Title: Universal graph states from the optical frequency comb

Date: May 01, 2008 10:10 AM

URL: http://pirsa.org/08050019

Abstract: One-way quantum computing allows any quantum algorithm to be implemented by the sole use of single-qubit measurements. The difficult part is to create a universal resource state on which the measurements are made. We propose to use continuous-variable (CV) entanglement in the optical frequency comb of a single optical parametric oscillator with a multimode pump to produce a very large CV graph state with a special 4-regular graph. This scheme is interesting because of its potential for scalability, although issues of error correction and fault tolerance are yet to be fully addressed. Other possible physical configurations that are achievable with this scheme are related to the existence of certain bipartite edge-weighted graphs with circulant support having orthogonal adjacency matrices. If the above description fails to move you, don\'t worry, there will be pretty pictures. Joint work with N. Menicucci and O. Pfister, and with S. Severini

Universal graph states from the optical frequency comb

Steven T. Flammia
Perimeter Institute

QIGT, 1 May 2008

Joint work with N. Menicucci & O. Pfister, and S. Severini

Universal graph states from the optical frequency comb

Steven T. Flammia
Perimeter Institute

QIGT, 1 May 2008

Joint work with N. Menicucci & O. Pfister, and S. Severini

Cluster states: qubits vs. continuous variables (CV)

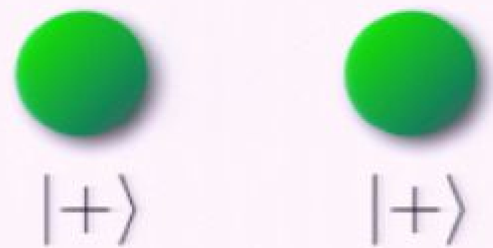
qubit cluster states

Raussendorf & Briegel PRL 2001

Cluster states: qubits vs. continuous variables (CV)

qubit cluster states

- prepare X eigenstates

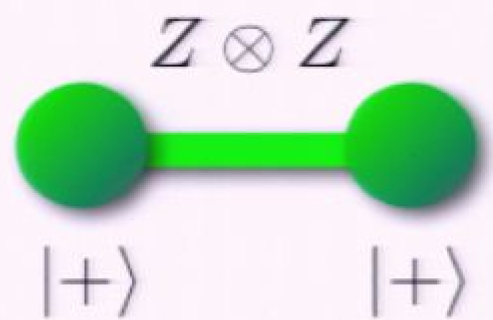


Raussendorf & Briegel PRL 2001

Cluster states: qubits vs. continuous variables (CV)

qubit cluster states

- prepare X eigenstates
- entangle neighbors with a Z-Z coupling

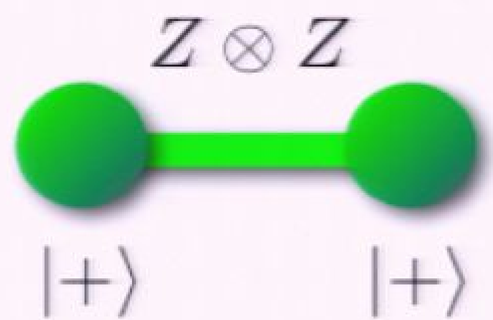


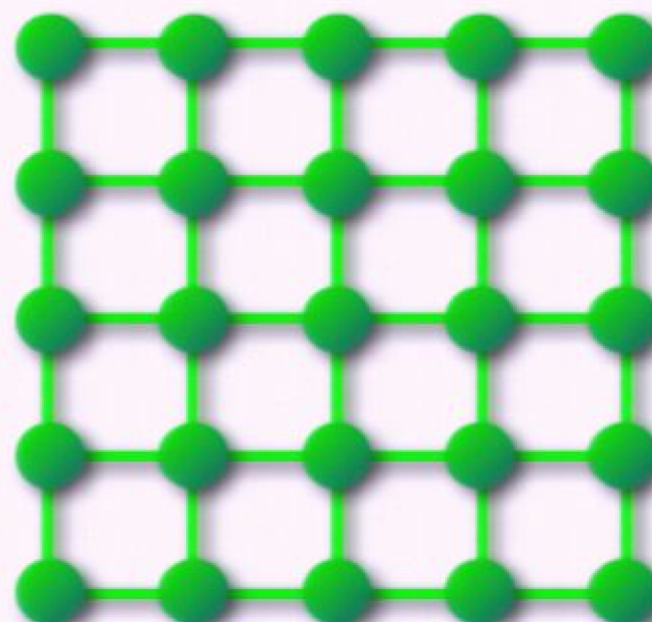
Raussendorf & Briegel PRL 2001

Cluster states: qubits vs. continuous variables (CV)

qubit cluster states

- prepare X eigenstates
- entangle neighbors with a Z-Z coupling
- arbitrary single-qubit measurements with feedforward on a large lattice for universality



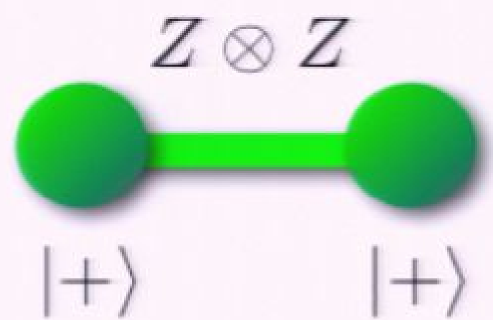


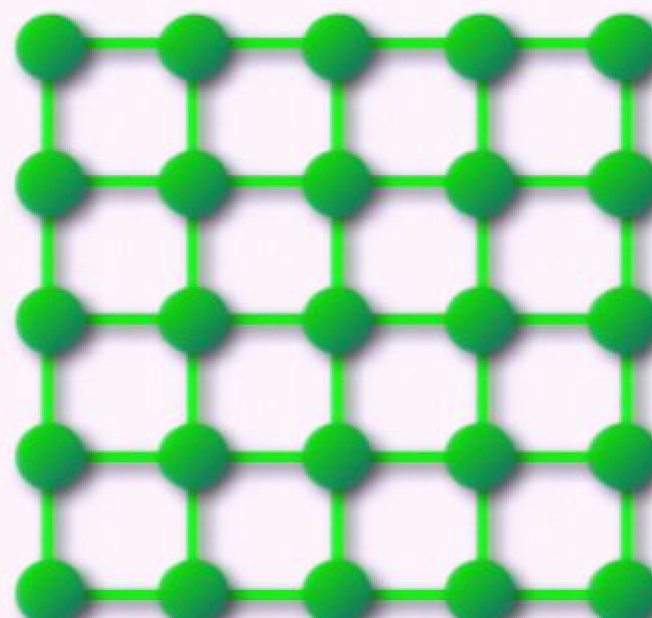
Raussendorf & Briegel PRL 2001

Cluster states: qubits vs. continuous variables (CV)

qubit cluster states

- prepare X eigenstates
- entangle neighbors with a Z-Z coupling
- arbitrary single-qubit measurements with feedforward on a large lattice for universality
- Clifford measurements can be done in any order





Raussendorf & Briegel PRL 2001

Cluster states: qubits vs. continuous variables (CV)

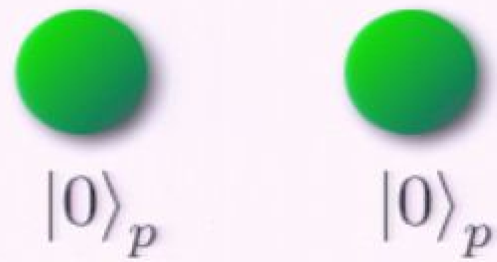
CV cluster states

Lloyd & Braunstein PRL 99; Zhang & Braunstein PRA 06; Menicucci et. al. PRL 06

Cluster states: qubits vs. continuous variables (CV)

CV cluster states

- prepare zero-momentum eigenstates



Lloyd & Braunstein PRL 99; Zhang & Braunstein PRA 06; Menicucci et. al. PRL 0

Cluster states: qubits vs. continuous variables (CV)

CV cluster states

- prepare zero-momentum eigenstates
- entangle neighbors with a q-q two-mode squeezing operation

Lloyd & Braunstein PRL 99; Zhang & Braunstein PRA 06; Menicucci et. al. PRL 06

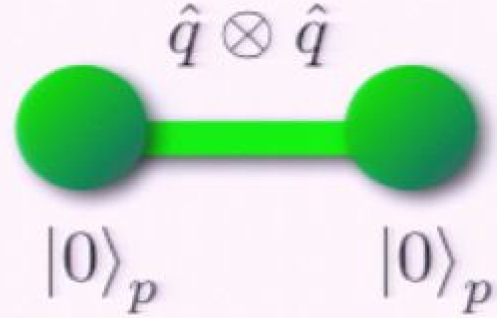
Cluster states: qubits vs. continuous variables (CV)

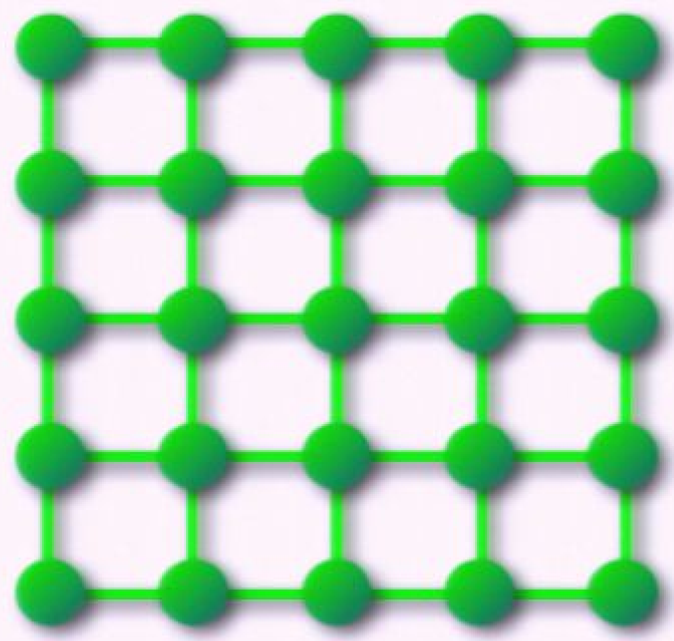
CV cluster states

- prepare zero-momentum eigenstates

- entangle neighbors with a q-q two-mode squeezing operation

- finite set of single-mode measurements with feedforward on a large lattice for universality



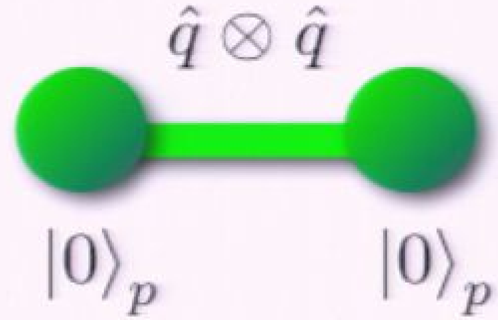


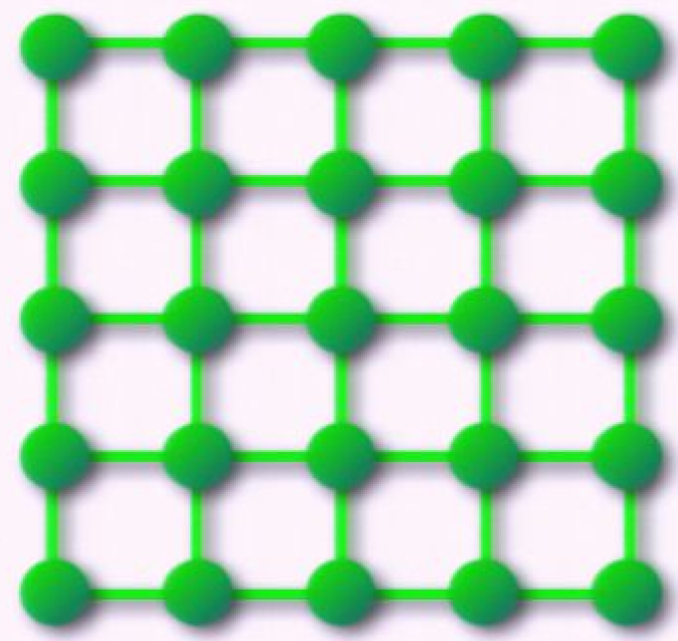
Lloyd & Braunstein PRL 99; Zhang & Braunstein PRA 06; Menicucci et. al. PRL 06

Cluster states: qubits vs. continuous variables (CV)

CV cluster states

- prepare zero-momentum eigenstates
- entangle neighbors with a q-q two-mode squeezing operation
- finite set of single-mode measurements with feedforward on a large lattice for universality
- Gaussian operations can be done in any order





Lloyd & Braunstein PRL 99; Zhang & Braunstein PRA 06; Menicucci et. al. PRL 06

Advantages of continuous-variable clusters

- unconditional state preparation

Gaussian transformations on the vacuum can be performed deterministically, so there is no need to do "fusion" of clusters. (Although this would be interesting...)

- well-established experimental infrastructure

While photon counting is still required, addressability is less of an issue compared to e.g. optical lattice schemes

Nielsen PRL 04; Browne & Rudolph PRL 05; Kok et. al. RMP 07; Kieling, Gross, Eisert J. Opt. Soc. Am. B 07; ...

Why this will NEVER work

- **finite squeezing** — Infinitely squeezed states are not physical. Finite squeezing effects will tend to degrade the cluster as the computation progresses.

- **error correction** — Decoherence isn't so much an issue, but photon loss is a problem. We need good CV error correcting codes.

- **fault tolerance** — Continuous variables will likely have NO threshold (they are like analog computers). Can we still do interesting things in this setting?

Gottesman, Kitaev, Preskill PRA 01; Glancy & Knill PRA 06

Why we're excited anyway

- simplicity of experiment

We use just a single optical cavity and O(1) modes, which is optimal

- scalability & addressability

Naturally large set of modes in the frequency comb. Use GKP encoding to achieve FT?

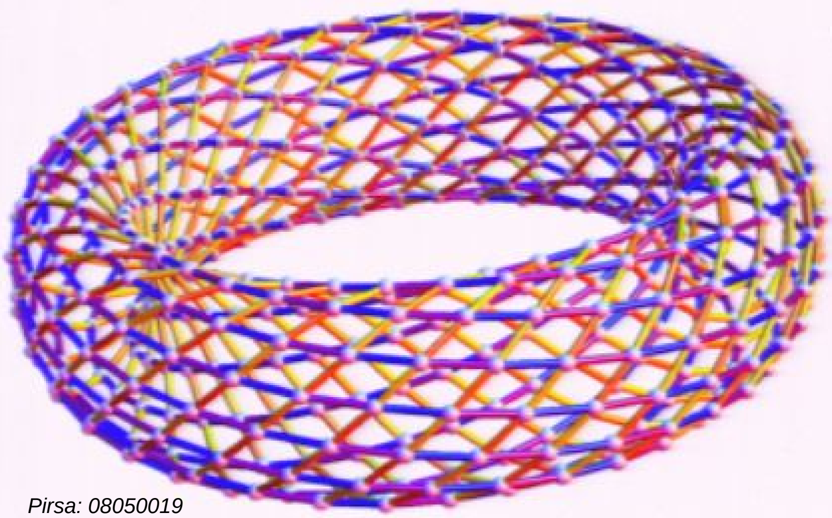
Why we're excited anyway

- simplicity of experiment

We use just a single optical cavity and O(1) modes, which is optimal

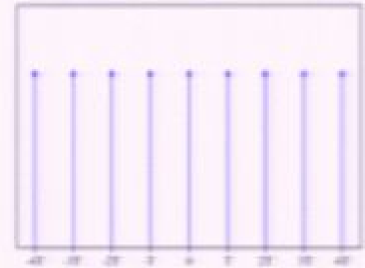
- scalability & addressability

Naturally large set of modes in the frequency comb. Use GKP encoding to achieve FT?



Pretty pictures, donut puns, Homer Simpson jokes, etc.

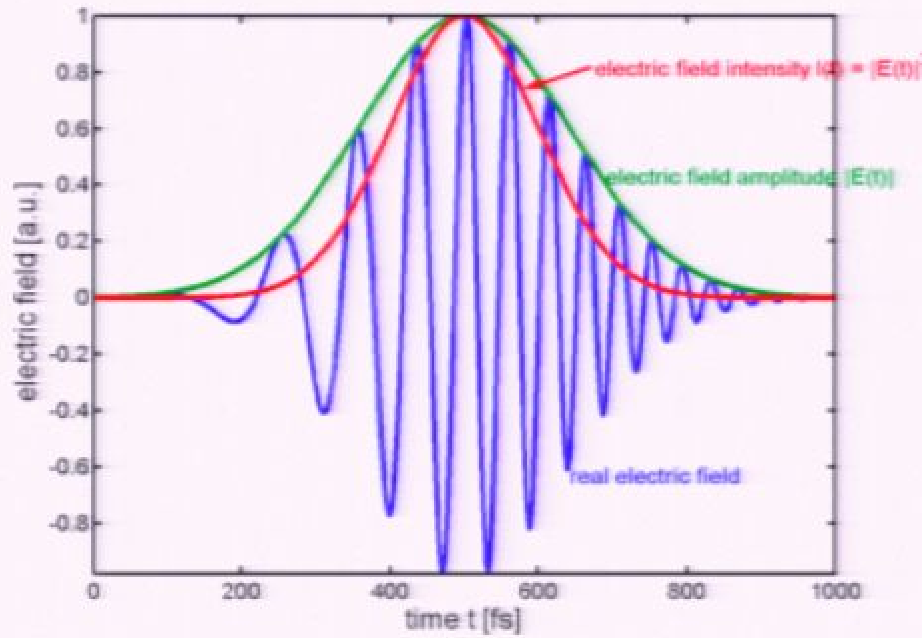
Optical frequency comb



- eigenmodes in an optical cavity yield very well-defined systems with high classical coherence

- inside the cavity is a linear gain medium

- why not look at the quantum case by using a *nonlinear* medium?



Optical parametric oscillators

In the interaction picture, the Hamiltonian is

$$\mathcal{H} = -i\hbar\kappa \sum_{m,n} G_{mn} \left(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n\right)$$

where:

κ Squeezing per time

$\hat{a}_n^\dagger$ Creation operator for mode *n*

G Symmetric matrix of couplings between modes

Optical parametric oscillators

In the interaction picture, the Hamiltonian is

$$\mathcal{H} = -i\hbar\kappa \sum_{m,n} G_{mn} \left(\hat{a}^\dagger_m \hat{a}^\dagger_n - \hat{a}_m \hat{a}_n \right)$$

where:

κ Squeezing per time

$\hat{a}^\dagger_n$ Creation operator for mode *n*

G Symmetric matrix of couplings between modes

example:

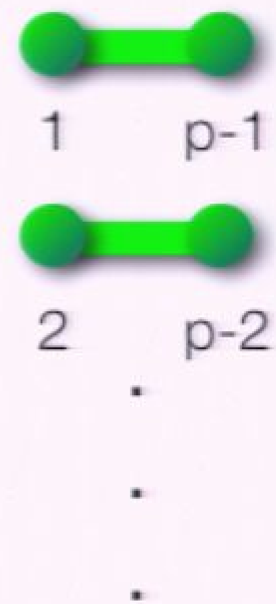
generating multiple entangled pairs

$$\omega_m + \omega_n = \omega_{\text{pump}} \quad \Rightarrow \quad G_{mn} = \begin{cases} +1 & \text{if } m + n = p \\ 0 & \text{otherwise} \end{cases}$$

Optical parametric oscillators

In the interaction picture, the Hamiltonian is

$$\mathcal{H} = -i\hbar\kappa \sum_{m,n} G_{mn} \left(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n \right)$$



where:

κ Squeezing per time

$\hat{a}_n^\dagger$ Creation operator for mode *n*

G Symmetric matrix of couplings between modes

example:

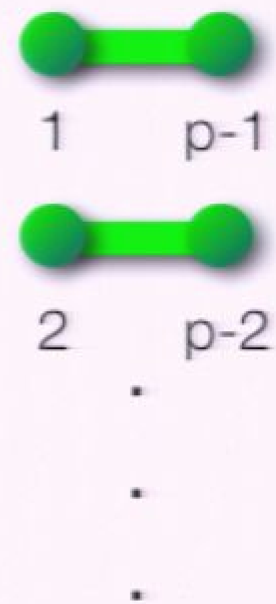
generating multiple entangled pairs

$$\omega_m + \omega_n = \omega_{\text{pump}} \quad \Rightarrow \quad G_{mn} = \begin{cases} +1 & \text{if } m+n=p \\ 0 & \text{otherwise} \end{cases}$$

Optical parametric oscillators

In the interaction picture, the Hamiltonian is

$$\mathcal{H} = -i\hbar\kappa \sum_{m,n} G_{mn} \left(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n \right)$$



where:

κ Squeezing per time

$\hat{a}_n^\dagger$ Creation operator for mode *n*

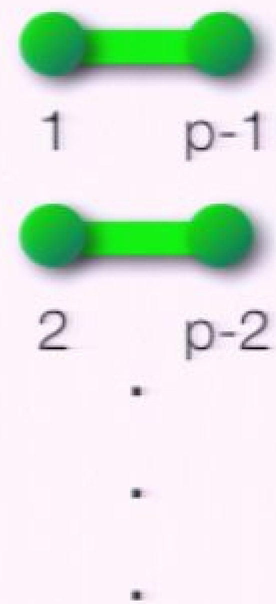
G Symmetric matrix of couplings between modes

$$G = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Optical parametric oscillators

In the interaction picture, the Hamiltonian is

$$\mathcal{H} = -i\hbar\kappa \sum_{m,n} G_{mn} \left(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n \right)$$



where:

κ Squeezing per time

$\hat{a}_n^\dagger$ Creation operator for mode *n*

G Symmetric matrix of couplings between modes

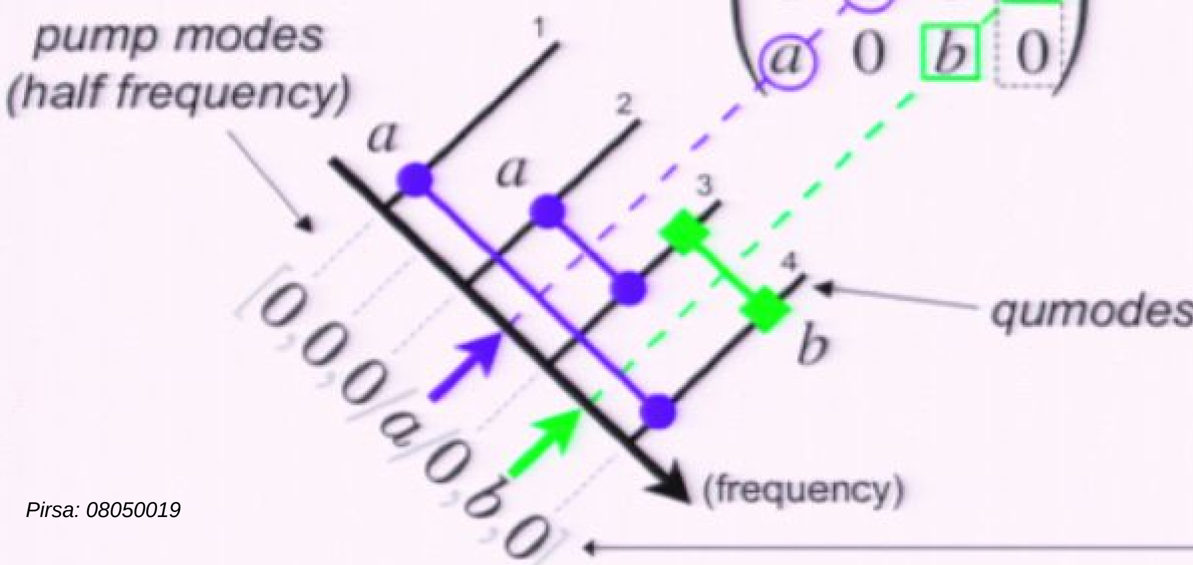
$$G = \begin{pmatrix} & & & 1 & \\ & & 1 & & \\ & 1 & & & \\ 1 & & & & \\ & & & & \end{pmatrix}$$

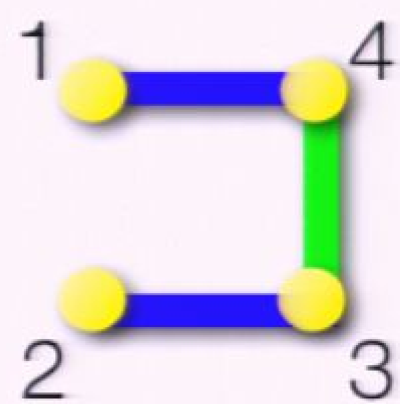
Multimode optical parametric oscillators

With more than one input, the Hamiltonian is

$$\mathcal{H} = i\hbar\kappa \sum_{p\in P} \sum_{m+n=p} G_{mn}(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n),$$

$$G = \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & a & 0 \\ 0 & a & 0 & b \\ a & 0 & b & 0 \end{pmatrix} = \underbrace{[0,0,0/a/0,b,0]}$$

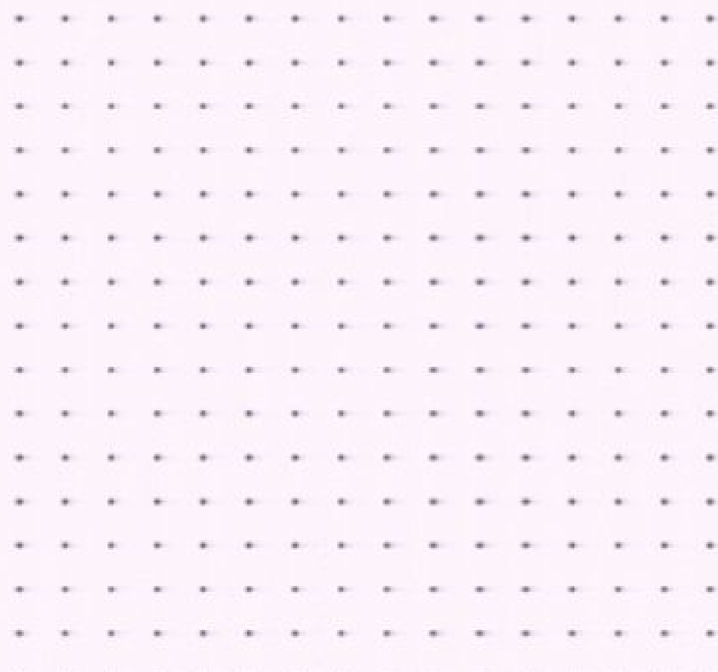




MOVIE S1: ultracompact experimental implementation of a graph quantum state

(Click to advance movie)

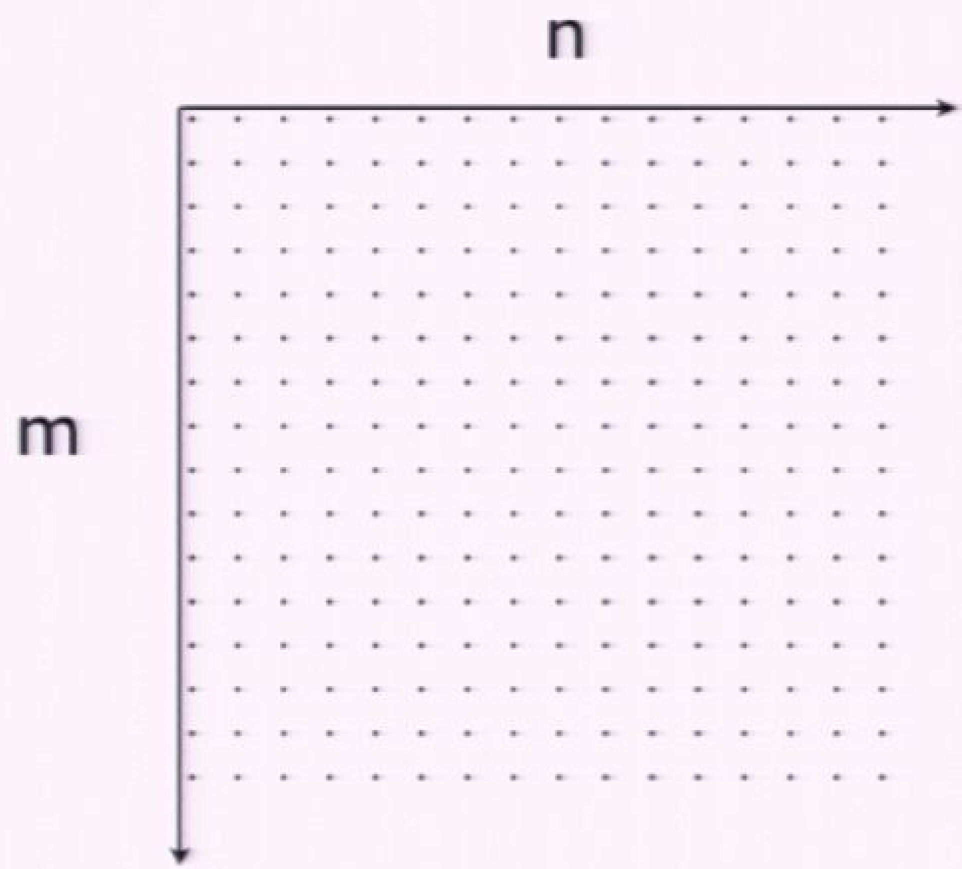
Consider the <u>adjacency matrix</u> A of a graph state to be created



MOVIE S1: ultracompact experimental implementation of a graph quantum state

(Click to advance movie)

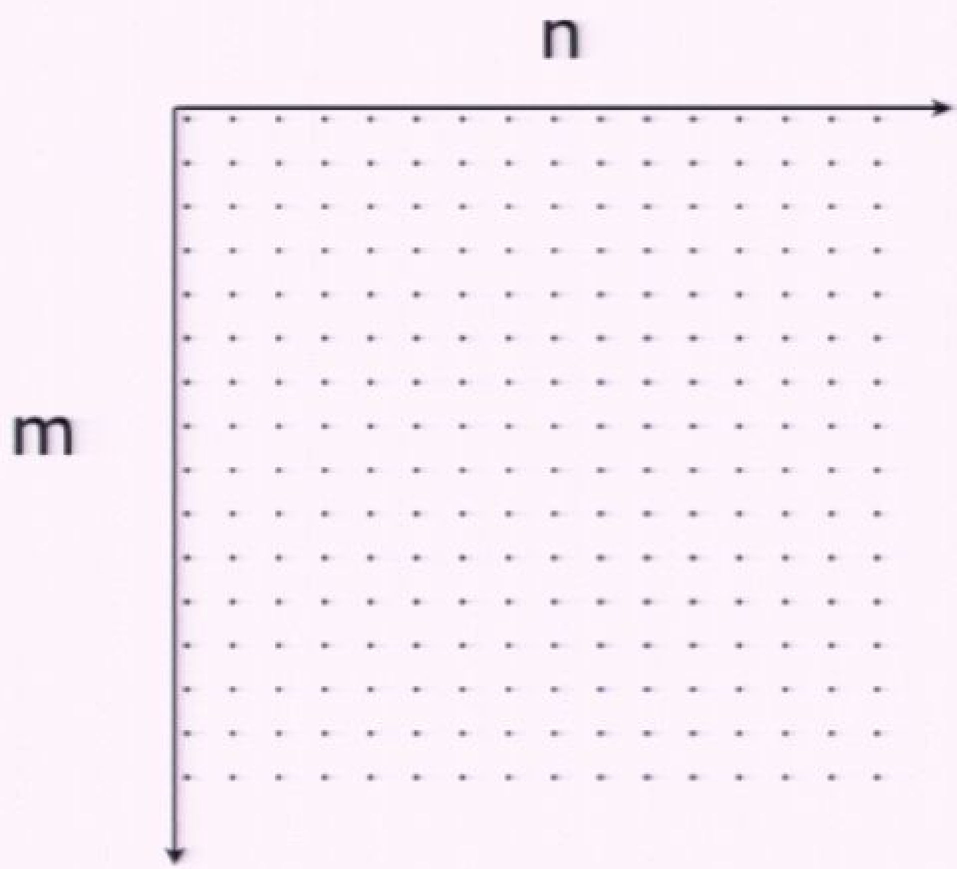
Consider the adjacency matrix A of a graph state to be created



MOVIE S1: ultracompact experimental implementation of a graph quantum state

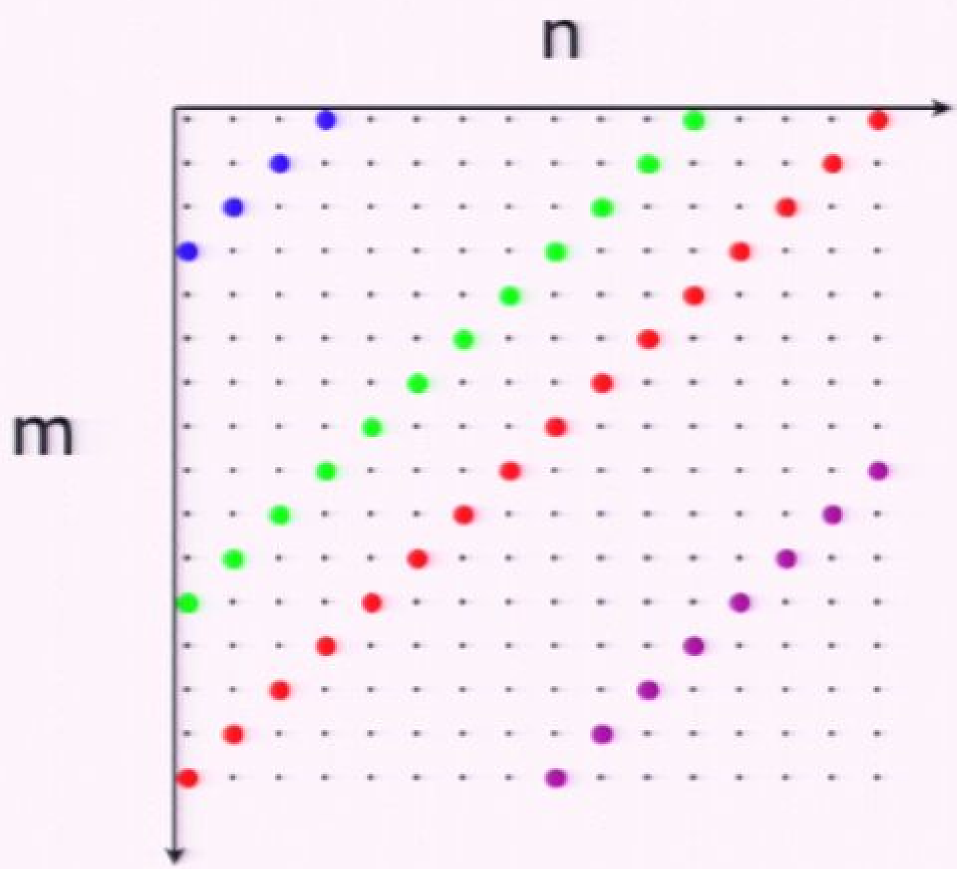
(Click to advance movie)

Consider the <u>adjacency matrix</u> A of a graph state to be created

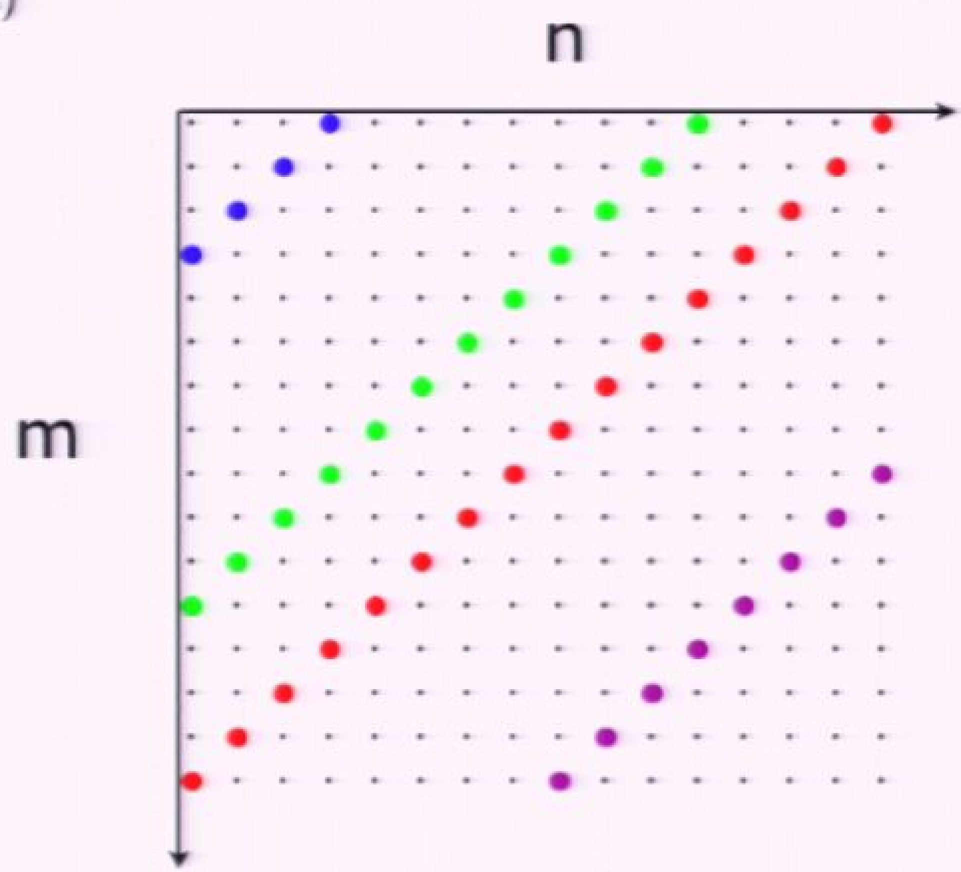


Matrix element A_{mn} gives the entangling strength of edge (m,n) between vertices m and n, i.e. between OFC qumodes of frequencies ω_m and ω_n.

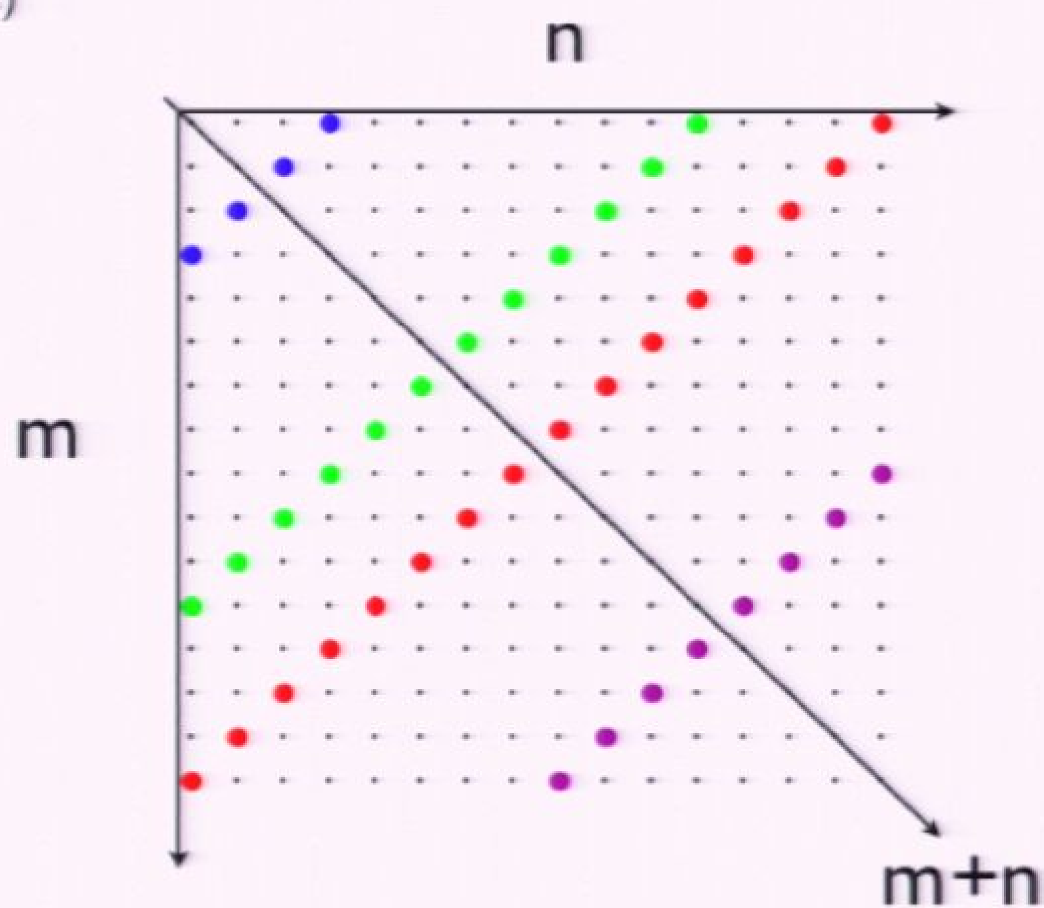
Moreover, we restrict A to Hankel matrices,

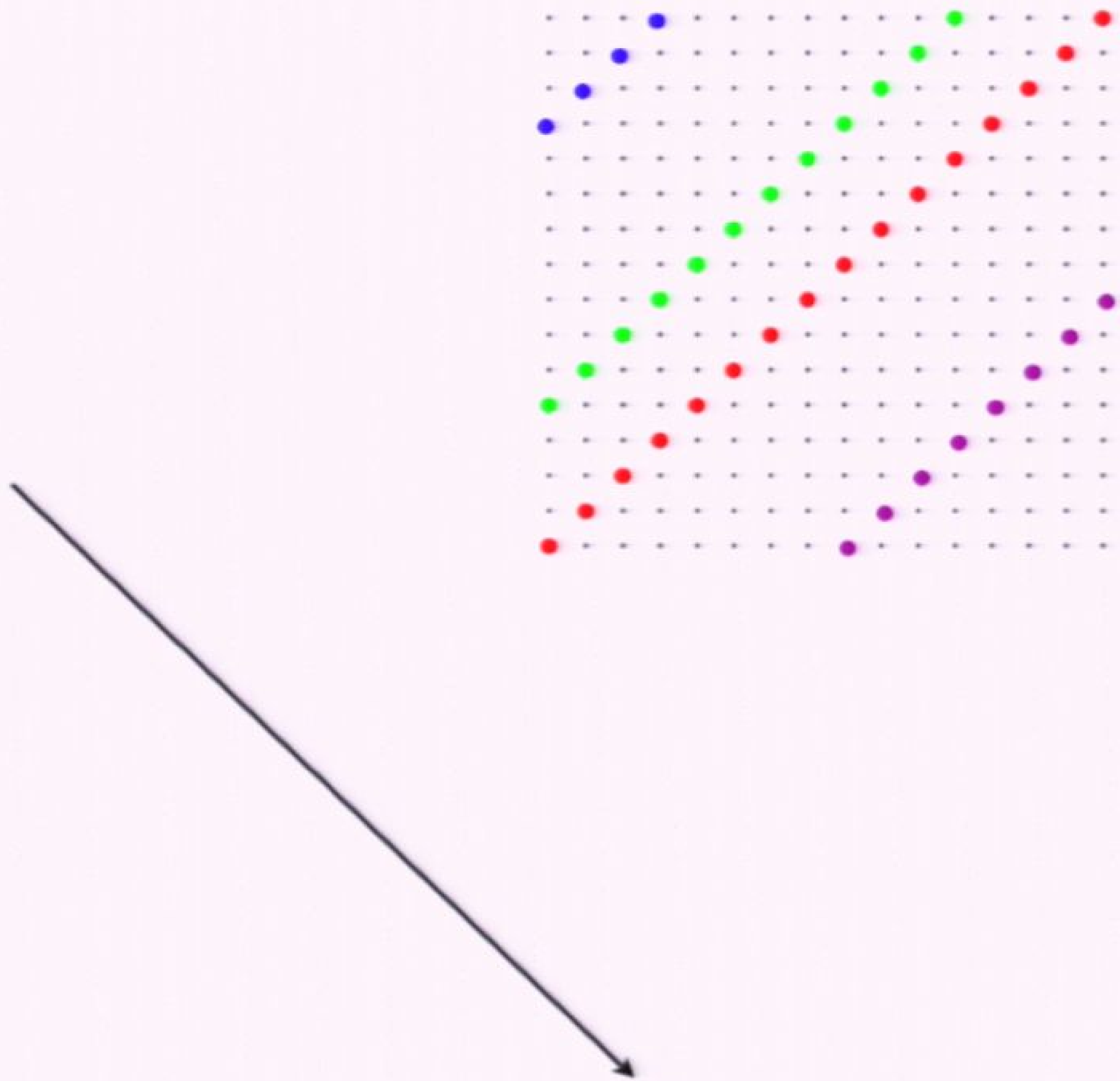


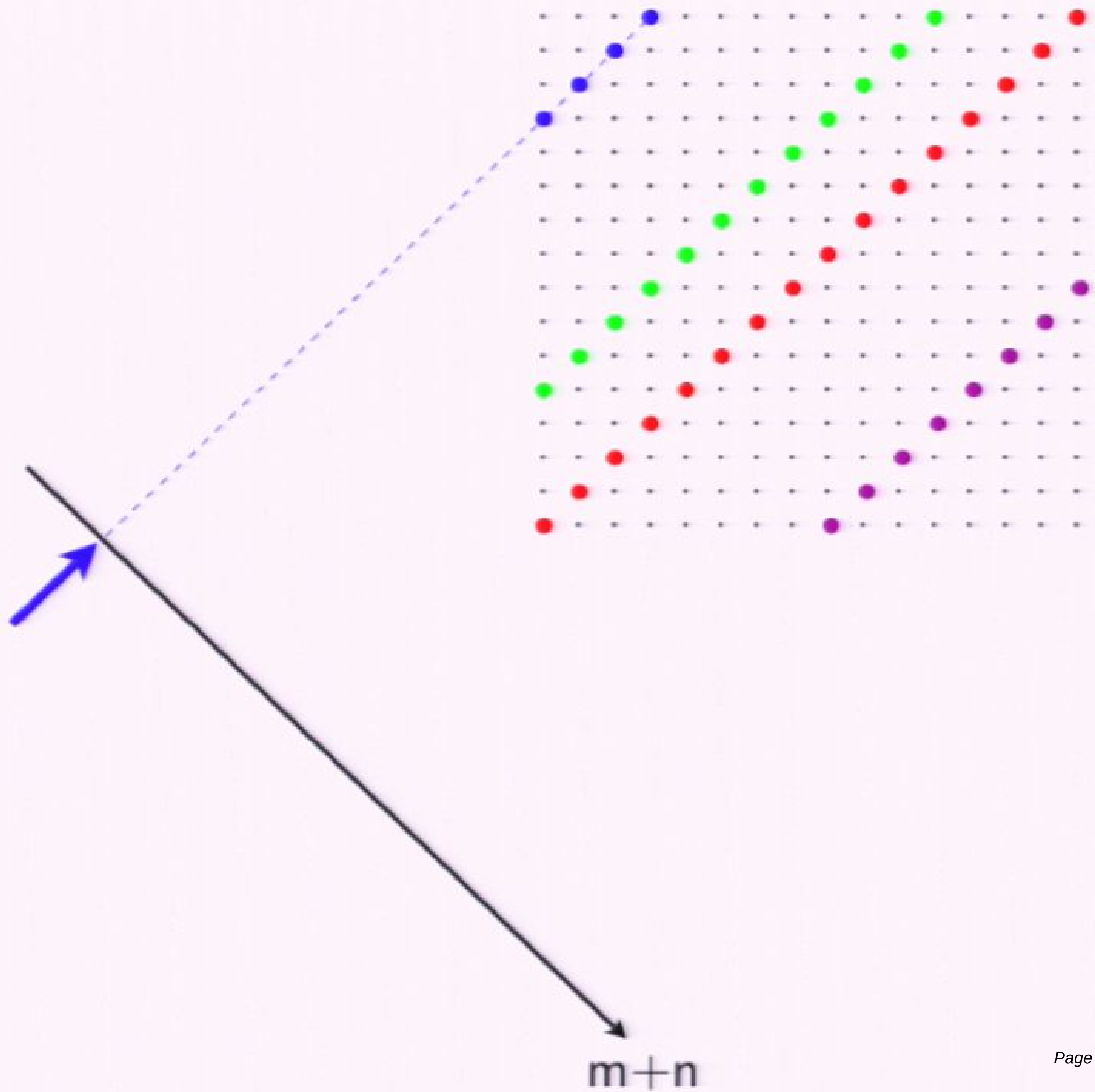
Moreover, we restrict A to Hankel matrices, whose skew diagonals are labeled by m+n (frequency sum index)

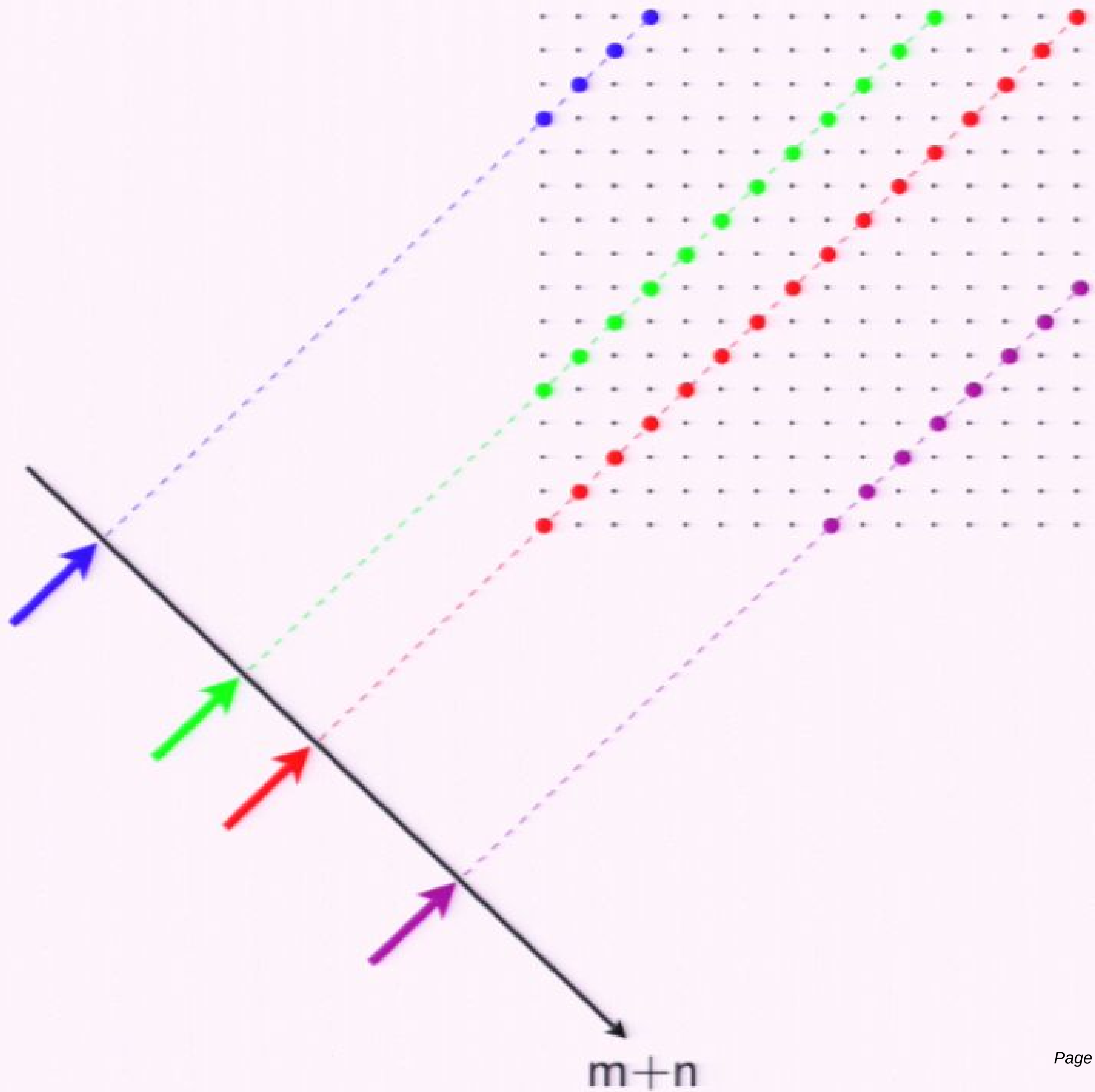


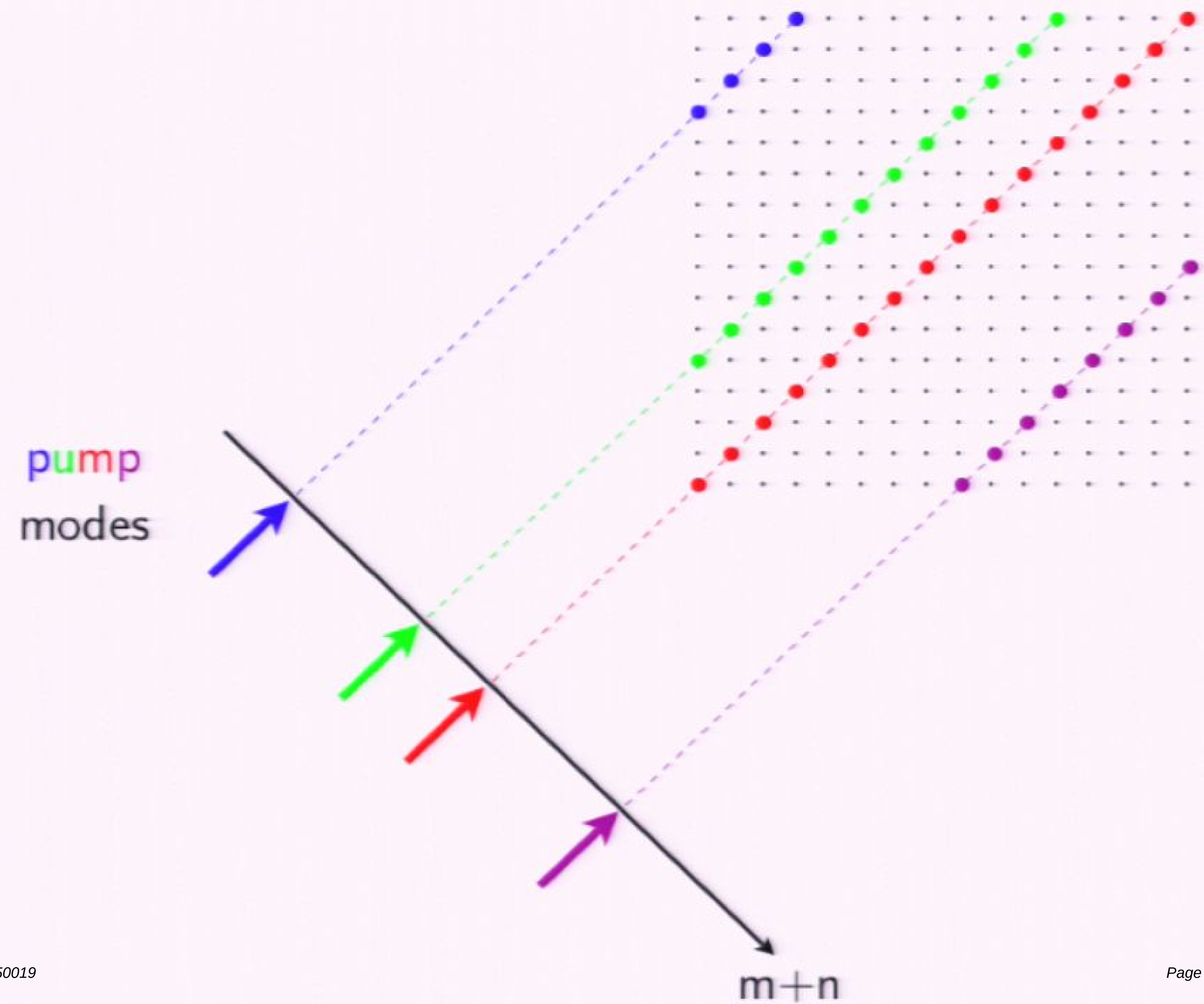
Moreover, we restrict A to Hankel matrices, whose skew diagonals are labeled by m+n (frequency sum index)

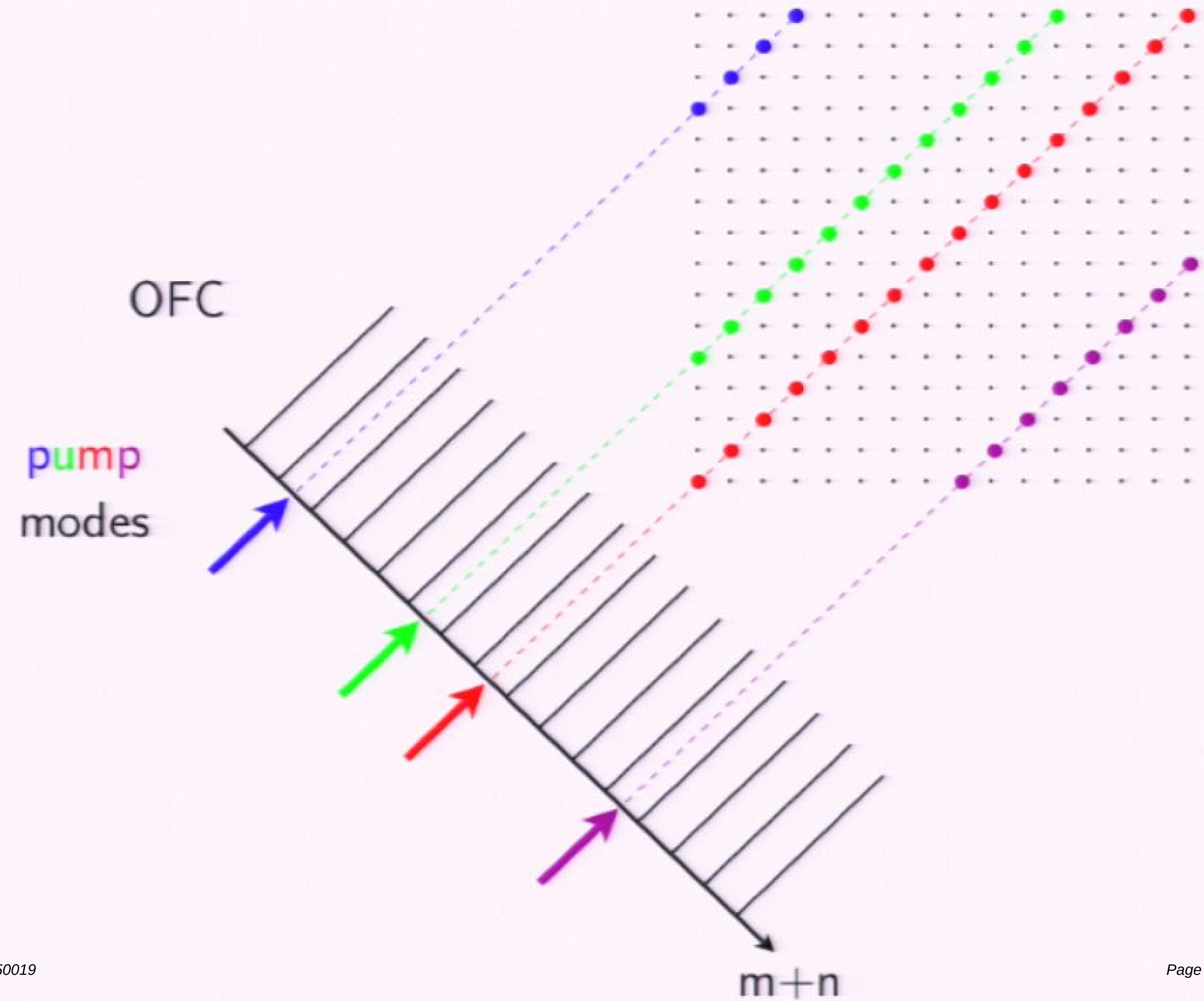



m+n


m+n

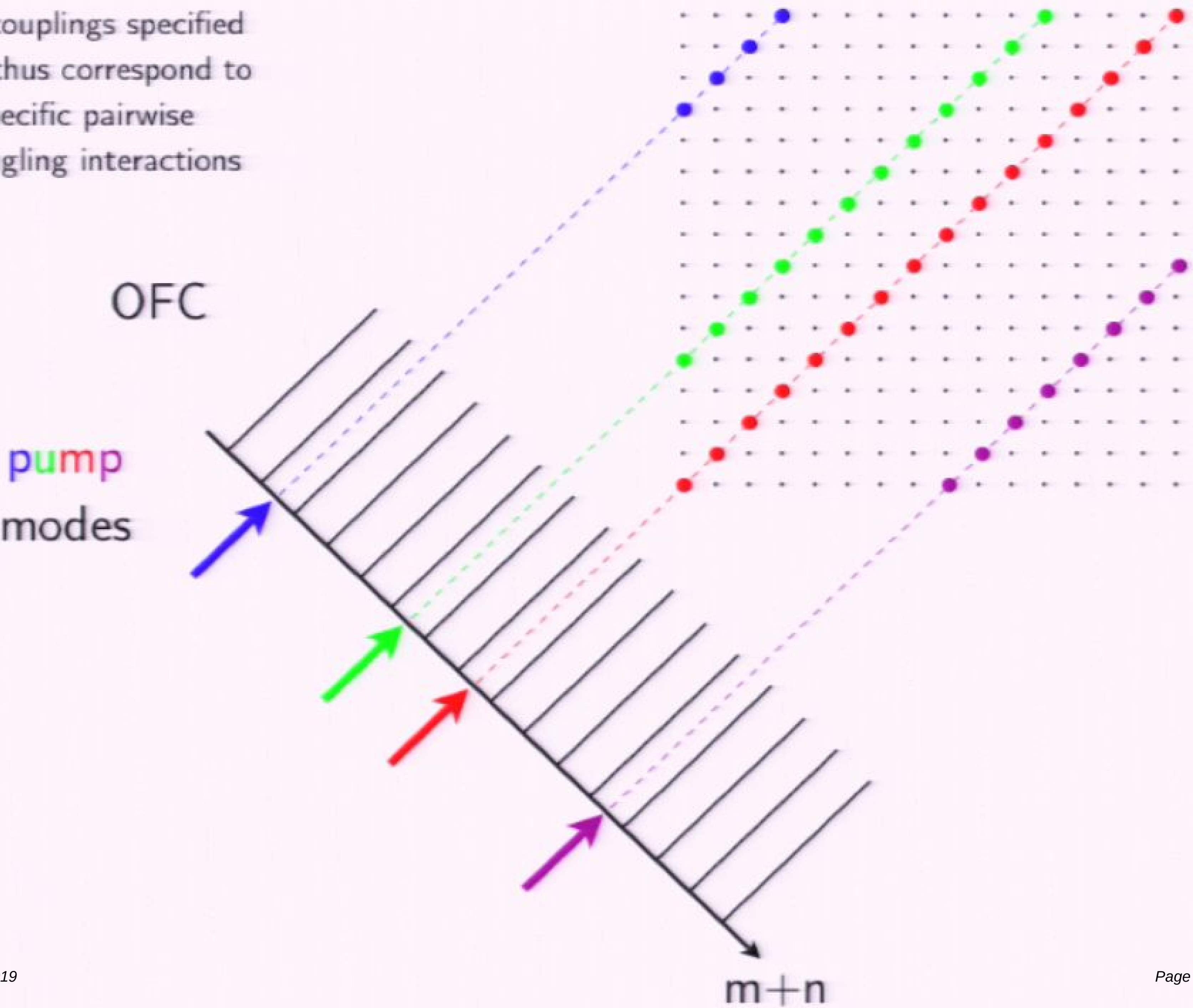

m+n


pump
modes
m+n



OFC
pump
modes
m+n

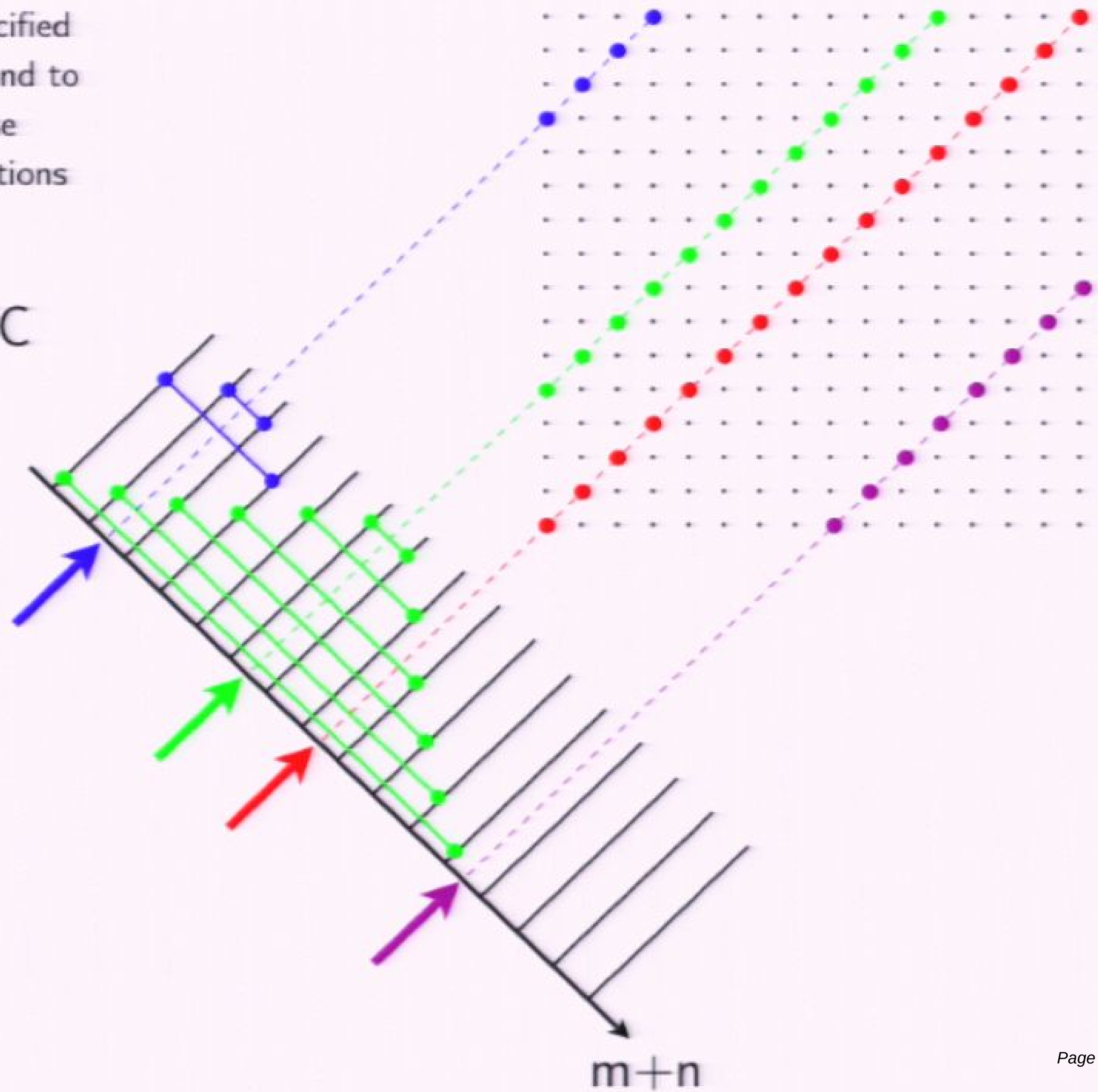
The couplings specified by A thus correspond to specific pairwise entangling interactions



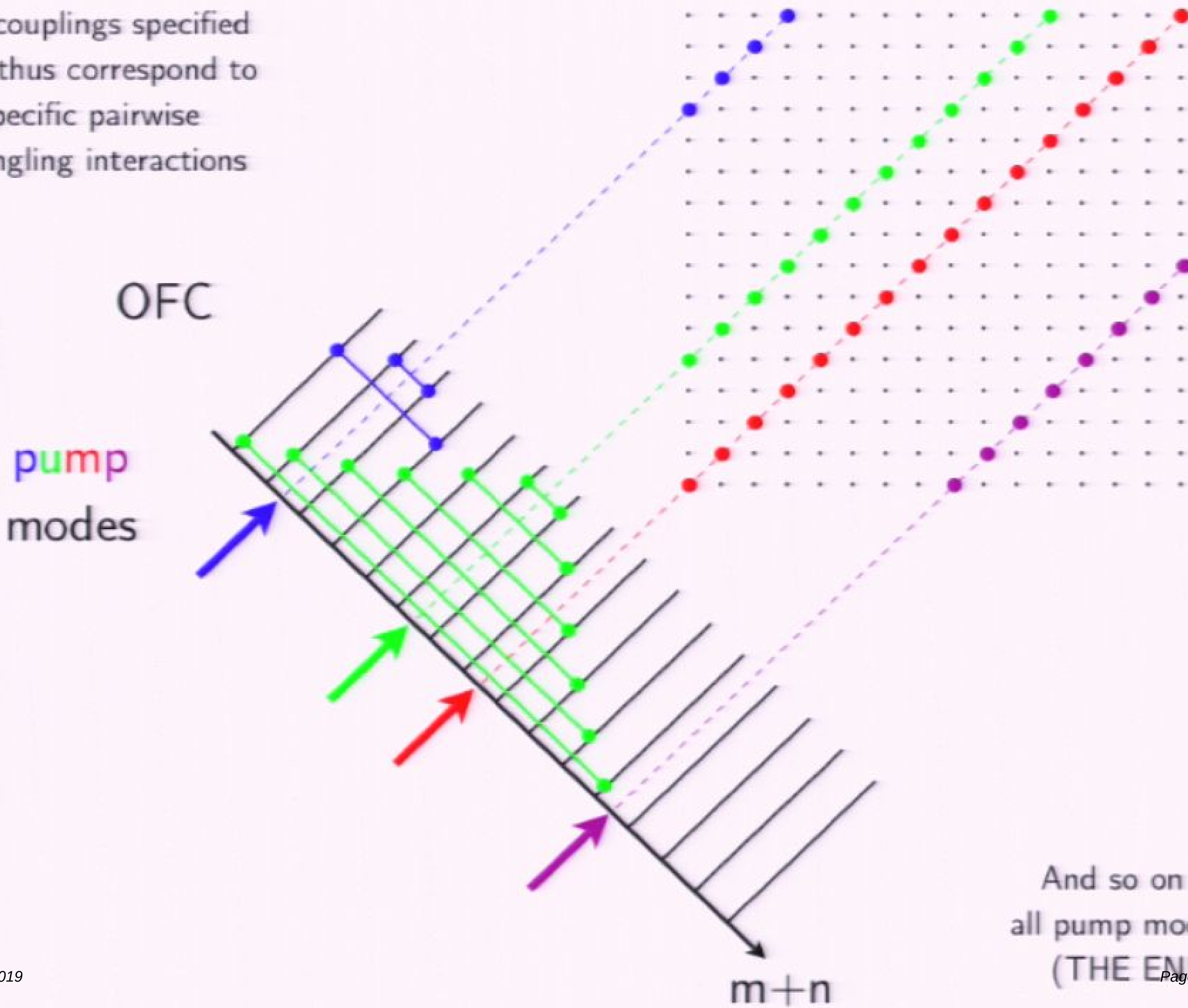
The couplings specified by A thus correspond to specific pairwise entangling interactions

OFC

pump modes



The couplings specified by A thus correspond to specific pairwise entangling interactions



And so on for all pump modes...
(THE END)

Relationship to the CV cluster state

A general cluster state with a (possibly weighted) adjacency matrix A satisfies the relation

$$\boldsymbol{p} - A\boldsymbol{q} \to 0$$

The arrow denotes the infinite-squeezing limit.

$\mathbf{q} = (q_1, \ldots, q_N)^T$
$\mathbf{p} = (p_1, \ldots, p_N)^T$ are quadratures of the field modes

What is the relationship between this graph state (labeled A) and the graph of couplings (G) in the OPO?

Relationship to the CV cluster state

$$\boldsymbol{p} - A\boldsymbol{q} \to 0 \qquad \star$$

Use the symplectic representation for Gaussian transformations on the vacuum

$$U = \exp(-it\mathcal{H}) \quad \Rightarrow \begin{pmatrix} e^{-rG} & 0 \\ 0 & e^{rG} \end{pmatrix}$$

$r = \kappa t$ is the total amount of squeezing

$\star$ becomes $$\begin{pmatrix} -A & I \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} \to 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

If G has no single-mode squeezing, then it can always be factored as a tensor product

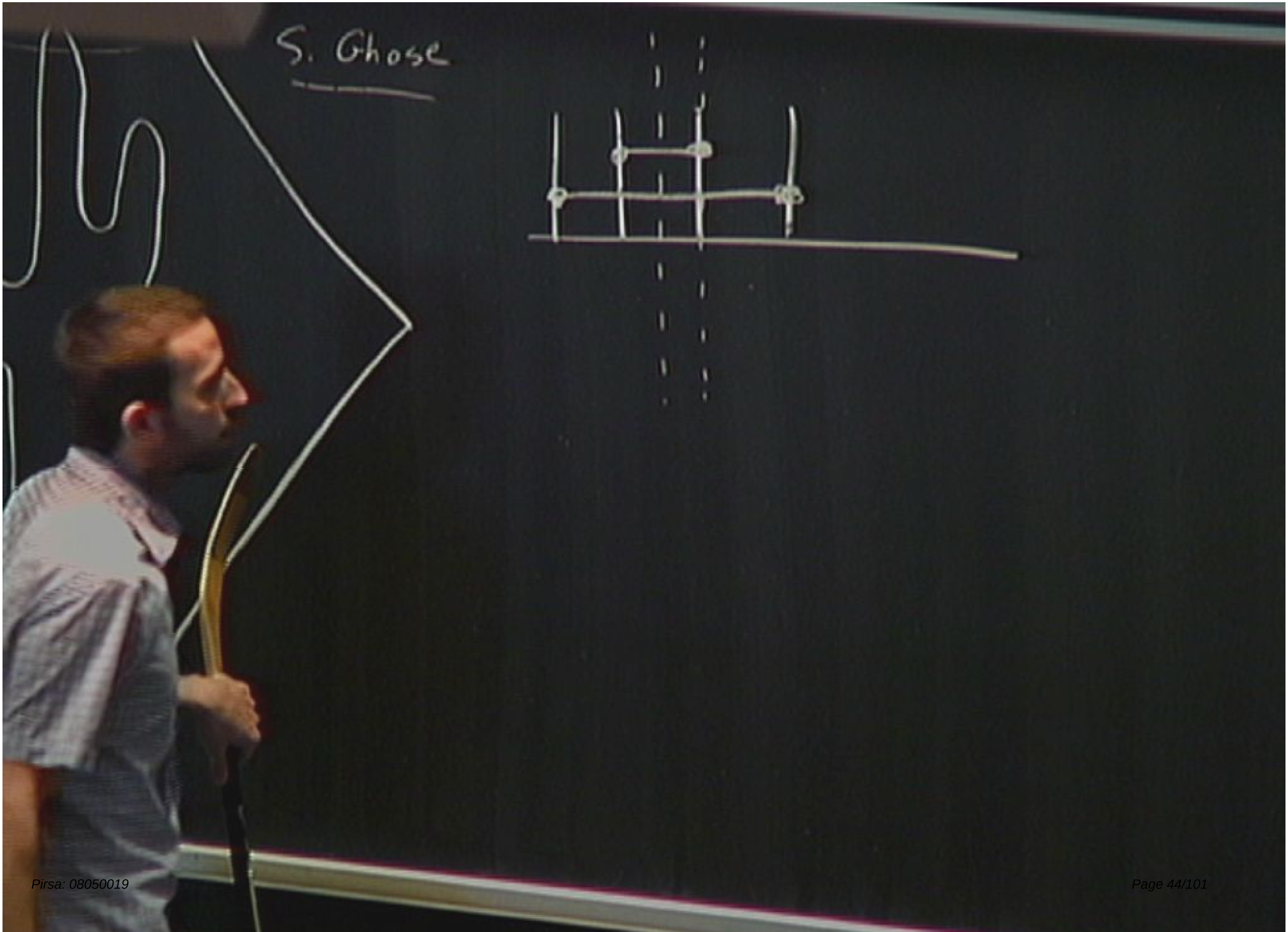
$$G = A_0 \otimes \sigma_x = A_0 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

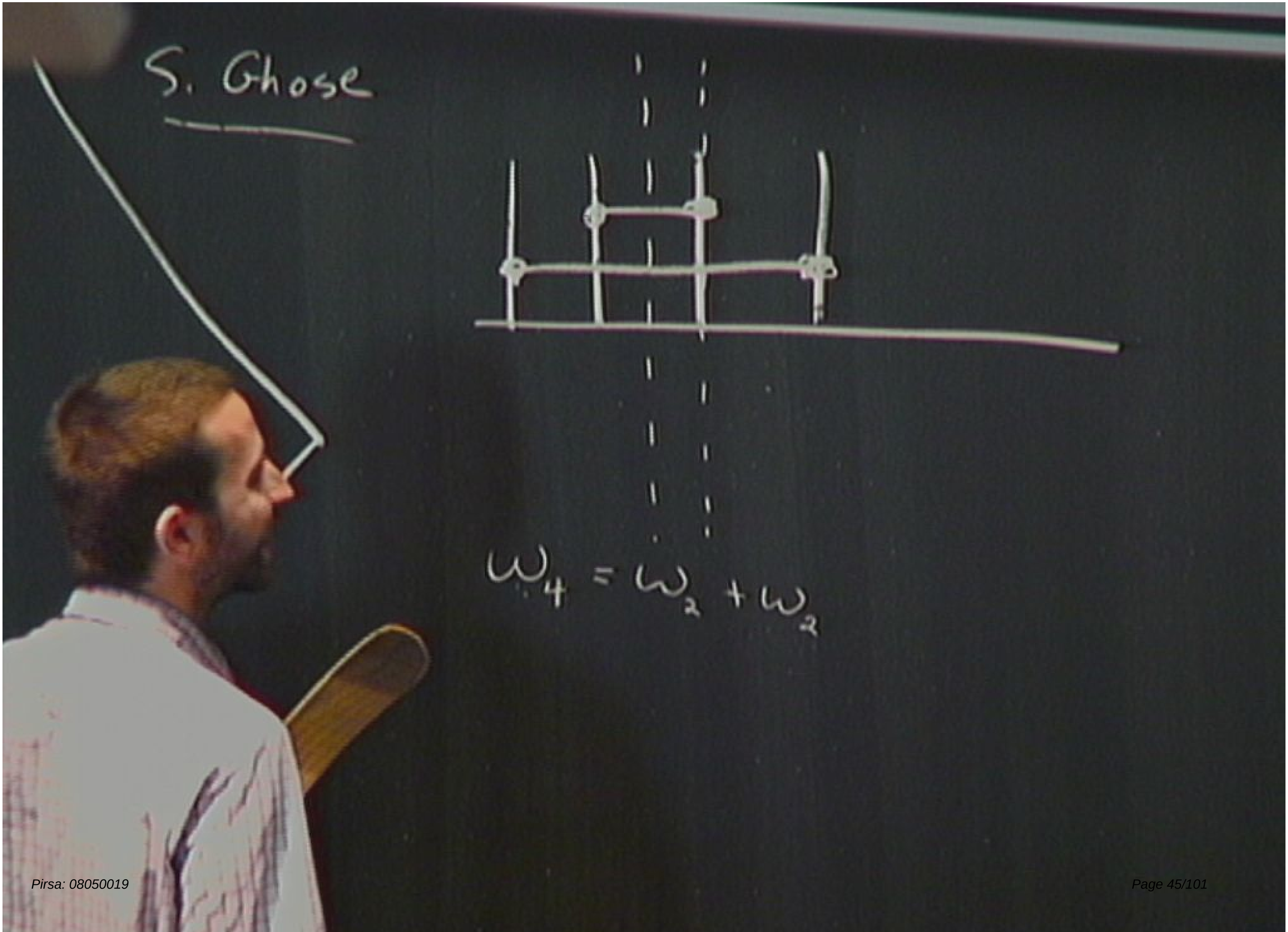
By reversing the tensor factor order, we see that G is bipartite

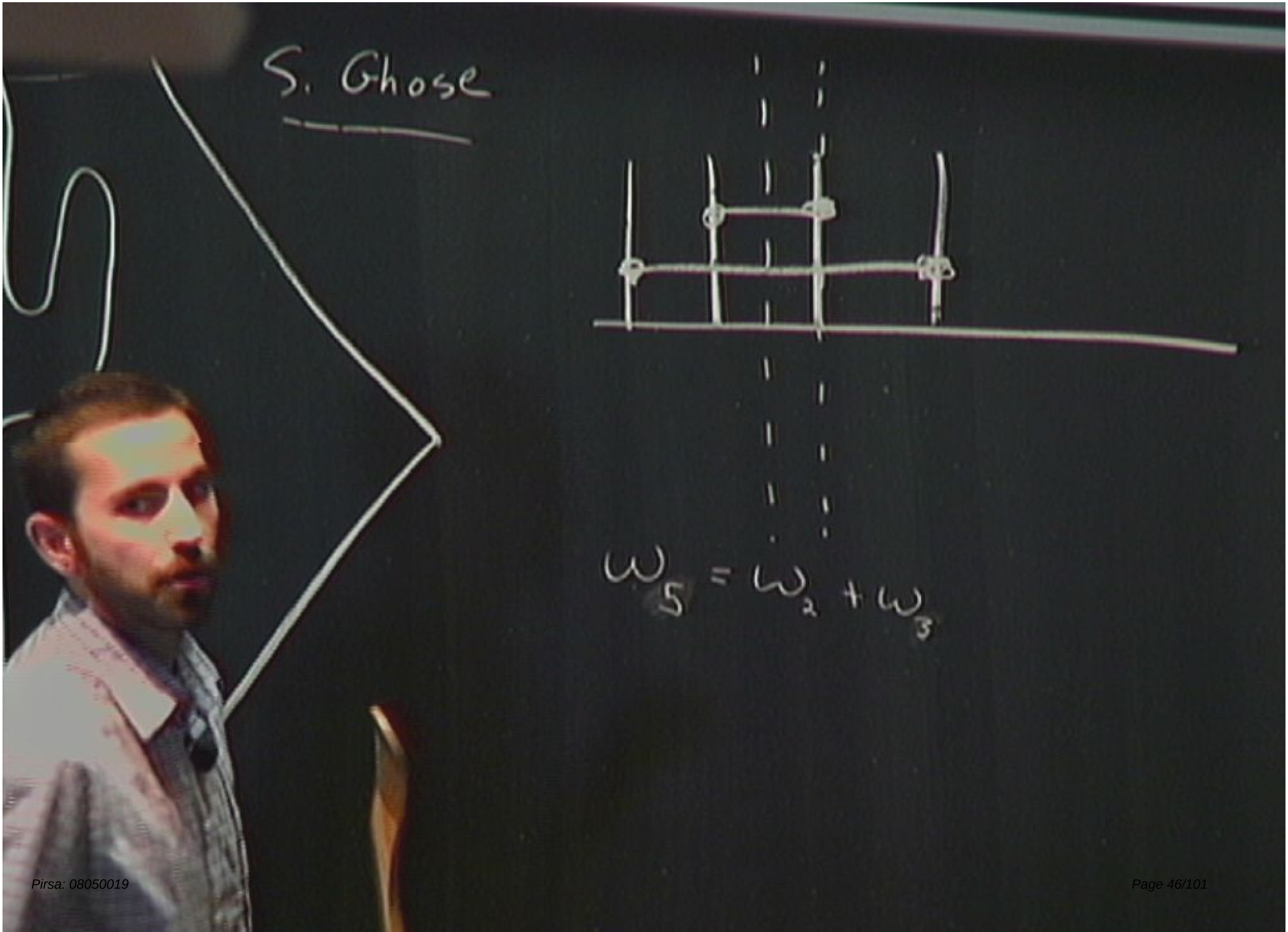
The factor matrix A_0 retains the Hankel property of G.

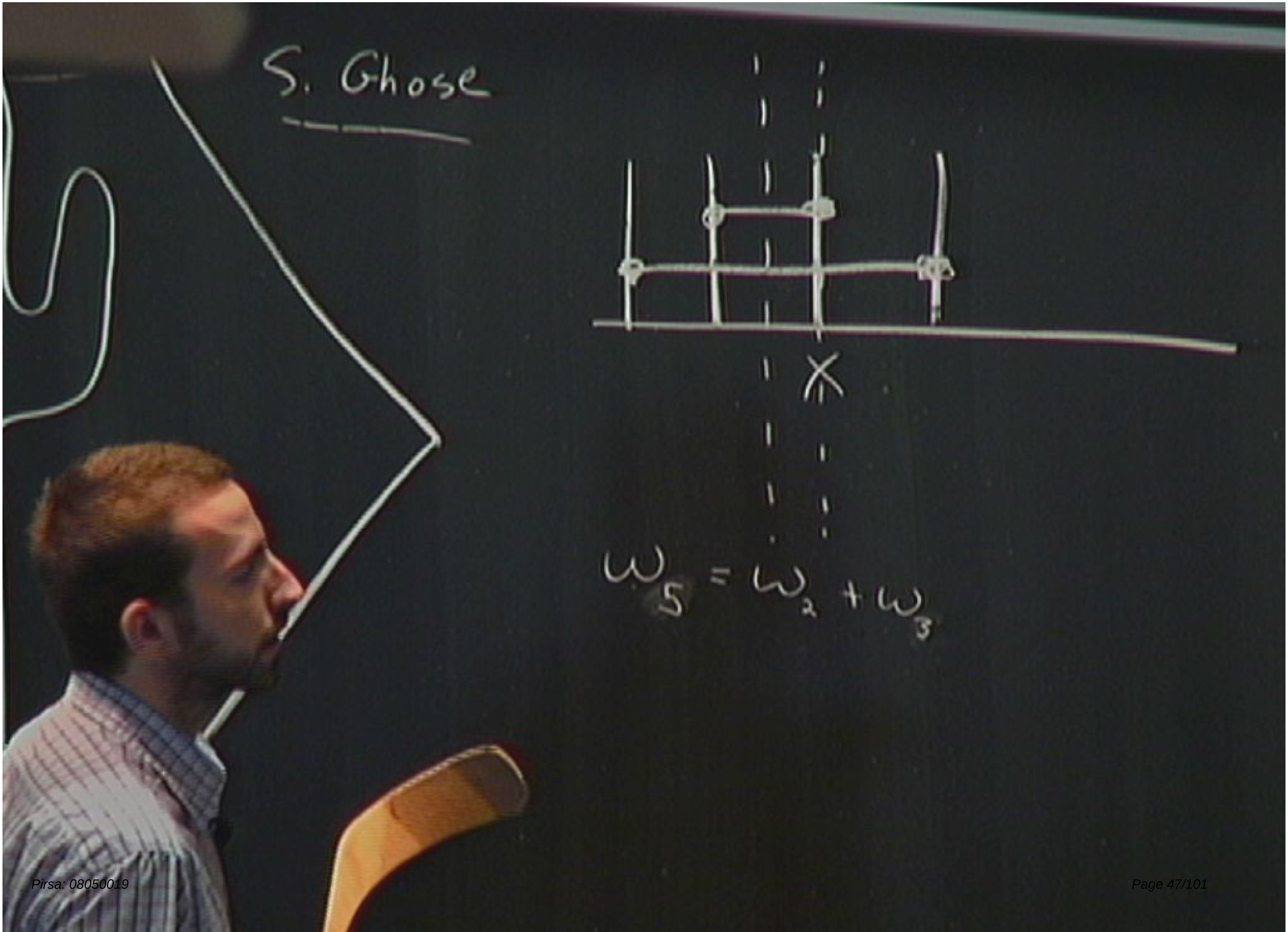
$$G \cong \sigma_x \otimes A_0$$

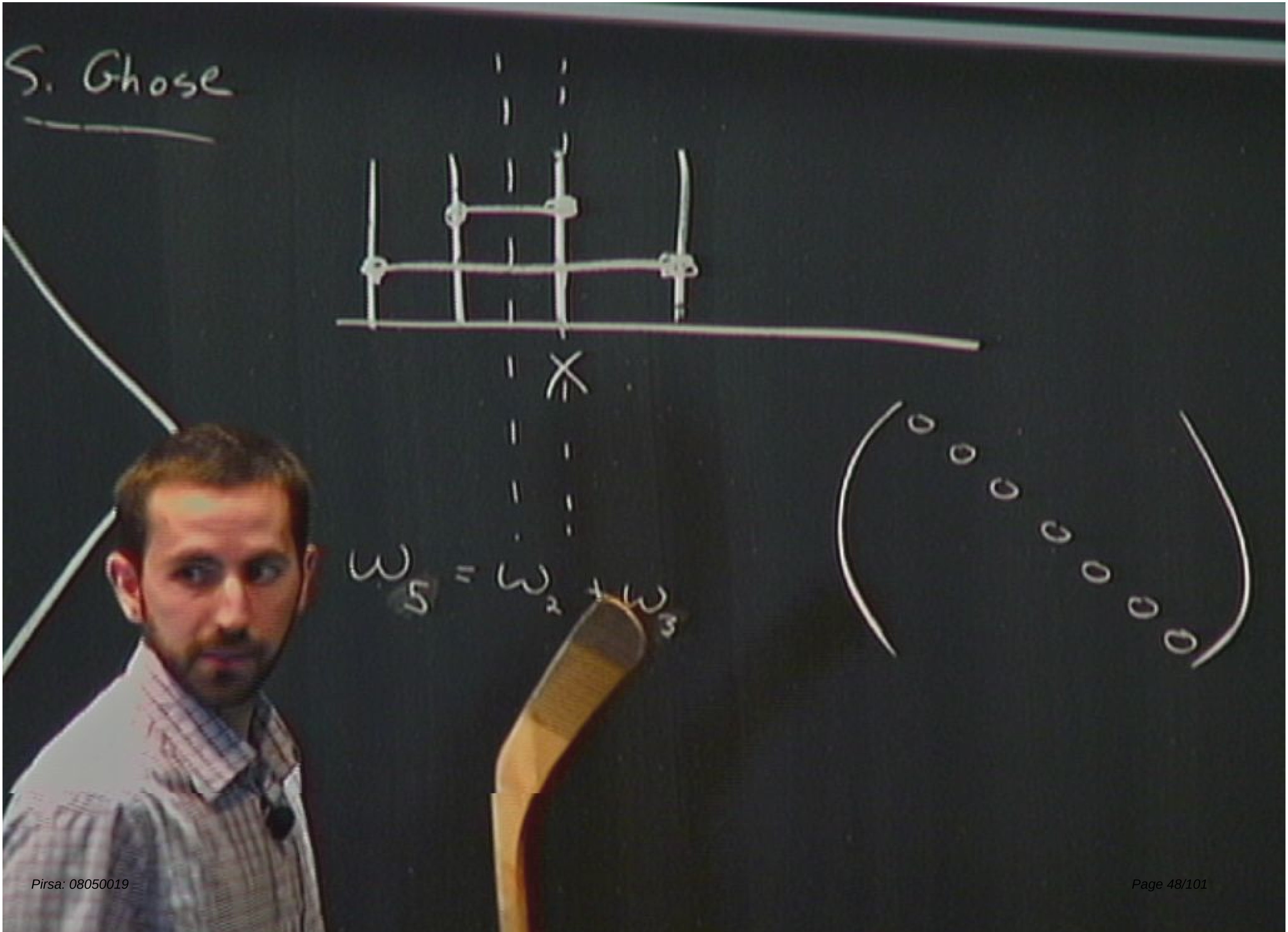
Menicucci, STF, Zaidi, Pfister PRA 2007

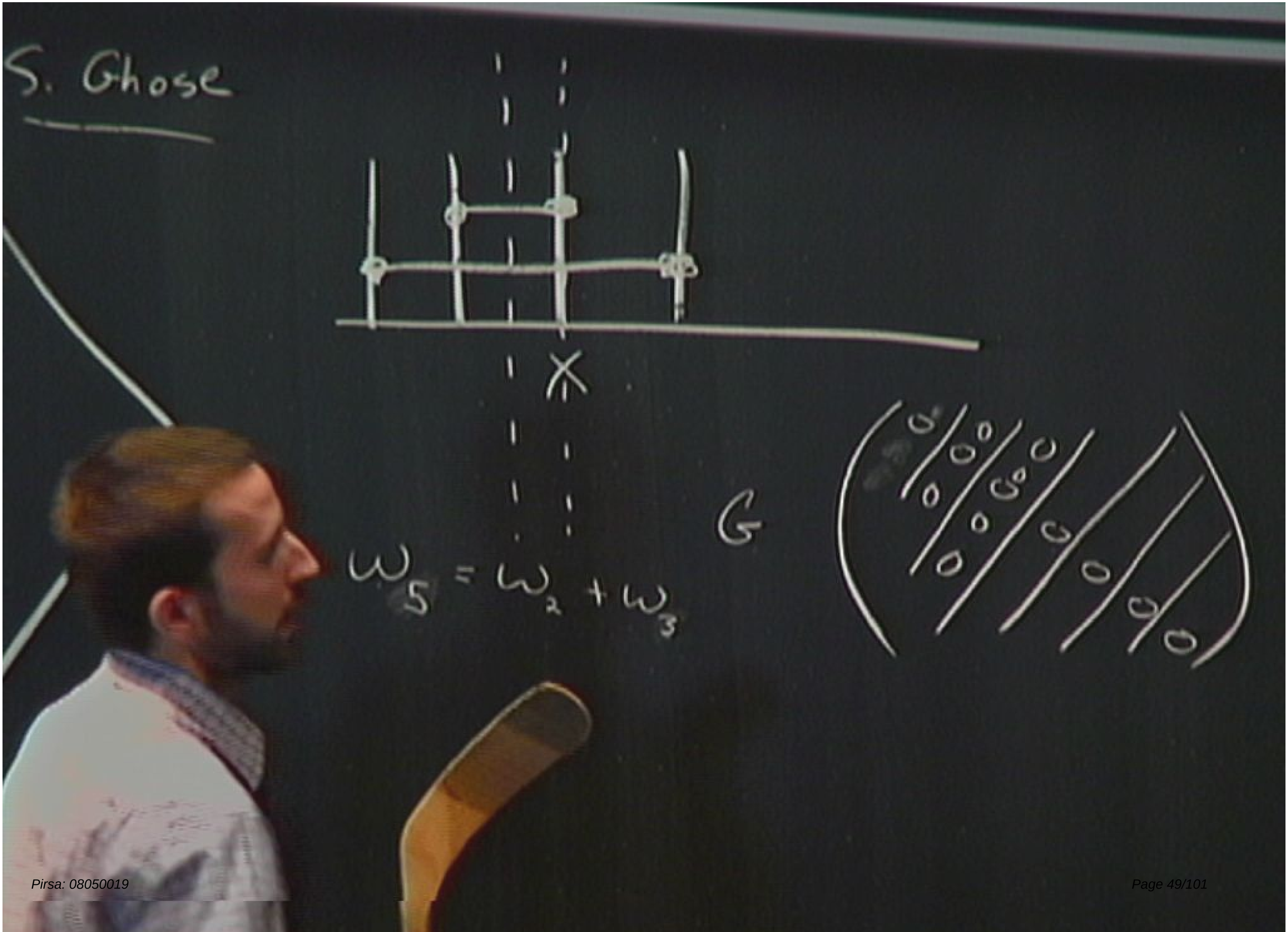

S. Ghose


S. Ghose
$\omega_4 = \omega_2 + \omega_2$


S. Ghose
ω5 = ω2 + ω3


S. Ghose
X
$\omega_5 = \omega_2 + \omega_3$


S. Ghose
ω5 = ω2 + ω3


S. Ghose
X
$\omega_5 = \omega_2 + \omega_3$
G

Relationship to the CV cluster state

If G has no single-mode squeezing, then it can always be factored as a tensor product

$$G = A_0 \otimes \sigma_x = A_0 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

By reversing the tensor factor order, we see that G is bipartite

The factor matrix A_0 retains the Hankel property of G.

$$G \cong \sigma_x \otimes A_0$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

If G has no single-mode squeezing, then it can always be factored as a tensor product

$$G = A_0 \otimes \sigma_x = A_0 \otimes \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$$

By reversing the tensor factor order, we see that G is bipartite

The factor matrix A_0 retains the Hankel property of G.

$$G \cong \sigma_x \otimes A_0$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

$$\boldsymbol{p} - A\boldsymbol{q} \to 0 \qquad \star$$

Use the symplectic representation for Gaussian transformations on the vacuum

$$U = \exp(-it\mathcal{H}) \quad \Rightarrow \begin{pmatrix} e^{-rG} & 0 \\ 0 & e^{rG} \end{pmatrix}$$

$r = \kappa t$ is the total amount of squeezing

$\star$ becomes $$\begin{pmatrix} -A & I \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} \to 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

If G has no single-mode squeezing, then it can always be factored as a tensor product

$$G = A_0 \otimes \sigma_x = A_0 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

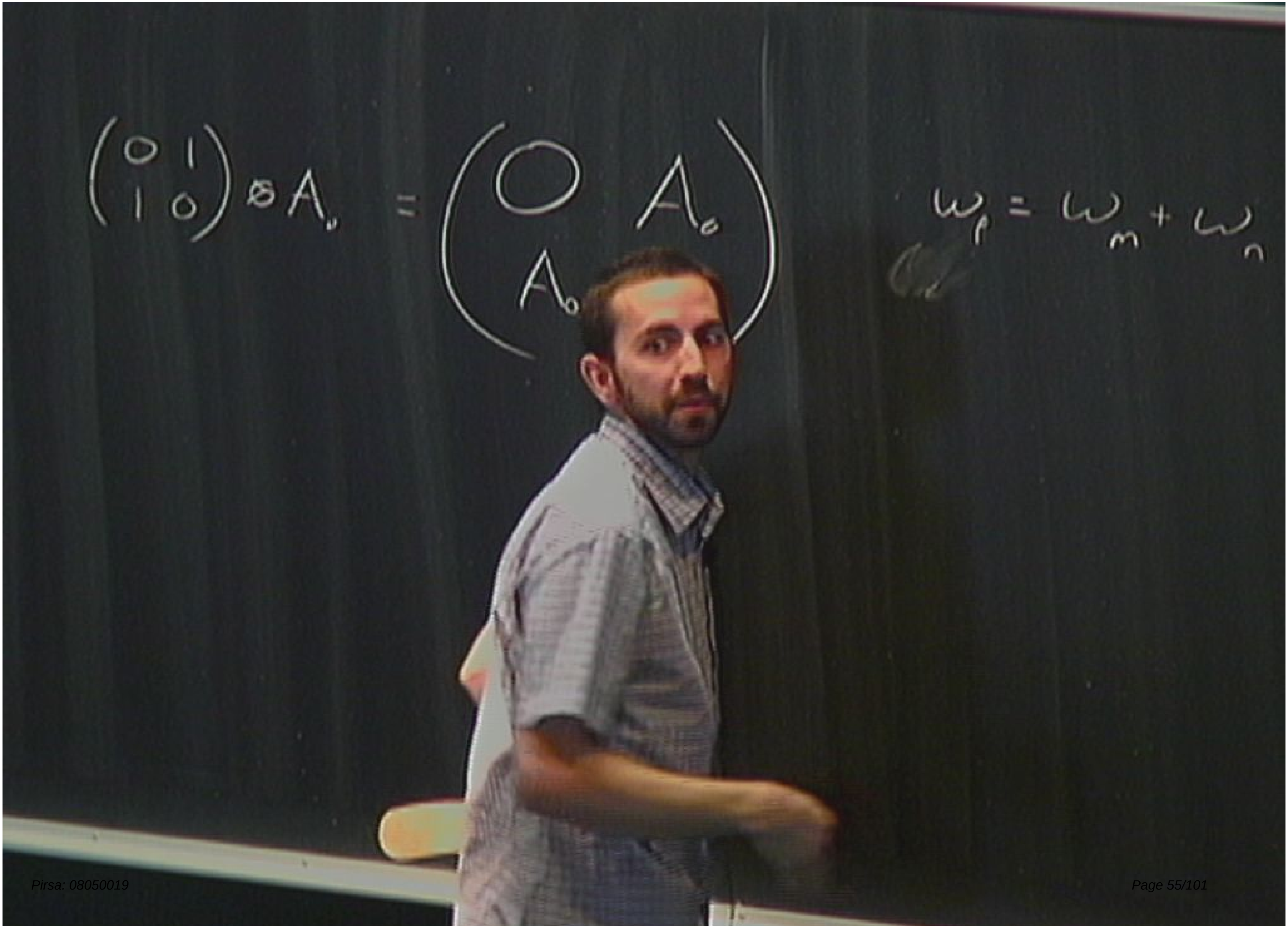
By reversing the tensor factor order, we see that G is bipartite

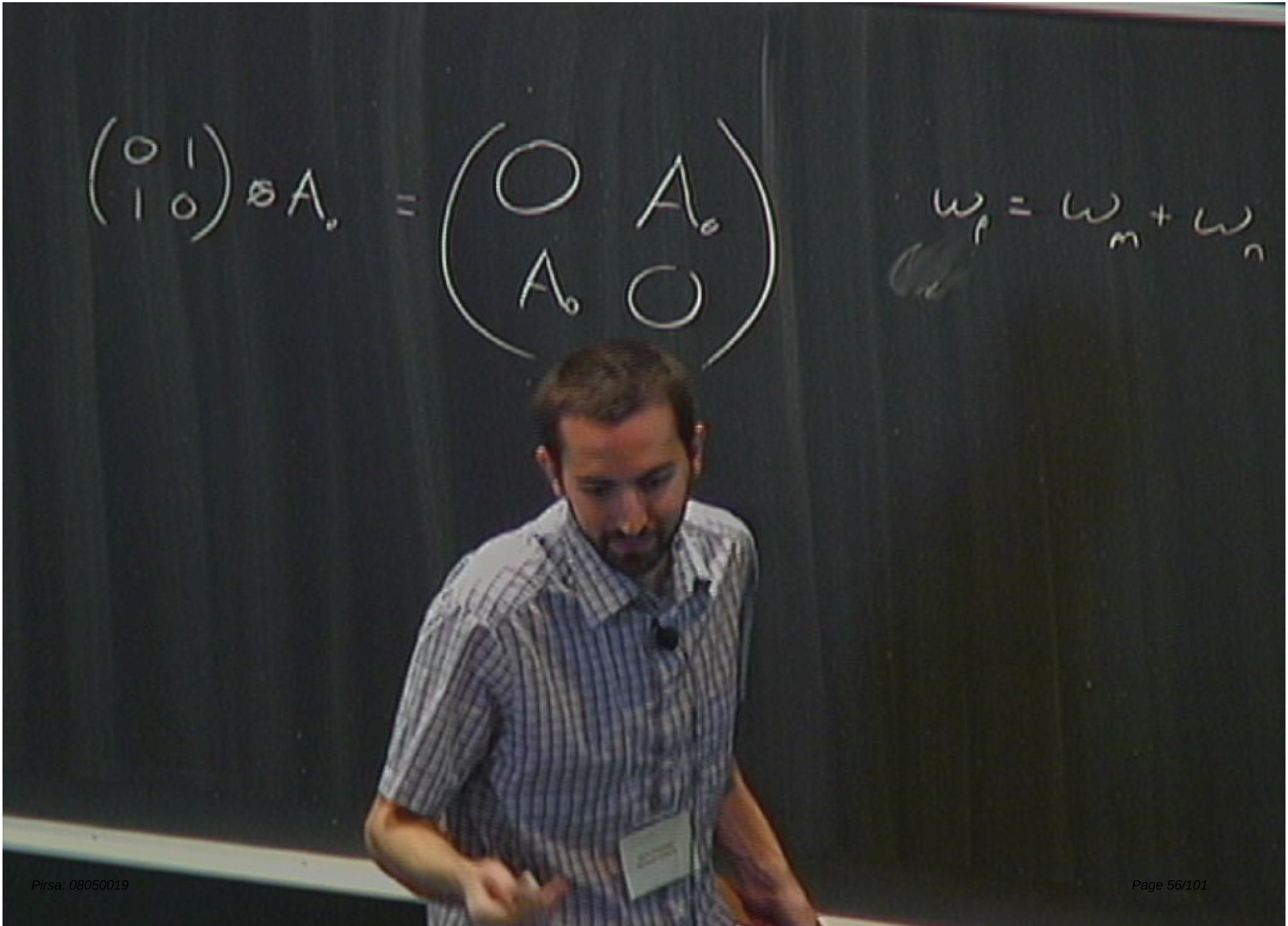
The factor matrix A_0 retains the Hankel property of G.

$$G \cong \sigma_x \otimes A_0$$

Menicucci, STF, Zaidi, Pfister PRA 2007

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes A_0 = \begin{pmatrix} 0 & A_0 \\ A_0 & 0 \end{pmatrix}$$


$\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes A_0 = \begin{pmatrix}0 & A_0\\ A_0 & \end{pmatrix}$
$\omega_p = \omega_m + \omega_n$


$\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes A_0 = \begin{pmatrix}0 & A_0\\ A_0 & 0\end{pmatrix}$
$\omega_p = \omega_m + \omega_n$

Relationship to the CV cluster state

In this reordered basis, the unitary matrix U becomes

$$U \cong \begin{pmatrix} 0 & e^{-rA_0} & 0 & 0 \\ e^{-rA_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{rA_0} \\ 0 & 0 & e^{rA_0} & 0 \end{pmatrix}$$

We also allow (experimentally trivial) phase shifts on half the modes, determined by the bipartite split of G

$$T \cong \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & -I & 0 & 0 \end{pmatrix},$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

In this reordered basis, the unitary matrix U becomes

$$U \cong \begin{pmatrix} 0 & e^{-rA_0} & 0 & 0 \\ e^{-rA_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{rA_0} \\ 0 & 0 & e^{rA_0} & 0 \end{pmatrix}$$

We also allow (experimentally trivial) phase shifts on half the modes, determined by the bipartite split of G

$$T \cong \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & -I & 0 & 0 \end{pmatrix} ,$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

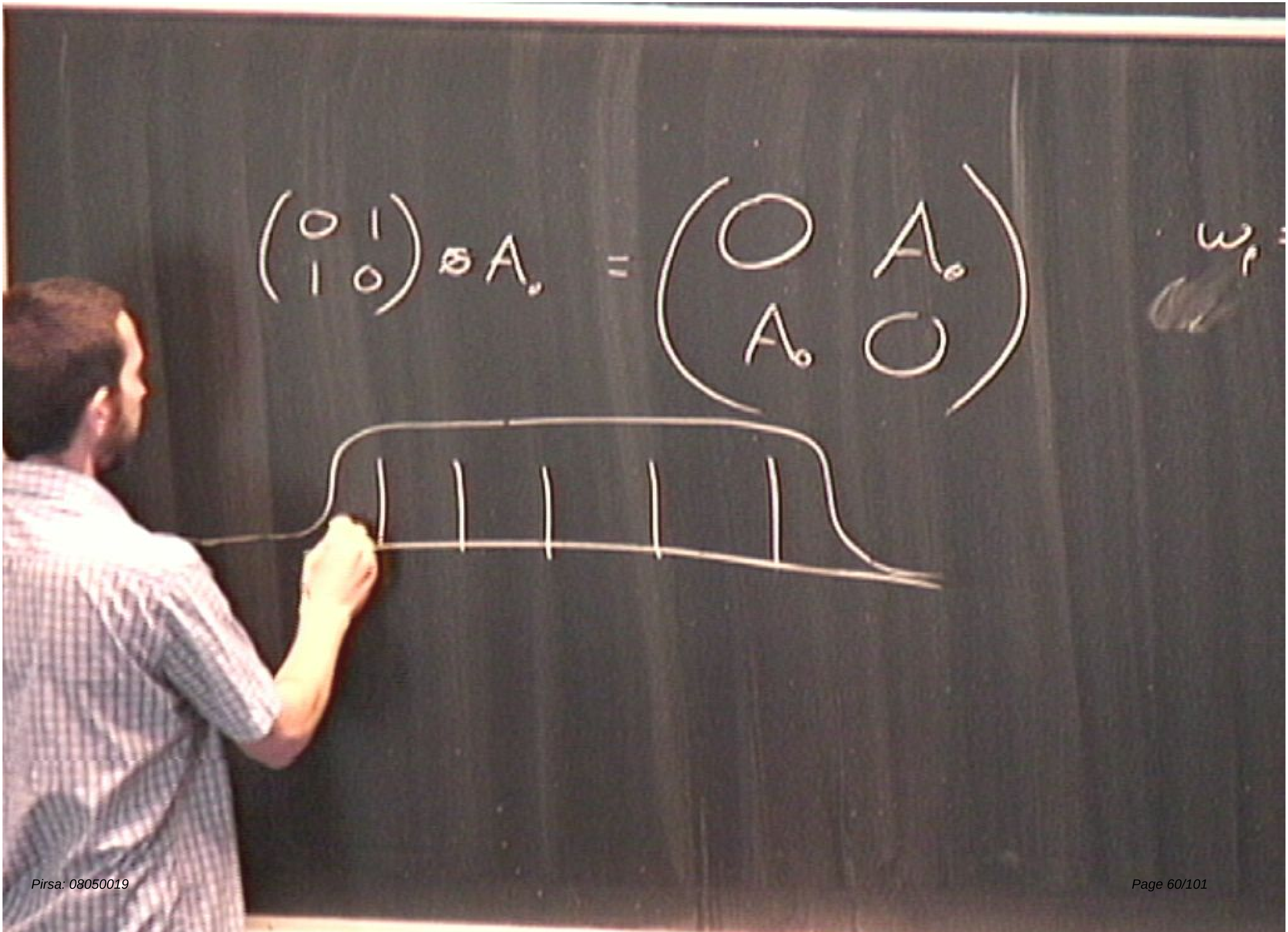
If we also assume that $G^2 = I$,
we can use the identity

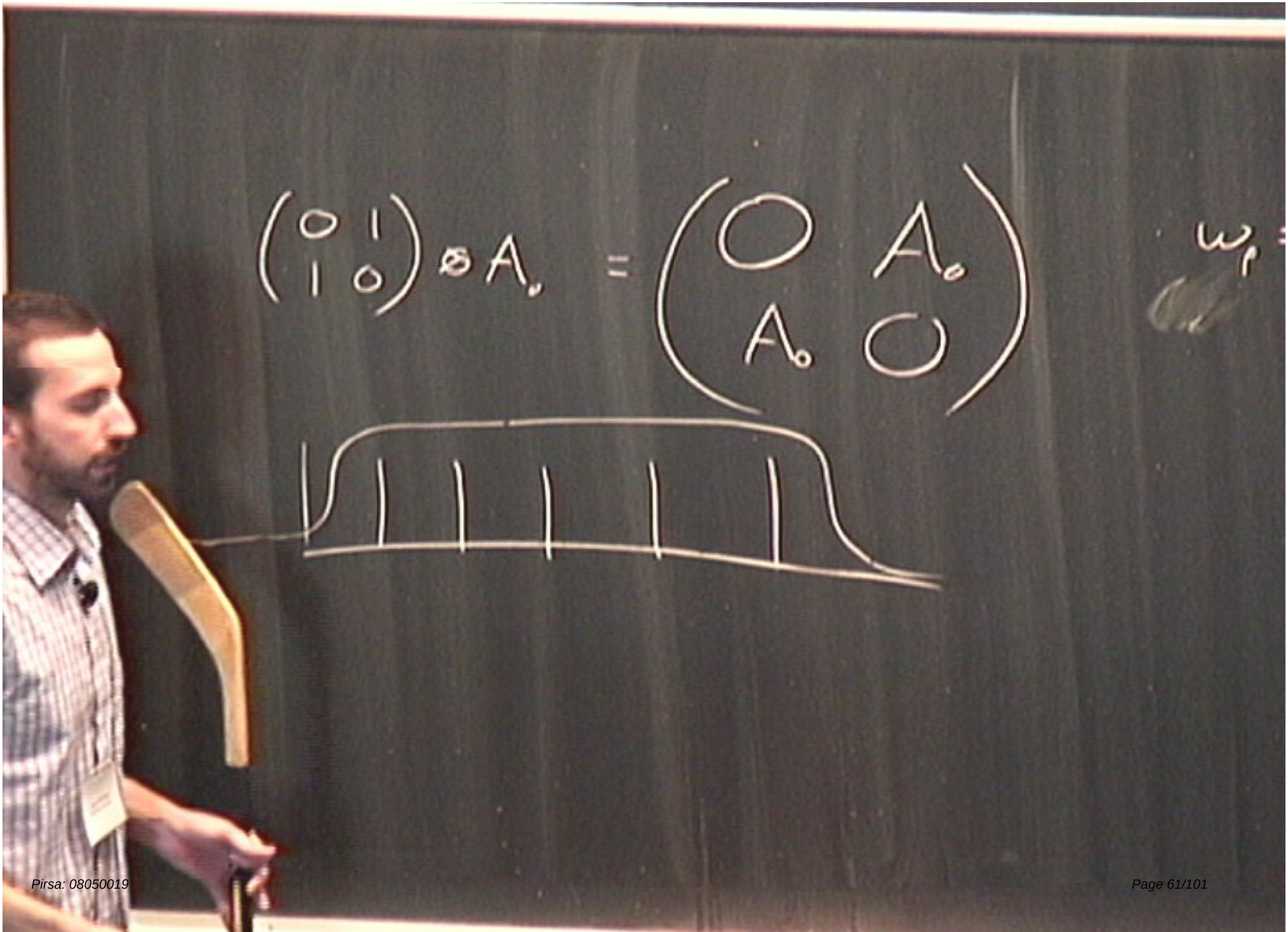
$$e^{\pm rG} = \cosh(r)I \pm \sinh(r)G$$

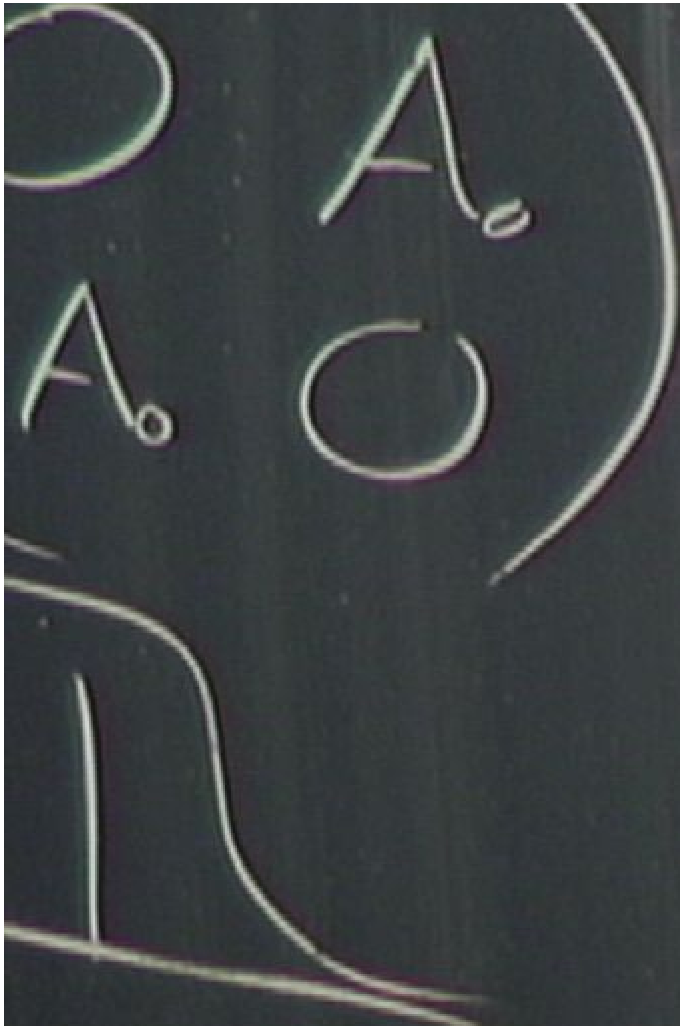
This has no physical motivation, but it simplifies things.

$$U \cong \cosh(r)\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix} + \sinh(r)\begin{pmatrix} 0 & -A_0 & 0 & 0 \\ -A_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_0 \\ 0 & 0 & A_0 & 0 \end{pmatrix}$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007


$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes A_0 = \begin{pmatrix} 0 & A_0 \\ A_0 & 0 \end{pmatrix}$
$\omega_f =$


$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes A_0 = \begin{pmatrix} 0 & A_0 \\ A_0 & 0 \end{pmatrix}$
$w_1 =$



$$\omega_p = \omega_m + \omega_n$$

$$G_{mn} = \begin{cases} \pm C \\ 0 \end{cases}$$

Relationship to the CV cluster state

Using the definitions of T and U as before, and the identity

$$\cosh(r) - \sinh(r) = e^{-r}$$

we find exponential convergence to a CV cluster state

$$\begin{pmatrix} -A & I \end{pmatrix} TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r \to \infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007

Relationship to the CV cluster state

Using the definitions of T and U as before, and the identity

$$\cosh(r) - \sinh(r) = e^{-r}$$

we find exponential convergence to a CV cluster state

$$\begin{pmatrix} -A & I \end{pmatrix} TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r\to\infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007

Relationship to the CV cluster state

Remarkably, assuming $G^2=I$ implies $G=A$!

we find exponential convergence
to a CV cluster state

$$\begin{pmatrix} -A & I \end{pmatrix} TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r\to\infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007

Relationship to the CV cluster state

In this reordered basis, the unitary matrix U becomes

$$U \cong \begin{pmatrix} 0 & e^{-rA_0} & 0 & 0 \\ e^{-rA_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{rA_0} \\ 0 & 0 & e^{rA_0} & 0 \end{pmatrix}$$

We also allow (experimentally trivial) phase shifts on half the modes, determined by the bipartite split of G

$$T \cong \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & -I & 0 & 0 \end{pmatrix} ,$$

Menicucci, STF, Zaidi, Pfister PRA 2007

Relationship to the CV cluster state

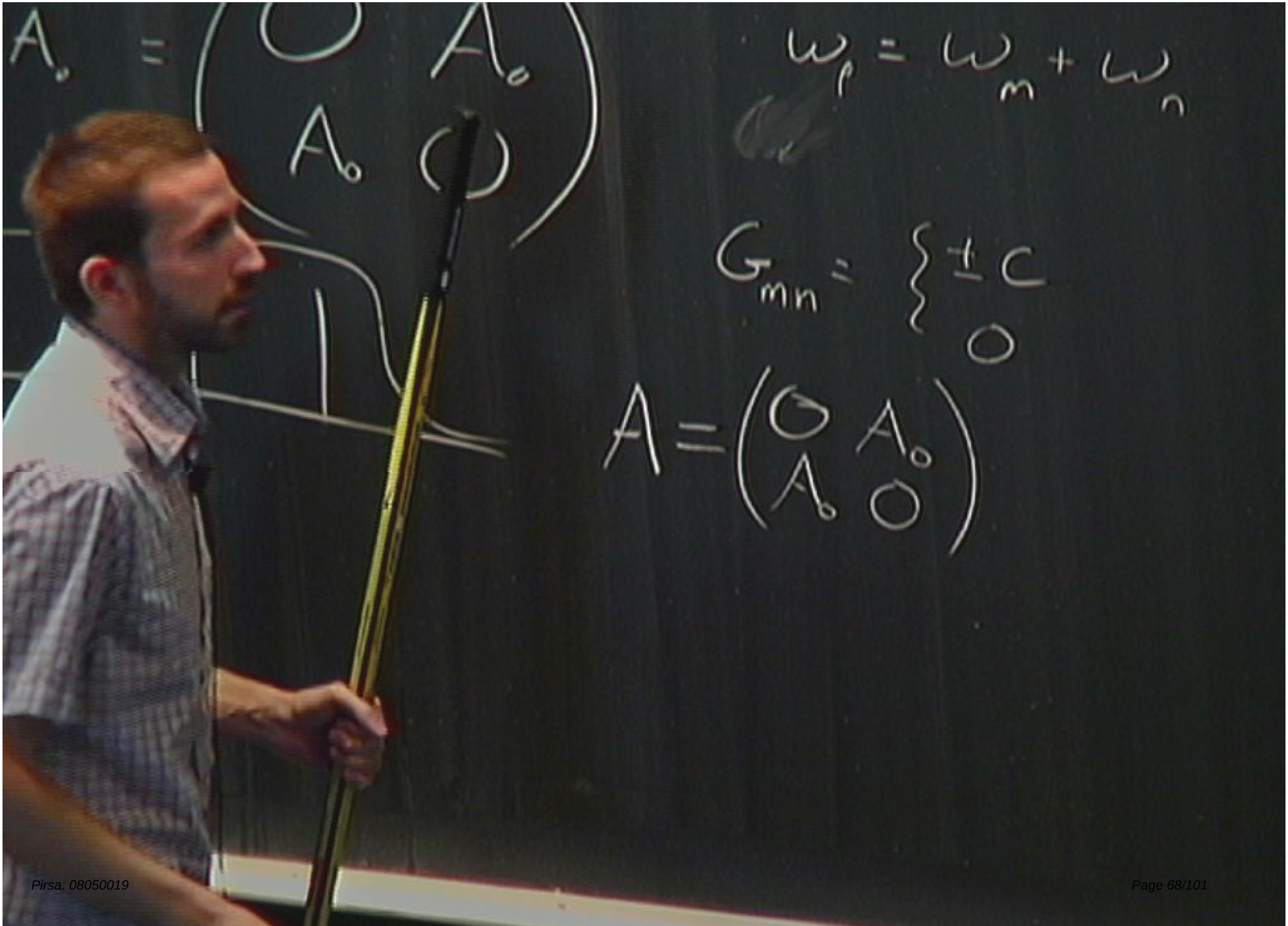
Using the definitions of T and U as before, and the identity

$$\cosh(r) - \sinh(r) = e^{-r}$$

we find exponential convergence to a CV cluster state

$$\begin{pmatrix} -A & I \end{pmatrix} TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r \to \infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007


$A_0 = \begin{pmatrix} 0 & A_0 \\ A_0 & 0 \end{pmatrix}$
$\omega_l = \omega_m + \omega_n$
$G_{mn} = \begin{cases} \pm c \\ 0 \end{cases}$
$A = \begin{pmatrix} 0 & A_0 \\ A_0 & 0 \end{pmatrix}$

Relationship to the CV cluster state

Remarkably, assuming $G^2=I$ implies $G=A$!

we find exponential convergence
to a CV cluster state

$$(-A \quad I)\, TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r\to\infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007

Relationship to the CV cluster state

Remarkably, assuming $G^2=I$ implies $G=A$!

we find exponential convergence
to a CV cluster state

$$\begin{pmatrix} -A & I \end{pmatrix} TU = -e^{-r} \begin{pmatrix} 0 & 0 & -I & A_0 \\ A_0 & I & 0 & 0 \end{pmatrix} \xrightarrow{r\to\infty} 0$$

Menicucci, STF, Zaidi, Pfister PRA 2007;
Zaidi et. al., Laser Phys. 2007

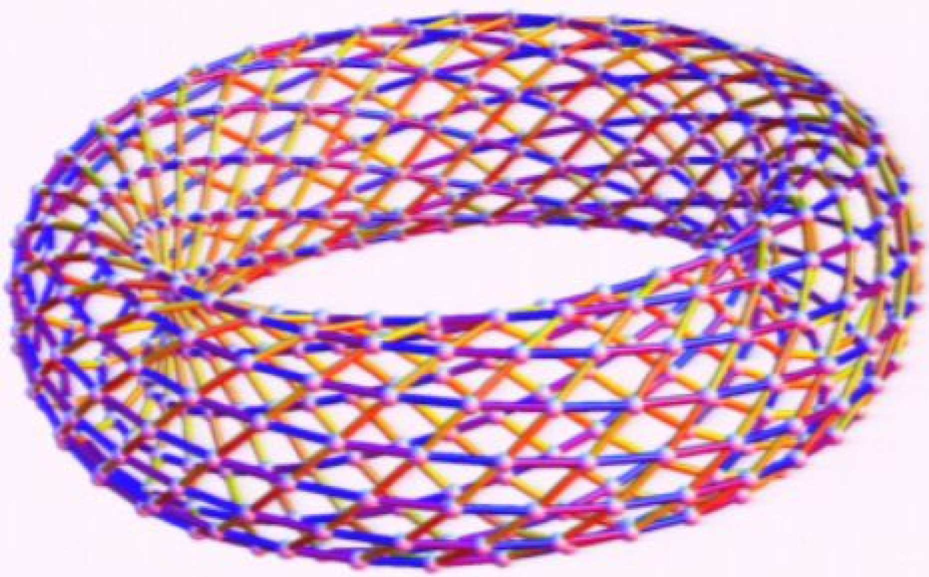
Mathematical construction

We seek an adjacency matrix A with the following properties:

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Mathematical construction

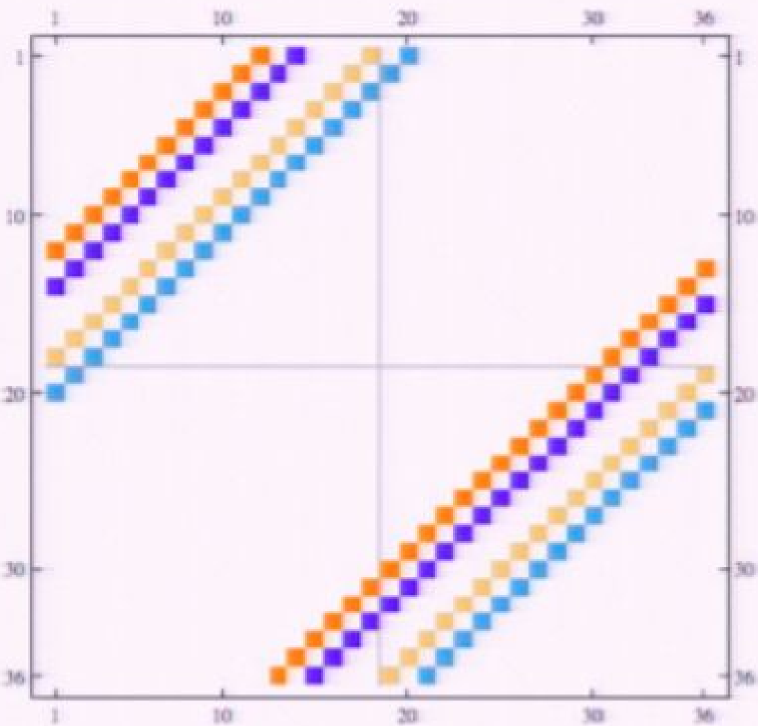
We seek an adjacency matrix A with the following properties:



- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Mathematical construction

A =



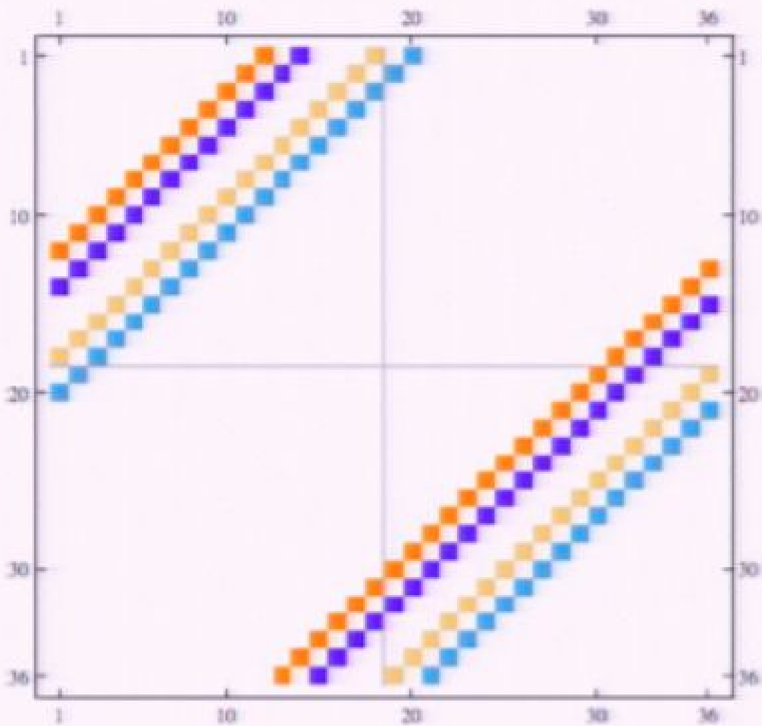
- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T = I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Orthogonality:

$$a^2+b^2+c^2+d^2 = 1\ ,\ ab+ad+cd = 0\ ,\ ac+bd = 0\ ,\ bc = 0$$

Mathematical construction

A =



- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Orthogonality:

No solution!

$$a^2+b^2+c^2+d^2 = I\ ,\ ab+ad+cd = 0\ ,\ ac+bd = 0\ ,\ bc = 0$$

Mathematical construction

Solution:

use projector-valued weights

$$\Pi_1 = \Pi_- \otimes \Pi_- \ , \quad \Pi_2 = \Pi_- \otimes \Pi_+ \ ,$$
$$\Pi_3 = \Pi_+ \otimes \Pi_- \ , \quad \Pi_4 = \Pi_+ \otimes \Pi_+ \ ,$$

where

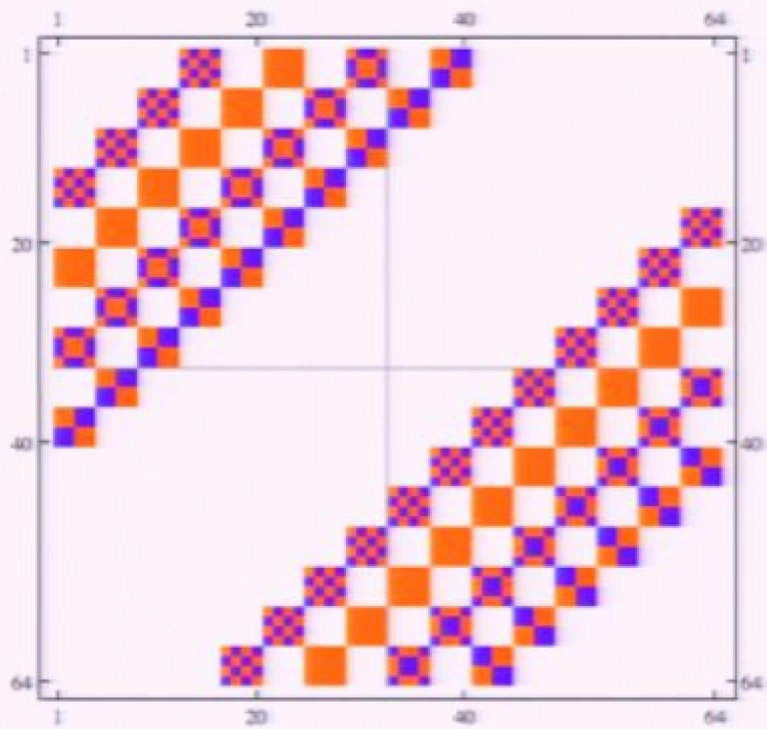
$$\Pi_\pm = \begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$

Orthogonality:

$a^2+b^2+c^2+d^2 = I$, $ab+ad+cd = 0$, $ac+bd = 0$, $bc = 0$

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Mathematical construction

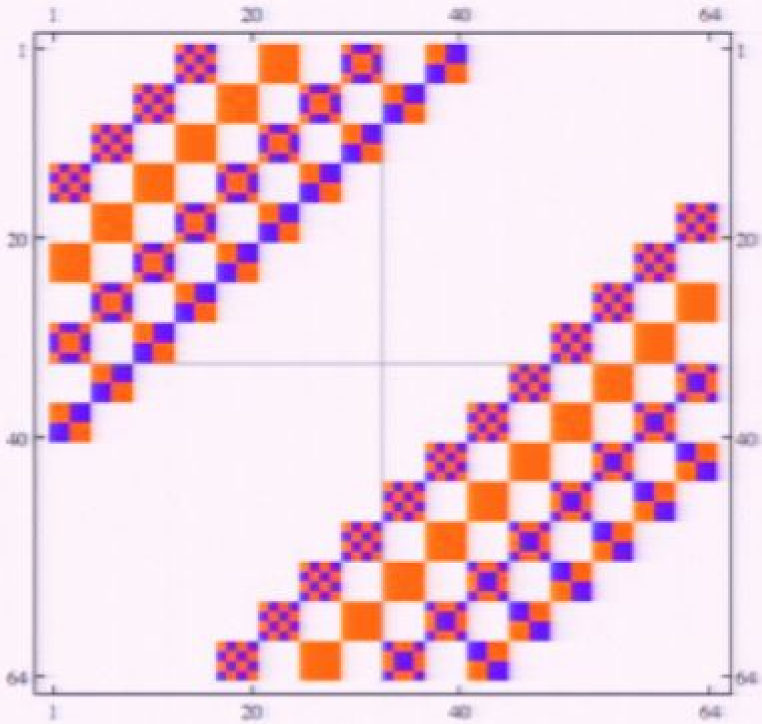
A = 

Problem: No longer Hankel, but block Hankel!

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

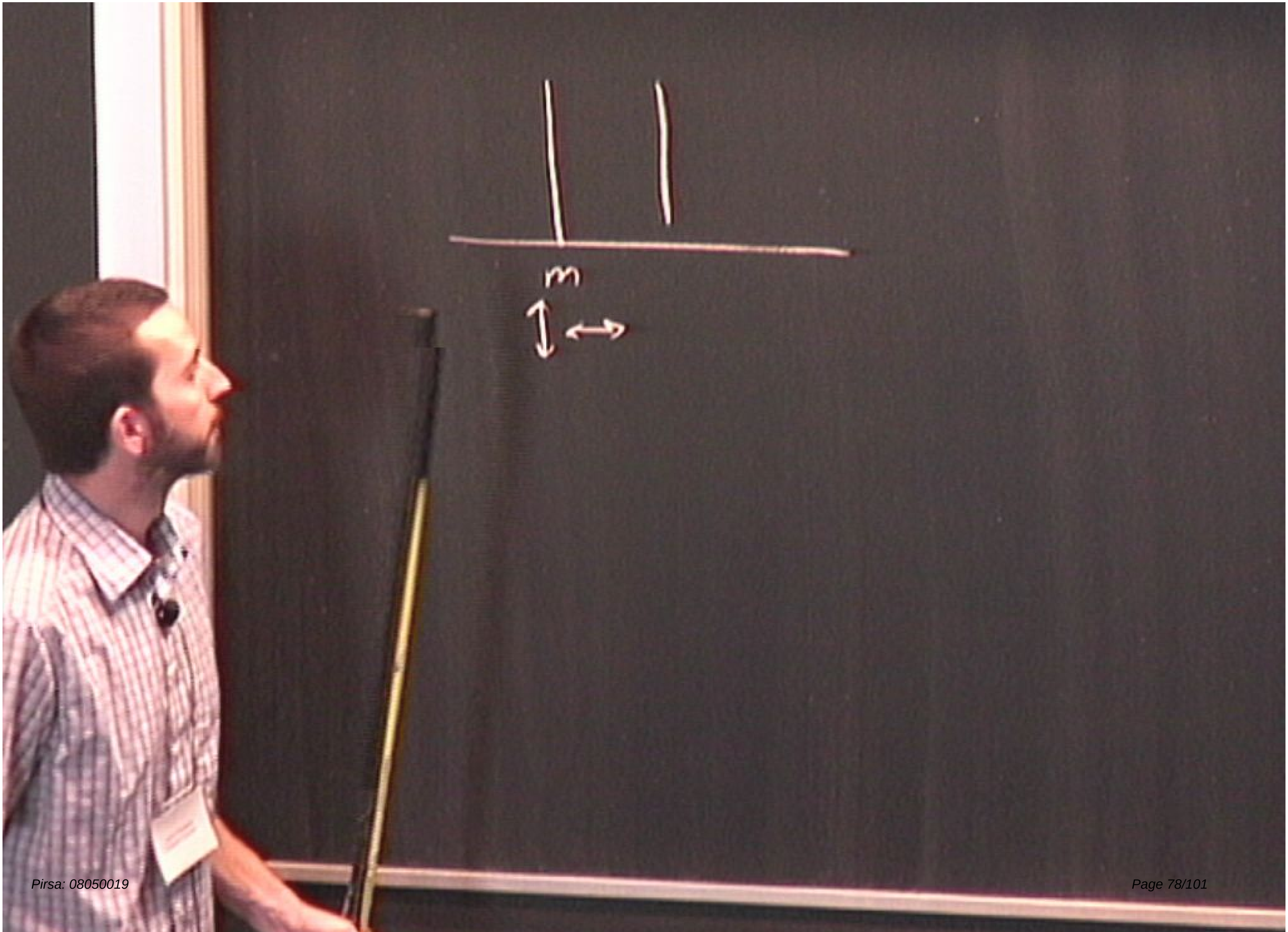
Mathematical construction

A =

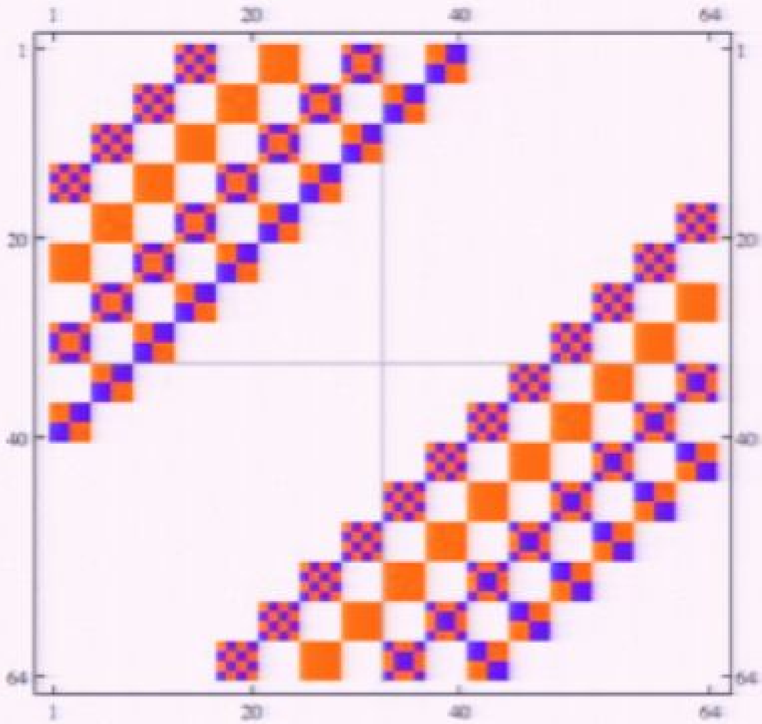


Solution: use polarization degrees of freedom, and twice as many pumps.

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing


m

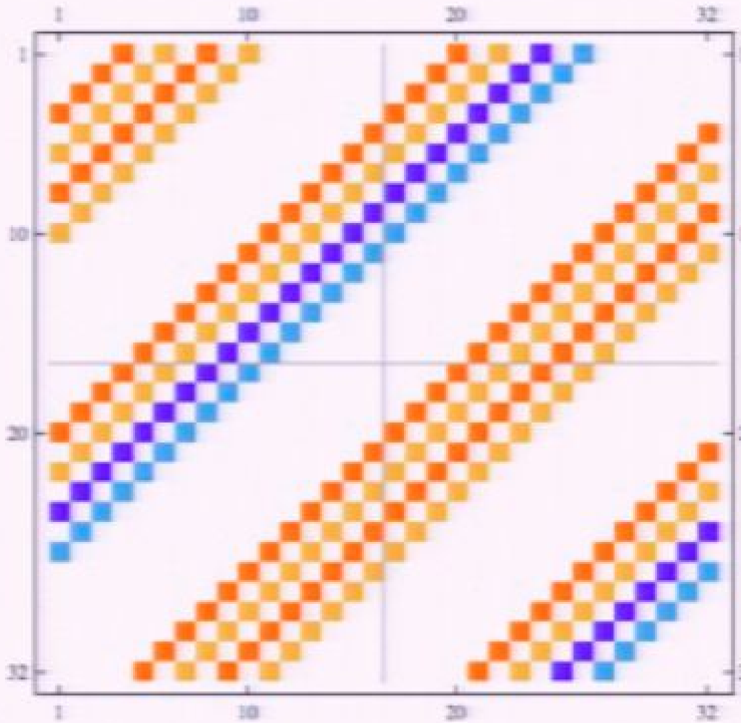
Mathematical construction

A = 

Solution: use polarization degrees of freedom, and twice as many pumps.

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

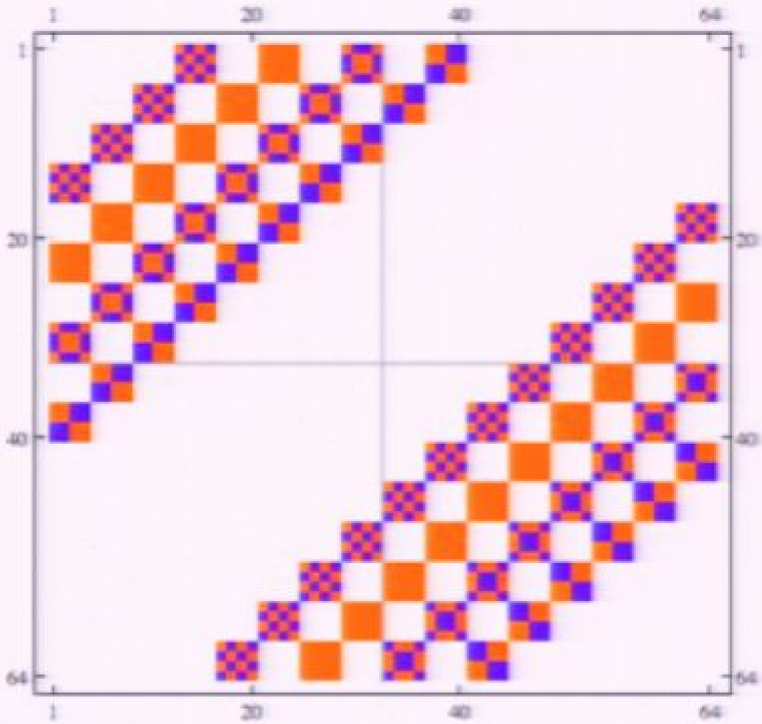
Mathematical construction

$A =$ 

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Solution: use polarization degrees of freedom, and twice as many pumps.

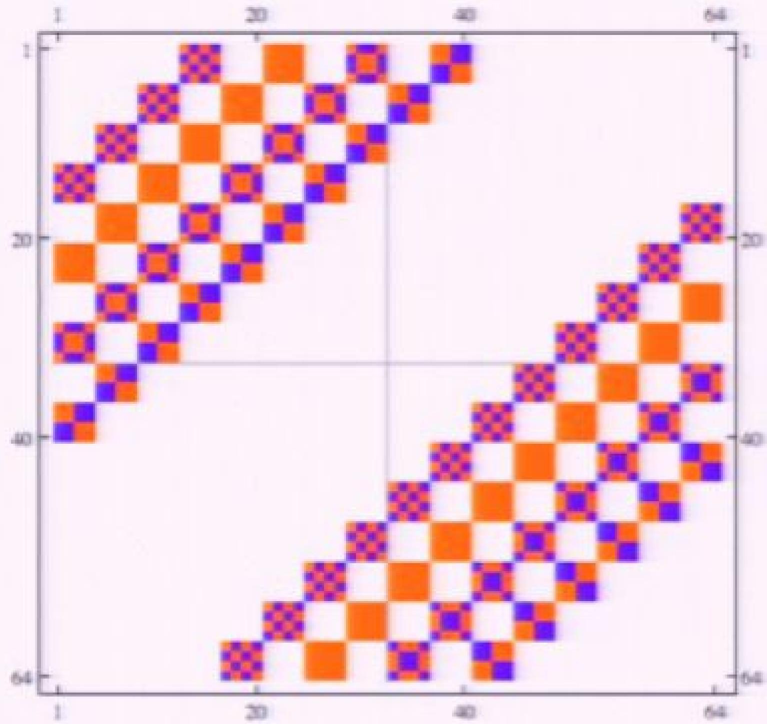
Mathematical construction

$A =$ 

Problem: No longer Hankel, but block Hankel!

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

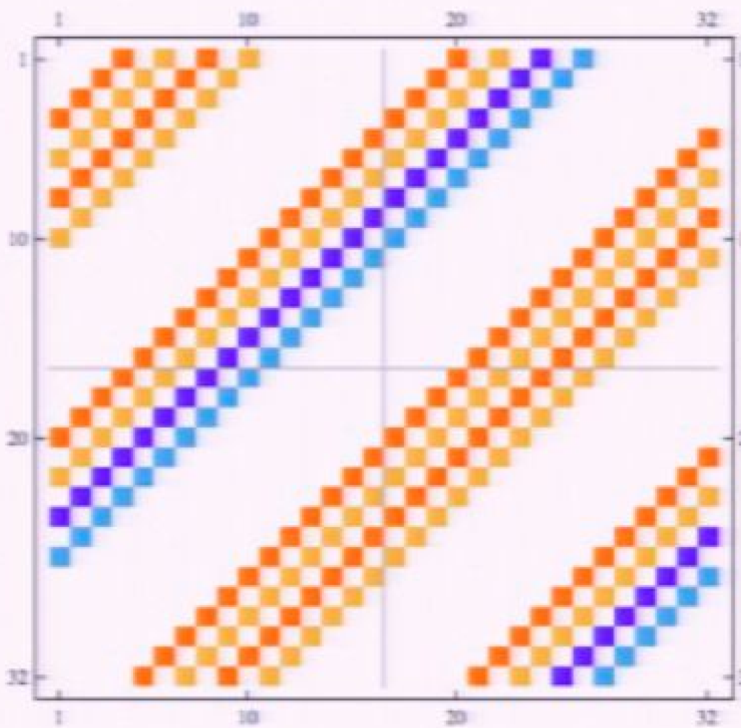
Mathematical construction

$A =$ 

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

Solution: use polarization degrees of freedom, and twice as many pumps.

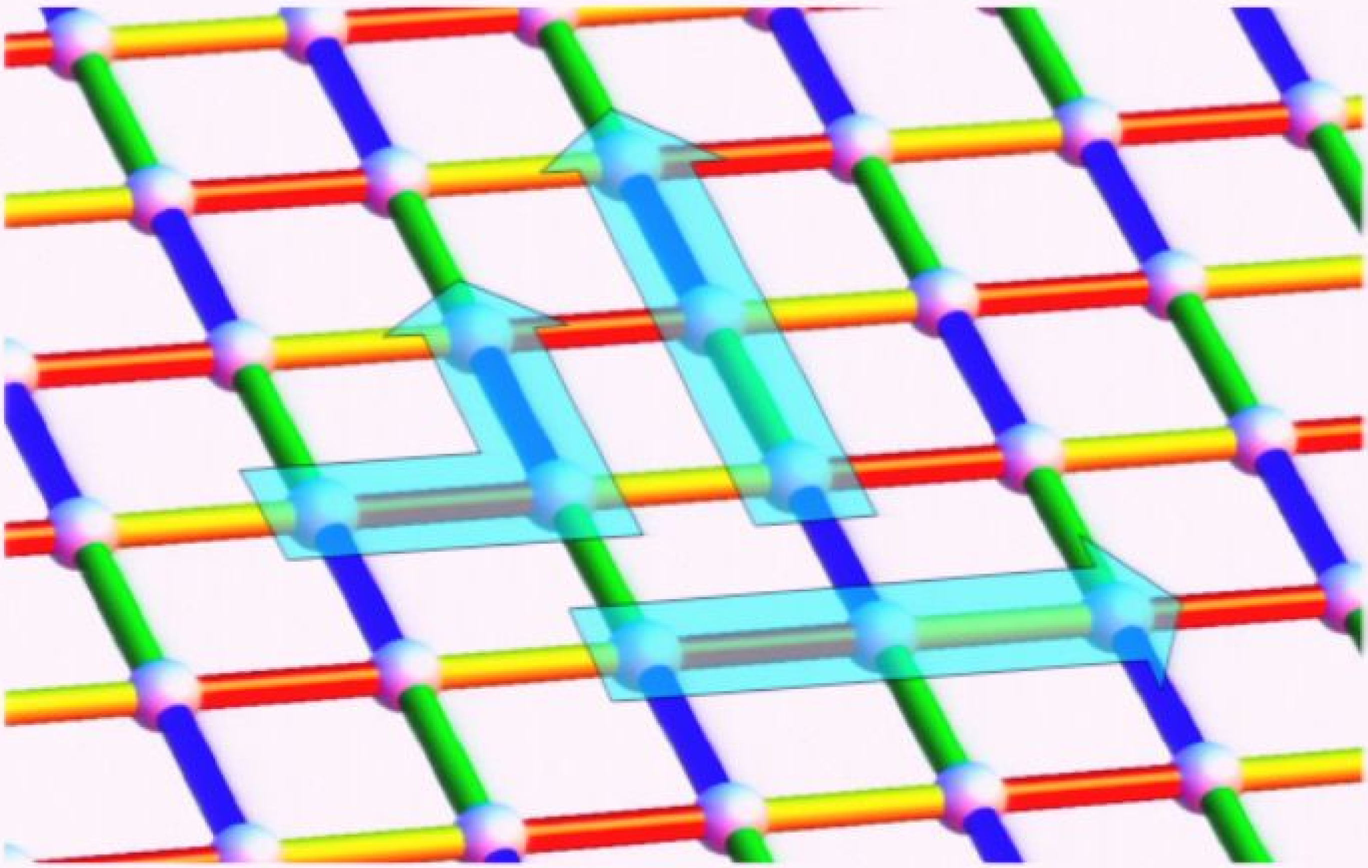
Mathematical construction

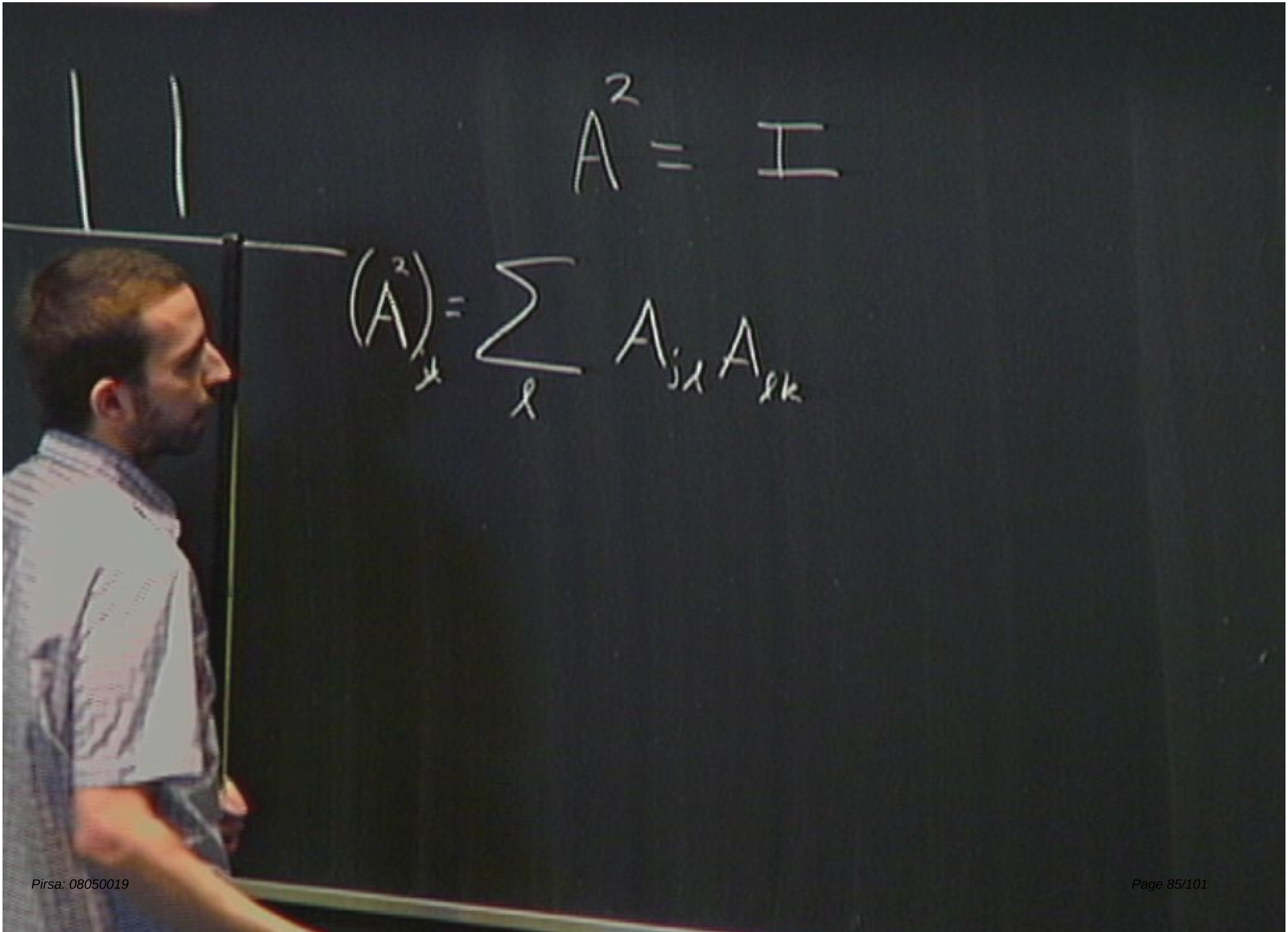
$A =$ 

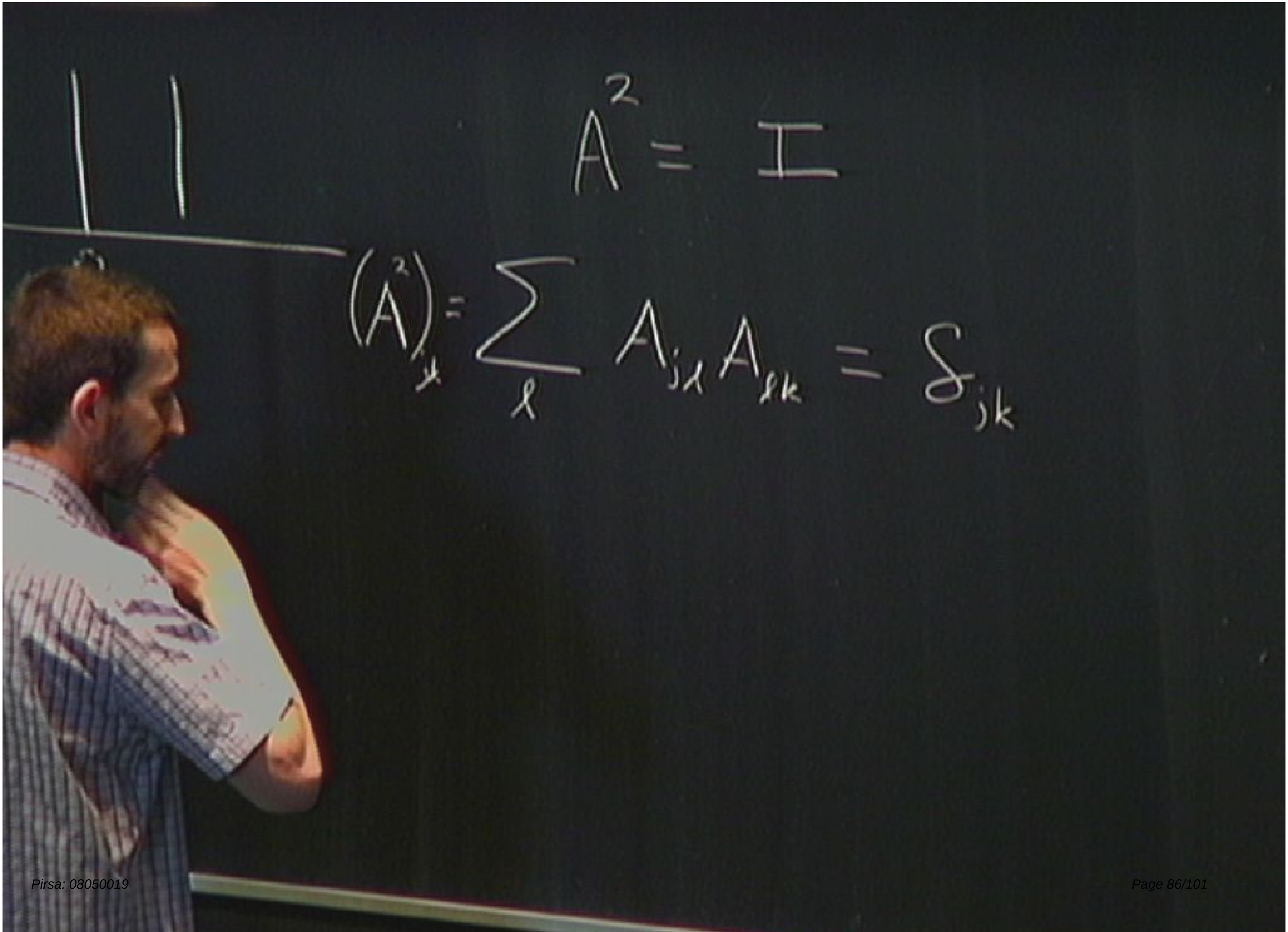
Solution: use polarization degrees of freedom, and twice as many pumps.

- the matrix elements of A are all +k,0,-k for some fixed k.
  constant strength interactions
- A is an orthogonal matrix; $AA^T=I$.
  simplifies the theory a lot
- A is Hankel, with a constant number of nonzero stripes
  photon energy conservation
- The graph of A is universal for cluster state quantum computation.
  we want to quantum compute!
- A is bipartite
  no single-mode squeezing

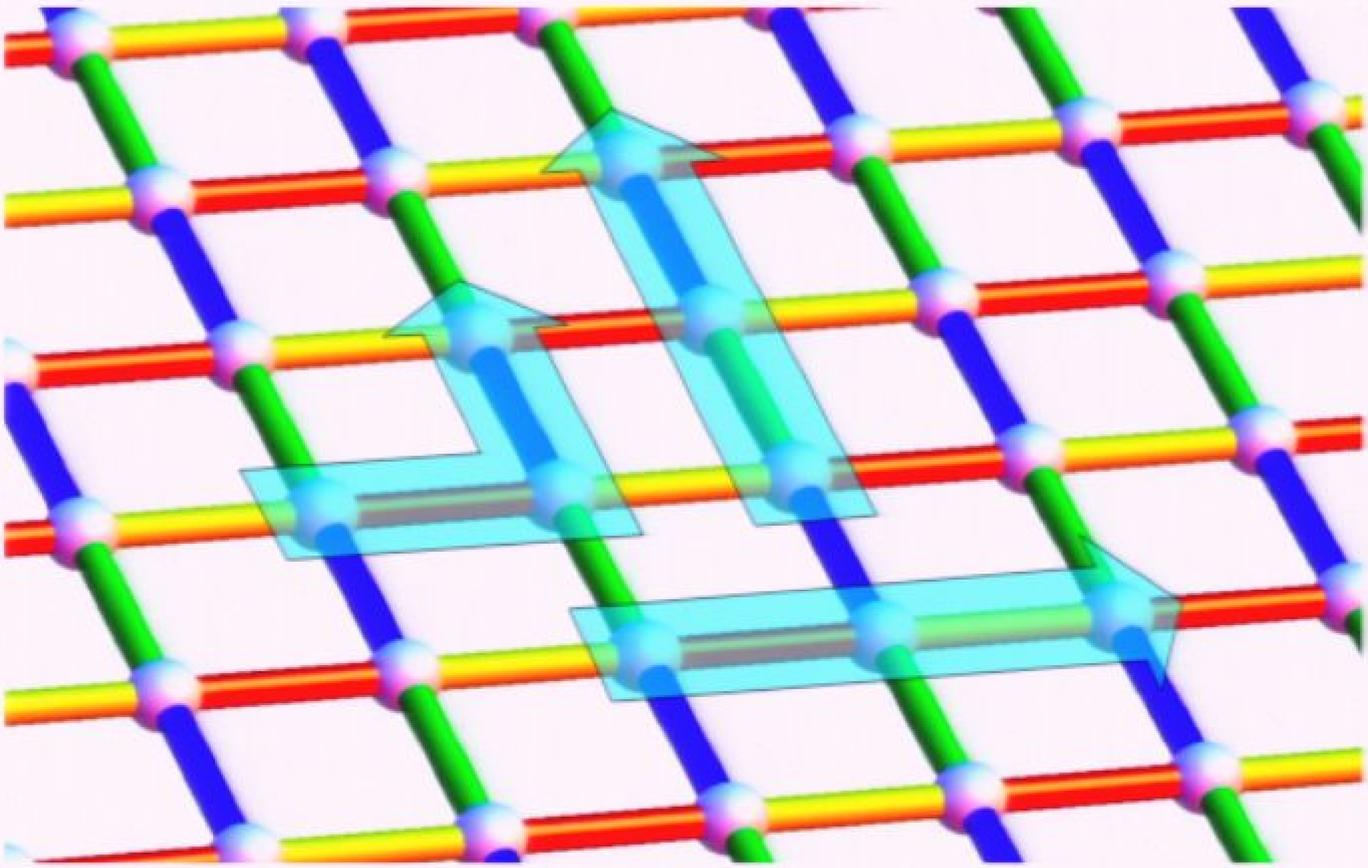
Mathematical construction



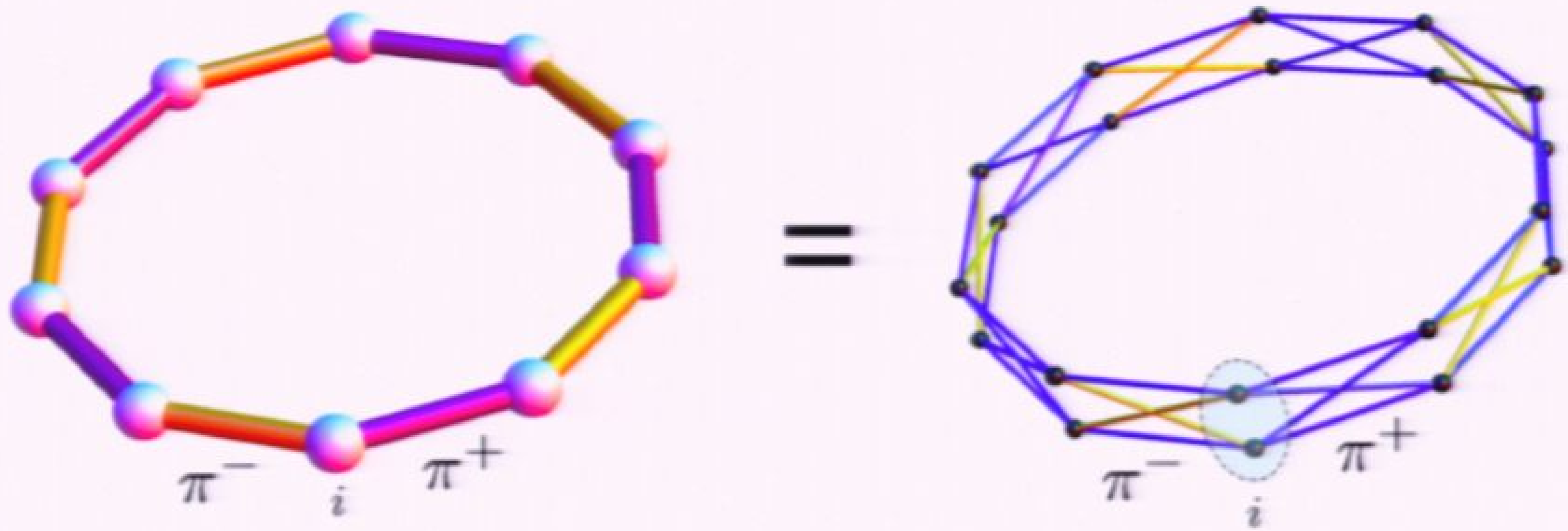

$A^2 = I$
$(A^2)_{jk} = \sum_l A_{jl} A_{lk}$


$A^2 = I$
$(A^2)_{jk} = \sum_{l} A_{jl} A_{lk} = \delta_{jk}$

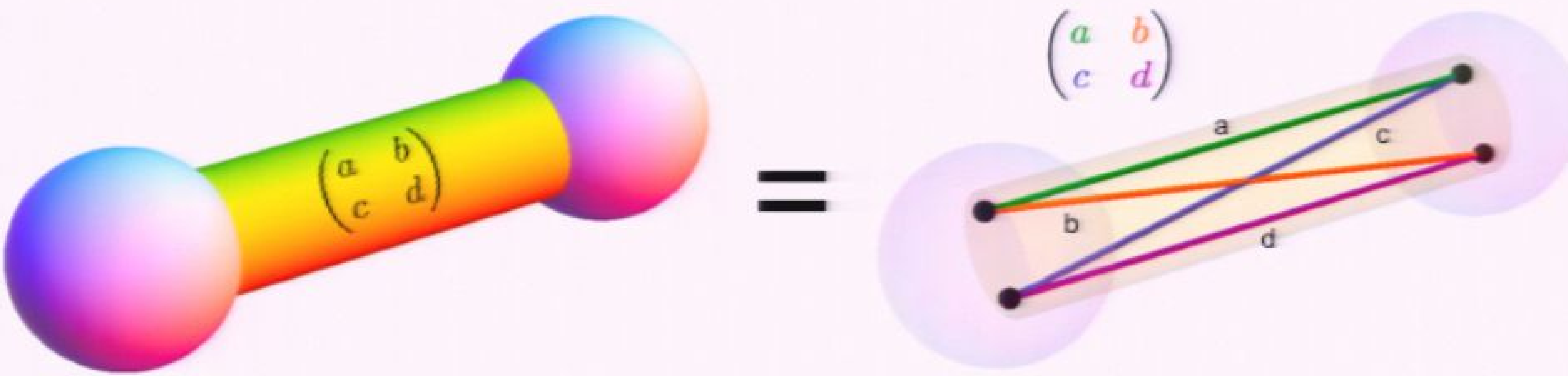
Mathematical construction



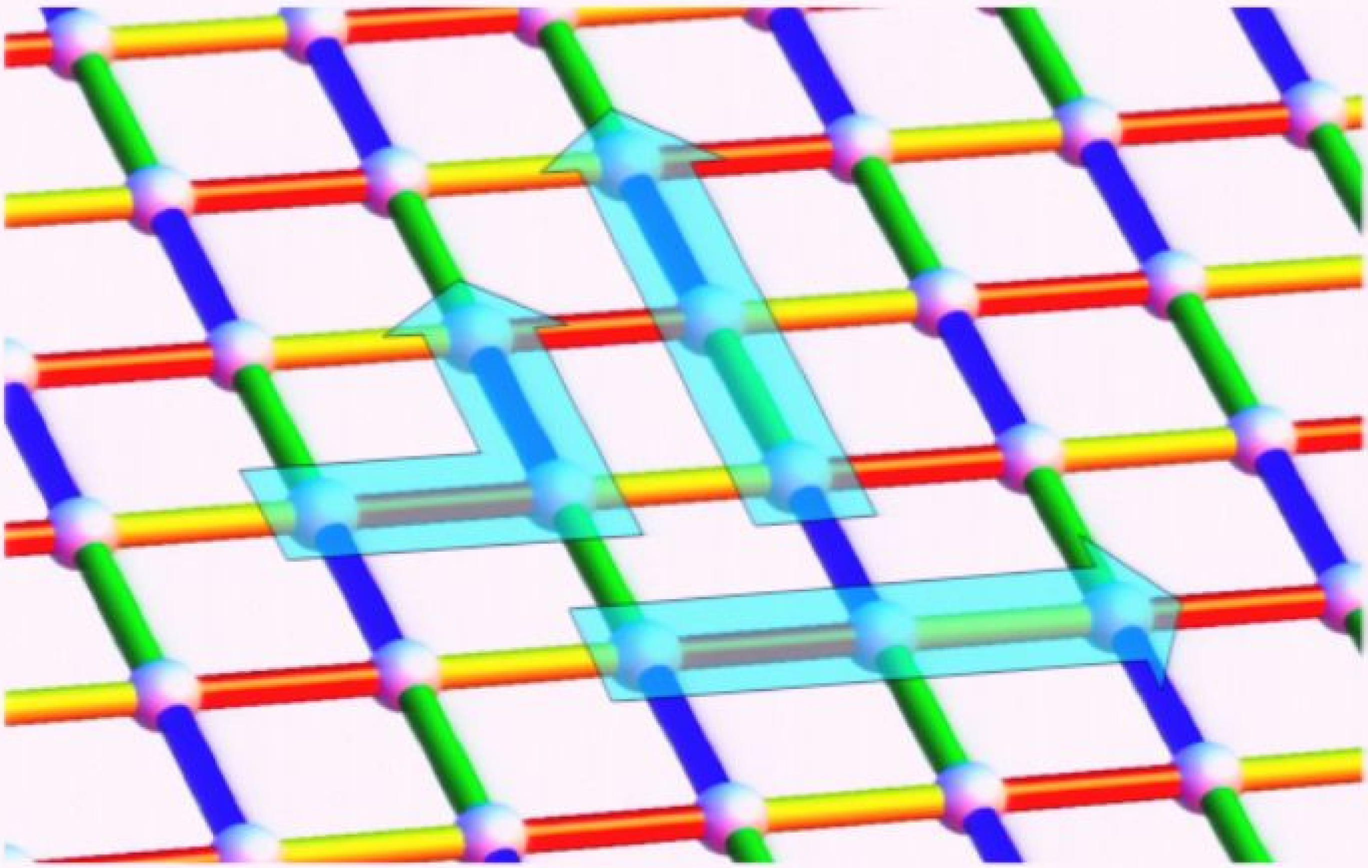
Mathematical construction



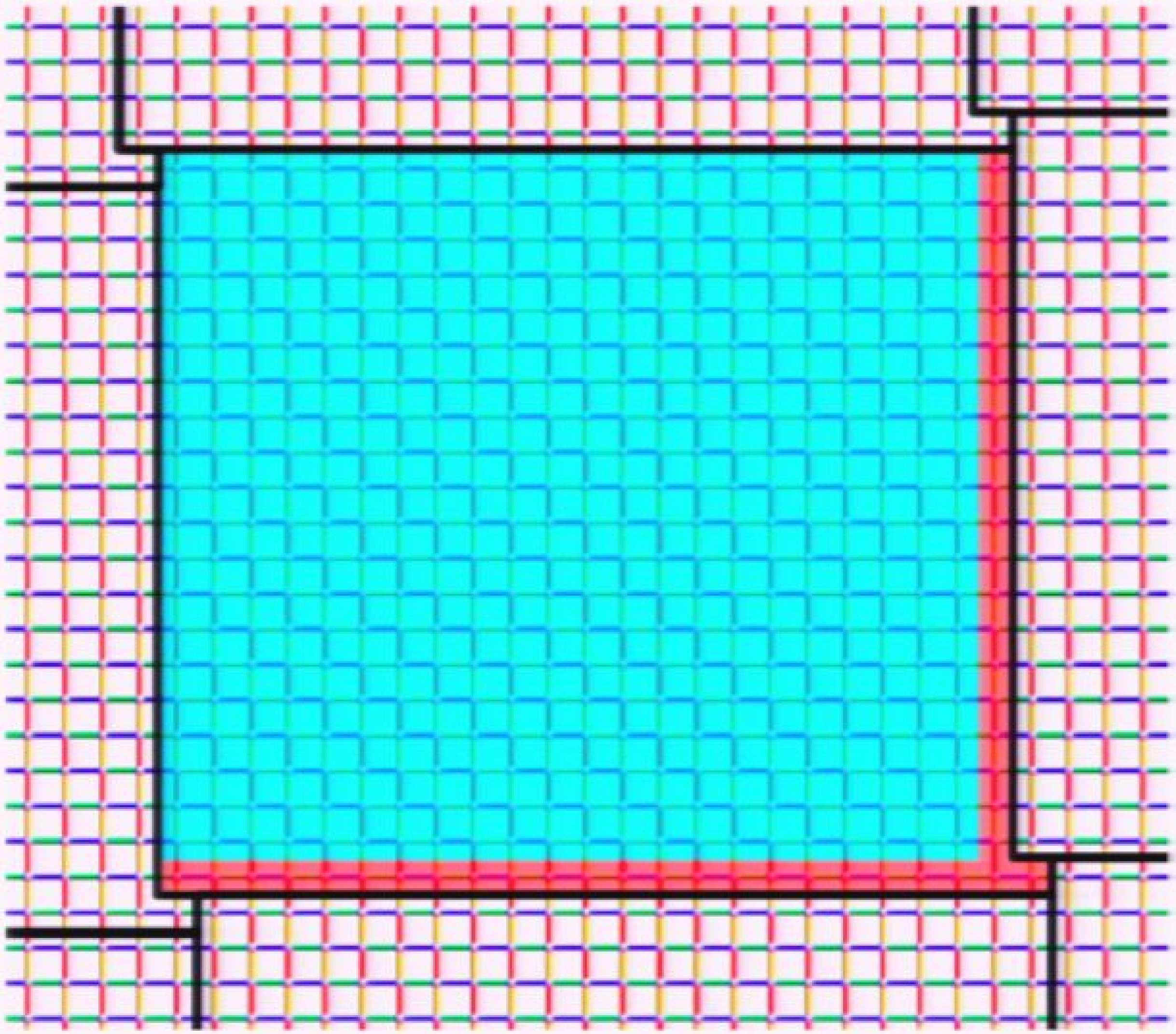
Mathematical construction



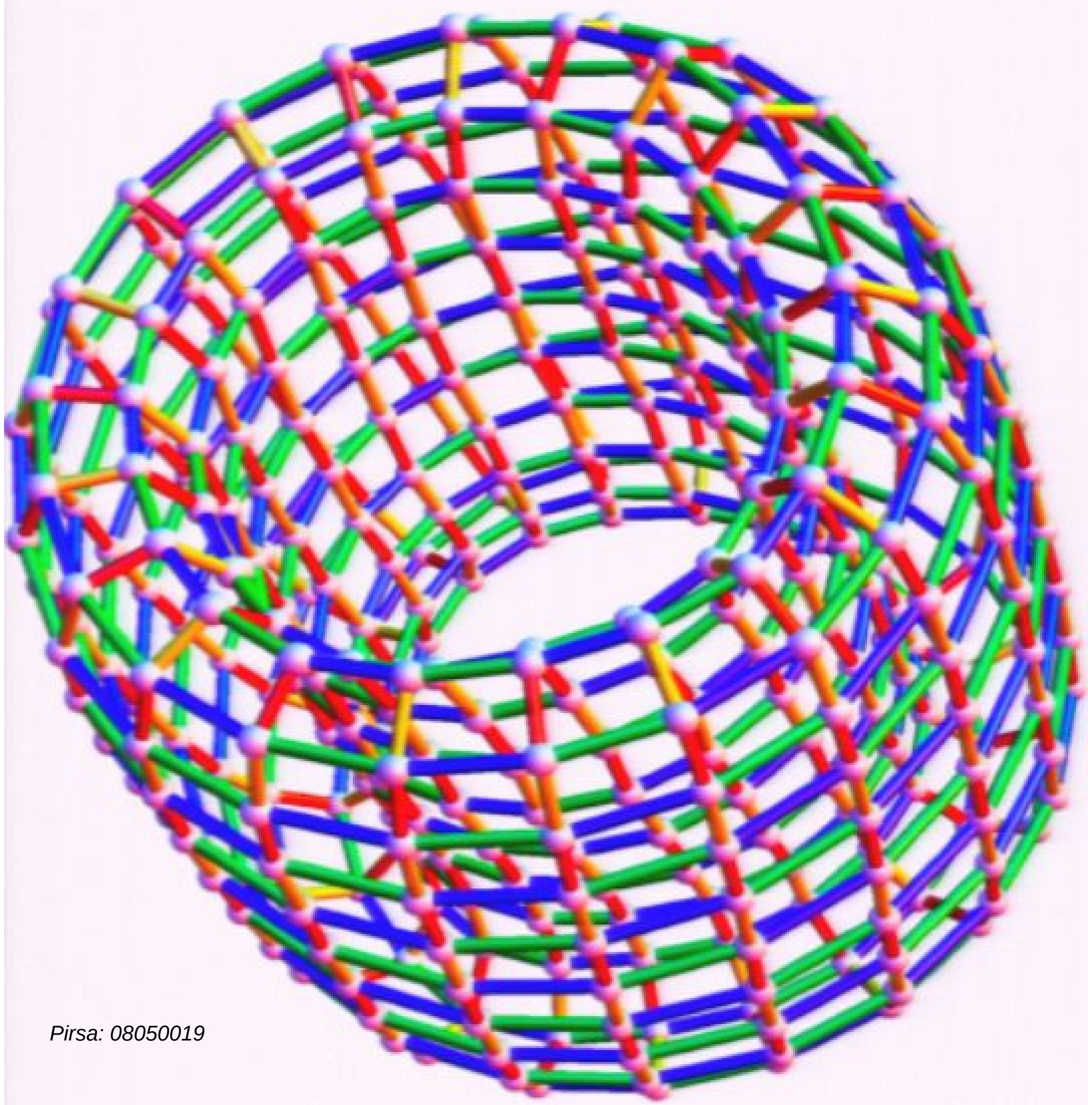
Mathematical construction



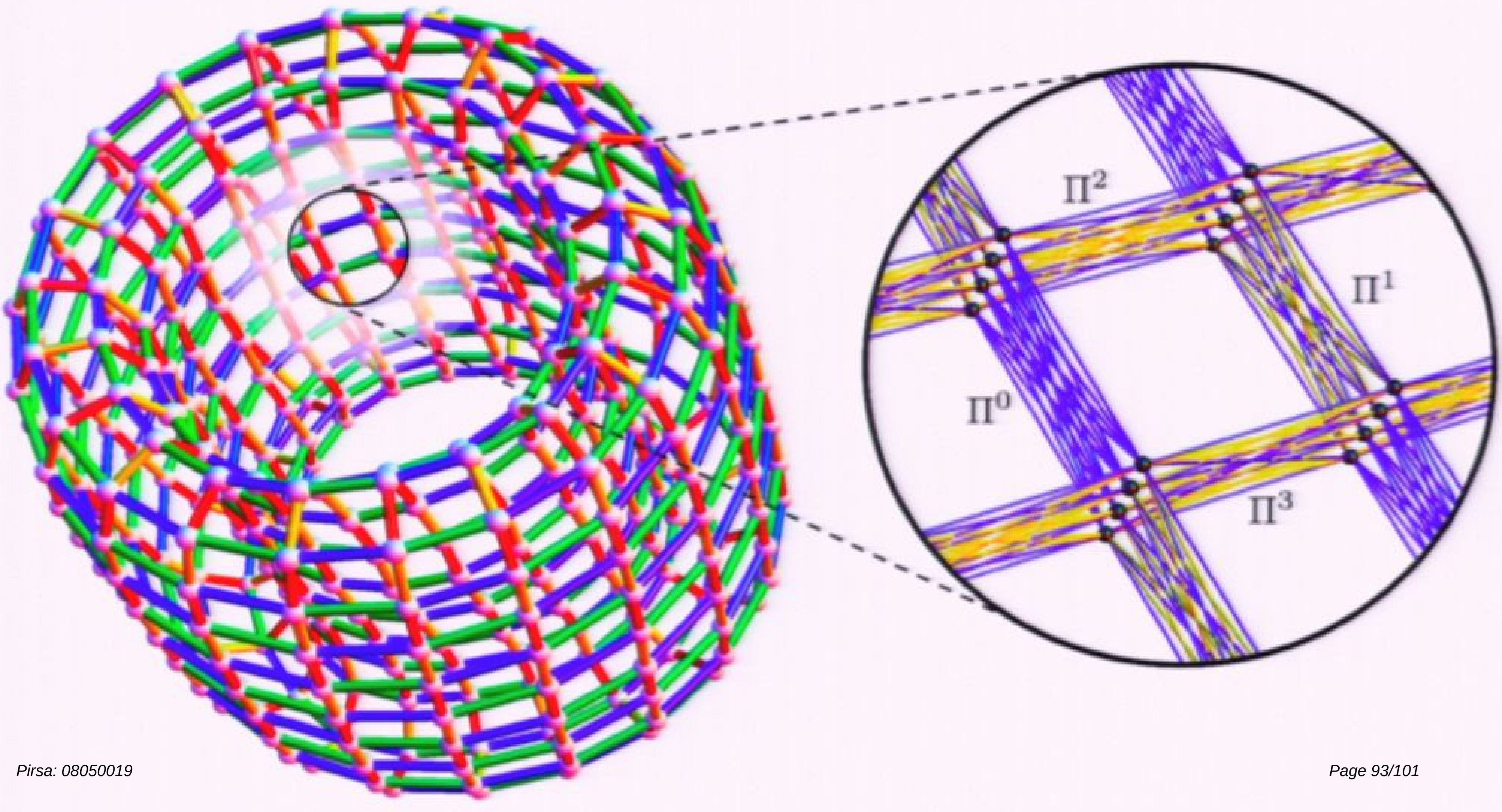
Mathematical construction



Mathematical construction

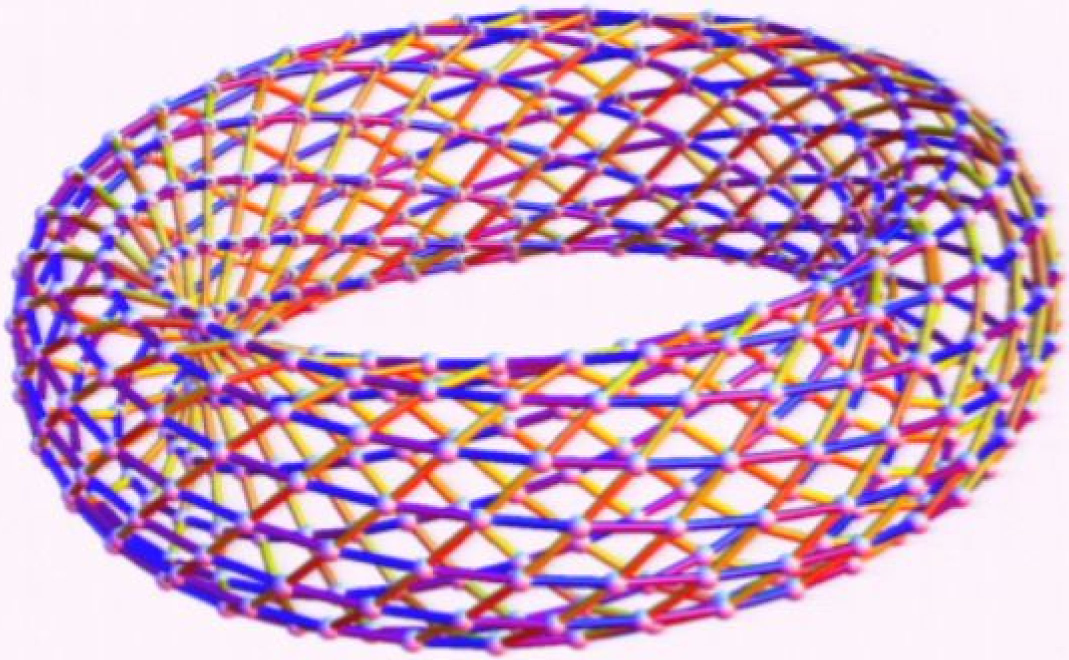


Mathematical construction

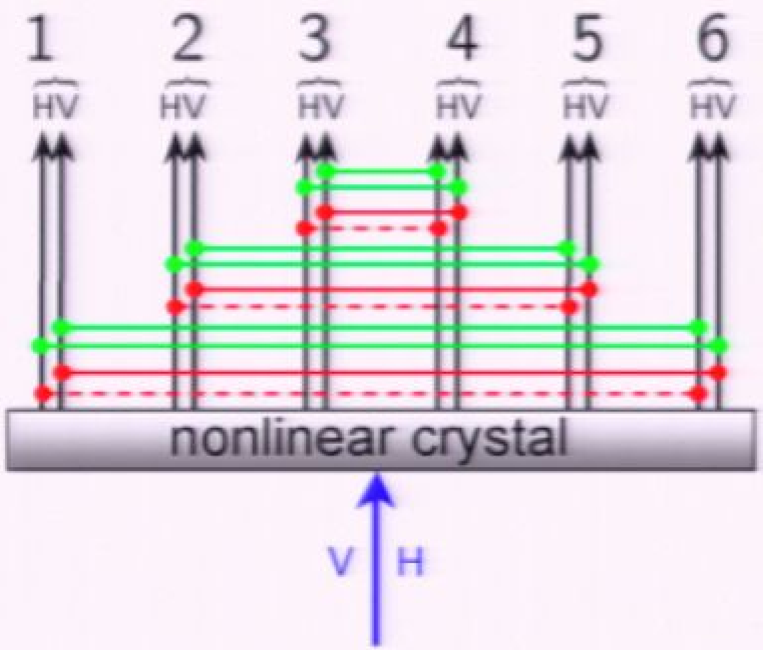


The final state

- global topology: twisted torus
- local structure: square lattice
- 4 modes per vertex induces a factor of 4 overhead (useful?)
- phasematching bandwidth is 10^4 times bigger than the free spectral range - natural large clusters!
- state preparation completed in one step, one cavity, and with a constant number of pumps (15)

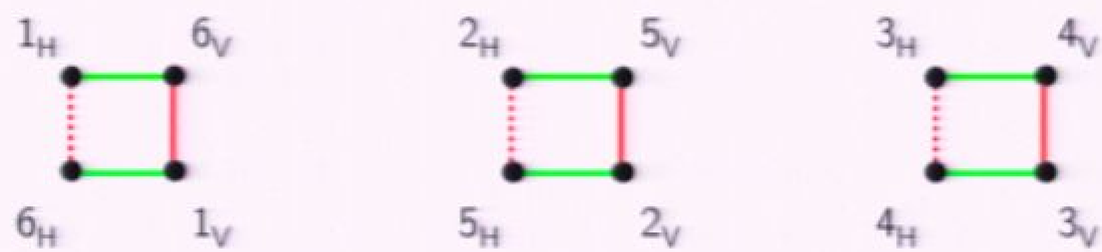


First steps: creating square clusters



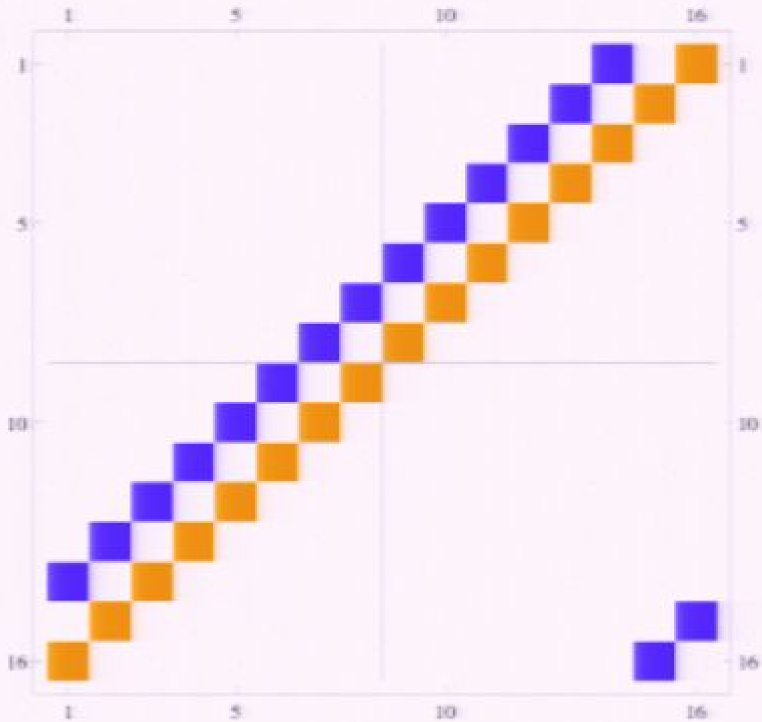
Single pump beam
(with polarization)

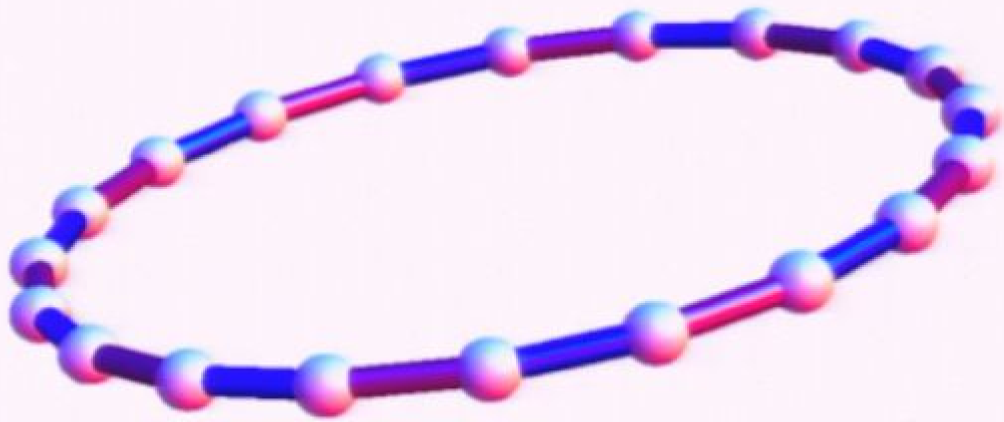
Many square clusters



Walther et. al. Nature 2005

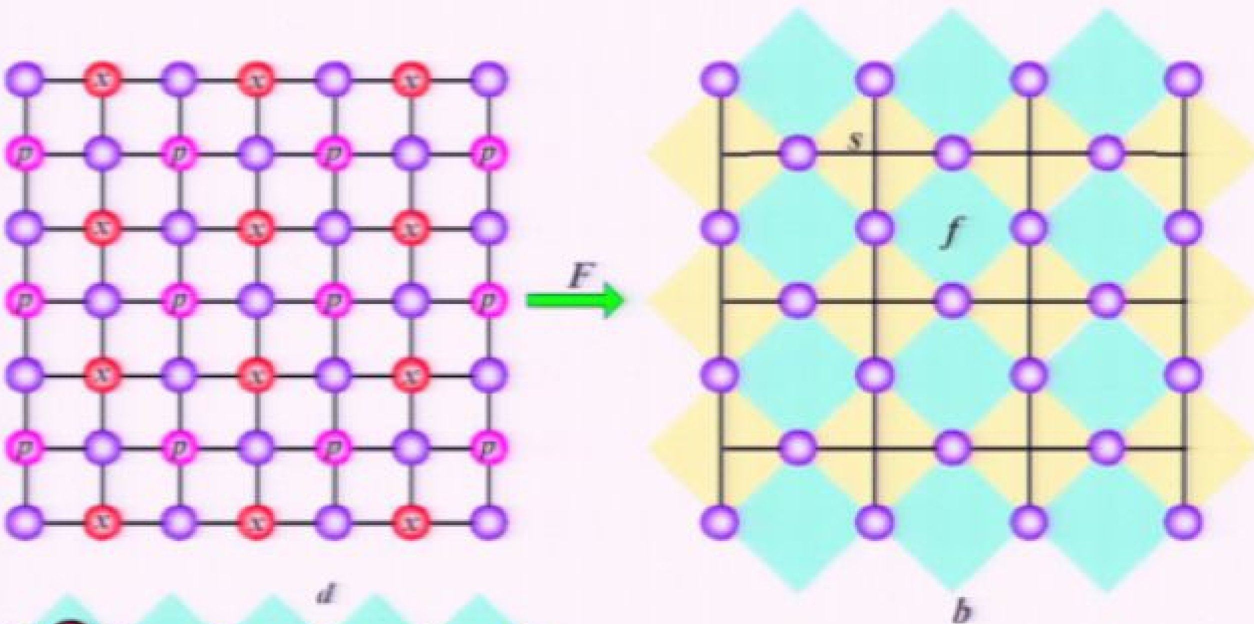
Next steps: creating ring clusters

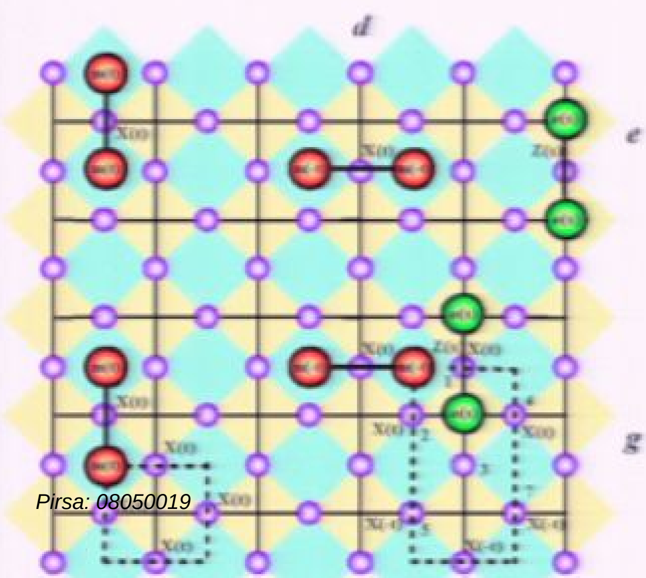




Long quantum wires are universal
for single mode tranformations

Irresponsible speculation



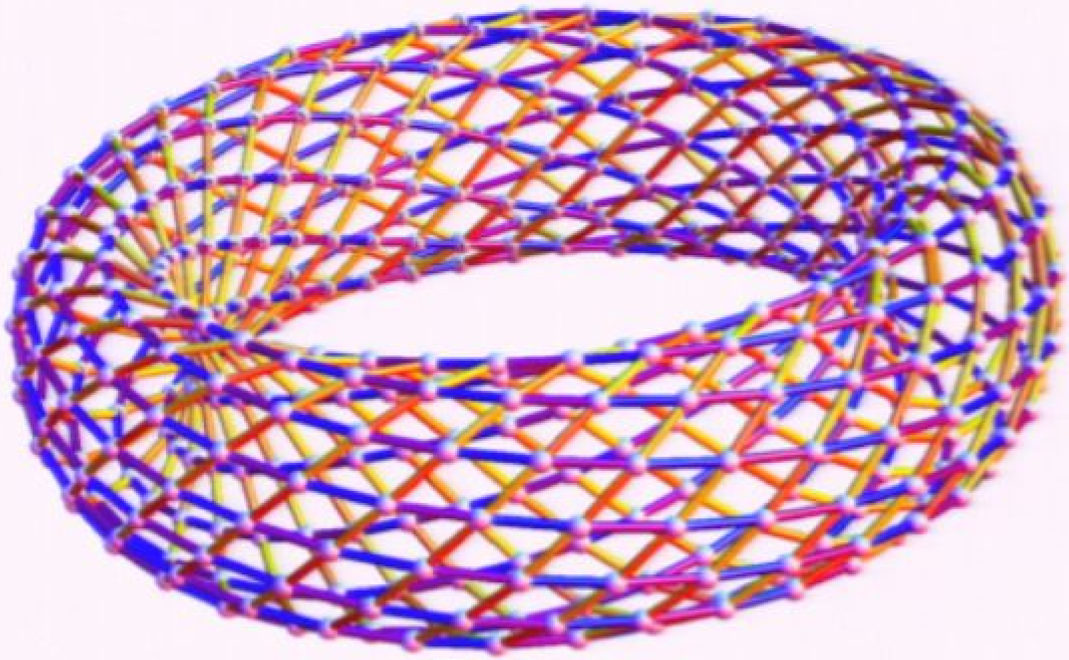


Zhang, Xie, Peng
arXiv:0711.0820

- measure x and p on certain nodes to create the toric code state
- Gaussian operations alone can create anyonic quasiparticles and braid them
- natural toroidal structure could give a natural system for quantum memory
- 4d toric code?

Open questions

- Fault tolerance for continuous variables?
- Utilize 4-fold redundancy for protection from errors?
- Easily convert to a discrete encoding? (GKP)
- How much squeezing is enough?



Collaborators & publications

theory

Steve Flammia - Perimeter Institute
Nick Menicucci - Princeton, U. Queensland
Hussain Zaidi - U. Virginia

experiment

Olivier Pfister - U. Virginia
Russell Bloomer - U. Virginia
Matthew Pysher - U. Virginia

papers

Menicucci et. al. Phys Rev A 76 010302(R) (2007).
Zaidi et. al., arXiv:0710.4980 (to appear in Laser Physics).
Flammia, Menicucci & Pfister, arXiv:0804.4468
Menicucci, Flammia & Pfister, in preparation.

m

$\updownarrow \leftrightarrow$

$$(A^2)_{jk} = \sum_{l} A_{jl} A_{lk} = \delta$$