Title: Universal resources for approximate and stochastic measurement-based quantum computation

Date: May 02, 2008 11:00 AM

URL: http://pirsa.org/08050021

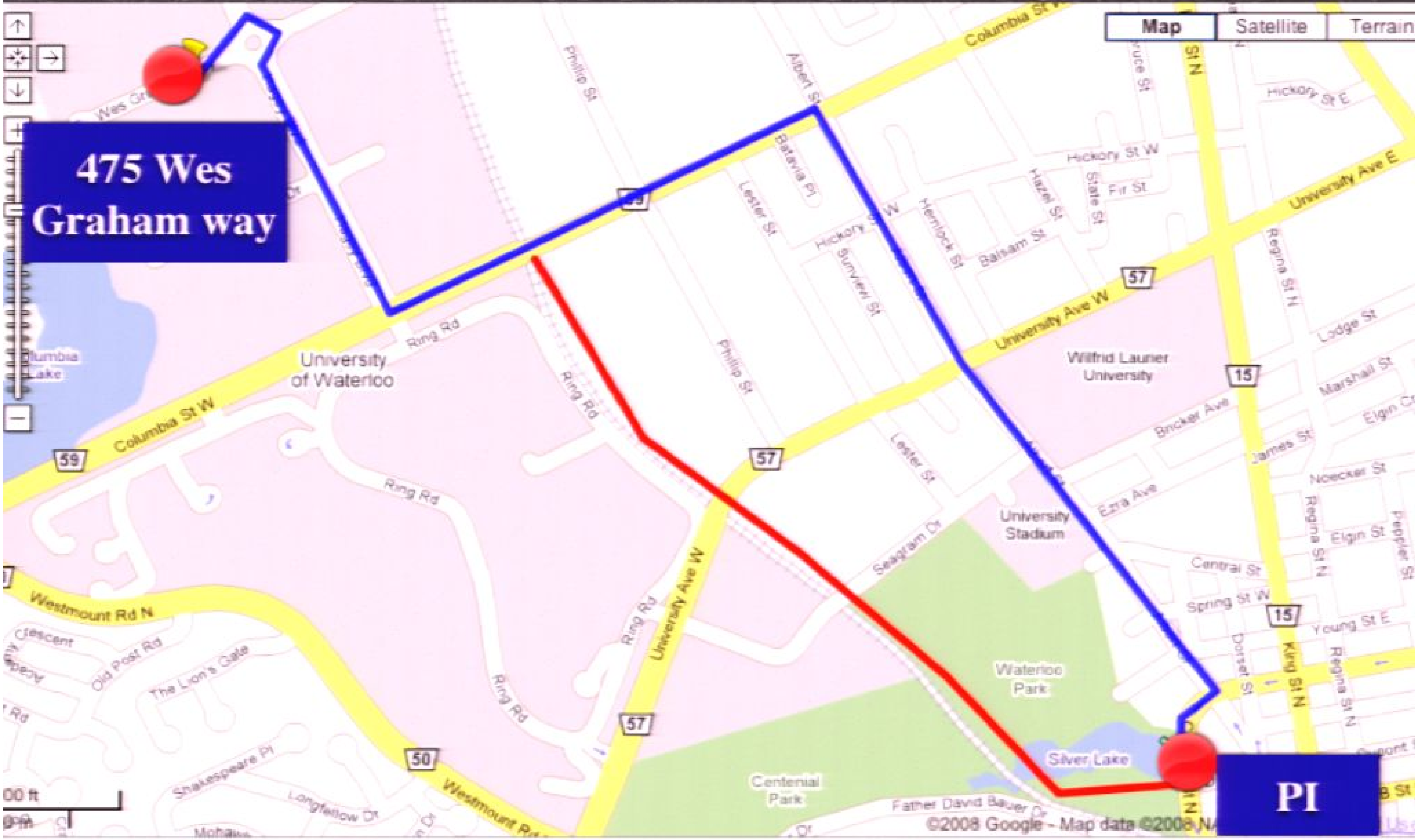
Abstract: We investigate which families of quantum states can be used as resources for approximate and/or stochastic universal measurement-based quantum computation, in the sense that single-qubit operations and classical communication are sufficient to prepare (with some fixed precision and/or probability) any quantum state from the initial resource. We find entanglement-based criteria for non-universality in the approximate and/or stochastic case. By applying them, we are able to discard some families of states as not universal also in this weaker sense. Finally, we show that any family $Sigma$ of states that is \'close\' to an (approximate and/or stochastic) universal family $Gamma$ is approximate and stochastic universal, and we prove that if $Gamma$ was efficiently universal then also $Sigma$ is.

Universal resources for approximate and stochastic measurement-based quantum computing

C
~~K~~aterina-E. Mora
(IQC)

Work in collaboration with
M. Piani, M. Van den Nest,
A. Miyake, W. Dür, H. Briegel

Welcome barbecue this afternoon!



475 Wes Graham way

PI

In the new temporary IQC building

Outline

- Measurement-based computation
  - Universal exact "state preparators"
  - Entanglement-based criteria
  - Examples

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  - Criteria in the approximate/stochastic case
  - Examples and "Go" results

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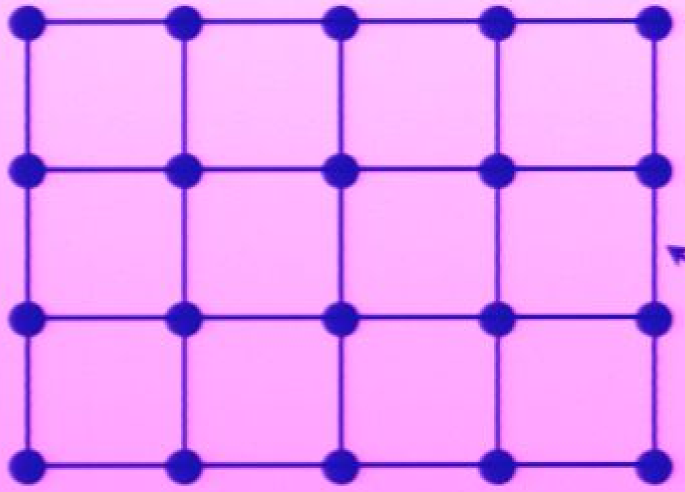
- Conclusions

The one-way model

Cluster state



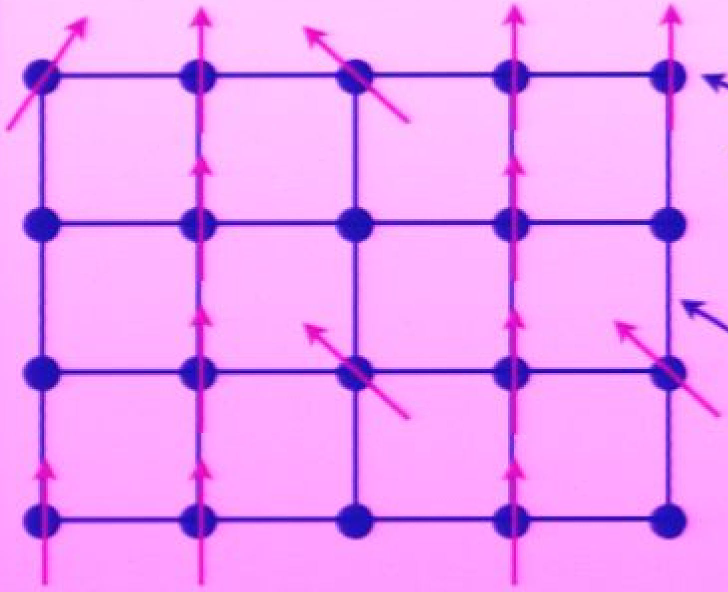
vertices=qubits

edges: c-phase gate between qubits

Remember
Dan Browne's talk!

The one-way model

Cluster state + single-qubit measurements



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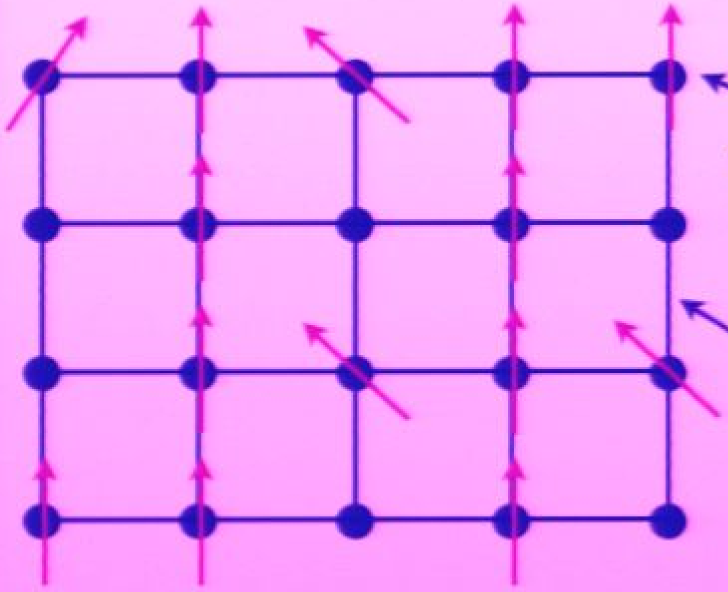

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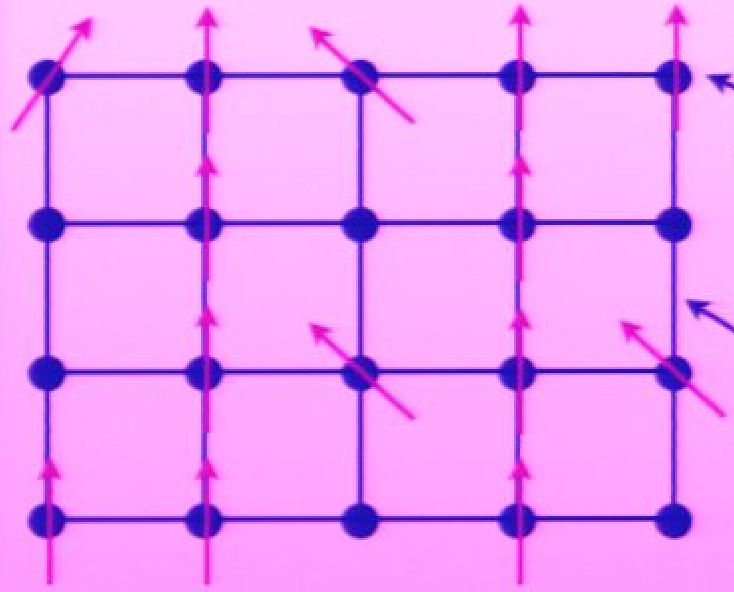

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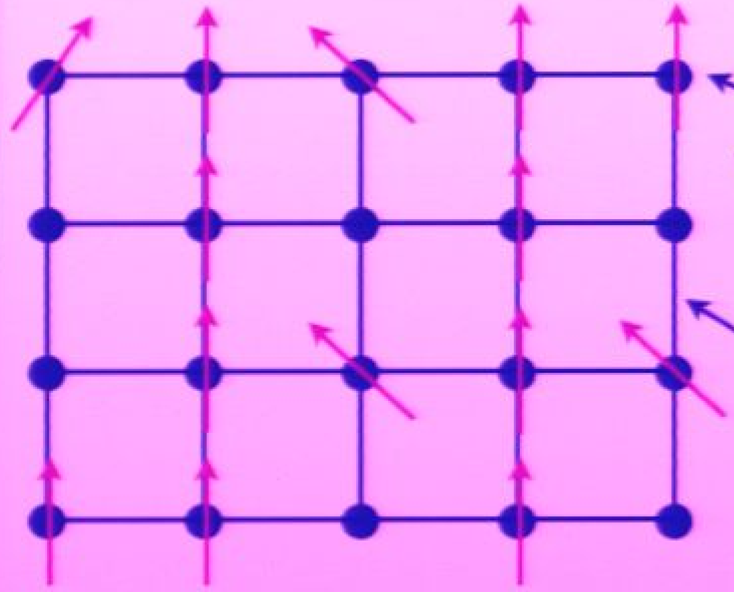

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⇨ Equivalent to the circuit model

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State preparation and exact universality (I)

For any state $|\psi\rangle$ there are:

- ✓ a cluster state $|C_{NxM}\rangle$ large enough
- ✓ a 1-qubit measurement pattern $\mathcal{M}$

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exact state preparation

The family $\Psi_C=\{|C_{NxM}\rangle\}$ of <u>cluster states</u> is universal for exact and deterministic quantum computation (state preparation)

State preparation and exact universality (II)

Just how "special" is the cluster state?

Cluster state + single-qubit measurements + classical communication

State preparation and exact universality (II)

Just how "special" is the cluster state?

Resource state + single-qubit measurements + classical communication

State preparation and exact universality (II)

Just how "special" is the cluster state?

Resource state + Local operations and classical communication

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Universal quantum computation
=
Universal state preparation

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$$\mathcal{E}(\Phi)=\sup_{|\varphi\rangle\in\Phi} \mathcal{E}(|\varphi\rangle)$$

Computing
$\mathcal{E}(\Phi)$ and $\mathcal{E}^*$ we prove
that some families are not
universal for MBQC

Some examples (I)

W states

$$\Phi_{\mathcal{W}} = \{|\mathcal{W}_{\mathcal{N}}\rangle\}_{\mathcal{N}} \qquad |\mathcal{W}_3\rangle \propto |100\rangle + |010\rangle + |001\rangle$$

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$$\Downarrow$$

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(graphs with bounded rank)

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$\mathcal{E}^*=\infty$ > $\mathcal{E}$ bounded in these cases

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Proof:

Sorry! not enough space in this small margin

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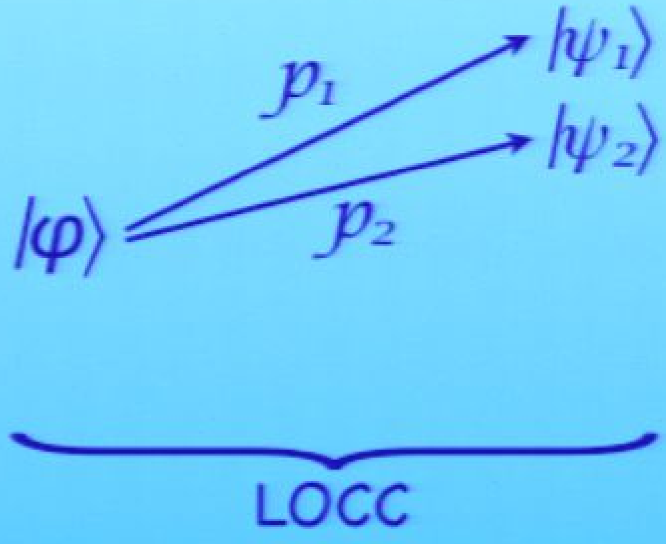

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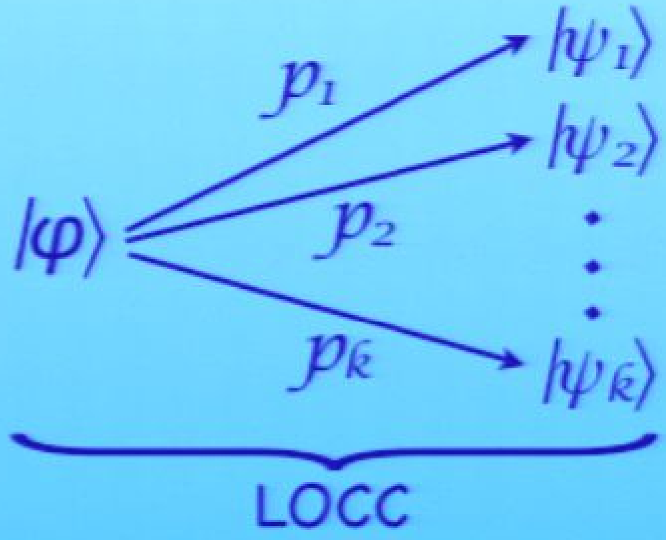

$|\varphi\rangle$

p_1 $|\psi_1\rangle$

p_2 $|\psi_2\rangle$

p_k $|\psi_k\rangle$

LOCC

ε-approximate:

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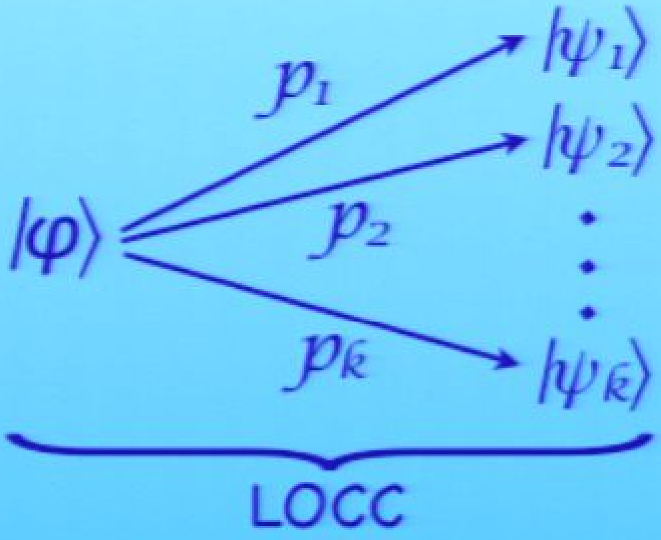

ε-approximate:

$$D(\psi, \psi_j) \leq \epsilon$$

δ-stochastic:

$$\sum_{j:\varepsilon-\mathrm{close}} p_j \geq 1 - \delta$$

Entanglement and universality
(The approximate and stochastic case)

Entanglement gave us criteria in the exact deterministic case.

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Recall!

$\mathcal{E}(\Phi)=\mathcal{E}^*$

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Idea: we are happy to go close to the state: we don't need to reach maximal entanglement

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(ε-measures of entanglement)

Consider a state ρ

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Consider a state ρ and a ball of radius ε around it

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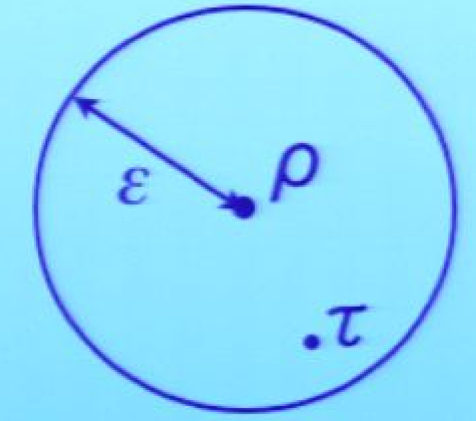

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$$\mathcal{E}_\varepsilon(\rho)=\min\{E(\tau)\ s.t.\ D(\rho,\tau)\leq\varepsilon\}$$

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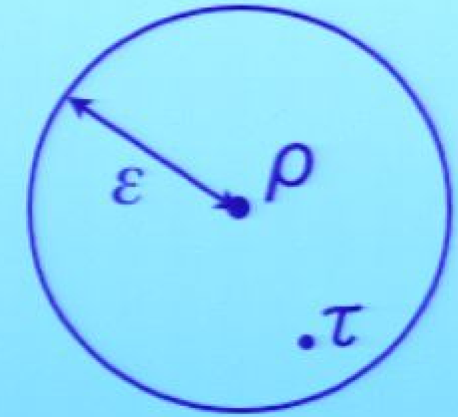

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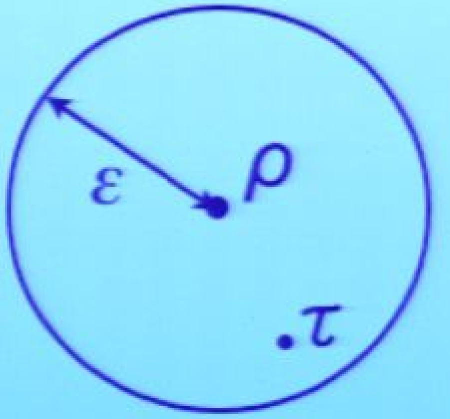

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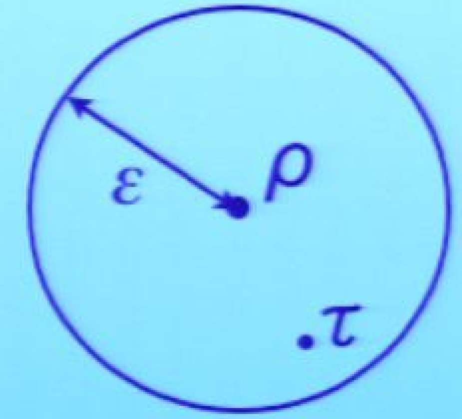

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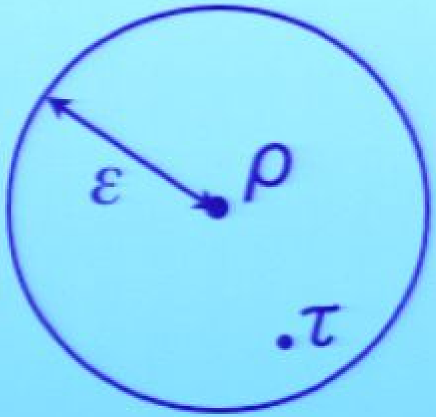

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✓ Measures the entanglement contained in a state that is known partially (as for imperfect preparation)

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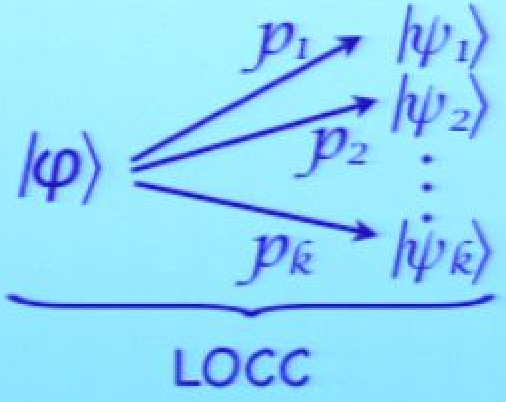
(The approximate/stochastic case)



$$\sum_{j:D(\psi,\psi_j)\leq\epsilon} p_j \geq 1-\delta$$

1) δ=0: *all $|\psi_j\rangle$ must be "close enough" to $|\psi\rangle$*

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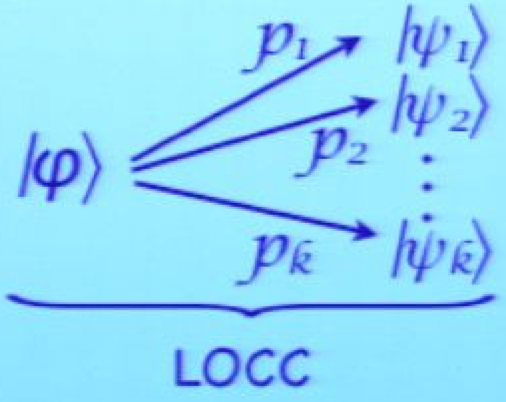

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We must be able to "reach" the minimum entanglement contained in a state close to $|\psi\rangle$

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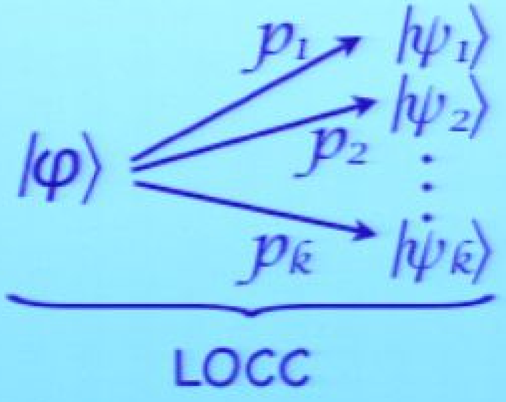

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$$\Rightarrow \quad \mathcal{E}(\Phi)\geq\mathcal{E}_\varepsilon^*$$

Exact case:
$\mathcal{E}(\Phi)\geq\mathcal{E}^*$
with $\mathcal{E}^*\geq\mathcal{E}_\varepsilon^*$

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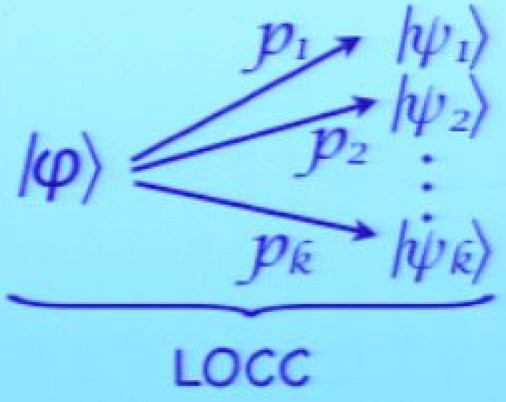

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Exact case:
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with $\mathcal{E}^* \ge\mathcal{E}^*_\varepsilon$

General case: if δ>0, less entanglement is sufficient (some of the outputs can be thrown away)

More examples

Computing $\mathcal{E}^*_\varepsilon$ is hard in general, but can be done!

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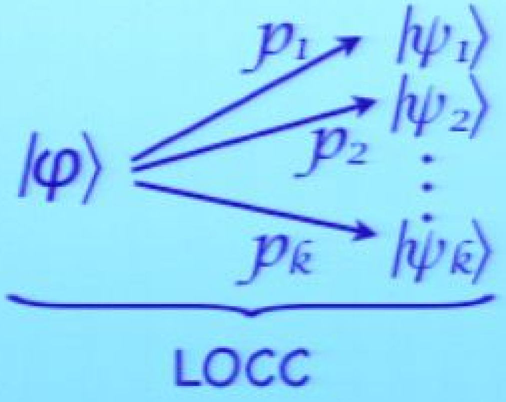

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1) δ=0: all $|\psi_j\rangle$ must be "close enough" to $|\psi\rangle$

$$\Downarrow$$

We must be able to "reach" the minimum entangleme ntained in a state close to $|\psi\rangle$

$$\Rightarrow \quad \mathcal{E}(\Phi)\ge\mathcal{E}^*_\varepsilon$$

Exact case:
$\mathcal{E}(\Phi)\ge\mathcal{E}^*$
with $\mathcal{E}^*\ge\mathcal{E}^*_\varepsilon$

General case: if δ>0, less entanglement is sufficient (some of the outputs can be thrown away)

$$\Rightarrow \quad \mathcal{E}(\Phi)\ge(1-\delta)\mathcal{E}^*_\varepsilon$$

More examples

Computing $\mathcal{E}_{\varepsilon}^{*}$ is hard in general, but can be done!

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W states

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More examples

Computing $\mathcal{E}^*_\varepsilon$ is hard in general, but can be done!

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$\Phi_{\mathcal{W}}=\{|\mathcal{W}_{\mathcal{N}}\rangle\}_{\mathcal{N}}$ $|W_3\rangle \propto |100\rangle + |010\rangle + |001\rangle$

Again the geometric measure

More examples

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$\mathcal{E}_\varepsilon^*(|\psi\rangle)\geq 1-f(\varepsilon)$ $\quad\mathcal{E}(\Phi_W) =1-1/e$

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Reasonable function
$f(\varepsilon) \to 0$ as $\varepsilon \to 0$

More examples

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W states
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In other
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More examples

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Previous examples
not ε-approximate
universal for some
values of ε

Efficiency and universality (I)

Different questions:

1) Can we prepare any state?

2) Is it very expensive?

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LOCC steps

! In the circuit model: $[O(poly(n \log 1/\varepsilon))]$

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polynomial resources

$|\varphi\rangle \in (\mathbb{C}^2)^{O(poly(n,\, 1/\delta, 1/\varepsilon))}$

polynomial time

$O(poly(n,1/\delta,1/\varepsilon))$
LOCC steps

In the circuit model: $[O(poly(n \log 1/\varepsilon))]$

Problem: want to treat δ and ε in the same way
no Solovay-Kitaev theorem for δ

Entanglement and efficiency

The efficiency requirement gives us stronger criteria:

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2') In the approximate/stochastic case:

Similar, but $\mathcal{E}_\varepsilon$ is enough: $\mathcal{E}_\varepsilon(|C_n\rangle)=\mathcal{E}(|\varphi_{k=O(poly(n))}\rangle)$

Entanglement and efficiency

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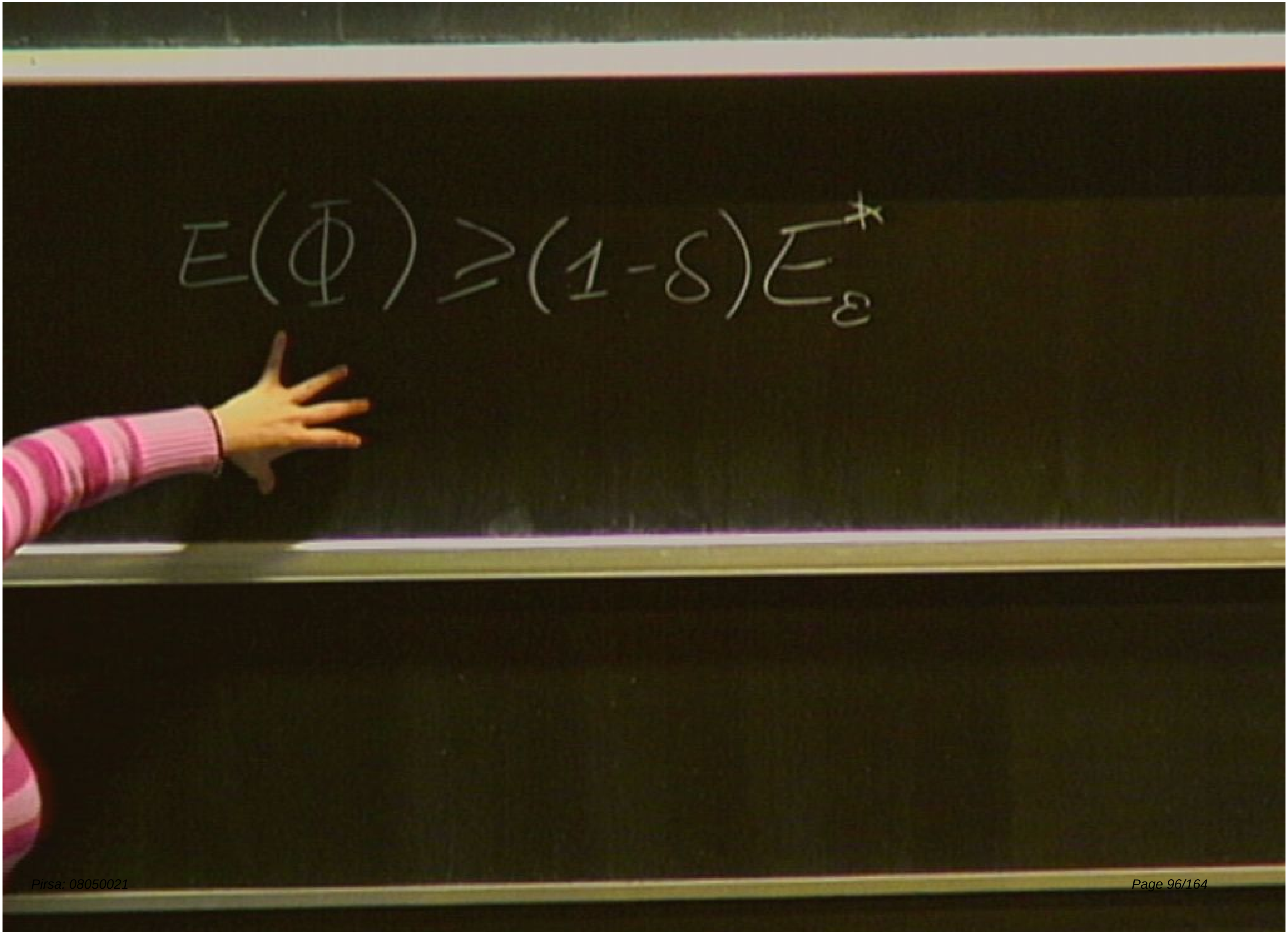
of

$\mathcal{E}(|C_n\rangle)=\mathcal{E}(|\varphi_{k=O(poly(n))}\rangle)$

2 mate/stochastic case:

is enough: $\mathcal{E}_\varepsilon(|C_n\rangle)=\mathcal{E}(|\varphi_{k=O(poly(n))}\rangle)$

If Φ is not approximate efficient universal (with δ=0) then δ>0 doesn't help

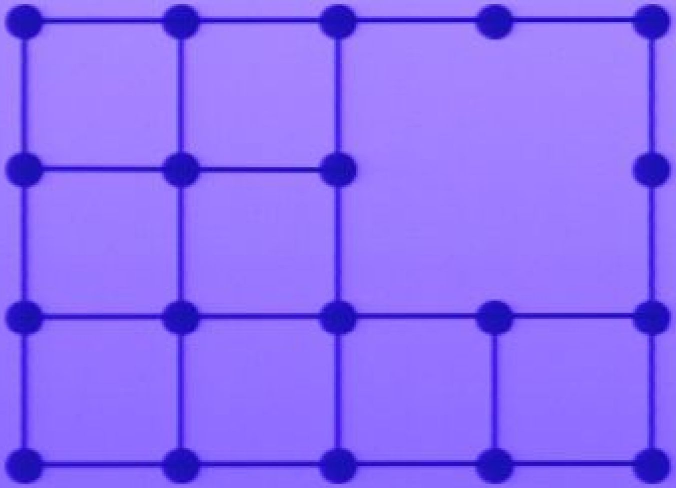


$$E(\Phi) \geq (1-\delta) E_\varepsilon^*$$

Good news (something works I)

(Cluster state with holes)

Suppose we want to use cluster states: $\Psi_C = \{|C_n\rangle\}_n$



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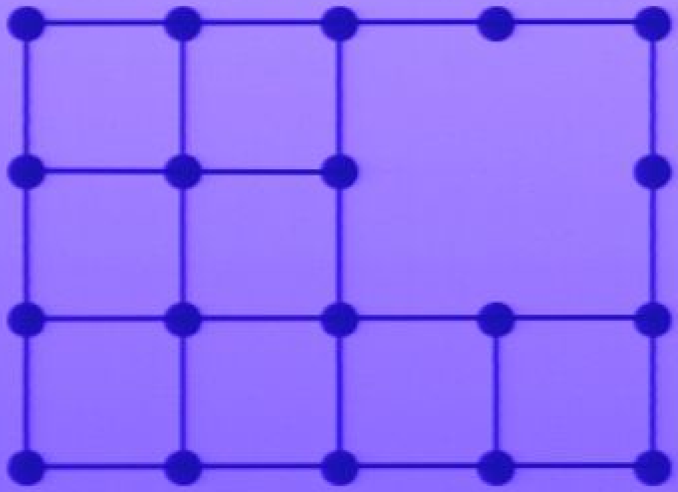

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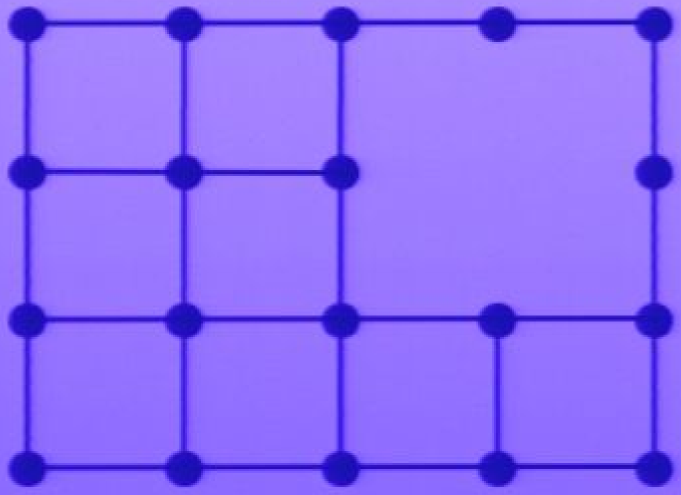

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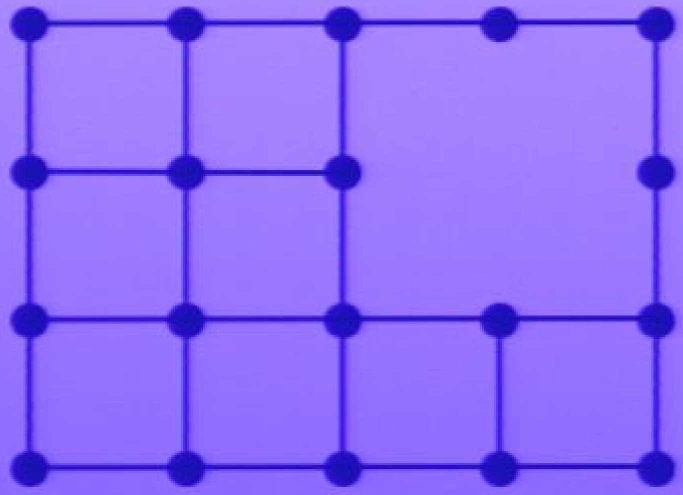

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Exact deterministic computation is <u>not possible</u>

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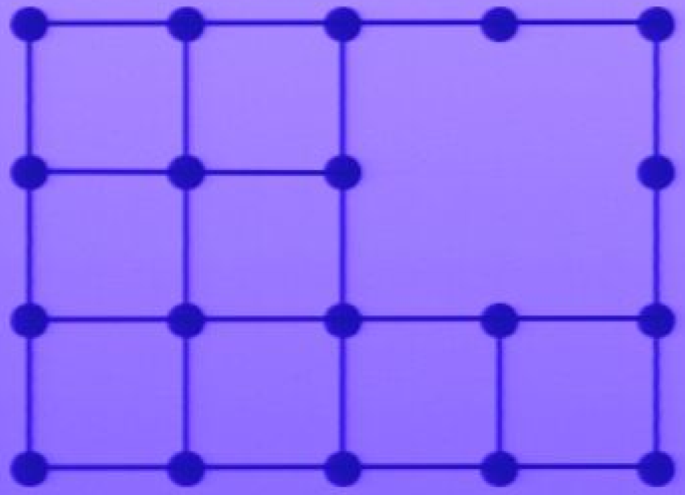

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Assume: we know where the holes are

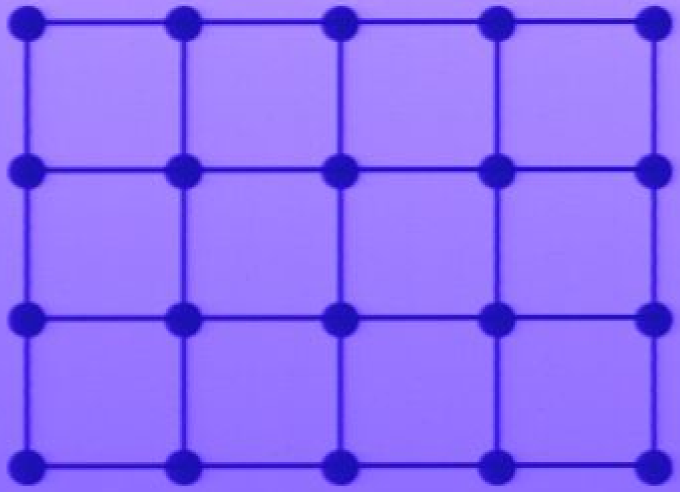
Exact deterministic computation is <u>not possible</u>

<u>Still useful!</u> Universal resource for exact δ-stochastic computation *for all δ >0*

Good news (something works II)

(Stability of universal resources)

Suppose we again want to use cluster states: $\Psi_C=\{|C_n\rangle\}_n$



Assume $\mathcal{D}(\sigma_{n,} C_n) \leq \eta$

The protocol does not increase the damage too much
(LOCC operations don't increase $\mathcal{D}$)

$\Rightarrow$ Most branches will still end up "close enough" to the desired output

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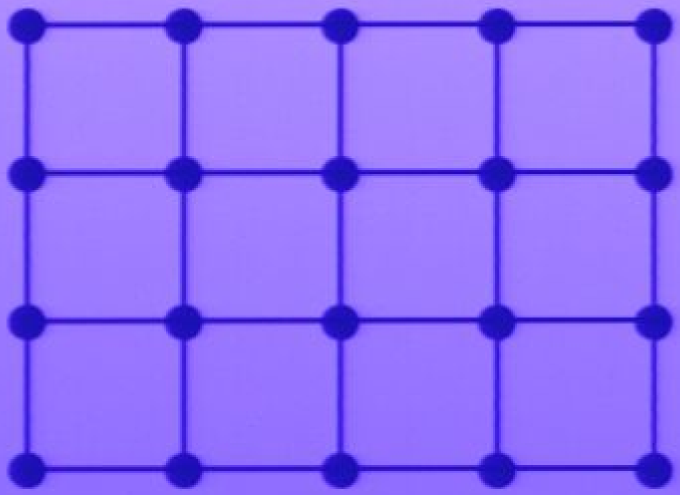

But we are in the real world!

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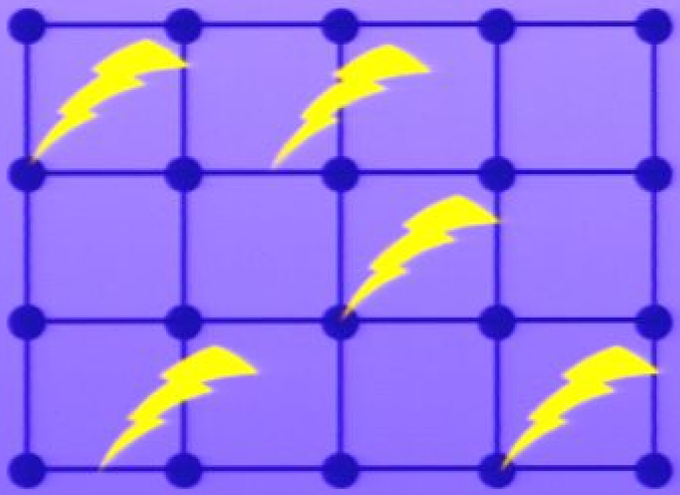

But we are in the real world!

Errors!!!!!!

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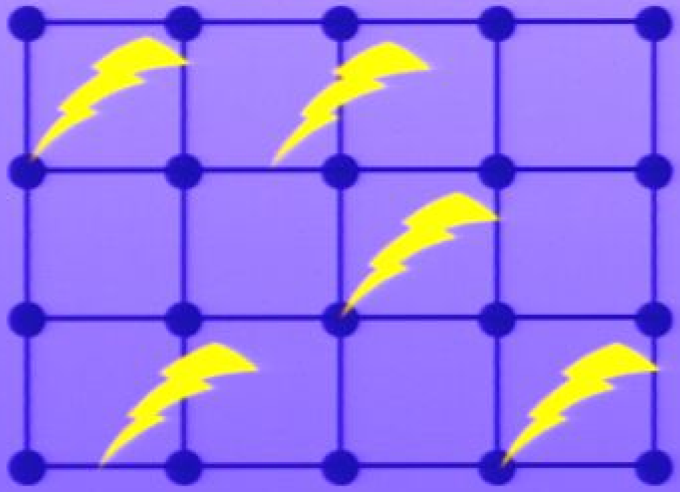

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Errors!!!!!!

⇓

We actually have $\Sigma_C=\{\sigma_n\rangle\}_n$

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Good news (something works IIb)

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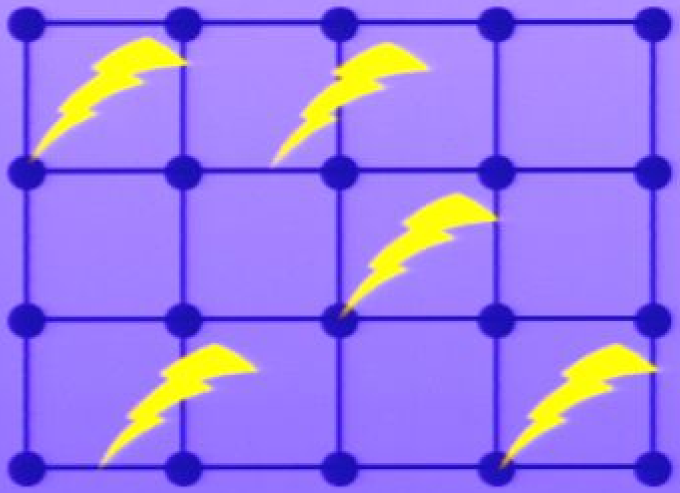

Cluster not so special: argument holds in general

Good news (something works IIb)

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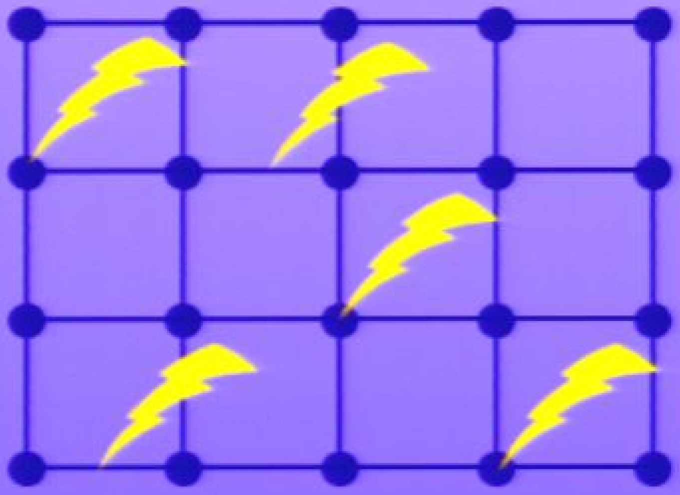

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↑

ε-approximate
δ-stochastic

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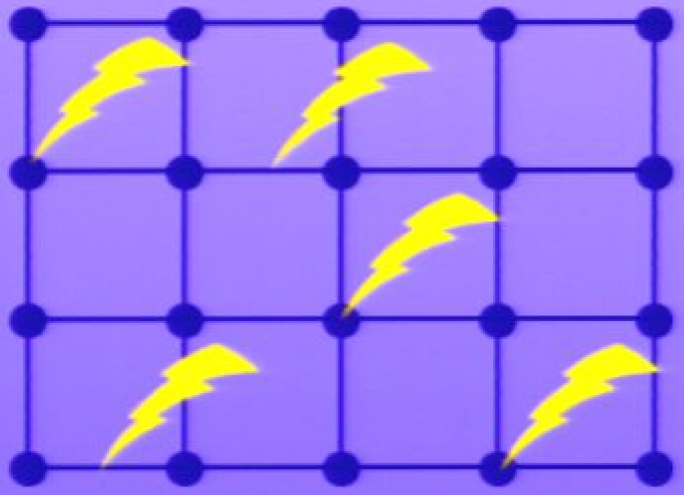

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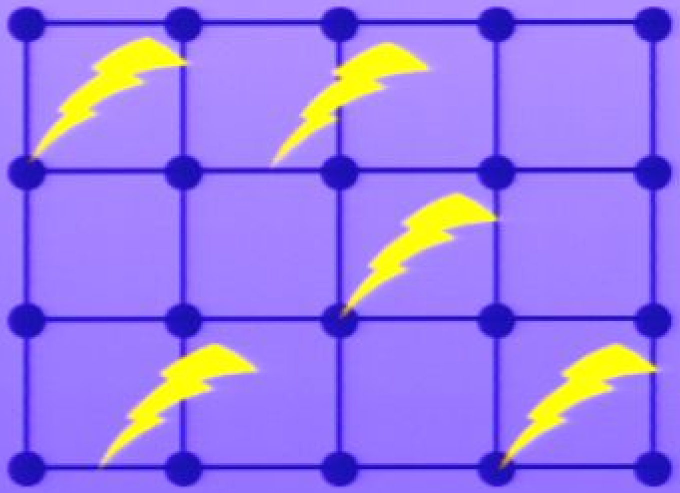

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δ-stochastic

$\Rightarrow$ Σ *is still approximate stochastic universal*

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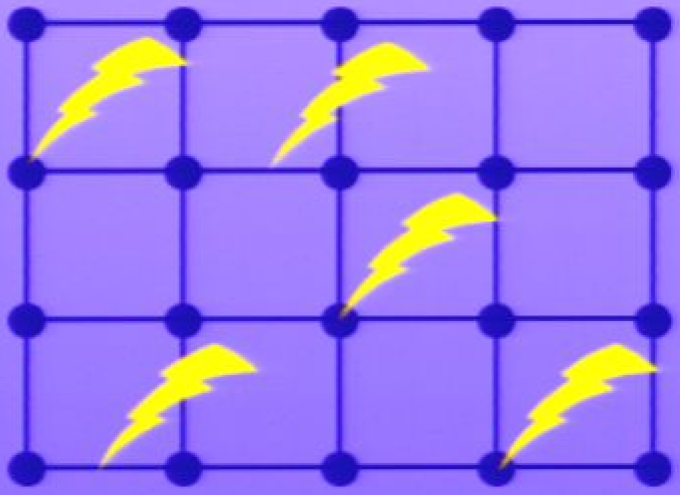

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New parameters δ', ε' *such that* $\delta'\varepsilon'\geq\delta+\varepsilon+\eta$

Good news (something works IIb)

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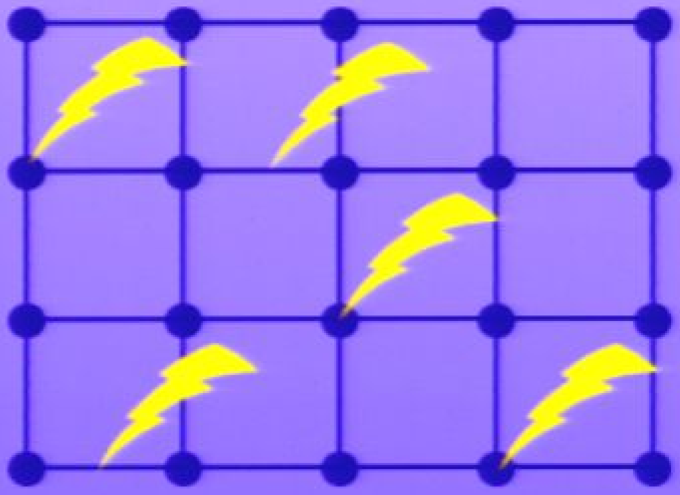

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ε-approximate
δ-stochastic

Universal resources are stable under perturbations

⇒ Σ is still approximate stochastic univer

New parameters δ', ε' such that $\delta'\varepsilon' \geq \delta+\varepsilon+\eta$

Conclusions

- Defined universality for quantum computation
  - Universal exact "state preparators"
  - Approximate and stochastic universality
  - Efficiency and universality
- Constructed entanglement-based criteria for URs
  - Criteria in the exact and deterministic case
  - ε-measures and generalized criteria
  - Examples of applications
- "Go" results for universal families
  - Deformed cluster
  - Approximate resources

Exact case

M. Van den Nest, A. Miyake,
W. Duer, H.-J. Briegel,
New J. Phys 9, 204 (2007)

Approximate/stochastic

C.M., M. Piani, and all above
On the arXiv (sooner or later)

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Thank you

Welcome barbecue this afternoon!

475 Wes Graham way

PI

Shuttle from PI leaving at 4.15pm (ask Mike!)

In the new temporary IOC building

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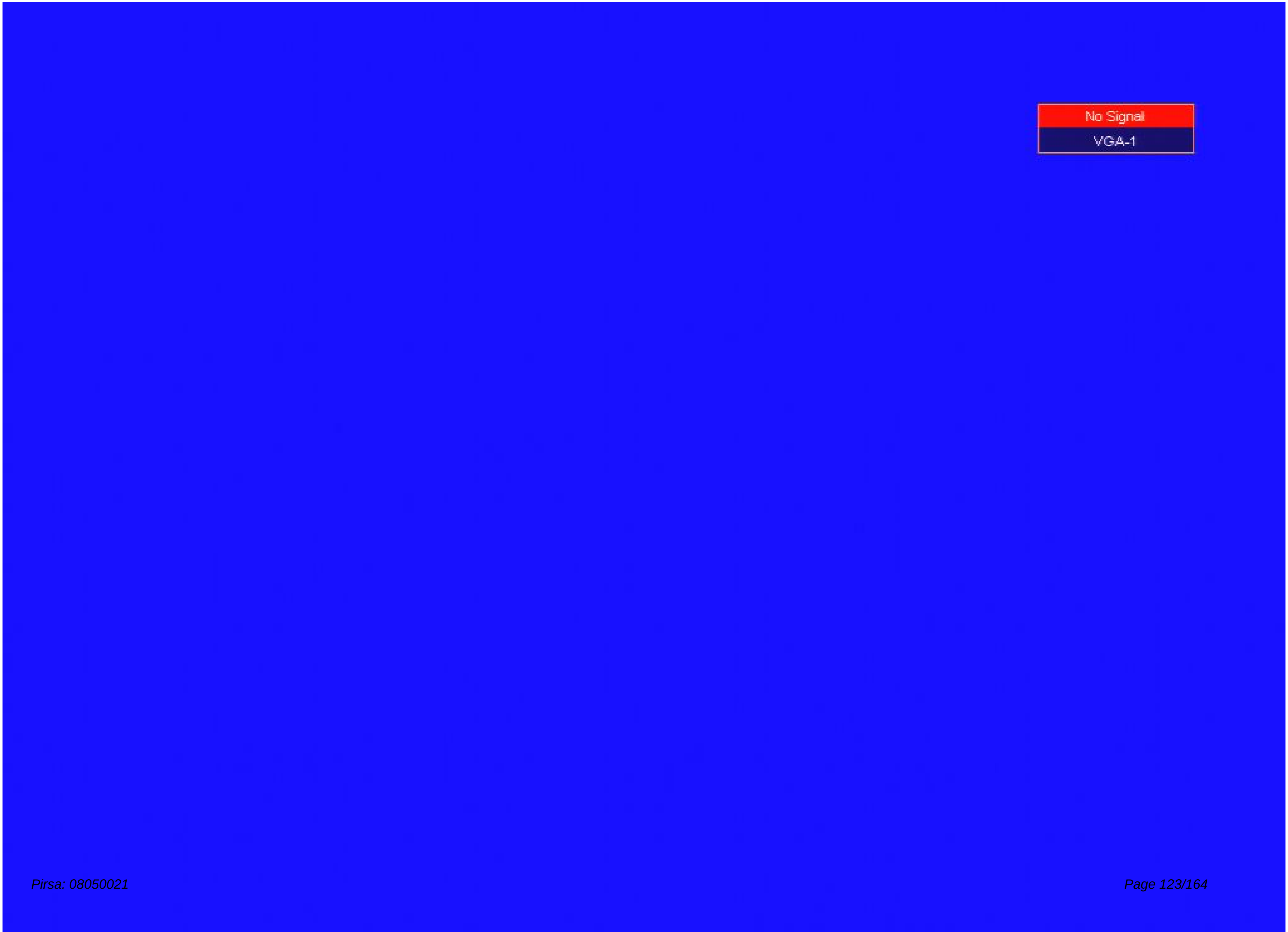
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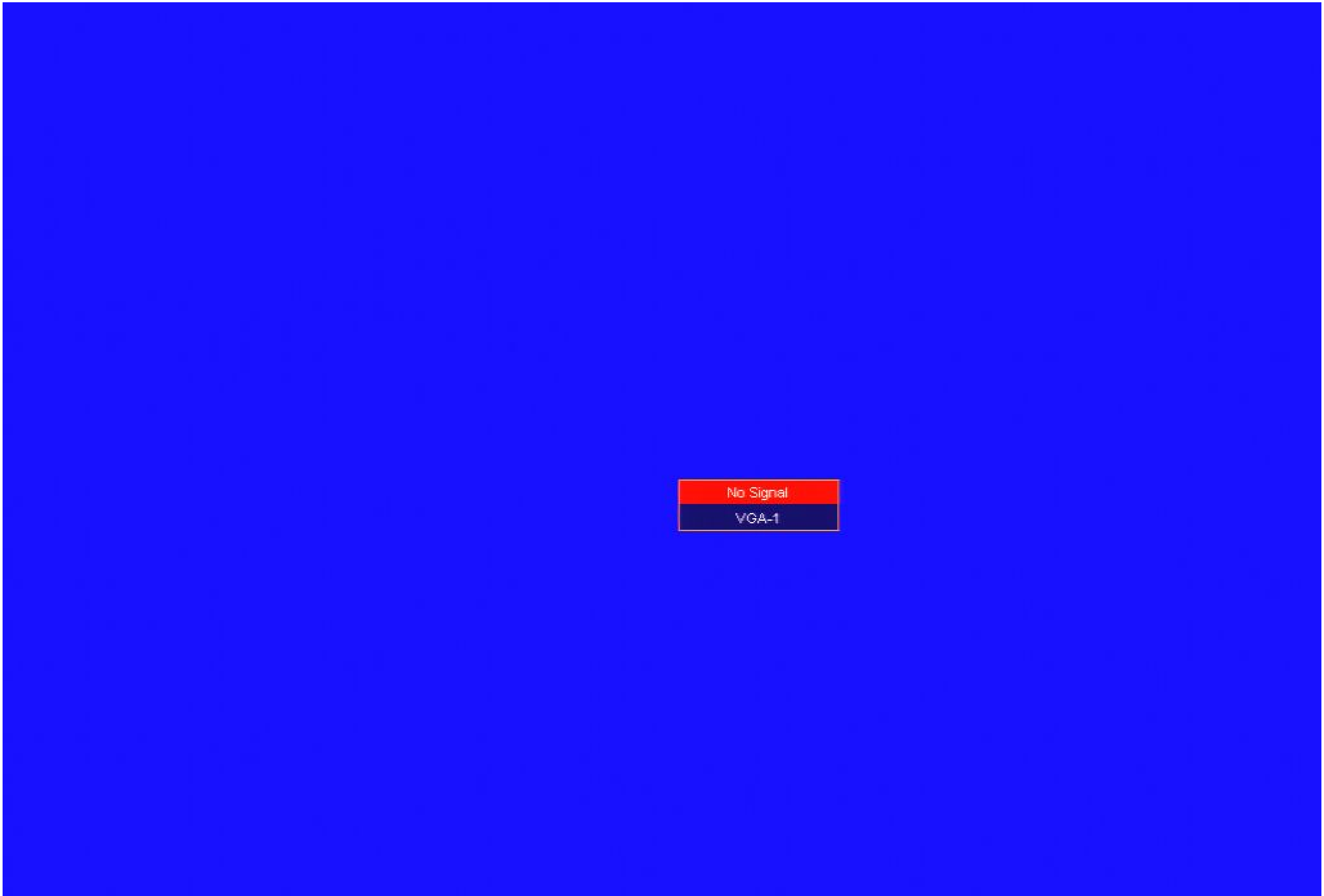

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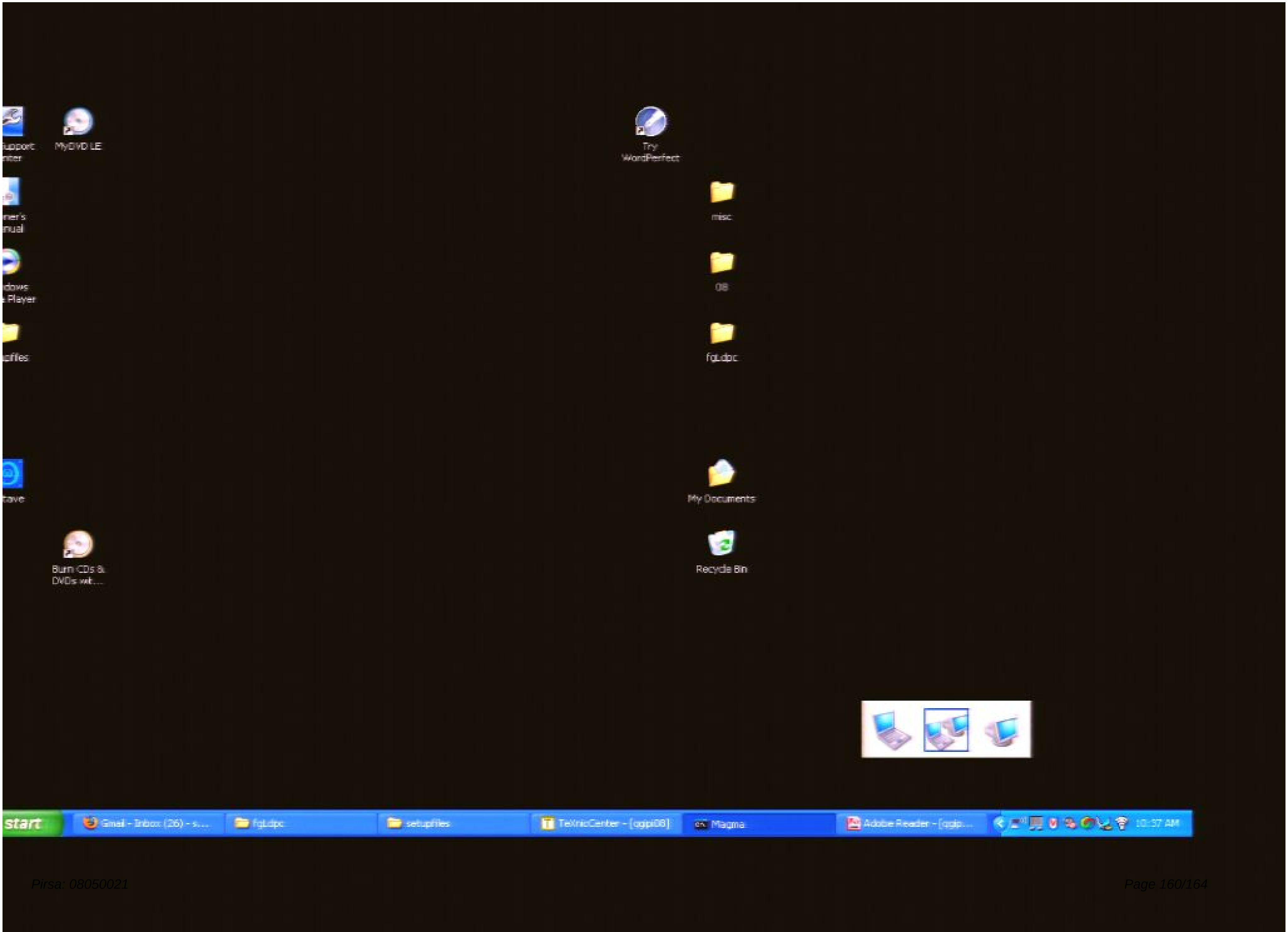
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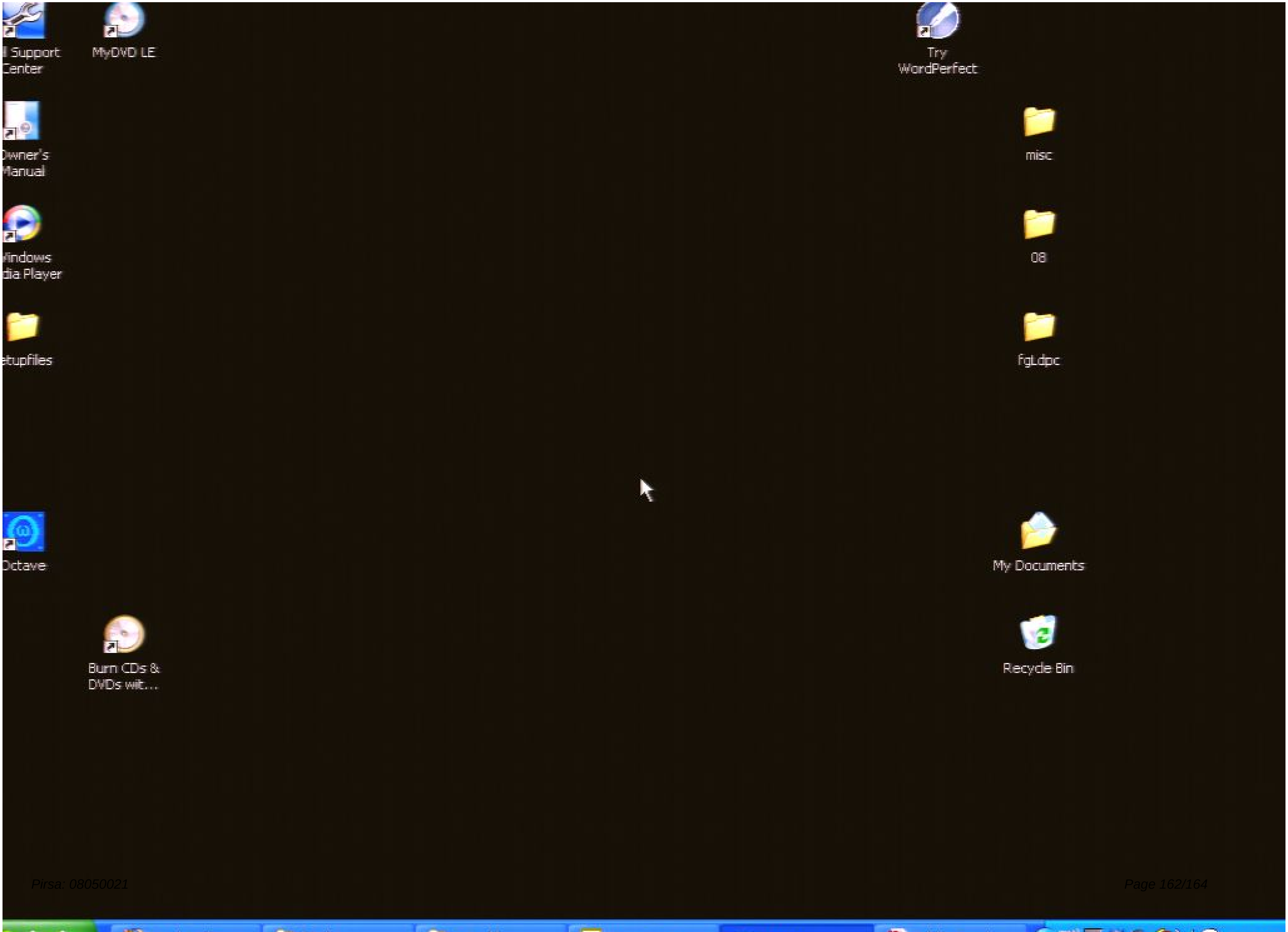
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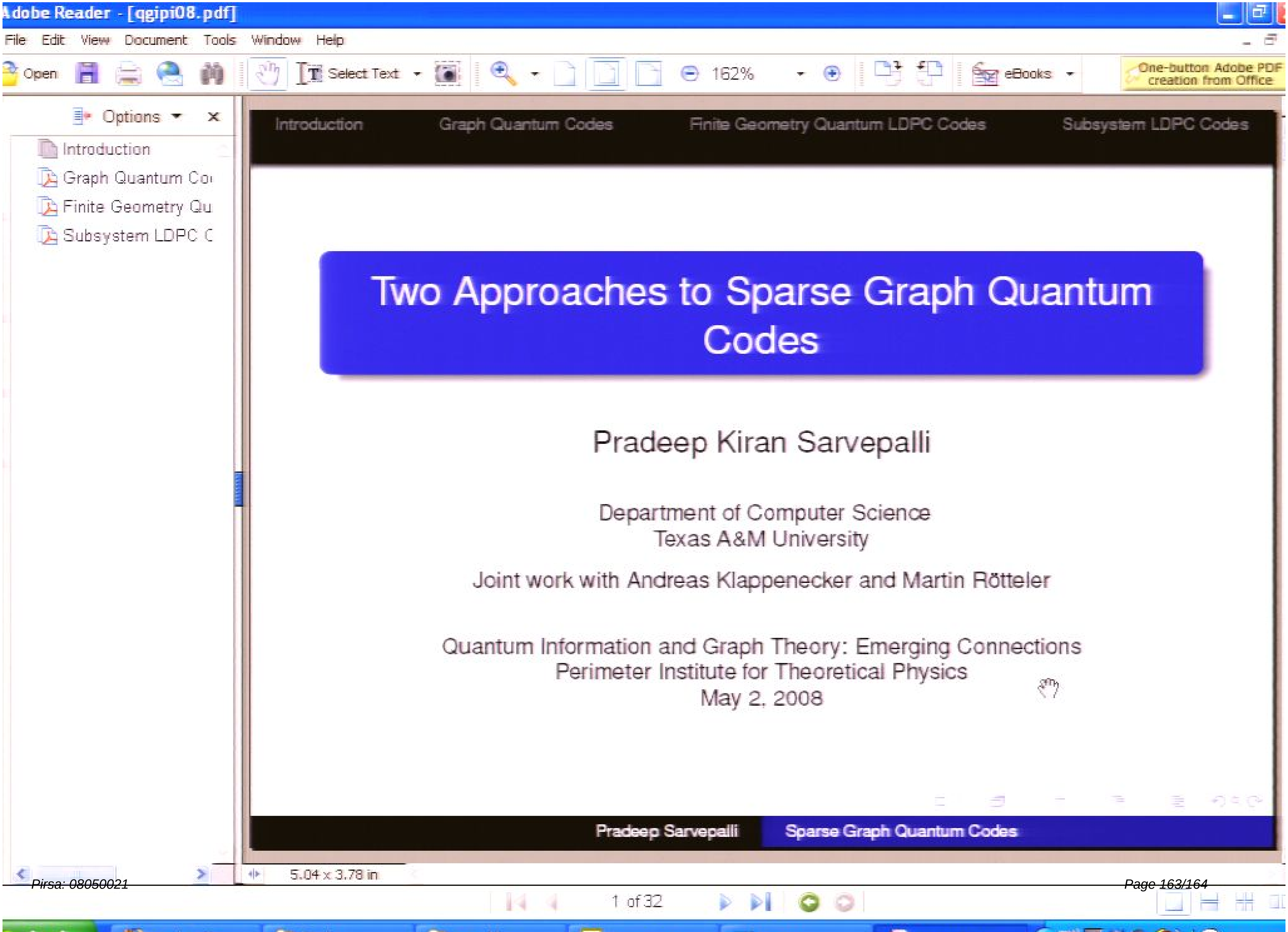

Two Approaches to Sparse Graph Quantum Codes

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Joint work with Andreas Klappenecker and Martin Rötteler

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