Title: Could be the quark masses dynamically generated within massless QCD?

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Abstract: Explorations of the possibility that the quark masses, and more generally the particle mass spectra, could be dynamically generated in the context of massless QCD will be presented. The basic idea is that the large degeneracy of the free massless QCD could lead to a large quark condensate and its corresponding mass. Under the presence of this very massive quark, the other five ones could acquire smaller masses as argued by Fritzsch in his Democratic Symmetry Breaking scheme. Further, the lepton and neutrinos could get their masses through their interaction with quarks mediated by radiative corrections. In this case the stronger electromagnetic corrections could imply the larger lepton masses with respect to the neutrino ones, since these are only weakly interacting with the quarks.

Could be the quark masses dynamically generated within massless QCD?

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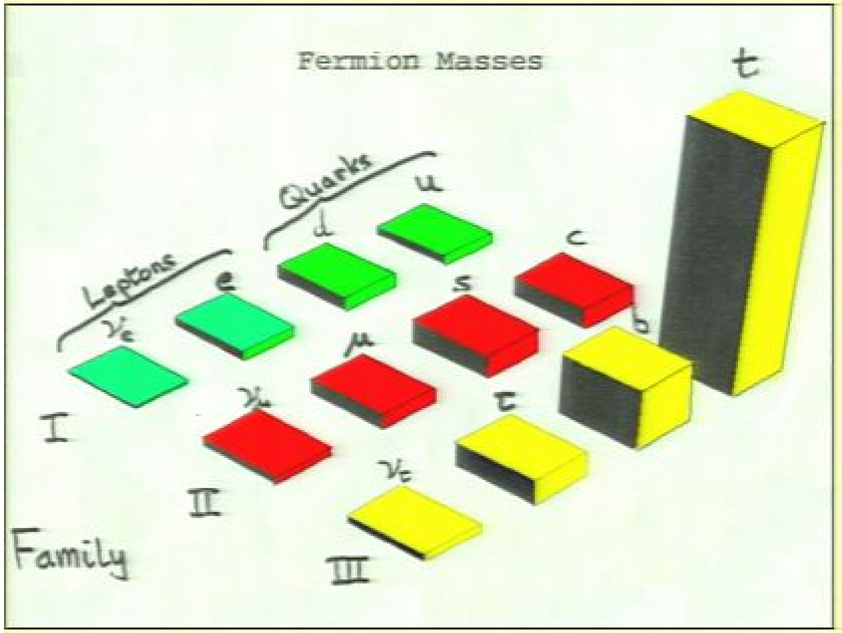
Explorations of the possibility that the quark masses, and more generally the particle mass spectra, could be dynamically generated in the context of massless QCD are presented. The basic idea is that the large degeneracy of the free massless QCD could lead to a large quark condensate and its corresponding mass. Under the presence of this very massive quark, the other five ones could acquire smaller masses as argued by Fritzsch in his Democratic Symmetry Breaking scheme. Further, the lepton and neutrinos could get their masses through their interaction with quarks mediated by radiative corrections. In this case the stronger electromagnetic corrections could imply the larger lepton masses with respect to the neutrino ones, since these are only weakly interacting with the quarks.

Introduction



Determining the causes of the wide range of values spanned by the **quark masses**, and more generally, the structure of the **lepton and quark mass spectrum** (illustrated in the figure on the left, but not at a correct scale) is one of the central problems of **High Energy Physics**.

For very high energy collisions the usual perturbative expansion for **QCD (PQCD)** produces good experimental predictions, thanks to the asymptotic freedom effect. However, the limitations of this standard Feynman diagram expansion in describing the low energy properties, by example in the Nuclear range, are recognized.

ı former works (**Mod. Phys. Lett. A10, 2413 (1995), Phys. Rev. D 62 074018 (2000), Eur. Phys. J. :23, 289 (2002), JHEP (04), 044 (2003), Eur. Phys. J. C47, 95 (2006), Eur. Phys. J. C47, 355 2006), Eur. Phys. J. C** to appear in 2008), we have been investigating the formulation and nplications of a modified version of the **PQCD**.

. general motivatation in the study of this problem is the possibility that the strong degeneration of ıe non-interacting vacuum (the state which is employed for the construction of the standard 'eynman rules of **PQCD**) could allow for modified rules being able to furnish useful non-erturbative results. The expectation is that the scheme could show similar merits as the so called 3ogoliubov shift" procedure in scalar field theories, which gives helpful information about non-erturbative effects in Bose condensation. However, historically the first motivation was the aim in eveloping a sort of improved "Savvidi Chromomagentic field model" having not the known ymmetry difficulties, which affected this helpful scheme. It was one the first models indicating the xistence of confinement in QCD.

$$|\phi\rangle = \exp \sum_a [C_1(p) A^{a+}_{p,1} A^{a+}_{p,1} + C_2(p) A^{a+}_{p,2} A^{a+}_{p,2} + C_3(p) \times (B^{a+}_p A^{L,a+}_p + i \bar{c}^{a+}_p c^{a+}_p)] |0\rangle,$$

The **BCS** like initial state for the derivation of the modified Feynman rules for the case of gluon condensation in the absence of quark pair condensation. (Phys. Rev. D 62 074018 (2000)

$$G^{ab}_{\mu\nu}(p) = \left(\frac{1}{p^2 + i\epsilon} - i\delta(p) C \right) \delta^{ab} g^{\mu\nu}$$

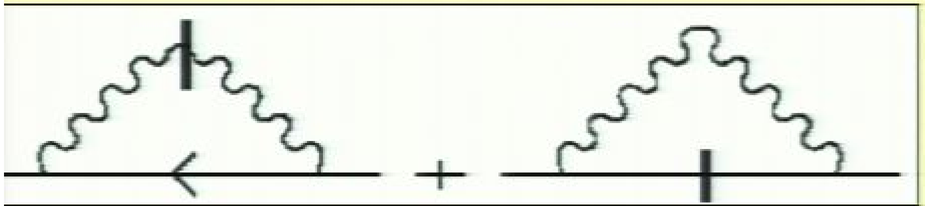
The originally proposed modified gluon propagator reflecting the condensation of zero momentum gluons, was reproduced after chosing appropriate values for the parameters in the initial **BCS** like state. (Mod. Phys. Lett. A10, 2413 (1995)),

$$|\phi^*\rangle = \lim_{p \to 0} \exp \left(\sum_{f_1 f_2} \tilde{C}_q^{f_1 f_2}(p) \overline{q}^{+}_{f_1}(p) q^{+}_{f_2}(p) \,|\, \phi \rangle \right)$$

The results for gluons motivated the idea of also considering the quarks as massless and search for the possibility of dynamically generate their masses, thanks to the condensation of quark pairs. For this purpose the **BCS** like initial state was generalized to include the quark pair condensates in massless QCD. (JHEP (04), 044 (2003))

$$G_q^{i_1 i_2; f_1 f_2}(p) = \left(-\frac{\gamma^\mu p_\mu \delta^{f_1 f_2}}{p^2 + i\epsilon} + i\delta(p) C^{f_1 f_2} \right) \delta^{i_1 i_2}$$

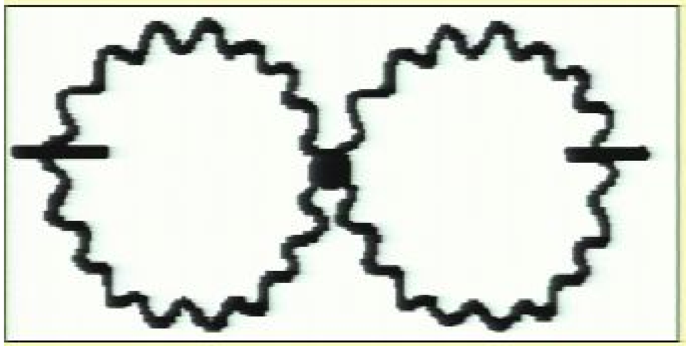
In this case a covariant **free-quark propagator** can be obtained in the form.



The quark self-energy in the lowest order in the power expansion in the condensate parameters

$$\frac{1}{p^2}\left(-p^2 p_\mu \gamma^\mu \left(1-\frac{M^2}{p^2}\right)\delta^{f_1 f_2}-\frac{4g^2 C_F}{(2\pi)^4}\,C^{f_1 f_2}\right)\Psi_i^{f_2}(p)=0\,.$$

Dyson equation for quarks with the above self-energies



$$\langle G^2\rangle = \frac{288 g^2 C^2}{(2\pi)^8},$$

The gluonic Lagrangian mean value in the simplest approximation

$$\langle g^2 G^2\rangle \cong 0.5\left(\mathrm{GeV}/c^2\right)^4$$

Estimated gluon condensate

$$g^2 C = 64.9394\left(\mathrm{GeV}/c^2\right)^2$$

A relation for the parameter ***C***

Quark q	$m_{qLow}^{Exp}(MeV)$	$m_{qUp}^{Exp}(MeV)$	$m_q^{Theo}(MeV)$
u	1.5	5	333
d	3	9	333
s	60	170	339–326–
c	1100	1400	1255
b	4100	4400	4233
[illegible]	168600	17900	173500

1) Disregarding the gluon condensate, the quark condensate matrix can be fixed in a diagonal form for producing the observable Lagrangian quark masses for the solutions of the Dyson equation.
2) After that, the solution of the same Dyson equation including the value of ***C*** furnished again the "constituent" values of 1/3 of the nucleon mass for the light quarks obtained before in Ref. Eur. Phys. J C23, 289 (2002).

The former results showed that the quark mass spectrum can be fixed in the first approximation of the proposed scheme, assumed the existence of the quark and gluon condensates. Therefore, the question arise about the possibility of their dynamical generation in the massless QCD. This research basically attempts to study this issue.

Elimination of the singularities of the modified expansion through dimensional regularization

The presence of the Dirac's Delta functions in the free propagators lead to singulaties in the Feynman dyagramas. One type of them is the appearance of factors in the form

$$\delta(0) = \int_E \frac{dp^D}{(2\pi)^D}.$$

Dirac's Delta at zero momentum

$$\begin{aligned} \int_E \frac{dp^D}{(2\pi)^D} &= \int_E \frac{dp^D}{(2\pi)^D} \frac{p^2}{p^2} \\ &= \int_0^\infty ds \int_E \frac{dp^D\ p^2}{(2\pi)^D} \exp(-s\ p^2) \end{aligned}$$

Representing one of the squared momenta as an integral

$$\begin{aligned} \int_E \frac{dp^D}{(2\pi)^D} \exp(-s\ p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \exp(-s\ f(\frac{D}{2})), \\ \int_E \frac{dp^D\ p^2}{(2\pi)^D} \exp(-s\ p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}}\ f(\frac{D}{2})\right] \exp(-s\ f(\frac{D}{2})). \end{aligned}$$

The dimensionally generalizaed Gaussian integrals furnish the representation:

$$\delta(0) = \int_0^\infty ds \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}}\ f(\frac{D}{2})\right] \exp(-s\ f(\frac{D}{2}))$$

$$\Gamma(z) = \int_0^\infty dt\ t^{z-1} \exp(-t),$$

Integral representation of the Gamma function.

$$\delta(0) = \frac{f(\frac{D}{2})^D}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} \Gamma(-\frac{D}{2}) + \Gamma(1 - \frac{D}{2}) \right],$$

Vanishing of the Dirac's Delta function evaluated at zero momentum.

The other kind of singularities appears when the momentum conservation and the Dirac's Delta functions associated to condensate arriving lines in other legs of a vertex, force a zero momentun value in a usual Feynman propagator attached to a leg.

$$\begin{aligned} D^{ab}_{g\mu\nu}(p,m) &= \frac{\theta(|\mathbf{p}|-\sigma)\delta^{ab}}{p^2+\mathrm{i}\varepsilon} \left[g_{\mu\nu} - \frac{(1-\alpha)p_\mu p_\nu}{p^2+\mathrm{i}\varepsilon} \right] \\ &\quad - \mathrm{i}C_g\, \delta^{ab}\delta(p), \qquad (44) \\ G^{f_1 f_2}_q(p,M,S) &= -\frac{\theta(|\mathbf{p}|-\sigma)\delta^{f_1 f_2} p_\mu \gamma^\mu}{p^2+\mathrm{i}\varepsilon} + \mathrm{i}\, \delta^{f_1 f_2} C_q\, \delta(p), \\ G^{ab}_{gh}(p) &= -\frac{\theta(|\mathbf{p}|-\sigma)\delta^{ab}}{p^2+\mathrm{i}\varepsilon}. \end{aligned}$$

In this case in Ref. **Eur. Phys. J C47, 355 (2006)**), it was argued that the infrared cutoff following from the Nakanishi approach for the quantization of the free gauge fields, leads to the vanishing of the mentioned sort of singularities in which a propagator is evaluated at zero momentum.

After considering the above defined rules, it follows that all the diagrams containing the mentioned two forms of singularities vanish. Then, for the rest of the graphs the limit δ–>0 could be taken, in order to normally evaluate the dimensionally regularized integrals.

$$\delta \to 0,$$
$$D \to 4$$

$$\Gamma(z) = \int_0^\infty dt\, t^{z-1} \exp(-t),$$

Integral representation of the Gamma function.

$$\delta(0) = \frac{f(\frac{D}{2})^D}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2}\Gamma(-\frac{D}{2}) + \Gamma(1-\frac{D}{2}) \right].$$

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$$G^{f_1 f_2}_q(p,M,S) = -\frac{\theta(|\mathbf{p}|-\sigma)\delta^{f_1 f_2} p_\mu \gamma^\mu}{p^2+i\varepsilon} + i\,\delta^{f_1 f_2} C_q\,\delta(p),$$

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$$\delta \to 0,$$

In Ref. **Eur. Phys. J C47. 95 (2006).** one important formal aspect was also addressed. It was argued that the modified expansion is gauge invariant, that is, independent of the value of gauge parameter selection. The Ward identities of the modified expansion coincide with the ones of PQCD. It became posible because the generating functional Z of the theory could be rewritten in the form:

$$
\begin{aligned}
Z = &\exp\left\{ \mathrm{i}\left[\frac{S^{g}_{ikl}}{3!\mathrm{i}^3}\frac{\delta^3}{\delta j_i \delta j_k \delta j_l} + \frac{S^{g}_{iklm}}{4!\mathrm{i}^4}\frac{\delta^4}{\delta j_i \delta j_k \delta j_l \delta j_m} \right.\right. \\
&\left.\left. + \frac{S^{gh}_{ikl}}{2!\mathrm{i}^3}\frac{\delta^3}{\delta \bar\eta_i \delta j_k \delta(-\eta_l)} + \frac{S^{q}_{ikl}}{\mathrm{i}^3}\frac{\delta}{\delta \bar\xi_i \delta j_k \delta(-\xi_l)} \right] \right\} \\
&\times Z_0\left[j,\eta,\bar\eta,\xi,\bar\xi\right] \\
= &\lim_{T\to\infty} \sum_{q,q^+} \langle \Psi | q^+ \rangle \int D\Phi \\
&\times \exp\left\{ \mathrm{i}\int_{-T}^{T}\int \mathrm{d}t\mathrm{d}\mathbf{x} \left(\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} - \frac{1}{2\alpha}\partial^\nu A^a_\nu \partial^\mu A^a_\mu \right.\right. \\
&\left.\left. - \mathrm{i}\partial_\nu \bar c^a D^{ab,\nu} c^b \right) \right\} \qquad (49) \\
&\times \exp\left\{ \mathrm{i}\int_{-T}^{T}\int \mathrm{d}t\mathrm{d}\mathbf{x} \; \left(\overline{\psi}(\mathrm{i}\gamma^\nu D_\nu - m)\psi + j_\mu A^\mu + \bar\xi c \right.\right. \\
&\left.\left. + \bar c \xi + \bar\eta \psi + \overline{\psi}\eta \right) \right\} \langle q^- | \Psi \rangle .
\end{aligned}
$$

In which the matrix elements appearing are gaussian functions of the fields components which are indixed by the symbol **q.** It is interesting that the only difference with the expression for the case of the PQCD is in the form of the Gaussian functions in both cases.

$$
\begin{aligned}
\widehat{A}^a{}_\mu(\pm T,\mathrm{x})\,|q^\pm\rangle &\equiv A^a_{q^\pm,\mu}(\mathrm{x})\;|q^\pm\rangle , \\
\widehat{c}^a(\pm T,\mathrm{x})\,|q^\pm\rangle &\equiv c^a_{q^\pm}(\mathrm{x})\,|q^\pm\rangle , \\
\widehat{\bar c}^a(\pm T,\mathrm{x})|q^\pm\rangle &\equiv \bar c^a_{q^\pm}(\mathrm{x})\,|q^\pm\rangle , \\
\widehat{\psi}^a(\pm T,\mathrm{x})\,|q^\pm\rangle &\equiv \psi^a_{q^\pm}(\mathrm{x})\;|q^\pm\rangle , \\
\widehat{\overline{\psi}}^a(\pm T,\mathrm{x})\,|q^\pm\rangle &\equiv \overline{\psi}^a_{q^\pm}(\mathrm{x})\;|q^\pm\rangle .
\end{aligned}
$$

The satisfaction fo the transversality condition for the one loop correction to the polarization operator was checked in Ref. **Eur. Phys. J C47, 95 (2006)** as well as the gauge parameter independence of the effective action up to order **g** squared.

Most recently, in Ref (**Eur. Phys. J C** to appear this year 2008**)** . it seems that an advance has been done in solving the difficult question related with how to implement the dimensional transmutation effect. It came from noticing that the new two constants of the scheme: the quark and gluon condensation parameters are always appearing in the expansion as mutiplied by a coupling constant factor **g**^2. Therefore after simply defining new two parameters as **g**^2 *Cq* and **g**^2 *Cg* the new expansion in powers of **g** seems to incorporate non-perturbative information after evaluationg corrections in powers of **g.** but exactly as functions of the other two parameters.

$$
\begin{aligned}
Z[j,\eta,\overline{\eta},\xi,\overline{\xi}] &= \frac{I[j,\eta,\overline{\eta},\xi,\overline{\xi}]}{I[0,0,0,0]} \\
I[j,\eta,\overline{\eta},\xi,\overline{\xi}] &= \frac{1}{\mathcal{N}}\int\int d\alpha d\overline{\chi} d\chi \exp[-\overline{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \times \mathcal{M}(\alpha,\overline{\chi},\chi) \\
&\quad \exp[\widetilde{V}^{int}[\frac{\delta}{\delta j},\frac{\delta}{\delta \overline{\eta}},\frac{\delta}{-\delta \eta},\frac{\delta}{\delta \overline{\xi}}\frac{\delta}{-\delta \xi},\alpha,\overline{\chi},\chi]]\times \\
&\quad \exp[\int \frac{dk}{(2\pi)^D} J^*(-k)S(k)J(k)],
\end{aligned}
$$

$$Z = \int e^{\bar{\theta}(0)\theta_i(0)\;\bar{\eta}(0)\eta(0)}$$

$$e^{i\varphi\cdot\theta + \frac{\dot{\varphi}\dot{\varphi}}{2}}$$

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I[j,\eta,\overline{\eta},\xi,\overline{\xi}] &= \frac{1}{\mathcal{N}}\int\int d\alpha d\overline{\chi} d\chi \exp[-\overline{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \times \mathcal{M}(\alpha,\overline{\chi},\chi) \\
& \exp[\widetilde{V}^{int}[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\xi}}\frac{\delta}{-\delta\xi},\alpha,\overline{\chi},\chi]]\times \\
& \exp[\int \frac{dk}{(2\pi)^D} J^*(-k)S(k)J(k)],
\end{aligned}
$$

$$Z = O\, e^{\bar{\psi}(0)\psi_x(0)\ \bar{\eta}(0)\eta(0)},$$

$$e^{k\varphi\cdot J + \left(\frac{\bar{\varphi}\cdot\varphi}{2}\right)}$$

The function M

$$\mathcal{M}(\alpha,\overline{\chi},\chi) = \frac{1}{\mathcal{N}}\ \exp[-\frac{1}{4}V^D F^a_{\mu\nu}(\alpha)F^a_{\mu\nu}(\alpha)] \times$$
$$\exp[-\frac{1}{2}Tr[Log[\mathbf{A}]] + Tr[Log[\mathbf{B}-\mathbf{D}\mathbf{A}^{-1}\mathbf{C}] + Tr[Log[\mathbf{G}]]],$$

Matrices **A, B, C, D** :

$$\begin{aligned}
\mathbf{A}^{(\mu,a),(\nu,b)}(\alpha) &\equiv -(\mathbf{D}^{ac}\mathbf{D}^{cb}\delta_{\mu\nu} - \frac{\mathbf{D}^{ac}_\mu\mathbf{D}^{cb}_\nu + \mathbf{D}^{ac}_\nu\mathbf{D}^{cb}_\mu}{2} + \frac{\delta^{ab}}{\alpha}k_\mu k_\nu),\\
\mathbf{B}^{(u,r),(v,s)}(\alpha,\overline{\chi},\chi) &\equiv \gamma_\mu(k_\mu\delta^{ij} + g\beta^a_\mu T^{ij}_a),\\
\mathbf{D}^{(u,r),(\nu,b)}(\overline{\chi},\chi) &\equiv -g(\frac{C^b_q}{(2\pi)^D})^{\frac{1}{2}}\gamma^{u\ q}_v T^{ij}_b\chi^{q,t},\\
\mathbf{C}^{(\mu,a),(v,s)}(\overline{\chi},\chi) &\equiv -g(\frac{C^b_q}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^{q,t}\gamma^{q\ v}_\mu T^{ts}_b,\\
\mathbf{G}^{ab}(\alpha) &= k_\mu\mathbf{D}^{ab}_\mu,\\
\mathbf{D}^{ab}_\mu &= k_\mu\delta^{ab} + gf^{abc}\ \beta^c_\mu,\\
\beta^a_\mu &= (\frac{2C^b_g}{(2\pi)^D})^{\frac{1}{2}}\alpha^a_\mu.
\end{aligned}$$

The inverse propagator

$$S^{-1} = \left\{ \begin{array}{ccc} \mathbf{A}/2 & \mathbf{C} & 0 \\ \mathbf{D} & \mathbf{B} & 0 \\ 0 & 0 & \mathbf{G} \end{array} \right\},$$

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\mathbf{B}^{(u,r),(v,s)}(\alpha,\overline{\chi},\chi) &\equiv \gamma_\mu(k_\mu\delta^{ij}+g\beta^a_\mu T^{ij}_a),\\
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\mathbf{G}^{ab}(\alpha) &= k_\mu\mathbf{D}^{ab}_\mu,\\
\mathbf{D}^{ab}_\mu &= k_\mu\delta^{ab}+gf^{abc}\ \beta^c_\mu,\\
\beta^a_\mu &= (\frac{2C^b_g}{(2\pi)^D})^{\frac{1}{2}}\alpha^a_\mu.
\end{aligned}$$

The inverse propagator

$$S^{-1}=\left\{\begin{array}{lll}\mathbf{A}/2 & \mathbf{C} & 0\\ \mathbf{D} & \mathbf{B} & 0\\ 0 & 0 & \mathbf{G}\end{array}\right\},$$

New parameters

$$
\begin{aligned}
g &= g, \\
m_g^2 &= g^2 C_g, \\
m_q^2 &= \left(g^2 C_q\right)^{\frac{2}{3}} .
\end{aligned}
$$

$$
\begin{aligned}
D[A] &= \frac{D-2}{2}, \quad D[\Psi] = \frac{D-1}{2} = D[\overline{\Psi}], \quad D[\chi] = \frac{D-2}{2} = D[\overline{\chi}], \\
D[g] &= 2 - \frac{D}{2}, \quad D[C_g] = D - 2, \quad D[C_q] = D - 1.
\end{aligned}
$$

$$
D[m_q^2] = \frac{2}{3}(4 - D + D - 1) = 2.
$$

$$
D[m_g^2] = (4 - D + D - s) = 2.
$$

Sample evaluation of the effective action for examining the dynamical generation of the gluon and quark condensates.

Being eliminated the mentioned singularities, in Ref. Eur. Phys. J C47, 355 (2006), we considered the evaluation of the effective potential, a quantity which should show a minimum at the exact ground state of physical system under consideration.

$$G^{ab}_{g\mu\nu}(p.m) = \frac{\delta^{ab}}{(p^2-m^2+i\epsilon)}(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2+i\epsilon}) + \frac{\beta\, p_\mu\ \ p_\nu}{(p^2+i\epsilon)^2}\delta^{ab}.$$
$$= \frac{\delta^{ab}}{(p^2-m^2+i\epsilon)}(g_{\mu\nu} - \frac{\beta\, p_\mu\, p_\nu\, m^2}{(p^2+i\epsilon)^2}). \quad \beta = 1.$$
$$G^{f_1 f_2}_q(p.M.S) = \frac{\delta^{f_1 f_2}}{\left(-p_\mu\gamma^\mu(1-\frac{M^2}{p^2}) \quad - \frac{S_{f_1}}{p^2}\right)}.$$
$$\chi^{ab}(p) = -\frac{\delta^{ab}}{p^2+i\epsilon}.$$
$$G^{ab}_m = -\frac{im^2}{g^2}\delta^{ab}\ \delta(p).$$
$$G_S = \frac{i4\pi^4 S_f}{g^2 C_F}\delta^{ab}\delta^{f_1 f_2}\delta(p).$$

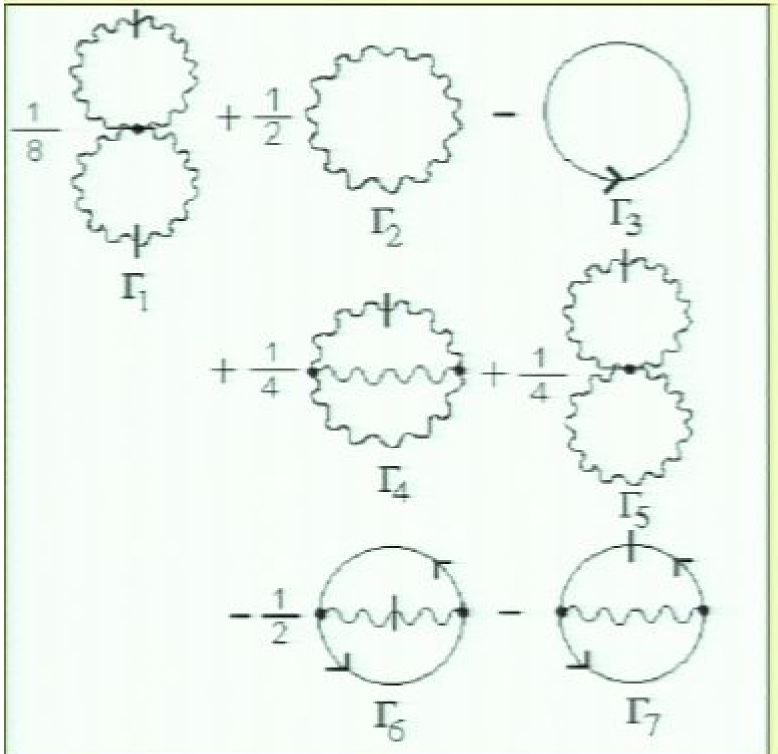
For this purpose we evaluated the one loop terms of the expansion for the Effective potential. However, in order to improve the result we included all the insertions of the above considered self-energies (in the lowest order in the condensate parameters) in the internal lines. This led to modified propagators similar to those employed in the previous work. Eur. Phys. J C23, 289 (2002)

$$-m^2 = m_g^2 = \frac{6g^2C}{(2\pi)^4},$$
$$S_f = \frac{g^2 C_F}{4\pi^4}\, C_f.$$
$$m^2 = f\ M^2. \quad f = \pm(\frac{3}{2})^2.$$
$$g^2 = g_o^2\ \ \mu^{2-\frac{D}{2}}.$$
$$D = 4 - 2\epsilon.$$

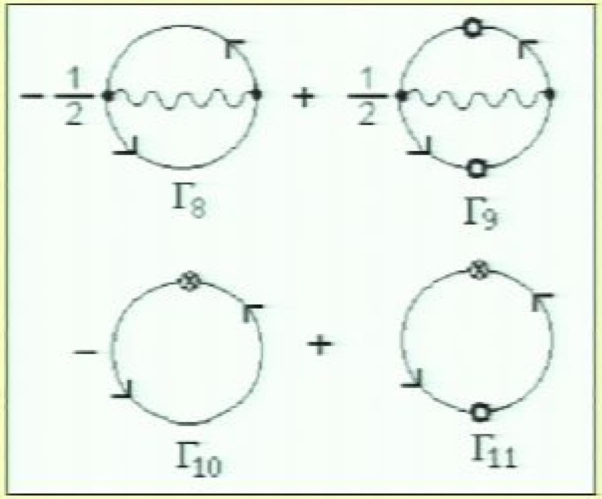
The constants appearing are: the "gluon mass" ***m***. the constant ***S*** defined in terms of a unique quark condensate parameter ***Cf***, reflecting the fact that only one quark type is assumed to condense in this calculation, in order to explore the possibility for the condensation of one very massive quark. The coupling **g** is expressed in terms of the mass scale parameter μ and the dimesional regularization parameter **D**.

The sign of **C** derived in previous works, is positive and produce a positive constituent mass ***M***. Thus the sign of ***f*** should be negative for consistency

The modified tree and one loop graphs of the effective action and potential



The one loop diagrams, including the ones coming from the two loop terms after a loop is cancelled by a Delta function.



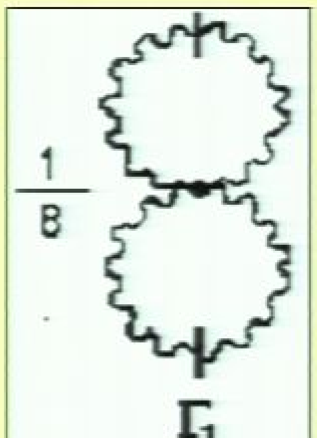
A particular two loop term and the counterterm. substractions

$$(Z_2 - 1) = -\frac{g^2 C_F}{(4\pi)^2}\frac{\beta}{\epsilon},$$

$$T_R = \frac{1}{2}, \quad C_G = N, \quad C_F = \frac{N^2-1}{N}$$

Quark renormalization parameter

Results for the various contributions



Modfied "tree" approximation

$$\boxed{\Gamma^{(0)} = -\frac{2m^4}{g^2} = -V^{(0)}}$$

$$\Gamma^{(1)}_{gf} = \Gamma^{(1)}_{g} + \Gamma^{(1)}_{f} + \Gamma^{(1)}_{S} = -V^{(1)}_{g} - V^{(1)}_{f} - V^{(1)}_{S}$$
$$= -\frac{i}{2}\, Tr\ \left[\log\left[G_g^{-1}(0)\, G_g^{-1}(m)\right]\right] + i\, Tr\ \left[\log\left[G_q^{-1}(0,0)\, G_q^{-1}(M,0)\right]\right] +$$
$$+ i\, Tr\ \left[\log\left[G_q^{-1}(0,0)\, G_q^{-1}(M,S)\right]\right] - i\, Tr\ \left[\log\left[G_q^{-1}(0,0)\, G_q^{-1}(M,0)\right]\right].$$

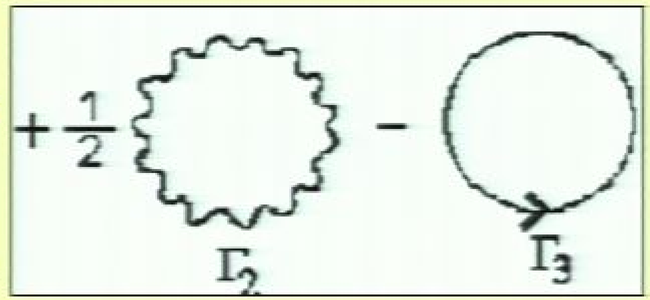
The "one loop" in terms of the propagators, dressed by the insertions of the modified tree self-energies.

$$\Gamma^{(1)}_{g} = -\frac{(N^2-1)(D-1)}{2}\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2}{p^2-m^2}\right],$$
$$\Gamma^{(1)}_{f} = 4N\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2}{p^2-M^2}\right].$$

The Minkowski space momentum p_o was Wick rotated to Euclidean space assuming that no poles are crossed. This means that the calculations done correspond to Euclidean field theory or what is the same: the zero temperature field theory.

$$V^{(1)}_{g}(m) = -\frac{(N^2-1)}{128\,\pi^2}\, m^4\left(-6\log\left(\frac{m^2}{4\pi\mu^2}\right) - 6\gamma + 5\right)$$
$$V^{(1)}_{f}(M) = \frac{3\,(N^2-1)}{128\,\pi^2}\, M^4\left(-2\log\left(\frac{M^2}{4\pi\mu^2}\right) - 2\gamma + 3\right)$$

M and *m* dependent terms



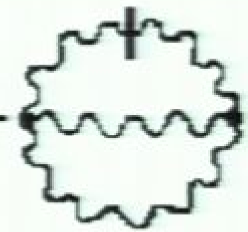
$$\Gamma^{(1)}_{S} = +i\, Tr\ \left[\log\left[G_q^{-1}(0,0)\, G_q^{-1}(M,S)\right]\right] -$$
$$-i\, Tr\ \left[\log\left[G_q^{-1}(0,0)\, G_q^{-1}(M,0)\right],\right]$$
$$= 2N\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2(p^2-M^2)^2}{p^2(p^2-M^2)^2-S^2}\right]$$

Quark condensate dependent term

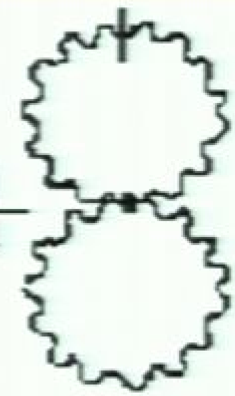
$$\Gamma^{(1)}_{S} = -V^{(1)}_{S} = \ 2\,N\int_E \frac{dp^D}{(2\pi)^D}\log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right]$$
$$= \ 4\,N\int^{\infty} \frac{dp\, p^3}{(4\pi)^2}\log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right].$$

The “one loop” diagrams “descending” from the “two loop” gluon ones

$$\Gamma_{2g}^{(1,1)} = \frac{(N-1)N\,\Gamma(1-\frac{D}{2})\left[3D^2+2\beta(D+11)(1-\frac{D}{2})\right]}{4(2\pi)^D(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}} + \\ + \frac{2(N-1)N\,\Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})}{(2\pi)^D(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})}(m^2)^{\frac{D}{2}}.$$

$+\frac{1}{4}$ 

$$\begin{aligned}\Gamma_{2g}^{(1.2)} &= -V_{2g}^{(1.2)} = \frac{(N^2-1)N(D-1)\ m^2}{2(2\pi)^D}\int\frac{dp^D}{(2\pi)^D i}\frac{1}{(p^2-m^2)}\left[1-\frac{\beta m^2}{p^2}\right], \\ &= -\frac{(N^2-1)N(D-1)\ m^2}{4(2\pi)^D}\int_E\frac{dp^D}{(2\pi)^D}\frac{1}{(p^2+m^2)}\left[1+\frac{\beta m^2}{p^2}\right]. \\ &= -\frac{(N-1)N\ (D-1)(D-\beta)\Gamma(1-\frac{D}{2})}{2(2\pi)^D(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}}.\end{aligned}$$

$\frac{1}{4}$ 

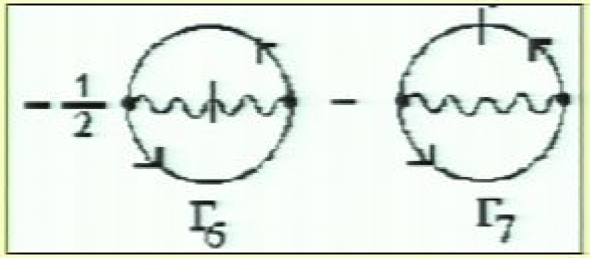
The result for these “one loop” graphs simplifies to

$$\boxed{\lim_{D\to 4}(V_{2g}^{(1.1)}+V_{2g}^{(1.2)}) = \frac{3f^2M^4}{8\pi^2}}$$

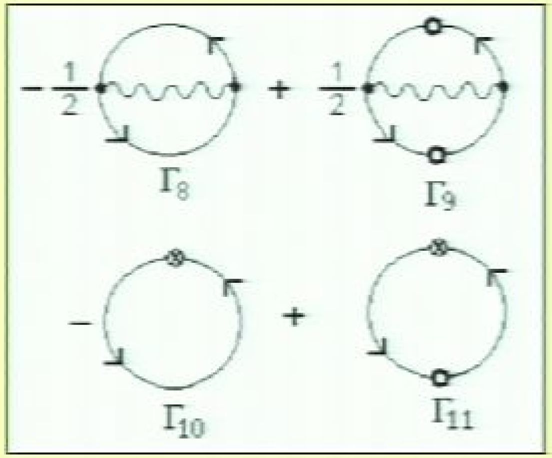
$$\Gamma_{2q}^{(1.1)} = -V_{2q}^{(1.1)} = \frac{(N^2-1)\ m^2}{3(4\pi)^2} \int_0^\infty dp\ p^3 \frac{(2\ p^2(\ p^2+M^2)^2+D\ S^2)}{(p^2(\ p^2+M^2)^2+S^2)^2},$$

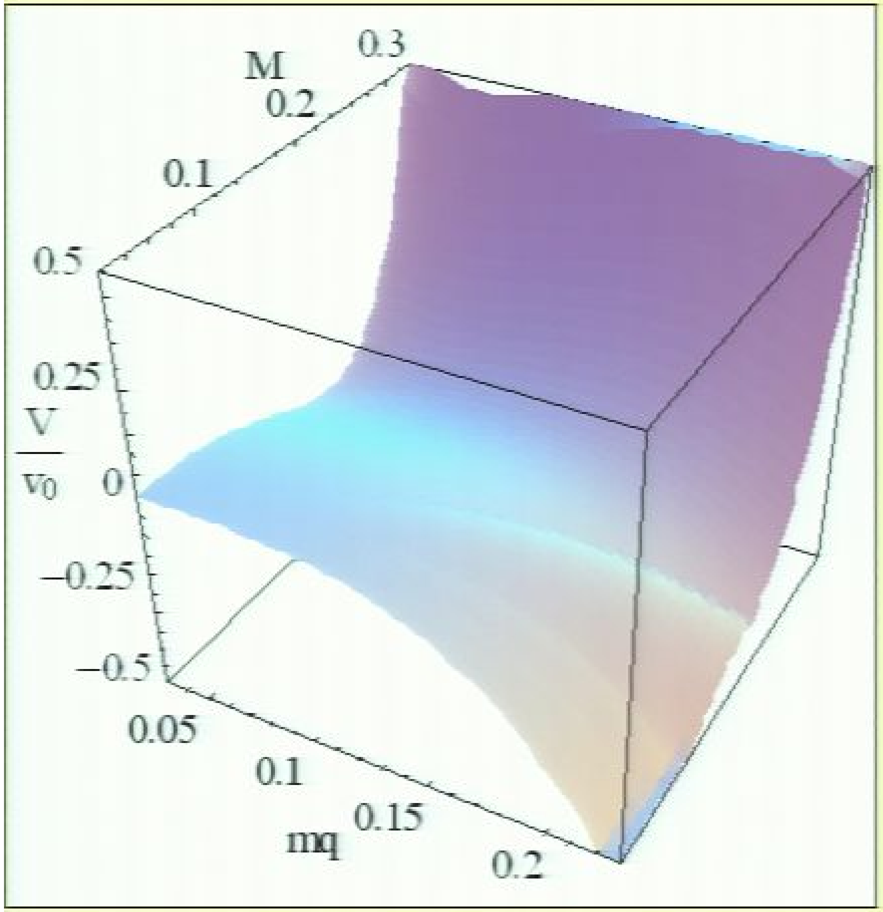
$$\Gamma_{2q}^{(1.2)} = -V_{2q}^{(1.2)} = -N\ S^2 \int_E \frac{dp}{(2\pi)^4} \frac{(D\ p^2+m^2-i\epsilon)}{(p^2(\ p^2+M^2)^2+S^2)(\ p^2+m^2-i\epsilon)}.$$

The results for the "one loop" diagrams "decending" from the "two loop" fermion ones



The below illustrated quark condensate dependent two loop contributions and the counterterms substractions were also evaluated. It allowed to check that the usual renormalization Z constants cancel the infinities of the evaluated terms.





The effective potential as a function of the parameters: ***M*** representing the gluon condensation and ***mq*** determining the quark oondensate.

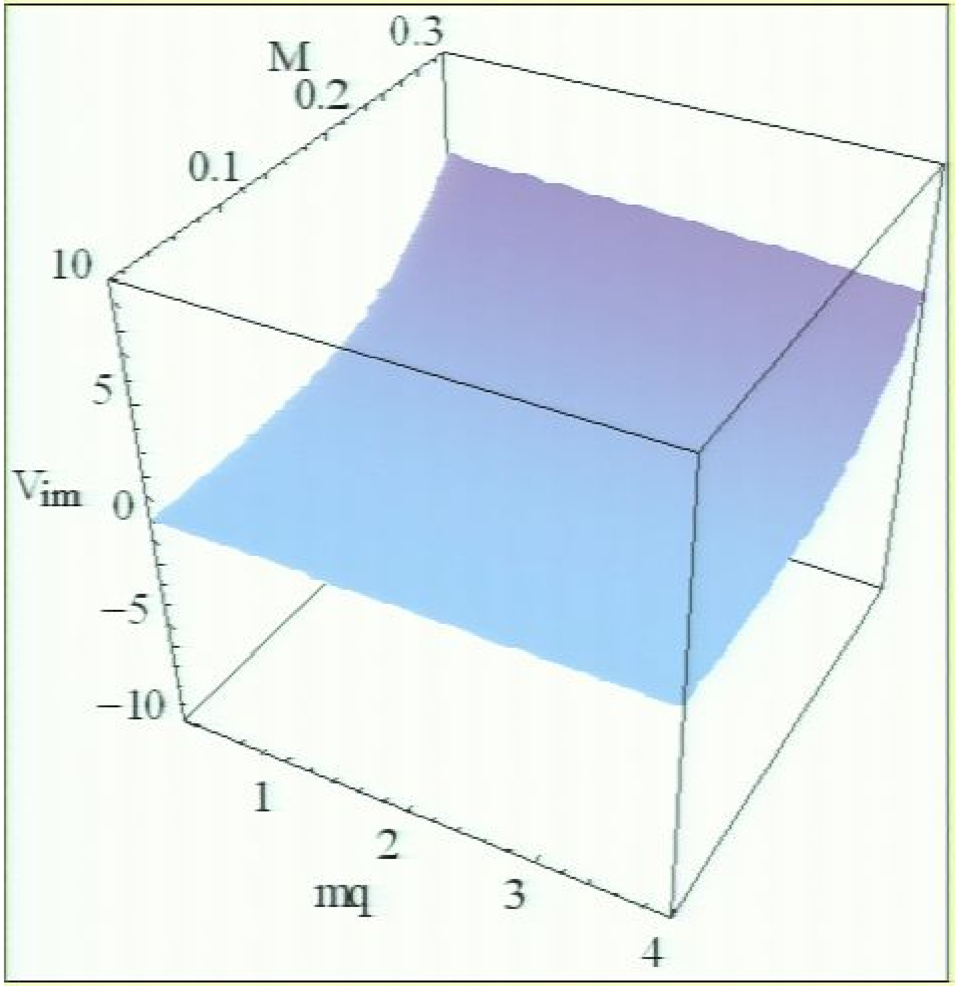
$$mq = \frac{S^{\frac{1}{3}}}{M}$$

The values of ***μ=6.8 GeV*** and ***g=2.7*** were chosen. This coupling g corresponds to

$$\alpha = \frac{g^2}{4\pi} = 0.597$$

which is an intermediate value for the strong coupling α suspected (P. Hoyer, Preprint NORDITA-2002-19 HE) to be a limit of the "running" coupling at low energies.

The picture shows that even at zero value of the gluon condensation, the system developes an instability with respect to the generation of the quark condensate which is not bounded within the considered approximation.



The imaginary part of the potential for the appropriate negative value of the parameter ***f*** **,** as a function of the same quantities ***M*** and ***mq***

1) Its can be observed that the imaginary part is small with respect to the real part of the potential for small values of the gluon condensation parameter ***M.***

2) It can be remarked that for the positive values of ***f*** the imaginary part of the efective potential vanish. Therefore (whenever propagators with positive values of the squares of ***M*** and ***m*** could be justified by constructing condensate states leading to them after connecting the interactions) the resulting expansion could be absent of the taquionic gluon mass. It is interesting to note that the gluon mass in this case turns to be near ***0.5 GeV*** , a value previously estimated by Cornwall and A. Soni .***Phys. Lett*** **120B. 431 (1983).**

Summary and possible extensions of the work

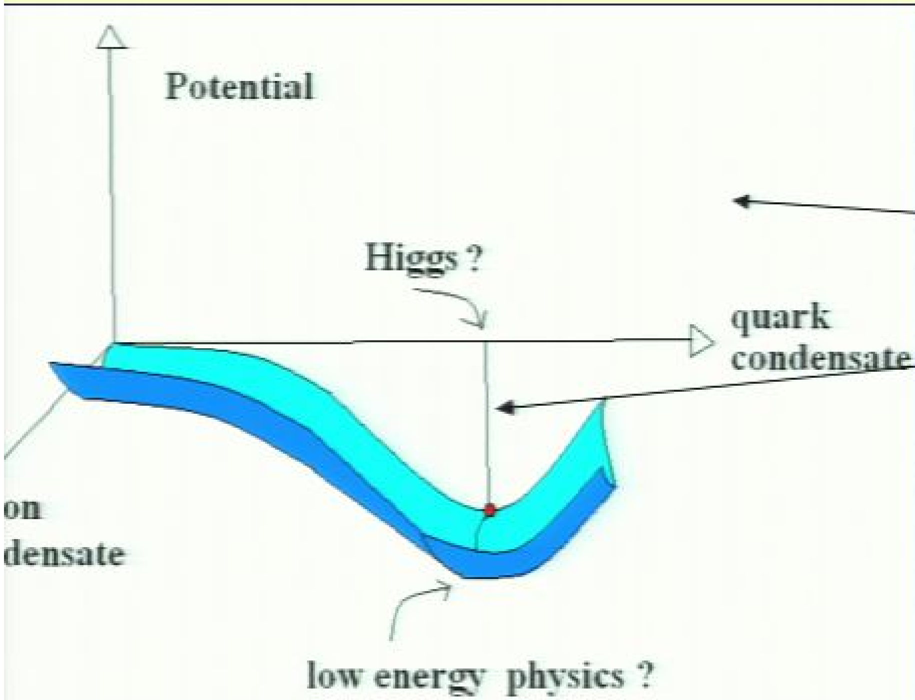
A modified perturbative expansion for QCD is proposed and its formulation and implication are being explored. Specifically, the main steps considered were:

1) The singularities associated to the modied perturbative expansion were eliminated by using the dimensional regularization scheme and a complementary Nakanishi regularization of the standard propagator at zero value of the momentum.

2) Another technical improvement is the incorporation in the scheme of the gauge paramater α, on which the physical quantities should be independent, since the renormalization procedure scale this parameter.

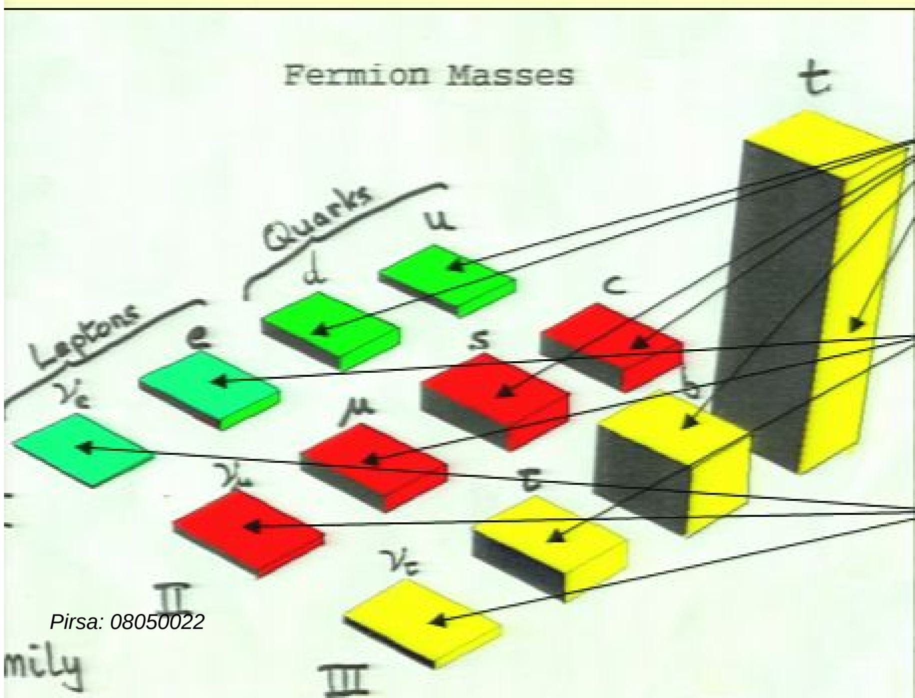
3) The renormalized generating functional has been also proposed, which incorporates on the physical quantities the dependence on the coupling as well as on the gluon and quark condensate parameters. An important issue is that it seems that the dimensional transmutation can be implemented through the reordering of the expansion thanks to the introduction of auxilliary constant gluon and fermion fields.

4) Simple contributions to the effective potential for the generation of quark and gluon condensates were evaluated. The results indicate the instability of the vacuum in the absence of these condensates. The instability upon the quark condensate generation does not produce a minimum of the potential at this step. This result opens the possibility for the production of heavy masses and large condensates after evaluating higher loop corrections.



Therefore, if the analysis turns to be correct, the following picture could perhaps arise:

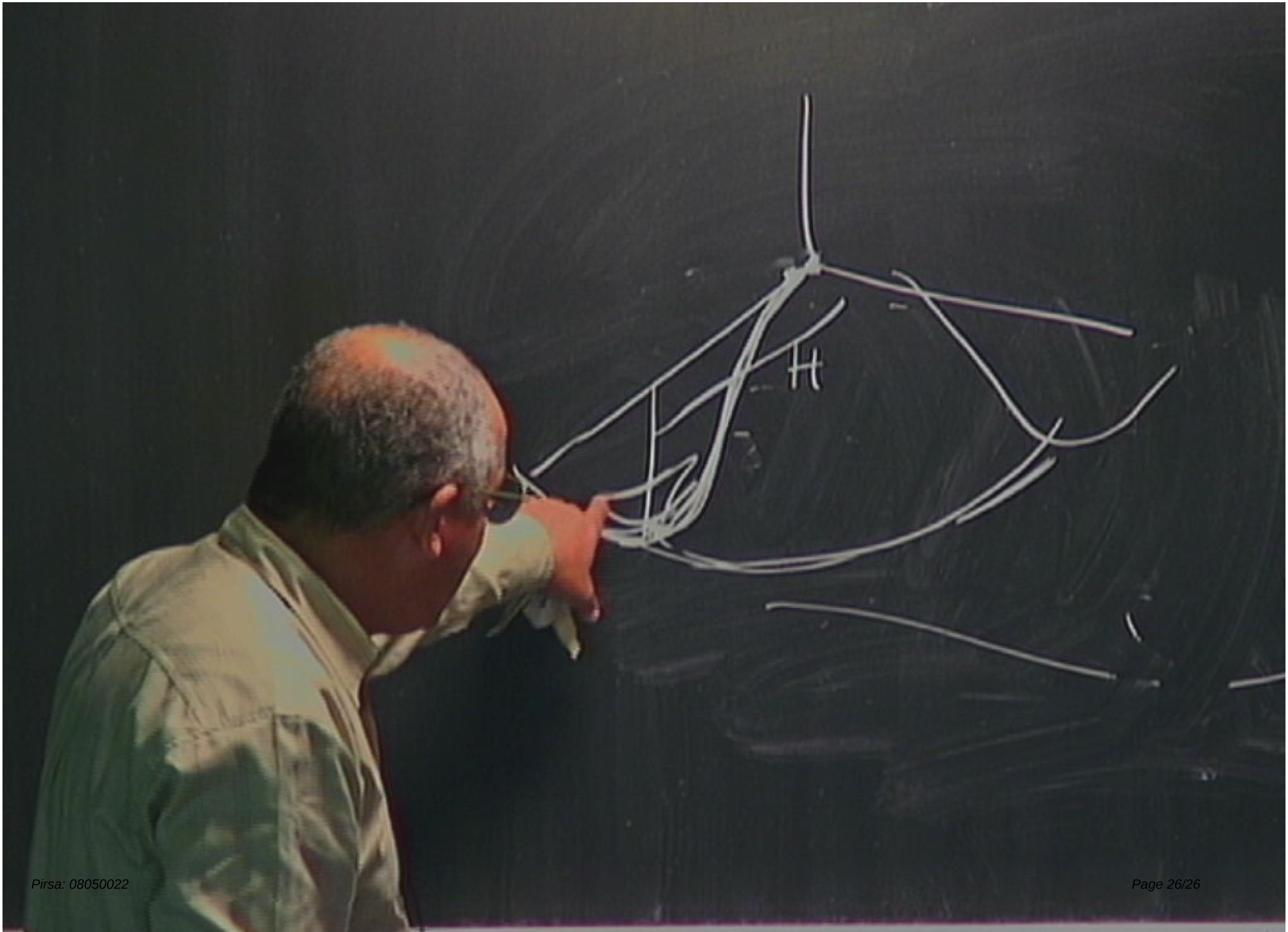
- A sort of the Top condensate model could be the effective action for massless QCD.
- The role of the Higgs field could be played by the Top quark condensate.
- The SM could be closed by generating all the masses within its proper context as follows:



The six quarks could get their masses thanks to the proposed flavour symmetry breaking.

The electron, muon and tau leptons, would receive their intermediate mass values due to radiative corrections mediated by their electromagnetic interactions with quarks.

Finally, the only weak interacting character of the three neutrinos with all the particles could determine their smaller masses.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