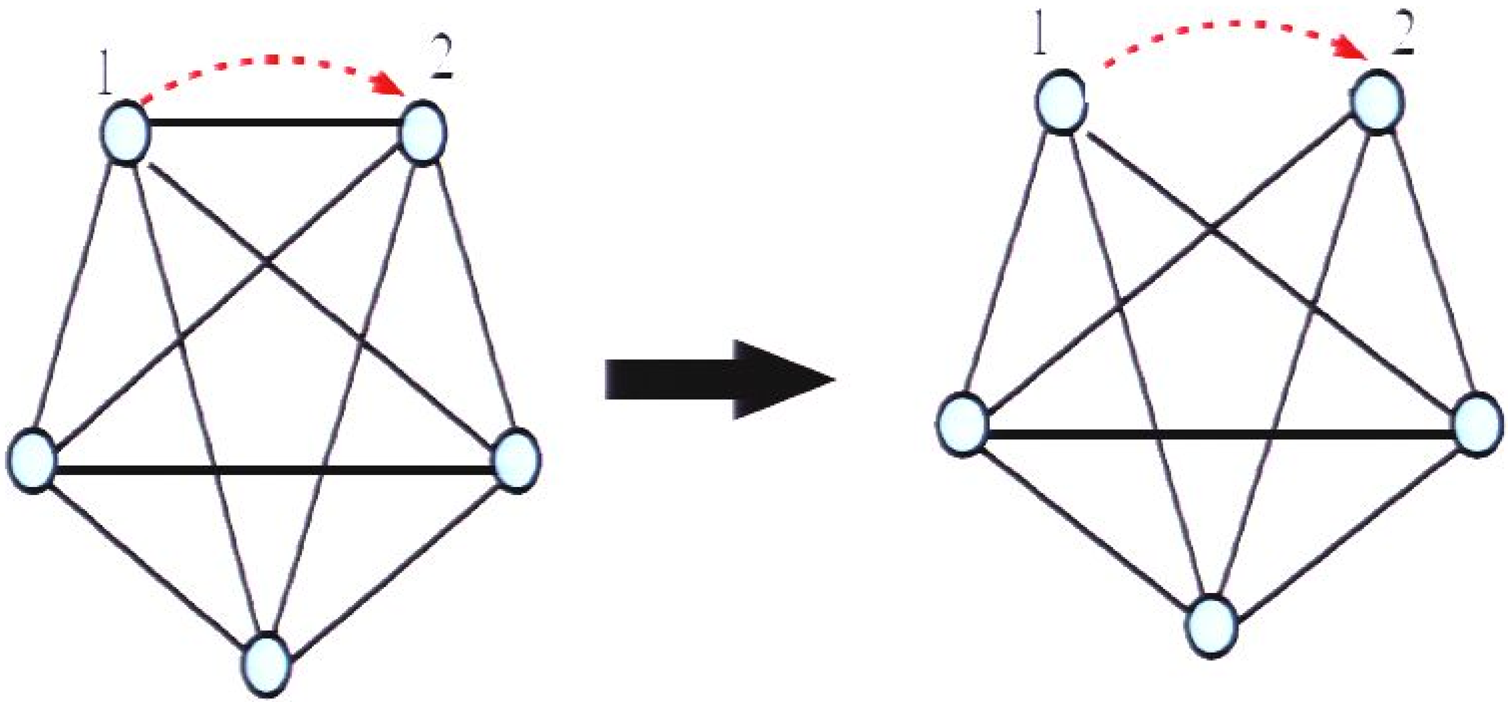

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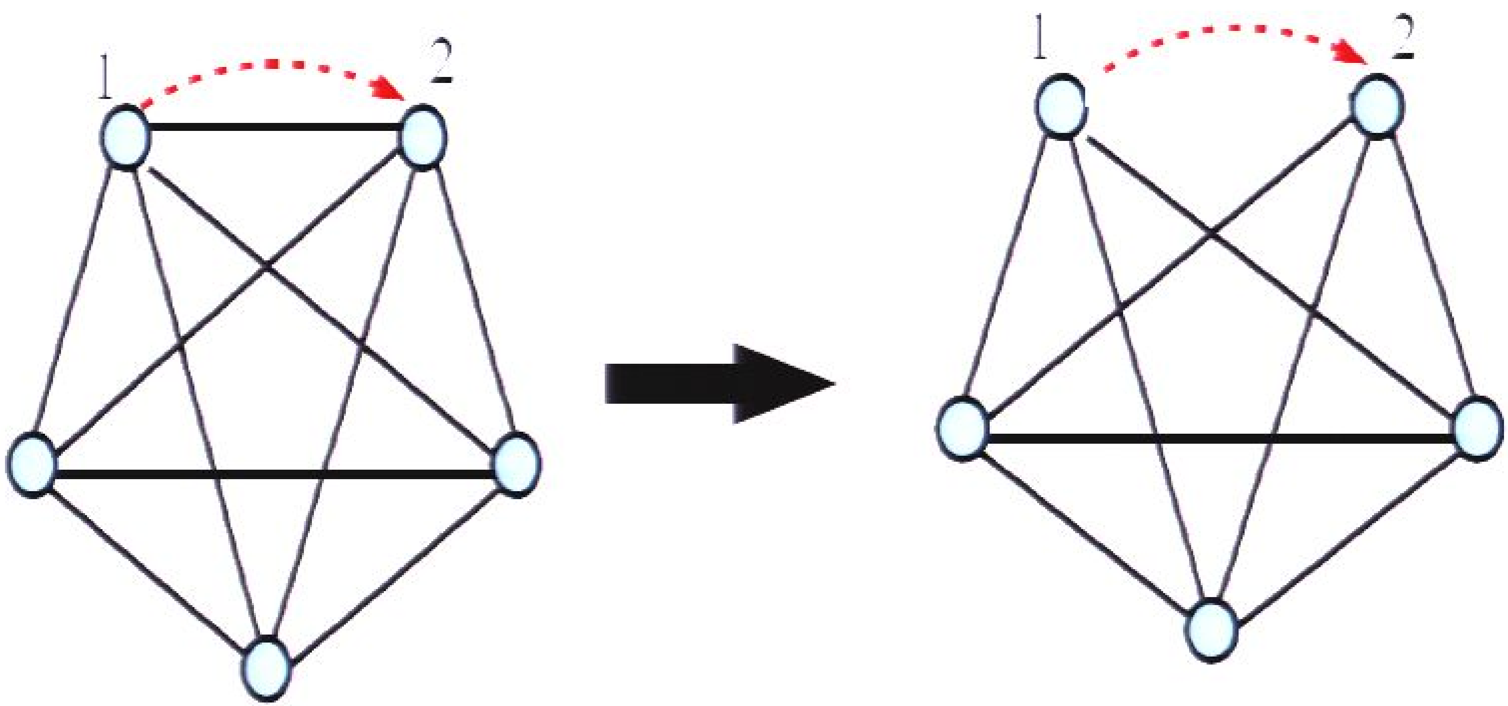

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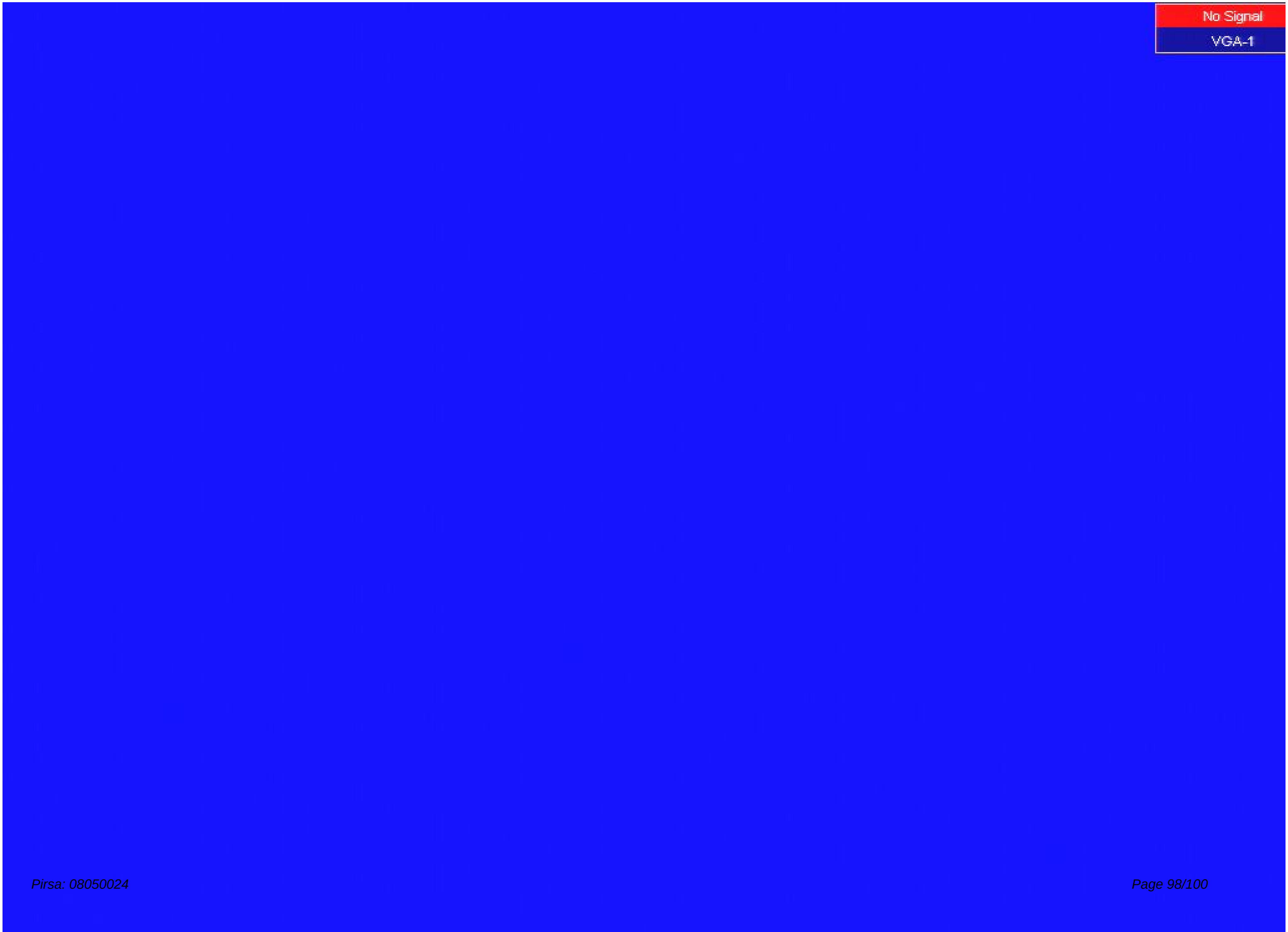
No Signal
VGA-1

No Signal
VGA-1

No Signal

VGA-1